India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2019 Mains Test Series

Mechancial Engineering Test No : 15

Section A

Q.1 (a) Solution:

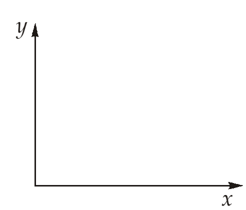
We need to find resultant of given 3 forces and its intersection with the AB.

$$\text{Resultant force, } \vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= 2\left(90\cos 30\hat{i} - 90\sin 30\hat{j}\right) + \left(-200\hat{j}\right)$$

$$= 90\sqrt{3}\hat{i} - 290\hat{j}$$

Where $\hat{i}$ and $\hat{j}$ are unit vector in x and y direction.



$$|R| = \sqrt{(90\sqrt{3})^2 + (-290)^2} = 329.24 \text{ kN}$$

Let's take C to be the point of intersection of resultant force and line AB which is at 'd' distance in x directions from A, then moment of these three forces will be zero at C.

So, M_c = –90 cos30° × (3.6 × 3) + 90sin30° × d – 90cos30° × (3.6 × 2) + 90sin30° ×

$$\left(d - \frac{6.3}{3 \times 3.6} \times 3.6\right) + 200 \times (d - 2.4)$$

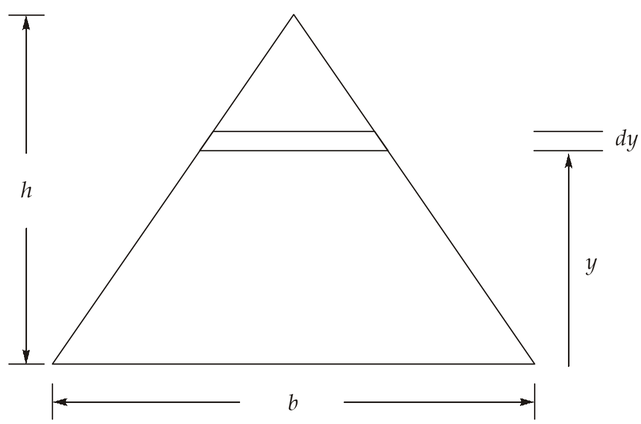
$$\Rightarrow \quad 0 = -45\sqrt{3} \times 3.6 \times 3 + 45d - 45\sqrt{3} \times 3.6 \times 2 + 45 \times (d - 2.1) + 200(d - 2.4)$$

$$\Rightarrow \quad d = 6.8188 \text{ m}$$

So, $\vec{R}$ will pass through C which is at 6.8188 m distance from A.

Q.1 (b) Solution:

Due to wind there will be moment at bottom of tree.



Consider a rectangular area of thickness dy at height y from bottom.

$$dM = dF \times y$$

$$M = \int_0^h P\left(\frac{b}{h}(h - y)\, dy\right) \times y$$

$$= \frac{Pb}{h}\int_0^h (hy - y^2)\, dy = \frac{Pb}{h}\left(\frac{hy^2}{2} - \frac{y^3}{3}\right)_0^h$$

$$M = \frac{Pb}{h} \times h^3\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{Pbh^2}{6}$$

$$= \frac{2 \times 10^3 \times 4 \times (10)^2}{6} = 133.33 \text{ kNm}$$

Using, $$\frac{M}{I} = \frac{\sigma}{r}$$

$$\Rightarrow \qquad \sigma = \frac{Mr}{\left(\frac{\pi r^4}{4}\right)} = \frac{4M}{\pi r^3} = \frac{4\times133.33\times10^3}{\pi\times(0.2)^3} = 21.2 \text{ MPa}$$

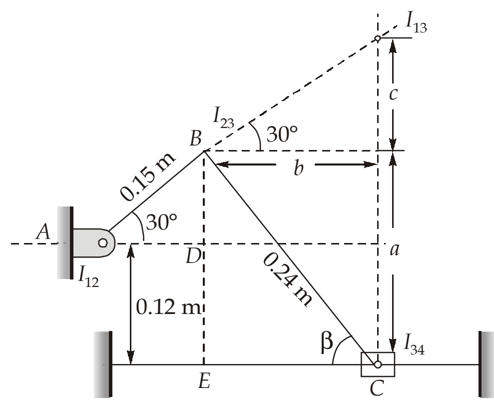
So, maximum stress at bottom will be 21.2 MPa.

Q.1 (c) Solution:

We note that $BD + DE = 0.24 \sin\beta$

$$0.15\sin30° + 0.12 = 0.24\sin\beta$$

$$\beta = 54.34°$$



$$a = 0.24\sin54.34°$$

$$a = 0.195 \text{ m}$$

$$b = 0.24\cos54.34°$$

$$b = 0.140 \text{ m}$$

$$c = b\tan30° = 0.140 \tan30° = 0.080 \text{ m}$$

$$I_{13}I_{23} = \frac{c}{\sin 30°} = \frac{0.080}{\sin 30°} = 0.16 \text{ m}$$

$$I_{13}I_{34} = a + c = 0.195 + 0.16 = 0.355 \text{ m}$$

Now that the instant centre for each bar has been found

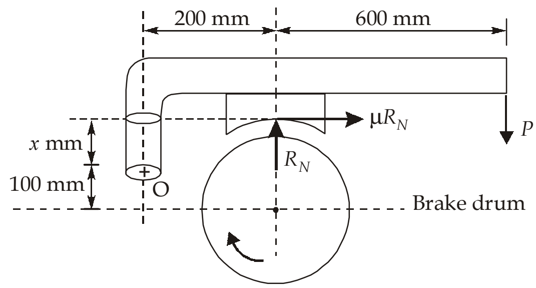
$$V_B = AB \times \omega_{AB} = 0.15 \times 3 = 0.45 \text{ m/s}$$

$$\omega_{BC} = \frac{V_B}{I_{13}I_{23}} = \frac{0.45}{0.16} = 2.8125 \text{ rad/s} \qquad \text{(anticlockwise)}$$

$$V_c = I_{13}I_{34} \times \omega_{BC} = 0.355 \times 2.8125 = 0.998 \text{ m/s} \simeq 1 \text{ m/s}$$

