DETAILED SOLUTIONS

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ESE 2020 : Prelims Exam
CLASSROOM TEST SERIES

GENERAL STUDIES & ENGG. APTITUDE

Test 1

Section A : Reasoning Aptitude

Section B : Engineering Mathematics

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DETAILED EXPLANATIONS

1. (c)

Nalni is the daughter of the only son of Gopi's grandfather. Hence, it's clear that Nalni is the sister of Gopi.

2. (d)

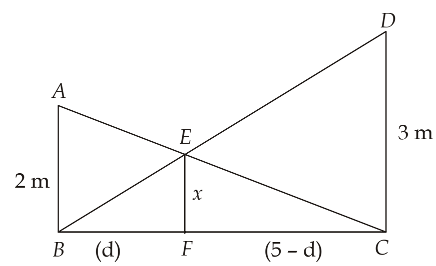
Let for class A: boys = x and girls = y

then for class B: boys = $x - 1$ and girls = $y - 2$

$$\frac{x}{y} = \frac{3}{4} \text{ and } \frac{(x-1)}{(y-2)} = \frac{4}{5}$$

Solving, we get $y = 12$

3. (a)



To find: $EF = x = ?$

In ΔABC and ΔEFC

$$\angle ACB = \angle ECF \text{ (common)}$$

$$\angle ABC = \angle EFC = 90°$$

$$\Rightarrow \quad \Delta ABC \simeq \Delta EFC$$

$$\Rightarrow \quad \frac{x}{2} = \frac{5-d}{5} \quad \text{...eq. (i)}$$

Similarly, $\Delta BCD \simeq \Delta BFE$

$$\Rightarrow \quad \frac{x}{3} = \frac{d}{5} \quad \text{...eq.(ii)}$$

Adding eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{x}{2} + \frac{x}{3} = \frac{5-d}{5} + \frac{d}{5}$$

$$\Rightarrow \quad \frac{3x+2x}{6} = \frac{5}{5}$$

$$\Rightarrow \quad 5x = 6 \times 1 = 6$$

$$\Rightarrow \quad x = \frac{6}{5} = 1.2 \text{ m}$$

4. (b)

$E \quad B \quad A \quad C \quad D$

Therefore, A is sitting in between B and C.

5. (d)

Here, the concept is of successive division

i.e. the number is first divided by 5 and it leaves remainder 2 and quotient is let x,

Therefore we have number $= 5x + 2$ ...(1)

and then the quotient x is divided by 7 and the remainder is 3

So, we have x in the form of $x = 7y + 3$

And then the quotient y is divided by 8 and the remainder is 4

So, we have y in the form of $y = 8z + 4$

putting this value of x and y in (1) above, we get

$$\text{number} = 5(7(8z + 4) + 3) + 2$$
$$\Rightarrow \quad \text{number} = 5(56z + 31) + 2$$
$$\Rightarrow \quad \text{number} = 280z + 157$$

When this number will be divided by 8, we will get remainder = 5 and quotient = $35z + 19$

When this quotient will be divided by 7, we will get remainder = 5 and quotient = $5z + 2$

When this quotient will be divided by 5, we will get remainder = 2

6. (c)

$$B - 3 = E \quad \text{...(i)}$$
$$B + 3 = D \quad \text{...(ii)}$$
$$A + B = D + E + 10 \quad \text{...(iii)}$$
$$B = C + 2 \quad \text{...(iv)}$$
$$A + B + C + D + E = 133 \quad \text{...(v)}$$

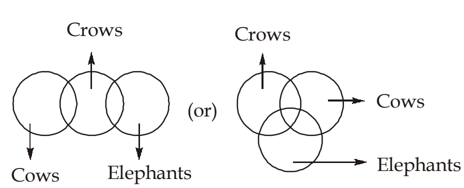
From (i) and (ii), we have : $2B = D + E$ ...(vi)

From (iii) and (vi), we have : $A = B + 10$ ...(vii)

Using (iv), (vi) and (vii) in (v), we get:

$$(B + 10) + B + (B - 2) + 2B = 133 \Rightarrow 5B = 125 \Rightarrow B = 25.$$

7. (d)



None of the two follows.

