OPSC-AEE 2020

Odisha Public Service Commission

Assistant Executive Engineer

Civil Engineering

Solid Mechanics

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY Publications

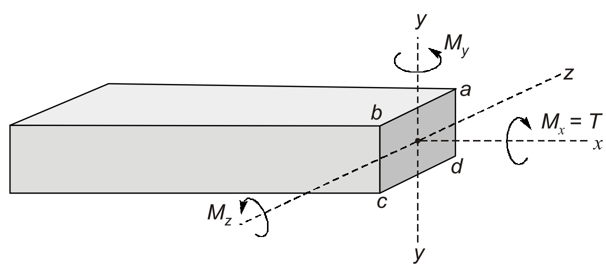
Solid Mechanics

Contents

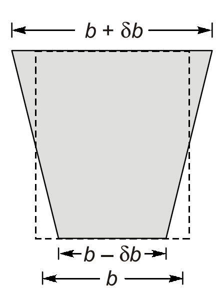
CHAPTER 6 Bending Stress

6.1 Effect of Bending

- In bending, the cross-sectional area *abcd* (as shown in figure) is rotated about transverse axes either *z-z* or *y-y*, whereas in twisting cross-sectional area rotates about longitudinal axis *x-x*.



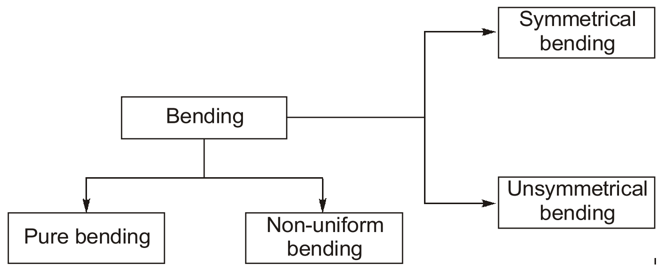
- If loading is in vertical plane i.e., *y*-direction then bending will be about horizontal transverse axis *z-z* and if loading is in horizontal direction i.e., *z*-direction then bending will be about vertical axis *y-y*.
- The transverse axis about which cross-section area rotates is called Neutral Axis (NA).
- Consider effect of vertical loading in *y*-direction, the bending will be about *z* axis. If loading produces sagging bending moment at any section, resulting this vertical cross-sectional area will be in compression above neutral axis and tension below neutral axis. Above neutral axis, due to longitudinal compression, lateral expansion occurs and tension below the neutral axis
- Above neutral axis due to longitudinal compression, lateral expansion occurs and below the natural axis due to longitudinal tension. Hence, a rectangular section before bending will become trapezoidal section after bending.



Effect of Bending

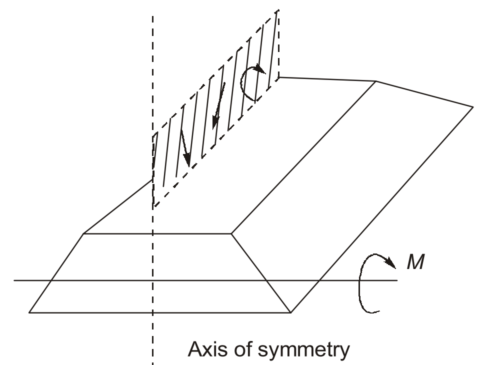
- Transverse deformations due to bending are small in magnitude and elastic in nature. Hence, for all practical purposes change may be neglected and area of cross-section still may be considered as rectangular.

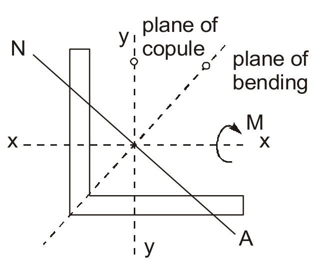
6.2 Bending Classification



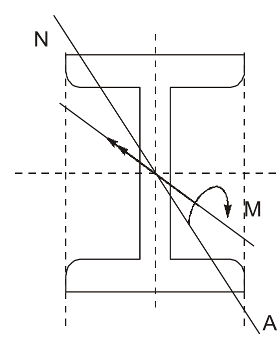
6.2.1 Symmetrical and Unsymmetrical Bending

- If the section posses a plane of symmetry and loading bending couple acts in that plane, the bending is called symmetrical bending.





- If the section is having no axis of symmetry 'or'. If section has axis of symmetry but load does not lie in that plane then the bending is called unsymmetrical bending.



- Plane perpendicular to the direction of couple is called plane of couple.
- Plane perpendicular to the neutral exist is called plane of bending.

