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ESE 2027 : Prelims Exam | CLASSROOM TEST SERIES | GS & ENGINEERING APTITUDE | Test 1

Section A : Reasoning & Aptitude [All Topics]

Section B : Engineering Mathematics [All Topics]

Answer Key

1.	(b)	11.	(b)	21.	(b)	31.	(c)	41.	(a)
2.	(c)	12.	(b)	22.	(b)	32.	(b)	42.	(b)
3.	(b)	13.	(b)	23.	(d)	33.	(c)	43.	(d)
4.	(c)	14.	(a)	24.	(a)	34.	(b)	44.	(c)
5.	(b)	15.	(b)	25.	(a)	35.	(d)	45.	(b)
6.	(a)	16.	(a)	26.	(c)	36.	(b)	46.	(b)
7.	(a)	17.	(b)	27.	(b)	37.	(b)	47.	(c)
8.	(d)	18.	(d)	28.	(b)	38.	(d)	48.	(b)
9.	(c)	19.	(b)	29.	(c)	39.	(a)	49.	(b)
10.	(b)	20.	(b)	30.	(b)	40.	(c)	50.	(c)

Section A : Reasoning & Aptitude

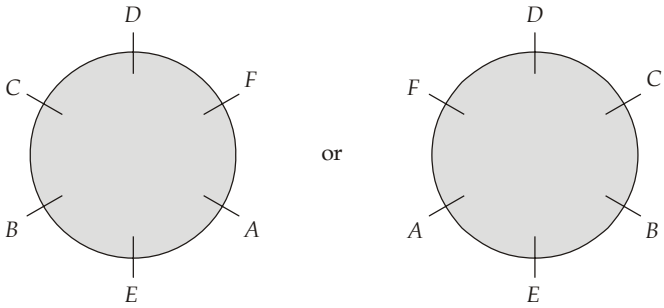
1. (b)

Since number of males equal that of females, there are three males and three females. Now, A, E and B are sons and hence males. So, C, D and F are females. D has one male and one female child. Since A has a child (B), A is a part of married couple. Thus, A and D form a couple having two children namly B and C (the remaining female who is girl). Hence A is the husband of D.

2. (c)

E is opposite D. A is between E and F. Since C is not the neighbour of E, so the only neighbouring blank position of E is occupied by B.

Thus we have two possible arrangements as below:

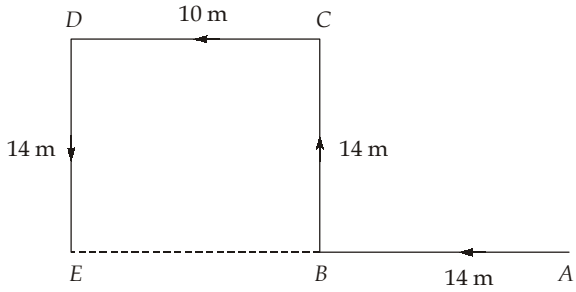


In each of the arrangements, F is opposite B.

3. (b)

The movements of Namita are as shown

A to B, B to C, C to D and D to E



Clearly, Namita's shortest distance from his initial position

$= AE = (AB + BE) = (AB + CD)$

= 14 + 10 = 24 m

4. (c)

CP of 80 apples = ₹240

$\therefore$ CP of 1 apple = ₹3

$\therefore$ CP of 20 apples = ₹60

To earn a profit of 20%,

$$\text{SP} = 120\% \text{ of } 240 = ₹288$$

If he sales $\frac{1}{4}$ of his apples i.e., 20 apples at cost price for ₹60 then to get same profit,

SP of remaining 60 apples = ₹(288 – 60) = ₹228

$$\therefore \quad \text{SP of 1 apple} = \frac{228}{60} = ₹3.80$$

5. (b)

Let the two numbers are x and y.