8. (c)

Distance travelled when the ball touches the floor 3rd time,

$h + 0.6h + 0.6h + 0.6 \times 0.6\text{ h} + 0.6 \times 0.6\text{h} = 292$

$$h + 2 \times 0.6 \times h + 2 \times 0.36 \times h = 292$$
$$h(1 + 1.2 + 0.72) = 292$$
$$\Rightarrow \quad 2.92h = 292$$
$$\Rightarrow \quad h = 100 \text{ cm}$$

9. (b)

Let total number of members be 100,

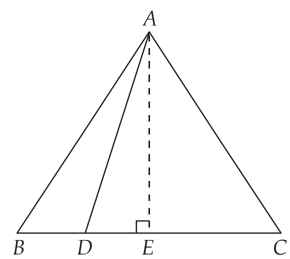
Then, number of members owning only 2 cars = 20

Number of members owning 3 cars = 40% of 80 = 32

Number of members owning only 1 car = 100 – (20 + 32) = 48

Thus, 48% of the total members own one car each.

10. (c)



Given: $AB = AC = 3$ cm and $BD = \frac{1}{2}CD$

AE is median.

To find: $AD = ?$

$$BD + CD = 3$$

$$\Rightarrow \quad BD + 2BD = 3BD = 3$$

$$\Rightarrow \quad BD = \frac{3}{3} = 1 \text{ cm}$$

Also, since AE is median

$$BE = CE = \frac{3}{2} \text{ cm}$$

$$\Rightarrow \quad DE = BE - DE = \frac{3}{2} - 1 = \frac{1}{2} \text{ cm}$$

Also, $$AE = \frac{\sqrt{3}}{2}a = \frac{3\sqrt{3}}{2} \text{ cm}$$

in ΔADE

$$\Rightarrow \quad (AD)^2 = (AE)^2 + (DE)^2$$

$$\Rightarrow \quad (AD)^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad (AD)^2 = \frac{27}{4} + \frac{1}{4} = \frac{28}{4}$$

$$\Rightarrow \quad AD = \sqrt{7} \text{ cm}$$

11. (c)

Here the common faces with 4 dots are in same positions. Hence 2 will be opposite to 5.

12. (b)

Minute hand does a relative gain of $5\frac{1}{2}^\circ$ over hour hand in 1 (each) minute.

So, in two minutes the relative gain $= 2\times 5\frac{1}{2}^\circ = 11^\circ$.

Alternative Solution:

Angle covered by the hour hand in 12 hours = 360°

$$\text{In 1 hour} = \frac{360^\circ}{12} = 30^\circ$$

$$\text{and in 1 minute} = \frac{30^\circ}{60} = \frac{1^\circ}{2}$$

Similarly, angle covered by minute hand in 1 hour = 360°

$$\text{In 1 minute} = \frac{360^\circ}{60} = 6^\circ$$

Every minute, the angle between the two hands changes by $= 6 - \frac{1}{2} = \frac{11^\circ}{2}$

From 7:45 A.M. to 7:47 A.M., i.e. in 2 minutes the angle between the two hands will change by

$$= 2\times\frac{11}{2} = 11^\circ$$

13. (c)

The alphabetical order = CCHJL

Number of words starting with C = 4! = 24

Number of words starting with H $= \frac{4!}{2} = 12$

Number of words starting with J $= \frac{4!}{2} = 12$

Total words till now = 24 + 12 + 12 = 48

First word starting with *L* (49^{th} in dictionary) = LCCHJ

Therefore, the 50^{th} word = LCCJH

14. (b)