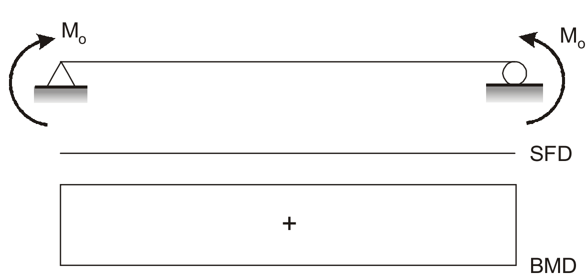
NOTE: Under unsymmetrical bending, plane of bending and plane of couple are not same whereas in case of symmetrical bending plane of couple and plane of bending are same.

6.2.2 Pure Bending

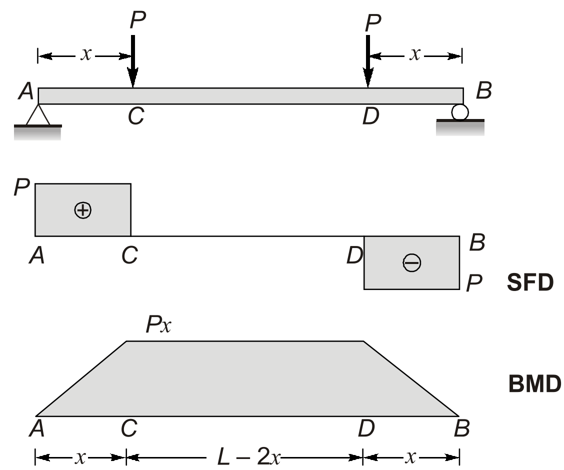
Bending of beam under constant bending moment is called pure bending.

or

If bending moment exist and S.F = 0, bending is called pure bending.



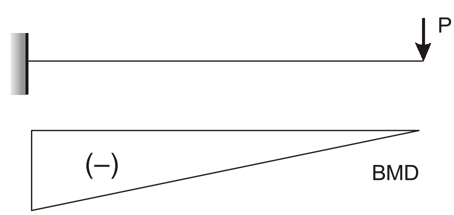
Pure bending throughout the beam.



Here portion CD is said to be in pure bending.

6.2.3 Non-uniform bending

Bending of beam in the presence of shear force is called non-uniform bending.



NOTE: In case of pure bending beam bends in he form of an are of circle.

6.3 Assumptions in Theory of Pure Bending

1. The plane section of the beam before bending remains plane after bending (i.e. strain variation from neutral axis is linear).
2. The material in the beam is homogeneous, isotropic and obeys Hooke's low.
3. Modulus of elasticity in tension and compression are equal.
4. Beam is initially straight and has a constant cross-section throughout its length. (i.e. beam is prismatic)
5. The plane of loading must contain a principal axis of the beam cross-section and the loads must be I^r (perpendicular) to the longitudinal axis of the beam.
6. Every layer of material is free to expand or contract longitudinally and laterally under stress and do not exert pressure upon each other. Thus, the Poisson's effect at the interface of the adjoining differently stressed fibers are ignored.
7. The section of the beam is symmetrical in the loading plane. If section is non-symmetrical then twisting and warping may occur apart from bending.
8. Radius of curvature is large compared to depth of beam.

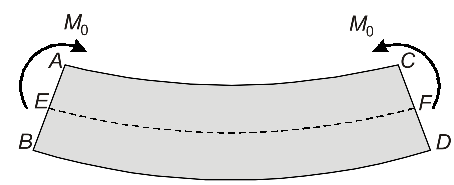
6.4 Neutral Axis

It is the axis about which the cross-section of the beam is in pure rotation.

or

It is the axis about which moment is applied.

- Consider a portion of beam *ABCD*, which is subjected to constant sagging bending moment M_0. The layer *AC* is in compression and layer *BD* is in tension. So, *AC* will contract and *BD* will elongate. But a layer *EF* also exists which is neither in compression nor in tension. Hence, the layer *EF*, does not undergo any change in length even after bending, is called **Neutral Layer**.
- The line of intersection of neutral layer with the plane of cross-section of the beam is called **Neutral Axis**. In pure bending, neutral axis of the beam always passes through the centroid of the cross-section.



NOTE : In plastic bending neutral axis passes through equal area axis. Equal area axis may be different from centroidal axis or may coincide with centroidal axis which depends upon shape of cross-section.

Example - 6.1 A prismatic beam of *T*-section (150 mm × 300 mm × 20 mm) is subjected to a sagging BM. Locate the location of neutral axis for the beam.