Radial component of acceleration = $BC \times \omega_{BC}^2 = 0.24 \times 2.8125^2 = 1.898 \text{ m/s}^2$

Q.1 (d) Solution:



Take moment about O:

$$F_t \times x + P \times 800 = R_N \times 200$$

$$\mu R_N \times x + P \times 800 = R_N \times 200 \quad \text{...(1)}$$

$$\text{Torque, } T = F_t \times \frac{D}{2}$$

$$T = \mu R_N \times \frac{D}{2}$$

$$360 = 0.3 \times R_N \times \frac{0.3}{2}$$

$$R_N = 8000 \text{ N}$$

From Eq. (1),

$$\mu R_N \times x + P \times 800 = R_N \times 200$$

$$0.3 \times 8000 \times 50 + P \times 800 = 8000 \times 200$$

$$P = 1850 \text{ N}$$

2. For self braking P should be zero.

So, from equation (1)

$$\mu R_N \times x + P \times 800 = R_N \times 200$$

$$0.3 \times 8000 \times x = 8000 \times 200$$

$$x = 666.66 \text{ mm}$$

Q.1 (e) Solution:

Let, d = Inner diameter of clutch

D = Outer diameter of clutch

$$\frac{d}{D} = x$$

We know that, for uniform wear theory

$$T = \frac{\pi\mu P_a d\left(D^2 - d^2\right)}{8}$$

$$T = \frac{\pi\mu P_a}{8}\left(D^2 d - d^3\right)$$

$$T = \frac{\pi\times\mu\times P_a}{8}\times D^3\left(\frac{D^2 d}{D^3} - \frac{d^3}{D^3}\right)$$

(multiplying and dividing by D^3)

$$T = \frac{\pi\times\mu\times P_a}{8}\times D^3\left(\frac{d}{D} - \frac{d^3}{D^3}\right) \qquad \left\{\text{Let, } \frac{d}{D} = x\right\}$$

$$T = \frac{\pi\times\mu\times P_a\times D^3}{8}\left(x - x^3\right)$$

$$T = \frac{\pi\times\mu\times Pa\times D^3}{8}\left(x(1-x)^2\right)$$

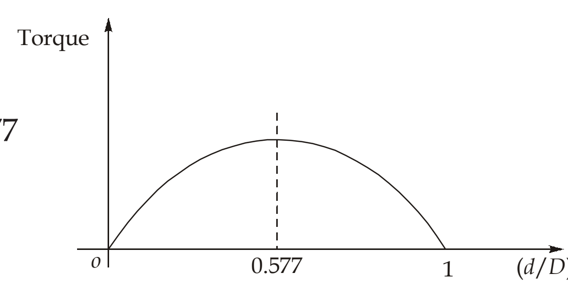
For maximum torque capacity,

$$\frac{d}{dx}\left(x - x^3\right) = 0 \qquad \{\text{only } x \text{ is variable here}\}$$

$$1 - 3x^2 = 0$$

$$x = \sqrt{\frac{1}{3}} = 0.577$$

So, $$\frac{d}{D} = 0.577$$



Q.2 (a) Solution:

As per given information

$$Z_B = 27,\ Z_C = 30,\ Z_D = 24,\ Z_E = 21$$

Tabular method:

action	arm A	Gear $D(24)$	Gear $B/C(27/30)$	Gear $E(21)$
A arm fixed $D+1$ rev	0	$+1$	$\frac{-24}{27}$	$\frac{+24}{27}\times\frac{30}{21}$
$D+x$ rev.	0	$+x$	$\frac{-24}{27}x$	$\frac{+24}{27}\times\frac{30}{21}x$
$+y$ rev. of arm	y	$y+x$	$y-\frac{24}{27}x$	$y+\frac{24}{27}\times\frac{30}{21}x$

As per given condition gear E is fixed,

$$y + \frac{24}{27} \times \frac{30}{21} x = 0$$

$$y = -\frac{24}{27} \times \frac{30}{21} x$$

Deriving shaft attached with arm A and driven shaft D,

$$\frac{N_A}{N_D} = \frac{y}{y + x}$$

$$\frac{N_A}{N_D} = \frac{-\frac{24}{27} \times \frac{30}{21} x}{\left[1 - \left(\frac{24}{27} \times \frac{30}{21}\right)\right] x} = 4.706$$

Assuming that there are no frictional losses and that the members are revolving at uniform speeds.

$$T_A + T_D + T_E = 0 \quad \text{...(1)}$$

$$\omega_A T_A + \omega_D T_D + \omega_E T_E = 0$$

But E is fixed, so that $\omega_E = 0$

$$\omega_A T_A + \omega_D T_D = 0$$

$$T_D = -T_A \times \frac{\omega_A}{\omega_D} = -80 \times 4.706 = -376.480 \text{ Nm}$$

Holding torque, $T_E = ?$

From equation (1)

$$T_A + T_D + T_E = 0$$

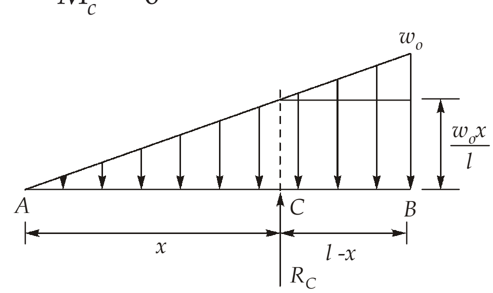
$$80 - 376.480 + T_E = 0$$

$$T_E = 296.480 \text{ Nm}$$

Q.2 (b) Solution:

For equilibrium moment about C will be zero.