Volume of tank = 150 × 120 × 100 = 1800000 cm^3

Volume of water in the tank = 1281600 cm^3

Volume to be filled in the tank = 1800000 – 1281600 = 518400 cm^3

Let the number of bricks to be placed in the tank = x

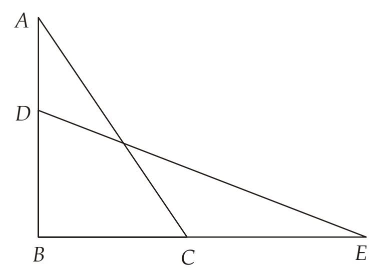
Volume of x bricks = $x \times 20 \times 6 \times 4 = 480x\ cm^3$

Each brick absorbs $\left(\frac{1}{10}\right)^{th}$ of its volume in water

$$x \text{ bricks will absorb} = \frac{480x}{10} = 48x\ cm^3$$

$$\therefore \quad 518400 + 48x = 480x$$
$$480x - 48x = 432x = 518400$$
$$x = \frac{518400}{432} = 1200$$

15. (d)



Original position of the ladder = $AC = 25\text{ft}$

Base = $BC = 7\text{ft}$

In ΔABC, using Pythagoras theorem

$$(AB)^2 = (AC)^2 - (BC)^2$$
$$\Rightarrow \quad (AB)^2 = (25)^2 - (7)^2$$
$$\Rightarrow \quad AB = \sqrt{625 - 49} = \sqrt{576}$$
$$\Rightarrow \quad AB = 24 \text{ ft}$$

After drawing out the base, the new position of ladder = ED = 25 ft

and $AD = x$ and $CE = 2x$

To find : $CE = ?$

In ΔDBE

$$DB = (24 - x) \text{ and } BE = (7 + 2x)$$
$$(DB)^2 + (BE)^2 = (ED)^2$$
$$(24 - x)^2 + (7 + 2x)^2 = (25)^2$$
$$(576 - 48x + x^2) + (49 + 28x + 4x^2) = 625$$
$$625 - 20x + 5x^2 = 625$$
$$5x^2 = 20x$$
$$x = \frac{20}{5} = 4$$
$$CE = 2 \times 4 = 8 \text{ ft}$$

None of the options include 8 in the interval.

16. (a)

Let's go step by step:

$$\text{First operation: } 3L - 1L = \frac{6}{3}L \text{ of wine left, total 4L;}$$

$$\text{Second operation: } \frac{6}{3}L - \left(\frac{6/3}{4}\right) = \frac{6}{3} - \frac{6}{12} = \frac{18}{12} = \frac{6}{4}L \text{ of wine left, total 5L;}$$

$$\text{Third operation: } \frac{6}{4}L - \left(\frac{6/4}{5}\right) = \frac{6}{4} - \frac{6}{20} = \frac{24}{20} = \frac{6}{5}L \text{ of wine left, total 6L;}$$

$$\text{Fourth operation: } \frac{6}{5}L - \left(\frac{6/5}{6}\right) = \frac{6}{5} - \frac{6}{30} = \frac{30}{30} = \frac{6}{6}L \text{ of wine left, total 7L;}$$

At this point it's already possible to see the pattern: $x = \dfrac{6}{n+2}$

$$n = 19$$

$$\Rightarrow \quad x = \frac{6}{(19+2)} = \frac{6}{21} = \frac{2}{7}L$$

17. (b)

$|a - b| + |b - c| - |c - a|$

We need to keep the value of $|c - a|$ minimum.

Let's take c = 18, a = 19

And b as 1

$$|a - b| + |b - c| - |c - a| = |19 - 1| + |1 - 18| - |18 - 19| = 18 + 17 - 1 = 34$$

18. (d)

a = Sum of an arithmetic sequence with first term, 15 and last term, 35 and common difference 2.

Number of odd numbers from 15 to 35, $n = \dfrac{(35-15)}{2} + 1 = 11$

$$a = \frac{11}{2}(15+35) = 275$$

b = number of even integers from 16 to 34 inclusive $= \dfrac{(34-16)}{2} + 1 = 10$

Therefore, $a - b = 265$

19. (a)

If these four lines are parallel, then we'll have 0 vertices.

