MADE EASY

MADE EASY

India's Best Institute for IES, GATE & PSUs

UPSC ESE 2019

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-I

EXAM DATE : 30-06-2019 | 09:00 AM to 12:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/ error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

Corporate Office : 44-A/1, Kalu Sarai, Near Hauz khas metro station, New Delhi-110016

☎ 011-45124612, 9958995830 www.madeeasy.in

Mechanical Engineering Paper-I Analysis
ESE 2019 Main Examination

Sl.	Subjects	Total Marks
1.	Thermodynamics	64
2.	Refrigeration and Air conditioning	84
3.	Heat Transfer	32
4.	IC Engine	32
5.	Fluid Mechanics	52
6.	Turbo Machinery	124
7.	Power Plant Engineering	60
8.	Renewable Sources of Energy	32
	Total	**480**

Scroll down for detailed solutions

Section A

Q.1 (a) A main pipe divides into two parallel pipes which again form as one pipe. The length and diameter of the first parallel pipe are 1000 m and 0.8 m respectively, while the length and diameter of the second parallel pipe are 1000 m and 0.6 m respectively. Find the rate of flow in each parallel pipe, if total flow in the main is 2.5 m^3/sec. The coefficient of friction for each parallel pipe is same and equal to 0.005.

[12 Marks]

Solution:

l_1 = 1000 m, d_1 = 0.8 m,, l_2 = 1000 m, d_2 = 0.6 m

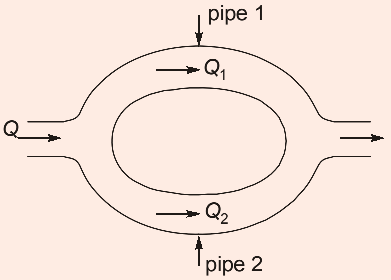
Total discharge, $Q = 2.5$ m^3/s

Also, $Q = Q_1 + Q_2 \Rightarrow 2.5 = Q_1 + Q_2$ ...(i)

Since pipe (i) and pipe (2) are in parallel,

$h_{f1} = h_{f1}$ where, h_f = head loss due to friction

$$\Rightarrow \quad \frac{f l_1 Q_1^2}{\frac{\pi^2 g}{8} \times d_1^5} = \frac{f l_2 Q_2^2}{\frac{\pi^2 g}{8} \times d_2^5}$$



$$\Rightarrow \quad \frac{1000 \times Q_1^2}{0.8^5} = \frac{1000 \times Q_2^2}{0.6^5}$$

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{0.8}{0.6}\right)^2$$

$$Q_1 = 2.0528 Q_2 \qquad \text{...(ii)}$$

From equation (i) and (ii), $2.5 = 2.0528 Q_2 + Q_2$

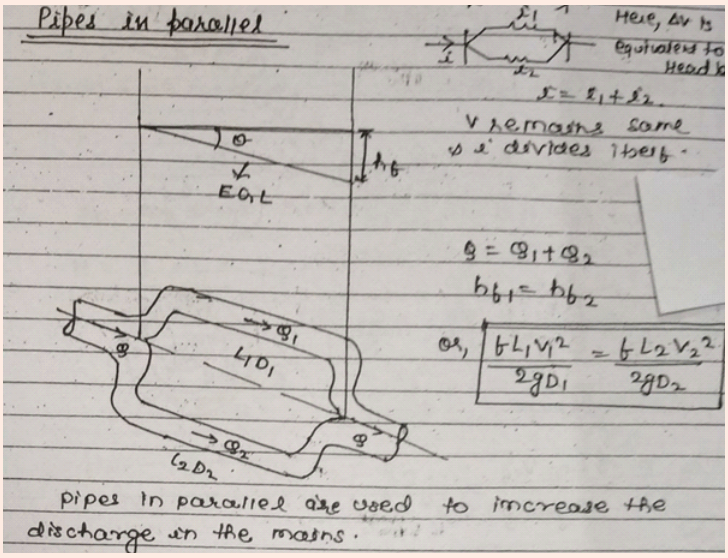
$2.5 = 3.0528 Q_2 \Rightarrow Q_2 = 0.81892$ m^3/s

Putting this value in equation (i),

$$Q_1 + 0.81892 = 2.5 \quad \Rightarrow Q_1 = 1.681 \text{ m}^3\text{/s}$$

MADE EASY Source

- **ESE Mains 2019 Workbook** (Q. 27, Page 210) discussed in Class
- **MADE EASY Classnotes**



Pipes in parallel

Here, Δv is equivalent to Head l

I = I₁ + I₂.

V remains same & I divides itself.

$Q = Q_1 + Q_2$

$h_{f_1} = h_{f_2}$

or, $\boxed{\frac{f L_1 V_1^2}{2gD_1} = \frac{f L_2 V_2^2}{2gD_2}}$

pipes in parallel are used to increase the discharge in the mains.

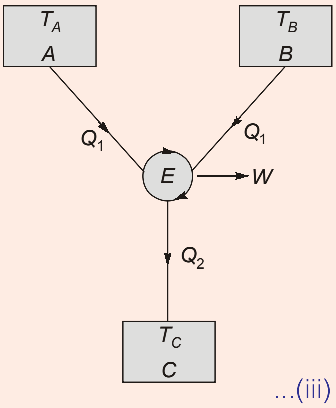
End of Solution

Q.1 (b) A reversible engine works between three thermal reservoirs, *A*, *B* and *C*. The engine absorbs an equal amount of heat from the thermal reservoirs *A* and *B* kept at temperatures T_A and T_B respectively, and rejects heat to the thermal reservoir *C* kept at temperature T_C. The efficiency of the engine is α times the efficiency of the reversible engine, which works between the two reservoirs *A* and *C*.

Prove that : $\dfrac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\dfrac{T_A}{T_C}$

[12 Marks]

Solution:



$$\eta = \alpha \cdot \eta_{AC} \qquad \text{...(i)}$$

$$\eta = \frac{W}{2Q_1}$$

$\{W = 2Q_1 - Q_2\}$

$$\eta = 1 - \frac{Q_2}{2Q_1} \qquad \text{...(ii)}$$

From equation (i), $\eta = \alpha\left(1 - \dfrac{T_C}{T_A}\right)$

From equation (ii), $\left(1 - \dfrac{Q_2}{2Q_1}\right) = \alpha\left(1 - \dfrac{T_C}{T_A}\right)$ ...(iii)

∵ *E* is an reversible engine.

$$\sum \frac{Q}{T} = 0$$

$$\frac{Q_1}{T_A} + \frac{Q_1}{T_B} = \frac{Q_2}{T_C}$$

$$\frac{Q_2}{Q_1} = \frac{T_C}{T_A} + \frac{T_C}{T_B}$$

Put value of $\dfrac{Q_2}{Q_1}$ in equation (iii),

$$1 - \frac{1}{2}\frac{T_C}{T_A} - \frac{1}{2}\frac{T_C}{T_B} = \alpha\left(1 - \frac{T_C}{T_A}\right)$$

$$1 - \frac{1}{2}\frac{T_C}{T_B} = \alpha - \alpha\frac{T_C}{T_A} + \frac{1}{2}\frac{T_C}{T_A}$$

$$(1 - \alpha) - \frac{1}{2}\frac{T_C}{T_B} = \frac{T_C}{T_A}\left(\frac{1}{2} - \alpha\right)$$

Multiply by $\dfrac{T_A}{T_C}$ in whole equation,

$$(1 - \alpha)\frac{T_A}{T_C} - \frac{1}{2}\frac{T_A}{T_B} = \frac{(1 - 2\alpha)}{2}$$

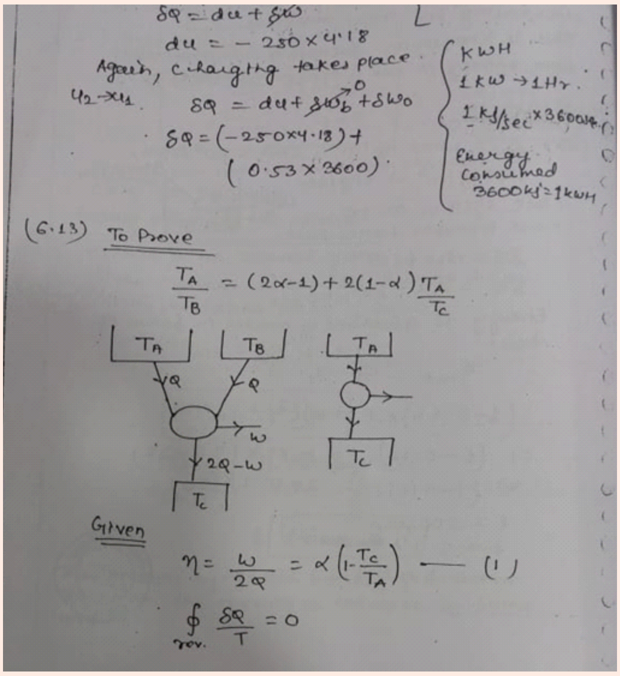
$$\frac{1}{2}\frac{T_A}{T_B} = \frac{(2\alpha - 1)}{2} + (1 - \alpha)\frac{T_A}{T_C}$$

Multiply by 2 in whole equation,

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C}$$

MADE EASY Source

- **Theory Book (2019 Edition):** Thermodynamics (Q. 5 Page 223)
- **MADE EASY Classnotes**



$\delta Q = du + \delta W$

$du = -250 \times 4.18$

Again, charging takes place.

$u_2 \to u_1$. $\delta Q = du + \cancel{\delta W_b}^{0} + \delta W_0$

$\delta Q = (-250 \times 4.18) + (0.53 \times 3600)$.

KWH

1 KW → 1 Hr.

1 KJ/sec × 3600 sec

Energy consumed

3600 kJ = 1 kWH

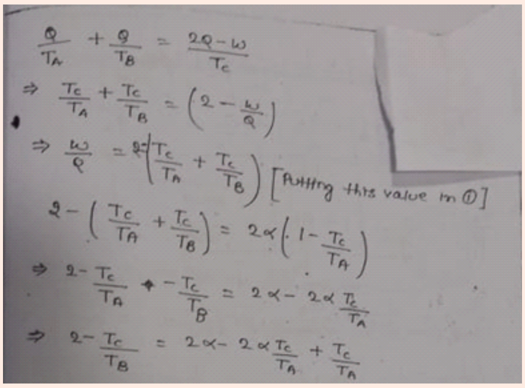
(6.13) To Prove

$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1-\alpha)\frac{T_A}{T_C}$

Given

$\eta = \frac{W}{2Q} = \alpha\left(1 - \frac{T_C}{T_A}\right)$ —— (1)

$\oint_{rev.} \frac{\delta Q}{T} = 0$



$\frac{Q}{T_A} + \frac{Q}{T_B} = \frac{2Q - W}{T_C}$

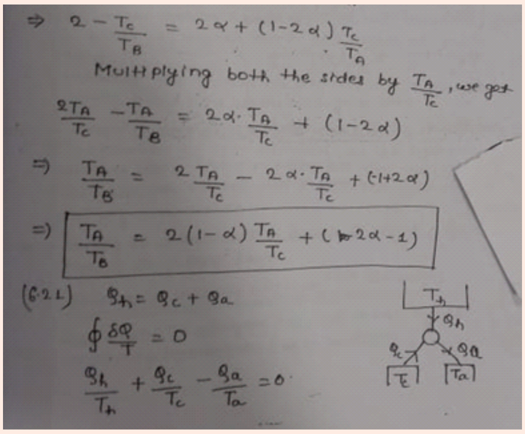
$\Rightarrow \frac{T_C}{T_A} + \frac{T_C}{T_B} = \left(2 - \frac{W}{Q}\right)$

$\Rightarrow \frac{W}{Q} = 2 - \left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right)$ [Putting this value in ①]

$2 - \left(\frac{T_C}{T_A} + \frac{T_C}{T_B}\right) = 2\alpha\left(1 - \frac{T_C}{T_A}\right)$

$\Rightarrow 2 - \frac{T_C}{T_A} \cancel{+} - \frac{T_C}{T_B} = 2\alpha - 2\alpha\frac{T_C}{T_A}$

$\Rightarrow 2 - \frac{T_C}{T_B} = 2\alpha - 2\alpha\frac{T_C}{T_A} + \frac{T_C}{T_A}$



$\Rightarrow 2 - \frac{T_C}{T_B} = 2\alpha + (1 - 2\alpha)\frac{T_C}{T_A}$

Multiplying both the sides by $\frac{T_A}{T_C}$, we get

$\frac{2T_A}{T_C} - \frac{T_A}{T_B} = 2\alpha \cdot \frac{T_A}{T_C} + (1 - 2\alpha)$

$\Rightarrow \frac{T_A}{T_B} = 2\frac{T_A}{T_C} - 2\alpha \cdot \frac{T_A}{T_C} + (-1 + 2\alpha)$

$\Rightarrow \boxed{\frac{T_A}{T_B} = 2(1-\alpha)\frac{T_A}{T_C} + (2\alpha - 1)}$

(6.21) $Q_h = Q_c + Q_a$

$\oint \frac{\delta Q}{T} = 0$

$\frac{Q_h}{T_h} + \frac{Q_c}{T_c} - \frac{Q_a}{T_a} = 0$

End of Solution

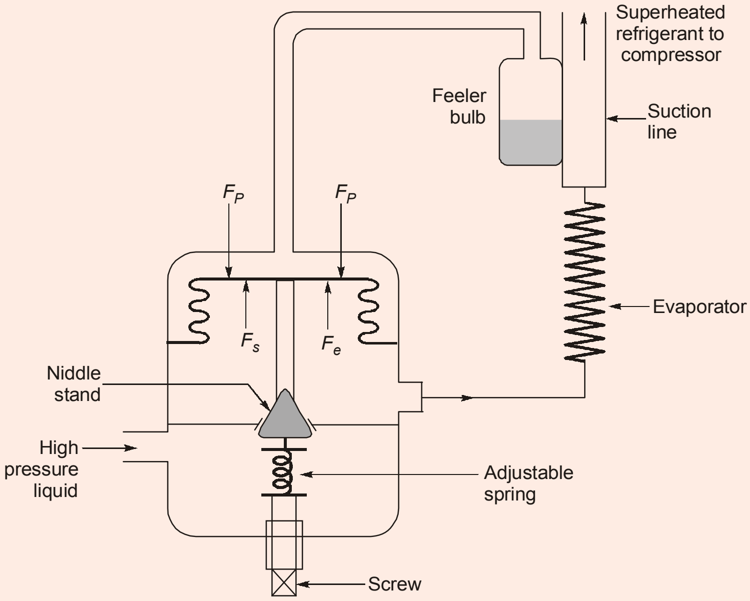
Q.1 (c) With the help of neat sketch, explain the working of a thermostatic expansion valve. How does it cope up with the variable load?

[12 Marks]

Solution:

(i)

Thermostatic expansion valve: A thermostatic expansion valve is used to maintain a constant degree of superheat at the exit of evaporator, hence it is most effective for dry evaporators in preventing the slugging of the compressors since it does not allow the liquid refrigerant to enter the compressor. The schematic diagram of the valve is given as below:



It consists of a feeler bulb that is attached to the evaporator exit tube so that it senses the temperature at the exit of evaporator. The feeler bulb and the narrow tube contains some fluid that is called power fluid. The power fluid may be same as the refrigerant or it may be different. In case if it is different from the refrigerant then the TEV is called TEV with cross charge. Let P_p is the pressure of power fluid, P_e is the saturation pressure corresponding to evaporator exit temperature and evaporator temperature is T_e then the purpose of TEV is to maintain a temperature $(T_e + \Delta T_s)$ at evaporator exit where ΔT_s is the degree of superheat. Feeler bulb senses the temperature $(T_e + \Delta T_s)$ and its pressure P_p is saturation pressure at this temperature. So force exerted on the top area A_b of bellows:

$$F_p = P_p A_b \quad \text{...(i)}$$

Force exerted by evaporator pressure from bottom side of bellows:

This is called external equalizer if the evaporator is large and has significant pressure drop otherwise it is known as TEV with internal equalizer.

The difference of forces F_p and F_e is exerted on the top of the middle which controls the opening of orifice and is equal to spring force F_s i.e.

$$F_s = (P_p - P_e)A_b$$

Also $$\Delta T_s \propto (P_p - P_e)A_b$$

As the compressor starts, P_e decreases so a positive spring force is applied on middle which opens the orifice and refrigerant flow starts.

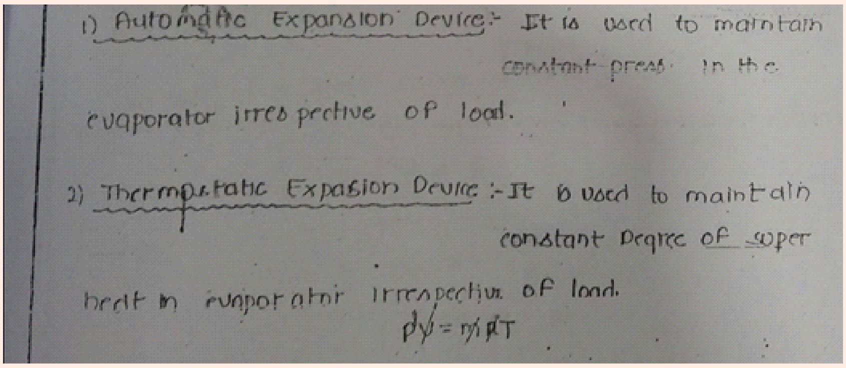
When the cooling load increases, the refrigerant evaporates at a faster rate in evaporator than the compressor can suck. As a result, the saturation pressure (p_0) correspond to the temperature at the exit end of the evaporator and the degree of superheat in evaporator increases. The increase in superheat causes the valve to open more and to allow more refrigerant to enter the evaporator. At the same time, the increase in suction pressure (p_0) also enables the compressor to deliver increased refrigerating capacity.

When the cooling load decreases, the refrigerant evaporates at a slower rate than the compressor is able to suck. As a result, the evaporator pressure drops and the degree of superheat decreases. The valve tends to close and the compressor delivers less refrigerating capacity at a decreased suction pressure.

Thus the thermostatic expansion valve, is capable of meeting variable load requirements.

MADE EASY Source

- **Theory Book (2019 Edition):** RAC (Page 31)
- **ESE 2019 Mains Test Series:** Exactly same as Q.4b(i) from Test-12
- **MADE EASY Classnotes**



1) Automatic Expansion Device:- It is used to maintain constant press. in the evaporator irrespective of load.

2) Thermostatic Expasion Device :- It is used to maintain constant Degree of super heat in evaporator irrespective of load.

$pV = mRT$

End of Solution

Q.1 (d) The fuel rod of a nuclear reactor is lagged with a tight fitting cladding material to prevent oxidation of the surface of the fuel rod by direct contact with the coolant. The heat generation occurs only in the fuel rod according to the following relation:

$q_g = q_0\left[1-\dfrac{r^2}{R^2}\right]$. Under steady state conditions, heat generated in the fuel rod is conducted through the cladding material and then dissipated to the coolant flowing around the cladding by convection.

Assuming that there is no contact resistance between the fuel rod and cladding, derive an expression for the heat flux through the fuel rod and cladding material.

[12 Marks]

Solution:

Fuel Rod:

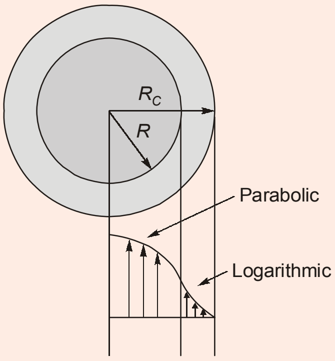
1 D heat conduction equation with heat generation in cylinder:

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\,\frac{r\cdot\partial T}{\partial r}\right)+\frac{q_g}{k_f} = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{q_g\cdot r}{k_f}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{q_o}{k_f}\left[1-\frac{r^2}{R^2}\right]r$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = -\frac{q_o}{k_f}\left[r-\frac{r^3}{R^2}\right]$$



$$r\frac{\partial T}{\partial r} = -\frac{q_o}{k_f}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]+C$$

$$\frac{\partial T}{\partial r} = -\frac{q_o}{k_f}\left[\frac{r}{2}-\frac{r^3}{4R^2}\right]+\frac{C}{r}$$

at $$r = 0,\ \frac{\partial T}{\partial r} = 0,\ C = 0$$

$$\frac{\partial T}{\partial r} = -\frac{q_o}{k_f}\left[\frac{r}{2}-\frac{r^3}{4R^2}\right]$$

Heat flux through fuel rod at any cross-section,

$$q_f'' = -\frac{k_f\,\partial T}{\partial r}$$

$$q_f'' = -k_f\left\{-\frac{q_o}{k_f}\left\{\frac{r}{2}-\frac{r^3}{4R^2}\right\}\right\}$$

$$q_f'' = q_o\left\{\frac{r}{2}-\frac{r^3}{4R^2}\right\}$$

At $$r = R$$

$$q_{f,R}'' = q_o\left\{\frac{R}{2}-\frac{R^3}{4R^2}\right\} = \frac{q_oR}{4}$$

Cladding:

Energy conservation:

$$\dot{E}_{\text{in}}-\dot{E}_{\text{out}}+\dot{E}_g = \dot{E}_{st}$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \qquad \left[\because\ \dot{E}_g = 0;\ \dot{E}_{st} = 0\right]$$

(Heat conducted from outer surface of rod) = (Heat conducted through cladding)

$$q_{f,R}''\times 2\pi RL = q_C''\times 2\pi rL$$

Where, q_C'' is heat flux through cladding at radius 'r'

$$\frac{q_o\times R}{4}\times\frac{R}{r} = q_C''$$

$\Rightarrow$ $$q_C'' = \frac{q_oR^2}{4r}$$

MADE EASY Source

- **Theory Book (2019 Edition):** HMT (Page 46)

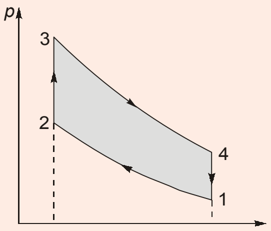
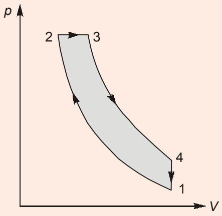
End of Solution

Q.1 (e) Compare compression ignition engine with spark ignition engine so far as the following points are concerned:

(i) Working cycle
(ii) Method of ignition
(iii) Method of fuel supply

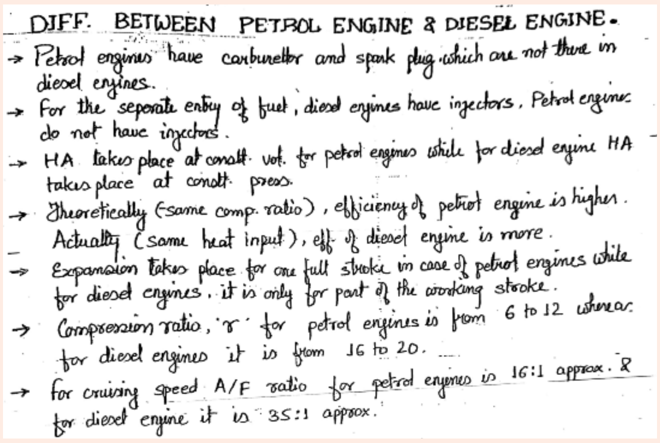
[12 Marks]

Solution:

SI Engine	CI Engine
• The ideal cycle for the SI engine is the Otto cycle	• The ideal cycle for the CI engine is the diesel cycle
• Heat addition is at constant volume.	• Heat addition is at constant pressure.
• It uses spark plug for ignition	• It does not use spark plug for ignition but self ignition take place due to compression of air
• It uses carburetter to supply air fuel mixture	• It uses fuel pump and injector to supply fuel.
• Lower compression ratio engine (6 to 10.5)	• High compression ratio engine (14 to 22)
	

MADE EASY Source

- **Theory Book (2019 Editon):** Exactly from IC Engine (Page 20)
- **MADE EASY Classnotes**



DIFF. BETWEEN PETROL ENGINE & DIESEL ENGINE.

→ Petrol engines have carburettor and spark plug which are not there in diesel engines.

→ For the seperate entry of fuel, diesel engines have injectors. Petrol engines do not have injectors.

→ HA takes place at constt. vol. for petrol engines while for diesel engine HA takes place at constt. press.

→ Theoretically (same comp. ratio), efficiency of petrol engine is higher. Actually (same heat input), eff. of diesel engine is more.

→ Expansion takes place for one full stroke in case of petrol engines while for diesel engines, it is only for part of the working stroke.

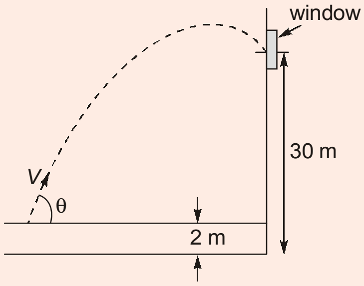
→ Compression ratio, 'r' for petrol engines is from 6 to 12 whereas for diesel engines it is from 16 to 20.

→ For cruising speed A/F ratio for petrol engines is 16:1 approx. & for diesel engine it is 35:1 approx.

End of Solution

Q.2 (a) A jet of water is discharging at 25 kg/sec from a nozzle of 25 mm diameter. The jet from the nozzle is directed towards a window of a building at a height of 30 m from the ground. Assuming the nozzle discharge to be at a height of 2 m from the ground, determine the greatest distance from the building where the fireman can stand, so that the jet can reach the window. [20 Marks]

Solution:



Given, mass flow rate, $\dot{m} = 25$ kg/s

Nozzle diameter, $d = 25$ mm

Mass flow rate, $\rho A V = 25$

$$V = \frac{25}{\rho \times A}$$

$$= \frac{25}{1000 \times \frac{\pi}{4} \times 0.025^2}$$

$$V = 50.929 \text{ m/s}$$

Let θ = angle of inclination,

For horizontal motion, $V_x = V\cos\theta$ m/s

$$a_x = 0 \text{ m/s}^2$$

$$x = V_x t$$

$\therefore$ $x = (V\cos\theta)t$ ...(i)

where t is time to reach window

$$V_y = V\sin\theta$$

$$a_y = -g \text{ m/s}^2$$

$\therefore$ $y = (V\sin\theta)t - \frac{1}{2}gt^2$ ...(ii)

From equation (i) and (ii), $y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$ equation of Trajectory

Substituting $y = 30 - 2 = 28$ m

$$28 = x\tan\theta - \frac{9.81}{2\times(50.929)^2}x^2\sec^2\theta$$

$$28 = x\tan\theta - (1.891 \times 10^{-3})x^2 \sec^2\theta$$

or $x\tan\theta - (1.891 \times 10^{-3})x^2 \sec^2\theta - 28 = 0$ ...(iii)

The maximum value of x is obtained by differentiating above equation with respect to θ and putting $\frac{dx}{d\theta} = 0$.

Differentiating equation (iii)

$$x\sec^2\theta + \tan\theta\left(\frac{dx}{d\theta}\right) - (1.891\times10^{-3})\times\left(x^2\times2\sec^2\theta\tan\theta + 2x\sec^2\theta\frac{dx}{d\theta}\right) = 0$$

Putting $\frac{dx}{d\theta} = 0$

$$x\sec^2\theta - (1.891 \times 10^{-3})x^2 \times 2\sec^2\theta\tan\theta = 0$$

$$1 - 3.782 \times 10^{-3}x\tan\theta = 0$$

or $x = \frac{264.41}{\tan\theta}$ ...(iv)

Substituting in equation (iii),

$$264.41 - \left[(1.891\times10^{-3})\frac{(264.41)^2}{\tan^2\theta}\sec^2\theta\right] - 28 = 0$$

$$264.41 - \frac{132.204}{\sin^2\theta} - 28 = 0$$

$$236.41 - \frac{132.204}{\sin^2\theta} = 0$$

$$\sin\theta = 0.7478$$

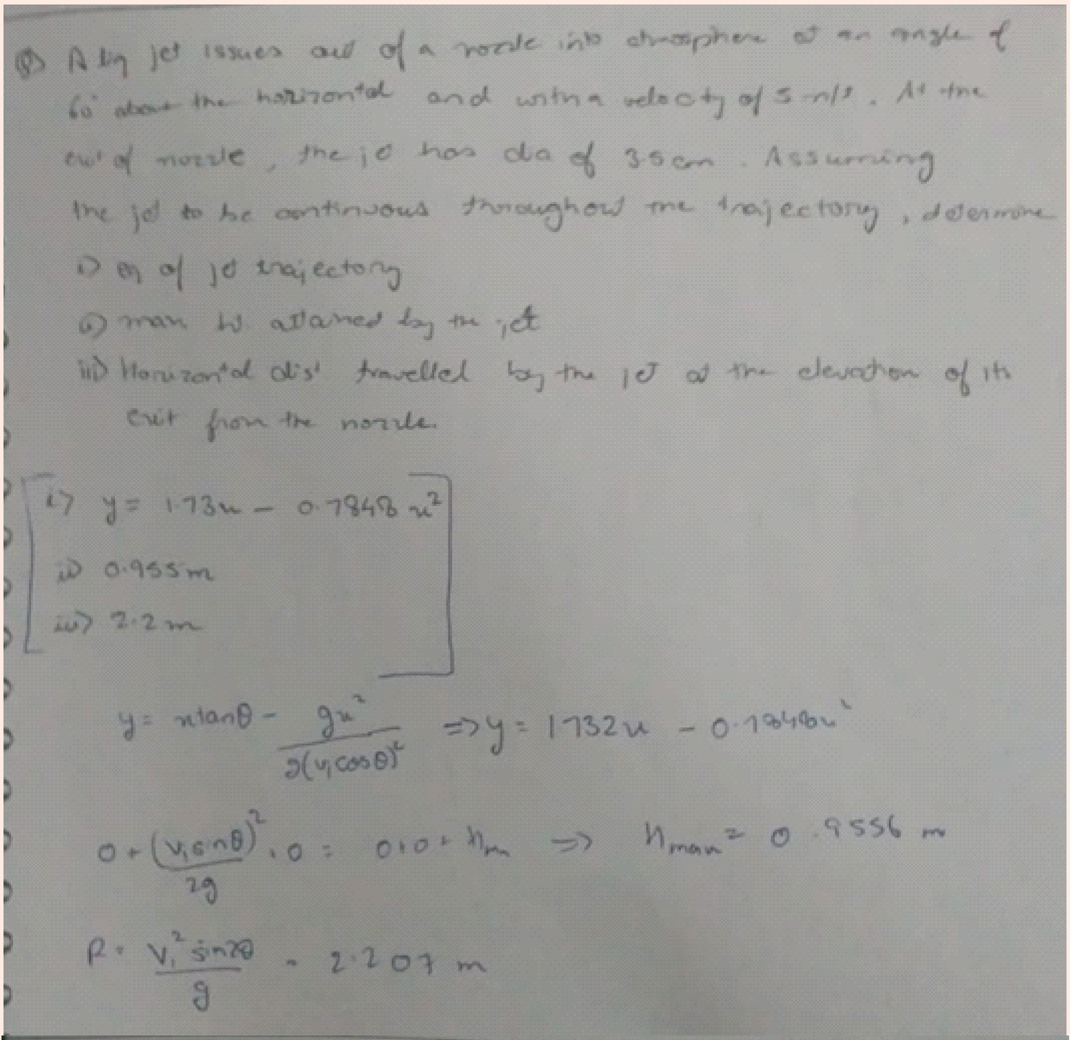
$$\theta = 48.4°$$

From equation (iv), $x = \frac{264.41}{\tan(48.4°)} = 234.754 \text{ m}$

Maximum horizontal distance = 234.754 m

MADE EASY Source

- **MADE EASY Classnotes:** Super Talent Batch



Q) A big jet issues out of a nozzle into atmosphere at an angle of 60° above the horizontal and with a velocity of 5 m/s. At the exit of nozzle, the jet has dia of 3.5 cm. Assuming the jet to be continuous throughout the trajectory, determine

i) eq of jet trajectory

ii) max ht attained by the jet

iii) Horizontal dist travelled by the jet at the elevation of its exit from the nozzle.

i) $y = 1.73x - 0.7848x^2$

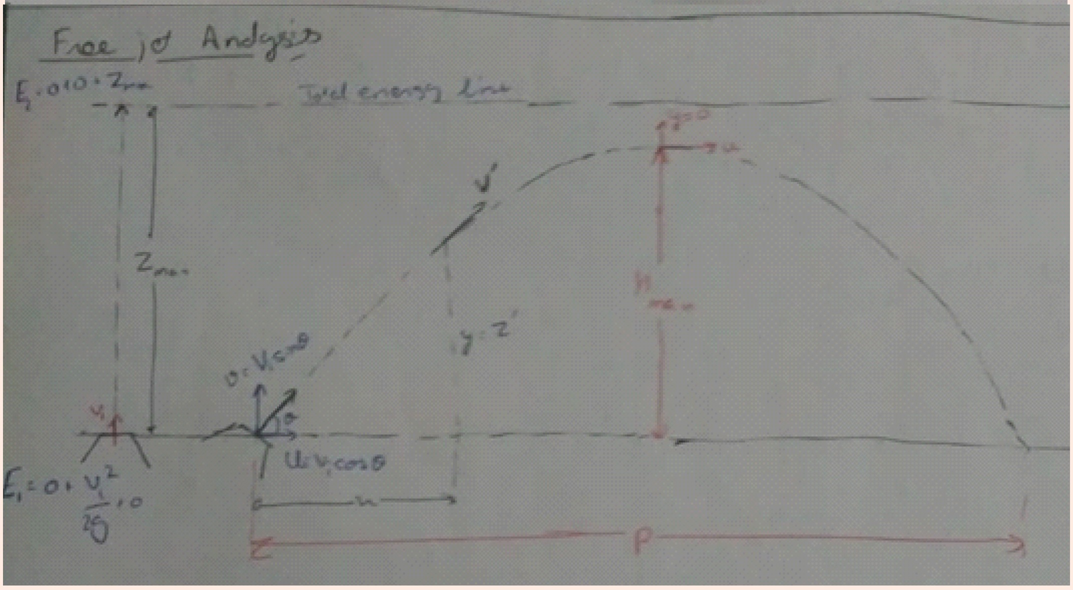
ii) 0.955 m

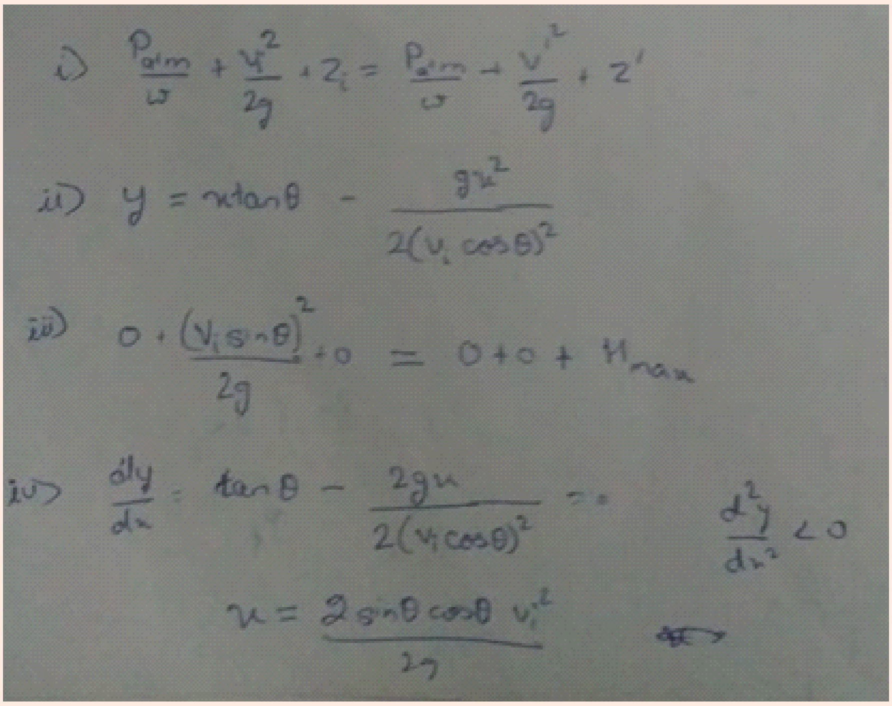
iii) 2.2 m

$$y = x\tan\theta - \frac{gx^2}{2(v_1\cos\theta)^2} \Rightarrow y = 1.732x - 0.7848x^2$$

$$0 + \frac{(v_1\sin\theta)^2}{2g} + 0 = 0 + 0 + h_m \Rightarrow h_{max} = 0.9556 \text{ m}$$

$$R = \frac{v_1^2 \sin 2\theta}{g} = 2.207 \text{ m}$$





i) $\frac{P_{atm}}{w} + \frac{v_i^2}{2g} + z_i = \frac{P_{atm}}{w} - \frac{v'^2}{2g} + z'$

ii) $y = x\tan\theta - \frac{gx^2}{2(v_i\cos\theta)^2}$

iii) $0 + \frac{(v_i\sin\theta)^2}{2g} + 0 = 0 + 0 + H_{max}$

iv) $\frac{dy}{dx} = \tan\theta - \frac{2gx}{2(v_i\cos\theta)^2} = 0$ $\qquad \frac{d^2y}{dx^2} < 0$

$x = \frac{2\sin\theta\cos\theta\, v_i^2}{2g}$

End of Solution

Q.2 (b) Two rigid tanks shown in Figure 2(b) each contain 10 kg of N_2 gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K, the process stops. Find the final (*P*, *T*) in both tanks and the work input to the heat pump, assuming constant heat capacities.

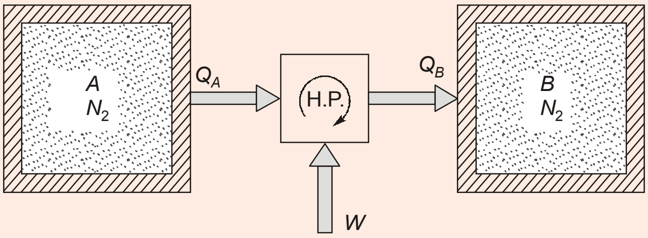
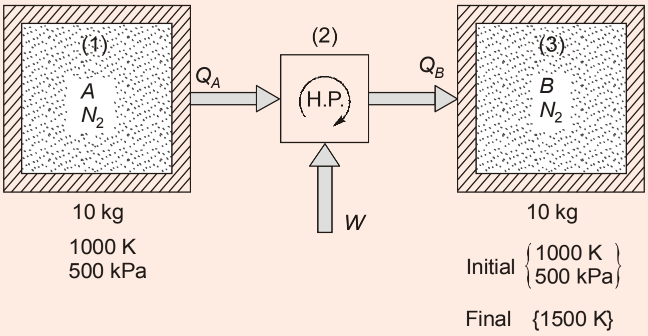


Figure 2 (b)

[20 Marks]

Solution:



Assumption:

(i) Specific heat is independent of temperature.