$$M_c = 0$$



$$\Rightarrow \quad \frac{1}{2}\times\left(\frac{w_o x}{l}\right)x\times\frac{x}{3} = \left(\frac{w_o x}{l}\right)(l-x)\times\frac{(l-x)}{2}+\frac{1}{2}\times(l-x)\left(w_0-\frac{w_0 x}{l}\right)\times\frac{2}{3}(l-x)$$

$$\Rightarrow \quad \frac{x^3}{6l} = \frac{x(l-x)^2}{2l}+\frac{(l-x)^3}{3l}$$

$$\Rightarrow \quad x^3 = 3x(l-x)^2+2(l-x)^3$$

$$\Rightarrow \quad x^3 = 3x(l^2+x^2-2lx)+2(l^3-x^3-3l^2x+3lx^2)$$

$$\Rightarrow \quad x^3 = 3xl^2+3x^3-6lx^2+2l^3-2x^3-6l^2x+6lx^2$$

$$\Rightarrow \quad 0 = 2l^3-3l^2x$$

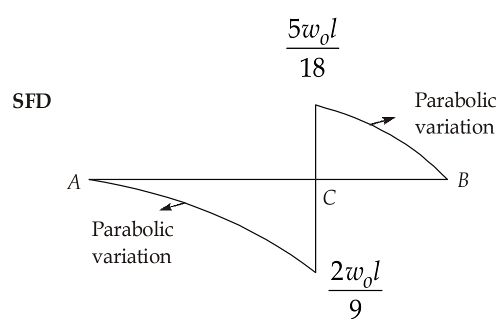
$$\Rightarrow \quad x = \frac{2l}{3}$$

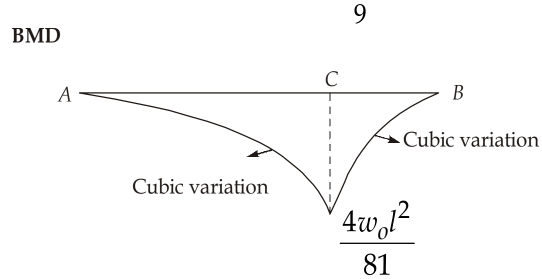
As $\quad \Sigma F_y = 0$

$$R_c = \frac{1}{2}w_o l$$

$$(\text{SF})_c = \frac{1}{2}\times\frac{w_o x}{l}\times x = \frac{w_o x^2}{2l} = \frac{w_o 4l^2}{2l\times 9}$$

$$SF = \frac{2w_o l}{9}$$





$$M_c = \frac{1}{2}\left(\frac{w_o x}{l}\right)x\times\frac{x}{3} = \frac{w_o}{6l}\left(\frac{2l}{3}\right)^3 = \frac{4w_o l^2}{81}$$

From BMD it is clear maximum moment is at C.

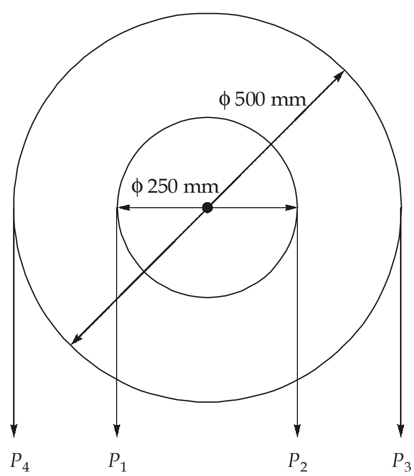
$$\text{Maximum stress, } \sigma_c = \frac{M_c}{I}\left(\frac{a}{2}\right) = \frac{\left(\frac{4w_o l^2}{81}\right)\left(\frac{a}{2}\right)}{\frac{a^4}{12}} = \frac{8w_o l^2}{27a^3} = \frac{w_o l^2}{(1.5a)^3}$$

Q.2 (c) Solution:

$$\text{Torque, } T = \frac{P\times 60}{2\pi N} = \frac{7.5\times 10^6 \times 60}{2\pi \times 360}$$

$$T = 198943.679 \text{ Nmm}$$

For pulley 1:



$$(P_1 - P_2) \times 125 = 198943.679$$

$$(P_1 - P_2) = 1591.55 \text{ N} \quad ...(1)$$

Also,
$$\frac{P_1}{P_2} = 2.5 \quad ...(2)$$

From equation (1) and (2)

$$P_1 = 2652.58 \text{ N}$$

and
$$P_2 = 1061.03 \text{ N}$$

Weight of pulley is given by;

$$W_1 = m_1 g = 10 \times (9.81) = 98.1 \text{ N}$$

Total downward force acting on pulley 1.

$$= P_1 + P_2 + W_1 = 2652.58 + 1061.03 + 98.1 = 3811.71 \text{ N}$$

For pulley 2:

$$(P_3 - P_4) \times 250 = 198943.679$$

and $$\frac{P_3}{P_4} = 2.5$$

By solving:

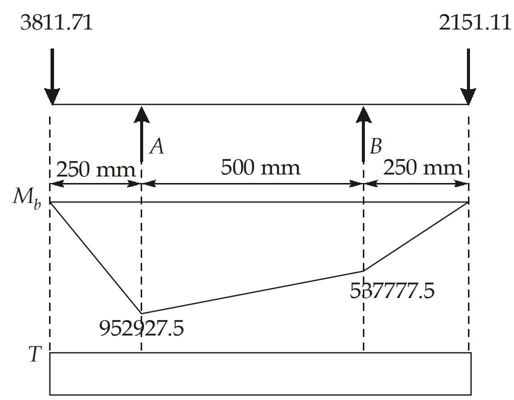
$$P_3 = 1326.29 \text{ N}$$

$$P_4 = 530.52 \text{ N}$$

Weight of pulley 2: $$W_2 = m_2 g = 30 \times 9.81 = 294.3 \text{ N}$$

Total downward force on pulley 2 = $P_3 + P_4 + W_2$

$$= 1326.29 + 530.52 + 294.3 = 2151.11 \text{ N}$$



Bending moment diagram,

$$(\text{B.M.})_A = 3811.71 \times 250 = 952927.5 \text{ N-mm}$$

$$(\text{B.M.})_B = 2151.11 \times 250 = 537777.5 \text{ N-mm}$$

Torque is constant, $T = 198943.68$ Nmm

As bending moment is maximum at A, so, critical point is A.

