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Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 15

Section-A

Q.1 (a) Solution:

(i) The characteristic equation from state model can be calculated as

$$|sI - A| = 0$$

Here, $$A = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$$

$\therefore$ $$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s+1 & 0 \\ 0 & s+3 \end{bmatrix}$$

and therefore,

$$|sI - A| = 0$$

$$(s + 1)(s + 3) = 0$$

$$s^2 + 4s + 3 = 0$$

On comparing the characteristic equation by standard second order characteristic equation, we get,

$$\omega_n^2 = 3$$

$\Rightarrow$ $$\omega_n = \sqrt{3} \text{ rad/sec}$$

(ii) $$2\xi\omega_n = 4$$

or $$\xi = \frac{4}{2\omega_n} = \frac{4}{2\sqrt{3}} = 1.15$$

$\therefore$ The system response is overdamped response.

(iii) $$\%M_{po} = 0 \quad \text{(for overdamped)}$$

(iv) $$\tau_s = \frac{4}{\xi\omega_n} = \frac{4}{2} = 2 \text{ sec}$$

Q.1 (b) Solution:

Maxwell's equations for time varying fields in differential forms:

(i) $\nabla \cdot \vec{D} = \rho$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = \dfrac{-\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{H} = \sigma\vec{E} + \dfrac{\partial \vec{D}}{\partial t} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

Maxwell's equations for time-varying fields in integral form:

(i) $\oiint_s \vec{D} \cdot d\vec{s} = \iiint_v \rho dv$

(ii) $\oiint_s \vec{B} \cdot d\vec{s} = 0$

(iii) $\oint_c \vec{E} \cdot d\vec{\ell} = -\iint_s \dfrac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

(iv) $\oint_c \vec{H} \cdot d\vec{\ell} = \iint_s \left(\sigma\vec{E} + \dfrac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s} = \iint_s \left(\vec{J} + \dfrac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s}$

Word statement of maxwell's equations are follows :

(i) The total electric flux density or total electric displacement ($\vec{D}$) through the surface enclosing a volume v is equal to the total charge within the volume.

(ii) The net magnetic flux emerging through any close surface is zero.

(iii) The electromotive force around a closed path is equal to the time derivative of the magnetic flux density or magnetic displacement $\vec{B}$ through any surface bounded by the path.

(iv) The magnetomotive force around a closed path is equal to the conduction current ($\vec{J}$) plus the time derivative of the electric flux density $\vec{D}$ through any surface bounded by the path.

Q.1 (c) Solution:

Bandwidth of the telephone line (B_T) = 3 kHz

(i) For polar signal with rectangular half-width pulses, minimum channel bandwidth required is $2R_b$.

So,
$$2R_b \le B_T$$
$$R_b \le 1.5 \text{ kbps}$$
$$R_{b(\max)} = 1.5 \text{ kbps}$$

(ii) For polar signal with rectangular full-width pulses, minimum channel bandwidth required is R_b.

So,
$$R_b \le B_T$$
$$R_b \le 3 \text{ kbps}$$
$$R_{b(\max)} = 3 \text{ kbps}$$

(ii) For polar signal using raised cosine pulses with a roll-off factor of 0.25, minimum channel bandwidth required is $\frac{R_b}{2}(1+0.25)$.

So,
$$\frac{R_b}{2}(1.25) \le B_T$$
$$\frac{R_b}{1.6} \le 3 \text{ kbps}$$
$$R_{b(\max)} = 3 \times 1.6 = 4.8 \text{ kbps}$$

(iv) For bipolar signal with rectangular half-width pulses, minimum channel bandwidth required is R_b.

So,
$$R_b \le B_T$$
$$R_b \le 3 \text{ kbps}$$
$$R_{b(\max)} = 3 \text{ kbps}$$

(v) For bipolar signal with rectangular full-width pulses, minimum channel bandwidth required is R_b.

So,
$$R_b \le B_T$$
$$R_b \le 3 \text{ kbps}$$
$$R_{b(\max)} = 3 \text{ kbps}$$

Q.1 (d) Solution:

(i) First of all we have to check whether the given system is linear or Non-linear. For this we assume,

$$y_1(n) = \begin{cases} x_1(n) & ; \quad n \geq 1 \\ 0 & ; \quad n = 0 \\ x_1(n+1) & ; \quad n \leq -1 \end{cases} \qquad \text{...(i)}$$

and
$$y_2(n) = \begin{cases} x_2(n) & ; \quad n \geq 1 \\ 0 & ; \quad n = 0 \\ x_2(n+1) & ; \quad n \leq -1 \end{cases} \qquad \text{...(ii)}$$

Let, $$x_3'(n) = a x_1(n) + b x_2(n) \qquad \text{...(iii)}$$

Let, $$y_3'(n) = T\{x_3'(n)\} \qquad \text{...(iv)}$$

Here, 'T' represents the transformation from $x[n]$ to $y[n]$.

Now, from equation (iv) and equation (iii) we have,

$$y_3'(n) = T\{a x_1(n) + b x_2(n)\}$$

$\therefore$ $$y_3'(n) = ay_1(n) + by_2(n) \qquad \text{(For linearity)} \qquad \text{...(v)}$$

Since, $$y_3'(n) = \begin{cases} x_3'(n) & ; \quad n \geq 1 \\ 0 & ; \quad n = 0 \\ x_3'(n+1) & ; \quad n \leq -1 \end{cases}$$

Now, $$x_3'(n+1) = ax_1(n+1) + bx_2(n+1)$$

and let, $$y_3(n) = ay_1(n) + by_2(n) \qquad \text{...(vi)}$$

Since, $$y_3(n) = y_3'(n)$$

So, the system is **linear**.

(ii) Since, $y(n) = x(n+1)$ for $n \leq -1$ so, we can say $y(n)$ depends on the future value of input sequence, so we may say that given system is not causal.

(iii) As given in the question,

$$\begin{aligned} y(n) &= x(n); \quad n \geq 1 \\ & 0; \quad n = 0 \\ & x(n+1); \quad n \leq -1 \end{aligned}$$

So,
$$\begin{aligned} y(n-k) &= x(n-k); \quad n-k \geq 1 \\ & 0; \quad n-k = 0 \\ & x(n-k+1); \quad n-k \leq -1 \end{aligned}$$

Now, $y(n, k) = T[x(n, k)]$

Since, $y_1(n, k) = T[x_1(n, k)]$

where, 'T' is the transformation from $x[n]$ to $y[n]$.

Let, $x_1(n, k) = x(n-k)$ i.e., shifted by 'k' unit.

So,
$$y_1(n, k) = T[x(n-k)]$$
$$= \begin{cases} x(n-k); & n \geq 1 \\ 0; & n = 0 \\ x(n-k+1); & n \leq -1 \end{cases}$$

Because, $y_1(n, k) \neq y(n-k)$

So, the system is **Time-variant.**

(iv) For this system when the input is bounded, the output is also bounded. So, the system is **stable**.

Q.1 (e) Solution:

(i) At 5 MHz, one clock cycle takes 0.2 μs.

To transfer one byte of information, one DMA cycle is required, and each DMA cycle needs three clock cycles.

So, the time required to transfer one byte = 0.6 μs.

(ii) The maximum data rate = 1/(0.6 μs) = 1.67×10^6 bytes/s

(iii) Two wait cycles consume 0.4 μs. So, it will take (0.4 + 0.6) μs = 1 μs to transfer one byte.

Hence, the actual data rate = 1/(1 μs) = 10^6 bytes/s

Q.2 (a) Solution

(i) The average power of the message signal, is

$$P_m = \int_{-W}^{W} S_M(f)\, df = 2S_0 f_0 \int_0^W \frac{(1/f_0)}{1+\left(\frac{f}{f_0}\right)^2} df$$

$$= 2S_0 f_0 \left[\tan^{-1}\left(\frac{f}{f_0}\right)\right]_0^W = 2S_0 f_0 \tan^{-1}\left(\frac{W}{f_0}\right)$$

The average power of the emphasized signal is,

$$P_y = \int_{-W}^{W} S_M(f)\,|H(f)|^2 = \int_{-W}^{W} S_0 K^2 df = 2S_0 K^2 W$$

To have $P_y = P_m$,

$$2S_0 K^2 W = 2S_0 f_0 \tan^{-1}\left(\frac{W}{f_0}\right)$$

$$K = \sqrt{\left(\frac{f_0}{W}\right)\tan^{-1}\left(\frac{W}{f_0}\right)} = \sqrt{\frac{1}{2}\tan^{-1}(2)} = 0.744$$

Note: While calculating $\tan^{-1}(2)$, calculator should be in radians mode, but not in degrees mode.