Now, by given condition,

$$\text{Sum of squares of two numbers} = 97$$

i.e. $x^2 + y^2 = 97$ ...(i)

$$\text{and square of their difference} = 25$$

i.e. $(x - y)^2 = 25$

$$(x^2 + y^2) - 2xy = 25$$

$97 - 2xy = 25$ using equation (i)

$2xy = 72$

$xy = 36$

6. (a)

Given that, $\log_{10} a = p$

$p \log_{10} a = p^2$

$\log_{10} a^p = p^2$ ...(i)

Similarly, $\log_{10} b = q$

$q \log_{10} b = q^2$

$\log_{10} b^q = q^2$ ...(ii)

On adding Equations (i) and (ii), we get

$$\log_{10} a^p + \log_{10} b^q = p^2 + q^2$$

$$\therefore \quad \log_{10} a^p b^q = p^2 + q^2 \qquad (\because \log x + \log y = \log xy)$$

7. (a)

Let the radius of circle is 'r' and the side of a square is 'a'. Since the perimeter is same,

$$2\pi r = 4a$$

$$a = \frac{\pi r}{2}$$

$$\therefore \quad \text{Area of square} = a^2 = \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} = \frac{9.86 r^2}{4} = 2.46\, r^2$$

$$\text{and area of circle} = \pi r^2 = 3.14\, r^2$$

Now let the side of equilateral triangle is x.

Since the perimeter of circle and triangle is same,

$$3x = 2\pi r$$

$$x = \frac{2\pi r}{3}$$

$$\therefore \quad \text{Area of equilateral triangle} = \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4} \times \frac{4\pi^2 r^2}{9}$$

$$= \frac{\pi^2 r^2}{3\sqrt{3}} = 1.89 r^2$$

Hence, Area of circle > Area of square > Area of equilateral triangle

8. (d)

$$\text{One day work of } A = \frac{1}{x}$$

$$\text{One day work of } B = \frac{1}{3x}$$

$$\therefore \quad \text{One day work of both A and B} = \frac{1}{x} + \frac{1}{3x} = \frac{4}{3x}$$

and given one day work of both A and B = $\frac{1}{12}$

$$\frac{4}{3x} = \frac{1}{12}$$

$$\therefore \quad x = 16$$

9. (c)

Let the number of students in each row = x

Total number of rows = y

Total number of students = xy

By given condition,

$$(x + 1)(y - 2) = xy$$

$$-2x + y = 2 \quad \ldots(i)$$

and $$(x - 1)(y + 3) = xy$$

$$3x + y = 3 \quad \ldots(ii)$$

By solving (i) and (ii),

$$x = 5, y = 12$$

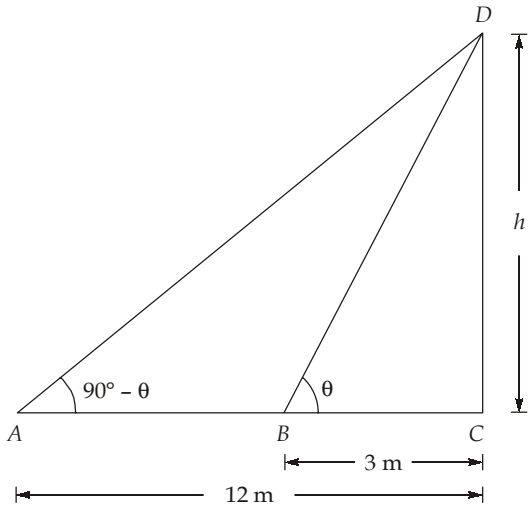
Number of students in each row = 5

Total number of row = 12

Total number of students = $12 \times 5 = 60$

10. (b)

Let h be the height of the tower and the angle of elevation from the first house be θ. Since the angles of elevation are complementary, the angle of elevation from the second house is $90° - \theta$.



We have, $\angle CBD = \theta$ and $\angle DAC = 90° - \theta$

$\therefore$ In $\triangle BCD$,

$$\tan\theta = \frac{CD}{BC} = \frac{h}{3}$$

and in $\triangle ACD$, $$\tan(90° - \theta) = \frac{CD}{AC}$$

$$\cot\theta = \frac{h}{12}$$

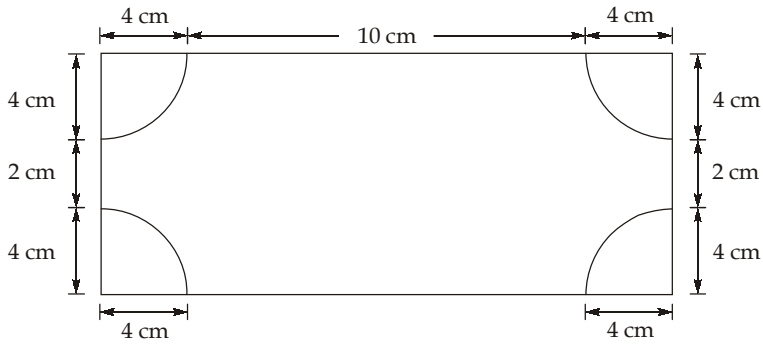
From equation (i) and (ii), we get

$$\tan\theta \cdot \cot\theta = \frac{h}{3} \times \frac{h}{12}$$

$$h^2 = 36$$

$$h = 6 \text{ m}$$

11. (b)