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{(M_b)^2 + (T)^2}$$

$$\frac{16}{\pi d^3}\sqrt{(952927.5)^2 + (198943.68)^2} = \frac{195}{3}$$

$$d = 42.41 \text{ mm}$$

Q.3 (a) Solution:

As per given information,

$$W = 440 \text{ N}$$

$$k = 35430 \text{ N/m}$$

$$c = 1240 \text{ Ns/m}$$

$$\text{Fluctuating pressure, } P = 4276\sin 30t$$

$$\text{Fluctuating force, } F = (P \times \text{area})$$

$$F = 4276\sin 30t \times (0.0516)$$

$$F = 220.64 \sin 30t \text{ N}$$

$$\text{Maximum force, } F_o = 220.64 \text{ N}$$

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{35430}{440/9.81}} = 28.1056 \text{ rad/s}$$

$$\xi = \frac{c}{2m\omega_n} = \frac{1240}{2 \times \frac{440}{9.81} \times 28.1056}$$

$$\xi = 0.4918 \text{ (hence this is case of underdamped system)}$$

The steady state amplitude

$$x = A = \frac{Fo/k}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{220.64/35430}{\sqrt{\left(1-\left(\frac{30}{28.1056}\right)^2\right)^2 + \left(2 \times 0.4918 \times \frac{30}{28.1056}\right)^2}}$$

$$A = X = 5.879 \times 10^{-3} \text{ m}$$

$$\text{The phase angle, } \phi = \tan^{-1}\left(\frac{2\xi\omega/\omega_n}{\left(1-(\omega/\omega_n)^2\right)}\right)$$

$$\phi = \tan^{-1}\left(\frac{2 \times 0.4918 \times 30/28.1056}{1-\left(\frac{30}{28.1056}\right)^2}\right) \quad \text{(IInd quadrant)}$$

$$\phi = (-82.439)$$

$$\phi = (180° - 82.439°) \times \frac{\pi}{180°}$$

$$\phi = 1.7027 \text{ radian}$$

Then the steady state displacement as a function of time

$$x = X\sin(\omega t - \phi)$$

$$x = 5.879 \times 10^{-3} \sin(30t - 1.7027) \text{ meter}$$

The force F_T transmitted to the base is the sum of the spring and damper forces.

$$F_T = (kx + c\dot{x})$$

$$F_T = kX\sin(\omega t - \phi) + c\omega X\cos(\omega t - \phi)$$

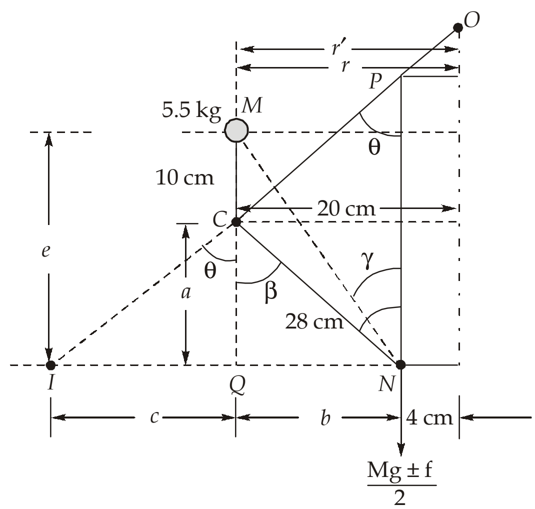
The maximum value of $F_T = \sqrt{(kX)^2 + (c\omega X)^2} = X\sqrt{k^2 + (c\omega)^2}$

$$= 5.879\times10^{-3}\sqrt{35430^2 + (1240\times30)^2}$$

$$F_T = 302.0184 \text{ N}$$

Q.3 (b) Solution:

As per given data



M = 90 kg; r = 20 cm = r', m = 5.5 kg;

$\Rightarrow$ $$h = \sqrt{28^2 - 20^2} = 19.59 \text{ cm}$$

$$a = \sqrt{28^2 - 16^2} = 22.978 \text{ cm}$$

$$e = (a + 10) \text{ cm}$$

$$e = 22.978 + 10 = 32.978 \text{ cm}$$

$$f = 0$$

$$k = \frac{\tan\beta}{\tan\theta}$$

$$\tan\beta = \frac{16}{22.978} \Rightarrow \beta = 34.85°$$

$$\tan\theta = \frac{20}{19.59} \Rightarrow \theta = 45.59°$$

$$\tan\gamma = \frac{16}{32.978}$$

$$\gamma = 25.88°$$

$$\beta - \gamma = 8.96°$$

$$k = \frac{16}{22.978} \times \frac{19.59}{20} = 0.682$$

$$\text{Length, MN} = ((10 + 22.978)^2 + 16^2)^{1/2} = 36.65 \text{ cm}$$

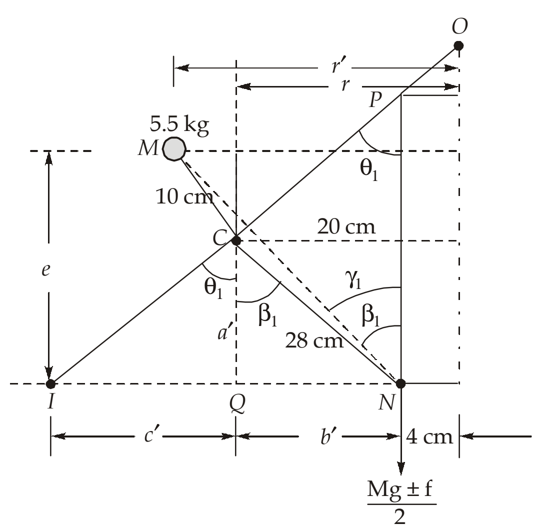
Given condition, $(r = r')$

1. $$N^2 = \frac{895}{h}\left(\frac{a}{e}\right)\left[\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right] \quad ...(1)$$

$$N^2 = \frac{895}{0.1959}\left(\frac{0.22978}{0.32978}\right)\left[\frac{2\times5.5\times9.81 + (90\times9.81)(1+0.682)}{2\times5.5\times9.81}\right]$$

