POSTAL Book Package 2021

Electronics Engineering

Conventional Practice Sets

Basic Electrical Engineering

Contents

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3 CHAPTER

Three Phase Induction Motors

Practice Questions : Level-I

Q.1 **A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1% at no-load and 3% at full-load. Find (i) synchronous speed, (ii) no-load speed (iii) full-load sped (iv) frequency of rotor current at standstill (v) Frequency of rotor current at full-load.**

Solution:

Number of poles, $P = 6$; Supply frequency, $f = 50$ Hz ; No-load slip, $s_0 = 1\%$ or 0.01

Full-load slip, $s_f = 3\%$ or 0.03

(i) Synchronous speed of motor, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1{,}000$ rpm

(ii) No-load speed, $N_0 = N_s(1 - s_0) = 1{,}000\,(1 - 0.01) = 990$ rpm

(ii) Full-load speed, $N_f = N_s(1 - s_f) = 1{,}000\,(1 - 0.03) = 970$ rpm

(iv) Frequency of rotor current at standstill = Supply frequency $f = 50$ Hz

(v) Frequency of rotor current at full-load, $f_r = s_f f = 0.03 \times 50 = 1.50$ Hz.

Note: When the rotor is at standstill, the motor is equivalent to a 3-phase transformer with secondary short circuited. So induced emf per phase E_2 in the rotor, when it is at standstill i.e. at the instant of starting is given by

$$E_2 = E_1 \times \frac{N_2}{N_1}$$

where E_1 is applied voltage per phase to primary i.e. stator winding, N_2 and N_1 are the numbers of turns per phase on rotor and stator respectively.

When the rotor starts running, the relative speed of the rotor with respect to stator flux i.e. slip s drops in direct proportion with the relative speed and the induced emf in rotor is given by sE_2.

Hence for slip s, the induced emf in the rotors is s times the induced emf in the rotor at standstill.

Q.2 **A 3-phase, 4-pole slip ring induction machine is connected to a 3-phase, 50 Hz supply. If the rotor terminals are shorted, the machine runs in clockwise direction seen from one end. Now the rotor terminals are opened and the machine is made to run with the help of a prime mover. What will be the frequency of the voltage across the slip rings, if**

(i) the machine is not running?

(ii) the machine is rotating at a speed of 1500 rpm in clockwise direction?

(iii) the machine is rotating at a speed of 1450 rpm in anti-clockwise direction?

(iv) the machine is rotating at a speed of 1550 rpm in clockwise direction?

Solution:

$$\text{Synchronous speed} = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Slip, } s = \frac{N_s - N}{N_s}$$

(i) $\because$ $s = 1$

Rotor frequency, $f = 50$ Hz

(ii) $\because$ $s = 0$,

Rotor frequency, $f_r = sf = 0$ (as $N = N_s$)

(iii) $$s = \frac{N_s - (-N)}{N_s} = \frac{1500 + 1450}{1500} = 1.967$$

$\therefore$ Rotor frequency, $f_r = sf = 98.33$ Hz

(iv) $$s = \frac{1500 - 1550}{1500} = -0.033$$

$\therefore$ Rotor frequency, $f_r = sf = 1.67$ Hz

Q.3 A 3-phase, 4-pole slip ring induction motor is connected to 3-phase, 50 Hz supply from the motor side through slip rings and the stator terminals are shorted. The machine is found to be running at 1440 rpm. Determine:

(i) The frequency of stator current.

(ii) The speed of rotor magnetic field with respect to rotor and its direction w.r.t. direction of rotation of rotor.

(iii) The speed of stator magnetic field with respect to stator and its direction w.r.t. direction of rotation of rotor.

(iv) The speed of stator magnetic field with respect to rotor magnetic field.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

(i) Frequency of stator current = 50 Hz

(ii) Speed of rotor magnetic field w.r.t. rotor = $(N_s - N) = 1500 - 1440 = 60$ rpm. Rotor magnetic field is in the same direction of rotation of rotor.

(iii) Speed of stator magnetic field w.r.t. stator = $N_s = 1500$ rpm. Stator magnetic field is in the same direction of rotation of rotor.