$$\text{Perimeter of the remaining portion} = \left(\frac{2\pi r}{4}\right)4 + 10 + 2 + 10 + 2$$

$$= 49.12 \text{ cm}$$

12. (b)

$$\text{For conical part, } r = \frac{6}{2} = 3 \text{ cm}, h = 4 \text{ cm}$$

$$l = \sqrt{h^2 + r^2} = 5 \text{ cm}$$

$$\text{Surface area of conical part} = \pi rl = 3.14 \times 3 \times 5$$

$$= 47.1 \text{ cm}^2$$

$$\text{For hemispherical part, } r = \frac{6}{2} = 3 \text{ cm}$$

$$\text{Surface area of hemispherical part} = 2\pi r^2$$

$$= 2 \times 3.14 \times 3 \times 3 = 56.52 \text{ sqcm}$$

$$\therefore \text{ Surface area of toy} = 47.1 + 56.52 = 103.62 \text{ sqcm}$$

13. (b)

- Two distinct lines intersect at only one point or do not intersect at all (if they are parallel). If two lines shared two or more points, they would be the exact same line (coincident), not distinct.
- Through a single point in a plane, infinitely many lines can be drawn in different directions.

14. (a)

Every triangle has a unique inscribed circle (incircle) tangent to all three of its sides.

Since, in congruent triangles corresponding sides and angles of one triangle are equal to that of other triangle. Their corresponding centroids coincide when the triangles are placed in corresponding positions, so $C_1 = C_2$.

The incircles have equal radii because the triangles have identical side lengths and angles, so $r_1 = r_2$.

15. (b)

The area of the triangle formed by joining the middle points of the sides of a triangle is equal to one-fourth area of the given triangle.

$$\text{Let area of } 15^{\text{th}}\ \Delta = A$$

$$\text{Then area of } 18^{\text{th}}\ \Delta = A \times \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) = \frac{A}{64}$$

$$\text{So, required ratio} = \frac{64}{1} = 64$$

16. (a)

$N = 1(9)^{17} \times 24^{71} + 3(3)^{92}$

- $(9)^{17}$: odd power of 9 (i.e. 9 at the unit place)
- 24^{71}: odd power of 4 (i.e. 4 at the unit place)
- $(3)^{92}$: $4n$ power of 3 (i.e. 1 at the unit place)

Unit digit of N: $9 \times 4 + 1 = 7$ (unit digit)

17. (b)

Number of vowels in the alphabet = 5

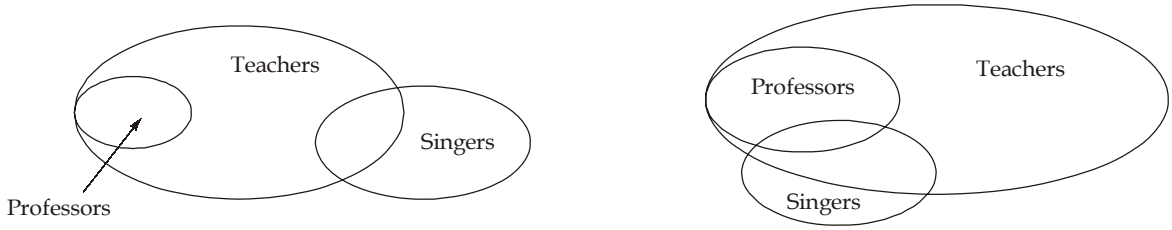
Number of consonants = 21

Number of non zero digits = 9

Chosen key can have as many options as the distinct values in the specific ring and the total number of possible combinations is the product of 3 numbers = 5 × 21 × 9 = 945

18. (d)

The possible Venn diagrams from the given statements can be drawn as below:



From a look at the Venn diagram, we get that it is not necessary that some professors are singers and not all singers are professors. Hence, none of the conclusions follow.