$\because$ Rigid takes, so, $V_f = V_i$

Now, for reversible process:

Entropy change of universe $= 0$

$$(\Delta S)_1 + (\Delta S)_2 + (\Delta S)_3 = 0 \quad \text{...(i)}$$

$$(\Delta S)_3 = mc_v \ln\left(\frac{T_{f3}}{T_{i3}}\right) + mR\ln\left(\frac{V_f}{V_i}\right)$$

$$(\Delta S)_3 = mc_v \ln\left(\frac{1500}{1000}\right)$$

$$(\Delta S)_3 = mc_v \ln(1.5)$$

Now, $$(\Delta S)_2 = 0 \quad \text{\{cyclic device\}}$$

$$(\Delta S)_1 = mc_v \ln\left(\frac{T_{f1}}{T_{i1}}\right) + mR\ln\left(\frac{V_f}{V_i}\right)$$

$$(\Delta S)_1 = mc_v \ln\left(\frac{T_{f1}}{1000}\right)$$

From equation (i),

$$mc_v \ln(1.5) + mc_v \ln\left(\frac{T_{f1}}{1000}\right) = 0$$

$$\ln\left(1.5 \times \frac{T_{f1}}{1000}\right) = 0$$

$$1.5 \times \frac{T_{f1}}{1000} = 1$$

$$T_{f1} = 666.67 \text{ K}$$

now, $$P_iV_i = m_i\ RT_i \quad \text{...(ii)}$$

$$P_fV_f = \text{m}_f\ RT_f \quad \text{...(iii)}$$

now divide equation (i) and (ii),

$$\frac{P_i}{P_f} = \frac{T_i}{T_f} \quad \text{...(iv)}$$

For tank (1), $$\frac{500}{P_{f_1}} = \frac{1000}{666.67}$$

$$P_{f_1} = 333.335 \text{ kPa}$$

For tank (3), from equation (iv),

$$\frac{500}{P_{f_3}} = \frac{1000}{1500}$$

$$P_{f_1} = 750 \text{ kPa}$$

Now, for work input to the pump,

specific heat at constant volume,

$$c_v = \frac{R}{\gamma - 1} = \frac{8.314}{28(1.4-1)} \quad \{\gamma = 1.4\}$$

$$c_v = 0.742 \text{ kJ/kgK}$$

Heat transfer to tank (3),

$$Q_B = mc_v(T_{f_3} - T_{i_3})$$
$$= 10 \times 0.742(1500 - 1000)$$
$$Q_B = 3710 \text{ kJ}$$

Heat transfer from tank (1),

$$Q_A = mc_v(T_{f_1} - T_{i_1})$$
$$= 10 \times 0.742(666.67 - 1000)$$
$$Q_A = -2473.31 \text{ kJ}$$

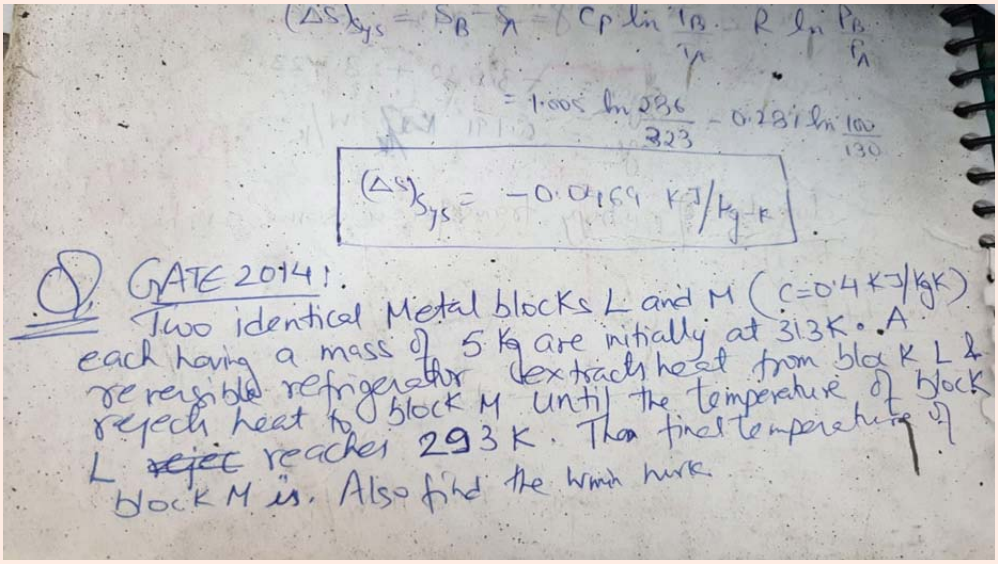
Negative sign indicates heat transfer from the tank.

Now, work input, $W = (Q_B - Q_A) = 3710 - 2473.31$

$$W = 1236.69 \text{ kJ}$$

MADE EASY Source

- **MADE EASY Classnotes**



$$(\Delta S)_{sys} = S_B - S_A = C_p \ln \frac{T_B}{T_A} - R \ln \frac{P_B}{P_A}$$

$$= 1.005 \ln \frac{286}{323} - 0.287 \ln \frac{100}{130}$$

$$\boxed{(\Delta S)_{sys} = -0.0469 \text{ kJ/kg-K}}$$

Q GATE 2014 :
Two identical Metal blocks L and M (c = 0.4 kJ/kgK)
each having a mass of 5 kg are initially at 313 K. A
reversible refrigerator extracts heat from block L &
reject heat to block M until the temperature of block
L ~~reject~~ reaches 293 K. The final temperature of
block M is. Also find the Wmin work

End of Solution

Q.2 (c) Water is flowing steadily over a smooth flat plate with a velocity of 2 m/sec. The length of the plate is 30 cm. Calculate

(i) The thickness of the boundary layer 10 cm from the leading edge of the plate.

(ii) The rate of growth of the boundary layer at 10 cm from the leading edge; and

(iii) The drag coefficient on one side of the plate.

Assume parabolic velocity profile.

Kinematic viscosity of water $\nu = 1.02 \times 10^{-6}$ m²/sec.

Derive the expressions used in the calculation.

[20 Marks]

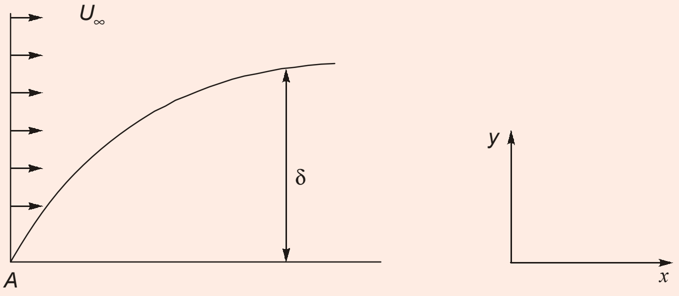
Solution:

Given,

$$U_\infty = 2 \text{ m/s}$$
$$L = 30 \text{ cm}$$
$$\nu = 1.02 \times 10^{-6} \text{ m}^2/\text{s}$$

It is given that velocity profile is parabolic

$$\frac{U}{U_\infty} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$



Boundary conditions

at $y = 0$, $U = 0$ $\therefore$ $a = 0$

at $y = \delta$, $U = U_\infty$,

$$1 = b + c \quad \text{...(i)}$$

$\therefore$ At $y = \delta$, $\tau = 0$,

So, $$\frac{dU}{dy} = 0$$

$$\frac{1}{U_\infty}\frac{dU}{dy} = \frac{b}{\delta} + \frac{2cy}{\delta^2}$$

$$0 = \frac{b}{\delta} + \frac{2c}{\delta}$$

$\Rightarrow$ $$b + 2c = 0 \quad \text{...(ii)}$$

From (i) and (ii), we get,

$$b = 2, \; c = -1$$

$\therefore$ The velocity profile is, $$\frac{U}{U_\infty} = \frac{2y}{\delta} - \left(\frac{y}{\delta}\right)^2$$

Using Von-Karman momentum integral equation

$$\frac{\tau_w}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x} \quad \text{...(i)}$$

where, τ_w = wall shear stress, θ = momentum thickness

$$\theta = \int_0^\delta \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy$$

$$\theta = \int_0^\delta \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right] dy$$

$$\theta = \int_0^\delta \left[2\left(\frac{y}{\delta}\right) - 4\left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right]dy$$

$$\theta = \int_0^\delta \left[2\left(\frac{y}{\delta}\right) - 5\left(\frac{y}{\delta}\right)^2 + 4\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right]dy$$

$$\theta = \frac{2}{\delta}\left(\frac{y}{2}\right)^2 - \frac{5}{\delta^2}\left(\frac{y^3}{3}\right) + \frac{4}{\delta^3}\left(\frac{y^4}{4}\right) - \frac{1}{\delta^4}\left(\frac{y^5}{5}\right)\Bigg|_0^\delta$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{1}{5}\delta = \frac{2}{15}\delta$$

So, $$\theta = \frac{2\delta}{15} \quad \text{...(ii)}$$

$$\tau_w = \mu \frac{dU}{dy}\bigg|_{y=0} = \mu \times U_\infty \left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]\bigg|_{y=0}$$

$$\tau_w = \frac{2\mu U_\infty}{\delta} \quad \text{...(iii)}$$

$$\frac{\tau_w}{pU_\infty^2} = \frac{\partial\theta}{\partial x}$$

$$\frac{2\mu U_\infty}{\delta} \times \frac{1}{\rho U_\infty^2} = \frac{\partial}{\partial x}\left(\frac{2}{15}\delta\right)$$

$$\frac{\mu}{\rho U_\infty \delta} = \left(\frac{1}{15}\right)\frac{\partial\delta}{\partial x}$$

$$\delta\partial\delta = \frac{15\mu}{\rho U_\infty^2}\partial x$$

Integrating above equation,

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U_\infty} + C$$

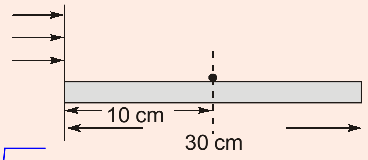
Boundary condition,
at $x = 0,\ \delta = 0 \Rightarrow C = 0$

$$\delta^2 = \frac{30\mu x}{\rho U_\infty}$$

$$\delta = \sqrt{30}\sqrt{\frac{\mu x}{\rho U_\infty}}$$

$$\delta = 5.48\sqrt{\frac{\mu x}{\rho U_\infty}}$$

or
$$\delta = \frac{5.48x}{\sqrt{\text{Re}_x}}$$



$$U_\infty = 2 \text{ m/s}$$

(i)
$$\delta_A = 5.48\sqrt{\frac{\mu x}{\rho U_\infty}} = 5.48\sqrt{\frac{\nu x}{U_\infty}}$$

$$= 5.48 \times \sqrt{\frac{1.02 \times 10^{-6} \times 0.1}{2}} = 1.2375 \times 10^{-3} \text{ m}$$

(ii) The rate of growth of the boundary layer,

$$\frac{d\delta}{dx} = \left(\sqrt{\frac{\nu}{U_\infty}}\right)\frac{d}{dx}(\sqrt{x})$$

$$\frac{d\delta}{dx} = \frac{1}{2}\sqrt{\frac{\nu}{U_\infty x}}$$

$$\frac{d\delta}{dx} = \frac{1}{2} \times \sqrt{\frac{1.02 \times 10^{-6}}{2 \times 0.1}} = 1.1291 \times 10^{-3}$$

(iii)
$$C_{fx} = \frac{\tau_{wx}}{\frac{1}{2}\rho U_\infty^2}$$

$$C_{fx} = \frac{2\mu U_\infty}{\delta} \times \frac{1}{\frac{1}{2}\rho U_\infty^2}$$

$$C_{fx} = \frac{4\mu}{\rho U_\infty \delta} = \frac{4\mu}{\rho U_\infty} \times \frac{1}{5.48}\sqrt{\frac{U_\infty}{\nu x}}$$

$$C_{fx} = 0.73\sqrt{\frac{\nu}{U_\infty x}}$$

Hence,
$$C_{fx} \propto \frac{1}{\sqrt{x}}$$

Drag coefficient,
$$C_D = \frac{1}{L}\int_0^L C_{fx}\, dx = \frac{1}{L}\int_0^L \frac{C}{\sqrt{x}} dx = \frac{2}{L} C\sqrt{L}$$

$$C_D = 2\left(\frac{2}{\sqrt{L}}\right) = 2 \times (C_{f_L})$$

$$C_D = 2 \times 0.73 \times \sqrt{\frac{\nu}{U_\infty L}}$$

$$C_D = 1.46 \times \sqrt{\frac{1.02 \times 10^{-6}}{2 \times 0.3}} = 1.9036 \times 10^{-3}$$

NEXT IAS
BIG LEARNINGS MADE EASY

An opportunity for
MADE EASY Ex- Students
who dare to dream big and make their mark
in the prestigious Civil Services

Live/Online
Classes also available

NEXT IAS announces
25% discount for MADE EASY Ex-Students
on the tuition fee of
General Studies Prelims-cum-Main Foundation Course
for CSE 2020

Batches commencing from

11th July, 2019	**8th July, 2019**
Old Rajinder Nagar Centre | 6 pm to 9 pm	**Saket Centre** | 6 pm to 9 pm

Optional Foundation Course
for CSE Main 2020

Subject offered
MATHEMATICS
SOCIOLOGY
PHILOSOPHY

Commences in
October / November, 2019

Venue:
Old Rajinder Nagar

Prelims Test Series
for CSE 2020

GENERAL STUDIES
& CSAT
Offline & Online
Test Series available

Commences in
September / October, 2019

Offline Venue:
Old Rajinder Nagar & Saket

Postal Package
for CSE 2020

GENERAL STUDIES
& CSAT

Includes :

- 17 books of General Studies & 6 books of CSAT covering Prelims & Main syllabus of CSE
- E-Subscription of Monthly Current Affairs Magazines of NEXT IAS is included.
- Online Subscription of General Studies Prelims Test Series(GS(30 Tests) + CSAT) for CSE 2019 is included.

NEXT IAS Centres

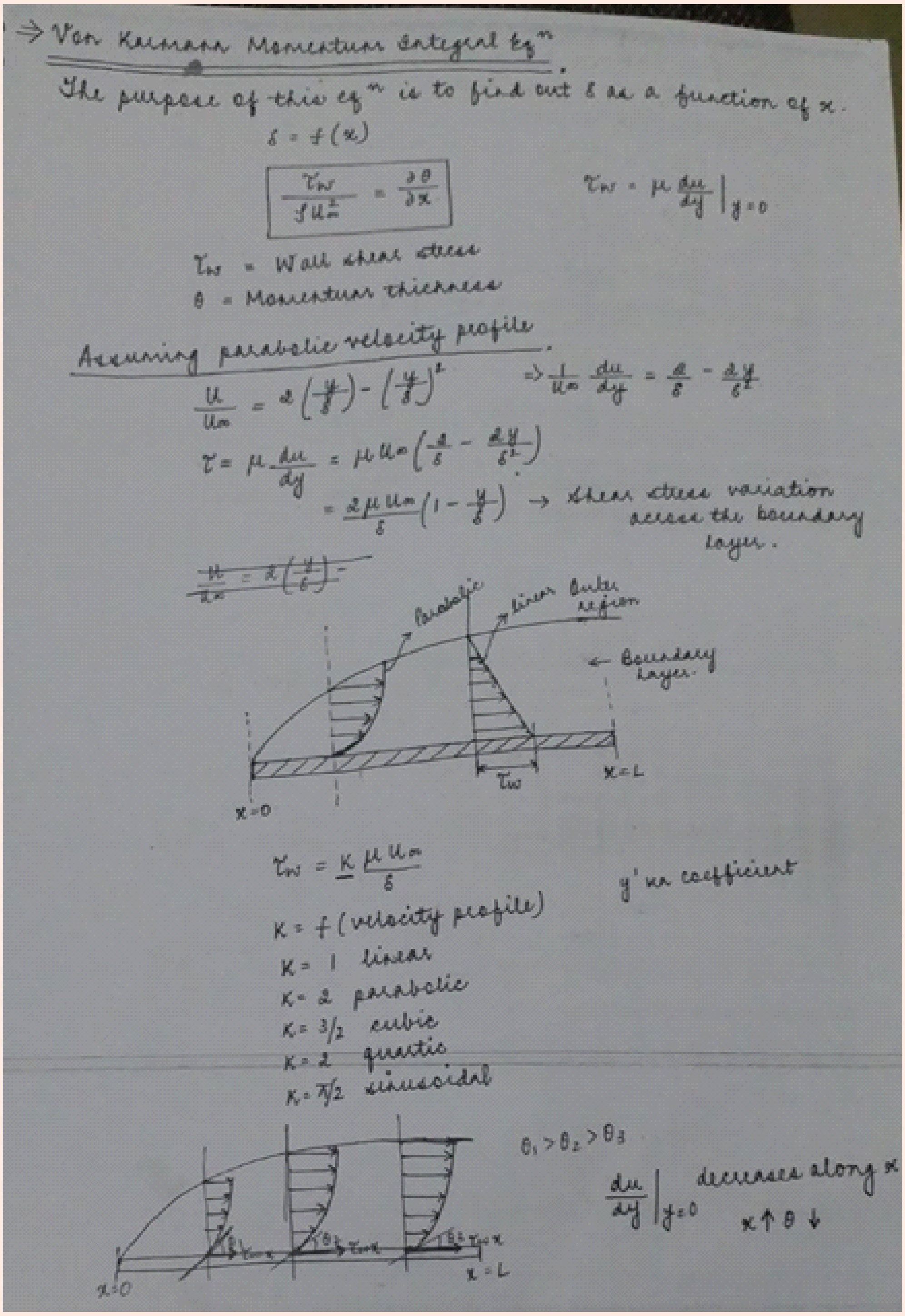
Old Rajinder Nagar Centre (Delhi)
Ph : 011-49858612, 8800338066

Saket Centre (Delhi)
Ph : 011-45124642, 8800776445

✉ **info@nextias.com**
🌐 **www.nextias.com**

MADE EASY Source

- **Covered in MADE EASY Class Notes**
- **Theory Book (2019 Edition):** Fluid Mechanics and Hydraulic Machinery (Page No. 269)
- **Covered in MADE EASY Practice Sheet**



→ Von Karmann Momentum Integral eqⁿ.

The purpose of this eqⁿ is to find out δ as a function of x.

$\delta = f(x)$

$$\boxed{\frac{\tau_w}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x}}$$

$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$

τ_w = Wall shear stress

θ = Momentum thickness

Assuming parabolic velocity profile.

$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ $\Rightarrow \frac{1}{U_\infty}\frac{du}{dy} = \frac{2}{\delta} - \frac{2y}{\delta^2}$

$\tau = \mu \frac{du}{dy} = \mu U_\infty\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right)$

$= \frac{2\mu U_\infty}{\delta}\left(1 - \frac{y}{\delta}\right)$ → Shear stress variation across the boundary layer.

~~$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) -$~~

$\tau_w = \frac{K \mu U_\infty}{\delta}$ y'ka coefficient

K = f (velocity profile)

K = 1 linear

K = 2 parabolic

K = 3/2 cubic

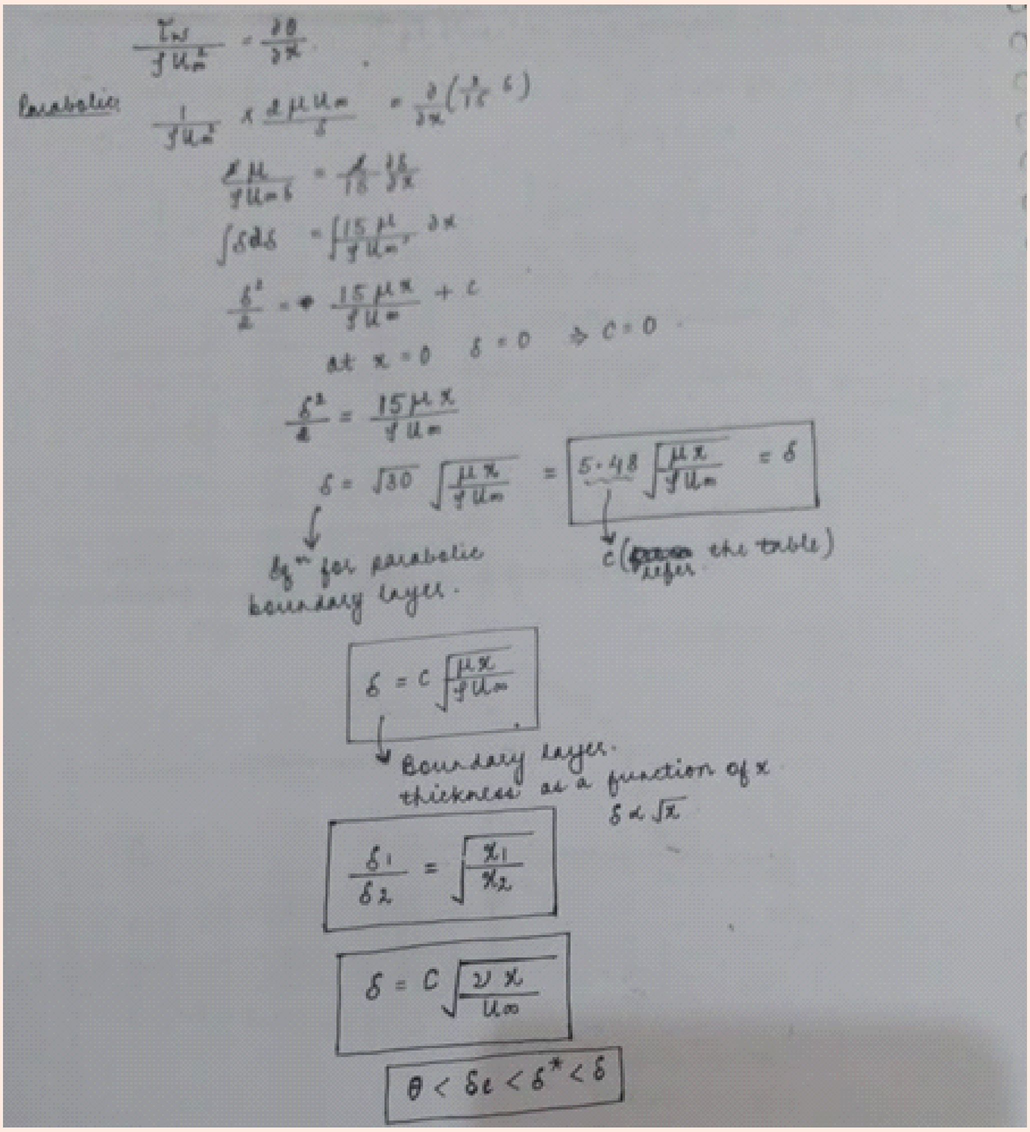
K = 2 quartic

K = π/2 sinusoidal

$\theta_1 > \theta_2 > \theta_3$

$\left.\frac{du}{dy}\right|_{y=0}$ decreases along x

x ↑ θ ↓



$$\frac{\tau_w}{\rho U_\infty^2} = \frac{\partial \theta}{\partial x}$$

Parabolic

$$\frac{1}{\rho U_\infty^2} \times \frac{2\mu U_\infty}{\delta} = \frac{\partial}{\partial x}\left(\frac{2}{15}\delta\right)$$

$$\frac{2\mu}{\rho U_\infty \delta} = \frac{2}{15}\frac{\partial \delta}{\partial x}$$

$$\int \delta\, d\delta = \int \frac{15\mu}{\rho U_\infty}\, \partial x$$

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U_\infty} + c$$

at $x = 0$ $\delta = 0$ $\Rightarrow c = 0$

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U_\infty}$$

$$\delta = \sqrt{30}\sqrt{\frac{\mu x}{\rho U_\infty}} = \boxed{5.48\sqrt{\frac{\mu x}{\rho U_\infty}} = \delta}$$

Eqn for parabolic boundary layer.

c (~~[illegible]~~ refer the table)

$$\boxed{\delta = c\sqrt{\frac{\mu x}{\rho U_\infty}}}$$

Boundary layer thickness as a function of x

$\delta \propto \sqrt{x}$

$$\boxed{\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}}$$

$$\boxed{\delta = C\sqrt{\frac{\nu x}{U_\infty}}}$$

$$\boxed{\theta < \delta_e < \delta^* < \delta}$$

End of Solution

Q.3 (a) A four-stroke cycle gasoline engine has six single-acting cylinders of 8 cm bore and 10 cm stroke. The engine is coupled to a brake having a torque radius of 40 cm. At 320 rpm, with all cylinders operating, the net brake load is 350 N. When each cylinder in turn is rendered inoperative, the average net brake load produced at the same speed by the remaining 5 cylinders is 250 N. Estimate the indicated mean effective pressure of the engine. With all cylinders operating, the fuel consumption is 0.33 kg/min; calorific value of fuel is 43 MJ/kg; the cooling water flow rate and temperature rise is 70 kg/min and 10°C respectively. On test, the engine is enclosed in a thermally and acoustically insulated box through which the output drive, water, fuel, air and exhaust connections pass. Ventilating air blown up through the box at the rate of 15 kg/min enters at 17°C and leaves at 62°C. Draw up a heat balance of the engine stating the items as a percentage of the heat input

[20 Marks]

Solution:

Given: 4-stroke engine,

Number of cylinders $(k) = 6$

Bore, $D = 8$ cm

Stroke, $L = 10$ cm

Speed, $N = 3200$ rpm

Net brake load, $W = 350$ N

Fuel consumption, $\dot{m}_f = 0.33$ kg/min

Calorific value, $CV = 43$ MJ/kg

Average brake load by 5 cylinder = 250 N

Brake radius, $r = 40$ cm

Brake power, $$BP = \frac{2\pi NT}{60} = \frac{2\pi N(Wr)}{60}$$

$$BP = \frac{2\pi \times 3200 \times 350 \times 0.4}{60} = 46914.45 \text{ W}$$

$$BP = 46.914 \text{ kW}$$

Brake power when 1 cylinder is in operative.

$$(BP)' = \frac{2\pi NT'}{60} = \frac{2\pi NW'r}{60}$$

$$(BP)' = \frac{2\pi \times 3200 \times 250 \times 0.4}{60}$$

$$(BP)' = 33.51 \text{ kW}$$

Indicated power $IP = 6 \times (BP - BP')$

$$IP = 6 \times (46.914 - 33.51) = 80.424 \text{ kW}$$

We know, $$IP = P_{\text{mep}} \times \frac{LAN \times k}{60 \times 2}$$

$$80.424 \times 10^3 = P_{\text{mep}} \times 0.1 \times \frac{\pi}{4} \times (0.08)^2 \times \frac{3200 \times 6}{120}$$

$$P_{\text{mep}} = 9.99 \text{ bar} \approx 10 \text{ bar}$$

Heat balance conditions

(i) Heat added,

$$\text{HA/min} = \dot{m}_f \times CV = 0.33 \times 43000 = 14191 \text{ kJ/min}$$

(ii) Brake power (equivalent)

$$BP_{\text{eq}} = BP \times 60 = 46.914 \times 60 = 2814.84 \text{ kJ/min}$$

(iii) Heat carried away by cooling water

$$q_w = \dot{m}_w \times 4.18 \times (\Delta T) = 70 \times 4.18 \times 10 = 2926 \text{ kJ/min}$$

(iv) Heat carried away by ventilation air,

$$q_{\text{air}} = \dot{m}_a \times c_{p_a}(T_2 - T_1) = 15 \times 1.005 \times (62 - 17)$$

$$= 678.375 \text{ kJ/min}$$

(v) Unaccounted losses [friction, radiation, other loses]

$$q_{\text{unc}} = 14190 - (2814.84 + 2926 + 678.375)$$

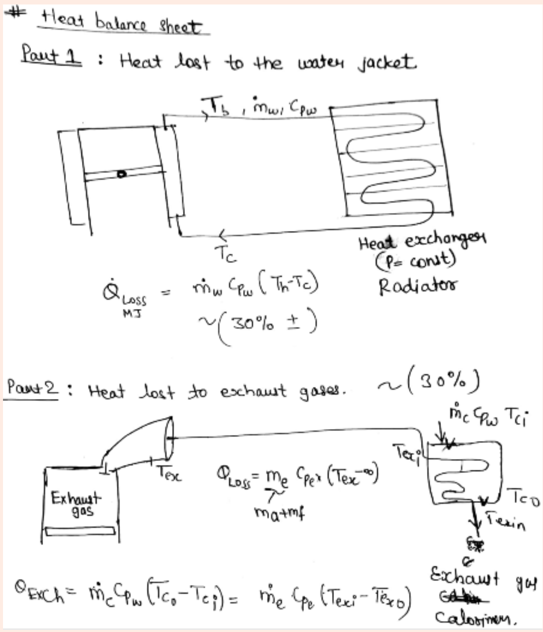
$$= 7770.785 \text{ kJ/min}$$

Heat Balance Sheet

	Heat Equivalent	Value (kJ/min)	Percentage
1.	Heat Added	14190	100
2.	BP equivalent	2814.84	19.83
3.	Heat carried away water	2926	20.62
4.	Heat carried away by ventilating air	678.375	4.78
5.	Unaccounted losses	7770.785	54.77

MADE EASY Source

- **ESE 2019 Mains Workbook:** Similar to Q. 20 of solved problems discussed in Class
- **MADE EASY Classnotes**



\# Heat balance sheet

Part 1 : Heat lost to the water jacket

$\dot{Q}_{\substack{Loss \\ MJ}} = \dot{m}_w c_{pw} (T_h - T_c)$

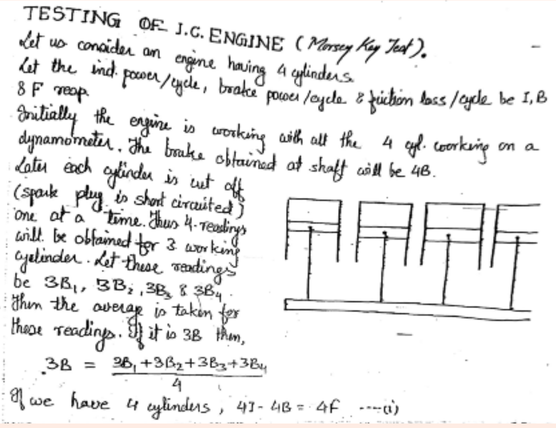
$\sim (30\% \pm)$

Part 2 : Heat lost to exhaust gases. $\sim (30\%)$

$Q_{Loss} = \underset{\substack{\sim \\ m_a + m_f}}{m_e} c_{pex} (T_{ex} - \infty)$

$Q_{ExCh} = \dot{m}_c c_{pw} (T_{c_o} - T_{c_i}) = \dot{m}_e c_{pe} (T_{exi} - T_{exo})$

Exhaust gas ~~Calo~~ Calorimeter.



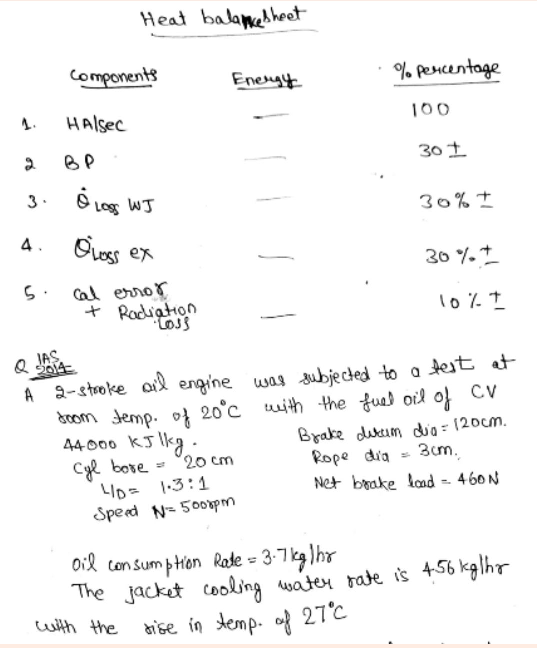
TESTING OF I.C. ENGINE (Morsey Key Test).

Let us consider an engine having 4 cylinders.
Let the ind. power/cycle, brake power/cycle & friction loss/cycle be I, B & F resp.

Initially the engine is working with all the 4 cyl. working on a dynamometer. The brake obtained at shaft will be 4B.
Later each cylinder is cut off (spark plug is short circuited) one at a time. Thus 4-readings will be obtained for 3 working cylinder. Let these readings be $3B_1$, $3B_2$, $3B_3$ & $3B_4$. Then the average is taken for these readings. If it is 3B then,

$$3B = \frac{3B_1 + 3B_2 + 3B_3 + 3B_4}{4}$$

If we have 4 cylinders, $4I - 4B = 4F$ ---(i)



Heat balance sheet

	Components	Energy	% percentage
1.	HA/sec	—	100
2	BP	—	30 ±
3.	$\dot{Q}_{Loss}$ WJ	—	30 % ±
4.	$\dot{Q}_{Loss}$ ex	—	30 % ±
5.	Cal error + Radiation Loss	—	10 % ±

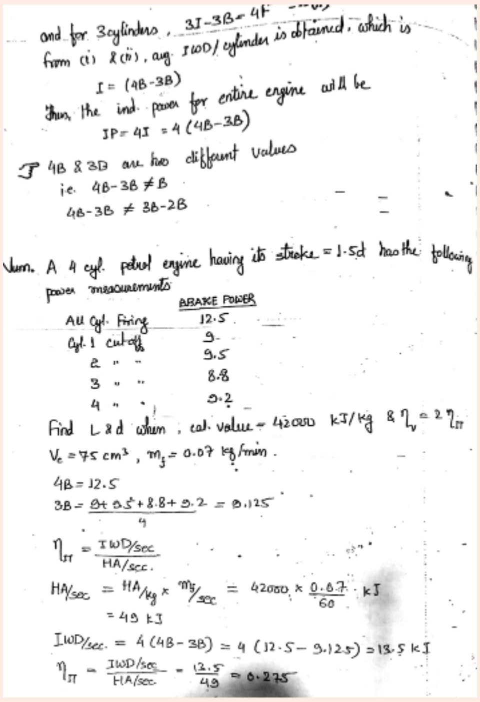
Q $\frac{IAS}{2014}$

A 2-stroke oil engine was subjected to a test at room temp. of 20°C with the fuel oil of CV 44000 kJ/kg.

Cyl bore = 20 cm
L/D = 1.3 : 1
Speed N = 500rpm

Brake drum dia = 120cm.
Rope dia = 3cm.
Net brake load = 460 N

Oil consumption Rate = 3.7 kg/hr
The jacket cooling water rate is 4.56 kg/hr with the rise in temp. of 27°C



and for 3 cylinders, $3I - 3B = 4F$ ---(ii)

from (i) & (ii), avg. IWD/cylinder is obtained, which is

$I = (4B - 3B)$

Thus, the ind. power for entire engine will be

$IP = 4I = 4(4B - 3B)$

If 4B & 3B are two different values

i.e. $4B - 3B \neq B$

$4B - 3B \neq 3B - 2B$

Num. A 4 cyl. petrol engine having its stroke = 1.5d has the following power measurements

	BRAKE POWER
All Cyl. Firing	12.5
Cyl. 1 cut off	9
2 " "	9.5
3 " "	8.8
4 " "	9.2

Find L & d when, cal. value = 42000 kJ/kg & $\eta_v = 2\eta_{IT}$

$V_c = 75\ cm^3$, $m_f = 0.07$ kg/min.

$4B = 12.5$

$3B = \frac{9 + 9.5 + 8.8 + 9.2}{4} = 9.125$

$\eta_{IT} = \frac{IWD/sec}{HA/sec}$

$HA/sec = HA/kg \times m_f/sec = 42000 \times \frac{0.07}{60}$ kJ

$= 49$ kJ

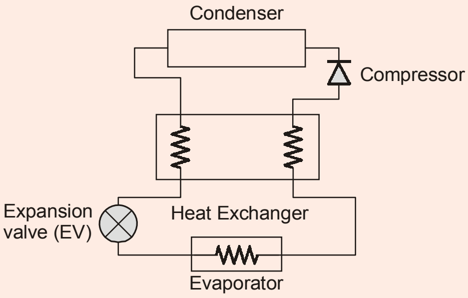
$IWD/sec = 4(4B - 3B) = 4(12.5 - 9.125) = 13.5$ kJ

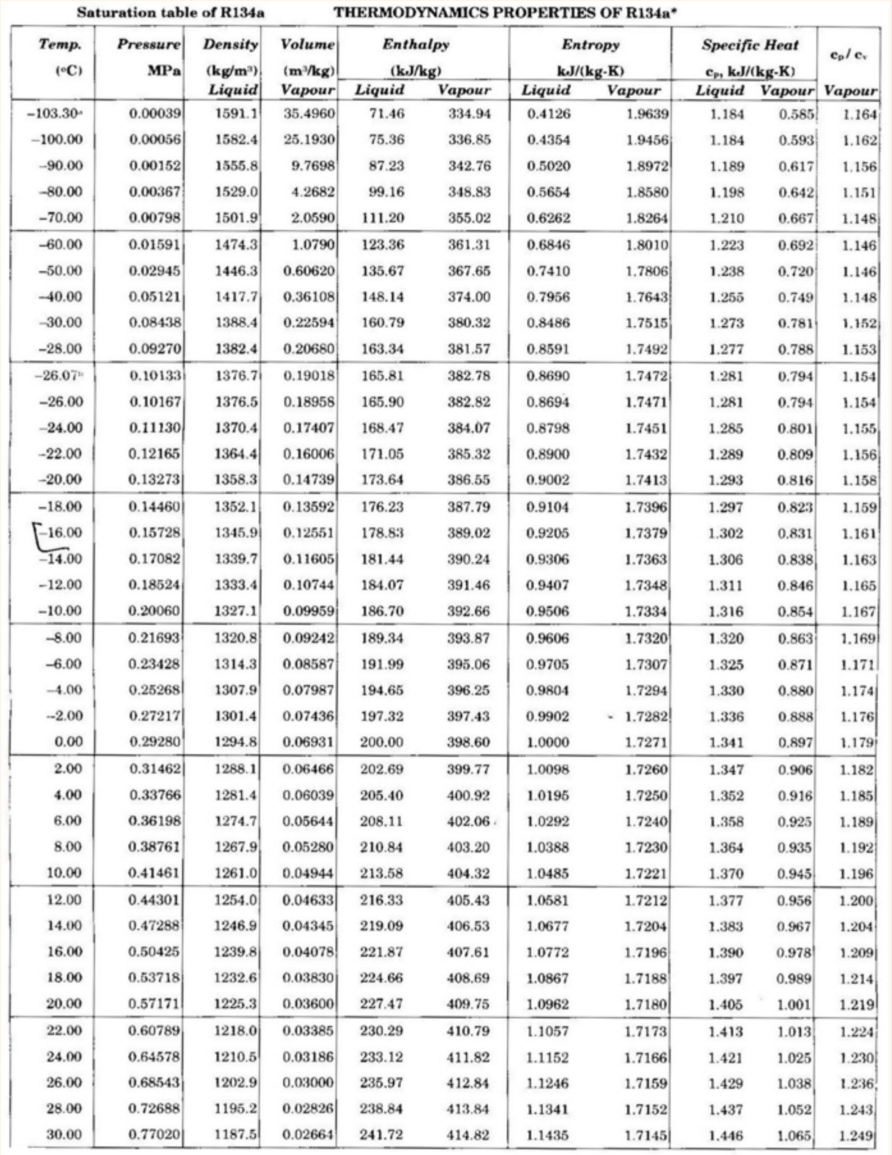
$\eta_{IT} = \frac{IWD/sec}{HA/sec} = \frac{13.5}{49} = 0.275$

End of Solution

Q.3 (b) A simple saturation refrigeration cycle uses R134a as refrigerant. The refrigeration system operates at 40°C condenser temperature and –16°C evaporation temperature respectively.

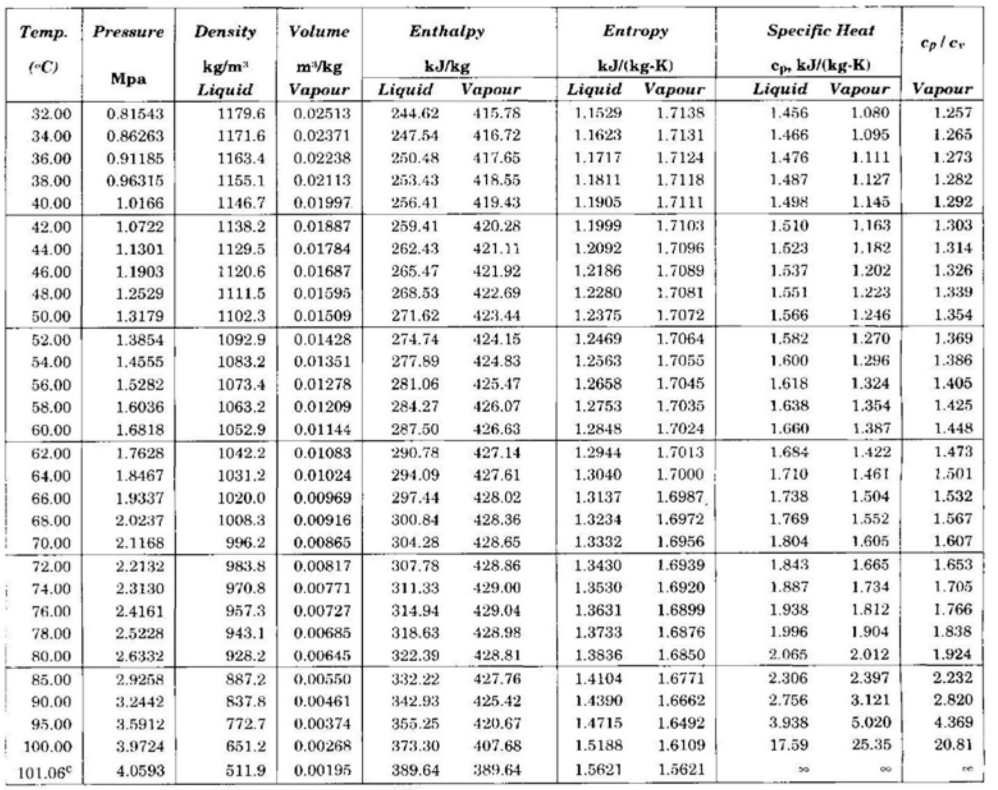
If a liquid vapour heat exchanger is installed in the above simple saturation refrigeration cycle, find the COP and power per ton of refrigeration. The outlet vapour of heat exchanger is 15°C temperature.