(iv) Speed of stator magnetic field w.r.t. rotor Magnetic field = zero

Note: Both the stator field and rotor field rotate in the airgap at same synchronous speed.

Q.4 A 4-pole, 50 Hz, 3-phase induction motor has blocked rotor reactance per phase which is four times the rotor resistance per phase. Find the speed at which maximum torque develops.

Solution:

For maximum torque, $$s_{max} = \frac{R}{X} = \frac{1}{4} = 0.25$$

$$\text{synchronous speed } n_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$$

$$\text{speed at maximum torque} = (1 - s_{max})\, n_s = (1 - 0.25) \times 1500 = 1125 \text{ rpm}$$

Q.5 A 12 pole, 3-phase, 600-V, 50-Hz, star -connected, induction motor has rotor-resistance and stand-still reactance of 0.03 and 0.5 ohm per phase respectively. Calculate : (a) Speed at maximum torque. (b) ratio of full-load torque to maximum torque, if the full-load speed is 495 rpm.

Solution:

For a 12-pole, 50 Hz motor,

$$\text{Synchronous speed} = 120 \times \frac{50}{12} = 500 \text{ rpm}$$

For $r = 0.03$ and $x = 0.5$ ohm, the slip for maximum torque is related as :

$$S_{mT} = a = \frac{r}{x} = \frac{0.03}{0.5} = 0.06$$

(a) Corresponding speed at maximum torque = $500(1 - s_{mT})$ = 470 rpm

(b) Full-load speed = 495 rpm

Slip at full-load, $$s = \frac{N_s - N}{N_s} = \frac{500 - 495}{500} = 0.01$$

$$\frac{\text{Full load torque}}{\text{Maximum torque}} = \frac{2as}{a^2 + s^2} = \frac{2 \times 0.06 \times 0.01}{0.06^2 + 0.01^2} = 0.324$$

Q.6 **The rotor resistance and standstill reactance per phase of a 3-Φ slip-ring induction motor are 0.05 Ω and 0.2 Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to give maximum torque at starting?**

Solution:

Let external resistance per phase added to the rotor circuit be r ohms.

Rotor resistance per phase, $R_2 = (0.05 + r)$

The starting torque will be maximum when

$$R_2 = X_2$$

$$0.05 + r = 0.20,\ r = 0.15\ \Omega$$

Q.7 **A 6-pole, 3Φ, 50 Hz induction motor runs on full load with a slip of 4 percent. Given the rotor standstill impedance per phase as (0.01 + j0.05) Ω, calculate the available maximum torque in terms of full-load torque. Also determine the speed at which the maximum torque occurs.**

Solution:

Synchronous speed, $$N_s = \frac{120f}{p} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = 4\% = 0.04$$

Slip at maximum torque, $$s_M = \frac{R_2}{X_2} = \frac{0.01}{0.05} = 0.2$$

$$N_M = (1 - s_M)\, N_s = (1 - 0.2) \times 1000 = 800 \text{ r.p.m.}$$

$$\frac{\tau_{max}}{\tau_{fl}} = \frac{s^2 + s_M^2}{2s\, s_M} = \frac{(0.04)^2 + (0.2)^2}{2 \times 0.04 \times 0.2} = 2.6$$

$\therefore$ $$\tau_{max} = 2.6\, \tau_{fl}.$$

Q.8 **A 3-phase slip-ring induction motor gives a reading of 60 V across slip rings when at rest with normal stator voltage applied. The rotor is star connected and has an impedance of (0.8 + j6) Ω per phase. find the rotor current when the machine is (a) at standstill with the slip-rings joined to a star-connected starter with a phase impedance of (4 + j3) Ω and (b) running normally with a 5% slip.**

Solution:

$$E_{20} = \text{e.m.f. induced per phase of the rotor at standstill} = \frac{60}{\sqrt{3}} = 34.64\,\text{V}$$

Total impedance of the rotor at standstill

$$Z_{20} = \text{impedance of rotor} + \text{impedance of starter}$$
$$= (0.8 + j6) + (4 + j3) = 4.8 + j9\ \Omega$$