(ii) For PM signal :

$$\Delta\phi_{max} = k_p |m(t)|_{max} = k_p \text{ rad} \quad ...(i)$$

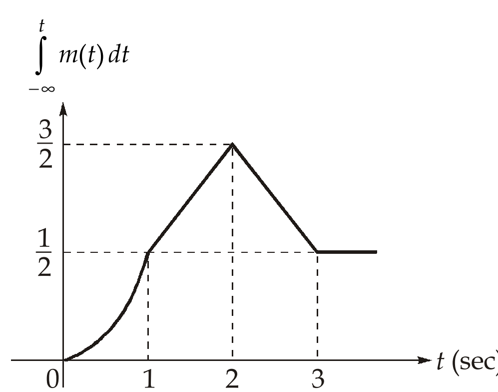
For FM signal :

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(t)\,dt$$

$$m(t) = tu(t) - (t-1)u(t-1) - 2u(t-2) + u(t-3)$$

$$\int_{-\infty}^{t} m(t)\,dt = \frac{t^2}{2}u(t) - \frac{(t-1)^2}{2}u(t-1) - 2(t-2)u(t-2) + (t-3)u(t-3)$$

The signal $\int_{-\infty}^{t} m(t)\,dt$ can be plotted as,



$$\Delta\phi_{max} = |\phi(t)|_{max} = 2\pi k_f \left(\frac{3}{2}\right) = 3\pi k_f \text{ rad} \quad ...(ii)$$

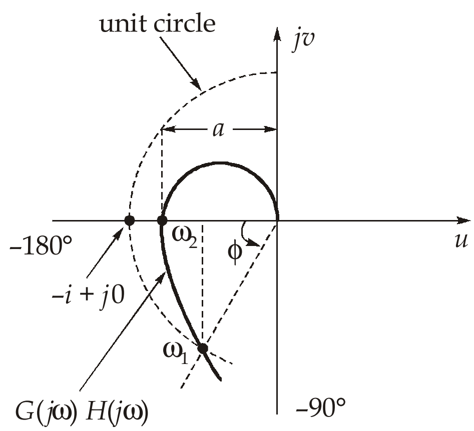
When the maximum phase deviation is same for both the modulated waveforms, From equations (i) and (ii),

$$k_p = 3\pi k_f$$

Q.2 (b) Solution:

(i) Gain margin from Nyquist diagram:

Consider a Nyquist plot as shown in below,



- Gain margin is the factor by which the system gain can be increased to drive it to the verge of instability.
- From the Nyquist diagram, the gain margin may be defined as the reciprocal of the gain at the frequency at which the phase angle becomes 180°. The frequency at which the phase angle is 180° is called the phase cross-over frequency (ω_2) represented as ω_{pc}.

In the diagram, we have,

$$GM = 1/a$$

Where $$a = |G(j\omega)\,H(j\omega)|_{\omega=\omega_2}$$

In decibels, $$\boxed{GM = 20\log\left(\frac{1}{a}\right) dB = -20\log a \text{ dB}}$$

- Phase margin from Nyquist diagram: Phase margin is defined as the amount of additional phase-lag at the gain cross-over frequency required to bring the system to the verge of instability.

In the Nyquist diagram, ω_1 is the gain cross-over frequency at which $|G(j\omega)\,H(j\omega)| = 1$. The additional phase-lag ϕ driving the system to the verge of instability is called the phase margin.

Thus, $$PM = \angle G(j\omega)\,H(j\omega)|_{\omega=\omega_1} + 180°$$

where the angle at ω_1 is measured negatively.

(ii) Since, $H(s) = 1$

The loop transfer function in frequency domain can be written as,

$$G(j\omega)\,H(j\omega) = \frac{e^{-j0.1\omega}}{1+j0.5\omega} = \frac{|1|\angle -0.1\omega}{\sqrt{(1+0.25\omega^2)}\angle \tan^{-1} 0.5\omega}$$

$$= \frac{1}{\sqrt{1+0.25\omega^2}}\angle(-0.1\omega - \tan^{-1} 0.5\omega)$$

Since at the phase crossover frequency ω_p the phase equals ±180°, we have

$$-0.1\,\omega_p - \tan^{-1} 0.5\,\omega_p = -\pi$$

or, $$0.1\omega_p + \left(\frac{\pi}{2} - \frac{1}{0.5\omega_p}\right) = \pi \qquad \left[\because \tan^{-1} x \cong \frac{\pi}{2} - \frac{1}{x} \text{ for } x > 1\right]$$

or, $$0.1\omega_p - \frac{1}{0.5\omega_p} = \frac{\pi}{2}$$

or, $$\frac{0.05\omega_p^2 - 1}{0.5\omega_p} = \frac{\pi}{2} = 1.571$$

or, $$0.05\omega_p^2 - 0.785\omega_p - 1 = 0; \qquad \text{...(i)}$$

Equation (i) yields two values of ω_p, namely, 16.9 rad/s and –1.18 rad/s.

The later value is rejected because the frequency cannot be negative.

The gain margin is calculated as,

$$\text{Gain margin} = \sqrt{1+0.25\omega_p^2} = 8.51$$

and $$[\text{Gain margin}]_{\text{dB}} = -20\log\frac{1}{\sqrt{1+0.25\omega_p^2}}$$

$$= -20\log\frac{1}{\sqrt{1+0.25\times 16.9^2}} = 18.6 \text{ dB}$$

Q.2 (c) Solution:

(i)

```
LXI H, 2060H
MOV C, M
LXI D, 0000H
```

```
LOOP:   INX H
        MOV A, M
        MOV B, A
        RLC
        JC SKIP
        RRC
        RRC
        JC SKIP
        MOV A, B
        ADD E
        MOV E, A
        JNC SKIP
        INC D
SKIP:   DCR C
        JNZ LOOP
        MOV A, E
        STA 3000H
        MOV A, D
        STA 3001H
        HLT
```

(ii) Memory mapped I/O:

In this scheme, there is only one address space. This address space is allocated to both memory and I/O devices. Some addresses are assigned to memories and some to I/O devices. The address for I/O devices is different from the addresses which have been assigned to memories. An I/O device is also treated as a memory location. In this scheme one address is assigned to each memory location and one address is assigned to each I/O device.

In this scheme, all data transfer instructions of the microprocessor can be used for transferring data from and to either memory or I/O devices. For example, MOV D, M instruction would transfer one byte of data from a memory location or an input device to the register D, depending on whether the address in the H-L register pair is assigned to a memory location or to an input device. If H-L contains address of a memory location, data will be transferred from that memory location to register D, while if H-L pair contains the address of an input device, data will be transferred from that input device to register D.

This scheme is suitable for systems using microprocessor with less number of instructions and which require less number of I/O devices. We can see this type of scheme in dedicated systems.

Advantages:

- No special instructions are required for communicating with I/O devices.
- Direct data transfer is possible between any register of the microprocessor and I/O devices.
- All the arithmetic and logical operations can be directly performed with I/O data.
- Special status/control signal, like IO/$\bar{M}$ in 8085 microprocessor, to distinguish between memory and I/O transfer is not required.

Disadvantages:

- Decoding 16-bit address for I/O device requires more hardware circuitry.
- By assigning some address space for I/O devices, we cannot use the system memory efficiently.

I/O mapped I/O:

Some CPUs provide one or more control lines (for example, IO/$\bar{M}$ line for 8085), the status of which indicates either memory or I/O operation. When the status of IO/$\bar{M}$ line is high, it points to I/O operation and when low, it points to memory operation. Thus, in this case, the same address may be assigned to both memory and an I/O device-depending on the status of IO/$\bar{M}$ line. This scheme is referred to as I/O mapped I/O scheme.

Here two separate address spaces exist-one space is meant exclusively for memory operations and the other for I/O operations. Usually, the space assigned for I/O is much smaller than memory space.

Advantages:

- As the address space assigned to I/O devices is less compared to memory space, circuitry required to decode the address of I/O devices is less complex.
- As separate address spaces are assigned to both I/O devices and memory, we can use the system memory efficiently.