Saturation table of R134a THERMODYNAMICS PROPERTIES OF R134a*

Temp. (°C)	Pressure MPa	Density (kg/m³)	Volume (m³/kg)	Enthalpy (kJ/kg)		Entropy kJ/(kg-K)		Specific Heat c_p, kJ/(kg-K)		c_p/c_v
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour	Vapour
–103.30^a	0.00039	1591.1	35.4960	71.46	334.94	0.4126	1.9639	1.184	0.585	1.164
–100.00	0.00056	1582.4	25.1930	75.36	336.85	0.4354	1.9456	1.184	0.593	1.162
–90.00	0.00152	1555.8	9.7698	87.23	342.76	0.5020	1.8972	1.189	0.617	1.156
–80.00	0.00367	1529.0	4.2682	99.16	348.83	0.5654	1.8580	1.198	0.642	1.151
–70.00	0.00798	1501.9	2.0590	111.20	355.02	0.6262	1.8264	1.210	0.667	1.148
–60.00	0.01591	1474.3	1.0790	123.36	361.31	0.6846	1.8010	1.223	0.692	1.146
–50.00	0.02945	1446.3	0.60620	135.67	367.65	0.7410	1.7806	1.238	0.720	1.146
–40.00	0.05121	1417.7	0.36108	148.14	374.00	0.7956	1.7643	1.255	0.749	1.148
–30.00	0.08438	1388.4	0.22594	160.79	380.32	0.8486	1.7515	1.273	0.781	1.152
–28.00	0.09270	1382.4	0.20680	163.34	381.57	0.8591	1.7492	1.277	0.788	1.153
–26.07^b	0.10133	1376.7	0.19018	165.81	382.78	0.8690	1.7472	1.281	0.794	1.154
–26.00	0.10167	1376.5	0.18958	165.90	382.82	0.8694	1.7471	1.281	0.794	1.154
–24.00	0.11130	1370.4	0.17407	168.47	384.07	0.8798	1.7451	1.285	0.801	1.155
–22.00	0.12165	1364.4	0.16006	171.05	385.32	0.8900	1.7432	1.289	0.809	1.156
–20.00	0.13273	1358.3	0.14739	173.64	386.55	0.9002	1.7413	1.293	0.816	1.158
–18.00	0.14460	1352.1	0.13592	176.23	387.79	0.9104	1.7396	1.297	0.823	1.159
–16.00	0.15728	1345.9	0.12551	178.83	389.02	0.9205	1.7379	1.302	0.831	1.161
–14.00	0.17082	1339.7	0.11605	181.44	390.24	0.9306	1.7363	1.306	0.838	1.163
–12.00	0.18524	1333.4	0.10744	184.07	391.46	0.9407	1.7348	1.311	0.846	1.165
–10.00	0.20060	1327.1	0.09959	186.70	392.66	0.9506	1.7334	1.316	0.854	1.167
–8.00	0.21693	1320.8	0.09242	189.34	393.87	0.9606	1.7320	1.320	0.863	1.169
–6.00	0.23428	1314.3	0.08587	191.99	395.06	0.9705	1.7307	1.325	0.871	1.171
–4.00	0.25268	1307.9	0.07987	194.65	396.25	0.9804	1.7294	1.330	0.880	1.174
–2.00	0.27217	1301.4	0.07436	197.32	397.43	0.9902	1.7282	1.336	0.888	1.176
0.00	0.29280	1294.8	0.06931	200.00	398.60	1.0000	1.7271	1.341	0.897	1.179
2.00	0.31462	1288.1	0.06466	202.69	399.77	1.0098	1.7260	1.347	0.906	1.182
4.00	0.33766	1281.4	0.06039	205.40	400.92	1.0195	1.7250	1.352	0.916	1.185
6.00	0.36198	1274.7	0.05644	208.11	402.06	1.0292	1.7240	1.358	0.925	1.189
8.00	0.38761	1267.9	0.05280	210.84	403.20	1.0388	1.7230	1.364	0.935	1.192
10.00	0.41461	1261.0	0.04944	213.58	404.32	1.0485	1.7221	1.370	0.945	1.196
12.00	0.44301	1254.0	0.04633	216.33	405.43	1.0581	1.7212	1.377	0.956	1.200
14.00	0.47288	1246.9	0.04345	219.09	406.53	1.0677	1.7204	1.383	0.967	1.204
16.00	0.50425	1239.8	0.04078	221.87	407.61	1.0772	1.7196	1.390	0.978	1.209
18.00	0.53718	1232.6	0.03830	224.66	408.69	1.0867	1.7188	1.397	0.989	1.214
20.00	0.57171	1225.3	0.03600	227.47	409.75	1.0962	1.7180	1.405	1.001	1.219
22.00	0.60789	1218.0	0.03385	230.29	410.79	1.1057	1.7173	1.413	1.013	1.224
24.00	0.64578	1210.5	0.03186	233.12	411.82	1.1152	1.7166	1.421	1.025	1.230
26.00	0.68543	1202.9	0.03000	235.97	412.84	1.1246	1.7159	1.429	1.038	1.236
28.00	0.72688	1195.2	0.02826	238.84	413.84	1.1341	1.7152	1.437	1.052	1.243
30.00	0.77020	1187.5	0.02664	241.72	414.82	1.1435	1.7145	1.446	1.065	1.249



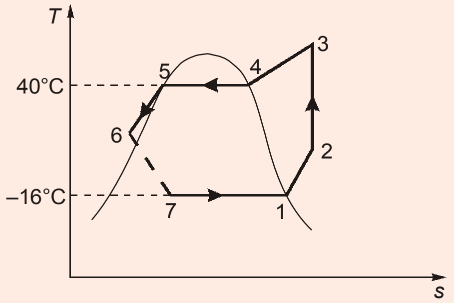
Temp. (°C)	Pressure Mpa	Density kg/m³ Liquid	Volume m³/kg Vapour	Enthalpy kJ/kg Liquid	Enthalpy kJ/kg Vapour	Entropy kJ/(kg-K) Liquid	Entropy kJ/(kg-K) Vapour	Specific Heat c_p, kJ/(kg-K) Liquid	Specific Heat c_p, kJ/(kg-K) Vapour	c_p/c_v Vapour
32.00	0.81543	1179.6	0.02513	244.62	415.78	1.1529	1.7138	1.456	1.080	1.257
34.00	0.86263	1171.6	0.02371	247.54	416.72	1.1623	1.7131	1.466	1.095	1.265
36.00	0.91185	1163.4	0.02238	250.48	417.65	1.1717	1.7124	1.476	1.111	1.273
38.00	0.96315	1155.1	0.02113	253.43	418.55	1.1811	1.7118	1.487	1.127	1.282
40.00	1.0166	1146.7	0.01997	256.41	419.43	1.1905	1.7111	1.498	1.145	1.292
42.00	1.0722	1138.2	0.01887	259.41	420.28	1.1999	1.7103	1.510	1.163	1.303
44.00	1.1301	1129.5	0.01784	262.43	421.11	1.2092	1.7096	1.523	1.182	1.314
46.00	1.1903	1120.6	0.01687	265.47	421.92	1.2186	1.7089	1.537	1.202	1.326
48.00	1.2529	1111.5	0.01595	268.53	422.69	1.2280	1.7081	1.551	1.223	1.339
50.00	1.3179	1102.3	0.01509	271.62	423.44	1.2375	1.7072	1.566	1.246	1.354
52.00	1.3854	1092.9	0.01428	274.74	424.15	1.2469	1.7064	1.582	1.270	1.369
54.00	1.4555	1083.2	0.01351	277.89	424.83	1.2563	1.7055	1.600	1.296	1.386
56.00	1.5282	1073.4	0.01278	281.06	425.47	1.2658	1.7045	1.618	1.324	1.405
58.00	1.6036	1063.2	0.01209	284.27	426.07	1.2753	1.7035	1.638	1.354	1.425
60.00	1.6818	1052.9	0.01144	287.50	426.63	1.2848	1.7024	1.660	1.387	1.448
62.00	1.7628	1042.2	0.01083	290.78	427.14	1.2944	1.7013	1.684	1.422	1.473
64.00	1.8467	1031.2	0.01024	294.09	427.61	1.3040	1.7000	1.710	1.461	1.501
66.00	1.9337	1020.0	0.00969	297.44	428.02	1.3137	1.6987	1.738	1.504	1.532
68.00	2.0237	1008.3	0.00916	300.84	428.36	1.3234	1.6972	1.769	1.552	1.567
70.00	2.1168	996.2	0.00865	304.28	428.65	1.3332	1.6956	1.804	1.605	1.607
72.00	2.2132	983.8	0.00817	307.78	428.86	1.3430	1.6939	1.843	1.665	1.653
74.00	2.3130	970.8	0.00771	311.33	429.00	1.3530	1.6920	1.887	1.734	1.705
76.00	2.4161	957.3	0.00727	314.94	429.04	1.3631	1.6899	1.938	1.812	1.766
78.00	2.5228	943.1	0.00685	318.63	428.98	1.3733	1.6876	1.996	1.904	1.838
80.00	2.6332	928.2	0.00645	322.39	428.81	1.3836	1.6850	2.065	2.012	1.924
85.00	2.9258	887.2	0.00550	332.22	427.76	1.4104	1.6771	2.306	2.397	2.232
90.00	3.2442	837.8	0.00461	342.93	425.42	1.4390	1.6662	2.756	3.121	2.820
95.00	3.5912	772.7	0.00374	355.25	420.67	1.4715	1.6492	3.938	5.020	4.369
100.00	3.9724	651.2	0.00268	373.30	407.68	1.5188	1.6109	17.59	25.35	20.81
101.06[c]	4.0593	511.9	0.00195	389.64	389.64	1.5621	1.5621	∞	∞	∞

[a]Triple point [b]NBP [c]Critical point

*Ashrae Handbook Fundamentals, 2005.

[20 Marks]

Solution:



$T_2 = 15°C$

Properties

T_{sat} (°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)	c_{pv} (kJ/kgK)	c_{pl} (kJ/kgK)
–16	178.83	389.02	0.9205	1.7379	0.831	–
40	256.41	419.43	1.1905	1.7111	1.145	1.498

Now,

Heat exchange in process1 - 2 = Heat exchange in process 5 - 6

$$c_{p_v}(T_2 - T_1) = c_{pl}(T_5 - T_6)$$

$$0.831[15 - (-16)] = 1.498(40 - T_6)$$

$$T_6 = 22.803°\text{C}$$

Now, enthalpy at (2),

$$h_2 = h_g + c_p(T_2 - T_1) = 389.02 + 0.831(15 + 16)$$

$$= 414.781 \text{ kJ/kg}$$

Now for enthalpy at (3), isentropic process 2 - 3,

$$s_2 = s_3$$

$$s_g + c_{pv_1}\ln\left(\frac{T_2}{T_1}\right) = s_g + c_{pv_4}\ln\left(\frac{T_3}{T_4}\right)$$

$$1.7379 + 0.831\ln\left(\frac{288}{257}\right) = 1.7111 + 1.145\ln\left(\frac{T_3}{313}\right)$$

$$T_3 = 348.02 \text{ K}$$

Now, enthalpy at (3),

$$h_3 = h_g + c_{pv4}(T_3 - T_4)$$

$$h_3 = 419.43 + 1.145(348.02 - 313) = 459.528 \text{ kJ/kg}$$

Now enthalpy at (6),

$$h_6 = h_f + c_{pl}(T_5 - T_6) = 256.41 - 1.498(40 - 22.803)$$

$$h_6 = 230.649 \text{ kJ/kg}$$

$$\text{COP} = \frac{h_1 - h_7}{h_3 - h_2} = \frac{389.02 - 230.649}{459.528 - 414.781} = 3.539$$

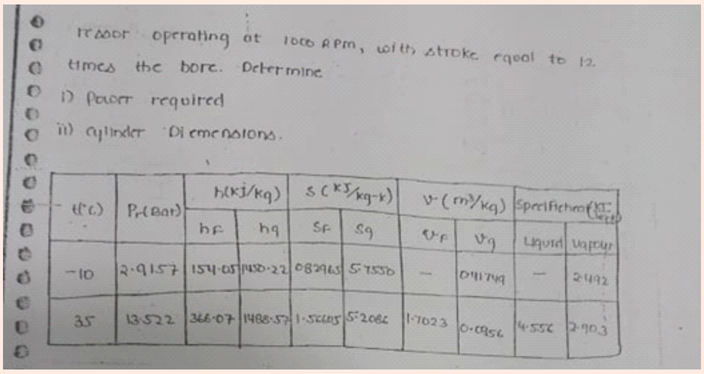
Now, $$\text{Power/TR} = \frac{\dot{m}(h_3 - h_2)}{\dot{m}(h_1 - h_7)/3.5} = \frac{3.5}{3.539}$$

Power per ton of refrigeration,

$$\frac{P}{TR} = 0.989 \text{ kW/TR}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Similar to Q.8(a) from Test-2
- **MADE EASY Classnotes**

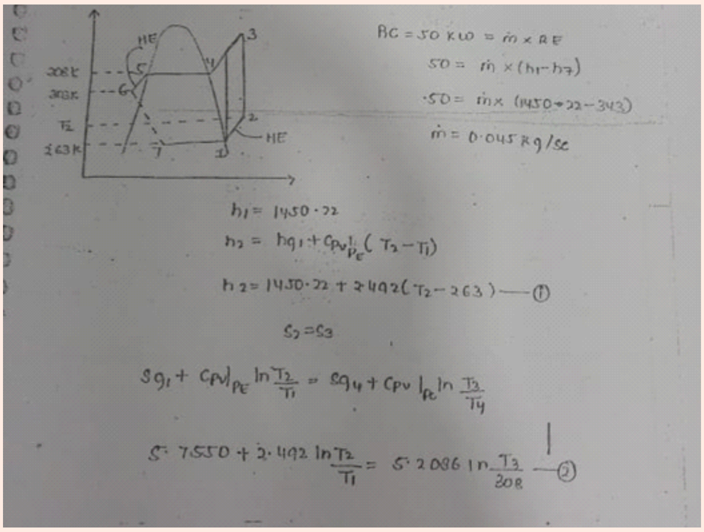


ressor operating at 1000 RPM, with stroke equal to 1.2 times the bore. Determine

i) Power required

ii) cylinder Dimensions.

t(°C)	P(Bar)	h(kJ/kg)		s (kJ/kg-K)		v (m³/kg)		Specific heat (kJ/kg-K)	
		hf	hg	sf	sg	vf	vg	Liquid	Vapour
−10	2.9157	154.05	1450.22	0.82965	5.7550	—	0.41749	—	2.492
35	13.522	366.07	1488.57	1.56605	5.2086	1.7023	0.0956	4.556	2.903



RC = 50 kW = $\dot{m}$ × RE

50 = $\dot{m}$ × ($h_1 - h_7$)

50 = $\dot{m}$ × (1450·22 − 343)

$\dot{m}$ = 0·045 kg/sec

h_1 = 1450·22

h_2 = $h_{g1} + C_{pv}|_{P_C}(T_2 - T_1)$

h_2 = 1450·22 + 2·492(T_2 − 263) —— ①

$S_2 = S_3$

$$s_{g1} + C_{pv}|_{P_E} \ln\frac{T_2}{T_1} = s_{g4} + C_{pv}|_{P_C} \ln\frac{T_3}{T_4}$$

$$5\cdot7550 + 2\cdot492 \ln\frac{T_2}{T_1} = 5\cdot2086 \ln\frac{T_3}{308} \quad \text{—— ②}$$

End of Solution

Q.3 (c) Moist air at 28°C DBT and 20.6 WBT and 101.325 kPa barometric pressure flows over a cooling coil and leaves it at a state of 10°C DBT and with specific humidity 7.046 gm/kg of dry air.

(i) If the air is required to offset a sensible heat gain of 2.35 kW and a latent heat gain of 0.31 kW in a space to be air-conditioned, calculate the mass of dry air which must be supplied to the room in order to maintain a DBT of 21°C in the room.

(ii) What will be the relative humidity in the room?

(iii) If a sensible heat gain diminishes by 1.175 kW but latent heat gain remains unchanged, at what temperature and moisture content must the air be supplied to the room?

Take specific capacity of air as 1.012 kJ/kg, K, latent enthalpy of water at 21°C is 2454 kJ/Kg. Show the processes on the psychrometric chart.

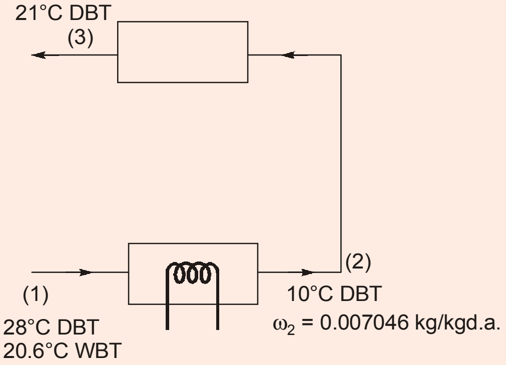
[20 Marks]

Solution:

(i)

Given, SH = 2.35 kW

LH = 0.31 kW



Latent heat, $$LH = (\omega_3 - \omega_2) \times \dot{m}_a \times h_{fg}$$

$$LH = (\omega_3 - 0.007046) \times \dot{m}_a \times 2454 \qquad \text{...(i)}$$

Sensible head, $$SH = \dot{m}_a \times c_{pa}(t_3 - t_2) + \omega \times \dot{m}_a \times c_{pv}(t_3 - t_2)$$

Now, $$= \dot{m}_a \times c_{pa}(21 - 10) + \omega \times \dot{m}_a \times c_{pv}(21 - 10)$$

Assume specific heat of water vapour, c_{pv} = 1.88 kJ/kgK

$$\text{SH} = \dot{m}_a[1.012 + 0.007046 \times 1.88] \times (11)$$

$$2.35 = \dot{m}_a \times 11.2777$$

$$\dot{m}_a = 0.2084 \text{ kg/s}$$

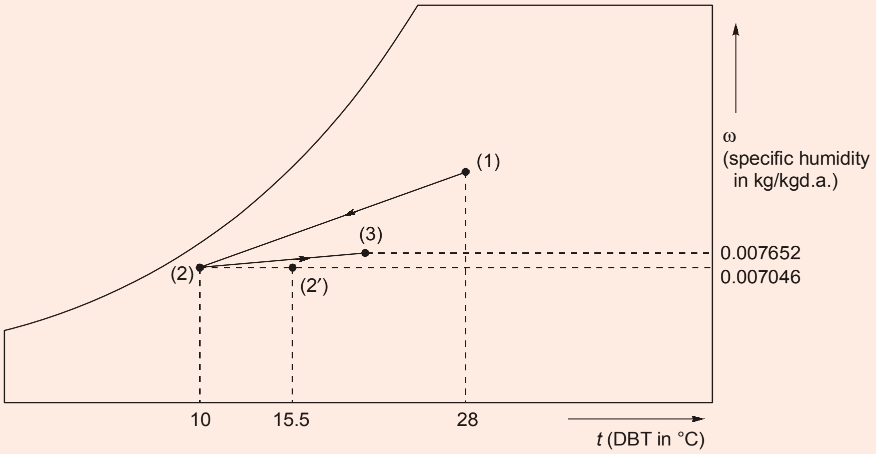
$$\dot{m}_a = 12.503 \text{ kg/min}$$

(ii)

Now, specific humidity at (3), from equation (i),

$$0.31 = (\omega_3 - 0.007046) \times 0.2084 \times 2454$$

$$\omega_3 = 0.007652 \text{ kg/kgd.a.}$$



From chart at 21°C DBT & 0.007652 kg/kgd.a. specific humidity,

Relative humidity, ϕ = 48%

(iii) Now heat gain diminishes by 1.175 kW

So, new LH = 0.31 kW

SH = 1.175 kW

Now, for same outlet condition,

$$\omega_3 = 0.007652 \text{ kg/kg.d.a}$$

$$\dot{m}_a = 0.2084 \text{ kg/s}$$

$$\text{SH} = \dot{m}_a \times c_{pa}(t_3 - t_2') + \omega \times \dot{m}_a \times c_{pv}(t_3 - t_2') \qquad \text{...(ii)}$$

now as latent heat remains unchanged new point (2′) inlet to the room remains on the horizontal line drawn from old point (2).

So, specific humidity also remains unchanged.

$$\omega_2' = \omega_2 = 0.007046 \text{ kg/kgd.a.}$$

$$1.175 = 0.2084 \times (21 - t_2')(1.012 + 0.007046 \times 1.88)$$

$$1.175 = 0.21366(21 - t_2')$$

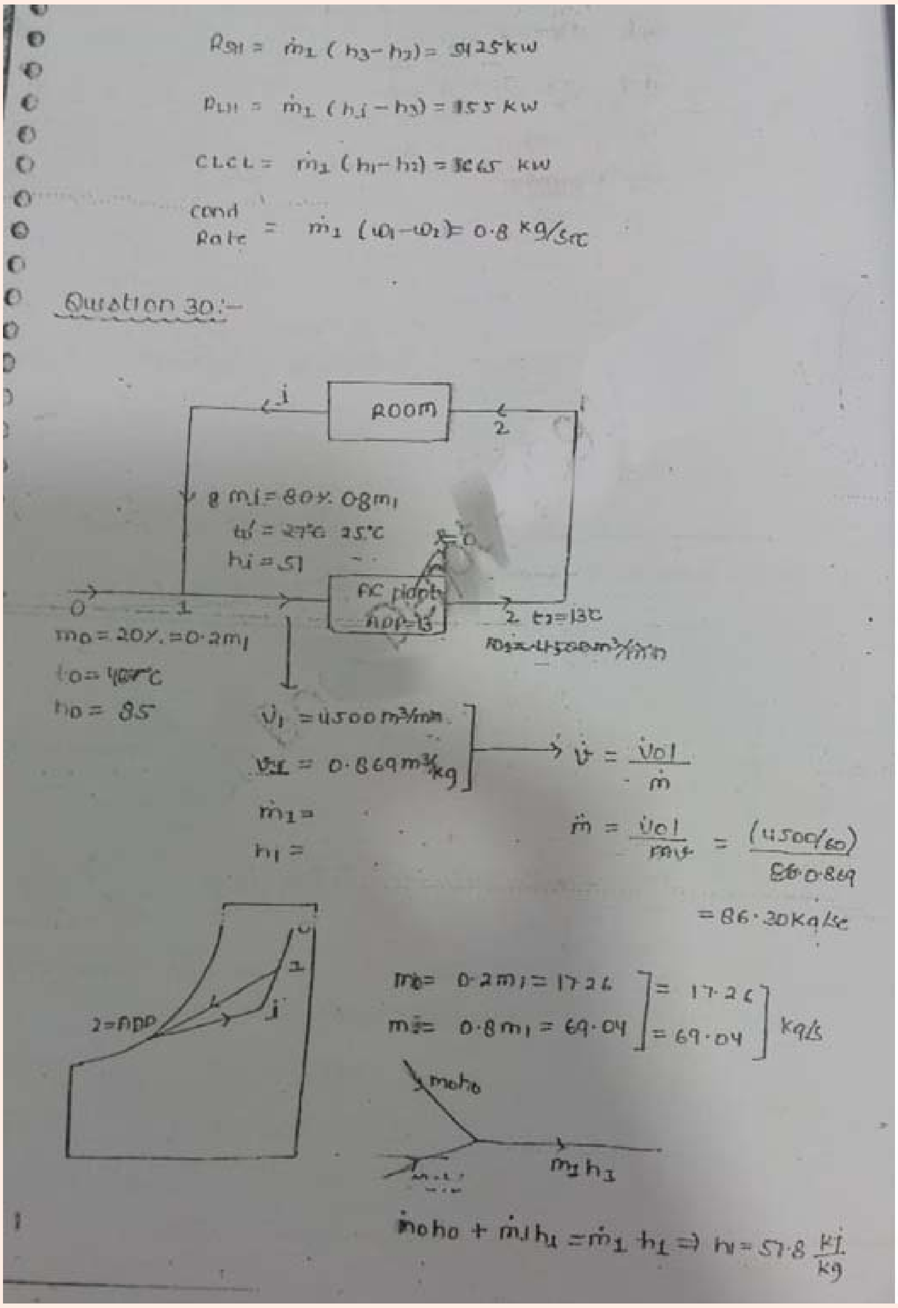
$$t_2' = 15.5°\text{C}$$

now, moisture content $= \omega_2' \times \dot{m}_a = 0.007046 \times 0.2084$

$= 0.001468$ kg $= 1.468$ gm

MADE EASY Source

- **MADE EASY Classnotes**



$R_{SH} = \dot{m}_1 (h_3 - h_2) = 9125$ kW

$R_{LH} = \dot{m}_1 (h_j - h_3) = 155$ kW

CLCL $= \dot{m}_1 (h_1 - h_2) = 3665$ kW

Cond Rate $= \dot{m}_1 (\omega_1 - \omega_2) = 0.8$ kg/sec

Question 30:-

$\dot{V}_1 = 4500\ m^3/min$

$v_1 = 0.869\ m^3/kg$ → $\dot{V} = \frac{\dot{V}ol}{\dot{m}}$

$\dot{m}_1 =$

$h_1 =$

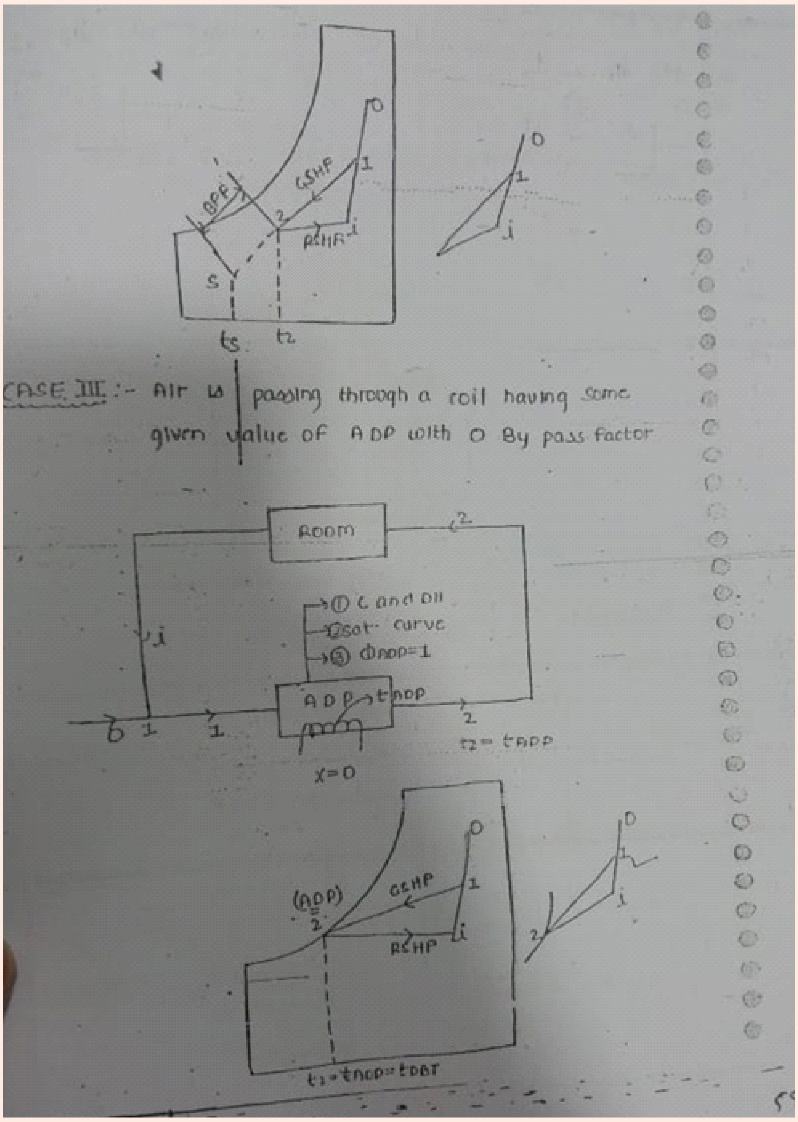
$\dot{m} = \frac{\dot{V}ol}{mv} = \frac{(4500/60)}{0.869}$

$= 86.30$ kg/sec

$m_0 = 0.2m_1 = 17.26$ $= 17.26$

$m_j = 0.8m_1 = 69.04$ $= 69.04$ kg/s

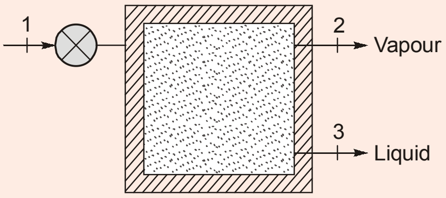
$\dot{m}_0h_0 + \dot{m}_jh_j = \dot{m}_1h_1 \Rightarrow h_1 = 57.8\ \frac{kJ}{kg}$



CASE III :- Air is passing through a coil having some given value of ADP with 0 By pass factor

End of Solution

Q.4 (a) A geothermal source provides 10 kg/s of hot water at 500 kPa, 150°C flowing into a flash evaporator that separates vapour and liquid at 200 kPa. Find the three fluxes of availability (inlet and two outlets) and the irreversibility rate. Take ambient temperature as 25°C.

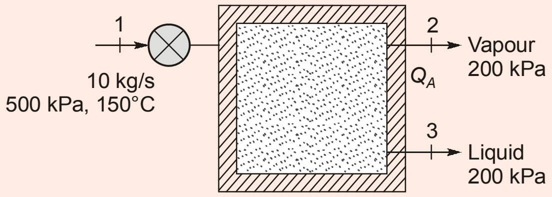


(Refer Table *A* placed at the end of booklet)

[20 Marks]

Solution:

$T_{amb} = 25°C$



Let us assume specific heat of water, c = 4.18 kJ/kgK

At 500 kPa and 500°C,

Enthalpy,
$$h_1 = h_f - C(\Delta T)$$
$$h_1 = 640.21 - 4.18(151.86 - 150)$$
$$h_1 = 632.435 \text{ kJ/kg}$$

Enthalpy of vapour at exit,
$$h_2 = 2706.63 \text{ kJ/kg}$$

Enthalpy of liquid at exit,
$$h_3 = 504.68 \text{ kJ/kg}$$

Now entropy of liquid at inlet,
$$s_1 = s_f - c\ln\left(\frac{T_f}{T_i}\right)$$

where, T_f = 151.86°C corresponding to 500 kPa
and T_i = 150°C corresponding to 500 kPa
$$= 1.8606 - 4.18\ln\left(\frac{424.86}{423}\right) = 1.8422 \text{ kJ/kgK}$$

Similarly, entropy of vapour at exit,
$$s_2 = 7.1271 \text{ kJ/kgK}$$

Entropy of liquid at exit,
$$s_3 = 1.5300 \text{ kJ/kgK}$$

Now, energy balance,
$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$
$$10 \times 632.435 = \dot{m}_2 \times (2706.63) + (10 - \dot{m}_2) \times 504.68$$
$$6324.35 - 10 \times 504.68 = \dot{m}_2 (2706.63 - 504.68)$$
$$\dot{m}_2 = 0.5802 \text{ kg/s}$$
$$\dot{m}_3 = 9.4198 \text{ kg/s}$$

Now, availability of liquid at inlet,
$$\Psi_1 = (h_1 - h_0) - T_0(s_1 - s_0) \quad \text{...(i)}$$

Now for ambient condition, T = 25°C

As saturated liquid enters to the flash evaporator so properties at 25°C and corresponding saturation pressure should be found out.

For enthalpy, h_0 by interpolation between 3 kPa and 24.08°C & 4 kPa and 28.96°C

$$\frac{h_0 - 101.03}{121.44 - 101.03} = \frac{25 - 24.08}{28.96 - 24.08}$$

$$h_0 = 104.877 \text{ kJ/kg}$$

Similarly for entropy s_0,

$$\frac{s_0 - 0.3545}{0.4226 - 0.3545} = \frac{25 - 24.08}{28.96 - 24.08}$$

$$s_0 = 0.3673 \text{ kJ/kgK}$$

Now, $\Psi_1 = (632.435 - 104.877) - 298[1.8422 - 0.3673]$

$$\Psi_1 = 88.0378 \text{ kJ/kg}$$

or $\Psi_1 = 88.0378 \times 10 \text{ kJ/s}$

$$\Psi_1 = 880.378 \text{ kW}$$

Now, availability of vapour at exit,

$$\Psi_2 = (h_2 - h_0) - T_0(s_2 - s_0)$$
$$= (2706.63 - 104.877) - 298(7.1271 - 0.3673)$$
$$= 587.3326 \text{ kJ/kg}$$

or $\Psi_2 = 0.5802 \times 587.3326 \text{ kJ/s} = 340.77 \text{ kW}$

Similarly availability of liquid at exit,

$$\Psi_3 = (h_3 - h_0) - T_0(s_3 - s_0)$$
$$= (504.68 - 104.877) - 298(1.5300 - 0.3673)$$
$$= 53.3184 \text{ kJ/kg}$$

or $\Psi_3 = 9.4198 \times 53.3184 \text{ kJ/s}$

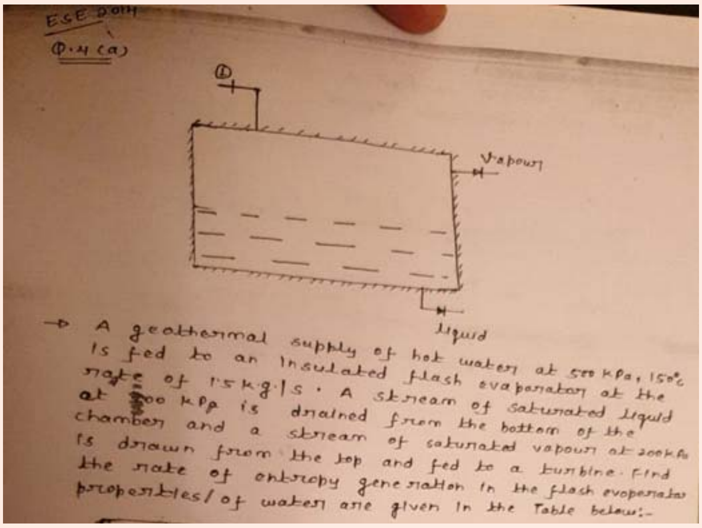
$$= 502.2486 \text{ kW}$$

Now, $\text{Irreversibility} = \Psi_1 - \Psi_2 - \Psi_3$

$$= 880.378 - 340.77 - 502.2486 = 37.3594 \text{ kW}$$

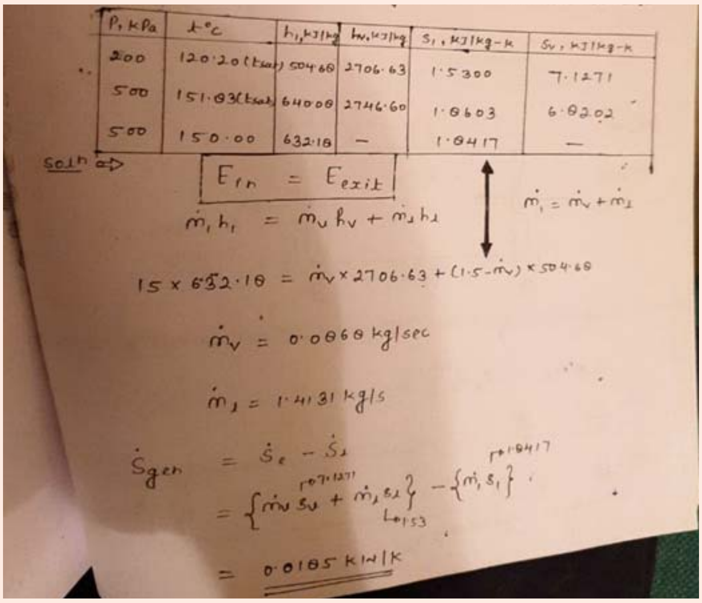
MADE EASY Source

- **MADE EASY Classnotes**



ESE 2014
Q.4 (a)

→ A geothermal supply of hot water at 500 kPa, 150°c
is fed to an insulated flash evaporator at the
rate of 1·5 kg/s. A stream of saturated liquid
at 200 kPa is drained from the bottom of the
chamber and a stream of saturated vapour at 200 kPa
is drawn from the top and fed to a turbine. Find
the rate of entropy generation in the flash evaporator
properties/ of water are given in the Table below:-



P, kPa	t °C	h_l, kJ/kg	h_v, kJ/kg	s_l, kJ/kg-K	s_v, kJ/kg-K
200	120.20 (tsat)	504.68	2706.63	1.5300	7.1271
500	151.83 (tsat)	640.08	2746.60	1.8603	6.8202
500	150.00	632.18	—	1.8417	—

Soln ⇨

$$\boxed{E_{in} = E_{exit}}$$

$$\dot m_1 h_1 = \dot m_v h_v + \dot m_l h_l \qquad \dot m_1 = \dot m_v + \dot m_l$$

$$1.5 \times 632.18 = \dot m_v \times 2706.63 + (1.5 - \dot m_v) \times 504.68$$

$$\dot m_v = 0.0868 \text{ kg/sec}$$

$$\dot m_l = 1.4131 \text{ kg/s}$$

$$\dot S_{gen} = \dot S_e - \dot S_i$$

$$= \{\dot m_v \overset{7.1271}{s_v} + \dot m_l \underset{1.53}{s_l}\} - \{\dot m_1 \overset{1.8417}{s_1}\}$$

$$= \underline{\underline{0.0185 \text{ kW/K}}}$$

End of Solution

Q.4 (b) Air at a mean velocity of 20 m/sec flows through a 2 cm diameter tube whose surface is maintained at 200°C. The temperature of air as it enters the tube at inlet is 20°C and leaves the tube at 180°C. Determine

(i) The length of the tube required to heat the water from 20°C to 180°C, and

(ii) The pumping power required to maintain the flow.

Assume $f = 0.3164/(\text{ReD})^{1/4}$.

Properties of air at the mean film temperature $\bar{T}_f$:

ρ = density = 0.8345 kg/m^3; specific heat = c_p = 1015 J/kg K; dynamic viscosity, $\mu = 2.3825 \times 10^{-5}$ kg/m.s; $Pr = 0.703$; thermal conductivity, $k = 0.034425$ W/mK.