19. (b)

$$\text{Sushil} + \text{Mukesh} = 200 \quad \ldots(1)$$
$$\text{Asim} + \text{Rajesh} = \text{Sushil} \quad \ldots(2)$$
$$\text{Mukesh} = 4\ \text{Rajesh} \quad \ldots(3)$$
$$\text{Rajesh} = \text{Asim} - 20$$
$$\Rightarrow \quad \text{Asim} = \text{Rajesh} + 20 \quad \ldots(4)$$

From equation (1) and (3),

$$\text{Sushil} + 4\ \text{Rajesh} = 200 \quad \ldots(5)$$

From equation (2) and (4),

$$\text{Rajesh} + \text{Rajesh} + 20 = \text{Sushil}$$
$$\text{Sushil} = 2\ \text{Rajesh} + 20 \quad \ldots(6)$$

From equation (5) and (6),

$$6\ \text{Rajesh} + 20 = 200$$
$$6\ \text{Rajesh} = 180$$
$$\text{Rajesh} = 30$$

Thus, from equation (4),

$$\text{Marks scored by Asim} = 30 + 20 = 50$$

20. (b)

The ratio of number of workers in each group is obtained by dividing the total wage ratio by the individual wage ratio.

$$\text{Ratio of number of men, women and children} = \frac{18}{6} : \frac{10}{5} : \frac{12}{3} = 3x : 2x : 4x$$

$$\therefore \quad (3x + 2x + 4x) = 18$$
$$x = 2$$

$$\text{Therefore, number of women} = 4$$

$$\text{Total wages of all women} = \frac{10}{18 + 10 + 12} \times 4000 = ₹1000$$

$$\therefore \quad \text{Daily wage of each women} = \frac{1000}{4} = ₹250$$

21. (b)

$$x+\frac{1}{x} = 2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}+2 = 4$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 2$$

$$\Rightarrow \quad x^4+\frac{1}{x^4}+2 = 4$$

$$\Rightarrow \quad x^4+\frac{1}{x^4} = 2$$

22. (b)

Let the quantity of wine in the cast originally be x litres.

Then, quantity of wine left in the cast after 5 operations

$$= \left[x\left(1-\frac{24}{x}\right)^5\right] \text{litres}$$

$$\therefore \quad \frac{x\left(1-\frac{24}{x}\right)^5}{x} = \frac{32}{32+211} = \frac{32}{243}$$

$$\Rightarrow \quad \left(1-\frac{24}{x}\right)^5 = \left(\frac{2}{3}\right)^5$$

$$\Rightarrow \quad x = 72 \text{ litres}$$

23. (d)

Let x, y, z, w be the filling rates for the inlet pipes. As per the given information,

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = \frac{1}{24}$$

$$\frac{1}{y}+\frac{1}{z}+\frac{1}{w} = \frac{1}{30}$$

$$\frac{1}{x}+\frac{1}{w} = \frac{1}{40}$$

Adding the 3 equations,

$$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right) = \frac{1}{24}+\frac{1}{30}+\frac{1}{40} = \frac{12}{120} = \frac{1}{10}$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{w} = \frac{1}{20}$$

$$\frac{1}{y}+\frac{1}{z} = \frac{1}{20}-\left(\frac{1}{x}+\frac{1}{w}\right) = \frac{1}{20}-\frac{1}{40} = \frac{1}{40}$$

So, second and third pipes together can fill the tank in 40 minutes.

24. (a)

Using statement I:

Let pipe C takes t hours to fill the tank, pipe A takes $3t$ hours and pipe B takes $1.5t$ hours.

At this stage, we have no more information to solve the question.

Hence, statement I is insufficient. Using the data given in the second statement.

Working together, they take 1 hour to fill the tank, using the information given in statement II,

Portion of the tank filled by pipe A and B together in hour = $\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$

$$\frac{1}{t}+\frac{1}{3t}+\frac{1}{1.5t} = 1$$

$$t = 2 \text{ hours}$$

Thus, pipe A takes 6 hours and pipe B takes 3 hours to fill the tank.

Thus, working together, they take 2 hours to fill the tank.

Therefore, both statements are needed to get the answer.