Disadvantages:

- Separate instructions are required for I/O communication.
- Direct data transfer is possible between I/O devices and accumulator only but not with other registers of microprocessor.
- Arithmetic and logical instructions cannot be performed directly with I/O data.
- Special status/control signal is required by the microprocessor to distinguish between I/O and memory transfer.

Q.3 (a) Solution:

(i)
$$\frac{E_b}{N_0} = \frac{P_R T_b}{N_0} = \frac{P_R L_c T_c}{N_0} = \frac{P_R L_c}{N_0 B}$$

$$= \left(\frac{P_R}{N}\right) L_c = (\text{SNR}) L_c$$

Processing gain, $L_c = \dfrac{(E_b/N_0)}{(\text{SNR})} = \dfrac{10}{10^{-2}} = 1000$

(ii) Received signal power,

$$P_R = (\text{SNR})N = 10^{-2}\,N$$

Noise power, $N = N_0 B$

$$N_0 = kT = 4.142 \times 10^{-21}\ \text{W/Hz}$$

$$B = 100\ \text{kHz} = 10^5\ \text{Hz}$$

So, $N = N_0 B = 4.142 \times 10^{-16}$ W

$$P_R = 4.142 \times 10^{-18}\ \text{W}$$

$$P_R = \frac{P_T G_T}{4\pi R^2}\left(\frac{\lambda^2}{4\pi}\right) G_R$$

Since, the receiver antenna is omnidirectional, $G_R = 1$.

So, $P_R = P_T G_T \left(\dfrac{\lambda}{4\pi R}\right)^2$

$$P_T = \left(\frac{4\pi R}{\lambda}\right)^2 \left(\frac{P_R}{G_T}\right)$$

$$[G_T] = 20\ \text{dB} \Rightarrow G_T = 100$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6} = 100\ \text{m}$$

So, $P_T = \left(\dfrac{4\pi\times2000\times1000}{100}\right)^2 \left(\dfrac{4.412\times10^{-18}}{100}\right)$ W

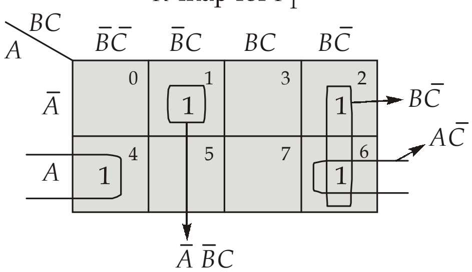
$$\approx 2.8 \times 10^{-9}\ \text{W} = 2.8\ \text{nW}$$

In decilogs, $[P_T] = 10\log_{10}(P_T) = -85.5$ dBW

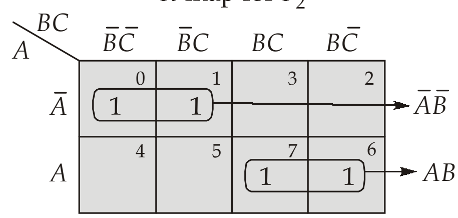
Q.3 (b) Solution

- By minimizing the given functions using K-map, we get,

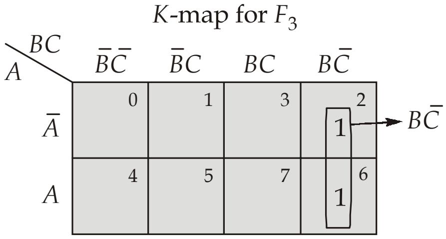
K-map for F_1



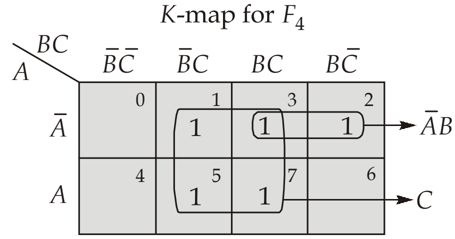
K-map for F_2



K-map for F_3



K-map for F_4

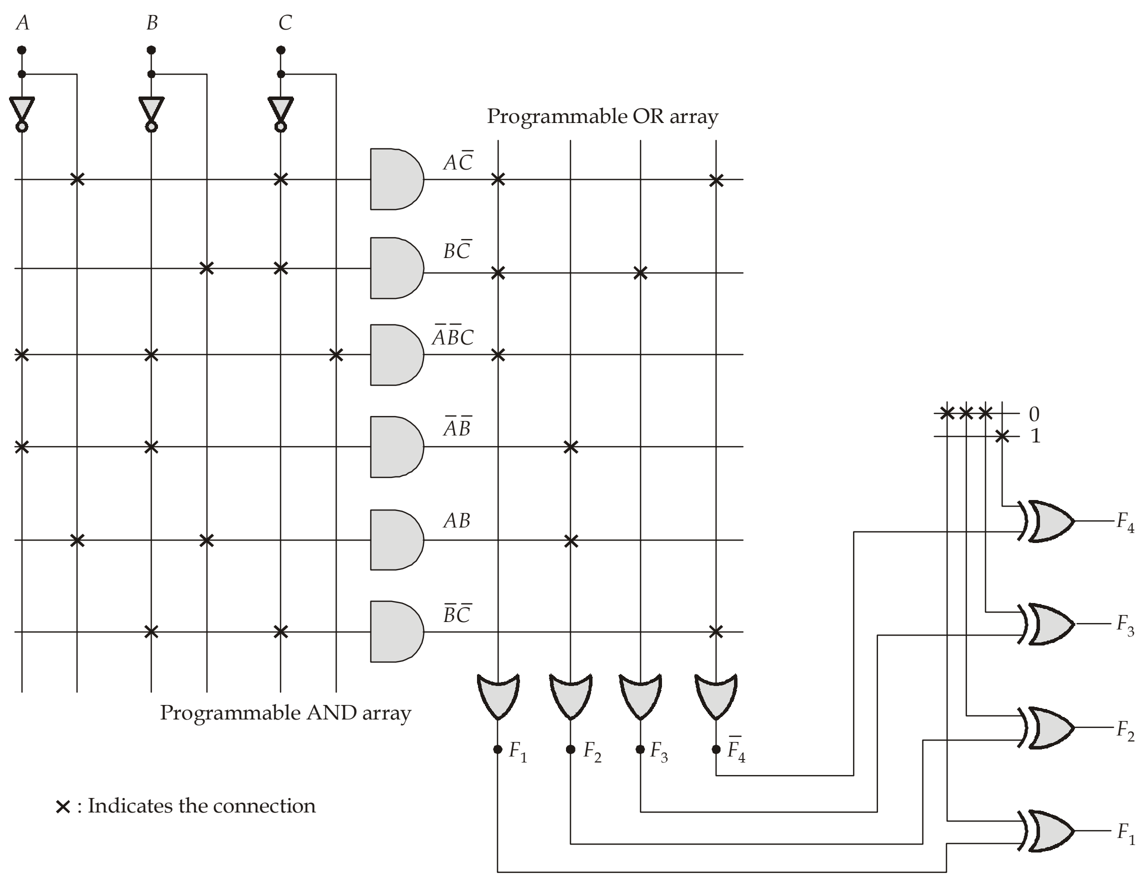


$$\left.\begin{aligned} F_1 &= A\bar{C} + B\bar{C} + \bar{A}\bar{B}C \\ F_2 &= \bar{A}\bar{B} + AB \\ F_3 &= B\bar{C} \\ F_4 &= C + \bar{A}B \end{aligned}\right\} \text{7 unique product terms} \Rightarrow \begin{array}{c}\text{design is not possible. because,} \\ \text{with the given PLA IC, only 6 unique} \\ \text{minterms are possible to form}\end{array}$$

$$\bar{F}_4 = (\overline{C + \bar{A}B}) = A\bar{C} + \bar{B}\bar{C}$$

$F_1, F_2, F_3, \bar{F}_4 \Rightarrow$ 6 unique product terms $\Rightarrow$ design is possible.