(a) Current at standstill,
$$I_{20} = \frac{E_{20}}{Z_{20}} = \frac{34.64\angle 0^\circ}{4.8 + j9} = \frac{34.64\angle 0^\circ}{10.2\angle 61.93^\circ} = 3.396\angle -61.93^\circ\ \text{A}$$

(b)
$$s = 5\% = 0.05\ \text{p.u.}$$

During normal running, the starting resistance are cut off:

$$I_{2s} = \frac{E_{2s}}{Z_{2s}} = \frac{sE_{20}}{R_2 + jsX_{20}}$$
$$= \frac{0.05 \times 34.64}{0.8 + j0.05 \times 6} = \frac{1.732}{0.8 + j0.3} = \frac{1.732}{0.8544\angle 20.56^\circ} = 2.027\angle -20.56\ \text{A}$$

Q.9 A 3-phase, 20 kW, 400 V, 4 pole, 50 Hz squirrel cage induction motor develops a torque of 100 N-m at a speed of 1400 rpm. If the motor is connected to a 30 Hz supply, keeping the same air-gap flux, find the supply voltage and the new speed for the same load torque.

Solution:

For same air-gap flux,
$$\frac{V_1}{f_1} = \frac{V_2}{f_2}$$
$$\frac{400}{50} = \frac{V_2}{30}$$
$$V_2 = 240\ \text{V}$$

Synchronous speed at 50 Hz,
$$N_{s1} = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500\ \text{rpm}$$

$\therefore$
$$\text{Slip } s_1 = \frac{1500 - 1400}{1500} = \frac{1}{15}$$

Synchronous speed at 30 Hz,
$$N_{s2} = \frac{120 f_2}{P} = \frac{120 \times 30}{4} = 900\ \text{rpm}$$

For low-slip,
$$T \propto \frac{sV^2}{f}$$

At constant torque,
$$\frac{s_1 V_1^2}{f_1} = \frac{s_2 V_2^2}{f_2}$$

$$\Rightarrow \quad \frac{400 \times 400}{15 \times 50} = \frac{s_2 \times 240 \times 240}{30}$$

$$\Rightarrow \quad s_2 = 0.11$$

$$\therefore \quad N_2 = N_{s2}(1 - s_2) = 900(1 - 0.11) = 801\ \text{rpm}$$

Q.10 A 50 kVA, 400 V, 3-phase, 50 Hz squirrel cage induction motor has full load slip of 5%. It is started using a tapped autotransformer. If the maximum allowable supply current at the time of starting is 100 A, then calculate the tap position and the ratio of starting torque to full load torque [Given: Stand-still impedance $Z_{01} = 0.866\ \Omega$/ph].

Solution :

$$Z_{01} = 0.866\ \Omega/\text{ph}$$

$$\text{Short circuit current} = \frac{400}{\sqrt{3}\times 0.866}$$

$$I_{sc} = 266.67\ \text{A}$$

$$\text{Rated full load current} = \frac{50{,}000}{\sqrt{3}\times 400} = 72.16\ \text{A}$$

If 'x' is the percentage tapping then

$$x^2 \times 266.67 = 100$$

$$x = \sqrt{\frac{100}{266.67}} = 0.6123$$

$$\frac{T_{st}}{T_{FL}} = \left(\frac{I_{st}}{I_{FL}}\right)^2 \times s_{FL}$$

$$= \left(\frac{x I_{SC}}{I_{FL}}\right)^2 s_{FL} = \left(\frac{0.6123\times 266.67}{72.16}\right)^2 \times 0.05 = 0.256$$

Q.11 If stator impedance of a three-phase induction motor is neglected, show from its equivalent circuit that maximum torque T_{em} per phase is given by

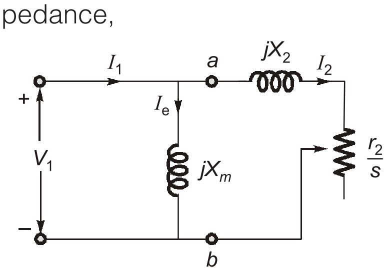
$$T_{em} = \frac{1}{\omega_s}\cdot\frac{V_1^2}{2x_2}$$

and hence show that, $$\frac{T_e}{T_{em}} = \frac{2}{\dfrac{s_{mT}}{s} + \dfrac{s}{s_{mT}}}$$

where s is any slip and s_{mT} = slip at which maximum torque occurs.