25. (a)

Let the least amount of wages be ₹x. Then,

$$x + (x + 20) + (x + 40) + (x + 60) = 4 \times 60$$

or, $$4x + 120 = 240$$

$$4x = 240 - 120 = 120$$

Therefore, $$x = ₹30$$

Section B : Engineering Mathematics

26. (c)

The line integral over the circular path C is given by

$$\int_C \vec{F}\,d\vec{r} = \int_C [(\sin y)\hat{i} + x(1+\cos y)\hat{j}][dx\hat{i} + dy\hat{j}]$$

$[\because z = 0, \text{ hence } dz = 0]$

$$\int_C \vec{F}\,d\vec{r} = \int_C \sin y\,dx + x(1+\cos y)dy$$

$$= \int_C \sin y\,dx + x\cos y\,dy + x\,dy$$

The parametric equations of given path $x^2 + y^2 = a^2$ are $x = a\cos\theta$, $y = a\sin\theta$

where θ varies from 0 to 2π

$\therefore$ $$\int_C \vec{F}\,d\vec{r} = \int_0^{2\pi} d[a\cos\theta\sin(a\sin\theta)] + \int_0^{2\pi} a\cos\theta\, a\cos\theta\, d\theta$$

$$= \int_0^{2\pi} d[a\cos\theta\sin(a\sin\theta)] + \int_0^{2\pi} a^2\cos^2\theta\, d\theta$$

$$= 0 + a^2 \int_0^{2\pi} \frac{1+\cos 2\theta}{2} d\theta = \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{a^2}{2} \times 2\pi = \pi a^2$$

27. (b)

$$\frac{dy}{dx} + y \tan x = \cos x$$

The given differential equation is of the form $\frac{dy}{dx} + Py = Q$.

Hence, it is a linear differential equation with

$$\text{I.F.} = e^{\int P.dx} = e^{\int \tan x} = e^{\int \frac{\sin x}{\cos x} dx} = e^{-\ln \cos x} = \frac{1}{\cos x}$$

The solution of the given differential equation is

$$y(\text{I.F.}) = \int Q(I.F.)\, dx + c$$

$$y \times \frac{1}{\cos x} = \int \cos x \times \frac{1}{\cos x} dx + c$$

$$y = (x + c)\cos x$$

Since, the curve passes through origin. Hence

$$0 = 0 + c$$

$$c = 0$$

$\therefore$ $$y = x \cos x$$

Hence, $$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\cos\frac{\pi}{2} = 0$$

28. (b)

For λ to be an eigenvalue of matrix A, there must exist a non-zero vector X (eigen vector) such that

$$AX = \lambda X$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$\Rightarrow$ $$\lambda = 1$$

The product of eigen values of a matrix is given by

$$\lambda_1\lambda_2\lambda_3 = \text{Determinant of A} = 12 - 3 = 9$$

$$\lambda_2\lambda_3 = \frac{9}{1} = 9$$

29. (c)

- The inverse of a square matrix, if it exists, is unique.
- If A and B are singular matrices of order n, then $|A| = 0$ and $|B| = 0$. Using the determinant property,

$$|AB| = |A||B| = 0$$

Since $|AB| = 0$, the matrix AB is also singular and of order n.

Thus, both the given statements are correct.

30. (b)

$$f(x) = (x+1)\tan^{-1}(e^{-2x})$$

On differentiating w.r.t. x, we get

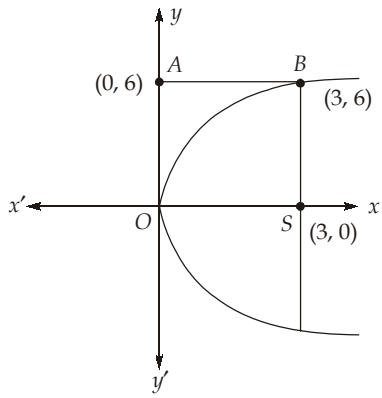
$$f'(x) = (x+1)\cdot\frac{1}{1+e^{-4x}}\cdot\{-2e^{-2x}\} + \tan^{-1}(e^{-2x})\cdot 1$$

$$\therefore \quad f'(0) = (0+1)\cdot\frac{1}{1+e^{0}}(-2e^{0}) + \tan^{-1}(e^{-0})$$

$$= -\frac{2}{2} + \tan^{-1}(1)$$

$$= -1 + \frac{\pi}{4} = \frac{\pi}{4} - 1$$

31. (c)