Implementation:



Q.3 (c) Solution:

```
#include <stdio.h>
void main(void)
{
        int i, j, n;
        printf("\nEnter the Value of N: ");
        scanf("%d", &n);
        for(i = 0; i <= 2*n; i++)
        {
                for(j = 0; j <= 2*n; j++)
                {
                        if(i <= n)
                                if(j < n-i || j > n+i)
                                        printf(" ");
                                else
```

```
                        printf("*");
                else
                        if(j< i–n || j > 3*n–i)
                                printf(" ");
                        else
                                print("*");
            }
            printf("\n");
        }
}
```

Output:

```
Enter the Value of N: 10
                    *
                   ***
                  *****
                 *******
                *********
               ***********
              *************
             ***************
            *****************
           *******************
          *********************
           *******************
            *****************
             ***************
              *************
               ***********
                *********
                 *******
                  *****
                   ***
                    *
```

4. (a)

Reducing the inner loop

$$G_{mot}(s) = \frac{10/s}{1+0.1\times 10/s} = \frac{10}{s+1}$$

from the forward path

$$\omega(s) = \left[\frac{10k}{s(s+1)}\right]E(s)$$

$$E(s) = V_r(s) - \omega(s)$$

$$E(s) = V_r(s) - \left[\frac{10k}{s(s+1)}\right]E(s)$$

for unit ramp input $\quad V_r(s) = \dfrac{1}{s^2}$

$\therefore \quad E(s) = \left[\dfrac{s(s+1)}{s(s+1)+10k}\right] \times \dfrac{1}{s^2}$

Steady-state error $\quad e_{ss} = \lim\limits_{s\to 0} sE(s) = \lim\limits_{s\to 0} \dfrac{(s+1)}{s(s+1)+10k}$

or $\quad e_{ss} = \dfrac{1}{10k} = 0.01$

$\Rightarrow \quad \boxed{k = 10}$

$$T(s) = \frac{\omega(s)}{V_r(s)} = \frac{\frac{10k}{s(s+1)}}{1+\frac{10k}{s(s+1)}}$$

$\Rightarrow \quad T(s) = \dfrac{10k}{s(s+1)+10k}$

To find $S_k^T \quad S_k^T = \dfrac{\partial T}{\partial k} \times \dfrac{k}{T}$

$\because \quad T(s) = \dfrac{10k}{s(s+1)+10k}$

$$\frac{\partial T}{\partial k} = \frac{[\{s(s+1)+(10k)\}10 - 10k(10)]}{[s(s+1)+10k]^2}$$

$\Rightarrow \quad \dfrac{\partial T}{\partial k} \times \dfrac{k}{T} = \dfrac{s(s+1)}{s(s+1)+10k}$

$\therefore \quad S_k^T = \dfrac{s(s+1)}{s(s+1)+10k}$

at $k = 10$

$$\boxed{S_{10}^T = \frac{s(s+1)}{s(s+1)+100}}$$

At low frequencies s = $j\omega$, ω tending to zero

$$S_{10}^T = 0$$

Q.4 (b) Solution:

- Let us assume that the input random variable is X and the output random variable is Y.
- The capacity C of a discrete memoryless channel can be defined as the maximum mutual information $I(X;Y)$ in any single use of the channel, where the maximization is over all possible input probability distributions on X[i.e., $P(x_i)$].

Mathematically it can be expressed as,

$$C = \max_{\{P(x_i)\}} I(X;Y)$$

Finding mutual information $I(X;Y)$ in terms of input probabilities:

Let us assume that the input probabilities are,

$$P(x_0) = \alpha_0,\ P(x_1) = \alpha_1 \text{ and } P(x_2) = \alpha_2$$

Mutual information, $I(X;Y) = H(Y) - H(Y \mid X)$

where, $$H(Y) = \text{Output entropy} = -\sum_{j=0}^{2} P(y_i)\log_2 P(y_i)$$

$$H(Y|X) = \text{Conditional entropy} = -\sum_{i=0}^{2}\sum_{j=0}^{2} P(x_i, y_j)\log_2 P(y_i|x_i)$$

The channel matrix of the given channel can be written as,

$$\left[P(y_j|x_i)\right] = \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.25 & 0.25 & 0.50 \end{bmatrix} \end{array}$$

The joint probability matrix can be given as,

$$\left[P(x_i, y_j)\right] = \left[P(x_i)\right]_{\text{diagonal}}\left[P(y_j|x_i)\right]$$

$$= \begin{bmatrix} \alpha_0 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_2 \end{bmatrix}\begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.25 & 0.50 & 0.25 \\ 0.25 & 0.25 & 0.50 \end{bmatrix}$$

$$= \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.50\alpha_0 & 0.25\alpha_0 & 0.25\alpha_0 \\ 0.25\alpha_1 & 0.50\alpha_1 & 0.25\alpha_1 \\ 0.25\alpha_2 & 0.25\alpha_2 & 0.50\alpha_2 \end{bmatrix} \end{array}$$

$$H(Y|X) = -[0.50\alpha_0 \log_2(0.50) + 0.25\alpha_0 \log_2(0.25) + 0.25\alpha_0 \log_2(0.25) + 0.25\alpha_1 \log_2(0.25) + 0.50\alpha_1 \log_2(0.50) + 0.25\alpha_1 \log_2(0.25) + 0.25\alpha_2 \log_2(0.25) + 0.25\alpha_2 \log_2(0.25) + 0.50\alpha_2 \log_2(0.50)]$$

$$= (\alpha_0 + \alpha_1 + \alpha_2)\,[0.50\log_2 2 + 0.25\log_2 4 + 0.25\log_2 4]$$

$$= (\alpha_0 + \alpha_1 + \alpha_2)\,[0.50 + 0.50 + 0.50]$$

$$H(Y|X) = 1.5(\alpha_0 + \alpha_1 + \alpha_2) \text{ bits/symbol} \quad \text{...(i)}$$

From the fundamental constraint of the probability,

$$\sum_{i=0}^{2} P(x_i) = \alpha_0 + \alpha_1 + \alpha_2 = 1$$

So, $H(Y|X) = 1.5$ bits/symbol

Output matrix, $$H(Y) = -\sum_{j=0}^{2} P(y_j)\log_2 P(y_j)$$

$$I(X;Y) = H(Y) - H(Y|X)$$

Here, $H(Y|X)$ is independent of α_i and $H(Y)$ depends on α_i

To maximize $I(X;Y)$ w.r.t. α_i, we have to maximize $H(Y)$ w.r.t. α_i.

From the fundamental properties of entropy, we know that, entropy will be maximum when all the symbols are equiprobable.

i.e., when, $$P(y_0) = P(y_1) = P(y_2) = \frac{1}{3}$$

The given channel is symmetric,

So, $$P(y_0) = P(y_1) = P(y_2) = \frac{1}{3} \quad \text{when } \alpha_0 = \alpha_1 = \alpha_2 = \frac{1}{3}$$

Hence the combination of input, for which the mutual information will be maximum is,

$$P(x_0) = P(x_1) = P(x_2) = \frac{1}{3}$$

For this combination of input probabilities,

$$H(Y) = -\sum_{j=0}^{2} P(y_j)\log_2 P(y_j)$$

$$= -3\left[\frac{1}{3}\log_2\left(\frac{1}{3}\right)\right] = \log_2(3) \text{ bits/symbol}$$

$$H(Y) = 1.585 \text{ bits/symbol} \qquad \text{...(ii)}$$

From equations (i) and (ii), the capacity of the given discrete memoryless channel is equal to,

$$C = [H(Y) - H(Y|X)]\Big|_{\text{Maximizing w.r.t. } P(x_i)}$$

$$= 1.585 - 1.5 = 0.085 \text{ bits/symbol}$$

Q.4 (c) Solution:

For the waveguide $a = 0.04$ m and $b = 0.03$ m

$$h = \frac{\pi}{a} = \frac{\pi}{0.04} = 78.5$$

$$\Rightarrow \qquad \beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\pi}{a}\right)^2} = 98.09 \text{ rad/m}$$

The fields TE_{10} mode, we get,

$$H_z = D\cos\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$E_x = 0$$

$$E_y = -\frac{j\omega\mu}{h^2}\left(\frac{\pi}{a}\right)D\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_x = \frac{j\beta}{h^2}(\pi/a)D\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_y = 0$$

Now it is given by $|E| = |E_y| = \dfrac{\omega\mu D}{h} = 50$ V/m

$$\Rightarrow \qquad D = \frac{50h}{\omega\mu} = \frac{50 \times h}{2\pi \times 6 \times 10^9 \times 4\pi \times 10^{-7}} = 82.82 \times 10^{-3}$$

Substituting for D we get the fields inside the waveguide as

$$E_y = -j50\sin\left(\frac{\pi x}{0.04}\right)e^{-j\beta z} \text{ V/m}$$

$$H_x = j0.1035\sin\left(\frac{\pi x}{0.04}\right)e^{-j\beta z} \text{ V/m}$$

$$H_z = 82.82\times10^{-3}\cos\left(\frac{\pi x}{0.04}\right)e^{-j\beta z} \text{ A/m}$$

Now the power density of the mode is

$$P = \frac{1}{2}\text{Re}\{E\times H^*\} = \frac{1}{2}\times50\times0.1035\times\sin^2\left(\frac{\pi x}{a}\right)$$

The total power carried by the mode can be obtained by integrating P over waveguide cross section.