[20 Marks]

Solution:

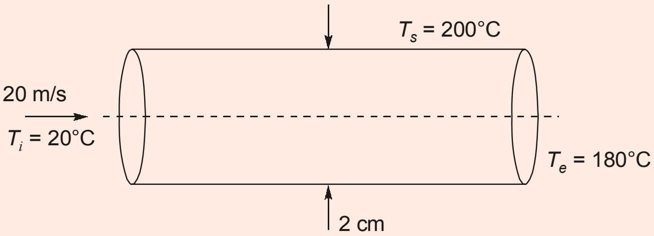
Assumptions:

(i) Steady flow

(ii) Fully developed flow

(iii) Constant wall temperature condition

(iv) Properties at mean temperature



Mean velocity of air, $V = 20$ m/s

Diameter of tube, $D = 0.02$ m

Reynolds number, $$Re_D = \frac{\rho VD}{\mu} = \frac{0.8345 \times 20 \times 0.02}{2.3825 \times 10^{-5}}$$

$$= 14010.493 > 4000$$

So, flow is turbulent,

Now, $$f = \frac{0.3164}{\text{Re}_D^{1/4}} = \frac{0.3164}{(14010.493)^{1/4}} = 0.02908$$

By Reynolds-Colburn Analogy,

$$St \cdot \text{Pr}^{2/3} = \frac{f'}{2} = \frac{f}{2 \times 4}$$

where, f' = Darcy's friction coefficient, f = friction factor

$$\frac{h}{\rho V c_p}\text{Pr}^{2/3} = \frac{f}{8}$$

$$\frac{h}{0.8345 \times 20 \times 1015} \times 0.703^{2/3} = \frac{0.02908}{8}$$

$$h = 77.8906 \text{ W/m}^2\text{-K}$$

Mass flow-rate of air, $$\dot{m} = \rho AV = 0.8345 \times \frac{\pi}{4} \times 0.02^2 \times 20$$

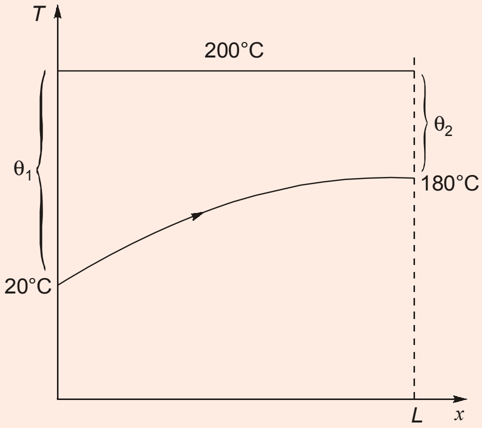
$$= 5.2433 \times 10^{-3} \text{ kg/s}$$

$$\theta_1 = 200 - 20 = 180°\text{C}$$

$$\theta_2 = 200 - 180 = 20°\text{C}$$

Logrithmic mean temperature difference,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{180 - 20}{\ln\left(\frac{180}{20}\right)} = 72.819°\text{C}$$



Energy balance, $\dot{m}\, c_p(T_e - T_i) = h(\pi DL)\theta_m$

$\Rightarrow$ $5.2433 \times 10^{-3} \times 1015 \times (180 - 20) = 77.8906 \times (\pi \times 0.02 \times L) \times 72.819$

Length of tube, $L = 2.389$ m

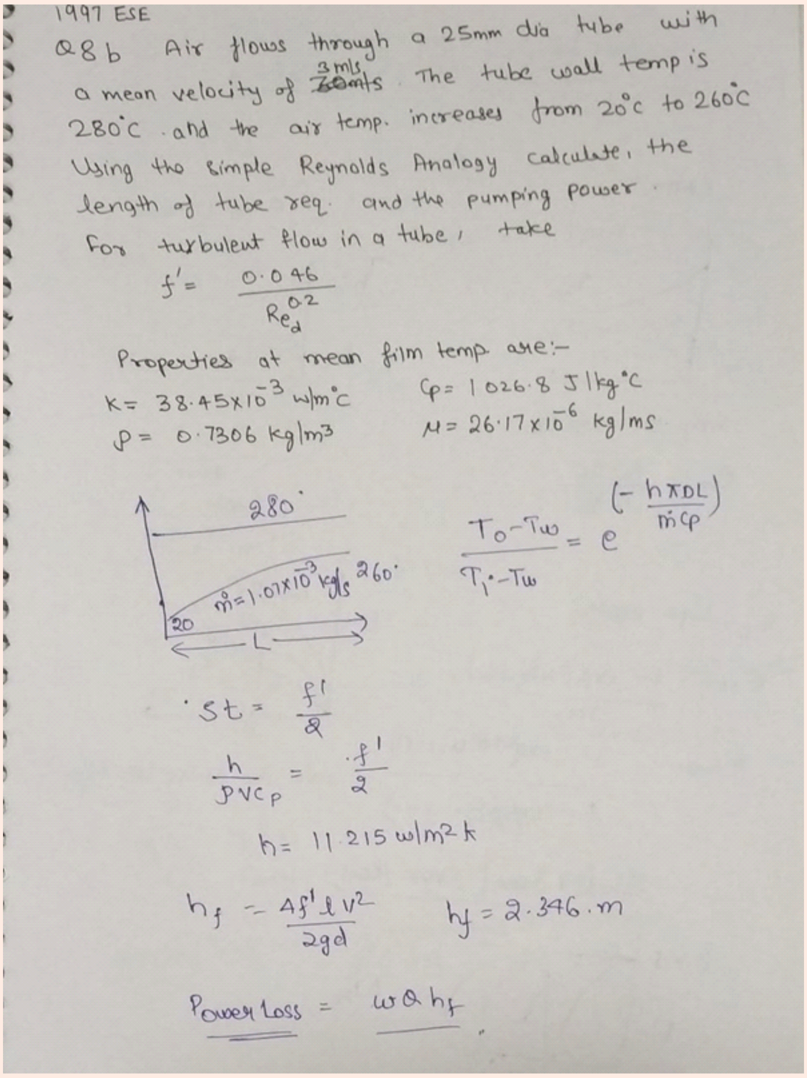
(ii) Head loss due to friction, $h_f = \dfrac{fLV^2}{D \times 2g} = \dfrac{0.02908 \times 2.389 \times 20^2}{0.02 \times 2 \times 9.81} = 70.828$ m

Pumping power, $p = \dot{m}gh_f = 5.2433 \times 10^{-3} \times 9.81 \times 70.828$

$= 3.643$ W

MADE EASY Source

- **ESE 2019 Mains Test Series:** Similar to Q.6(c), Test-3
- **MADE EASY Classnotes**



1997 ESE

Q8b Air flows through a 25mm dia tube with a mean velocity of ~~30m/s~~ 3 m/s. The tube wall temp is 280°C and the air temp. increases from 20°c to 260°C Using the simple Reynolds Analogy calculate, the length of tube req. and the pumping power. For turbulent flow in a tube, take

$f' = \dfrac{0.046}{Re_d^{0.2}}$

Properties at mean film temp are:–

$k = 38.45 \times 10^{-3}$ w/m°c $C_p = 1026.8$ J/kg°c

$\rho = 0.7306$ kg/m³ $\mu = 26.17 \times 10^{-6}$ kg/ms

280°

$\dot{m} = 1.07 \times 10^{-3}$ kg/s   260°

20

L

$\dfrac{T_o - T_w}{T_i - T_w} = e^{\left(-\frac{h\pi DL}{\dot{m}C_p}\right)}$

$St = \dfrac{f'}{2}$

$\dfrac{h}{\rho V C_p} = \dfrac{f'}{2}$

$h = 11.215$ w/m²k

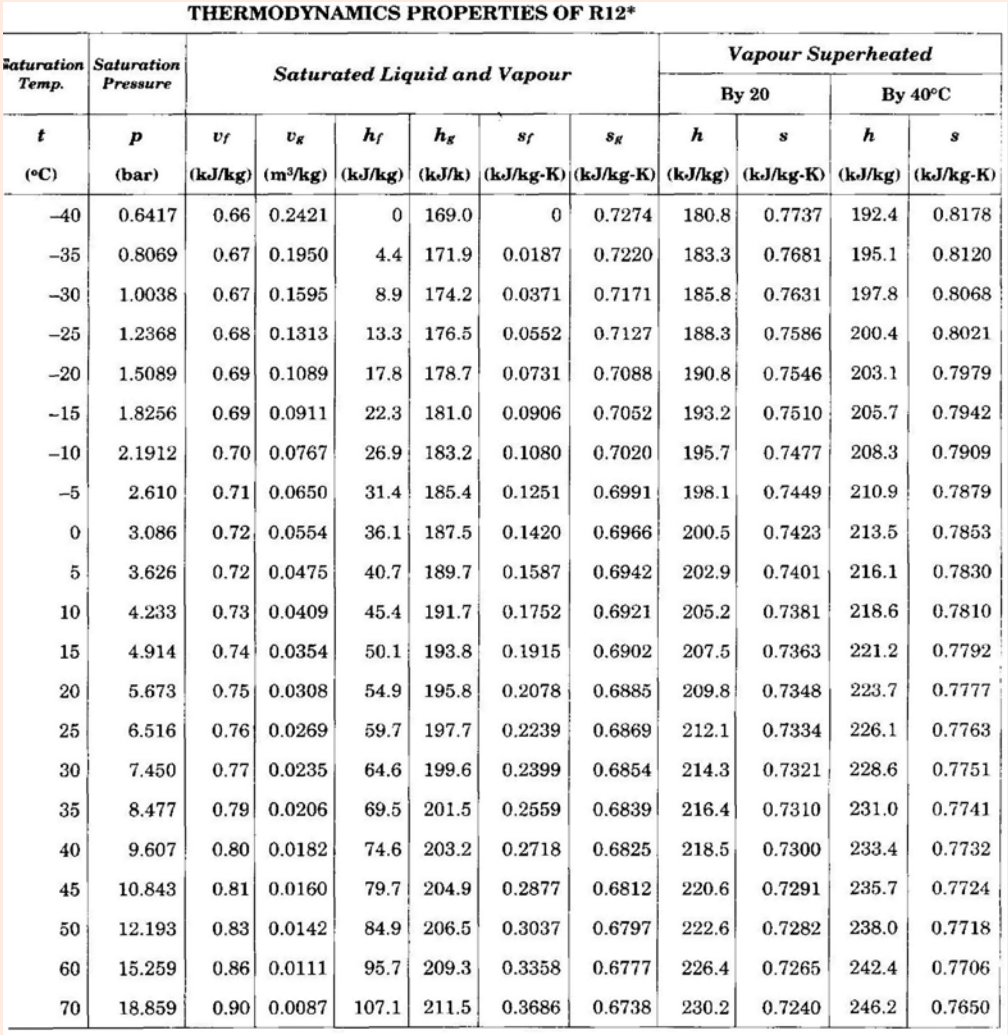
$h_f = \dfrac{4f'lV^2}{2gd}$ $h_f = 2.346$ m

Power Loss = wQh_f

End of Solution

Q.4 (c) A single-cylinder, single-acting reciprocating compressor using R12 as refrigerant has a bore 80 mm and stroke 60 mm. The compressor runs at 1450 rpm. If the condensing temperature is 40°C, find the performance characteristics of the compressor when the suction temperature is –10°C. Specific heat of vapour at 40°C is 0.759 kJ/kg K.

Assume the simple cycle of operation and no clearance.

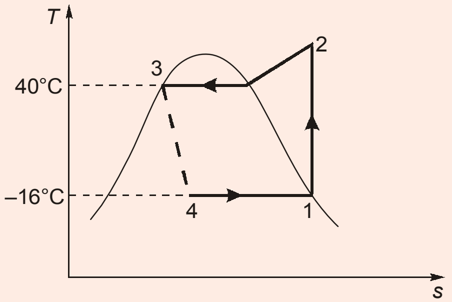


THERMODYNAMICS PROPERTIES OF R12*

Saturation Temp.	*Saturation Pressure*	*Saturated Liquid and Vapour*						*Vapour Superheated*			
								By 20		By 40°C	
t (°C)	p (bar)	v_f (kJ/kg)	v_g (m³/kg)	h_f (kJ/kg)	h_g (kJ/k)	s_f (kJ/kg-K)	s_g (kJ/kg-K)	h (kJ/kg)	s (kJ/kg-K)	h (kJ/kg)	s (kJ/kg-K)
–40	0.6417	0.66	0.2421	0	169.0	0	0.7274	180.8	0.7737	192.4	0.8178
–35	0.8069	0.67	0.1950	4.4	171.9	0.0187	0.7220	183.3	0.7681	195.1	0.8120
–30	1.0038	0.67	0.1595	8.9	174.2	0.0371	0.7171	185.8	0.7631	197.8	0.8068
–25	1.2368	0.68	0.1313	13.3	176.5	0.0552	0.7127	188.3	0.7586	200.4	0.8021
–20	1.5089	0.69	0.1089	17.8	178.7	0.0731	0.7088	190.8	0.7546	203.1	0.7979
–15	1.8256	0.69	0.0911	22.3	181.0	0.0906	0.7052	193.2	0.7510	205.7	0.7942
–10	2.1912	0.70	0.0767	26.9	183.2	0.1080	0.7020	195.7	0.7477	208.3	0.7909
–5	2.610	0.71	0.0650	31.4	185.4	0.1251	0.6991	198.1	0.7449	210.9	0.7879
0	3.086	0.72	0.0554	36.1	187.5	0.1420	0.6966	200.5	0.7423	213.5	0.7853
5	3.626	0.72	0.0475	40.7	189.7	0.1587	0.6942	202.9	0.7401	216.1	0.7830
10	4.233	0.73	0.0409	45.4	191.7	0.1752	0.6921	205.2	0.7381	218.6	0.7810
15	4.914	0.74	0.0354	50.1	193.8	0.1915	0.6902	207.5	0.7363	221.2	0.7792
20	5.673	0.75	0.0308	54.9	195.8	0.2078	0.6885	209.8	0.7348	223.7	0.7777
25	6.516	0.76	0.0269	59.7	197.7	0.2239	0.6869	212.1	0.7334	226.1	0.7763
30	7.450	0.77	0.0235	64.6	199.6	0.2399	0.6854	214.3	0.7321	228.6	0.7751
35	8.477	0.79	0.0206	69.5	201.5	0.2559	0.6839	216.4	0.7310	231.0	0.7741
40	9.607	0.80	0.0182	74.6	203.2	0.2718	0.6825	218.5	0.7300	233.4	0.7732
45	10.843	0.81	0.0160	79.7	204.9	0.2877	0.6812	220.6	0.7291	235.7	0.7724
50	12.193	0.83	0.0142	84.9	206.5	0.3037	0.6797	222.6	0.7282	238.0	0.7718
60	15.259	0.86	0.0111	95.7	209.3	0.3358	0.6777	226.4	0.7265	242.4	0.7706
70	18.859	0.90	0.0087	107.1	211.5	0.3686	0.6738	230.2	0.7240	246.2	0.7650

[20 Marks]

Solution:



$L = 60$ mm

$D = 80$ mm

$N = 1450$ rpm

Clearance ratio, $C = 0$

Assuming isentropic compression

$$s_1 = s_2$$

$$s_1 = s_{g@-10°C} = 0.702 \text{ kJ/kgK}$$

$$s_2 = (s_g)_{@40^\circ\text{C}} + c_{pv}\ln\left(\frac{T_2}{313}\right)$$

$$0.702 = 0.6825 + 0.759\ln\left(\frac{T_2}{313}\right)$$

$$T_2 = 321.14 \text{ K} \qquad \text{Compressor discharge temperature}$$

$$h_2 = h_{g@40^\circ} + c_{pv}(T_2 - 313)$$

$$h_2 = 203.2 + 0.759 \times (321.14 - 313)$$

$$= 209.37 \text{ kJ/kg}$$

Since, clearance ratio is zero, So $\eta_v = 100\%$

Mass flow rate,

$$\eta_v = \frac{\dot{m}\nu_1}{\frac{\pi}{4}D^2LN}$$

$$\dot{m} = \frac{\eta_v}{\nu_1}\left(\frac{\pi}{4}D^2LN\right)$$

where $\nu_1 = v_{g@-10^\circ\text{C}} = 0.0767 \text{ m}^3/\text{kg}$

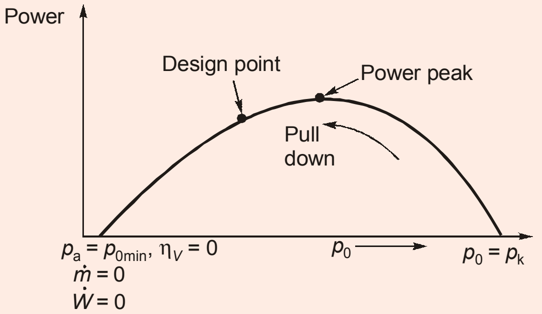
$$\dot{m} = \frac{1}{0.0767} \times \left(\frac{\pi}{4} \times 0.08^2 \times 0.06 \times 1450\right) = 5.7 \text{ kg/min}$$

Power input to compressor,

$$P = \dot{m}(h_2 - h_1) = 5.7 \times (209.37 - 183.2)$$

$$= 149.169 \text{ kJ/min}$$

or $= 2.486$ kW



Performance characteristics of a reciprocating compressor as a function of evaporator pressure

Generally refrigeration systems operate on the left-hand side of this curve. But just after starting, the compressor passes through the power peak. The compressor motors are, therefore, oversized to enable them to take the peak load during pull-down. The starting current is more than the running current.

The effect of the discharge pressure can similarly be analysed. At constant suction pressure, an increase in the discharge pressure will cause a reduction in the volumetric efficiency due to higher compression ratio. The mass of refrigerant circulated will thus be reduced. At the same time the specific work will increase. But there is a continuous increase in the power consumption and power per ton. The capacity will be decreased due to decrease in the mass flow and slight decrease in the refrigerating effect.

End of Solution

Section B

Q.5 (a) A single-cylinder, single-acting, square reciprocating pump has piston diameter and stroke length of 300 mm. The pump is placed such that the vertical distance between the center-line of the pump and sump level is 5 m. The water is being delivered at a height of 22 m above the centerline of the pump. The suction and delivery pipes are 8 m and 28 m long respectively, and diameter of both the pipes is 150 mm. If the pump is running at 30 rpm and coefficient of friction for suction and delivery pipes is 0.005, estimate the theoretical power required to drive the pump (kW).

[12 Marks]

Solution:

Suction heat of pump, $h_s = 5$ m
Delivery heat of pump, $h_d = 22$ m
Length of suction pump, $l_s = 8$ m
Length of delivery pipe, $l_d = 28$ m
Diameter of suction pipe, $d_s = 150$ mm
Diameter of delivery pipe, $d_d = 150$ mm
$N = 30$ rpm
Stroke length, $L = 300$ mm $= 0.3$ m
Piston diameter, $D = 300$ mm $= 0.3$ m

Pistion area, $A = \frac{\pi}{4} \times 0.3^2 = 0.07068 \text{ m}^2$

Theoretical discharge, $Q = \frac{ALN}{60} = \frac{0.07068 \times 0.3 \times 30}{60} = 0.010603 \text{ m}^3\text{/s}$

Maximum velocity in suction pipe,

$$V_{s,\,\max} = \frac{A}{a_s} \times \omega \times r$$

$$= \frac{0.07068}{\frac{\pi}{4} \times 0.15^2} \times \left(\frac{2\pi \times 30}{60}\right) \times 0.15 = 1.8848 \text{ m/s}$$

$$h_{fs,\,\max} = \frac{f l_s V^2_{s,\max}}{d_s \times 2g} = \frac{(4 \times 0.005) \times 8 \times 1.8848^2}{0.15 \times 2 \times 9.81}$$

$$= 0.1931 \text{ m}$$

Since, $d_s = d_d$

Maximum velocity in delivery pipe,

$$V_{d,\max} = V_{s,\max} = 1.8848 \text{ m/s}$$

$$h_{fd,\,\max} = \frac{f l_d V^2_{d,\max}}{d_d \times 2g} = \frac{(4 \times 0.005) \times 28 \times 1.8848^2}{0.15 \times 2 \times 9.81} = 0.6758 \text{ m}$$

Theoretical power required,

$$P = \rho g Q\left[h_s + h_d + \frac{2}{3} h_{fs,\max} + \frac{2}{3} h_{fd,\max}\right]$$

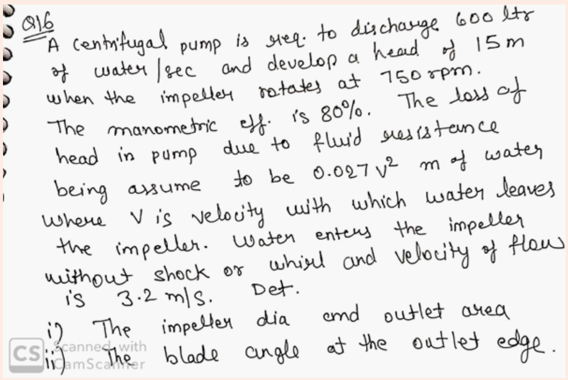
$$= 1000\times 9.81\times\left[5+22+\frac{2}{3}\times 0.1931+\frac{2}{3}\times 0.6758\right]\times 0.010603$$

$$= 0.6758 \text{ m}$$

$$P = 2.868 \times 10^3 \text{ W} = 2.868 \text{ kW}$$

MADE EASY Source

- **Covered in MADE EASY Class Notes**
- **ESE 2019 Mains Workbook:** Similar to Q.49 discussed in Class

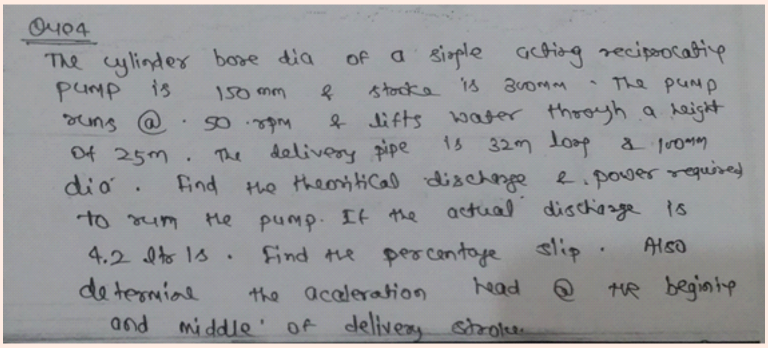


Q16

A centrifugal pump is req. to discharge 600 ltr of water/sec and develop a head of 15 m when the impeller rotates at 750 rpm. The manometric eff. is 80%. The loss of head in pump due to fluid resistance being assume to be $0.027 V^2$ m of water where V is velocity with which water leaves the impeller. Water enters the impeller without shock or whirl and velocity of flow is 3.2 m/s. Det.

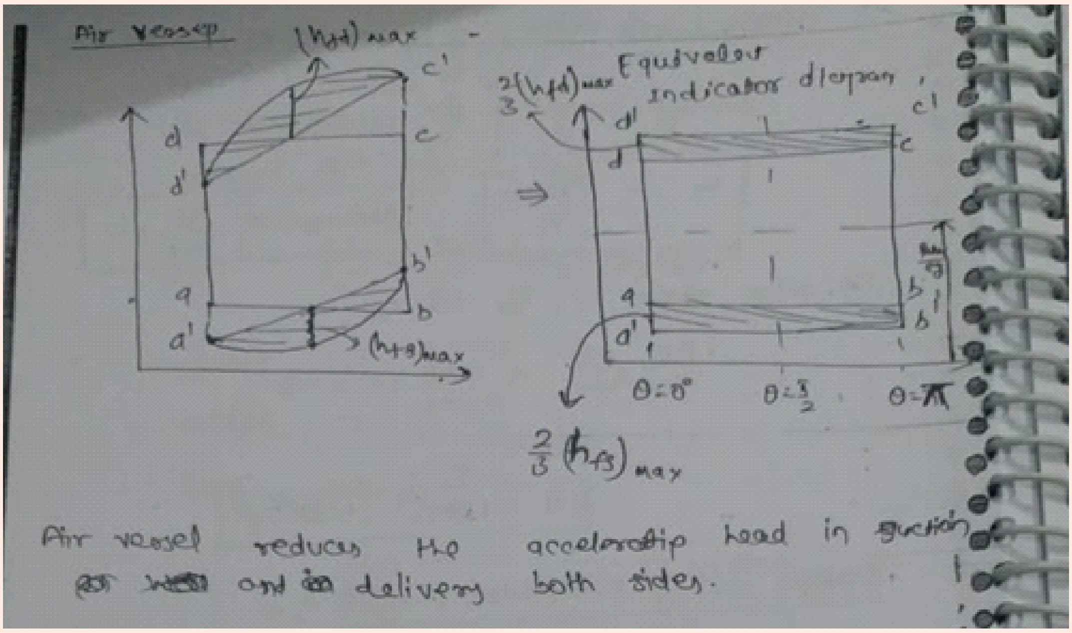
i) The impeller dia and outlet area

ii) The blade angle at the outlet edge.

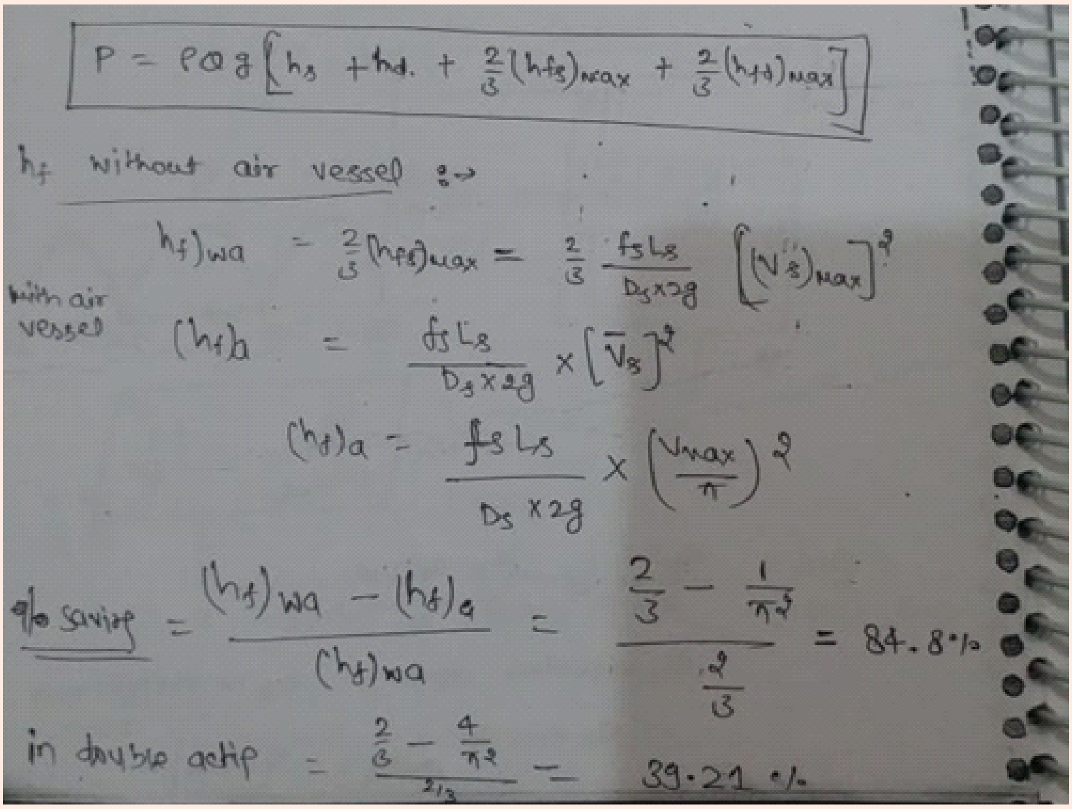


Q404

The cylinder bore dia of a single acting reciprocating pump is 150 mm & stroke is 300 mm. The pump runs @ 50 rpm & lifts water through a height of 25 m. The delivery pipe is 32 m long & 100 mm dia. Find the theoritical discharge & power required to run the pump. If the actual discharge is 4.2 ltr/s. Find the percentage slip. Also determine the acceleration head @ the begining and middle of delivery stroke.



Air vessel reduces the acceleration head in suction and delivery both sides.



$$\boxed{P = \rho Q g\left[h_s + h_d + \frac{2}{3}(h_{fs})_{max} + \frac{2}{3}(h_{fd})_{max}\right]}$$

h_f without air vessel :→

$$h_f)_{wa} = \frac{2}{3}(h_{fs})_{max} = \frac{2}{3}\,\frac{f_s L_s}{D_s \times 2g}\left[(V_s')_{max}\right]^2$$

with air vessel

$$(h_f)_a = \frac{f_s L_s}{D_s \times 2g} \times \left[\bar{V}_s\right]^2$$

$$(h_f)_a = \frac{f_s L_s}{D_s \times 2g} \times \left(\frac{V_{max}}{\pi}\right)^2$$

$$\text{\% saving} = \frac{(h_f)_{wa} - (h_f)_a}{(h_f)_{wa}} = \frac{\frac{2}{3} - \frac{1}{\pi^2}}{\frac{2}{3}} = 84.8\%$$

$$\text{in double actin} = \frac{\frac{2}{3} - \frac{4}{\pi^2}}{2/3} = 39.21\%$$

End of Solution

Q.5 (b) Show that the diagram work per unit mass of steam for maximum blading efficiency of a 50% reaction stage is V_b^2, where V_b is the mean blade velocity.

[12 Marks]

Solution:

Gross Stage Efficiency and Optimum Value of ρ for Parson's Reaction Turbine

- Work done/kg of steam

$$w = uV_w \text{ kJ/kg} = u\left[V_1\cos\alpha_1 + V_{r_2}\cos\beta_2 - u\right]$$

$$= u\left[V_1\cos\alpha_1 + V_1\cos\alpha_1 - u\right]$$

$$= \left[2uV_1\cos\alpha_1 - u^2\right]$$

($\because$ For Parson's turbine: $V_{r_2} = V_1$ and $\beta_2 = \alpha_1$)

$$= u[2V_1\cos\alpha_1 - u]$$

$$w = V_1^2\left[2\rho\cos\alpha_1 - \rho^2\right] \qquad \left(\because \text{Blade speed ratio, } \rho = \frac{u}{V_1}\right)$$

Blade or Diagram Efficiency of Parson's Turbine : (η_b)

$$\eta_b = \frac{\text{Work done/sec}}{\text{Energy input/sec}} \qquad \text{...(i)}$$

- Work done/sec $= uV_w = V_1^2(2\rho\cos\alpha_1 - \rho^2)$ ...(ii)
- Energy input to fixed blade $= \dfrac{V_1^2}{2}$

and energy input to moving blade

$$= \frac{V_{r_2}^2 - V_{r_1}^2}{2}$$

So, Total energy input $= \dfrac{V_1^2}{2} + \dfrac{V_{r_2}^2 - V_{r_1}^2}{2}$

$$= \frac{V_1^2}{2} + \frac{V_1^2 - V_{r_1}^2}{2}$$

For Parson's turbine, $V_{r_2} = V_1$

$$= V_1^2 - \frac{V_{r_1}^2}{2} = V_1^2 - \frac{1}{2}\left[V_1^2 + u^2 - 2V_1 u\cos\alpha_1\right]$$

$$= V_1^2 - \frac{V_1^2}{2}\left[1 + \frac{u^2}{V_1^2} - 2\times\frac{u}{V_1}\cos\alpha_1\right]$$

$$= -\frac{V_1^2}{2}\left[1 + \rho^2 - 2\rho\cos\alpha_1\right] + V_1^2$$

$$= \frac{V_1^2}{2}\left[-1 - \rho^2 + 2\rho\cos\alpha_1 + 2\right]$$

$$= \frac{V_1^2}{2}\left[1 - \rho^2 + 2\rho\cos\alpha_1\right] \quad \text{...(iii)}$$

So from Eqs. (i), (ii) and (iii), we get

$$\eta_b = \frac{V_1^2(2\rho\cos\alpha_1 - \rho^2)}{\frac{V_1^2}{2}\left[1 - \rho^2 + 2\rho\cos\alpha_1\right]}$$

$$\Rightarrow \quad \eta_b = \frac{2(2\rho\cos\alpha_1 - \rho^2)}{1 - \rho^2 + 2\rho\cos\alpha_1} \quad \text{...(iv)}$$

For maximum efficiency $\dfrac{\partial\eta_b}{\partial\rho} = 0$

On solving, $\rho_{opt} = \cos\alpha_1$

Since $\rho_{opt} = \cos\alpha_1$

So $(\eta_b)_{max} = \dfrac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}$

(Work done)$_{max}$ Corresponding to Maximum Blading Efficiency

Substituting $\rho_{opt} = \cos\alpha_1$ in equation (ii)

$$W = V_1^2\left[2\rho\times\rho - \rho^2\right] = \rho^2 V_1^2 = \frac{u^2}{V_1^2}\times V_1^2$$

$$\Rightarrow \qquad \boxed{W_{\max} = u^2}$$

where, u is the mean blade velocity, but in given question mean blade velocity is denoted by V_b

$$W_{\max} = V_b^2$$

MADE EASY Source

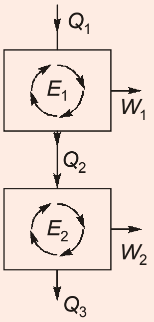
- **Theory Book (2019 Edition):** Power Plant (Page 91)

End of Solution

Q.5 (c) Derive an expression for efficiency of a combined cycle where two thermodynamic cycles are coupled in series. The expression should be derived in terms of efficiencies of the coupled cycles. Conventional notations may be used.

[12 Marks]

Solution:



then,
$$\eta_1 = 1 - \frac{Q_2}{Q_1} \text{ and } \eta_2 = 1 - \frac{Q_3}{Q_2}$$

$$Q_1 = \frac{Q_2}{1-\eta_1} \qquad Q_3 = (1-\eta_2)Q_2$$

For combined cycle,

$$\eta = 1 - \frac{Q_3}{Q_1} = 1 - \frac{(1-\eta_2)Q_2}{\frac{Q_2}{(1-\eta_1)}}$$

$$\eta = 1 - (1-\eta_1)(1-\eta_2) = \eta_1 + \eta_2 - \eta_1\eta_2$$

MADE EASY Source

- **Theory Book (2019 Edition):** Power Plant (Page 64)

End of Solution

Q.5 (d) Explain with neat sketch how solar absorption refrigeration system works for space cooling.

[12 Marks]

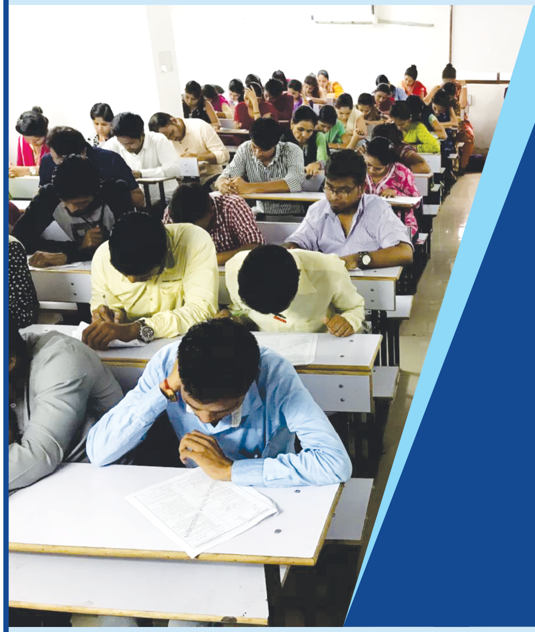
Solution:

A simple vapour absorption system, consists of a condenser, an expansion device and an evaporator as in the vapour compression system and in addition, an absorber, a pump, a generator and a pressure reducing valve to replace the compressor. The schematic representation of the system is shown in figure in which various components of the system are arranged according to their pressures and temperatures. The refrigerating effect is shown as Q_0 at temperature T_0 and the heat rejected in the condenser as Q_C at temperature $T_C = T_K$ of the environment. The compressor work is replaced by the heat supplied in the generator Q_h plus pump work W_P. Cooling must be done in the absorber to remove the latent heat of the refrigerant vapour as it changes into the liquid state by absorption by the weak solution. Heat rejected in the absorber be Q_A at absorber temperature $T_A = T_K$. Then the energy balance of the system,

$$Q_0 + W_P + Q_h = Q_C + Q_A$$

MADE EASY

India's Best Institute for IES, GATE & PSUs



ESE 2020 PRELIMS

CLASSROOM TEST SERIES

Stream : CE, ME, EE, E&T

- **Paper-I : 13 Tests**
  GS & Engineering Aptitude
- **Paper-II : 13 Tests**
  Engineering Discipline

Total
26 Tests

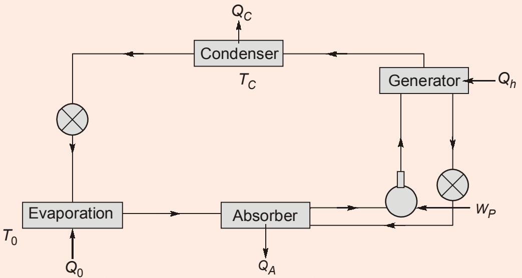
Fee Structure		
Current Session Students (ESE 2020) **Nil**	Ex. MADE EASY Students **Rs. 1,000 + GST**	Non MADE EASY Students **Rs. 1,400 + GST**

Commencing from 29th September, 2019

Admission Open from 10th July, 2019

Register online at : www.madeeasy.in

The pump work $W_P = -\int vdP$ is very small compared to compressor work in the vapour compression system, as the specific volume v of the liquid is extremely small compared to that of the vapour ($v_f << v_g$). The energy consumption of the system is mainly in the generator in the form of heat supplied Q_h.



Schematic representation of simple vapour absorption system

In the vapour-absorption system, the function of the compressor is accomplished in a three step process by the use of the absorber, pump and generator. Functions of these devices are given below:

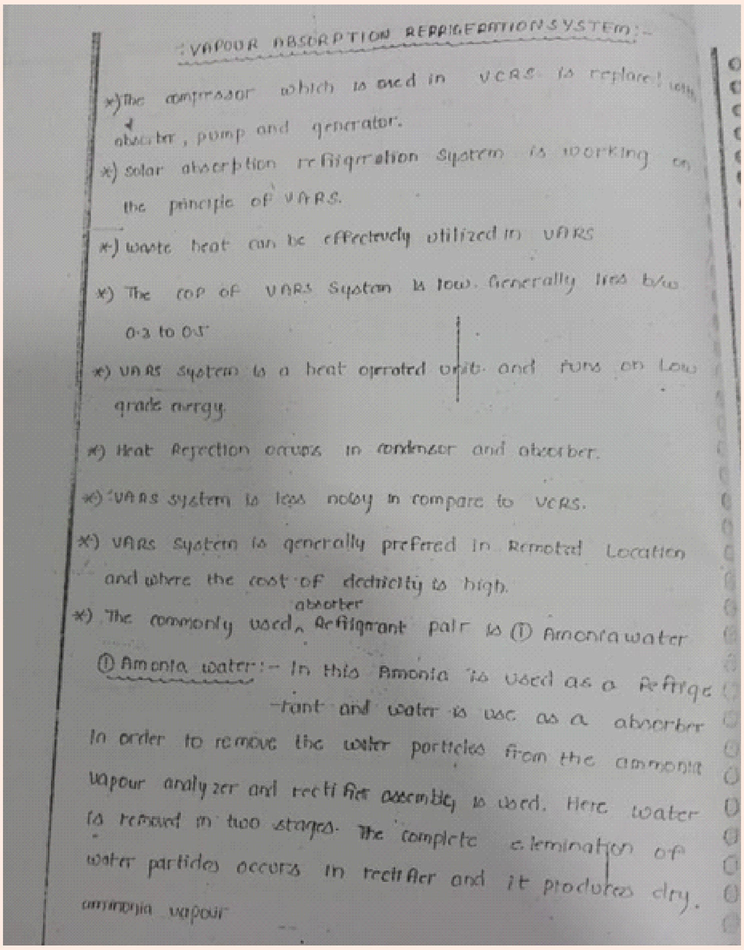
(i) **Absorber:** Absorption of the refrigerant vapour by its weak or poor solution in a suitable absorbent or adsorbent, forming strong or rich solution of refrigerant in absorbent/adsorbent.

(ii) **Pump:** Pumping of the rich solution raising its pressure to the condenser pressure.

(iii) **Generator:** Distillation of the vapour from the rich solution leaving the poor solution for recycling.

MADE EASY Source

- **ESE 2019 Mains Test Series:** Exactly same as Q.6c(i) Test- 1
- **MADE EASY Classnotes**



:VAPOUR ABSORPTION REFRIGERATION SYSTEM:-

*) The compressor which is used in VCRS is replaced with absorber, pump and generator.

*) solar absorption refrigeration system is working on the principle of VARS.

*) waste heat can be effectively utilized in VARS

*) The COP of VARS System is low. Generally lies b/w 0.2 to 0.5

*) VARS system is a heat operated unit and runs on Low grade energy.

*) Heat Rejection occurs in condensor and absorber.