Equation of curve is

$$y^2 = 12x$$

The line $y = 6$ intersects the curve at,

$$36 = 12x$$

$$\Rightarrow \quad x = 3$$

$$\therefore \quad \text{Required area} = \int_0^3 (6 - \sqrt{12x})\,dx$$

$$= [6x]_0^3 - \sqrt{12}\left[\frac{2x^{3/2}}{3}\right]_0^3$$

$$= [6\times 3] - \frac{\sqrt{12}\times 2\times\sqrt{27}}{3}$$

$$= 18 - 12 = 6 \text{ Square unit}$$

32. (b)

A leap year has 366 days, in which 2 days may be any one of the following pairs :

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday).

$$\therefore \quad \text{Required probability} = \frac{3}{7}$$

33. (c)

Given that Mean = $np = 4$ and Variance = $npq = 3$

$$q = \frac{npq}{np} = \frac{3}{4}$$

Thus, $$p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \quad n = 16$$

We know that in a binomial distribution, the value of r for which $P(X = r)$ is maximum is known as mode of binomial distribution. The mode r satisfies the inequality :

$$(n+1)p - 1 \le r \le (n+1)p$$

$$\frac{17}{4} - 1 \le r \le \frac{17}{4}$$

$$\frac{13}{4} \le r \le \frac{17}{4}$$

$$3.25 \le r \le 4.25$$

Since r is an integer, $$r = 4$$

34. (b)

Case I, Let a white ball is drawn from the first bag and placed unseen in the second bag.

Probability (black drawn from second bag) = $\frac{{}^5C_1}{{}^8C_1} \times \frac{{}^4C_1}{{}^7C_1} = \frac{20}{56}$

Case II, Let a black ball is drawn from the first bag and placed unseen in the second bag.

Probability (black drawn from second bag) = $\frac{{}^3C_1}{{}^8C_1} \times \frac{{}^5C_1}{{}^7C_1} = \frac{15}{56}$

$\therefore$ Required probability = $\frac{20}{56} + \frac{15}{56} = \frac{35}{56}$

35. (d)

There is only one 3rd-order minor. In order that the rank may be 2, we must have

$$\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$$

Consider, $\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix}$

Using $R_1 \to R_1 - R_3$

$$\begin{vmatrix} 1+x & 0 & -1-x \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix}$$

Using $C_3 \to C_3 + C_1$

$$\begin{vmatrix} 1+x & 0 & 0 \\ 1 & 7+x & 7 \\ 2 & 5 & 5+x \end{vmatrix}$$

$$= (1+x)\begin{vmatrix} 7+x & 7 \\ 5 & 5+x \end{vmatrix} = (1+x)\{(7+x)(5+x) - 35\}$$

$$= (1+x)(x^2 + 12x)$$

$$= x(1+x)(x+12)$$

$\therefore$ $x = 0, -1, -12$ for the determinant to be zero.

When $x = 0$, the matrix $= \begin{bmatrix} 3 & 5 & 2 \\ 1 & 7 & 6 \\ 2 & 5 & 3 \end{bmatrix}$

Clearly, a minor $\begin{vmatrix} 3 & 5 \\ 1 & 7 \end{vmatrix} \neq 0$, so, the rank = 2 .

When $x = -1$, the matrix $= \begin{bmatrix} 2 & 5 & 2 \\ 1 & 6 & 6 \\ 2 & 5 & 2 \end{bmatrix}$

Clearly, a minor $\begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix} \neq 0$. So, the rank = 2.

When $x = -12$, the matrix $= \begin{bmatrix} -9 & 5 & 2 \\ 1 & -5 & 6 \\ 2 & 5 & -9 \end{bmatrix}$

Clearly, a minor $\begin{vmatrix} -9 & 5 \\ 1 & -5 \end{vmatrix} \neq 0$, so, the rank = 2

$\therefore$ The matrix has the rank 2 if $x = 0, -1, -12$.