The total power in the mode is

$$W = \int_{x=0}^{a}\int_{y=0}^{b} P\,dx\,dy = 2.588b\int_0^a \sin^2\left(\frac{\pi x}{a}\right)dx$$

$$W = 2.588b\times\frac{a}{2} = 1.55\times10^{-3} \text{ Watts}$$

Q.5 (a) Solution:

(i) $$t_3 - t_1 = \frac{2500 \text{ m}}{2\times10^8 \text{ m/s}} = 12.5\ \mu\text{s}$$

So, station B detects the collision at,

$$t_3 = t_1 + 12.5\ \mu\text{s} = 12.5\ \mu\text{s}$$

(ii) $$t_4 - t_2 = \frac{2500 \text{ m}}{2\times10^8 \text{ m/s}} = 12.5\ \mu\text{s}$$

So, station A detects the collision at,

$$t_4 = t_2 + 12.5\ \mu\text{s} = 3 + 12.5\ \mu\text{s} = 15.5\ \mu\text{s}$$

(iii) Before detection of the collision, station A sends t_4R_b = 155 bits

(iv) Before detection of the collision, station B sends $(t_3 - t_2)R_b$ = 95 bits

Q.5 (b) Solution:

It is hierarchical because access to each item is only through one path beginning at the top. One could not access the cost without knowing the part name.

If it were relational model, it would be in tables as shown below. The tables would be connected by the SUPPLIER ID number.

Part Name	Colour	Cost	Supplier ID

Supplier ID	Supplier Address	Supplier Phone

The relational model would be better because any item can be accessed quickly in a variety of ways. For example, the cost of all items from a particular supplier could be listed.

Q.5 (c) Solution:

- The characteristic equation of the given system is,

$$1 + G(s) = 0$$
$$s(s+1)(6s^2+5s+1)+K = 0$$
$$s(6s^3+11s^2+6s+1)+K = 0$$
$$6s^4+11s^3+6s^2+s+K = 0$$

- The Routh table for the characteristic equation of the given system can be formed as follows:

$$\begin{array}{c|ccc}
s^4 & 6 & 6 & K \\
s^3 & 11 & 1 & 0 \\
s^2 & \dfrac{66-6}{11} = \dfrac{60}{11} & K & 0 \\
s^1 & \dfrac{\frac{60}{11}-11K}{(60/11)} = \dfrac{60-121K}{60} & 0 & 0 \\
s^0 & K & 0 & 0
\end{array}$$

- For the system to be stable, all the elements in the first column of the Routh table should have same sign. For this the following conditions should be satisfied:

$$\frac{60-121K}{60} > 0$$
$$121K < 60$$
$$K < \frac{60}{121} \quad \text{...(i)}$$
$$K > 0 \quad \text{...(ii)}$$

- From equations (i) and (ii), the required range of K for which the system will be stable is $0 < K < \frac{60}{121}$.
- To produce the sustained oscillations, the system should be marginally stable. For this in the primary Routh table of the system, there should be a row of zeros.
- From the above Routh table, when $60 - 121K = 0$, all the elements in s^1 row will be zero. So, the system will be marginally stable when $K = \frac{60}{121}$.
- When $K = \frac{60}{121}$, the auxiliary equation of the system can be given as,

$$A(s) = \frac{60}{11}s^2 + K = 0$$

$$A(s) = \frac{60}{11}s^2 + \frac{60}{121} = 0$$

$$A(s) = s^2 + \frac{1}{11} = 0$$

- The roots of the auxiliary equation gives the frequency of sustained oscillations for a marginally stable system, which can be calculated as follows:

$$s^2 = -\frac{1}{11}$$

$$s = \pm j\frac{1}{\sqrt{11}} = \pm j\omega_n$$

- The frequency of sustained oscillations is,

$$\omega_n = \frac{1}{\sqrt{11}}\text{ rad/sec}$$

Q.5 (d) Solution:

The radiation intensity U is defined as

$$U = r^2 \times \text{Average power radiated by antenna}$$

$$= r^2 \times P_{av}$$

$$= r^2 \times A_0 \frac{\sin^2\theta}{r^2} = A_0 \sin^2\theta$$

The maximum radiation is directed along,

$$\theta = \frac{\pi}{2}$$

$$\therefore \qquad U_{max} = A_0 \sin^2\left(\frac{\pi}{2}\right) = A_0$$

$\therefore$ Total radiated power,

$$P_{rad} = \oiint U \cdot d\Omega$$

$$= A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi$$

$$= A_0 \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta \, d\phi$$

On solving the above equation, we have,

$$P_{rad} = A_0\left(\frac{8\pi}{3}\right)$$

$\therefore$ Maximum directivity = $D = \dfrac{4\pi U_{max}}{P_{rad}}$

$$D = \frac{4\pi A_0}{\frac{8\pi}{3} A_0} = \frac{3}{2}$$

5. (e) Solution

Given that, $X(t)$ is a WSS process.

So, the following two conditions should be satisfied by $X(t)$.

Condition - 1 : $E[X(t)]$ should be independent of t.

Condition - 2 : $E[X(t+\tau)X(t)]$ should be a function of τ only.

Check for condition - 1:

$$E[X(t)] = E[A\cos t + B\sin t]$$

$$= E[A\cos t] + E[B\sin t]$$

$$= E[A]\cos t + E[B]\sin t$$

$E[X(t)]$ will be independent of t, when and only $E[A] = E[B] = 0$.

Check for condition - 2:

$$E[X(t+\tau)\,X(t)] = E\Big[\big(A\cos(t+\tau)+B\sin(t+\tau)\big)\big(A\cos t+B\sin t\big)\Big]$$

$$= E\Big[A^2\cos(t+\tau)\cos t + AB\sin(t+\tau)\cos t + AB\cos(t+\tau)\sin t + B^2\sin(t+\tau)\sin t\Big]$$

$$= E\left[\frac{A^2}{2}\cos(\tau)+\frac{A^2}{2}\cos(2t+\tau)+\frac{AB}{2}\sin(2t+\tau)+\frac{AB}{2}\sin(\tau)+\frac{AB}{2}\sin(2t+\tau)\right.$$

$$\left.-\frac{AB}{2}\sin(\tau)+\frac{B^2}{2}\cos(\tau)-\frac{B^2}{2}\cos(2t+\tau)\right]$$

$$= E\left[\left(\frac{A^2+B^2}{2}\right)\cos(\tau)+\left(\frac{A^2-B^2}{2}\right)\cos(2t+\tau)+AB\sin(2t+\tau)\right]$$

$$= E\left[\frac{A^2+B^2}{2}\right]\cos(\tau)+E\left[\frac{A^2-B^2}{2}\right]\cos(2t+\tau)+E[AB]\sin(2t+\tau)$$

If the above equation is to be a function of "τ" only, then

$$E\left[\frac{A^2-B^2}{2}\right] = 0 \Rightarrow E[A^2] = E[B^2]$$

and $\quad E[AB] = 0$

So, if $X(t)$ is a WSS process, then the conditions to be satisfied are,

$$E[A] = E[B] = 0$$
$$E[AB] = 0$$
$$E[A^2] = E[B^2]$$

Q.6 (a) Solution:

(i)
$$P_t = 18 \text{ W}$$
$$G_t = 36 \text{ dB (or) } 10^{3.6} = 3981$$

The effective aperture area of the earth station antenna can be given by,

$$A_e = \eta\frac{\pi d^2}{4} = 0.62\times\frac{\pi\times(3)^2}{4} = 4.3825 \text{ m}^2$$