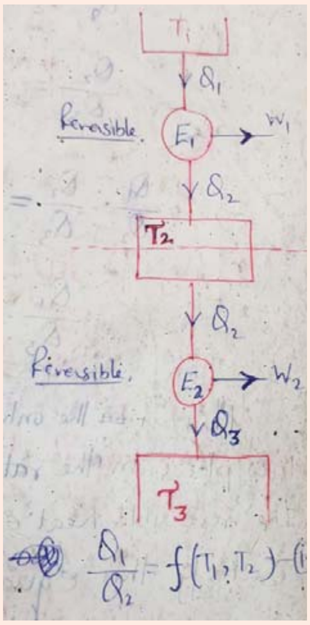
*) VARS system is less noisy in compare to VCRS.

*) VARS system is generally prefered in Remoted Location and where the cost of electricity is high.

*) The commonly used absorber Refrigerant pair is ① Amonia water

① Amonia water:- In this Amonia is used as a Refrigerant and water is use as a absorber

In order to remove the water particles from the ammonia vapour analyzer and rectifier assembly is used. Here water is remared in two stages. The complete elemination of water particles occurs in rectifier and it produces dry ammonia vapour



End of Solution

Q.5 (e) How do fuel cells work? Explain the principle with the help of a sketch.

[12 Marks]

Solution:

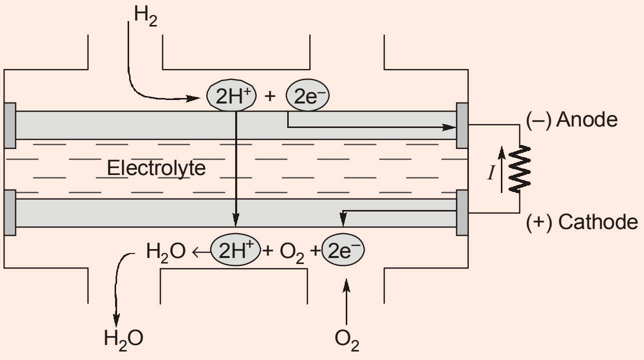
Fuel Cell:

A fuel cell is an electrochemical device which converts chemical energy of the fuel into electricity without undergoing combustion cycles. Unlike conventional combustion route i.e. fuel → heat → work → electricity, fuel cell directly convert fuel to electricity. Hence, efficiency of fuel cells is not limited by Carnot cycle or Second Law of Thermodynamics. Theoretically, a fuel cell may be 100% efficient.

Although fuel cell has two electrodes separated by an electrolyte similar to batteries yet they are different than batteries. In fuel cell, continuous supply of fuel is required to produce electricity and there is no as such charging and discharging like in the case of batteries.

Fuel cells mainly consist of four parts viz:

1. Fuel
2. Electrode
3. Electrolyte
4. Oxidant



($H_2 - O_2$) Fuel cell

Working Principle

The principle of operation of a fuel cell is similar to electrolysis but in reverse. A schematic diagram of a fuel cell working on hydrogen-oxygen fuel is shown in figure below.

In a fuel cell, fuel (hydrogen in this case) is supplied to the negative electrode (anode) and oxygen or air is supplied to the positive electrode.

A catalyst on the porous anode causes hydrogen molecules to dissociate into hydrogen ions and electrons. The H^+ ion migrates through the electrolyte, usually an acid to the cathode. At cathode H^+ ion reacts with electrons supplied by an external circuit and with oxygen to form water.

Reaction at anode: $H_2 \rightarrow 2H^+ + 2e^-$

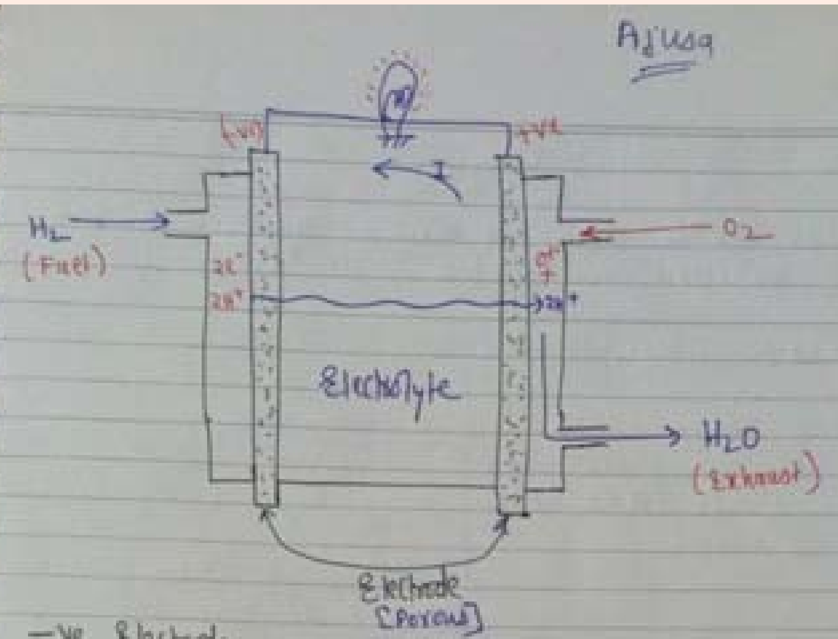
Reaction at cathode: $(2H^+) + (2e^-) + \frac{1}{2}O_2 \rightarrow H_2O$

As a consequence electrical current flow from cathode to anode i.e. in the opposite direction of flow of electrons.

Fuel cells produce typically 0.7 to 0.8 volts. Fuel cells are connected in series to get useful working voltage.

MADE EASY Source

- **From MADE EASY Class Notes**



-ve Electrode

$$H_2 \longrightarrow 2H^+ + 2e^-$$

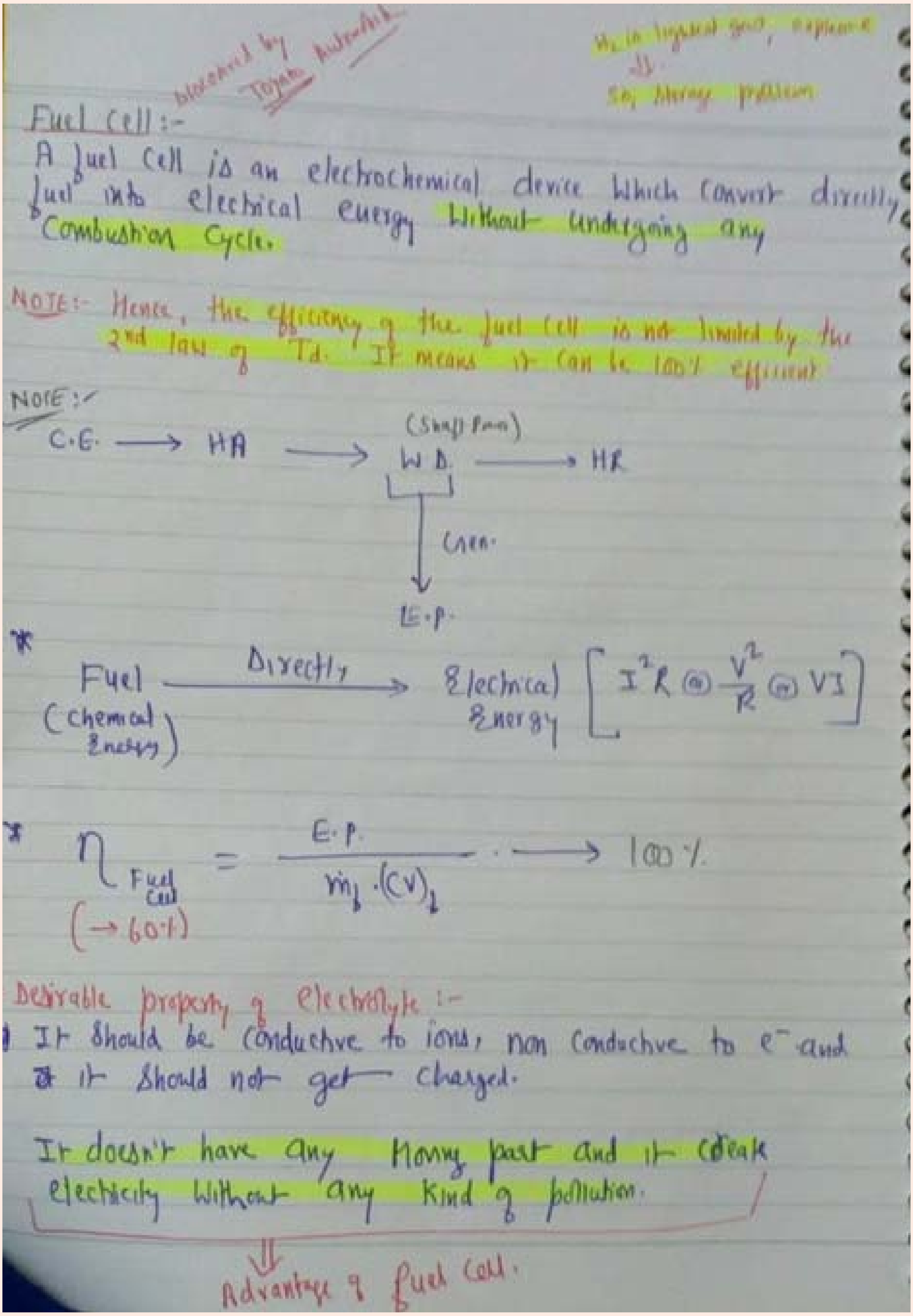
+ve Electrode

$$\frac{1}{2}O_2 + 2e^- \longrightarrow O^{2-}$$

$$2H^+ + O^{2-} \longrightarrow H_2O$$

Working principle:-

In fuel cell, generally the fuel supplied is Hydrogen to the -ve electrode, which dissociates the Hydrogen molecule into H^+ ion and electron (e^-) due to the Catalytic action of porous ~~electron~~ electrode; only the H^+ ion can migrate through electrolyte. It means e^- travelled by the external circuit to reach the +ve electrode.



blocked by Toyota Automobile

H_2 in liquid form, explosive
⇓
So, storage problem

Fuel Cell :-

A fuel cell is an electrochemical device which convert directly fuel into electrical energy without undergoing any combustion cycle.

NOTE :- Hence, the efficiency of the fuel cell is not limited by the 2nd law of TD. It means it can be 100% efficient.

NOTE :-

$$\text{C.E.} \longrightarrow \text{HA} \longrightarrow \overset{\text{(Shaft Power)}}{\text{W.D.}} \longrightarrow \text{HR}$$

$$\text{W.D.} \xrightarrow{\text{Gen.}} \text{E.P.}$$

* $$\underset{\text{(Chemical Energy)}}{\text{Fuel}} \xrightarrow{\text{Directly}} \text{Electrical Energy} \left[I^2R \text{ or } \frac{V^2}{R} \text{ or } VI \right]$$

* $$\eta_{\text{Fuel Cell}} = \frac{\text{E.P.}}{\dot{m}_f \cdot (CV)_f} \longrightarrow 100\%$$

$(\rightarrow 60\%)$

Desirable property of electrolyte :-

1 It should be conductive to ions, non conductive to e^- and
2 it should not get charged.

It doesn't have any moving part and it create electricity without any kind of pollution.
⇓
Advantage of fuel cell.

Q.6 (a) A centrifugal pump has an impeller diameter at outlet as 1 m and delivers 1.5 m^3/s of water against a head of 100 m. The impeller is running at 1000 rpm. The width of the impeller is 85 mm. If the manometric efficiency is 85%, determine the type of impeller (forward, radial or backward curved), and the blade angle at outlet. Draw velocity triangle at outlet.

[20 Marks]

Solution:

Impeller diameter at outlet, $D_2 = 1$ m

$N = 1000$ rpm

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1 \times 1000}{60} = 52.36 \text{ m/s}$$

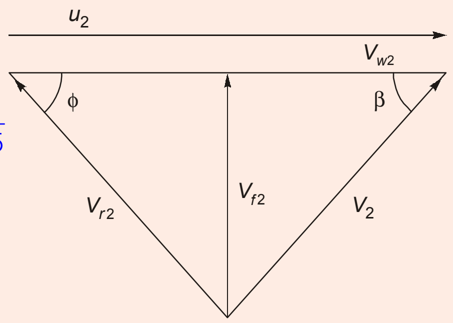
Manometric head, $H_m = 100$ m

$$\text{Manometric efficiency, } \eta_m = \frac{gH_m}{V_{w2}u_2}$$

$$0.85 = \frac{9.81 \times 100}{V_{w2} \times 52.36}$$

$$V_{w2} = \frac{9.81 \times 100}{52.36 \times 0.85}$$

$$V_{w2} = 22.04 \text{ m/s}$$



Velocity Triangle at outlet

Since, $u_2 > V_{w2}$,
the type of impeller is backward curved.

Discharge, $Q = 1.5$ m^3/s

Width of impeller, $B_2 = 85$ mm

$$Q = \pi D_2 B_2 V_{f2}$$

$$1.5 = \pi \times 1 \times 0.085 \times V_{f2}$$

Flow component of velocity at outlet

$$V_{f2} = 5.6172 \text{ m/s}$$

From Velocity triangle at outlet,

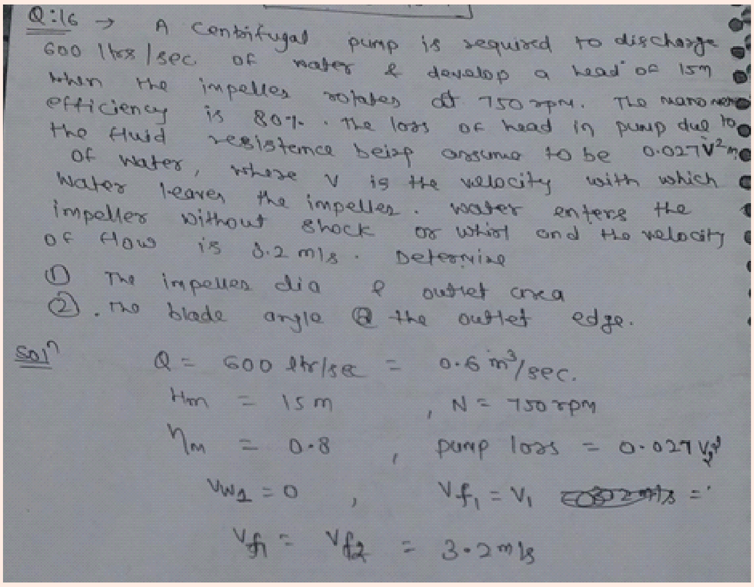
$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\tan\phi = \frac{5.6172}{52.36 - 22.04}$$

$$\phi = 10.496°$$

MADE EASY Source

- **From MADE EASY Mains Batch Class Notes**
- **ESE 2019 Mains Workbook:** Similar to Q.59 (Page 271)



Q:16 → A Centrifugal pump is required to discharge 600 ltrs/sec of water & develop a head of 15m when the impeller rotates at 750 rpm. The manometric efficiency is 80%. The loss of head in pump due to the fluid resistence being assume to be $0.027V^2$ m of water, where V is the velocity with which water leaves the impeller. water enters the impeller without shock or whirl and the velocity of flow is 3.2 m/s. Determine

① The impeller dia & outlet area

②. The blade angle @ the outlet edge.

Soln

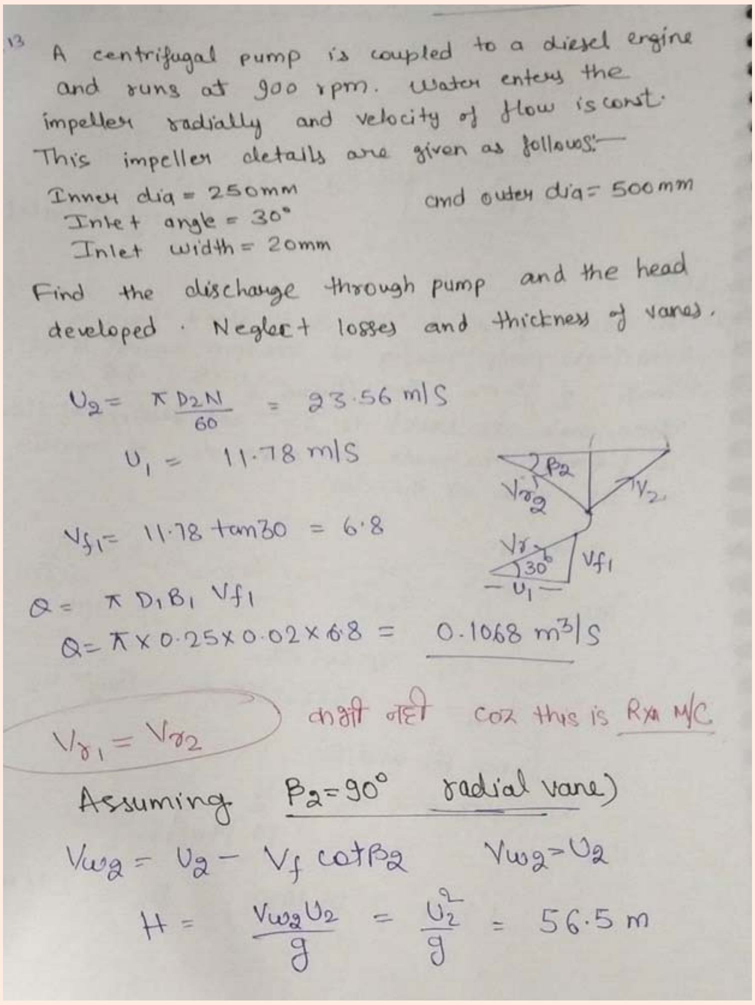
$Q = 600$ ltr/sec $= 0.6\ m^3/sec.$

$H_m = 15\,m$, $N = 750$ rpm

$\eta_m = 0.8$, pump loss $= 0.027V_2^2$

$V_{w_1} = 0$, $V_{f_1} = V_1$ ~~3.2 m/s~~ ='

$V_{f_1} = V_{f_2} = 3.2\ m/s$



13 A centrifugal pump is coupled to a diesel engine and runs at 900 rpm. Water enters the impeller radially and velocity of flow is const. This impeller details are given as follows:—

Inner dia = 250mm and outer dia = 500mm
Inlet angle = 30°
Inlet width = 20mm

Find the discharge through pump and the head developed. Neglect losses and thickness of vanes.

$U_2 = \pi \frac{D_2 N}{60} = 23.56\ m/s$

$U_1 = 11.78\ m/s$

$V_{f_1} = 11.78 \tan 30 = 6.8$

$Q = \pi D_1 B_1 V_{f1}$

$Q = \pi \times 0.25 \times 0.02 \times 6.8 = \underline{0.1068\ m^3/s}$

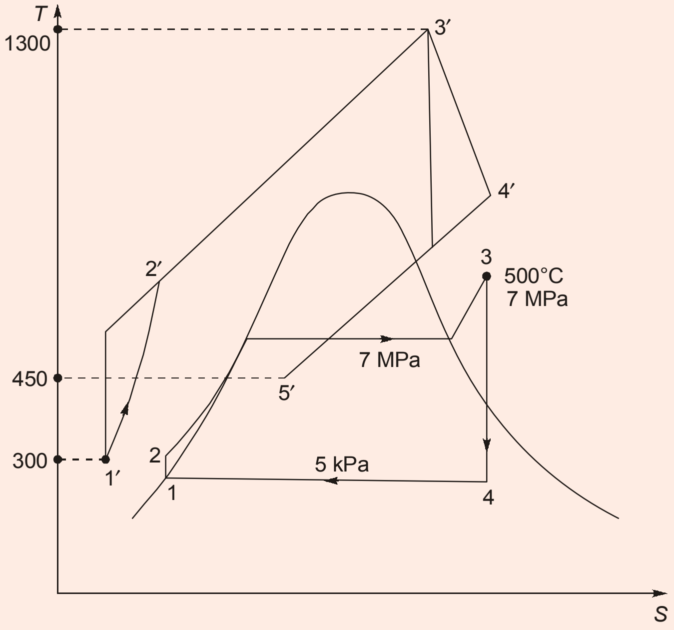
$V_{r_1} = V_{r_2}$ कभी नहीं coz this is Rxn M/C

Assuming $\underline{\beta_2 = 90°}$ radial vane)

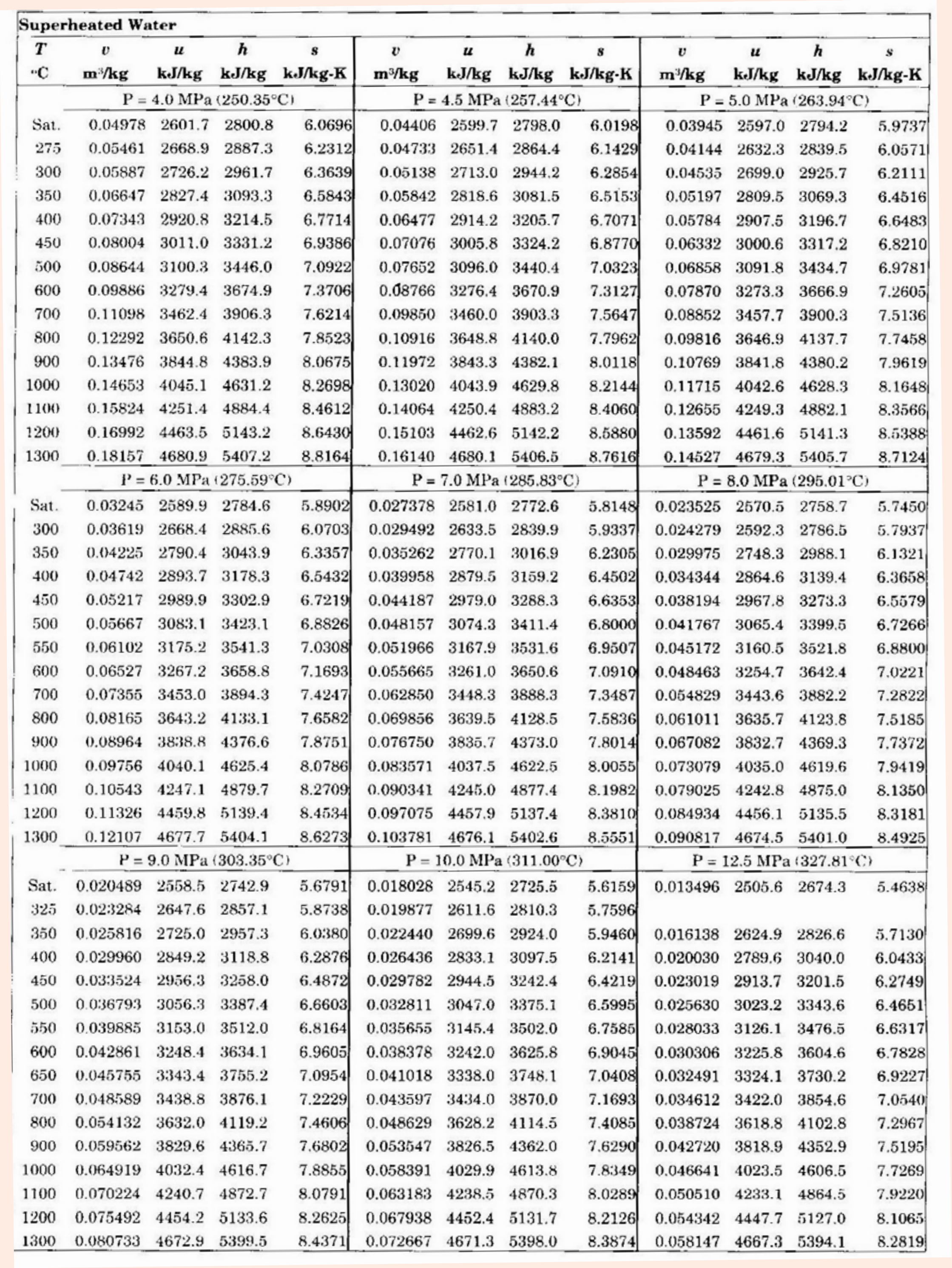
$V_{w_2} = U_2 - V_f \cot\beta_2$ $V_{w_2} = U_2$

$H = \frac{V_{w_2} U_2}{g} = \frac{U_2^2}{g} = 56.5\ m$

Q.6 (b) Consider the combined gas steam power cycle shown in the figure. The topping cycle is a gas turbine cycle that has a pressure ratio of 8. Air enters the compressor at 300 K and the turbine at 1300 K. The isentropic efficiency of the compressor is 80 percent, and that of the gas turbine is 85 percent. The bottoming cycle is a sample ideal Rankine cycle operating between the pressure limits of 7 MPa and 5 kPa. Steam is heated in a heat exchanger by the exhaust gases to a temperature of 500°C. The exhaust gases leave the heat exchanger at 450 K. Determine (i) The ratio of the mass flow rates of the steam and the combustion gases. (ii) The thermal efficiency of the combined cycle.

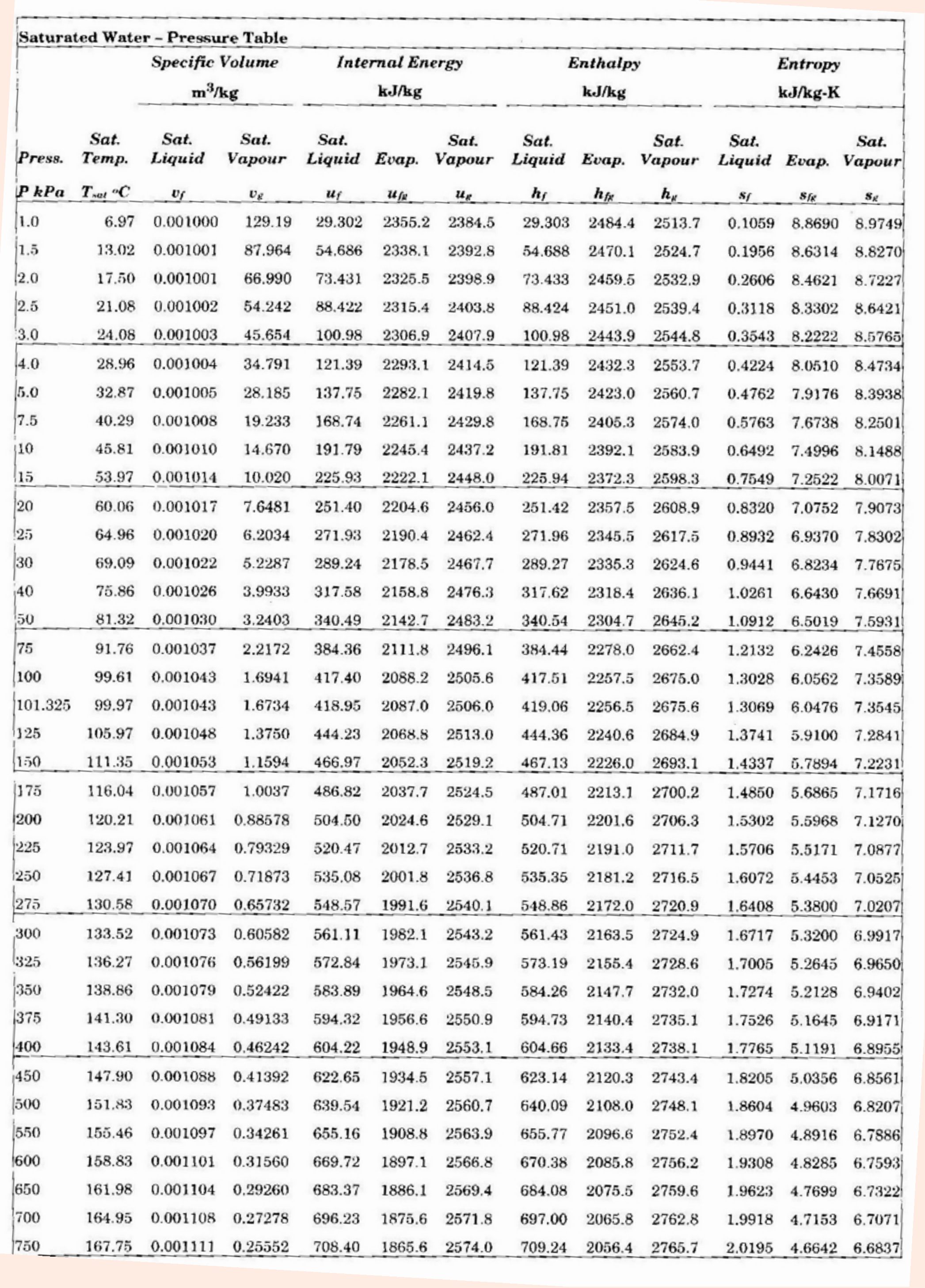


Assume specific heat of gas as 1.005 kJ/kgK



Superheated Water

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
°C	m³/kg	kJ/kg	kJ/kg	kJ/kg-K	m³/kg	kJ/kg	kJ/kg	kJ/kg-K	m³/kg	kJ/kg	kJ/kg	kJ/kg-K
	P = 4.0 MPa (250.35°C)				P = 4.5 MPa (257.44°C)				P = 5.0 MPa (263.94°C)			
Sat.	0.04978	2601.7	2800.8	6.0696	0.04406	2599.7	2798.0	6.0198	0.03945	2597.0	2794.2	5.9737
275	0.05461	2668.9	2887.3	6.2312	0.04733	2651.4	2864.4	6.1429	0.04144	2632.3	2839.5	6.0571
300	0.05887	2726.2	2961.7	6.3639	0.05138	2713.0	2944.2	6.2854	0.04535	2699.0	2925.7	6.2111
350	0.06647	2827.4	3093.3	6.5843	0.05842	2818.6	3081.5	6.5153	0.05197	2809.5	3069.3	6.4516
400	0.07343	2920.8	3214.5	6.7714	0.06477	2914.2	3205.7	6.7071	0.05784	2907.5	3196.7	6.6483
450	0.08004	3011.0	3331.2	6.9386	0.07076	3005.8	3324.2	6.8770	0.06332	3000.6	3317.2	6.8210
500	0.08644	3100.3	3446.0	7.0922	0.07652	3096.0	3440.4	7.0323	0.06858	3091.8	3434.7	6.9781
600	0.09886	3279.4	3674.9	7.3706	0.08766	3276.4	3670.9	7.3127	0.07870	3273.3	3666.9	7.2605
700	0.11098	3462.4	3906.3	7.6214	0.09850	3460.0	3903.3	7.5647	0.08852	3457.7	3900.3	7.5136
800	0.12292	3650.6	4142.3	7.8523	0.10916	3648.8	4140.0	7.7962	0.09816	3646.9	4137.7	7.7458
900	0.13476	3844.8	4383.9	8.0675	0.11972	3843.3	4382.1	8.0118	0.10769	3841.8	4380.2	7.9619
1000	0.14653	4045.1	4631.2	8.2698	0.13020	4043.9	4629.8	8.2144	0.11715	4042.6	4628.3	8.1648
1100	0.15824	4251.4	4884.4	8.4612	0.14064	4250.4	4883.2	8.4060	0.12655	4249.3	4882.1	8.3566
1200	0.16992	4463.5	5143.2	8.6430	0.15103	4462.6	5142.2	8.5880	0.13592	4461.6	5141.3	8.5388
1300	0.18157	4680.9	5407.2	8.8164	0.16140	4680.1	5406.5	8.7616	0.14527	4679.3	5405.7	8.7124
	P = 6.0 MPa (275.59°C)				P = 7.0 MPa (285.83°C)				P = 8.0 MPa (295.01°C)			
Sat.	0.03245	2589.9	2784.6	5.8902	0.027378	2581.0	2772.6	5.8148	0.023525	2570.5	2758.7	5.7450
300	0.03619	2668.4	2885.6	6.0703	0.029492	2633.5	2839.9	5.9337	0.024279	2592.3	2786.5	5.7937
350	0.04225	2790.4	3043.9	6.3357	0.035262	2770.1	3016.9	6.2305	0.029975	2748.3	2988.1	6.1321
400	0.04742	2893.7	3178.3	6.5432	0.039958	2879.5	3159.2	6.4502	0.034344	2864.6	3139.4	6.3658
450	0.05217	2989.9	3302.9	6.7219	0.044187	2979.0	3288.3	6.6353	0.038194	2967.8	3273.3	6.5579
500	0.05667	3083.1	3423.1	6.8826	0.048157	3074.3	3411.4	6.8000	0.041767	3065.4	3399.5	6.7266
550	0.06102	3175.2	3541.3	7.0308	0.051966	3167.9	3531.6	6.9507	0.045172	3160.5	3521.8	6.8800
600	0.06527	3267.2	3658.8	7.1693	0.055665	3261.0	3650.6	7.0910	0.048463	3254.7	3642.4	7.0221
700	0.07355	3453.0	3894.3	7.4247	0.062850	3448.3	3888.3	7.3487	0.054829	3443.6	3882.2	7.2822
800	0.08165	3643.2	4133.1	7.6582	0.069856	3639.5	4128.5	7.5836	0.061011	3635.7	4123.8	7.5185
900	0.08964	3838.8	4376.6	7.8751	0.076750	3835.7	4373.0	7.8014	0.067082	3832.7	4369.3	7.7372
1000	0.09756	4040.1	4625.4	8.0786	0.083571	4037.5	4622.5	8.0055	0.073079	4035.0	4619.6	7.9419
1100	0.10543	4247.1	4879.7	8.2709	0.090341	4245.0	4877.4	8.1982	0.079025	4242.8	4875.0	8.1350
1200	0.11326	4459.8	5139.4	8.4534	0.097075	4457.9	5137.4	8.3810	0.084934	4456.1	5135.5	8.3181
1300	0.12107	4677.7	5404.1	8.6273	0.103781	4676.1	5402.6	8.5551	0.090817	4674.5	5401.0	8.4925
	P = 9.0 MPa (303.35°C)				P = 10.0 MPa (311.00°C)				P = 12.5 MPa (327.81°C)			
Sat.	0.020489	2558.5	2742.9	5.6791	0.018028	2545.2	2725.5	5.6159	0.013496	2505.6	2674.3	5.4638
325	0.023284	2647.6	2857.1	5.8738	0.019877	2611.6	2810.3	5.7596				
350	0.025816	2725.0	2957.3	6.0380	0.022440	2699.6	2924.0	5.9460	0.016138	2624.9	2826.6	5.7130
400	0.029960	2849.2	3118.8	6.2876	0.026436	2833.1	3097.5	6.2141	0.020030	2789.6	3040.0	6.0433
450	0.033524	2956.3	3258.0	6.4872	0.029782	2944.5	3242.4	6.4219	0.023019	2913.7	3201.5	6.2749
500	0.036793	3056.3	3387.4	6.6603	0.032811	3047.0	3375.1	6.5995	0.025630	3023.2	3343.6	6.4651
550	0.039885	3153.0	3512.0	6.8164	0.035655	3145.4	3502.0	6.7585	0.028033	3126.1	3476.5	6.6317
600	0.042861	3248.4	3634.1	6.9605	0.038378	3242.0	3625.8	6.9045	0.030306	3225.8	3604.6	6.7828
650	0.045755	3343.4	3755.2	7.0954	0.041018	3338.0	3748.1	7.0408	0.032491	3324.1	3730.2	6.9227
700	0.048589	3438.8	3876.1	7.2229	0.043597	3434.0	3870.0	7.1693	0.034612	3422.0	3854.6	7.0540
800	0.054132	3632.0	4119.2	7.4606	0.048629	3628.2	4114.5	7.4085	0.038724	3618.8	4102.8	7.2967
900	0.059562	3829.6	4365.7	7.6802	0.053547	3826.5	4362.0	7.6290	0.042720	3818.9	4352.9	7.5195
1000	0.064919	4032.4	4616.7	7.8855	0.058391	4029.9	4613.8	7.8349	0.046641	4023.5	4606.5	7.7269
1100	0.070224	4240.7	4872.7	8.0791	0.063183	4238.5	4870.3	8.0289	0.050510	4233.1	4864.5	7.9220
1200	0.075492	4454.2	5133.6	8.2625	0.067938	4452.4	5131.7	8.2126	0.054342	4447.7	5127.0	8.1065
1300	0.080733	4672.9	5399.5	8.4371	0.072667	4671.3	5398.0	8.3874	0.058147	4667.3	5394.1	8.2819

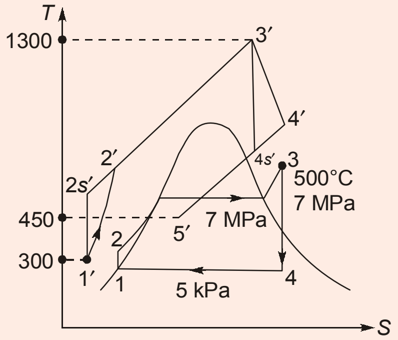


Saturated Water – Pressure Table

		Specific Volume m³/kg		Internal Energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/kg-K		
Press.	Sat. Temp.	Sat. Liquid	Sat. Vapour	Sat. Liquid	Evap.	Sat. Vapour	Sat. Liquid	Evap.	Sat. Vapour	Sat. Liquid	Evap.	Sat. Vapour
P kPa	T_{sat} °C	v_f	v_g	u_f	u_{fg}	u_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
175	116.04	0.001057	1.0037	486.82	2037.7	2524.5	487.01	2213.1	2700.2	1.4850	5.6865	7.1716
200	120.21	0.001061	0.88578	504.50	2024.6	2529.1	504.71	2201.6	2706.3	1.5302	5.5968	7.1270
225	123.97	0.001064	0.79329	520.47	2012.7	2533.2	520.71	2191.0	2711.7	1.5706	5.5171	7.0877
250	127.41	0.001067	0.71873	535.08	2001.8	2536.8	535.35	2181.2	2716.5	1.6072	5.4453	7.0525
275	130.58	0.001070	0.65732	548.57	1991.6	2540.1	548.86	2172.0	2720.9	1.6408	5.3800	7.0207
300	133.52	0.001073	0.60582	561.11	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3200	6.9917
325	136.27	0.001076	0.56199	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2645	6.9650
350	138.86	0.001079	0.52422	583.89	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2128	6.9402
375	141.30	0.001081	0.49133	594.32	1956.6	2550.9	594.73	2140.4	2735.1	1.7526	5.1645	6.9171
400	143.61	0.001084	0.46242	604.22	1948.9	2553.1	604.66	2133.4	2738.1	1.7765	5.1191	6.8955
450	147.90	0.001088	0.41392	622.65	1934.5	2557.1	623.14	2120.3	2743.4	1.8205	5.0356	6.8561
500	151.83	0.001093	0.37483	639.54	1921.2	2560.7	640.09	2108.0	2748.1	1.8604	4.9603	6.8207
550	155.46	0.001097	0.34261	655.16	1908.8	2563.9	655.77	2096.6	2752.4	1.8970	4.8916	6.7886
600	158.83	0.001101	0.31560	669.72	1897.1	2566.8	670.38	2085.8	2756.2	1.9308	4.8285	6.7593
650	161.98	0.001104	0.29260	683.37	1886.1	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322
700	164.95	0.001108	0.27278	696.23	1875.6	2571.8	697.00	2065.8	2762.8	1.9918	4.7153	6.7071
750	167.75	0.001111	0.25552	708.40	1865.6	2574.0	709.24	2056.4	2765.7	2.0195	4.6642	6.6837

[20 Marks]

Solution:



Topping Cycle

Pressure ratio, $r_p = 8$, $T_1' = 300$ K

$$\frac{T_{2s'}}{T_1'} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_{2s'}}{300} = (8)^{\frac{1.4-1}{1.4}}$$

$$T_{2's} = 543.434 \text{ K}$$

$$T_{2'} = T_{1'} + \frac{T_{2s'} - T_1}{\eta_C} = 300 + \frac{543.434 - 300}{0.8} = 604.293 \text{ K}$$

$$T_{3'} = 1300 \text{ K}$$

$$\frac{T_{4s'}}{T_{3'}} = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{4s'} = 717.658 \text{ K}$$

$$T_{4'} = T_{3'} - \eta_T(T_{3'} - T_{4s'}) = 1300 - 0.85(1300 - 717.658)$$

$$T_{4'} = 805.009 \text{ K}$$

Bottoming cycle

From steam table,

At 500°C, 7 MPa (superheated)

$$h_3 = 3411.4 \text{ kJ/kg}$$

$$s_3 = 6.8 \text{ kJ/kgK}$$

At 5 kPa (saturated water)

$$v_f = 0.001005 \text{ m}^3\text{/kg}$$

$$h_1 = h_f = 137.75 \text{ kJ/kg}$$

$$s_1 = s_f = 0.4762 \text{ kJ/kgK}$$

$$h_{fg} = 2423 \text{ kJ/kg}$$

$$s_{fg} = 7.9176 \text{ kJ/kgK}$$

Since process 3 - 4 is isentropic,

$$s_3 = s_4$$

$\Rightarrow$ $$6.8 = s_f + x_4 s_{fg} = 0.4762 + x_4 \times 7.9176$$

$$x_4 = 0.7987$$

$$h_4 = h_f + x_4 h_{fg}$$

$$= 137.75 + 0.7987 \times 2423 = 2073 \text{ kJ/kg}$$

Pump work, $$w_p = v_f(p_2 - p_1)$$

$$= 0.001005 \times (7000 - 5) = 7.0299 \text{ kJ/kg}$$

Also, $$w_p = h_2 - h_1$$

$$7.0299 = h_2 - 137.75$$

$$h_2 = 144.78 \text{ kJ/kg}$$

In heat exchanger,

$$\text{Heat rejected by gas} = \text{heat taken by steam}$$

$$\Rightarrow \quad \dot{m}_g C_{pg}(T_{4'} - T_{5'}) = \dot{m}_s(h_3 - h_2)$$

$$\Rightarrow \quad \dot{m}_g \times 1.005(805.009 - 450) = \dot{m}_s(3411.4 - 144.78)$$

Ratio of mass flow rate of steam to that of gas,

$$\frac{\dot{m}_s}{\dot{m}_g} = 0.1092$$

Net work in topping cycle,

$$W_T = \dot{m}_g C_{pg}\left[(T_{3'} - T_{4'}) - (T_{2'} - T_{1'})\right]$$

$$= \dot{m}_g \times 1.005\left[(1300 - 805.009) - (604.293 - 300)\right]$$

$$W_T = 191.65\,\dot{m}_g \text{ kW}$$

Net work in bottoming cycle,

$$W_B = \dot{m}_s\left[(h_3 - h_4) - w_p\right]$$

$$= \dot{m}_s\left[(3411.4 - 2073) - 7.0299\right]$$

$$W_B = 1331.37\,\dot{m}_s \text{ kW}$$

Total heat supplied,

$$Q_s = \dot{m}_g C_{pg}\left[T_{3'} - T_{2'}\right]$$

$$= \dot{m}_g \times 1.005 \times \left[1300 - 604.293\right]$$

$$Q_s = 699.186\,\dot{m}_g \text{ kW}$$

Thermal efficiency of combined cycle,

$$\eta = \frac{\text{Total work}}{\text{Total heat supplied}} = \frac{W_T + W_B}{Q_s}$$

$$= \frac{191.65\,\dot{m}_g + 1331.37\,\dot{m}_s}{699.186\,\dot{m}_g}$$

$$= \frac{191.65}{699.186} + \frac{1331.37}{699.186} \times 0.1092 \qquad \left[\because \frac{\dot{m}_s}{\dot{m}_g} = 0.1092\right]$$

$$\eta = 0.4820 = 48.20\%$$

MADE EASY Source

- **ESE 2019 Mains Workbook:** Power Plant Q. 22 (Page. 157) discussed in Class

End of Solution

Q.6 (c) What is Betz limit for wind turbines? Derive an expression for Betz limit for wind turbines.