36. (b)

For a valid probability density function,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 c(4x - 2x^2)\,dx = 1$$

$$c\int_0^2 (4x - 2x^2)\,dx = 1$$

$$c\left[\frac{4x^2}{2} - \frac{2x^2}{3}\right]_0^2 = 1$$

$$\frac{8}{3}c = 1$$

$$c = \frac{3}{8}$$

37. (b)

$$\lim_{x\to 5} \frac{x^2 - 4x - 5}{x^2 + 2x - 35} = \frac{0}{0}$$

Applying L-Hospital rule,

$$\lim_{x\to 5} \frac{2x - 4}{2x + 2} = \frac{10 - 4}{10 + 2} = \frac{6}{12} = \frac{1}{2}$$

38. (d)

The characteristic equation of the matrix P is given by $|P - \lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 2\lambda = 0$

By Cayley-Hamilton theorem, every square matrix satisfies it's characteristic equation. Thus,

$$P^2 - 2P = 0 \quad \ldots(i)$$

Multiplying both sides by P^6, we get

$$P^8 - 2P^7 = 0 \quad \ldots(ii)$$

Multiplying both sides of (i) by P^4, we get

$$P^6 - 2P^5 = 0$$

$$2P^6 - 4P^5 = 0 \quad \ldots(iii)$$

Multiplying both sides of (i) by P^2, we get

$$P^4 - 2P^3 = 0$$

$$3P^4 - 6P^3 = 0 \quad \ldots(iv)$$

Now, $P^8 - 2P^7 + 2P^6 - 4P^5 + 3P^4 - 6P^3 + 2P^2$

$$= 2P^2 = 2(2P) = 4P$$

39. (a)

$z^2 - 4 = 0 \Rightarrow z = 2, -2$ are the singular points

$z = 2$ lies inside $C : |z - 1.5| = 1$

$z = -2$ lies outside $C : |z - 1.5| = 1$

Using Cauchy's Integral Theorem, if $f(z)$ is analytic on and inside a simple closed curve C and a is any point inside C, then

$$\oint_c \frac{f(z)}{z-a} dz = 2\pi i f(a)$$

where, $f(z) = \dfrac{5z^3 + 3z^2}{z+2}$

We have $$\int_C \frac{5z^3 + 3z^2}{z^2 - 4} dz = \int_C \frac{5z^3 + 3z^2}{(z+2)(z-2)} dz$$

$$= \int_C \frac{\left(\dfrac{5z^3 + 3z^2}{z+2}\right)}{(z-2)} dz = 2\pi i f(2)$$

$$= 2\pi i \left\{ \frac{5(2)^3 + 3(2)^2}{2+2} \right\} = 2\pi i \left(\frac{52}{4} \right) = 26\pi i$$

40. (c)

Comparing the differential equation with the standard form $Mdx + Ndy = 0$, we get

$$2xy\,dx + x^2\,dy = 0$$

Considering option (c), $M = 2xy, N = x^2$

$$\frac{\partial M}{\partial y} = 2x, \quad \frac{\partial N}{\partial x} = 2x$$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, thus it is exact differential equation.

Hence, option (c) is correct.

41. (a)

$$f(x) = x^3 - 15x^2 + 40$$

$$f'(x) = 3x^2 - 30x = 0$$

$$= 3(x^2 - 10x) = 0$$

$$x = 0, 10 \text{ are critical points}$$

But x = 10 does not belong to [–1, 7]

x = –1, 7 are end points.

$$f''(x) = 6x - 30$$

$$f''(0) = -30$$

Since $f''(0) < 0$,

$f(x)$ has a maximum at $x = 0$

At end points, $f(-1) = 24$ and $f(7) = -352$

∴ The minimum value of $f(x)$ in [–1, 7] is –352.

42. (b)

$$\text{div } \vec{u} = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial y}(y^2) = 3x^2 + 2y = \phi \text{ (say)}$$

$$\text{Grad } \phi = \vec{\nabla}\phi = 6x\hat{i} + 2\hat{j}$$

$$\text{At } (4,2,0), \vec{\nabla}\phi = 24\hat{i} + 2\hat{j}$$

The directional derivative of $\text{div } \vec{u}$ at the point (4, 2, 0) in the direction of the vector $4\hat{i} - 3\hat{j}$ is

$$(\vec{\nabla}\phi)\cdot\frac{\vec{a}}{|\vec{a}|} = (24\hat{i} + 2\hat{j})\cdot\frac{(4\hat{i} - 3\hat{j})}{\sqrt{4^2 + 3^2}} = \frac{96 - 6}{5} = 18$$