If no two of the four are parallel, then each distinct pair of lines will give a vertex, thus total of ${}^4C_2$ = 6 vertices.

20. (d)

The relative speed of the two trains is 30 + 40 = 70 miles per hour. Therefore 1 hour before they meet, they must be 70 miles apart (in the final 1 hour they will cover 70 miles to meet).

21. (c)

In a regular hexagon three diagonals pass through the centre.

G is the centre making the total number of points = 7.

To form a triangle, we need 3 points at a time.

Therefore, total number of possible triangles = ${}^7C_3 = 35$

But since three diagonals pass through the centre, G will be collinear in three cases.

Therefore total number of triangles that can be formed using vertices from amongst these 7 points = 35 – 3 = 32.

22. (d)

Any number ending in 7 when raised to a power will have the following pattern 7,9,3,1 as the units digit and any number ending in 2 when raised to a power will have the following pattern 2, 4, 8, 6 as the units digit.

Now 97^{275} means we divide 275 by 4 and compare it against the pattern 275th power will have 3 as the units digit.

32^{44} means we divide 44 by 4 and compare it against the pattern 44th power will have 6 as the units digit.

Thus we have 3 – 6. The trick is that you have to imagine the normal subtraction and get 1 as the carry over thus it is actually 13 – 6 = 7.

23. (a)

$$(x+1)(x+9)+8 = 0$$
$$x^2+10x+17 = 0$$

The roots of the equation are a and b,

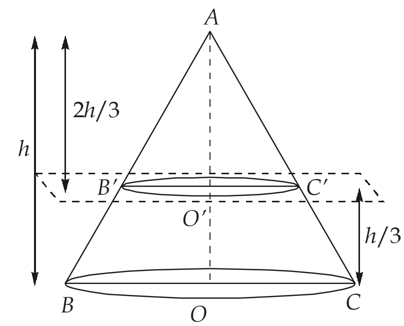
$$\therefore \quad a+b = -10$$
$$ab = 17$$

Now, $$(x+a)(x+b)-8 = 0$$
$$\Rightarrow \quad x^2+(a+b)x+ab-8 = 0$$
$$\Rightarrow \quad x^2-10x+9 = 0$$
$$\Rightarrow \quad (x-1)(x-9) = 0$$

Therefore, roots of $(x+a)(x+b)-8=0$ are 1 and 9.

24. (b)

The plane cuts the cone at a height h/3 from the base as shown below.



Let R be the radius of the base of the cone. Then, the volume of the original cone is $V = \pi R^2 h/3$

If we look at the figure, AO′B′ and AOB, we can see similar triangles.

We know AO′ = $\frac{2h}{3}$ and $AO = h$

Applying properties of similar triangles

$$\frac{O'B'}{OB} = \frac{AO'}{AO} = \frac{2h/3}{h} = \frac{2}{3}$$

$$OB = R$$

$$O'B' = \frac{2}{3}\ OB = \frac{2}{3}R$$

The height and the radius of the smaller cone are therefore, $\frac{2h}{3}$ and $\frac{2R}{3}$ respectively.

So its volume = $\frac{1}{3}\pi\left(\frac{2R}{3}\right)^2 \frac{2h}{3} = \frac{8V}{27}$

Volume of the frustum = Total volume – volume of smaller cone

$$= \left(V - \frac{8V}{27}\right) = \frac{19V}{27}$$

Ratio of volume of smaller cone and frustum is = $\frac{8V}{27} : \frac{19V}{27}$ = 8 : 19

Therefore required ratio is 8 : 19.

25. (d)

Possibilities where M is ahead of N

$$= 4! + 3 \times 3! + 2 \times 3! + 3!$$
$$= 24 + 18 + 12 + 6 = 60$$

Alternative solution:

Arrangement of runners in 1st to 5th position = 5! = 120

M can either be ahead or behind N.