[20 Marks]

Solution:

Betz limit: Theoretically, the maximum power extracted by a turbine rotor is 59.3% of the total wind energy in the area swept by the rotor, this is known as Betz limit.

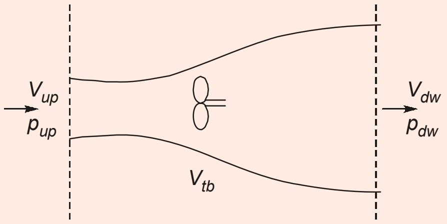
Power coefficient,
$$C_p = \frac{P_{\max}}{P_{\text{total}}} = 0.593$$

Maximum theoretical efficiency (also known as power coefficient C_p) is the ratio of maximum power output to total power available in the wind. Its maximum value is 0.593, which is known as Betz limit (after the name of the engineer who first derived this relationship.)

Let us assume, wind turbine is installed in the flow path of wind.

Let V_{up} and p_{up} be velocity and pressure upstream of the turbine and V_{dw} and p_{dw} be velocity and pressure downstream of the turbine. Also V_{tb} be wind velocity at turbine.

Bernoulli's equation can be applied between upstream side and downstream side



$$(p_{up} - p_{dw}) = \frac{1}{2}\rho\left[V_{up}^2 - V_{dw}^2\right]$$

Force on the rotor
$$F = A_s\,[p_{up} - p_{dw}]$$

[Where A_s is area swept by rotor perpendicular to wind direction]

Also from linear momentum
$$F = \dot{m}\left[V_{up} - V_{dw}\right]$$

Combining these equations:

$$A_s \times \frac{1}{2}\rho[V_{up}^2 - V_{dw}^2] = (\rho A_s \times V_{tb})\,(V_{up} - V_{dw})$$

$$\Rightarrow \qquad V_{tb} = \frac{V_{up} + V_{dw}}{2}$$

Now power output of wind turbine is given by:

$$P_{\text{turbine}} = \frac{1}{2}\dot{m}V^2$$

$$= \frac{1}{2}\dot{m}[V_{up}^2 - V_{dw}^2] = \frac{1}{2}\rho A_s V_{tb}[V_{up}^2 - V_{dw}^2]$$

$$= \frac{1}{2}\rho A_s\left[\frac{V_{up} + V_{dw}}{2}\right][V_{up}^2 - V_{dw}^2]$$

Maximum power is obtained if $\dfrac{dP_{\text{turbine}}}{dV_{dw}} = 0$

$$\frac{dP_{\text{turbine}}}{dV_{dw}} = \frac{1}{4}\rho A_s[V_{up}^2 - 3V_{dw}^2 - 2V_{up}V_{dw}] = 0$$

$$\Rightarrow \qquad 3V_{dw}^2 + 2V_{dw}V_{up} - V_{up}^2 = 0$$

$$(3V_{dw} - V_{up})(V_{dw} + V_{up}) = 0$$

$$\Rightarrow \qquad V_{dw} = \frac{V_{up}}{3} \text{ and } V_{dw} = -V_{up}$$

In order to get power $V_{dw} < V_{up}$ and hence taking $V_{dw} = \dfrac{V_{up}}{3}$

$$(P_{\text{turbine}})_{\max} = \frac{1}{2}\rho A_s V_{tb}\left[V_{up}^2 - \frac{V_{up}^2}{9}\right]$$

$$= \frac{1}{2}\rho A_s \left(\frac{3V_{up} + V_{up}}{6}\right)\left(\frac{8V_{up}^2}{9}\right)$$

$$= \frac{1}{2}\rho A_s \left[\frac{16}{27}\right] V_{up}^3$$

$$(P_{\text{turbine}})_{\max} = 0.593\,\rho\left(\frac{1}{2}A_s V_{up}^3\right) \qquad \text{...(i)}$$

The available wind power is given by:

$$(P_{\text{wind}})_{\text{available}} = \frac{1}{2}\rho A_s V_{up}^3 \qquad \text{...(ii)}$$

From equation (ii) and (ii)

$$(P_{\text{turbine}})_{\max} = 0.593\,(P_{\text{wind}})_{\text{available}}$$

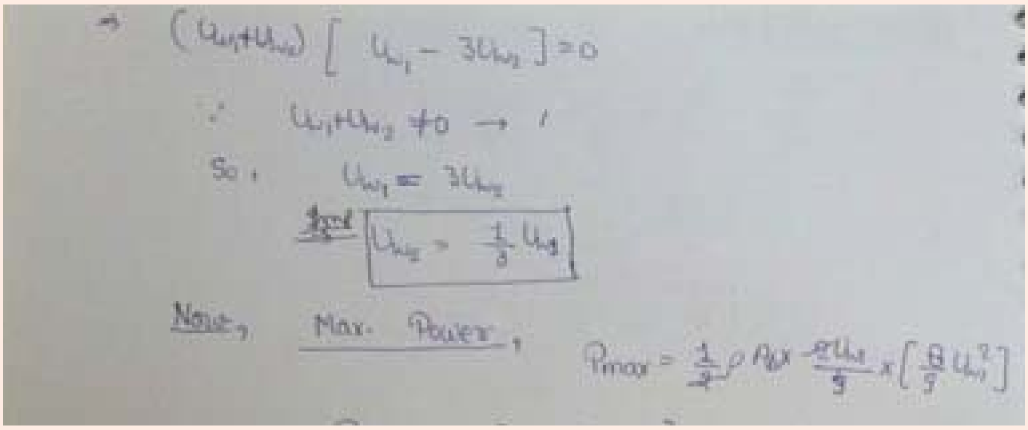
The factor '0.593' is known as coefficient of power (C_p) i.e.

$$= \frac{P_{\text{turbine}}}{P_{\text{wind}}}$$

The value of C_p i.e. 0.593 is known as Lanchester - Betz limit which indicates only 59.3% of available power can be converted by wind turbine at maximum.

MADE EASY Source

- **From MADE EASY Class Notes**
- **ESE 2019 Mains Workbook:** Unsolved Q.3 discussed in Class



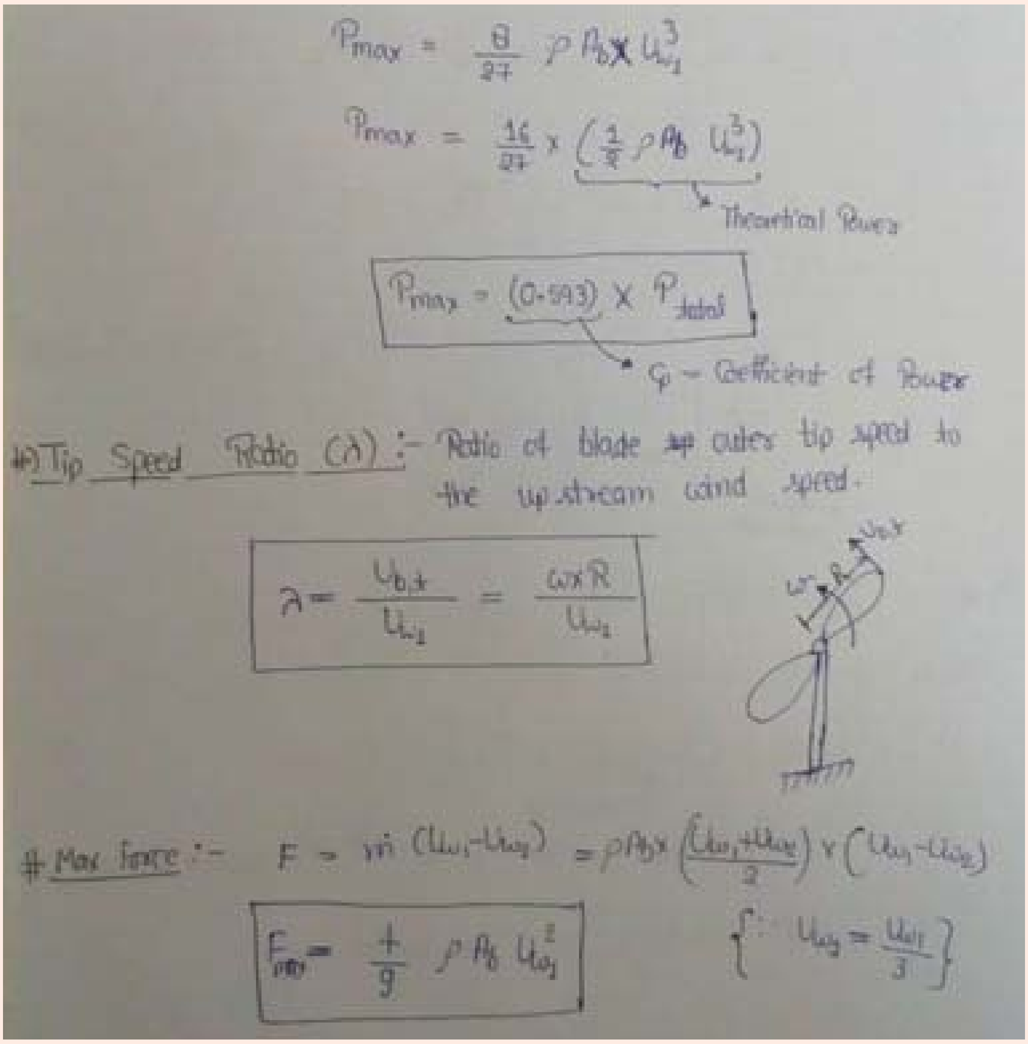
$\Rightarrow (u_{w_1}+u_{w_2})\left[u_{w_1} - 3u_{w_2}\right] = 0$

$\because\ u_{w_1}+u_{w_2} \neq 0 \rightarrow$

So, $u_{w_1} = 3u_{w_2}$

And $\boxed{u_{w_2} = \frac{1}{3}u_{w_1}}$

Now, Max. Power, $P_{max} = \frac{1}{2}\rho A_s \times \frac{2u_{w_1}}{3} \times \left[\frac{8}{9}u_{w_1}^2\right]$



$$P_{max} = \frac{8}{27}\rho A_b \times U_{w_1}^3$$

$$P_{max} = \frac{16}{27} \times \underbrace{\left(\frac{1}{2}\rho A_b U_{w_1}^3\right)}_{\text{Theoretical Power}}$$

$$\boxed{P_{max} = \underbrace{(0.593)}_{C_p\text{ - Coefficient of Power}} \times P_{total}}$$

Tip Speed Ratio (λ) :- Ratio of blade outer tip speed to the upstream wind speed.

$$\boxed{\lambda = \frac{U_{bt}}{U_{w_1}} = \frac{\omega \times R}{U_{w_1}}}$$

Max force :- $F = \dot{m}(U_{w_1} - U_{w_2}) = \rho A_b \times \left(\frac{U_{w_1}+U_{w_2}}{2}\right) \times \left(U_{w_1} - U_{w_2}\right)$

$$\boxed{F_{max} = \frac{4}{9}\rho A_b U_{w_1}^2} \qquad \left\{\because U_{w_2} = \frac{U_{w_1}}{3}\right\}$$

End of Solution

Q.7 (a) A Pelton turbine with a wheel diameter of 1.5 m, operating with four nozzles, produces 16 MW of power. The turbine is running at 400 rpm and operating under a gross head of 300 m. Water is supplied through penstock of length 2 km. The coefficient of friction in penstock is 0.004. There is 10% of head loss taking place in the penstock. If the velocity coefficient is 0.97, blade velocity coefficient is 0.9, overall efficiency is 0.84 and Pelton bucket deflects the jet by 165°, determine

(i) Discharge through the turbine (m^3/s)
(ii) Penstock diameter (m)
(iii) Jet diameter (m)
(iv) Hydraulic efficiency of the turbine

Draw velocity triangles.

[20 Marks]

MADE EASY

MADE EASY

India's Best Institute for IES, GATE & PSUs

GS & Engineering Aptitude Improvement Program

for ESE 2020 Preliminary Examination

100 Days Course

Classroom Course

Live/Online Course also available

6-7 days/week • 4 hours classes /day

Regular Classroom Course : Fee Structure

Subject	Hours	Fee for Non MADE EASY Students	Fee for Ex. MADE EASY Students
GS (8 Subjects)	200 Hrs.	₹20,000	₹15,000
GS + Maths + Reasoning & Aptitude (10 Subjects)	300 Hrs.	₹25,000	₹18,000

Live/Online Classes : Fee Structure

Subject	Hours	Fee for Non MADE EASY Students	Fee for Ex. MADE EASY Students
GS (8 Subjects)	200 Hrs.	₹16,000	₹12,000
GS + Maths + Reasoning & Aptitude (10 Subjects)	300 Hrs.	₹20,000	₹14,500

Batches commencing from **20th Aug, 2019**

Class timing **5.00 PM - 9:00 PM**

Venue

Hundred Million IGNOU main Road Saket,
Saidulajab Extension, New Delhi-110030

Ph: 9958995830 • www.madeeasy.in

Solution:

Gross head, $H_g = 300$ m

Net head, $H = H_g - h_f$

where h_f = head loss due to friction in penstock

$= 300 - 0.1 \times 300 = 300 - 30 = 270$ m

(i) As we know, $P = \eta_0 \times \rho g Q H$

$\Rightarrow$ $16 \times 10^6 = 0.84 \times 1000 \times 9.81 \times Q \times 270$

Discharge through turbine,

$Q = 7.1913$ m^3/s

(ii) Length of penstock, $L = 2000$ m

Coefficient of friction, $f' = 0.004$

Friction factor, $f = 4f' = 4 \times 0.004 = 0.016$

Now,
$$h_f = \frac{fLQ^2}{\frac{\pi^2 g}{8} D^5}$$

$$0.1 \times 300 = \frac{0.016 \times 2000 \times 7.1913^2}{\frac{\pi^2 \times 9.81}{8} \times D^5}$$

Penstock diameter, $D = 1.3544$ m

(iii) Velocity coefficient, $C_v = 0.97$

Velocity of jet at inlet, $V_1 = C_v\sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 270} = 70.599$ m/s

Now,
$$Q = n \times \left(\frac{\pi}{4} d^2 \times V_1\right)$$

$$7.1913 = 4 \times \left(\frac{\pi}{4} \times d^2 \times 70.599\right)$$

Jet diameter, $d = 0.18$ m

(iv) $N = 400$ rpm

Wheel diameter, $D = 1.5$ m

$$4 = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 400}{60} = 31.416 \text{ m/s}$$

$V_{r1} = V_1 - u = 70.599 - 31.416 = 39.183$ m/s

$V_{r2} = kV_{r1}$, where blade velocity coefficient, $k = 0.9$

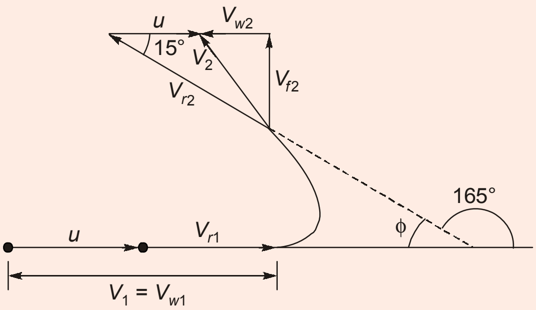
$= 0.9 \times 39.183 = 35.265$ m/s

Jet deflection angle, $\delta = 165°$

Vane angle at outlet, $\phi = 180° - \delta = 180 - 165 = 15°$

$V_{r2} \cos\phi = 35.265 \times \cos 15 = 34.063$ m/s

Since, $V_{r2} \cos\phi > u$, velocity diagram:



$$V_{w2} = V_{r2}\cos\phi - u$$

$$= 34.063 - 31.416 = 2.647 \text{ m/s}$$

$$V_{w1} = V_1 = 70.599 \text{ m/s}$$

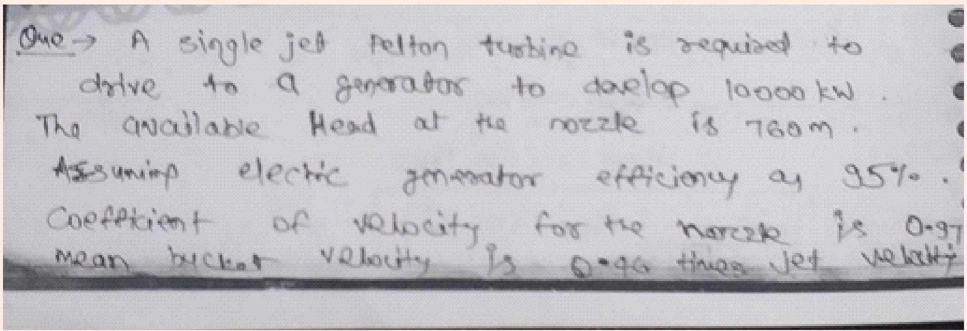
Hydraulic efficiency,
$$\eta_h = \frac{\text{Runner power}}{\text{Water power}} = \frac{\rho Q[V_{w1} + V_{w2}]u}{\rho Q g H}$$

$$= \frac{[70.599 + 2.647] \times 31.416}{9.81 \times 270}$$

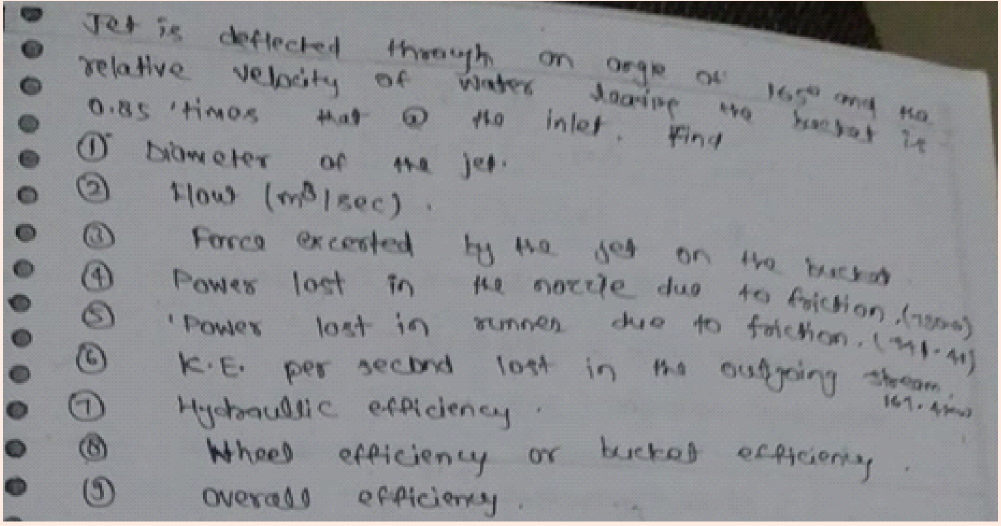
$$\eta_h = 0.86876 \text{ or } 86.876\%$$

MADE EASY Source

- **ESE 2019 Workbook (Prelims):** Fluid Machines (Similar to Q. 78) discussed in Class
- **ESE 2019 Mains Test Series:** Similar to Q.8(b) from Test-10
- **MADE EASY Class Notes**

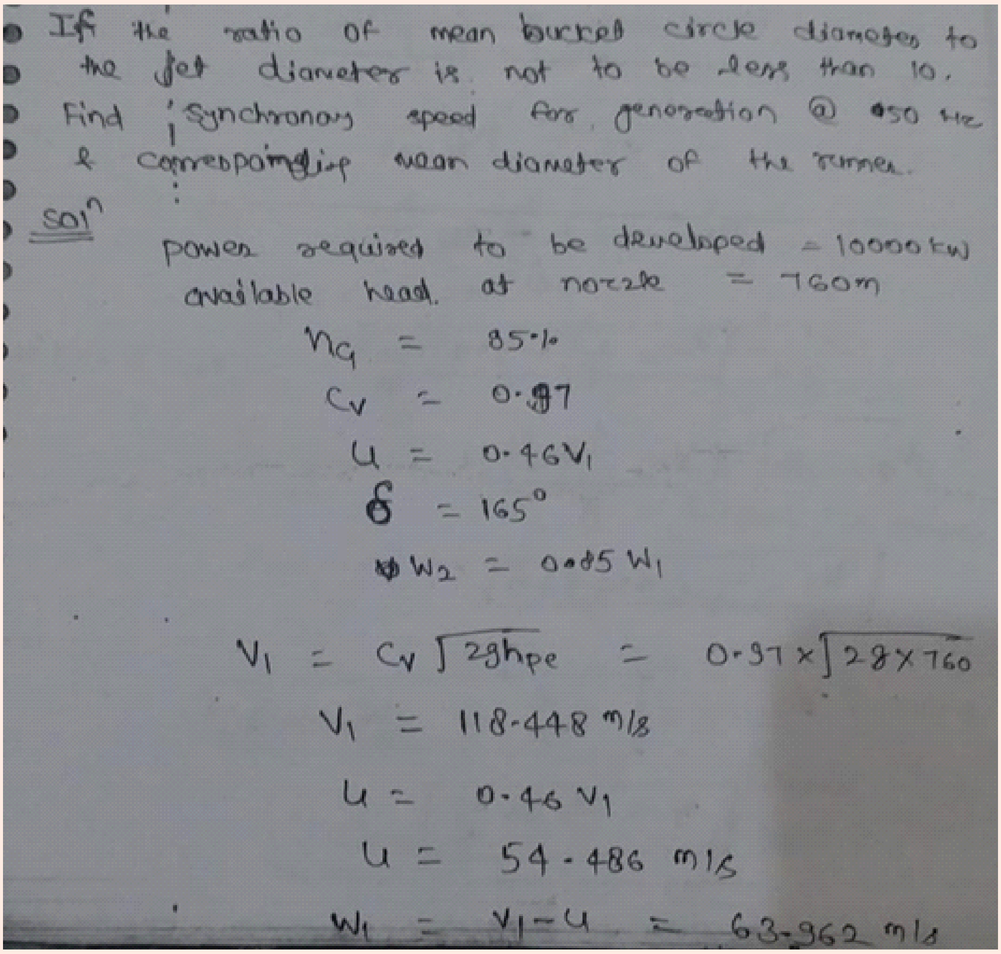


Que → A single jet Pelton turbine is required to drive to a generator to develop 10000 kW. The available Head at the nozzle is 760 m. Assuming electric generator efficiency as 95%. Coefficient of velocity for the nozzle is 0.97 mean bucket velocity is 0.46 times jet velocity



Jet is deflected through an angle of 165° and the relative velocity of water leaving the bucket is 0.85 times that @ the inlet. Find

① Diameter of the jet.
② Flow (m³/sec).
③ Force excerted by the jet on the bucket
④ Power lost in the nozzle due to friction. (780 kW)
⑤ Power lost in runner due to friction. (341.41)
⑥ K.E. per second lost in the outgoing stream 167.4 kW
⑦ Hydraulic efficiency.
⑧ Wheel efficiency or bucket efficiency.
⑨ Overall efficiency.



If the ratio of mean bucket circle diameter to the jet diameter is not to be less than 10. Find synchronous speed for generation @ 50 Hz & corresponding mean diameter of the runner.

Sol^n

power required to be developed = 10000 kW
available head at nozzle = 760 m

$\eta_g = 85\%$

$C_v = 0.97$

$u = 0.46 V_1$

$\delta = 165^\circ$

$W_2 = 0.85 W_1$

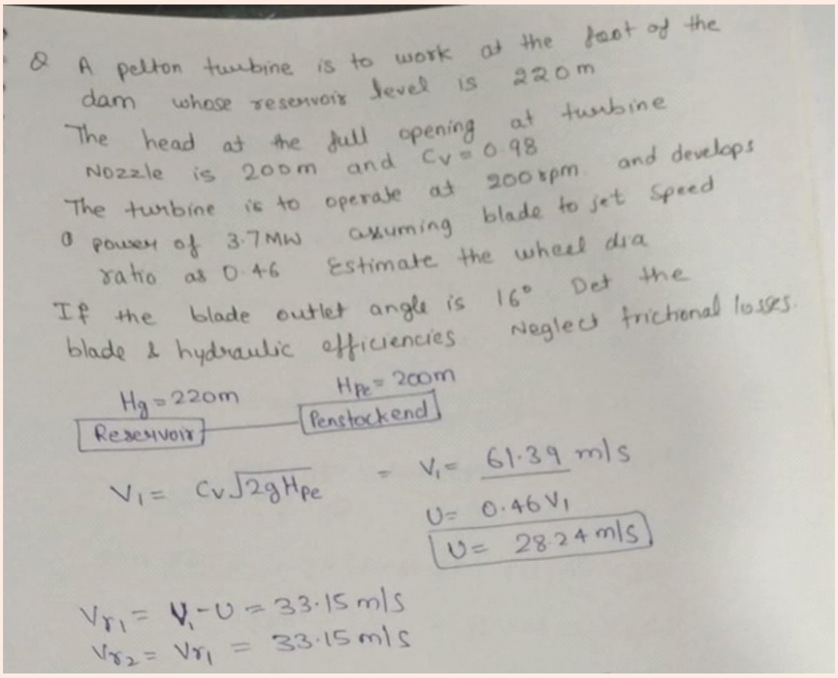
$V_1 = C_v\sqrt{2gh_{pe}} = 0.97\times\sqrt{2g\times 760}$

$V_1 = 118.448$ m/s

$u = 0.46 V_1$

$u = 54.486$ m/s

$W_1 = V_1 - u = 63.962$ m/s



Q A pelton turbine is to work at the foot of the dam whose reservoir level is 220 m
The head at the full opening at turbine Nozzle is 200 m and $C_v = 0.98$
The turbine is to operate at 200 rpm and develops a power of 3.7 MW assuming blade to jet Speed ratio as 0.46 Estimate the wheel dia
If the blade outlet angle is 16° Det the blade & hydraulic efficiencies Neglect frictional losses.

$H_g = 220$m Reservoir — Penstock end $H_{pe} = 200$m

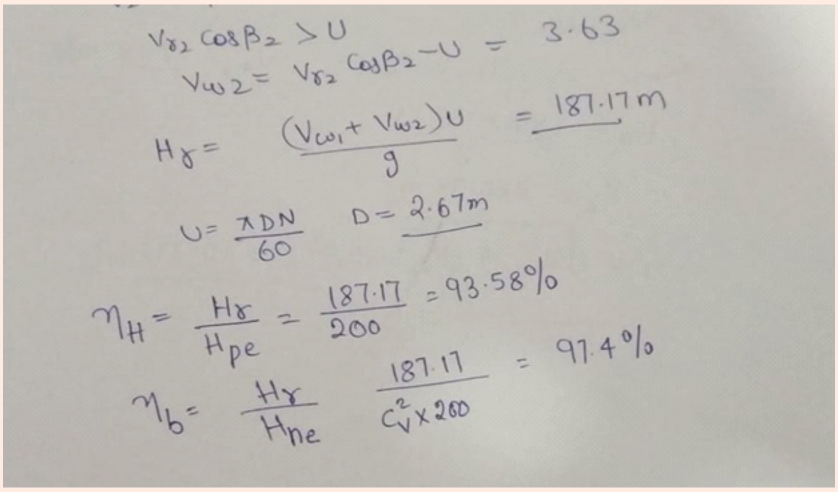
$V_1 = C_v\sqrt{2gH_{pe}}$ = $V_1 = 61.39$ m/s

$U = 0.46 V_1$

$\boxed{U = 28.24 \text{ m/s}}$

$V_{r1} = V_1 - U = 33.15$ m/s

$V_{r2} = V_{r1} = 33.15$ m/s



$V_{r_2} \cos\beta_2 > U$

$V_{w_2} = V_{r_2} \cos\beta_2 - U = 3.63$

$H_r = \frac{(V_{w_1} + V_{w_2})U}{g} = \underline{187.17\,m}$

$U = \frac{\pi D N}{60} \quad D = \underline{2.67m}$

$\eta_H = \frac{H_r}{H_{pe}} = \frac{187.17}{200} = 93.58\%$

$\eta_b = \frac{H_r}{H_{ne}} \quad \frac{187.17}{C_v^2 \times 200} = 97.4\%$

End of Solution

Q.7 (b) What do you mean by compounding in steam and gas turbines ? What are the various methods of compounding in steam and gas turbines? Explain all the methods with neat sketch.

[20 Marks]

Solution:

Compounding of Steam Turbines

If the steam is expanded from the boiler pressure to condenser pressure in single stage then rotor speed exceeded high within the very less time which creates practical complicates.

One row of nozzle followed by one row of blades is called a **stage of turbine**. If steam is allowed to expand from boiler to condenser in a single row of nozzle, the velocity at exit from nozzle is very large. For example, single stage impulse turbine called **De Laval turbine** have high rotational speeds (N). Such large rotational speeds are not properly utilized. It entails large frictional losses and high centrifugal stresses.

Compounding is a method for reducing the rotational speed of the impulse turbine to practical limits.

There are different methods of compounding:

(i) Velocity compounded impulse turbine
(ii) Pressure compounded impulse turbine
(iii) Pressure Velocity compounded impulse turbine

Velocity Compounded Impulse Turbine

In this type of arrangement, velocity gained from the exit of the nozzles is splitted up into many drops through several rows of moving blades and hence named as velocity compounded impulse turbine.

It is also called **curtis staging**. In velocity compounding, all the pressure drop, and enthalpy drop of steam takes place in a single row of nozzles and resultant kinetic energy of steam is absorbed by wheel in number of row of moving blades with guide blades in between two rows.

The kinetic energy of steam jet at nozzle exit is partially converted to shaft work in first row of moving blades with velocity decreasing from V_1 to V_2. The existing steam jets are deflected by stationary guide blades to the next moving blade where part of kinetic energy in converted to shaft work.

Velocity compounded stage is used to give lower blade speed ratio and better utilization of kinetic energy of steam.

When exit from the 2nd row is axial then kinetic energy carried by steam is minimum and therefore efficiency becomes maximum compared to the simple impulse turbine the leaving velocity is small and it is about 2% of initial total available energy of steam.

Velocity of blade (u) is same for all rows because of being on same shaft.

$$\frac{V_{r2}}{V_{r1}} = \frac{V_3}{V_2} = \frac{V_{r4}}{V_{r3}} = K_b$$

where, K_b is blade friction factor

(a) **Stationary Nozzle** : Steam expanded through Nozzle from boiler (inlet) pressure to condenser pressure,

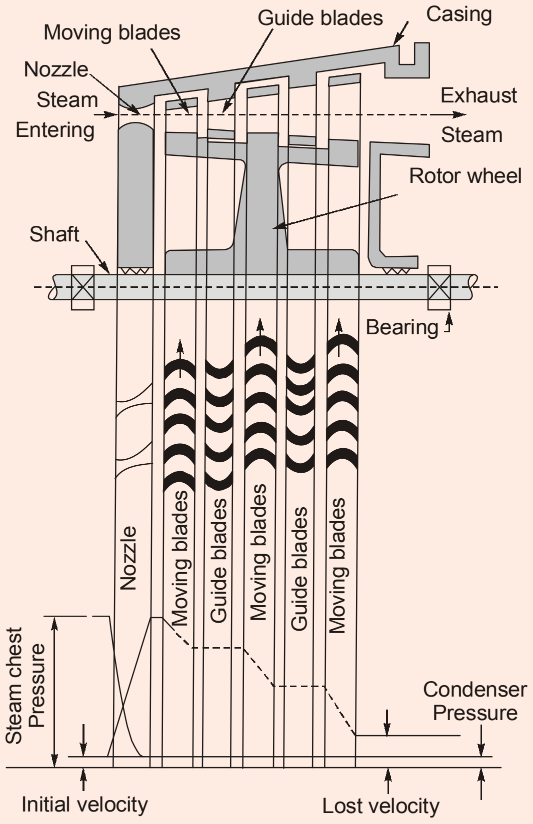
During expansion in nozzle, pressure drops and velocity increases hence KE of steam increases.

(b) **Moving blades** : Moving blades absorbs a portion of available kinetic energy (KE) and velocity decreases while moving over moving blades.

(c) **Fixed blades or guide blades**: Re-direct the steam without changing its velocity to the next row of moving blades where again work is done and steam leaves the turbine with low velocity

Advantages : Low initial cost since lesser number of stages.

Disadvantages : Efficiency is low and used for driving small machines.



Diagrammatic arrangement of Velocity Compound Impulse Turbine

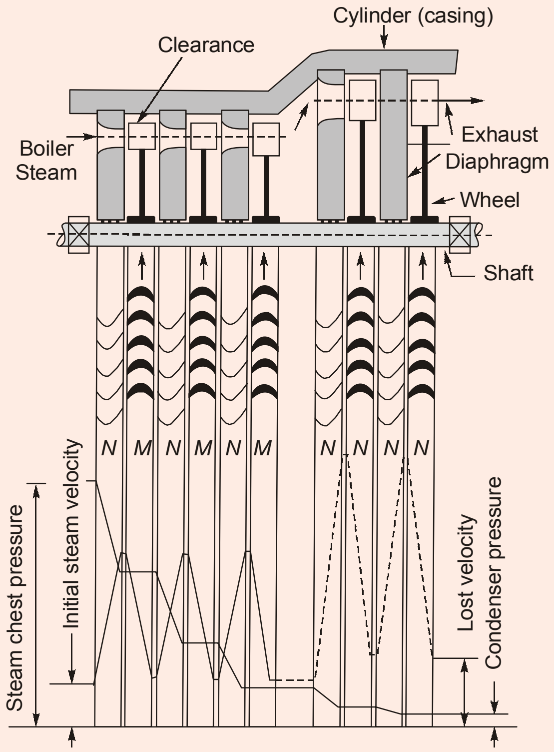
Pressure Compounded Impulse Turbine

It is also called **Rateau staging**. It corresponds to putting a number of simple impulse stages in series. The total enthalpy drop is divided equally among the stages. The pressure drop occur only in nozzle. There is no pressure drop while steam flows through the blades. The kinetic energy of steam increases in nozzle at expense of pressure drop and it is absorbed by blade in each stage.

- In this, fixed nozzles are in between the rings of moving blades.
- Steam expands partially in I^{st} set of nozzle after which velocity increases and hence KE. increase. This KE. of steam is absorbed by the moving blades of I^{st} stage.
- Then steam expands partially in second set of nozzles after which K.E. again increases which absorbed by the II^{nd} stage of moving blades
- This is repeated in next stage also and finally steam leaves the turbine **at very low velocity compared to the De Laval turbine.**
- In the arrangement, whole pressure drop is splitted into a series of smaller pressure drops across several stages of impulse turbine and hence named as pressure compounded impulse turbine.
- Pressure compounding is used in **Rateau and Zoelly turbines.**

Advantages: Most efficient since speed ratio remains constant.

Disadvantages: Costly due to large number of stages.



Diagrammatic Arrangement of Pressure Compounded Impulse Turbine

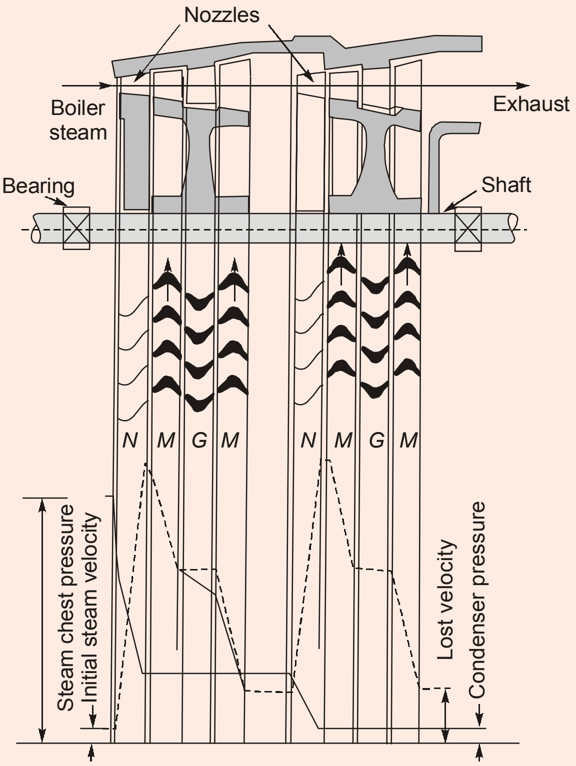
Pressure Velocity Compounded Impulse Turbine

It is combination of velocity and pressure compounding.

- In diagram it is shown that velocity drop is achieved by two rows of moving blades hence it is velocity compounded and pressure drop is divided in small drops is two sets of nozzles, hence it is pressure compounded.
- In the set of nozzles, there is decrease in pressure due to which, steam velocity increases, and there is no drop is pressure in the rows of moving and fixed blades.

Only velocity drop takes place in the rows of moving blades and there is slight drop is velocity due to friction in the fixed blades.

- Due to very low efficiency, now a days, it is very rarely used.



Diagrammatic Arrangement of pressure velocity compounded impulse turbine

MADE EASY Source

- **ESE 2019 Mains Test Series:** Test-3 (Similar Q. 1d(i))

End of Solution

Q.7 (c) A reaction steam turbine having diameter of 1400 mm is rotating at 3000 rpm. The turbine stages are designed in such a fashion that the enthalpy drop in both, rotor and stator, is same in each stage. If the speed ratio is 0.7 and angle at outlet is 20°, draw velocity triangles and determine degree of reaction, blade angle at inlet and diagram efficiency.