So, the power received by the earth station can be given by,

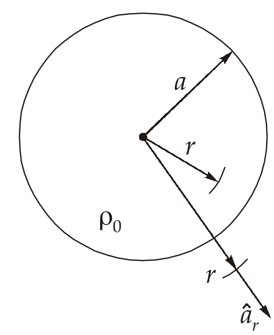
$$P_r = \frac{P_t G_t A_e}{4\pi r^2} = \frac{18\times 3981\times 4.3825}{4\pi\times(35760\times 10^3)^2} \text{ W}$$

$$= 1.954\times 10^{-11} \text{ W} = 19.54 \text{ pW}$$

(ii)
$$\left(\frac{N_0}{C}\right)_{\text{overall}} = \left(\frac{N_0}{C}\right)_{\text{up}} + \left(\frac{N_0}{C}\right)_{\text{down}}$$
$$= 10^{-10} + 10^{-8.7} = 2.0952 \times 10^{-9}$$
$$\left[\frac{C}{N_0}\right]_{\text{overall}} = 10\log_{10}\left(\frac{C}{N_0}\right)_{\text{overall}} = -10\log_{10}\left(\frac{N_0}{C}\right)_{\text{overall}}$$
$$= -10\log_{10}(2.0952 \times 10^{-9}) \approx 86.8 \text{ dBHz}$$

Q.6 (b) Solution:

(i) Apply Gauss's law for,



$$r \le a$$
$$\oint_S \vec{D}\cdot\vec{dS} = \int_v \rho_v \, dv$$
$$D\cdot 4\pi r^2 = \frac{4}{3}\pi r^3 \rho_0$$
$$D = \frac{r\rho_0}{3}$$
$$\vec{E} = \frac{r\rho_0}{3\varepsilon_0}\hat{a}_r \text{ V/m} \qquad 0 \le r \le a$$

For $r \ge a$, from Gauss'a law
$$D\cdot 4\pi r^2 = \frac{4}{3}\pi a^3 \rho_0$$
$$D = \frac{a^3\rho_0}{3r^2}$$
$$\vec{E} = \frac{\rho_0 a^3}{3\varepsilon_0 r^2}\hat{a}_r \text{ V/m} \qquad r \ge a$$

(ii) The potential at any can be obtained from the relation
$$V = -\int \vec{E}\cdot\vec{dl}$$

For $r \ge a$,
$$V = -\int \frac{\rho_0 a^3}{3\varepsilon_0 r^2}dr = \frac{\rho_0 a^3}{3\varepsilon_0 r} + A$$

at $r = \infty$, let $V = 0$, so $A = 0$

$$V = \frac{\rho_0 a^3}{3\varepsilon_0 r} \qquad \text{for } r \ge a$$

For $r \le a$, $$V = -\int \vec{E}\cdot\vec{dl} = -\int \frac{\rho_0 r}{3\varepsilon_0} dr = -\frac{\rho_0 r^2}{6\varepsilon_0} + B$$

at $r = a$, $$V = \frac{\rho_0 r^2}{3\varepsilon_0}$$

$$\frac{\rho_0 a^2}{3\varepsilon_0} = -\frac{\rho_0 a^2}{6\varepsilon_0} + B$$

$$B = \frac{\rho_0 a^2}{2\varepsilon_0}$$

$$V = \frac{\rho_0}{6\varepsilon_0}(3a^2 - r^2)$$

(iii) The energy stored is given by,

$$W = \frac{1}{2}\int_v \varepsilon_0 E^2 dv$$

Considering both regions, we have

$$W = \frac{1}{2}\int_{\substack{v \\ r\le a}} \varepsilon_0 E^2 dv + \frac{1}{2}\int_{\substack{v \\ r \ge a}} \varepsilon_0 E^2 dv$$

Substitute the value of E in both regions

$$W = \frac{\rho_0^2}{18\varepsilon_0}\int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$+\frac{\rho_0^2 a^6}{18\varepsilon_0}\int_{r=a}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{r^4} r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \frac{\rho_0^2 a^5}{18\varepsilon_0 5}4\pi + \frac{\rho_0^2 a^6}{18\varepsilon_0}\left(-\frac{1}{r}\right)\Bigg|_a^{\infty} 4\pi$$

$$= \frac{2\pi\rho_0^2 a^5}{45\varepsilon_0} + \frac{2\pi\rho_0^2 a^5}{9\varepsilon_0}$$

$$W = \frac{4\pi\rho_0^2 a^5}{15\varepsilon_0}\ \text{J}$$

Q.6 (c) Solution:

(i) Total run time = 60 sec

15000 page faults need 15000 × 2 msec = 30 sec

So, the actual run time of the program without any faults = 60 sec – 30 sec = 30 sec.

When the available memory is doubled, page faults will be halved and page fault overhead will be 15 sec.

So, the total run time will be 30 + 15 = 45 sec.

(ii) An address on a paging system is a logical page number and an offset. The physical page is found by searching a table based on the logical page number to produce a physical page number. Because the operating system controls the contents of this table, it can limit a process to accessing only those physical pages allocated to the process. There is no way for a process to refer to a page it does not own because the page will not be in the page table. To allow such access, an operating system simply needs to allow entries for non-process memory to be added to the process's page table. This is useful when two or more processes need to exchange data—they just read and write to the same physical addresses (which may be at varying logical addresses). This makes for very efficient inter-process communication.

Q.7 (a) Solution:

The overall transfer function si determined as

$$\frac{C(s)}{R(s)} = \frac{\frac{5s+\beta}{s} \times \frac{1}{s(s+4)}}{1+\frac{5s+\beta}{s^2(s+4)} \times 1}$$

or
$$\frac{C(s)}{R(s)} = \frac{5s+\beta}{s^3+4s^2+5s+\beta}$$

Therefore, the original characteristic equation is

$$s^3 + 4s^2 + 5s + \beta = 0$$

and therefore, modified transfer function is

$$G_1(s)\,H_1(s) = \frac{\beta}{s^3+4s^2+5s}$$

or
$$G_1(s)\,H_1(s) = \frac{\beta}{s(s^2+4s+5)}$$

The poles of $G_1(s)H_1(s)$ are $s = 0$, and $s = \frac{-4 \pm \sqrt{4^2 - 4\times 5}}{2} = -2 \pm j1$

The number of root contour branches is three. The root contour exists on entire negative real axis. The root contours start (β = 0) at $s = 0$ and $s = -2 \pm j1$.

The angle of asymptotes are determine below:

$$\angle \text{Asy} = \frac{(2k+1)}{P-Z} \times 180°; \quad k = 0, 1, 2$$

(i) $$\frac{(2\times 0+1)}{3-0}\times 180° = 60°$$

(ii) $$\frac{(2\times 1+1)}{3-0}\times 180° = 180°$$

(iii) $$\frac{(2\times 2+1)}{3-0}\times 180° = 300°$$

The asymptotes intersection on real axis at:

$$x = \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{(0-2+j1-2-j1)-0}{3-0} = -1.33$$

The characteristic equation with respect to root contour is given by:

$$1 + G_1(s)\, H_1(s) = 0$$

or $$1+\frac{\beta}{s^3+4s^2+5s} = 0$$

or $$s^3 + 4s^2 + 5s + \beta = 0$$

The Routh's tabulation for above equation is formed below:

s^3	1	5
s^2	4	β
s^1	$\frac{20-\beta}{4}$	
s^0	β	

The root contour intersects imaginary axis for β = 20, the point of intersection can be calculated by solving

$$4s^2 + 20 = 0$$

i.e., $$s = \pm j\sqrt{5}$$

Breakaway:

$$\because \qquad 1+\frac{\beta}{s(s^2+4s+5)} = 0$$

$$\therefore \qquad \beta = -s(s^2 + 4s + 5) = -(s^3 + 4s^2 + 5s)$$

and
$$\frac{d\beta}{ds} = -(3s^2 + 8s + 5)$$

to determine breakaway point, put $\frac{d\beta}{ds} = 0.$

i.e.,
$$3s^2 + 8s + 5 = 0$$

or
$$s^2+\frac{8}{3}s+\frac{5}{3} = 0$$

or
$$s^2 + 2.66s + 1.66 = 0$$

$$\therefore \qquad s = \frac{-2.66 \pm \sqrt{2.66^2 - 4\times 1.66}}{2} = -1.01, -1.66$$

The two breakaway points $s = -1.01$ and $s = -1.66$ lie on the root contour.