Hence possibilities where M is ahead of $N = \frac{1}{2} \times 120 = 60$

26. (c)

Given: $$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$A^2 = I$$

By Cayley Hamilton theorem

$$\lambda^2 = 1$$

$\Rightarrow$ $\lambda = \pm 1$ are eigen values

$$|A| = -1$$
$$-\alpha^2 - \beta\gamma = -1$$
$$1 - \alpha^2 - \beta\gamma = 0$$

Alternative:

Given: $$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$\therefore$ $$A^2 = A.A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$$

Given that $A^2 = I$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ $$\alpha^2 + \beta\gamma = 1$$
$$1 - \alpha^2 - \beta\gamma = 0$$

27. (b)

Given,
$$4x_4 + 13x_5 = 46 \quad ...(1)$$
$$2x_1 + 5x_2 + 5x_3 + 2x_4 + 10x_5 = 161 \quad ...(2)$$
$$2x_3 + 5x_4 + 3x_5 = 61 \quad ...(3)$$
$$4x_4 + 5x_5 = 30 \quad ...(4)$$
$$2x_1 + 3x_2 + 2x_3 + 1x_4 + 5x_5 = 81 \quad ...(5)$$

Solving (1) and (4)
$$x_5 = 2$$
$$x_4 = 5$$
Putting in (3) we get
$$2x_3 + 25 + 6 = 61$$
$$x_3 = 15$$

Alternative:

The matrix form of the equation is

$$[A|B] = \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 4 & 13 & 46 \\ 2 & 5 & 5 & 2 & 10 & 161 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 5 & 30 \\ 2 & 3 & 2 & 1 & 5 & 81 \end{array}\right]$$

Rewriting it as below

$$[A|B] = \left[\begin{array}{ccccc|c} 2 & 5 & 5 & 2 & 10 & 161 \\ 2 & 3 & 2 & 1 & 5 & 81 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 13 & 46 \\ 0 & 0 & 0 & 4 & 5 & 30 \end{array}\right]$$

Applying, $R_2 \to R_1 - R_2$ and $R_5 \to R_4 - R_5$

$$\left[\begin{array}{ccccc|c} 2 & 5 & 5 & 2 & 10 & 161 \\ 0 & 2 & 3 & 1 & 5 & 80 \\ 0 & 0 & 2 & 5 & 3 & 61 \\ 0 & 0 & 0 & 4 & 13 & 46 \\ 0 & 0 & 0 & 0 & 8 & 16 \end{array}\right]$$

$$\begin{bmatrix} 2 & 5 & 5 & 2 & 10 \\ 0 & 2 & 3 & 1 & 5 \\ 0 & 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 4 & 13 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 161 \\ 80 \\ 61 \\ 46 \\ 16 \end{bmatrix}$$

Now, we get
$$8x_5 = 16$$
$$x_5 = 2$$
and
$$4x_4 + 13x_5 = 46$$
$$x_4 = 5$$
Similarly,
$$2x_3 + 5x_4 + 3x_5 = 61$$
$$2x_3 + 25 + 6 = 61$$
$$x_3 = 15$$

28. (c)

$\because$ One of the eigen value is 0,

$\therefore$ Determinant of matrix is equal to 0.

So, $$B_{11} B_{22} - B_{12} B_{21} = 0$$

29. (b)

Here, $$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

Applying, $$R_3 \to R_3 - 2R_2$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

$\therefore$ $$\text{Rank}[A] = 2 \text{ and rank } [A|B] = 3$$

Since rank (A) < rank $(A|B)$, the given system of equations is inconsistent, and hence there is no solution.

30. (b)

Statements 1 and 3 are correct.

- For the orthogonal martix $|A|$ = + 1 or – 1.
- For a $n \times n$ matrix, inverse exists only if rank = n.