[20 Marks]

Solution:

Degree of reaction, $$R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stator}} + \Delta h_{\text{rotor}}}$$

Since, $\Delta h_{\text{stator}} = \Delta h_{\text{rotor}}$

$$R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{rotor}} + \Delta h_{\text{rotor}}} = \frac{1}{2} = 50\%$$

Diameter, $D = 1400$ mm, $N = 3000$ rpm

$$u = \frac{\pi DN}{60} = \frac{\pi \times 1.4 \times 3000}{60} = 219.911\,\text{m/s}$$

Speed ratio, $\rho = \dfrac{u}{V_1}$

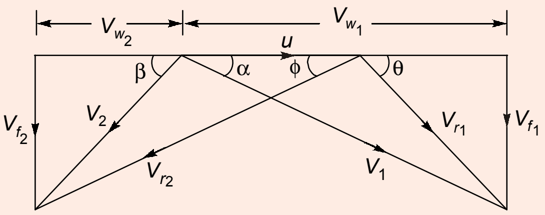
$\Rightarrow$ $0.7 = \dfrac{219.911}{V_1}$

$V_1 = 314.159$ m/s

For 50% reaction turbine,

Guide blade angle, α = Blade angle at outlet, ϕ

So, $\alpha = \phi = 20°$



Velocity Triangles

From inlet velocity triangle, $V_{f1} = V_1\sin\alpha = 314.159 \times \sin 20 = 107.448$ m/s

$V_{w1} = V_1\cos\alpha = 314.159 \times \cos 20 = 295.213$ m/s

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u} = \frac{107.448}{295.213 - 219.911}$$

Blade angle at inlet, $\theta = 54.976°$

$$V_2 = V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{107.448}{\sin 54.976} = 131.207\,\text{m/s}$$

Jet angle at outlet, $\beta = \theta = 54.976°$

$V_{w2} = V_2\cos\beta = 131.207 \times \cos 54.976° = 75.303$ m/s

$V_{r2} = V_1 = 314.159$ m/s

Diagram efficiency,
$$\eta_d = \frac{(V_{w1} + V_{w2})u}{\left[\frac{V_1^2}{2} + \frac{V_{r2}^2 - V_{r1}^2}{2}\right]}$$

$$= \frac{(295.213 + 75.303) \times 219.911}{\frac{1}{2}\left[314.159^2 + 314.159^2 - 131.207^2\right]}$$

$= 0.9044 = 90.44\%$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Test-5 (Similar Q. 7b(ii))

End of Solution

MADE EASY

India's Best Institute for IES, GATE & PSUs

General Awareness and General Intelligence & Reasoning Courses

for

SSC (JE/CGL)

Classroom Course

Live/Online Course also available

90 Days Course

6-7 days/week • 3 hours classes /day

Batches commencing
from **20th Aug, 2019**

Class timing
8.00 AM - 11:00 AM

Venue
Hundred Million IGNOU main Road Saket,
Saidulajab Extension, New Delhi-110030

Regular Classroom Course : Fee Structure

Subject	Hours	Fee for all
General Awareness	160 Hrs.	₹10,000
General Intelligence & Reasoning	50 Hrs.	₹4,000
Combined	210 Hrs.	₹12,000

Live/Online Classes : Fee Structure

Subject	Hours	Fee for all
General Awareness	160 Hrs.	₹8,000
General Intelligence & Reasoning	50 Hrs.	₹3,200
Combined	210 Hrs.	₹10,000

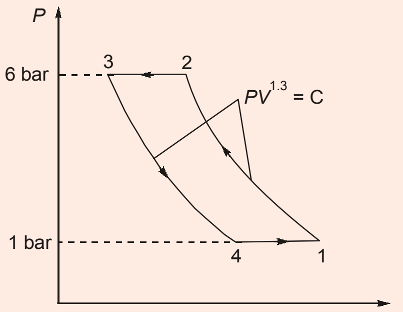
Ph: 8851176822, 8851176823 • www.ssc.madeeasy.in

Q.8 (a) A single-stage air compressor delivers air at 6 bar. The pressure and temperature at the end of suction are 1 bar and 27°C. It delivers 1.5 m^3 of free air per minute when the compressor is running at 350 rpm. The clearance volume is 5% of stroke volume. The free air conditions are 1.013 bar and 15°C. The index of compression and expansion is 1.3. Find

- (i) The volumetric efficiency,
- (ii) Bore and stroke of cylinder if both are equal,
- (iii) The power required if the mechanical efficiency is 80%.

[20 Marks]

Solution:



Given: Single Stage Reciprocating Compressor.

Suction	Delivery
P = 1 bar	P = 6 bar
T = 300 K	

Clearance ratio, C = 0.05

V_{fad} = 1.5 m^3/min at 1.013 bar and 288 K

Actual volume at compressor inlet,

$$\dot{m} = \left(\frac{PV}{RT}\right)_{\text{suction}} = \left(\frac{PV}{RT}\right)_{\text{FAD}}$$

[as mass flow rate is same]

$$\frac{1 \times V_s}{300} = \frac{1.013 \times 1.5}{288} \quad \Rightarrow V_s = 1.5828 \text{ m}^3/\text{min}$$

(i) Volumetric efficiency
$$\eta_v = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.05 - 0.05\left(\frac{6}{1}\right)^{\frac{1}{1.3}}$$
$$\eta_v = 0.8516 \text{ or } 85.16\%$$

(ii) Bore and stroke,
$$\eta_v = \frac{\dot{V}_s}{\frac{\pi}{4}D^2LNk}$$

for single cylinder k = 1, since D = L given

$$\therefore \quad D^3 = \frac{\dot{V}_s}{\eta_v\left(\frac{\pi}{4}N\right)} = \frac{1.5828}{0.8516 \times \frac{\pi}{4} \times 350} = 6.7613 \times 10^{-3}$$

$$D = 0.18909 \text{ m or } D = L = 18.909 \text{ cm}$$

(iii) Power required if η_m = 0.8

Theoretical power,
$$P_{\text{th}} = \frac{n}{n-1}P_1V_s\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$
$$P_{\text{th}} = \left(\frac{1.3}{1.3-1}\right) \times 100 \times 1.5828 \times \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right]$$
$$P_{\text{th}} = 351.22 \text{ kJ/min}$$

Actual power required,
$$P_{\text{act}} = \frac{P_{\text{th}}}{\eta_m} = \frac{351.22}{0.8} = 439.025 \text{ kJ/min}$$

So, $P_{act} = 439.025$ kJ/min

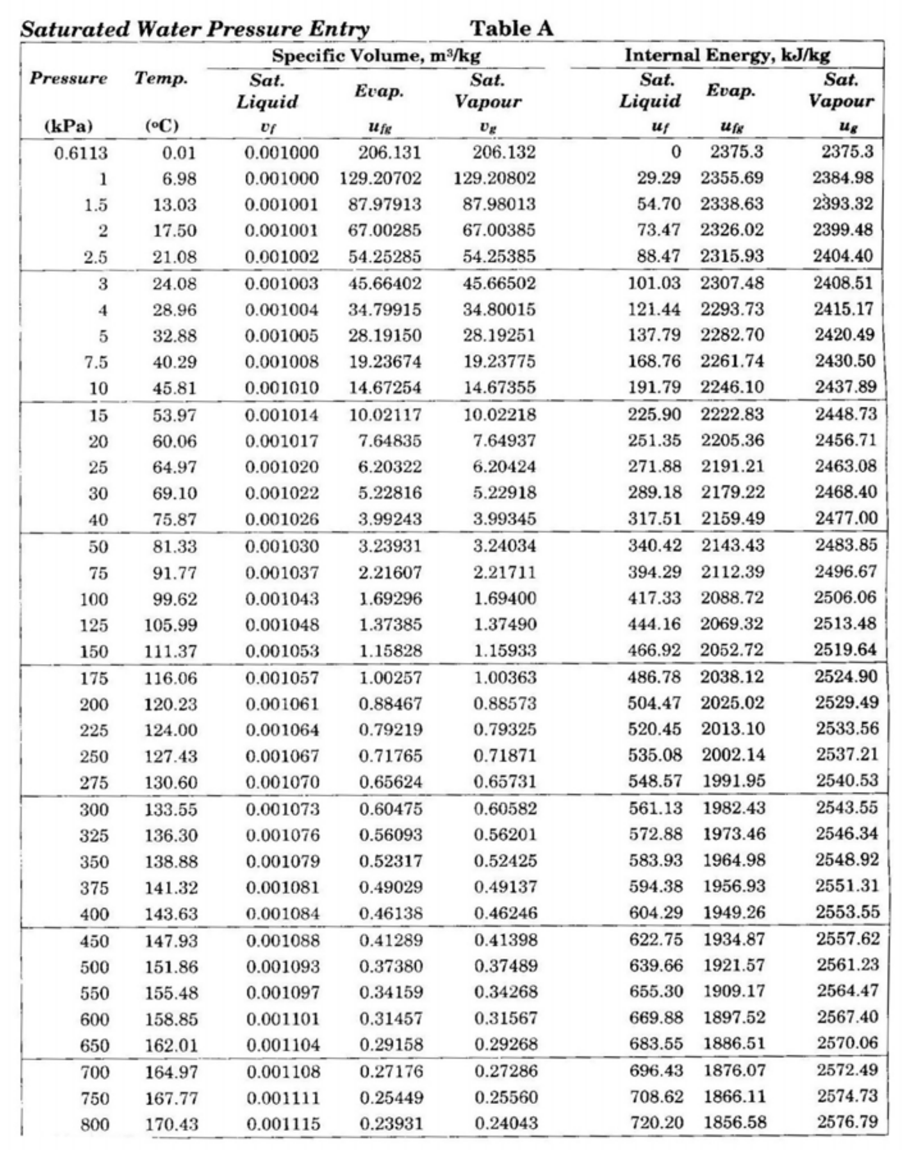
or $P_{act} = 7.317$ kW

MADE EASY Source

- **ESE 2019 Mains Test Series:** Test-2 (Similar Q. 6c)

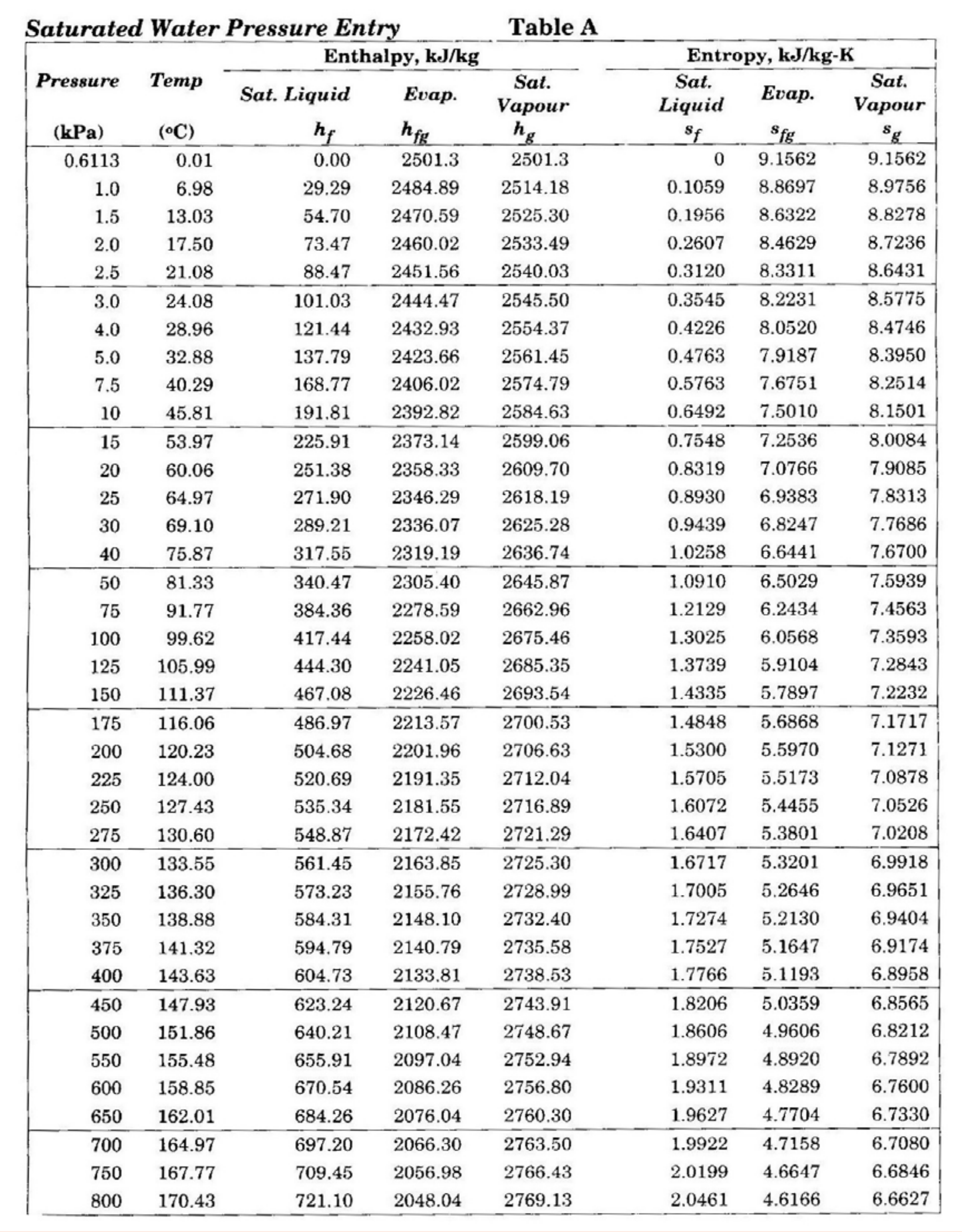
End of Solution

Q.8 (b) Consider an ideal steam regenerative cycle in which steam enters the turbine at 3 MPa, 300°C and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa and supplied to an open feed water heater. The feed water leaves the heater as saturated liquid. The appropriate pumps are used for the water leaving the condenser and feed water heater. If the mass flow rate through the boiler is 1 kg/s, determine the amount of steam extracted (kg/s), the total pump work (kW) and total turbine work (kW). Draw the schematic of this set-up.



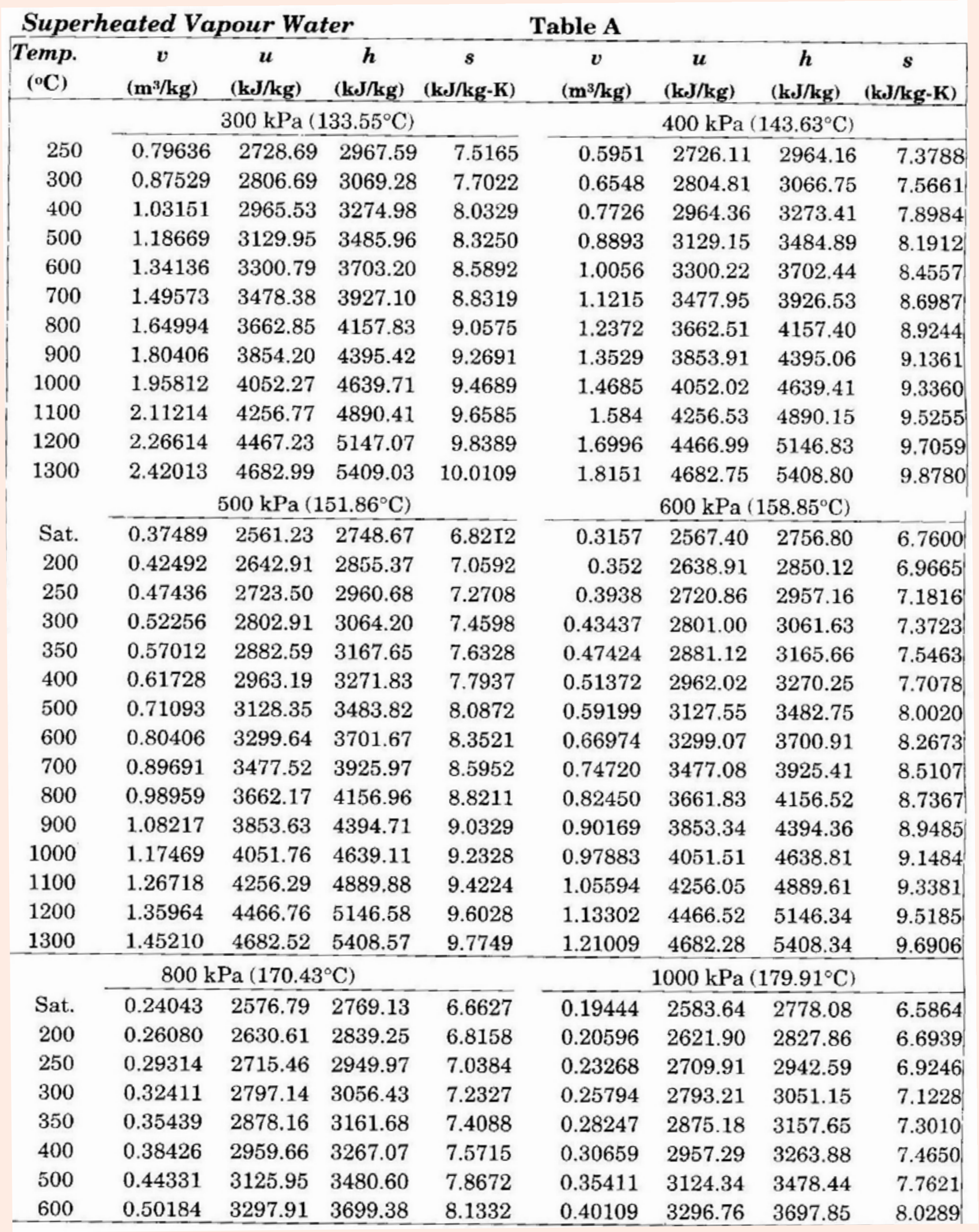
Saturated Water Pressure Entry **Table A**

Pressure	*Temp.*	Specific Volume, m³/kg			Internal Energy, kJ/kg		
		Sat. Liquid	*Evap.*	*Sat. Vapour*	*Sat. Liquid*	*Evap.*	*Sat. Vapour*
(kPa)	(°C)	v_f	u_{fg}	v_g	u_f	u_{fg}	u_g
0.6113	0.01	0.001000	206.131	206.132	0	2375.3	2375.3
1	6.98	0.001000	129.20702	129.20802	29.29	2355.69	2384.98
1.5	13.03	0.001001	87.97913	87.98013	54.70	2338.63	2393.32
2	17.50	0.001001	67.00285	67.00385	73.47	2326.02	2399.48
2.5	21.08	0.001002	54.25285	54.25385	88.47	2315.93	2404.40
3	24.08	0.001003	45.66402	45.66502	101.03	2307.48	2408.51
4	28.96	0.001004	34.79915	34.80015	121.44	2293.73	2415.17
5	32.88	0.001005	28.19150	28.19251	137.79	2282.70	2420.49
7.5	40.29	0.001008	19.23674	19.23775	168.76	2261.74	2430.50
10	45.81	0.001010	14.67254	14.67355	191.79	2246.10	2437.89
15	53.97	0.001014	10.02117	10.02218	225.90	2222.83	2448.73
20	60.06	0.001017	7.64835	7.64937	251.35	2205.36	2456.71
25	64.97	0.001020	6.20322	6.20424	271.88	2191.21	2463.08
30	69.10	0.001022	5.22816	5.22918	289.18	2179.22	2468.40
40	75.87	0.001026	3.99243	3.99345	317.51	2159.49	2477.00
50	81.33	0.001030	3.23931	3.24034	340.42	2143.43	2483.85
75	91.77	0.001037	2.21607	2.21711	394.29	2112.39	2496.67
100	99.62	0.001043	1.69296	1.69400	417.33	2088.72	2506.06
125	105.99	0.001048	1.37385	1.37490	444.16	2069.32	2513.48
150	111.37	0.001053	1.15828	1.15933	466.92	2052.72	2519.64
175	116.06	0.001057	1.00257	1.00363	486.78	2038.12	2524.90
200	120.23	0.001061	0.88467	0.88573	504.47	2025.02	2529.49
225	124.00	0.001064	0.79219	0.79325	520.45	2013.10	2533.56
250	127.43	0.001067	0.71765	0.71871	535.08	2002.14	2537.21
275	130.60	0.001070	0.65624	0.65731	548.57	1991.95	2540.53
300	133.55	0.001073	0.60475	0.60582	561.13	1982.43	2543.55
325	136.30	0.001076	0.56093	0.56201	572.88	1973.46	2546.34
350	138.88	0.001079	0.52317	0.52425	583.93	1964.98	2548.92
375	141.32	0.001081	0.49029	0.49137	594.38	1956.93	2551.31
400	143.63	0.001084	0.46138	0.46246	604.29	1949.26	2553.55
450	147.93	0.001088	0.41289	0.41398	622.75	1934.87	2557.62
500	151.86	0.001093	0.37380	0.37489	639.66	1921.57	2561.23
550	155.48	0.001097	0.34159	0.34268	655.30	1909.17	2564.47
600	158.85	0.001101	0.31457	0.31567	669.88	1897.52	2567.40
650	162.01	0.001104	0.29158	0.29268	683.55	1886.51	2570.06
700	164.97	0.001108	0.27176	0.27286	696.43	1876.07	2572.49
750	167.77	0.001111	0.25449	0.25560	708.62	1866.11	2574.73
800	170.43	0.001115	0.23931	0.24043	720.20	1856.58	2576.79



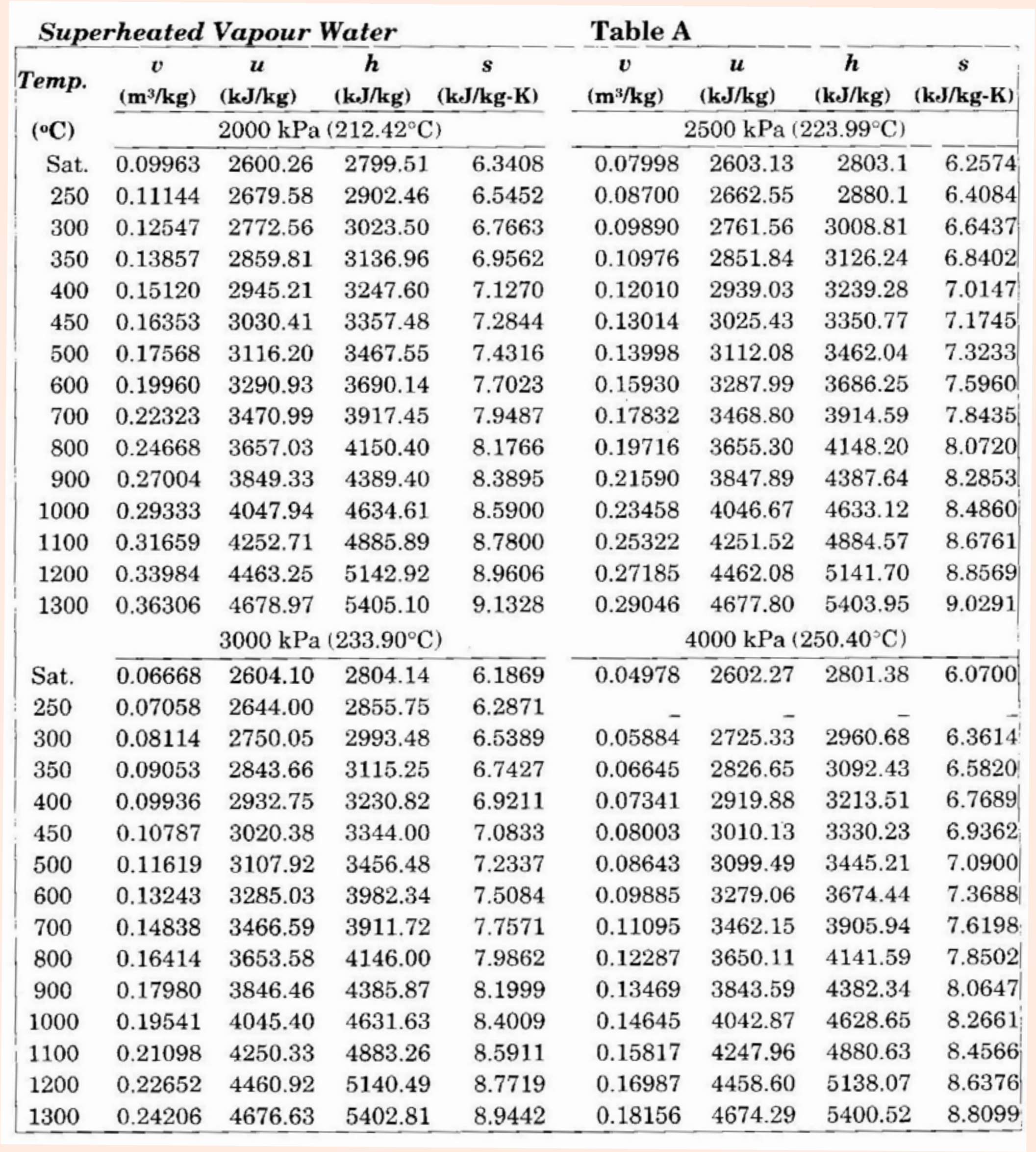
Saturated Water Pressure Entry **Table A**

Pressure	Temp	Enthalpy, kJ/kg			Entropy, kJ/kg-K		
		Sat. Liquid	Evap.	Sat. Vapour	Sat. Liquid	Evap.	Sat. Vapour
(kPa)	(°C)	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.6113	0.01	0.00	2501.3	2501.3	0	9.1562	9.1562
1.0	6.98	29.29	2484.89	2514.18	0.1059	8.8697	8.9756
1.5	13.03	54.70	2470.59	2525.30	0.1956	8.6322	8.8278
2.0	17.50	73.47	2460.02	2533.49	0.2607	8.4629	8.7236
2.5	21.08	88.47	2451.56	2540.03	0.3120	8.3311	8.6431
3.0	24.08	101.03	2444.47	2545.50	0.3545	8.2231	8.5775
4.0	28.96	121.44	2432.93	2554.37	0.4226	8.0520	8.4746
5.0	32.88	137.79	2423.66	2561.45	0.4763	7.9187	8.3950
7.5	40.29	168.77	2406.02	2574.79	0.5763	7.6751	8.2514
10	45.81	191.81	2392.82	2584.63	0.6492	7.5010	8.1501
15	53.97	225.91	2373.14	2599.06	0.7548	7.2536	8.0084
20	60.06	251.38	2358.33	2609.70	0.8319	7.0766	7.9085
25	64.97	271.90	2346.29	2618.19	0.8930	6.9383	7.8313
30	69.10	289.21	2336.07	2625.28	0.9439	6.8247	7.7686
40	75.87	317.55	2319.19	2636.74	1.0258	6.6441	7.6700
50	81.33	340.47	2305.40	2645.87	1.0910	6.5029	7.5939
75	91.77	384.36	2278.59	2662.96	1.2129	6.2434	7.4563
100	99.62	417.44	2258.02	2675.46	1.3025	6.0568	7.3593
125	105.99	444.30	2241.05	2685.35	1.3739	5.9104	7.2843
150	111.37	467.08	2226.46	2693.54	1.4335	5.7897	7.2232
175	116.06	486.97	2213.57	2700.53	1.4848	5.6868	7.1717
200	120.23	504.68	2201.96	2706.63	1.5300	5.5970	7.1271
225	124.00	520.69	2191.35	2712.04	1.5705	5.5173	7.0878
250	127.43	535.34	2181.55	2716.89	1.6072	5.4455	7.0526
275	130.60	548.87	2172.42	2721.29	1.6407	5.3801	7.0208
300	133.55	561.45	2163.85	2725.30	1.6717	5.3201	6.9918
325	136.30	573.23	2155.76	2728.99	1.7005	5.2646	6.9651
350	138.88	584.31	2148.10	2732.40	1.7274	5.2130	6.9404
375	141.32	594.79	2140.79	2735.58	1.7527	5.1647	6.9174
400	143.63	604.73	2133.81	2738.53	1.7766	5.1193	6.8958
450	147.93	623.24	2120.67	2743.91	1.8206	5.0359	6.8565
500	151.86	640.21	2108.47	2748.67	1.8606	4.9606	6.8212
550	155.48	655.91	2097.04	2752.94	1.8972	4.8920	6.7892
600	158.85	670.54	2086.26	2756.80	1.9311	4.8289	6.7600
650	162.01	684.26	2076.04	2760.30	1.9627	4.7704	6.7330
700	164.97	697.20	2066.30	2763.50	1.9922	4.7158	6.7080
750	167.77	709.45	2056.98	2766.43	2.0199	4.6647	6.6846
800	170.43	721.10	2048.04	2769.13	2.0461	4.6166	6.6627



Superheated Vapour Water **Table A**

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	300 kPa (133.55°C)				400 kPa (143.63°C)			
250	0.79636	2728.69	2967.59	7.5165	0.5951	2726.11	2964.16	7.3788
300	0.87529	2806.69	3069.28	7.7022	0.6548	2804.81	3066.75	7.5661
400	1.03151	2965.53	3274.98	8.0329	0.7726	2964.36	3273.41	7.8984
500	1.18669	3129.95	3485.96	8.3250	0.8893	3129.15	3484.89	8.1912
600	1.34136	3300.79	3703.20	8.5892	1.0056	3300.22	3702.44	8.4557
700	1.49573	3478.38	3927.10	8.8319	1.1215	3477.95	3926.53	8.6987
800	1.64994	3662.85	4157.83	9.0575	1.2372	3662.51	4157.40	8.9244
900	1.80406	3854.20	4395.42	9.2691	1.3529	3853.91	4395.06	9.1361
1000	1.95812	4052.27	4639.71	9.4689	1.4685	4052.02	4639.41	9.3360
1100	2.11214	4256.77	4890.41	9.6585	1.584	4256.53	4890.15	9.5255
1200	2.26614	4467.23	5147.07	9.8389	1.6996	4466.99	5146.83	9.7059
1300	2.42013	4682.99	5409.03	10.0109	1.8151	4682.75	5408.80	9.8780
	500 kPa (151.86°C)				600 kPa (158.85°C)			
Sat.	0.37489	2561.23	2748.67	6.8212	0.3157	2567.40	2756.80	6.7600
200	0.42492	2642.91	2855.37	7.0592	0.352	2638.91	2850.12	6.9665
250	0.47436	2723.50	2960.68	7.2708	0.3938	2720.86	2957.16	7.1816
300	0.52256	2802.91	3064.20	7.4598	0.43437	2801.00	3061.63	7.3723
350	0.57012	2882.59	3167.65	7.6328	0.47424	2881.12	3165.66	7.5463
400	0.61728	2963.19	3271.83	7.7937	0.51372	2962.02	3270.25	7.7078
500	0.71093	3128.35	3483.82	8.0872	0.59199	3127.55	3482.75	8.0020
600	0.80406	3299.64	3701.67	8.3521	0.66974	3299.07	3700.91	8.2673
700	0.89691	3477.52	3925.97	8.5952	0.74720	3477.08	3925.41	8.5107
800	0.98959	3662.17	4156.96	8.8211	0.82450	3661.83	4156.52	8.7367
900	1.08217	3853.63	4394.71	9.0329	0.90169	3853.34	4394.36	8.9485
1000	1.17469	4051.76	4639.11	9.2328	0.97883	4051.51	4638.81	9.1484
1100	1.26718	4256.29	4889.88	9.4224	1.05594	4256.05	4889.61	9.3381
1200	1.35964	4466.76	5146.58	9.6028	1.13302	4466.52	5146.34	9.5185
1300	1.45210	4682.52	5408.57	9.7749	1.21009	4682.28	5408.34	9.6906
	800 kPa (170.43°C)				1000 kPa (179.91°C)			
Sat.	0.24043	2576.79	2769.13	6.6627	0.19444	2583.64	2778.08	6.5864
200	0.26080	2630.61	2839.25	6.8158	0.20596	2621.90	2827.86	6.6939
250	0.29314	2715.46	2949.97	7.0384	0.23268	2709.91	2942.59	6.9246
300	0.32411	2797.14	3056.43	7.2327	0.25794	2793.21	3051.15	7.1228
350	0.35439	2878.16	3161.68	7.4088	0.28247	2875.18	3157.65	7.3010
400	0.38426	2959.66	3267.07	7.5715	0.30659	2957.29	3263.88	7.4650
500	0.44331	3125.95	3480.60	7.8672	0.35411	3124.34	3478.44	7.7621
600	0.50184	3297.91	3699.38	8.1332	0.40109	3296.76	3697.85	8.0289



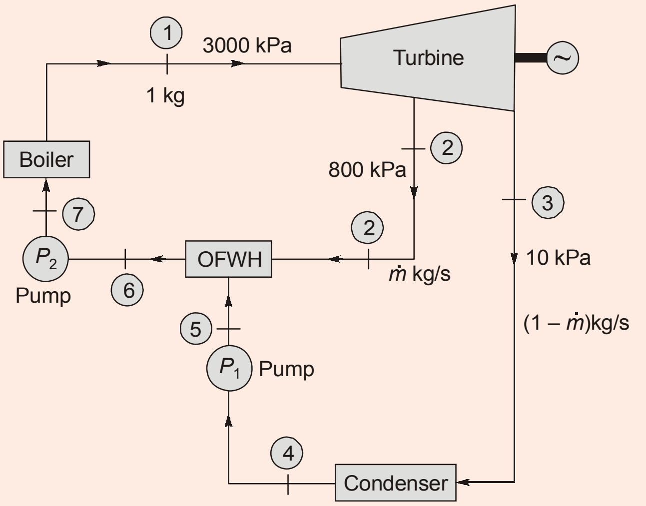
Superheated Vapour Water **Table A**

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	2000 kPa (212.42°C)				2500 kPa (223.99°C)			
Sat.	0.09963	2600.26	2799.51	6.3408	0.07998	2603.13	2803.1	6.2574
250	0.11144	2679.58	2902.46	6.5452	0.08700	2662.55	2880.1	6.4084
300	0.12547	2772.56	3023.50	6.7663	0.09890	2761.56	3008.81	6.6437
350	0.13857	2859.81	3136.96	6.9562	0.10976	2851.84	3126.24	6.8402
400	0.15120	2945.21	3247.60	7.1270	0.12010	2939.03	3239.28	7.0147
450	0.16353	3030.41	3357.48	7.2844	0.13014	3025.43	3350.77	7.1745
500	0.17568	3116.20	3467.55	7.4316	0.13998	3112.08	3462.04	7.3233
600	0.19960	3290.93	3690.14	7.7023	0.15930	3287.99	3686.25	7.5960
700	0.22323	3470.99	3917.45	7.9487	0.17832	3468.80	3914.59	7.8435
800	0.24668	3657.03	4150.40	8.1766	0.19716	3655.30	4148.20	8.0720
900	0.27004	3849.33	4389.40	8.3895	0.21590	3847.89	4387.64	8.2853
1000	0.29333	4047.94	4634.61	8.5900	0.23458	4046.67	4633.12	8.4860
1100	0.31659	4252.71	4885.89	8.7800	0.25322	4251.52	4884.57	8.6761
1200	0.33984	4463.25	5142.92	8.9606	0.27185	4462.08	5141.70	8.8569
1300	0.36306	4678.97	5405.10	9.1328	0.29046	4677.80	5403.95	9.0291
	3000 kPa (233.90°C)				4000 kPa (250.40°C)			
Sat.	0.06668	2604.10	2804.14	6.1869	0.04978	2602.27	2801.38	6.0700
250	0.07058	2644.00	2855.75	6.2871	–	–	–	–
300	0.08114	2750.05	2993.48	6.5389	0.05884	2725.33	2960.68	6.3614
350	0.09053	2843.66	3115.25	6.7427	0.06645	2826.65	3092.43	6.5820
400	0.09936	2932.75	3230.82	6.9211	0.07341	2919.88	3213.51	6.7689
450	0.10787	3020.38	3344.00	7.0833	0.08003	3010.13	3330.23	6.9362
500	0.11619	3107.92	3456.48	7.2337	0.08643	3099.49	3445.21	7.0900
600	0.13243	3285.03	3982.34	7.5084	0.09885	3279.06	3674.44	7.3688
700	0.14838	3466.59	3911.72	7.7571	0.11095	3462.15	3905.94	7.6198
800	0.16414	3653.58	4146.00	7.9862	0.12287	3650.11	4141.59	7.8502
900	0.17980	3846.46	4385.87	8.1999	0.13469	3843.59	4382.34	8.0647
1000	0.19541	4045.40	4631.63	8.4009	0.14645	4042.87	4628.65	8.2661
1100	0.21098	4250.33	4883.26	8.5911	0.15817	4247.96	4880.63	8.4566
1200	0.22652	4460.92	5140.49	8.7719	0.16987	4458.60	5138.07	8.6376
1300	0.24206	4676.63	5402.81	8.9442	0.18156	4674.29	5400.52	8.8099

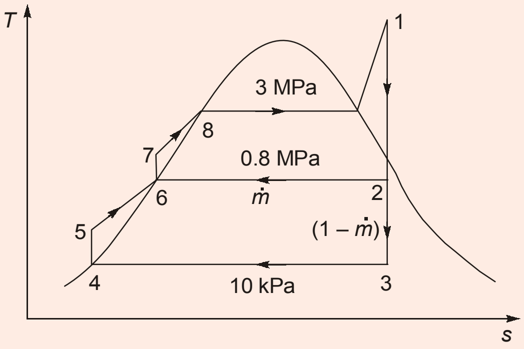
[20 Marks]

Solution:

Ideal Regenerative Steam Cycle



State 1 (Turbine inlet)
3 MPa, 300°C
superheated steam
h_1 = 2993.48 kJ/kg
s_1 = 6.5389 kJ/kgK



Assumptions

1. There are no pressure losses in pipings and equipments
2. Turbine and pumps have 100% isentropic efficiency.
3. No heat loss in OFWH.

Process 1 → 2 isentropic

$$s_1 = s_2$$

$$6.5389 = [s_f + x_2 s_{fg}]_{@0.8\text{MPa}}$$

At 0.8 MPa (From table)

$$s_f = 2.0461 \text{ kJ/kgK}$$

$$s_{fg} = 4.6166 \text{ kJ/kgK}$$

$$h_f = 721.10 \text{ kJ/kg}$$

$$h_{fg} = 2048.04 \text{ kJ/kg}$$

$$v_f = 0.001115 \text{ m}^3\text{/kg}$$

$$6.5389 = 2.0461 + x_2(4.6166)$$

$$x_2 = 0.97318$$

(dryness fraction of steam entering OFWH from turbine)

$$h_2 = h_f + x_2 h_{fg}$$

$$h_2 = 721.1 + 0.97318(2048.04)$$

$$h_2 = 2714.21 \text{ kJ/kg}$$

enthalpy of steam entering OFWH from turbine

Process 1 - 3 isentropic

$$s_1 = s_3$$

$$6.5389 = [s_f + x_3 s_{fg}]_{@10\text{ kPa}}$$

At 10 kPa (From table)

$$s_f = 0.6492\text{ kJ/kgK}$$
$$s_{fg} = 7.5010\text{ kJ/kgK}$$
$$h_f = 191.81\text{ kJ/kg}$$
$$h_{fg} = 2392.82\text{ kJ/kg}$$
$$v_f = 0.001010\text{ m}^3\text{/kg}$$
$$6.5389 = 0.6492 + x_3(7.501)$$
$$x_2 = 0.78518$$

(dryness fraction of steam entering condenser)

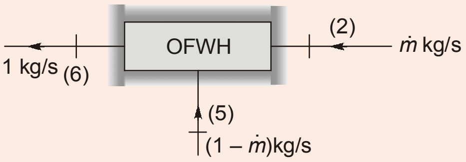
$$h_3 = h_f + x_3 h_{fg}$$
$$h_3 = 191.81 + 0.78518 \times (2392.82)$$
$$h_3 = 2070.6\text{ kJ/kg}$$

enthalpy of steam entering condenser

Let $\dot{m}$ be the mass of steam per second extracted from turbine for regeneration.

So, $(1 - \dot{m})$ kg/s mass flow through condenser.