Solution:

In pure bending, neutral axis of the beam always passes through the centroid of the cross-section.

Let $\bar{y}$ = depth of centroid from top of the cross-section.

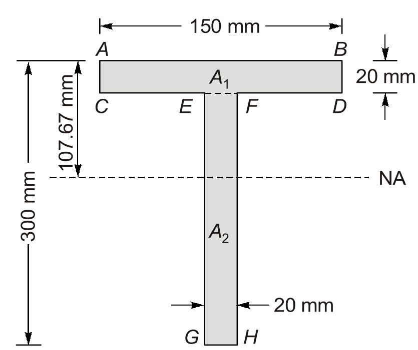
A_1 = (150 × 20) mm2

$\bar{y}_1$ = 10 mm

A_2 = $(280 \times 20)\text{mm}^2$, $\bar{y}_2$ = 160 mm

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$= \frac{(150\times20\times10)+(280\times20\times160)}{(150\times20)+(280\times20)} = 107.67 \text{ mm}$$



$\bar{y}$ = 107.67 mm

6.5 Equation of Pure Bending

6.5.1 Flexure formula

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where, M = Bending moment

σ = Bending stress at a distance y from NA

$\frac{1}{R}$ = Curvature

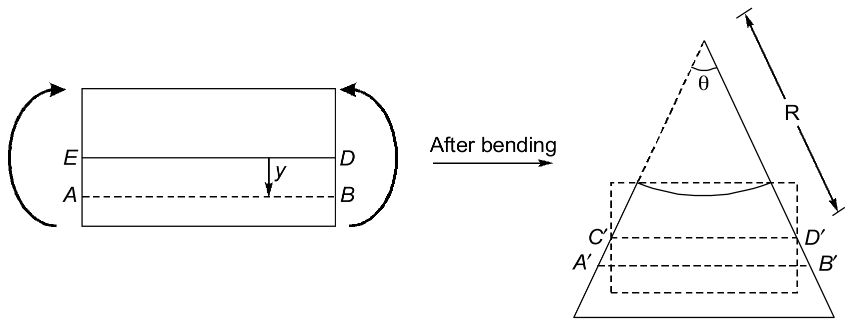
I = MOI of cross-section about NA

E = Young's modulus of elasticity

NOTE : The flexure formula will actually be applicable for pure bending only. However, it can be used for non-uniform bending also without much error.

6.5.2 Derivation of Flexure Formula

Consider a section of beam subjected to pure bending as shown below:



$$\text{Stain in } AB(\epsilon_{AB}) = \frac{A'B' - AB}{AB} \Rightarrow \frac{(R+y)\theta - R\theta}{R\theta} \qquad [\because AB = CD = C'D' = R\theta]$$

$$\epsilon_{AB} = \frac{y}{R} \Rightarrow \frac{\sigma}{E} = \frac{y}{R} \qquad ...(1)$$

Now consider a strip of area 'dA' at distance 'y' from N.A.

$$dF = \sigma dA$$

$$dM = \sigma dA \times y$$

$$dM = \frac{Ey^2}{R} dA$$

Integrating $$\int dM = \frac{E}{R}\int y^2 dA$$

$$M = \frac{E}{R} I$$

$$\frac{M}{I} = \frac{E}{R} \qquad ...(2)$$

From eq. (1) and eq. (2)

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

6.6 Section Modulus (Z)

Section modulus is the ratio of moment of inertia of the beam cross-section about neutral axis to the distance of extreme fibre from the neutral axis. It is denoted by 'z'.

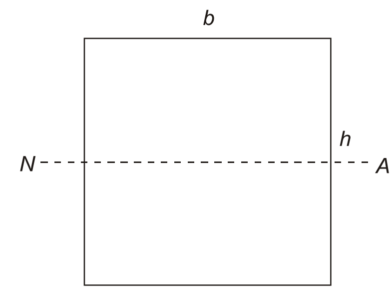
$$Z = \frac{I_{NA}}{y_{max}}$$

- If section modulus of an area of cross-section is known, then maximum bending stress may be obtained by,

$$\sigma_{max} = \frac{M}{Z}$$

- Section modulus also represents the strength of the section. If it is more then section is stronger.