$$N^2 = 46991.1812$$

$$N = 216.7744 \text{ rpm}$$



$$\sin\gamma_1 = \frac{20}{36.65}$$

$$\gamma_1 = 33.072°$$

$$m = 5.5 \text{ kg}$$

$$M = 90 \text{ kg}$$

$$\beta_1 = 33.072° + 8.96° = 42.032°$$

$$e = 30.711 \text{ cm}$$

$$\cos\beta_1 = \frac{a}{28}$$

$$\cos 42.032° = \frac{a'}{28}$$

$$a' = 20.797 \text{ cm}$$

$$b' = \sqrt{28^2 - 20.797^2}$$

$$b' = 18.747 \text{ cm}$$

$$r' = 24 \text{ cm}$$

$$r = b' + 4 \text{ cm} = 22.747 \text{ cm}$$

$$\sin\theta_1 = \frac{22.747}{28}$$

$$\Rightarrow \quad \theta_1 = 54.33°$$

$$\tan\theta_1 = \frac{c'}{a'}$$

$$\tan 54.33° = \frac{c'}{20.797}$$

$$c' = 28.974 \text{ cm}$$

Taking moment about I,

$$\omega^2 = \frac{2mg(c' + r - r') + (Mg \pm f)(b' + c')}{2mr'e}$$

$$\omega^2 = \frac{2\times5.5\times9.81(0.28974 + 0.22747 - 0.24) + (90\times9.81)(0.18747 + 0.28974)}{2\times5.5\times0.24\times0.30711}$$

$$\omega^2 = 556.56$$

$$\Rightarrow \quad N = \sqrt{\frac{556.56\times60^2}{(2\pi)^2}} = 225.282 \text{ rpm}$$

Q.3 (c) Solution:

Given: d = 60 cm, r = 30 cm = 0.3 m, t = 2 cm

$$\in_1 = 255 \times 10^{-6}$$

$$\in_2 = 60 \times 10^{-6}$$

(a) $$\gamma_{max} = \in_1 - \in_2 \qquad \text{(In-plane)}$$

$$= 195 \times 10^{-6}$$

From Hooke's law for shearing stress and strain, we have

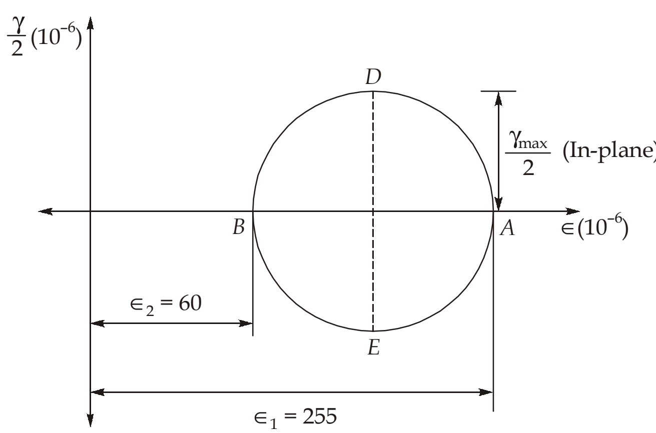
$$\tau_{max} = G\gamma_{max}$$
$$= 77.2 \times 10^9 \times 195 \times 10^{-6}$$
$$= 15.054 \text{ MPa}$$

Note that this is maximum in plane shear stress as we calculated maximum in plane strain.

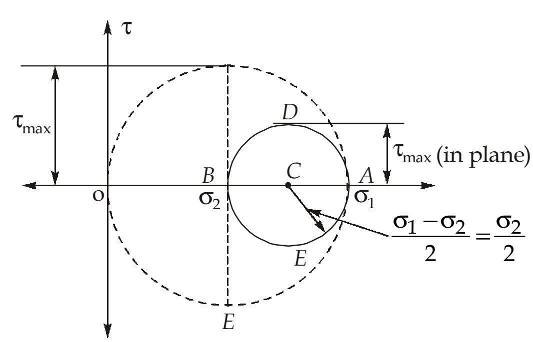
As we know $\quad \tau_{max(in\ plane)} = \dfrac{Pr}{4t}$

$$\Rightarrow \quad 15.054 = \frac{P \times 30}{4 \times 2}$$

$$\Rightarrow \quad P = 4.0144 \text{ MPa}$$



(b) For a thin walled cylinder we know



$$\sigma_1 = \frac{Pr}{t} = 60.216 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = 30.108 \text{ MPa}$$

$$\tau_{max} = \frac{Pr}{2t} = 30.108 \text{ MPa} \qquad \text{(Max. absolute shear stress)}$$

Maximum shearing stress corresponding to a rotation of 45° about longitudinal axis.

Alternative Solution;

Given: Diameter of cylinder, d = 60 cm = 0.6 m

Wall thickness, t = 2 cm = 0.02 m

$$\in_{transverse} = \in_1 = 255 \times 10^{-6}$$

$$\in_{longitudinal} = \in_2 = 60 \times 10^{-6}$$

Shear modulus, G = 77.2 GPa

We have to first find the gauge pressure. Let's take gauge pressure to be P and tank to be thin cylinder vessel, then we have.

$$\sigma_{hoop} = \sigma_1 = \frac{Pd}{2t}$$

$$\sigma_{longitudinal} = \sigma_2 = \frac{Pd}{4t}$$

Let's take ν to be the Poisson's ratio, then,

$$\in_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} = \frac{Pd}{2tE} - \frac{\nu Pd}{4tE}$$

$$= \frac{Pd}{4tE}(2-\nu) \qquad ...(1)$$

$$\in_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E} = \frac{Pd}{4tE} - \frac{\nu Pd}{2tE}$$

$$= \frac{Pd}{4tE}(1-2\nu) \qquad ...(2)$$

Dividing eq. (1) and eq. (2)

$$\frac{\in_1}{\in_2} = \frac{2-\nu}{1-2\nu}$$

$$\Rightarrow \quad \frac{255}{60} = \frac{2-\nu}{1-2\nu}$$

$$\Rightarrow \quad 17 - 34\nu = 8 - 4\nu$$

$$\Rightarrow \quad 9 = 30\nu$$

$$\nu = 0.3$$

Given, G = 77.2 GPa

$$\frac{E}{2(1+\nu)} = 77.2 \text{ GPa}$$

$$E = 77.2 \times 2 \times (1.3) = 200.72 \text{ GPa}$$

Using eq. (1)

$$\epsilon_1 = \frac{Pd}{4tE}(2-\nu)$$

$$255 \times 10^{-6} = \frac{P \times 0.6}{4 \times 0.02 \times 200.72 \times 10^9}(2-0.3)$$

$\Rightarrow$ $$P = 4.0144 \text{ MPa}$$

(ii) Now in second part we have to find principal stresses and maximum shearing stess. We know in the given plane as there is no shearing force σ_1 and σ_2 are itself principal stresses.