Energy Balance at open feed water heater (OFWH)



$$\dot{m}h_2 + (1 - \dot{m})h_5 = h_6$$
$$h_5 = (h_f)_{@10\text{ kPa}} + (800 - 10) \times v_{f@10\text{ kPa}}$$
$$h_5 = 191.81 + 790 \times 0.00101$$
$$h_5 = 192.6079\text{ kJ/kg}$$
$$h_5 = h_{f@0.8\text{ MPa}} = 721.10\text{ kJ/kg}$$
$$\therefore\ \dot{m}(2714.21) + (1 - \dot{m})(192.6079) = 721.10$$
$$\dot{m} = 0.20958\text{ kg/s}$$

mass of steam extracted from Turbine for regeneration

Total Turbine Work, $W_T = 1 \times (h_1 - h_2) + (1 - \dot{m}) \times (h_2 - h_3)$

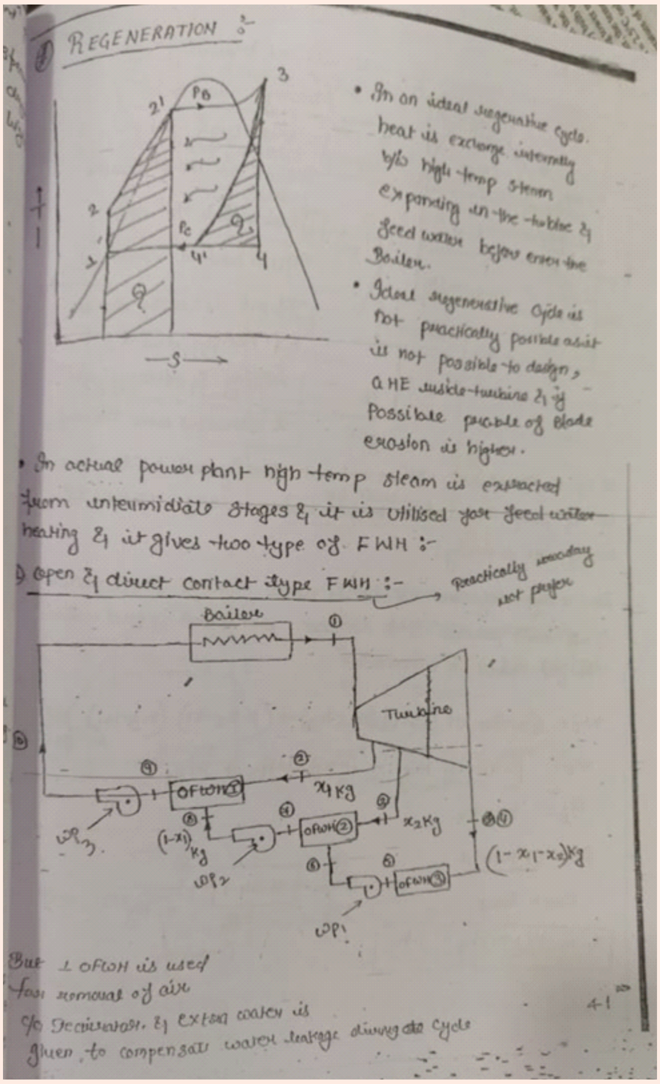
$$W_T = (2993.48 - 2714.21) + (1 - 0.20958) \times (2714.21 - 2070.6)$$
$$W_T = 787.992\text{ kW} \approx 788\text{ kW}$$

Total pump work, $W_P = W_{P1} + W_{P2} = (1 - \dot{m})(\Delta P_1) \times v_{f4} + 1 \times (\Delta P_2) \times v_{f6}$

$$= (1 - 0.20958) \times (800 - 10) \times 0.00101$$
$$+ 1 \times (3000 - 800) \times 0.001115$$
$$W_P = 3.083\text{ kW}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Similar to Q.7(a), Test-14
- **MADE EASY Mains Class Notes**
- **ESE 2019 Mains Workbook:** Similar to Q.14, Page 143 discussed in Class
- **MADE EASY Classnotes**



④ REGENERATION :-

- In an ideal regenerative cycle heat is exchange internally b/w high-temp steam expanding in the turbine & feed water before enter the Boiler.
- Ideal regenerative cycle is not practically possible as it is not possible to design a HE inside turbine & if possible proble of blade erosion is higher.

- In actual power plant high-temp steam is extracted from intermidiate stages & it is utilised for feed water heating & it gives two type of FWH :-

1) Open & direct contact type FWH :- → practically nowaday not prefer

But 1 OFWH is used for removal of air i/o Deaerator. & extra water is given to compensate water leakage during the cycle

End of Solution

Q.8 (c) A Brayton cycle works between 1 bar, 300 K and 5 bar 1250 K. There are two stages of compression with perfect inter-cooling and two stages of expansion. The work out of first expansion stage is being used to drive the two compressors. The air from the first stage turbine is again heated to 1250 K and expanded. Calculate the power output of free power turbine and cycle efficiency without and with a perfect heat exchanger and compare them. Also calculate the percentage improvement in the efficiency because of the addition of heat exchangers.

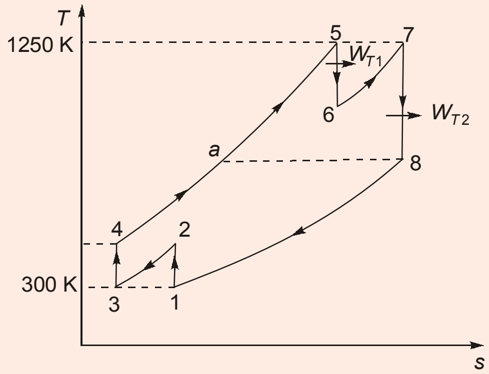
[20 Marks]

Solution:

Given:

$$P_1 = 1 \text{ bar}$$
$$T_1 = 300 \text{ K}$$
$$P_4 = P_5 = 5 \text{ bar}$$
$$T_5 = T_7 = 1250 \text{ K}$$

Turbine-1 is only used to give power to both compressors.



Assumption:

1. Air is working fluid
2. c_p, c_v, γ remains constant throughout.
3. No pressure losses in combustion chamber.
4. Isentropic efficiency of turbine and compressors is 100%.

In perfect intercooling, pressure ratio across two compressor should be same and it is also the same condition for minimum work.

Since perfect intercooling is present

$\therefore$ $$W_{c1} = W_{c2}$$

and intermediate pressure,

$$P_3 = P_2 = \sqrt{P_1 P_4}$$

$$P_3 = P_2 = \sqrt{1 \times 5} = \sqrt{5} \text{ bar}$$

$$W_{c1} = W_{c2} = c_p T_1 \left\{ (r_{pc})^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

$$= 300\, c_p \left[(\sqrt{5})^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

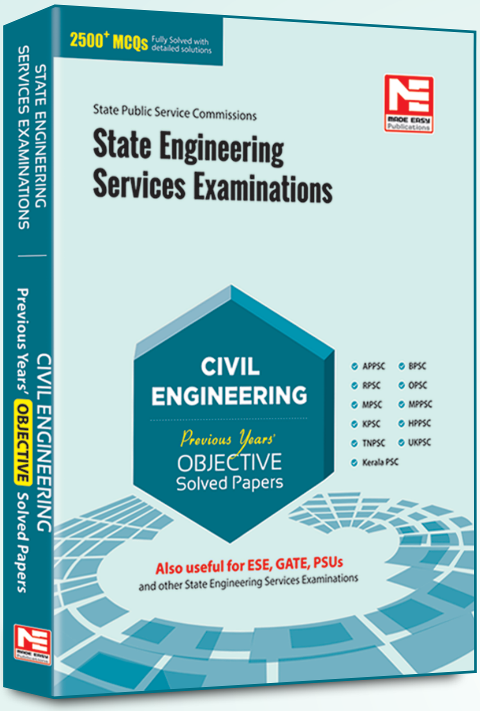
$$W_{c1} = W_{c2} = 77.5496 c_p \text{ kJ/kg}$$

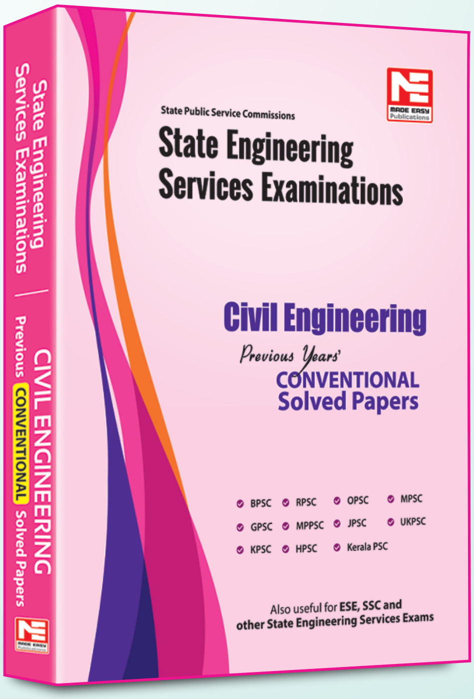
Total compressor work,
$$W_{TC} = 2 \times 77.5496 c_p$$
$$W_{TC} = 155.1 c_p \text{ kJ/kg}$$

$$\frac{T_4}{T_3} = (\sqrt{5})^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 377.55 \text{ K}$$

Since turbine, 1 give power to both compressors.

$\therefore$ $$W_{T1} = W_{TC}$$
$$c_p(T_5 - T_6) = 155.1 c_p$$


2500+ MCQs Fully Solved with detailed solutions
State Public Service Commissions
State Engineering Services Examinations
CIVIL ENGINEERING
Previous Years' OBJECTIVE Solved Papers
APPSC BPSC RPSC OPSC MPSC MPPSC KPSC HPPSC TNPSC UKPSC Kerala PSC
Also useful for ESE, GATE, PSUs
and other State Engineering Services Examinations


State Public Service Commissions
State Engineering Services Examinations
Civil Engineering
Previous Years' CONVENTIONAL Solved Papers
BPSC RPSC OPSC MPSC GPSC MPPSC JPSC UKPSC KPSC HPSC Kerala PSC
Also useful for ESE, SSC and other State Engineering Services Exams

$$T_6 = 1250 - 155.1 = 1094.9 \text{ K}$$

Pressure ratio across turbine 1

$$r_{\text{PT1}} = \left(\frac{T_5}{T_6}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1250}{1094.9}\right)^{\frac{1.4}{0.4}} = 1.59$$

Since total pressure ratio is 5, so pressure ratio across free power turbine is

$$r_{\text{PT2}} = \frac{5}{1.59} = 3.14465$$

Power of free power turbine

$$W_{T2} = c_p(T_7 - T_8)$$

$$\frac{T_7}{T_8} = (r_{PT2})^{\frac{\gamma-1}{\gamma}} = (3.14465)^{0.4/1.4}$$

$$T_8 = \frac{1250}{1.3872} = 901.1\text{K}$$

$\therefore$ $$W_{T2} = c_p \times (1250 - 901.1) = 348.9c_p \text{ kJ/kgK}$$

For air, $$c_p = 1.005 \text{ kJ/kgK}$$

So, power output of free power turbine

$$W_{T2} = 348.9 \times 1.005 = 350.644 \text{ kJ/kg}$$

Heat added

(i) Without regeneration

$$\begin{aligned} Q_1 &= c_p(T_5 - T_4) + c_p(T_7 - T_6) \\ &= c_p(1250 - 377.5 + 1250 - 1094.9) \\ &= 1027.55c_p \text{ kJ/kg} \end{aligned}$$

(ii) With perfect regeneration $[T_a = T_8]$

$$\begin{aligned} Q_2 &= c_p(T_5 - T_a) + c_p(T_7 - T_6) \\ &= c_p(1250 - 901.1 + 1250 - 1094.9) \\ &= 504c_p \text{ kJ/kg} \end{aligned}$$

Efficiency

(i) Without regeneration

$$\eta_1 = \frac{W_{T1} + W_{T2} - W_{TC}}{Q_1} \qquad W_{T1} = W_{TC}$$

$$\eta_1 = \frac{W_{T2}}{Q_1} = \frac{348.9c_p}{1027.55c_p} \times 100$$

$$\eta_1 = 33.97\%$$

(ii) With perfect regeneration

$$\eta_2 = \frac{W_{T1} + W_{T2} - W_{TC}}{Q_2} \qquad W_{T1} = W_{TC}$$

$$\eta_2 = \frac{W_{T2}}{Q_2} = \frac{348.9c_p}{504c_p} \times 100 = 69.22\%$$

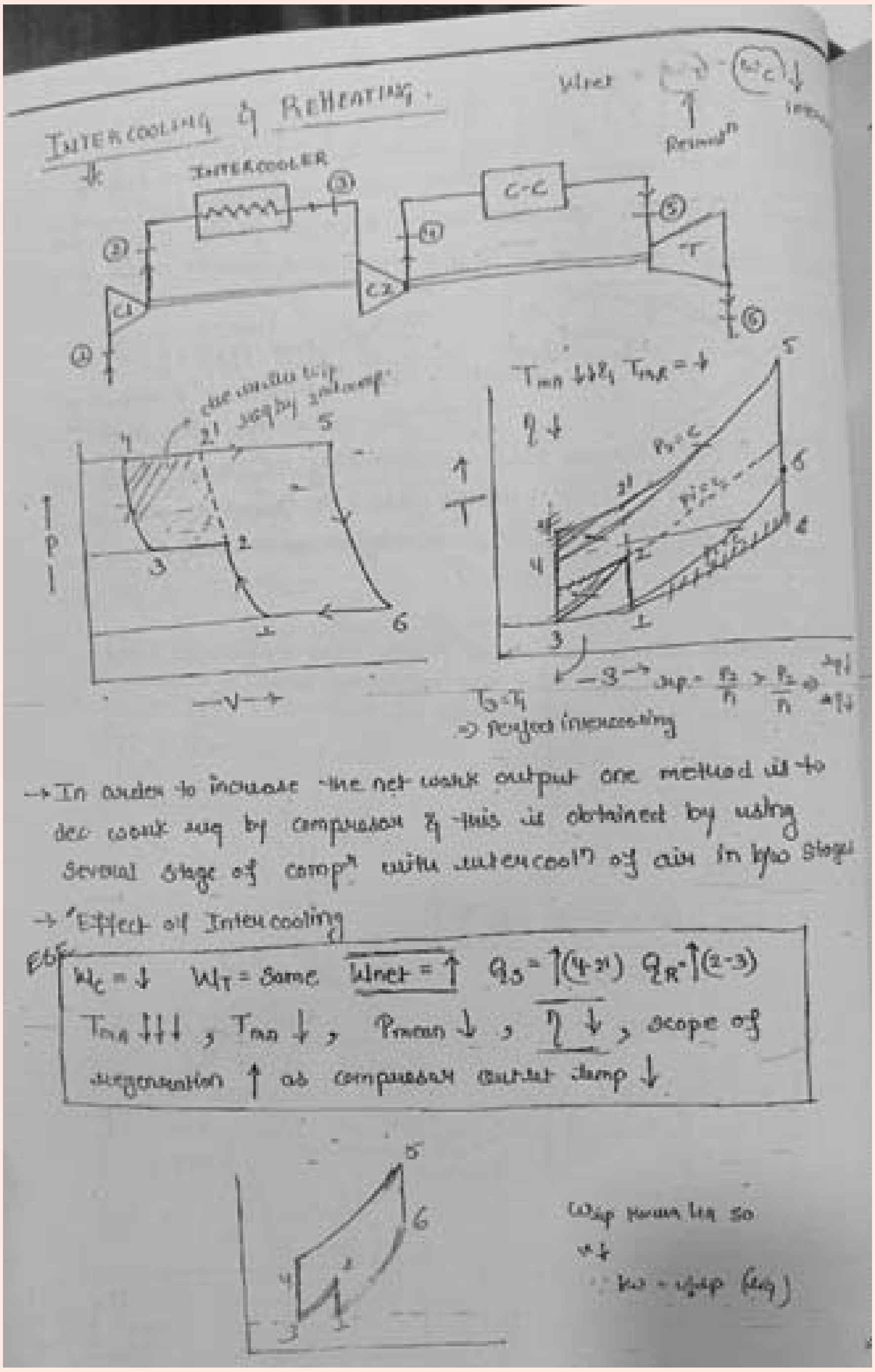
% improvement in efficiency,

$$= \frac{\eta_2 - \eta_1}{\eta_1} \times 100 = \frac{69.22 - 33.97}{33.97} \times 100$$

$$= 103.768\%$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Similar to Q.6(c), Test-14
- **MADE EASY Classnotes**



→ In order to increase the net work output one method is to dec work req by compressor & this is obtained by using several stage of compⁿ with intercoolⁿ of air in two stage

→ Effect of Intercooling

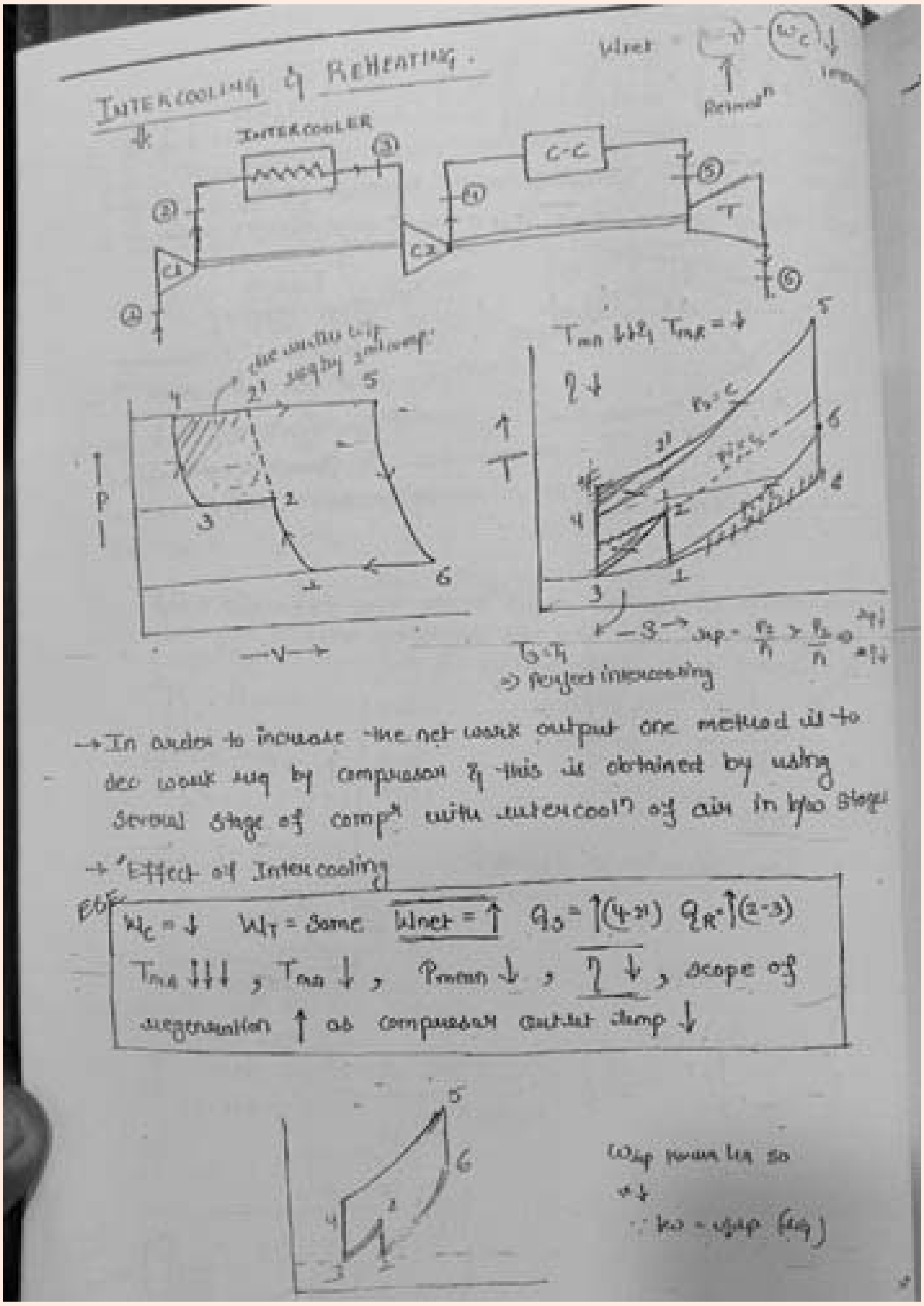
ESE

W_C = ↓ W_T = Same $\boxed{W_{net} = \uparrow}$ Q_S = ↑(4-2') Q_R = ↑(2-3)

T_{ma} ↓↓↓ , T_{mr} ↓ , P_{mean} ↓ , η ↓ , scope of regeneration ↑ as compressor outlet temp ↓

Wsp remain less so

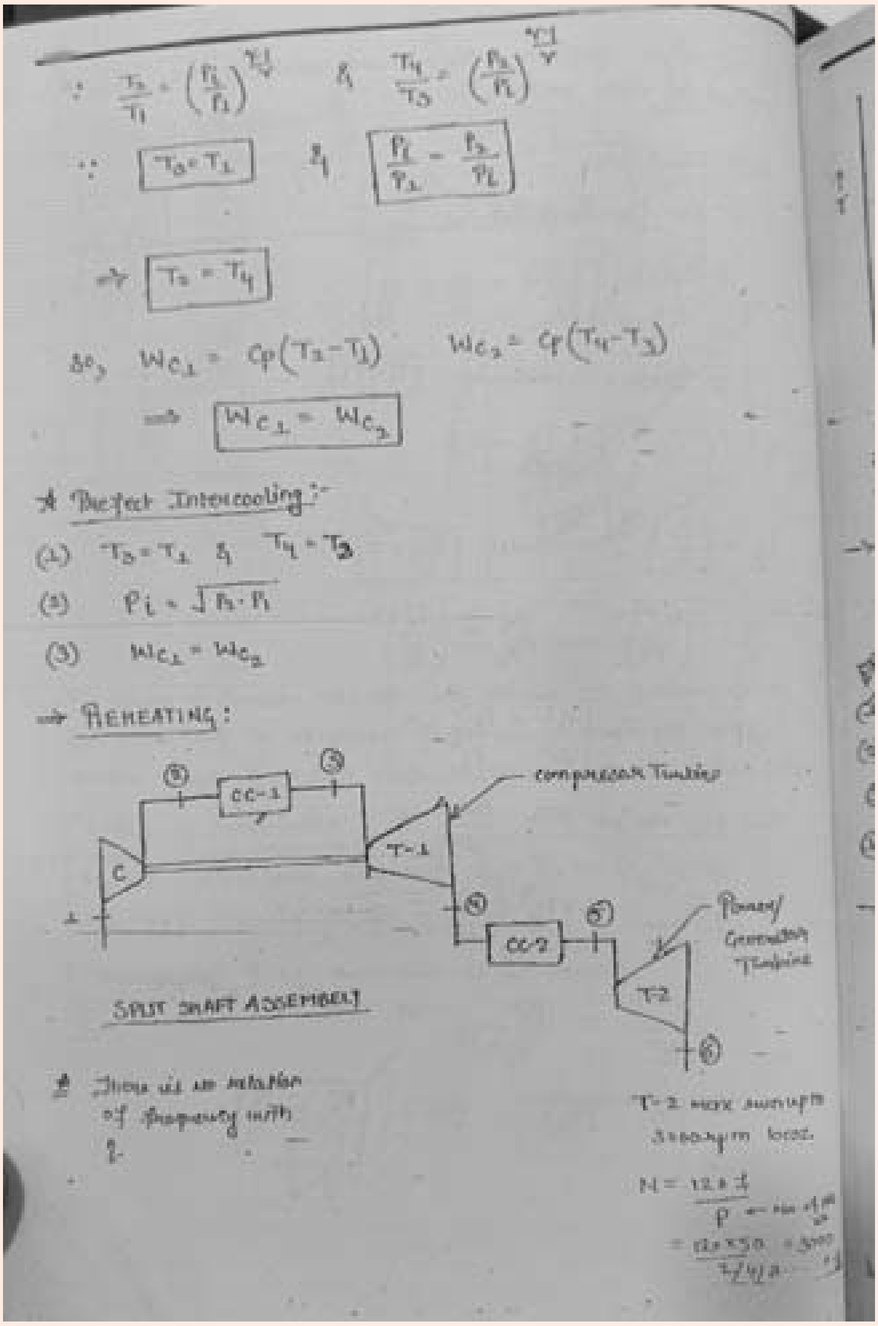
η ↓



→ In order to increase the net work output one method is to dec work req by compressor & this is obtained by using several stage of compⁿ with intercoolⁿ of air in two stage

→ Effect of Intercooling

W_c = ↓ W_T = Same Wnet = ↑ q_s = ↑(4-2') q_R ↑(2-3)
Tmin ↓↓↓, Tmax ↓, Pmean ↓, η ↓, scope of regeneration ↑ as compressor outlet temp ↓



∴ $\frac{T_2}{T_1} = \left(\frac{P_i}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ & $\frac{T_4}{T_3} = \left(\frac{P_2}{P_i}\right)^{\frac{\gamma-1}{\gamma}}$

∵ $\boxed{T_3 = T_1}$ & $\boxed{\frac{P_i}{P_1} = \frac{P_2}{P_i}}$

→ $\boxed{T_2 = T_4}$

So, $W_{c_1} = C_p(T_2 - T_1)$ $W_{c_2} = C_p(T_4 - T_3)$

⇒ $\boxed{W_{c_1} = W_{c_2}}$

★ Perfect Intercooling :-

(1) $T_3 = T_1$ & $T_4 = T_2$

(2) $P_i = \sqrt{P_2 \cdot P_1}$

(3) $W_{c_1} = W_{c_2}$

→ REHEATING :

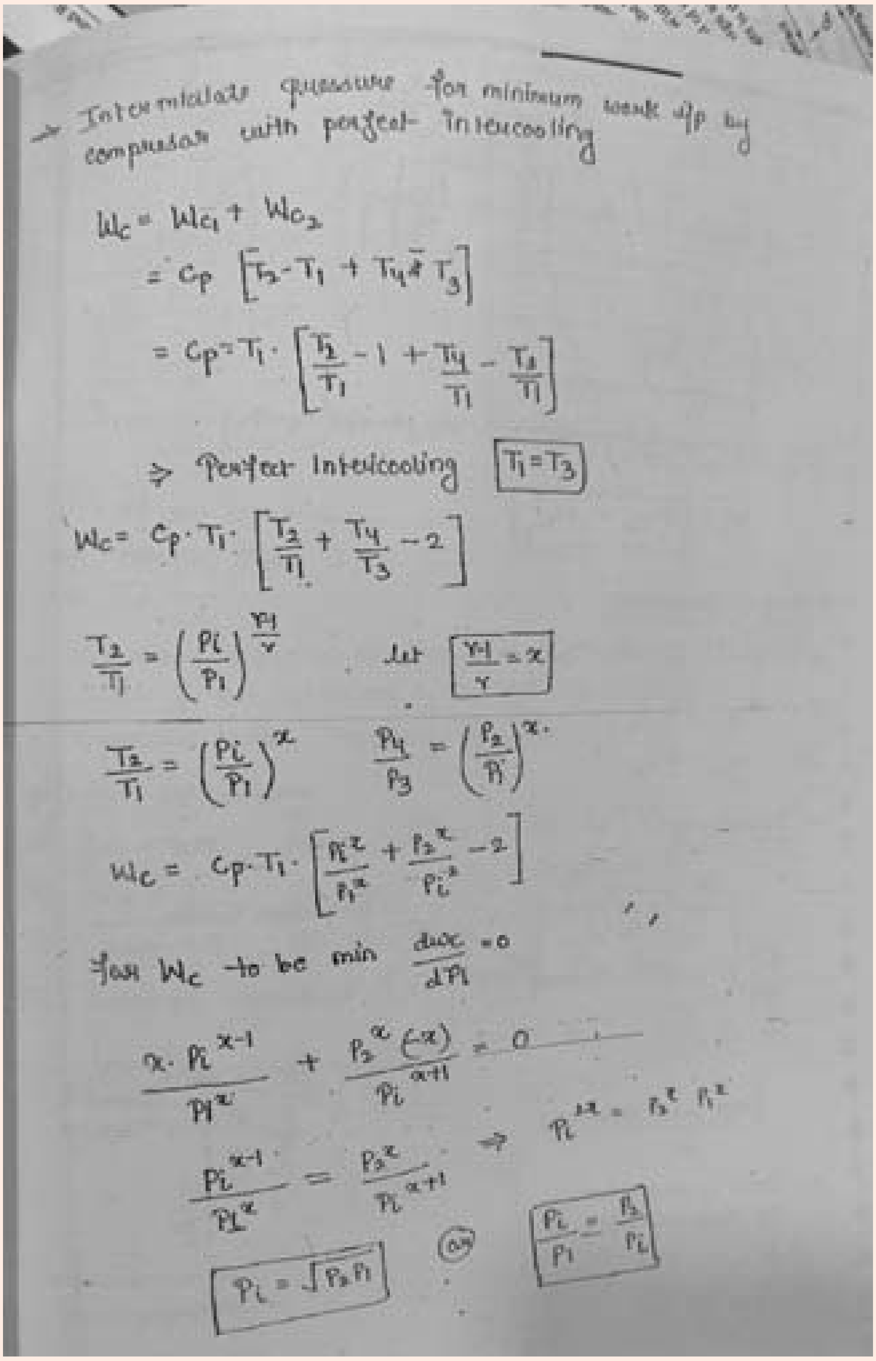
SPLIT SHAFT ASSEMBLY

★ There is no relation of frequency with η.

T-2 here runs upto 3000 rpm bcoz.

$N = \frac{120 f}{P}$ ← no. of poles

$= \frac{120 \times 50}{2} = 3000$



→ Intermidiate pressure for minimum work i/p by compressor with perfect Intercooling

$W_c = W_{c_1} + W_{c_2}$

$= C_p \left[T_2 - T_1 + T_4 - T_3\right]$

$= C_p \cdot T_1 \cdot \left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - \frac{T_3}{T_1}\right]$

⇒ Perfect Intercooling $\boxed{T_1 = T_3}$

$W_c = C_p \cdot T_1 \cdot \left[\frac{T_2}{T_1} + \frac{T_4}{T_3} - 2\right]$

$\frac{T_2}{T_1} = \left(\frac{P_i}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ let $\boxed{\frac{\gamma-1}{\gamma} = x}$

$\frac{T_2}{T_1} = \left(\frac{P_i}{P_1}\right)^x$ $\frac{P_4}{P_3} = \left(\frac{P_2}{P_i}\right)^x$

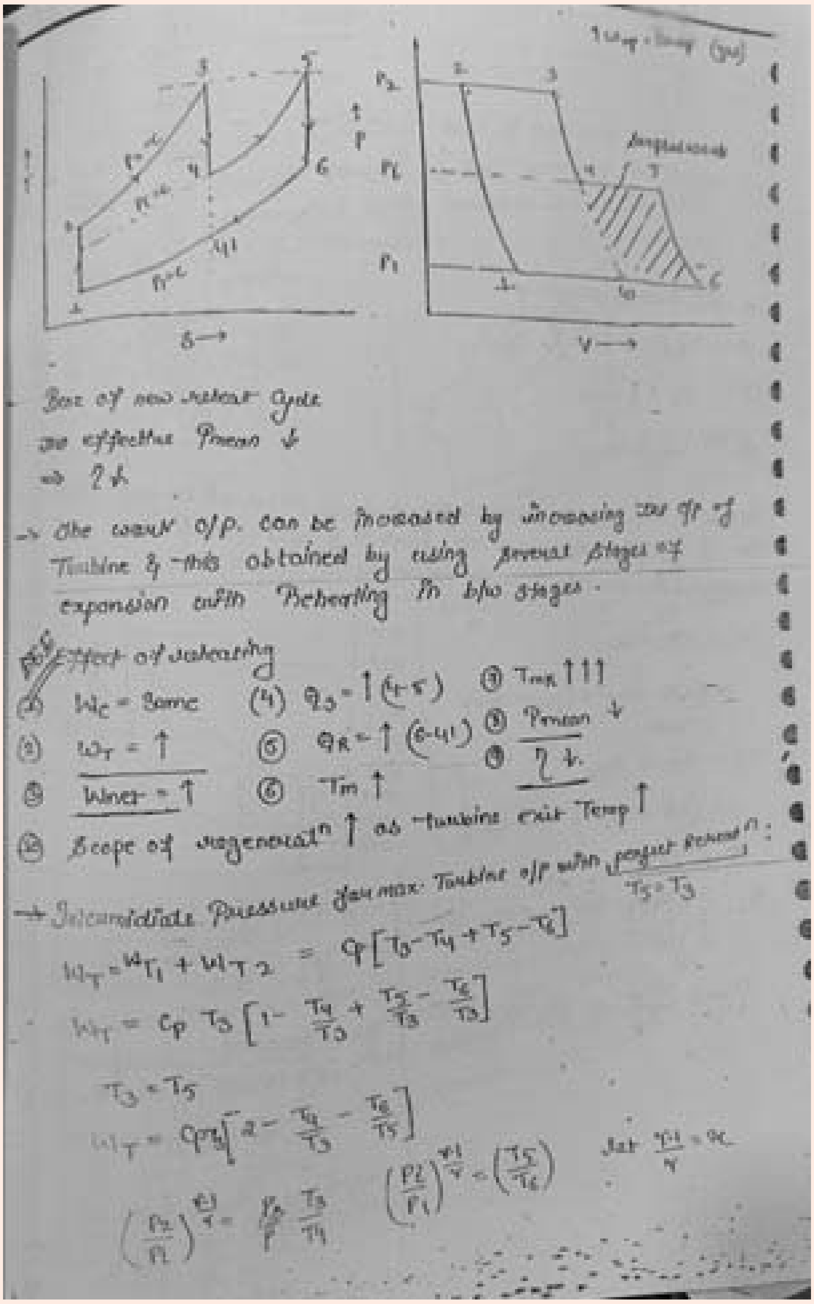
$W_c = C_p \cdot T_1 \cdot \left[\frac{P_i^x}{P_1^x} + \frac{P_2^x}{P_i^x} - 2\right]$

for W_c to be min $\frac{dW_c}{dP_i} = 0$

$\frac{x \cdot P_i^{x-1}}{P_1^x} + \frac{P_2^x(-x)}{P_i^{x+1}} = 0$

$\frac{P_i^{x-1}}{P_1^x} = \frac{P_2^x}{P_i^{x+1}} \Rightarrow P_i^{2x} = P_2^x P_1^x$

$\boxed{P_i = \sqrt{P_2 P_1}}$ or $\boxed{\frac{P_i}{P_1} = \frac{P_2}{P_i}}$



Bcz of new reheat cycle
the effective Pmean ↓
⇒ η ↓

→ The work o/p. can be increased by increasing the o/p of Turbine & this obtained by using several stages of expansion with Reheating in b/w stages.

Effect of reheating

① W_C = Same
② $W_T = \uparrow$
③ $W_{net} = \uparrow$
(4) $q_S = \uparrow$ (4-5)
⑤ $q_R = \uparrow$ (6-4')
⑥ $T_m \uparrow$
⑦ $T_{max} \uparrow\uparrow\uparrow$
⑧ $P_{mean} \downarrow$
⑨ $\eta \downarrow$
⑩ Scope of regeneratⁿ ↑ as turbine exit Temp ↑

→ Intermediate Pressure for max. Turbine o/p with perfect Reheatⁿ: $T_5 = T_3$

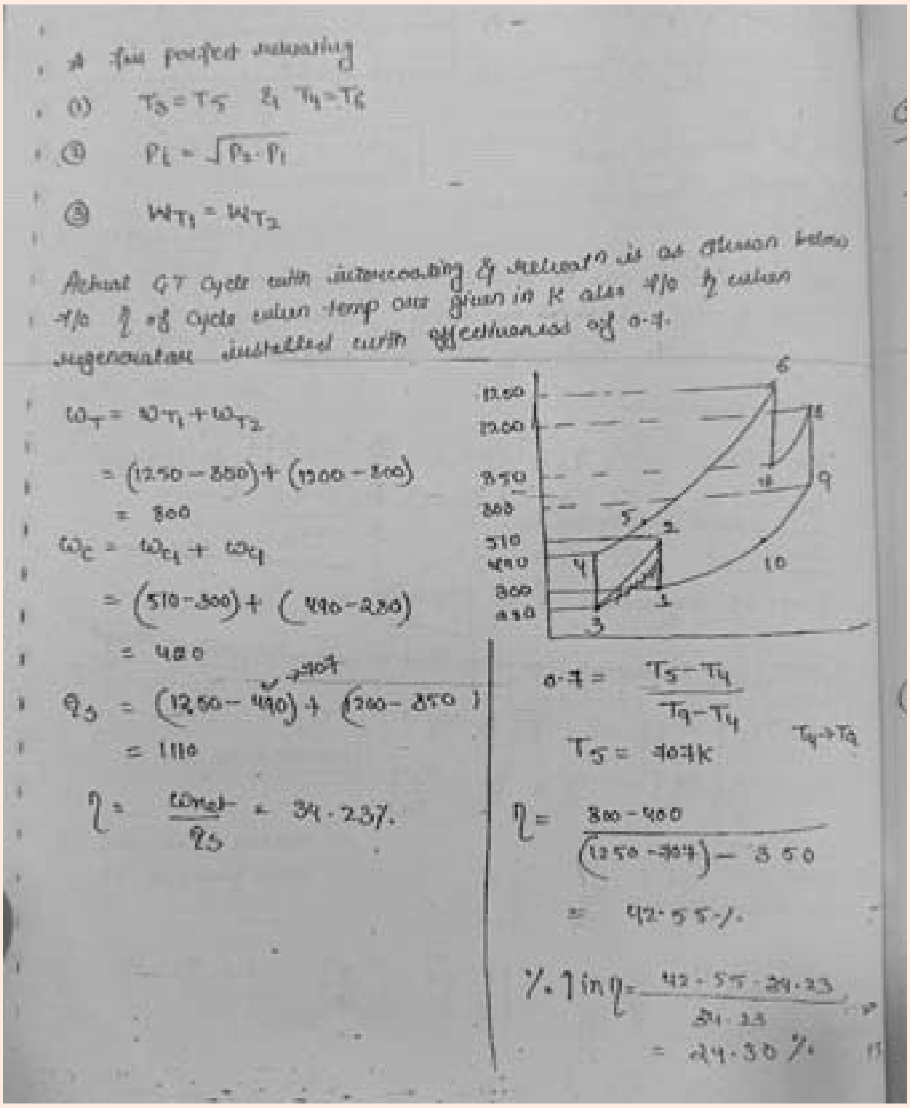
$$W_T = W_{T_1} + W_{T_2} = c_p[T_3 - T_4 + T_5 - T_6]$$

$$W_T = c_p T_3\left[1 - \frac{T_4}{T_3} + \frac{T_5}{T_3} - \frac{T_6}{T_3}\right]$$

$$T_3 = T_5$$

$$W_T = c_p T_3\left[2 - \frac{T_4}{T_3} - \frac{T_6}{T_5}\right]$$

$$\left(\frac{P_2}{P_i}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4} \qquad \left(\frac{P_i}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_5}{T_6}\right) \qquad \text{let } \frac{\gamma-1}{\gamma} = x$$



* for perfect intercooling

(1) $T_3 = T_5$ & $T_4 = T_6$

(2) $P_i = \sqrt{P_2 \cdot P_1}$

(3) $W_{T_1} = W_{T_2}$

Actual GT cycle with intercooling & reheatⁿ is as shown below. t/o η of cycle when temp are given in K also t/o η when regenerator installed with effectiveness of 0.7.

$w_T = w_{T_1} + w_{T_2}$

$= (1250 - 850) + (1200 - 800)$

$= 800$

$w_C = w_{C_1} + w_{C_2}$

$= (510 - 300) + (490 - 230)$

$= 480$

$q_S = (1250 - \overset{707}{490}) + (1200 - 850)$

$= 1110$

$\eta = \frac{w_{net}}{q_S} = 34.23\%$

$0.7 = \frac{T_5 - T_4}{T_9 - T_4}$

$T_5 = 707\,K$

$T_4 \rightarrow T_9$

$\eta = \frac{800 - 480}{(1250 - 707) - 350}$

$= 42.55\%$

$\% \uparrow \text{in } \eta = \frac{42.55 - 34.23}{34.23}$

$= 24.30\%$

End of Solution