(a) Rectangular section



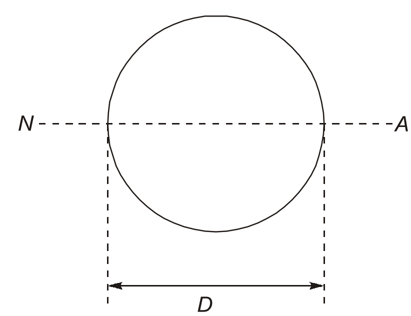
$$I_{NA} = \frac{bh^3}{12} \text{ (moment of inertia about NA)}$$

$$y_{max} = \frac{h}{2}$$

$$Z = \frac{I_{NA}}{y_{max}} = \frac{\left(\frac{bh^3}{12}\right)}{\frac{h}{2}}$$

$$Z = \frac{bh^2}{6}$$

(b) Solid circular section



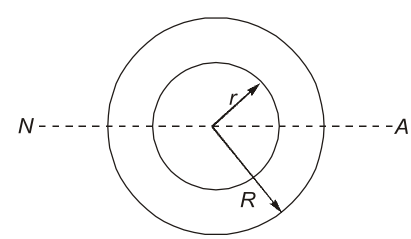
$$I_{NA} = \frac{\pi D^4}{64}$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I_{NA}}{y_{max}} = \frac{\left(\frac{\pi D^4}{64}\right)}{\frac{D}{2}}$$

$$Z = \frac{\pi D^3}{32}$$

(c) Hollow circular section

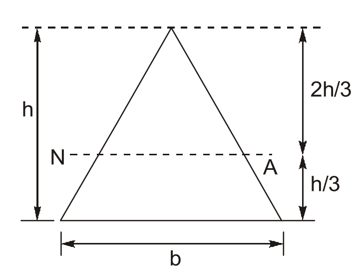


$$I_{NA} = \frac{\pi(R^4 - r^4)}{4},$$

$$y_{max} = R$$

$$Z = \frac{\pi(R^4 - r^4)}{4R}$$

(d) Triangular section



$$I_{NA} = \frac{bh^3}{36}$$

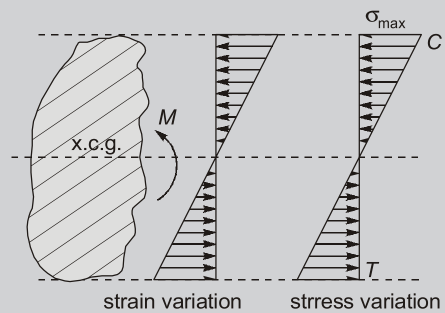
$$y_{max} = \frac{2h}{3}$$

$$Z = \frac{I_{NA}}{y_{max}} = \frac{\left(\frac{bh^3}{36}\right)}{\left(\frac{2h}{3}\right)} = \frac{bh^2}{24}$$

$$Z = \frac{bh^2}{24}$$

NOTE

For any cross-section:



6.7 Moment of Resistance of the Section (MOR)

Moment of resistance is the maximum bending moment that the section can resist without any point in the cross-section having stress greater than the permissible stress.

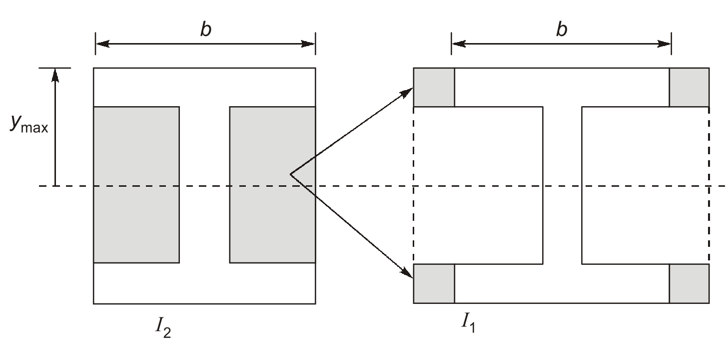
$$\text{MOR} = \sigma_{per} Z$$

$$\sigma_{per} = \text{permissible bending stress}$$

- For designing a beam of uniform section, the section should be select such that the moment of resistance of the section becomes equal to the applied maximum bending moment.

6.8 Justification for Use of I-section as a Beam

- Moment of resistance of the section is more when Z is more.
- But $Z = \dfrac{I}{y_{max}}$, hence for a given y, Z is more if I is more also I is more if more area is located away form NA.



$$(I_2 > I_1)$$
$$Z_2 > Z_1$$
$$(\text{MOR})_2 > (\text{MOR})_1$$

- In case of I-section more area is located away from NA leading to increase in moment of inertia (I) and hence increase in Z-value thus more moment can be resisted by I-section as compared to the rectangular section of same area and same overall depth.
- In case of I-section more than 80% of the moment is resisted by the flanges.
- From the above justification one can easily argue that out of rectangular and circular section of same cross-section area rectangular section is more efficient in bending.