31. (a)

Given,

$$\frac{d^2y}{dx^2} + \frac{6dy}{dx} + 9y = 5e^{3x}$$

$$(D^2 + 6D + 9)y = 5e^{3x}$$

Auxiliary equation is $m^2 + 6m + 9 = 0$

$$(m+3)^2 = 0$$

$$m = -3, -3$$

$$\text{Complementary function} = (c_1 + c_2x)e^{-3x}$$

$$\text{Particular integral} = \frac{1}{D^2 + 6D + 9} 5e^{3x} = \frac{5e^{3x}}{(3)^2 + 6(3) + 9} = \frac{5e^{3x}}{36}$$

The complete solution is,

$$y = (c_1 + c_2x)e^{-3x} + \frac{5e^{3x}}{36}$$

32. (c)

Given equation: $\sin x \dfrac{dy}{dx} + 2y = \tan^3 \dfrac{x}{2}$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{\sin x} y = \frac{\tan^3 \frac{x}{2}}{\sin x}$$

This is linear form of $\dfrac{dy}{dx} + Py = Q$

$$\therefore \quad P = \frac{2}{\sin x}$$

$$\text{Integrating factor} = e^{\int P dx} = e^{\int \frac{2}{\sin x} dx}$$

$$= e^{2\int \text{cosec}\, x dx}$$

$$= e^{2 \ln \tan \frac{x}{2}} = \tan^2 \frac{x}{2}$$

33. (b)

Given $\quad \dfrac{dy}{dx} + \dfrac{x}{y} = 0$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\int y dy = -\int x dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

$x^2 + y^2 = 2c$ Represents family of circles.

34. (b)

If z is function of x alone, the solution will be $z = A\sin x + B\cos x$, where A and B are constants. Since z is a function of x and y, A and B can be arbitrary functions of y. Hence the solution of the given equation is

$$z = f(y)\sin x + \phi(y)\cos x$$

$$\frac{\partial z}{\partial x} = f(y)\cos x - \phi(y)\sin x$$

When $\quad x = 0;\ z = e^y$

$\therefore \quad \phi(y) = e^y$

When $\quad x = 0,\ \dfrac{\partial z}{\partial x} = 1$

$\therefore \quad f(y) = 1$

Hence the desired solution is,

$$z = \sin x + e^y \cos x.$$

Alternate solution:

$$\frac{\partial^2 z}{\partial x^2} + z = 0$$

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$z = e^0 [A\cos x + B\sin x]$$

$$z(0) = A + 0$$

$$e^y = A$$

$$\frac{\partial z}{\partial x} = -A\sin x + B\cos x$$

At $x = 0$

$$\frac{\partial z}{\partial x} = 1$$

$$B = 1$$

$$z = e^y \cos x + \sin x$$

35. (a)

Given,

$$\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}} = \frac{(\cos 12\theta + i\sin 12\theta)(\cos(-20\theta) + i\sin(-20\theta))}{(\cos 12\theta + i\sin 12\theta)(\cos(-20\theta) + i\sin(-20\theta))}$$

$$= \frac{(\cos\theta + i\sin\theta)^{12} (\cos\theta + i\sin\theta)^{-20}}{(\cos\theta + i\sin\theta)^{12} (\cos\theta + i\sin\theta)^{-20}} = 1$$

36. (d)

$$f(z) = ze^{1/z^2} = z\left\{1 + \frac{1}{1!}z^{-2} + \frac{1}{2!}z^{-4} + \frac{1}{3!}z^{-6} + \ldots\right\}$$

$$= z + z^{-1} + \frac{z^{-3}}{2} + \frac{z^{-5}}{6} + \ldots.\infty$$

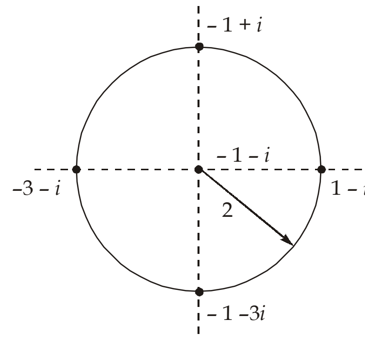
Since, there are infinite number of terms in the negative powers of z, therefore $z = 0$ is an essential singularity of $f(z)$.