Angle of departure from complex poles:

The angle of departure from the pole $(-2 + j1)$ is calculated below:

$$\phi_{(-2+j1)} = 180^\circ - (\theta_{p_2} - \theta_{p_3})$$

$$= 180^\circ - \left\{(+90^\circ) + \left(90^\circ + \tan^{-1}\frac{2}{1}\right)\right\}$$

$$= -\tan^{-1}\frac{2}{1} = -63.43^\circ$$

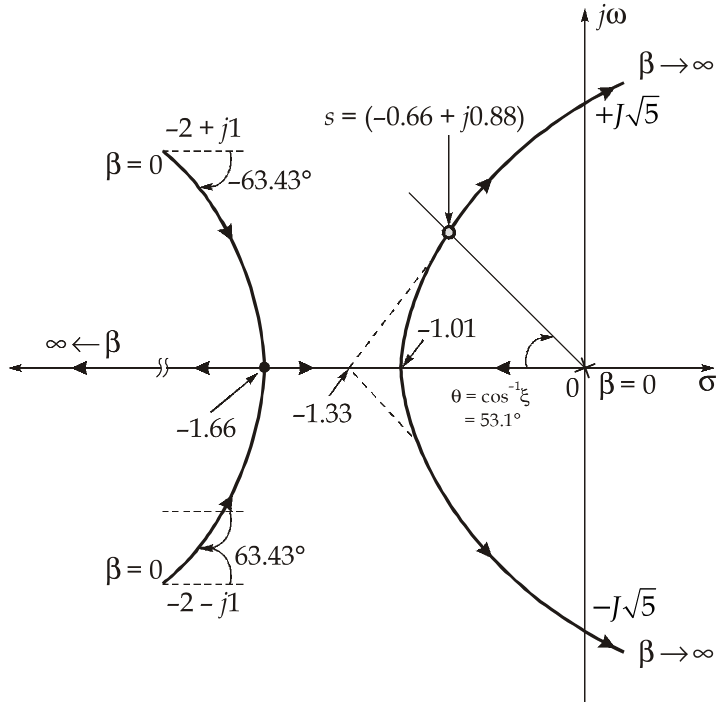
The angle of departure from the pole $(-2 - j1)$ is calculated below:

$$\phi_{(-2-j1)} = 180^\circ - (\theta_{p_1} + \theta_{p_3})$$

$$= 180^\circ - \left\{(-90^\circ) + \left(-90^\circ - \tan^{-1}\frac{2}{1}\right)\right\}$$

$$= (360^\circ - 296.57^\circ) = 63.43^\circ$$

As per the data calculated above the root contour is plotted as shown in below figure.



Q.7 (b) Solution:

For free space, $\rho_v = 0$, $\sigma = 0$, $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$.

$\therefore$ The Maxwell's equation for free space becomes,

$$\nabla \cdot \vec{D} = 0 = \nabla \cdot \varepsilon_0 \vec{E} = \nabla \cdot E_s \text{ (In phasor)} \quad ...(i)$$

$$\nabla \cdot \vec{B} = 0 = \nabla \cdot \mu_0 \vec{H} = \nabla \cdot H_s \text{ (In phasor)} \quad ...(ii)$$

$$\nabla \times \vec{H} = \sigma E + \varepsilon_0 \frac{\partial E}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = j\omega\varepsilon_0 E_s \quad ...(iii)$$

and
$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial}{\partial t}\vec{H} = -j\omega\mu_0 H_s \quad ...(iv)$$

where,
$$\vec{E}_s = \frac{50}{\rho} e^{j\beta z}\, \hat{a}_\phi \text{ and } \vec{H} = \frac{H_0}{\rho} e^{j\beta z}\, \hat{a}_\rho$$

from equation (i) and (ii), we get,

$$\nabla \cdot E_s = \frac{1}{\rho}\frac{\partial}{\partial \phi} E_\phi = 0$$

$$\nabla \cdot H_s = \frac{1}{\rho}\frac{\partial}{\partial \rho} \rho H_\rho = 0$$

Now, $$\nabla \times H_s = \nabla \times \left(\frac{H_0}{\rho} e^{j\beta z} \hat{a}_\rho \right) = \frac{jH_0\beta}{\rho} e^{j\beta z} \hat{a}_\phi \quad \text{...(v)}$$

From equation (ii), (v) and (iii), we have,

$$\frac{jH_0\beta}{\rho} e^{j\beta z} \hat{a}_\phi = j\omega\varepsilon_0 \frac{50}{\rho} e^{j\beta z} \hat{a}_\phi$$

or $$H_0\beta = 50\omega\varepsilon_0 \quad \text{...(vi)}$$

Similarly from equation (ii) and (iv), we have

$$-j\beta \frac{50}{\rho} e^{j\beta z} \hat{a}_\rho = -j\omega\mu_0 \frac{H_0}{\rho} e^{j\beta z} \hat{a}_\rho$$

or $$\frac{H_0}{\beta} = \frac{50}{\omega\mu_0} \quad \text{...(vii)}$$

from equation (vi) and (vii),

$$H_0^2 = (50)^2 \times \frac{\varepsilon_0}{\mu_0}$$

or $$H_0 = \pm 50 \sqrt{\frac{\varepsilon_0}{\mu_0}} = \pm \frac{50}{120\pi} = \pm 0.1326$$

also, $$\beta^2 = \omega^2 \mu_0 \varepsilon_0$$

or $$\beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c} = \frac{10^6}{3 \times 10^8} = 3.33 \times 10^{-3} \text{ rad/m}$$

Q.7 (c) Solution

We can determine the lattice coefficients corresponding to the given FIR filter by computing all the lower-degree polynomials of forward predictor $A_m(z)$ and its reciprocal polynomial $B_m(z)$ as follows:

$$A_4(z) = H(z) = 1 + 2.88z^{-1} + 3.4048z^{-2} + 1.74z^{-3} + 0.4z^{-4}$$
$$B_4(z) = z^{-4}A_4(z^{-1}) = 0.4 + 1.74z^{-1} + 3.4048z^{-2} + 2.88z^{-3} + z^{-4}$$

hence, $K_4 = 0.40$.

$$A_3(z) = \frac{A_4(z) - K_4B_4(z)}{1 - K_4^2}$$

$$= \frac{1}{0.84}\left[(1-0.16) + (2.88-0.696)z^{-1} + (3.4048-1.36192)z^{-2} + (1.74-1.152)z^{-3} + (0.4-0.4)z^{-4}\right]$$

$$= 1 + 2.6z^{-1} + 2.432z^{-2} + 0.7z^{-3}$$

$$B_3(z) = z^{-3}A^3(z^{-1}) = 0.7 + 2.432z^{-1} + 2.6z^{-2} + z^{-3}$$

Hence, $K_3 = 0.70$.

$$A_2(z) = \frac{A_3(z) - K_3B_3(z)}{1-K_3^2}$$

$$= \frac{1}{0.51}\left[(1-0.49)+(2.6-1.7024)z^{-1}+(2.432-1.82)z^{-2}+(0.7-0.7)z^{-3}\right]$$

$$= 1 + 1.76z^{-1} + 1.2z^{-2}$$

$$B_2(z) = z^{-2}A_2(z^{-1}) = 1.2 + 1.76z^{-1} + z^{-2}$$

Hence, $K_2 = 1.20$.

$$A_1(z) = \frac{A_2(z) - K_2B_2(z)}{1-K_2^2}$$

$$= -\frac{1}{0.44}\left[(1-1.44)+(1.76-2.112)z^{-1}+(1.2-1.2)z^{-2}\right]$$

$$= 1 + 0.8z^{-1}$$

$$B_1(z) = z^{-1}A_1(z^{-1}) = 0.8 + z^{-1}$$

Hence, $K_1 = 0.80$.