$$\sigma_1 = \frac{Pd}{2t} = \frac{4.0144 \times 10^6 \times 0.6}{2 \times 0.02} = 60.216 \text{ MPa}$$

$$\sigma_2 = \frac{Pd}{4t} = 30.108 \text{ MPa}$$

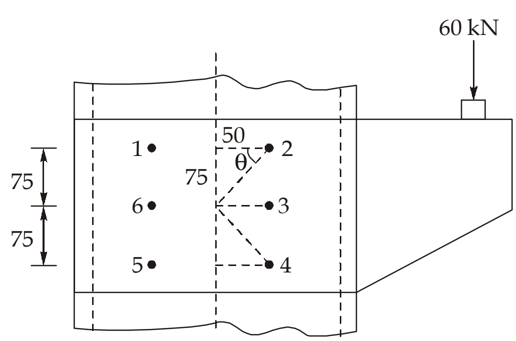
Also as we neglect radial stress due to pressure, we get

$$\sigma_3 = 0$$

Maximum shear stress, (in plane 12) = $\dfrac{\sigma_1 - \sigma_2}{2}$ = 15.054 MPa

Maximum shear stress(abs.) = $\dfrac{\sigma_1 - \sigma_3}{2}$ = 30.108 MPa

Q.4 (a) Solution:



Primary force at each bolt:

$$P_1' = P_2' = P_3' = P_4' = P_5' = P_6' = \frac{60}{6} = 10 \text{ kN}$$

As radius from centre of gravity is higher for 2 and 4 rivet and they are near to load P. So 2 and 4 rivets subjected to maximum force.

$$r_2 = \sqrt{75^2 + 50^2}$$

$$r_2 = 90.138 \text{ mm}$$

$$\tan\theta = \frac{75}{50}$$

$$\theta = 56.31°$$

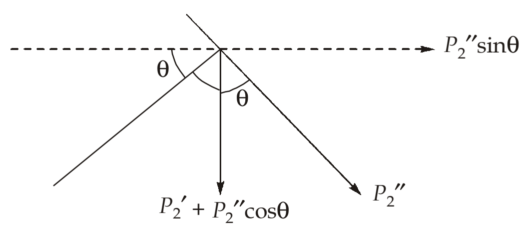
Secondary force due to moment at 2 (or 4);

$$P \times e = \frac{P_2''}{r_2}\left(r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2 + r_6^2\right)$$

$$60 \times 200 = \frac{P_2''}{90.138}\left(90.138^2 \times 4 + 50^2 \times 2\right)$$

$$P_2'' = 28.844 \text{ kN}$$

Resultant force on rivet 2:



$$P_2 = \sqrt{\left(P_2' + P_2''\cos\theta\right)^2 + \left(P_2''\sin\theta\right)^2}$$

$$P_2 = \sqrt{(10 + 28.844\cos 56.31°)^2 + (28.844 \times \sin 56.31°)^2}$$

$$P_2 = 35.383 \text{ kN}$$

Diameter of rivets:

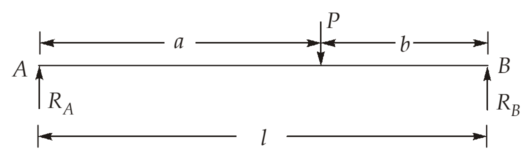
$$P_4 = \frac{\pi}{4} \times d^2\,\tau$$

$$d = \sqrt{\frac{4 \times 35.383 \times 10^3}{\pi \times 150}} = 17.33 \text{ mm} \simeq 18 \text{ mm}$$

Q.4 (b) Solution:

Different methods can be used for this problem, the given solution is using singularity or Macualay method.

FBD of the beam,



By taking moments at A and B, we get

$$R_B = \frac{Pa}{l} \text{ and } R_A = \frac{Pb}{l}$$

Using singularity method.

$$EI\frac{d^2y}{dx^2} = \frac{Pbx}{l} - P(x-a) \qquad \text{...(1)}$$

On integrating,

$$EI\frac{dy}{dx} = \frac{Pbx^2}{2l} - \frac{P(x-a)^2}{2} + c_1 \qquad \text{...(2)}$$

On integrating again,

$$EIy = \frac{Pbx^3}{6l} - \frac{P(x-a)^3}{6} + c_1 x + c_2 \qquad \text{...(3)}$$

Using boundary condition, $x = 0$, $y = 0$

$$c_2 = 0$$

Using another boundary condition $x = l$, $y = 0$

So,
$$0 = \frac{Pbl^2}{6} - \frac{Pb^3}{6} + c_1 l \qquad (\text{as, } l - a = b)$$

$$\Rightarrow \qquad c_1 = \frac{-Pb}{6l}(l^2 - b^2) = -\frac{Pab}{6l}(l+b)$$

Now we have slope and deflection equation so we can find slope and deflection at different location,

1.
$$EI\frac{dy}{dx} = \frac{Pbx^2}{2l} - \frac{P(x-a)^2}{2} - \frac{Pab}{6l}(l+b)$$

At A, $x = 0$

So, $$\theta_A = \left(\frac{dy}{dx}\right)_A = \frac{-Pab}{6EIl}(l+b)$$

At B, $x = l$

So, $$\theta_B = \left(\frac{dy}{dx}\right)_B = \frac{1}{EI}\left[\frac{Pbl}{2} - \frac{Pb^2}{2} - \frac{Pab}{6l}(l+b)\right]$$

$$= \frac{Pb}{2EI}\left(l - b - \frac{a}{3l}(l+b)\right)$$

$$= \frac{Pb}{2EI}\left(a - \frac{a}{3l}(l+b)\right)$$

$$= \frac{Pab}{2EI}\left(\frac{3l - l - b}{3l}\right) = \frac{Pab}{6EI\,l}(2l - b) = \frac{Pab}{6EI\,l}(l+a)$$

2. As given $a > b$, we can observe that maximum deflection will be in the region $x \in (o, a)$. For maximum deflection, $\frac{dy}{dx} = 0$.