37. (a)

The poles of $f(z) = \dfrac{z-3}{z^2 + 2z + 5}$ are given by $z^2 + 2z + 5 = 0$

i.e. $\quad z = -1 \pm 2i$

Here only the pole, $z = -1 - 2i$ lies inside the circle c : $|z + 1 + i| = 2$.



Therefore, $f(z)$ is analytic within c except at this pole.

$$\text{Residue } f(-1-2i) = \lim_{z\to -1-2i} \frac{(z+1+2i)(z-3)}{z^2+2z+5}$$

$$= \lim_{z\to -1-2i} \frac{z-3}{z+1-2i} = \frac{-4-2i}{-4i} = \frac{1}{2} - i$$

Hence by Residue theorem,

$$\int_c f(z)\,dz = 2\pi i \,\text{Res}\, f(-1-2i) = 2\pi i\left(\frac{1}{2} - i\right)$$
$$= \pi(2+i)$$

38. (c)

Given, $\lim\limits_{x\to 0} \dfrac{\log x}{\cot x}$; $\dfrac{\infty}{\infty}$ Form

Applying L' Hospital's rule.

$$\lim_{x\to 0} \frac{1/x}{-\text{cosec}^2 x} = -\lim_{x\to 0} \frac{\sin^2 x}{x} \quad ; \quad \frac{0}{0} \text{ form}$$

Again applying L' Hospital's rule.

$$= -\lim_{x\to 0} \frac{2\sin x \cos x}{1} = 0$$

39. (b)

Given, $\displaystyle\int_0^a \frac{x^7}{\sqrt{(a^2-x^2)}}\,dx$

Put $x = a\sin\theta$

$dx = a\cos\theta d\theta$

Changing limits:

when $x = 0$, $\theta = 0$, where $x = a$, $\theta = \dfrac{\pi}{2}$

$$\therefore \quad \int_0^{\pi/2} \frac{a^7 \sin^7\theta}{a\cos\theta} a\cos\theta d\theta = a^7 \int_0^{\pi/2} \sin^7\theta d\theta$$

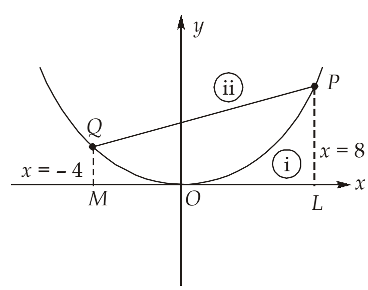
$$= \frac{a^7 (n-1)(n-3)....2}{n(n-2)....3}$$

$$= a^7 \frac{6\times4\times2}{7\times5\times3} = \frac{16}{35}a^7$$

NOTE: • When n is odd, $\int_0^{\pi/2} \sin^n x dx = \frac{(n-1)(n-3)(n-5)....2}{n(n-2)(n-4)....3}$

• When n is even, $\int_0^{\pi/2} \sin^n x dx = \frac{(n-1)(n-3)(n-5)....1}{n(n-2)(n-4)....2} \frac{\pi}{2}$

40. (c)



Given, parabola is, $x^2 = 8y$
and the straight line is, $x - 2y + 8 = 0$

The required area POQ = $\begin{pmatrix}\text{area bounded by straight line \&} \\ x\text{-axis from } x = -4 \text{ to } x = 8\end{pmatrix} - \begin{pmatrix}\text{area bounded by parabola \&} \\ x\text{-axis from } x = -4 \text{ to } x = 8\end{pmatrix}$

$$= \int_{-4}^{8} \frac{x+8}{2} dx - \int_{-4}^{8} \frac{x^2}{8} dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2} + 8x\right|_{-4}^{8} - \frac{1}{8}\left|\frac{x^3}{3}\right|_{-4}^{8}$$

$$= \frac{1}{2}\left|(32+64)-(-24)\right| - \frac{1}{24}(512+64)$$

$$= \frac{1}{2}[96+24] - \frac{1}{24}(576) = 36 \text{ square unit}$$

41. (b)

$f(x) = 0$ is the root of the solution.

Clearly the line, $f(x) = 0$ intersects at 4 distinct points in $0 < x < 6$.