The required lattice coefficients corresponding to the given FIR system are:

$$K_1 = 0.80,\ K_2 = 1.20,\ K_3 = 0.70 \text{ and } K_4 = 0.40$$

Q.8 (a) Solution

Given data:

Power emitted by the source, $P_S = 1$ mW (–30 dB or 0 dBm)

Bit rate, $R_b = 2$ Gbps

Detector sensitivity, $\frac{P_{R(min)}}{R_b} = -14$ dBm/Gbps

Source coupling loss, $L_{SC} = 2$ dB

Detector coupling loss, $L_{DC} = 1$ dB

Splice loss, $L_S = 0.25$ dB/splice

Attenuation of the cable, $\alpha = 1.5$ dB/km

System margin, $P_{margin} = 5$ dB

(i) Calculation of the maximum possible length of the link without any repeater :

• Minimum detectable power by the detector is,

$$P_{R(min)} = (\text{Detector sensitivity}) \times R_b = -28 \text{ dBm} = -58 \text{ dB}$$

$$\text{Available power} = P_S - P_{R(min)} = (-30) - (-58) \text{ dB} = 28 \text{ dB}$$

• Total loss is, $L = L_{SC} + L_{DC} + L_S$ (number of splices) $+ \alpha l$ ...(i)

Number of splices required $= \left(\dfrac{l}{2.5} - 1\right)$

l = length of the link (in km)

So, $$L = 2 + 1 + (0.25)\left(\frac{l}{2.5} - 1\right) + 1.5l \text{ dB}$$

$$L = \left(\frac{11}{4} + \frac{16}{10}l\right) \text{dB}$$

• Total loss + System margin $\leq P_S - P_{R(min)}$ ...(ii)

So, $$\left(\frac{11}{4} + \frac{16}{10}l\right) + 5 \leq 28$$

$$\frac{16}{10}l \leq \left(28 - 5 - \frac{11}{4}\right)$$

$$\frac{16}{10}l \leq \frac{81}{4}$$

$$l \leq 12.65 \text{ km}$$

So, $$l_{max} = 12.65 \text{ km}$$

But this is only an approximated value of the maximum length of the link. Because, the value of number of splices to be used is always an integer. We should go for a trial and error method to get the exact integer value of the number of splices and the corresponding maximum length of the link possible.

Number of splices required, $m = \left(\dfrac{l}{2.5} - 1\right) = 4.06$

• When $m = 4$ is chosen,

From equations (i) and (ii), we get,

Total loss, $$L = 2 + 1 + (4 \times 0.25) + 1.5l \text{ dB}$$

$$= 4 + 1.5\,l \text{ dB}$$

$$4 + 1.5l + 5 \leq 28 \qquad \because P_{margin} = 5 \text{ dB}$$

$$1.5l \leq 19$$

$$l \leq 12.67 \text{ km}$$

$$l_{max} = 12.67 \text{ km}$$

But, by using 4 splices, the maximum length of the link that can be constructed is $(4 + 1) \times 2.5$ km = 12.5 km only, which is less than 12.67 km.

• When $m = 5$ is chosen,

From equations (i) and (ii), we get,

Total loss, $L = 2 + 1 + (5 \times 0.25) + 1.5l \text{ dB} = 4.25 + 1.5\, l \text{ dB}$

$$4.25 + 1.5l + 5 \le 28 \qquad \because P_{margin} = 5 \text{ dB}$$

$$1.5l \le 18.75$$

$$l \le 12.5 \text{ km}$$

$$l_{max} = 12.5 \text{ km}$$

So, the maximum length of the link possible with no repeater and 5 splices is 12.5 km.

• The best choice for the number of splices (m) is 4. Because, with both $m = 4$ and $m = 5$, the maximum length possible is 12.5 km only. So, if we select $m = 4$, then losses can be reduced compared to the case with $m = 5$ and more power will be available at the detector input after 12.5 km.

• So, the maximum possible length of the link without any repeater is 12.5 km.

(ii) To calculate the minimum number of repeaters required to construct a link with a length of 100 km:

• Let, the number of repeaters required = n

• Then, $(n + 1)(12.5) \ge 100$

$$n + 1 \ge \left\lceil \frac{100}{12.5} \right\rceil \qquad \because n \text{ is always an integer}$$

$$n \ge 7$$

So, $n_{min} = 7$

• Minimum 7 repeaters are required to construct the link with a length of 100 km.

Q.8 (b) Solution:

(i) Processor speed = 600 MHz = 6×10^8 cycles/sec

$$\text{Transfer time} = \frac{20{,}000 \text{ bytes}}{10 \text{ MBps}} = \frac{20}{10 \times 1000} \text{ sec}$$

$$= \frac{20}{10 \times 1000} \times 6 \times 10^8 \text{ cycles} = 12 \times 10^5 \text{ cycles}$$

Total cycles consumed = 1200000 + 300 + 900 cycles = 1201200 cycles

$$\% \text{ of processor time consumed} = \frac{1201200}{6 \times 10^8} \times 100 \approx 0.2\%$$

(ii) In programmed I/O mode,

Data transfer rate = 10 kBps

$$1 \text{ Byte takes} = \frac{1}{10 \times 1000} \text{ sec} = 100\ \mu\text{sec}$$

In interrupt mode CPU takes 4 μsec, since transfer time between other components is negligible.

$$\text{Performance gain} = \frac{100}{4} = 25$$

Q.8 (c) Solution

The probability density function of the input variable R can be given by,

$$f_R(r|s_0) = \frac{1}{2}e^{-|r+1|} \text{ ; When "0" is transmitted.}$$

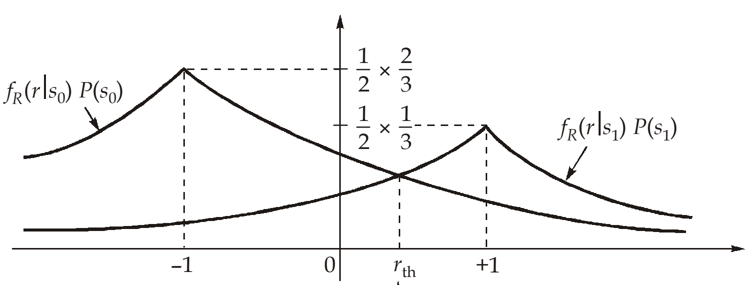
$$f_R(r|s_1) = \frac{1}{2}e^{-|r-1|} \text{ ; When "1" is transmitted.}$$

The rule to decide an optimum threshold value using MAP criteria is as follows:

$$f_R(r|s_0)P(s_0) \underset{H_1}{\overset{H_0}{\gtrless}} f_R(r|s_1)P(s_1)$$

Given that, $P(s_0) = \frac{2}{3}$ and $P(s_1) = \frac{1}{3}$.

The optimum threshold value can be decided by plotting the functions $f_R(r|s_0)P(s_0)$ and $f_R(r|s_1)P(s_1)$ on the same axis as shown below.



At $r = r_{th}$,

$$f_R(r_{th}|s_0)P(s_0) = f_R(r_{th}|s_1)P(s_1)$$

$$\frac{1}{2}\times\frac{2}{3}e^{-|r_{th}+1|} = \frac{1}{2}\times\frac{1}{3}e^{-|r_{th}-1|}$$

$$2e^{-(r_{th}+1)} = e^{-(1-r_{th})}$$

$$e^{(r_{th}+1)}\ e^{(r_{th}-1)} = 2$$

$$e^{2r_{th}} = 2$$

$$r_{th} = \frac{1}{2}\ln(2) \approx 0.35$$

To determine the probability of error:

$$P_e = P(r_0 | s_1)\, P(s_1) + P(r_1 | s_0) P(s_0)$$

$$P(r_0 | s_1) = \int_{-\infty}^{r_{th}} f_R(r | s_1)\, dr = \int_{-\infty}^{0.35} \frac{1}{2} e^{-|r-1|}\, dr$$

$$= \frac{1}{2}\int_{-\infty}^{0.35} e^{-(1-r)}\, dr = \frac{e^{-1}}{2}\left[e^{r}\right]_{-\infty}^{0.35} = \frac{1}{2}e^{-0.65}$$

$$P(r_1 | s_0) = \int_{r_{th}}^{\infty} f_R(r | s_0)\, dr = \int_{0.35}^{\infty} \frac{1}{2} e^{-|r+1|}\, dr$$

$$= \frac{1}{2}\int_{0.35}^{\infty} e^{-(r+1)}\, dr = \frac{e^{-1}}{2}\left[-e^{-r}\right]_{0.35}^{\infty} = \frac{1}{2}e^{-1.35}$$

So,

$$P_e = \left(\frac{1}{3}\times\frac{1}{2}e^{-0.65}\right) + \left(\frac{2}{3}\times\frac{1}{2}e^{-1.35}\right) = 0.1734$$

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