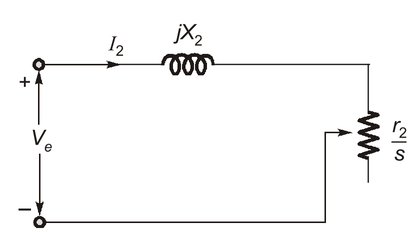
Solution:

Equivalent circuit without stator impedance,



Induction motor equivalent circuit

By applying Thevenins theorem across ab



$$V_e = V_1$$

$$\therefore \quad I_2 = \frac{V_1}{\dfrac{r_2}{s} + jX_2}$$

Total torque, $$T_e = \frac{m}{\omega_s}\cdot\frac{V_1^2}{\left(\dfrac{r_2}{s}\right)^2 + (x_2)^2}\cdot\frac{r_2}{s}$$

where, m = number of stator phases

From the maximum power transfer theorem, the torque will be maximum when impedance r_2/s becomes equal to the magnitude of impedance seen by r_2/s towards the voltage source v_e.

$$\frac{r_2}{s_{mT}} = x_2 \Rightarrow s_{mT} = \frac{r_2}{x_2}$$

Maximum torque,
$$T_{em} = \frac{mV_1^2}{\omega_s}\frac{1}{(x_2)^2+(x_2)^2}\cdot(x_2) = \frac{mV_1^2}{2\omega_s x_2}$$

$$\frac{T_e}{T_{em}} = \frac{m}{\omega_s}\cdot\frac{V_1^2}{\left(\frac{r_2}{s}\right)^2+x_2^2}\cdot\frac{r_2}{s}\times\frac{2\omega_s x_2}{mV_1^2} = \frac{\frac{r_2}{s}}{\left(\frac{r_2}{s}\right)^2+x_2^2}\times 2x_2$$

$$\frac{T_e}{T_{em}} = \frac{\frac{r_2}{s}}{\left(\frac{r_2}{s}\right)^2+\left(\frac{r_2}{s_{mT}}\right)^2}\cdot 2\times\frac{r_2}{s_{mT}} \qquad \left(as\ x_2 = \frac{r_2}{s_{mT}}\right)$$

$$= \frac{\frac{2}{s}}{\frac{1}{s^2}+\frac{1}{s_{mT}^2}}\times\frac{1}{s_{mT}} = \frac{2}{\frac{s_{mT}}{s}+\frac{s}{s_{mT}}}$$

Q.12 A 230 V, 20 hp, 60 Hz, 6 pole, 3-phase induction motor driving a constant torque load at rated frequency, rated voltage and rated horse-power, has a speed of 1175 rpm and an efficiency of 92.1%. Determine the new operating speed if a system disturbance causes 10% drop in voltage and 6% drop in frequency. Assume that friction, windage and stray power losses remain constant.

Solution :

Using the relation,
$$T \propto \frac{sV_0^2}{N_s} \qquad ...(i)$$

$$N_{s_1} = \frac{120\times 60}{6} = 1200 \text{ rpm}$$

$$s_1 = \frac{1200-1175}{1200} = 0.0208$$

For 6% drop in frequency, the synchronous speed becomes

$$N_{s_2} = \frac{120\times 0.94\times 60}{6} = 1128 \text{ rpm}$$

As the load torque remain same

$$\frac{s_1 V_1^2}{N_{s_1}} = \frac{s_2 V_2^2}{N_{s_2}}$$

or
$$\frac{0.0208\,V_1^2}{1200} = \frac{s_2(0.9V_1)^2}{1128}$$

$\Rightarrow$
$$s_2 = 0.024$$

Also
$$s_2 = \frac{N_{s_2}-N_2}{N_{s_2}}$$

or
$$0.024 = \frac{1128-N_2}{1128}$$

$\Rightarrow$
$$N_2 \simeq 1100 \text{ rpm}$$