So, using slope equation we get

$$0 = \frac{Pbx^2}{2l} - \frac{Pab(l+b)}{6l}$$

$$\Rightarrow \quad x = \sqrt{\frac{a(l+b)}{3}} \quad \text{(You can observe this } x \text{ is less than } a\text{)}$$

Putting this value of x in elastic curve or deflection curve, we get

$$EIy_{\max} = \frac{Pb}{6l}\left(\frac{a(l+b)}{3}\right)\sqrt{\frac{a(l+b)}{3}} - \frac{Pab}{6l}(l+b)\sqrt{\frac{a(l+b)}{3}}$$

$$EIy_{\max} = \frac{Pab}{6l}(l+b)\sqrt{\frac{a(l+b)}{3}}\left(\frac{1}{3} - 1\right)$$

$$\Rightarrow \quad y_{\max} = \frac{-Pab(l+b)}{9lEI}\sqrt{\frac{a(l+b)}{3}}$$

By comparing we can observe that,

$$k = -9$$

Where minus sign shows the deflection in downward direction.

Q.4 (c) Solution:

$$m_w = 0.7 \text{ kg}; r = 0.04 \text{ m}, m_f = 0.3 \text{ kg}$$

$$N = 3000 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60} = 314.16 \text{ rad/s}$$

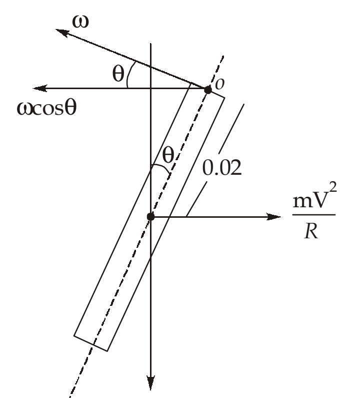
$$\text{Vehicle speed} = 60 \text{ km/h} = 60 \times \frac{5}{18} = 16.67 \text{ m/s}$$

Radius of circular path, R =100 m

$$\omega_p = \frac{V}{R} = \frac{16.67}{100} = 0.1667 \text{ rad/s}$$

Assuming wheel as a disc,

$$I_\omega = \frac{1}{2}Mr^2 = \frac{1}{2} \times 0.7 \times 0.04^2 = 560 \times 10^{-6} \text{ kgm}^2$$



$$I\omega\omega_p \cos\theta = 0.02 \sin\theta \times mg + \frac{mV^2}{R} \times 0.02 \cos\theta$$

$$\cos\theta \left(I\omega\omega_p - \frac{mV^2}{R} \times 0.02 \right) = 0.02 \times mg\sin\theta$$

$$\tan\theta = \left(\frac{I\omega\omega_p - \frac{mV^2}{R} \times 0.02}{0.02mg} \right) = 0$$

$$\tan\theta = \frac{560 \times 10^{-6} \times 0.1667 \times 314.16 - \frac{0.3 \times 16.67^2 \times 0.02}{100}}{0.02 \times 0.3 \times 9.81}$$

$$\tan\theta = 0.2149$$

$$\theta = 12.133°$$

Section B

Q.5 (a) Solution:

Given,

$$\text{Initial diameter, } d_1 = 200 \text{ mm}$$

$$\text{Initial height, } h_1 = 120 \text{ mm}$$

$$\text{Final height, } h_2 = 90 \text{ mm}$$

$$\text{Coefficient of friciton, } \mu = 0.25$$

$$\text{Average pressure, } p_{avg.} = \sigma_f\left(1+\frac{2\mu r_2}{3h_2}\right)$$

{where, r_2 is final radius and h_2 is final height}

$$\text{True strain, } \epsilon_1 = \ln\left(\frac{h_o}{h_1}\right) = \ln\left(\frac{120}{90}\right) = 0.28768$$

$$\text{Flow stress, } \sigma_f = k\epsilon_1^n = 1000\epsilon_1^{0.17} = 1000(0.28768)^{0.17}$$

$$= 809.123 \text{ MPa}$$

By volume conservation,

$$\frac{\pi}{4}d_1^2 h_1 = \frac{\pi}{4}d_2^2 h_2$$

$$(200)^2 \times 120 = d_2^2 \times (90)$$

$$\text{Final diameter, } d_2 = 230.94 \text{ mm}$$

$$\text{Average pressure, } p_{avg.} = \sigma_f\left(1+\frac{2\mu r_2}{3h_2}\right)$$

$$= 809.123\left(1+\frac{2\times 0.25\times 230.94}{2\times 3\times 90}\right)$$

$$p_{avg.} = 982.140 \text{ MPa}$$

$$\text{Upsetting force, } F = (p_{avg.}) \times (\pi r_2^2)$$

$$= (982.140)\times\pi\times\left(\frac{230.94}{2}\right)^2$$

$$F = 41139745.75 \text{ N}$$

Upsetting force at the end of the stroke, F = 41.1397 MN

Hence, upsetting force required at the end of the stroke is 41.1397 MN

Q.5 (b) Solution:

1. **Accumulator (ACC) register**:

- It is an 8-bit special purpose register that is a part of the ALU. It is also identified as register A.
- In arithmetic and logical operations the accumulator may store the operand, execute an instruction with the help of other registers, and memory and finally store the result of the operation. In the former case it acts as a source, and in the latter a destination.

2. **General purpose registers**:

- The 8085 microprocessor contains six -bit general purpose registers. These are identified as *B*, *C*, *D*, *E*, *H* and *L*.
- These registers are used in microprocessor for temporary storage of operands or intermediate data in calculations.
- These registers can be used either simply for storage of 8-bit data or in pairs for storage of 16 bit data. When used in pairs, only selected combination can be used for pairing, i.e., B-C, D-E and H-L. When registers are used in pairs the high order byte resides in the first register and low order byte in the second register.

3. **Stack pointer(SP)**:

- It is a 16 bit special function register.
- The stack is a sequence of memory locations set aside by a programmer to store/retrieve the contents of accumulator, flags, program counter and general purpose registers during the execution of a program. Any portion of the memory can be used as a stack.
- In this register, data is stored temporarily on first come and last go basis.

4. **Program counter (PC)**:

- It is a 16-bit special purpose register and is used to hold the memory address of the next instruction to be executed.
- The contents of the PC are automatically updated by the microprocessor during the execution of an instruction so that at the end of execution it points to the address of the next instruction in the memory.
- The microprocessor uses the PC for sequencing the execution of instructions.