42. (a)

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x^4 - 3x + 1$$

$$f'(x) = 4x^3 - 3$$

Given, $x_0 = 0$

Therefore, $f(x_0) = 0^4 - 3 \times 0 + 1 = 1$

$$f'(x_0) = 4 \times 0^3 - 3 = -3$$

Hence, $x_1 = 0 - \frac{1}{-3} = \frac{1}{3}$

43. (d)

Bisection, Regula-falsi, Secant and Newton -Raphson methods are used to solve non-linear algebraic and transcendental equations.

44. (b)

Laplace transform of $\cosh(bt) = \frac{s}{s^2 - b^2}$

45. (d)

The Fourier coefficient $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx dx = \frac{2}{\pi}\int_{0}^{\pi} x \sin nx dx$

(x sinnx is an even function on $[-\pi, \pi]$)

$$= \frac{2}{\pi}\left[-x\left(\frac{\cos nx}{n}\right) + \left(\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi \cos n\pi}{n}\right] = \frac{2}{n}(-1)^{n+1}$$ Put $n = 3$

$$b_3 = \frac{2}{3}(-1)^4 = \frac{2}{3}$$

46. (d)

Taylor series expansion of a function $f(x)$ about $x = 0$ is given by

$$f(x) = f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+...$$

$$\text{Coefficient of } x^2 = \frac{f''(0)}{2!}=\frac{f''(0)}{2}$$

Given:

$$f(x) = \cos^2 x$$
$$f'(x) = -\sin(2x)$$
$$f''(x) = -2\cos(2x)$$
$$f''(0) = -2\cos(0) = -2$$

$$\text{Therefore coefficient of } x^2 = \frac{f''(0)}{2}=\frac{-2}{2}=-1$$

47. (b)

The probability that A can solve the problem = $\frac{1}{2}$.

The probability that A cannot solve the problem.

$$= 1-\frac{1}{2}=\frac{1}{2}$$

Similarly the probability that B and C cannot solve the problem are $\left(1-\frac{3}{4}\right)$ and $\left(1-\frac{1}{4}\right)$.

The probability that A, B and C cannot solve the problem = $\left(1-\frac{1}{2}\right)\times\left(1-\frac{3}{4}\right)\times\left(1-\frac{1}{4}\right)=\frac{3}{32}$

The probability that the problem will be solved is = $1-\frac{3}{32}=\frac{29}{32}$

48. (d)

Here there are three types of families.

Case I: For, zero child family.

Probability of a family having no child (boys) = 0.2

Case II: For one child family

Boy	Girl
0	1
1	0

In this case probability of a family having no boy = 0.3 × 0.5 = 0.15

Case III:

Boy	Girl
0	2
1	1
2	0

In this case probability of a family having no boy = $0.5 \times \frac{1}{3}$ = 0.167

Considering all three cases,

Probability of a family having no boy = 0.2 + 0.15 + 0.167 = 0.517

49. (a)

$p = 1\% = 0.01$, $n = 100$, $m = np = 100 \times 0.01 = 1$

$$P(r) = \frac{e^{-m}\cdot(m)^r}{r!} = \frac{e^{-1}(1)^r}{r!} = \frac{e^{-1}}{r!}$$

P(4 or more faulty condensers)

$$= P(4) + P(5) + \dots P(100)$$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right] = 1 - \frac{8}{3}e^{-1}$$

50. (a)

Given,

$$f(x) = 3x^3 - 7x^2 + 5x + 6$$

$$f'(x) = 9x^2 - 14x + 5$$

$$f''(x) = 18x - 14$$

$$f'(x) = 0$$

$$9x^2 - 14x + 5 = 0$$

$$x = 1, 0.55$$

For $x = 1$, $f''(1) = 18 - 14 = 4 > 0$ (local minima)

For $x = 0.55$

$f''(0.55) = -4.1 < 0$ (local maxima)

Minimum $\{f(0), f(1), f(2)\}$

Minimum $\{6, 7, 12\} = 6$

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