43. (d)

Given : $\frac{dy}{dx} = \frac{\sin x}{y + 2}$

$$\int (y + 2)\, dy = \int \sin x\, dx$$

$$\frac{y^2}{2}+2y = -\cos x + C$$

$$y^2+4y = -2\cos x + 2C$$

Using the given initial condition:

$$y(0) = 0, \text{ we get}$$

$$0 = -2 + 2C$$

$$C = 1$$

$$y^2+4y = -2\cos x + 2$$

$\therefore$ The general solution is $y(y+4) = 2(1-\cos x)$

44. (c)

$$\text{PI} = \frac{1}{D^3+1}\cos(2x-1) \qquad \left[\frac{1}{f(D^2)}\cos ax = \frac{1}{f(-a^2)}\cos ax\right]$$

$$= \frac{1}{D(-4)+1}\cos(2x-1) = \frac{1+4D}{(1-4D)(1+4D)}\cos(2x-1)$$

$$= 1+4D\times\frac{1}{1-16D^2}\cos(2x-1)$$

Put $D^2 = -2^2 = -4$

$$= (1+4D)\times\frac{1}{1-16(-4)}\cos(2x-1)$$

$$= \frac{1}{65}\left[\cos(2x-1)+4D\cos(2x-1)\right]$$

$$= \frac{1}{65}\left[\cos(2x-1)-8\sin(2x-1)\right]$$

$\Rightarrow$ Option (c) is correct.

45. (b)

The total area under the curve for any pdf is always equal to 1, hence

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$\int_0^{\infty}\frac{e^{-2x}}{k}dx = 1$$

$$\frac{1}{k}\left(\frac{e^{-2x}}{(-2)}\right)_0^{\infty} = 1$$

$$-\frac{1}{2k}\left(e^{-2x}\right)_0^{\infty} = 1$$

$$-\frac{1}{2k}(0-1) = 1$$

$\therefore$ $$k = 0.5$$

46. (b)

For Newton Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We have to find $x = \sqrt{28} \Rightarrow x^2 - 28 = 0$

Consider $f(x) = x^2 - 28$

$f'(x) = 2x$

$$x_1 = 5.6 - \left.\frac{x^2 - 28}{2x}\right|_{x_0}$$

$$= 5.6 - \frac{5.6^2 - 28}{2 \times 5.6} = 5.30$$

47. (c)

- If the first derivative is zero (a critical point) and the second derivative is strictly negative (concave downward), $x = C$ is a point of local maximum.
- If the first derivative is zero (a critical point) and the second derivative is strictly positive (concave upward), $x = C$ is a point of local minimum.

Thus, both the statements are correct.

48. (b)

$$f(x) = \frac{x^3}{3} - x$$

We first find the first and second derivative.

$$f'(x) = \frac{3x^2}{3} - 1 = x^2 - 1 \text{ and } f''(x) = 2x$$

For a local maxima at $x = c$, $f'(c) = 0$ and $f''(c) < 0$.
Putting $f'(x) = x^2 - 1 = 0$ gives $x = 1$ or -1
For $x = -1$ only, $f''(x) < 0$ which means maximum value of the function exists for $x = -1$.

49. (b)

For the second-order linear partial differential equation

$$A\frac{\partial^2\phi}{\partial x^2} + B\frac{\partial^2\phi}{\partial x \partial y} + C\frac{\partial\phi}{\partial x} + E\frac{\partial\phi}{\partial y} + F\phi = 0$$

to be parabolic, $B^2 - 4AC = 0$. On comparing the given equation, we get $A = 2$ and $C = 2$. Thus,

$$B^2 - 4AC = 0$$
$$B^2 - 4 \times 2 \times 2 = 0$$
$$B^2 - 16 = 0$$
$$B^2 = 16$$

50. (c)

For a valid probability distribution function, the sum of all individual probabilities must equal 1 i.e. $\sum P(x_i) = 1$. Thus,

$$P(0) + P(1) + P(2) + P(3) = 1$$
$$k + 2k + 3k + 4k = 1$$
$$k = 0.1$$
$$P(x < 2) = P(0) + P(1) = k + 2k = 0.3$$
$$P(x \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 0.6$$
$$P(x < 2) + P(x \leq 2) = 0.9$$

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