5. **Instruction register:**

- During the execution of a program, microprocessor addresses some memory, which supplies an 8-bit data of instruction code to the data bus which gets stored in the register called the instruction register.
- The instruction register holds the op-code (operation code or instruction code) of the instruction which is being decoded and executed.

Q.5 (c) Solution:

If, $C = ₹250,\ C_h = 10\%$ of C

$$EOQ = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times 15000\times 2500}{250\times 0.1}} = 1732.051 \text{ units}$$

This calculated EOQ do not fall under offered lot size of 1-1499, so we have to go to next unit price.

If $C = ₹235,\ C_h = 0.1 \times 235 = ₹23.5$

$$EOQ = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times 15000\times 2500}{23.5}} = 1786.474 \text{ units}$$

$$TIC(EOQ) = \sqrt{2DC_0C_h} = \sqrt{2\times 15000\times 2500\times 23.5} = ₹41982.14$$

$$\text{Total cost, (TC)} = TIC + D \times C = 41982.14 + 15000 \times 235$$

$$= ₹3566982.14$$

$$\text{Total cost, } C(Q = 2500) = D\times C + \frac{Q}{2}\times C_h + \frac{D}{Q}\times C_0$$

$$= (15000\times 225) + \left(\frac{2500}{2}\times 225\times 0.1\right) + \left(\frac{15000}{2500}\times 2500\right)$$

$$= 3375000 + 28125 + 15000$$

$$TC = ₹3418125$$

$$\text{Total cost, } C(Q = 5000) = D\times C + \frac{Q}{2}\times C_h + \frac{D}{Q}\times C_0$$

$$= (15000\times 220) + \frac{15000}{5000}\times 2500 + \frac{5000}{2}\times 22$$

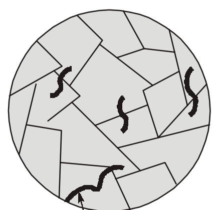
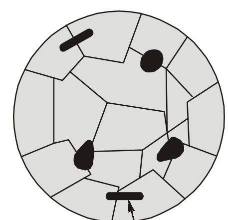
$$= 3300000 + 7500 + 55000 = ₹3362500$$

As evident from total inventory cost which is minimum for Q = 5000, best lot size for ordering is 5000 and above.

Q.5 (d) Solution:

Cast Iron: Cast iron are defined as the iron-carbon alloys that contain carbon between 2.1% and 6.67%. However commercial cast irons normally contain less than 4.5% of carbon. They can be easily melted because of the high percent of carbon and can be castable to required shape because of their fluidity.

Gray cast iron microstructure consists of graphite flakes dispersed throughout the metal matrix. The tips of the graphite flakes are sharp and pointed, and may act as the sites for crack initiation, when an external tensile stress is applied.

	Gray cast iron	**Malleable cast iron**
(i) Composition and heat treatment	C - 2.5 – 4% Si - 1.3% Mn - 0.7 – 0.8% No special heat treatment required.	C - 2.3 – 2.7% Si - 1–1.7% Mn - <0.55% Requires initial rapid cooling for formation of white cast iron, later, white cast iron is held at 800-900°C for more graphite formation, which leads to formation of malleable cast iron.
(ii)	 Graphite flakes	 Graphite clusters
(iii) Mechanical characteristics	→Weak in tension, but excellent compressive strength. →Good machinability as graphite flakes offer lubrication and chip breaking. →Very good damping characteristics as graphite flakes absorb energy. →Good resistance to adhesive wear.	→Increased ductility →Higher corrosion resistance →Excellent impact strength →Higher tensile strength
(iv) Application	As damper machine base materials, engine blocks of automobiles	General engineering equipments and others including. connecting rods, transmission gears and other heavy duty services.

Q.5 (e) Solution:

Given: $$P = 4 + 0.8L - 0.1L^2$$

For maximum power, $$\frac{dP}{dL} = 0$$

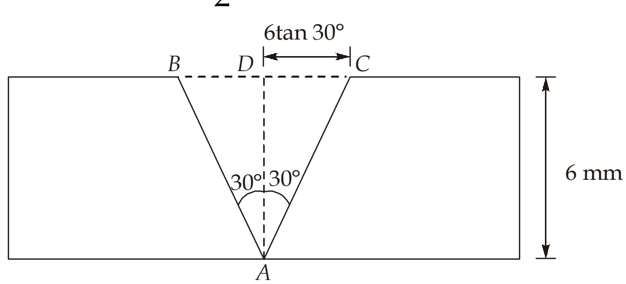
$$0.8 - 0.2L = 0$$

$$L = 4 \text{ mm}$$

Maximum power, $$P = 4 + 0.8 \times 4 - 0.1 \times 4^2 = 5.6 \text{ kW}$$

Efficiency, $$\eta = 75\%$$

Area of bead, $$A = \frac{1}{2} \times BC \times AD$$



$$= \frac{1}{2} \times (2 \times 6 \tan 30°) \times 6 = 20.7846 \text{ mm}^2$$

$$\text{Volume of weld bead} = (A \times l)$$

$$= (20.7846 \times 800) = 16627.68 \text{ mm}^3 = 16.62768 \text{ cm}^3$$

$$\text{Weight of weld bead} = V \times \rho$$

$$= (16.62768 \times 7880 \times 10^{-3}) \text{ gram}$$

$$= 131.026 \text{ gram}$$

$$\text{Heat required for melting} = 131.026 \times 1440 = 188677.44 \text{ Joule}$$

$$= 188.67744 \text{ kJ}$$

$$\text{Time required for welding} = \frac{188.67744}{0.75 \times 5.6} = 44.9232 \text{ s}$$

$$\text{Speed of welding} = \frac{800}{44.9232} = 17.808 \text{ mm/s}$$

Result: Speed of welding will be 17.808 mm/s

Q.6 (a) Solution:

Hydraulic actuators: An actuator wherein hydraulic energy is used to impart motion is called hydraulic actuators. All systems involving high loads are operated by hydraulic actuators in which oil pressure is applied on mechanical actuator to give an output in terms of rotary or linear motion. Basic example of hydraulic actuator is steering gear of ship in which hydraulic pressure is used to move the rudder actuator. It is based on principle of Pascal's law.

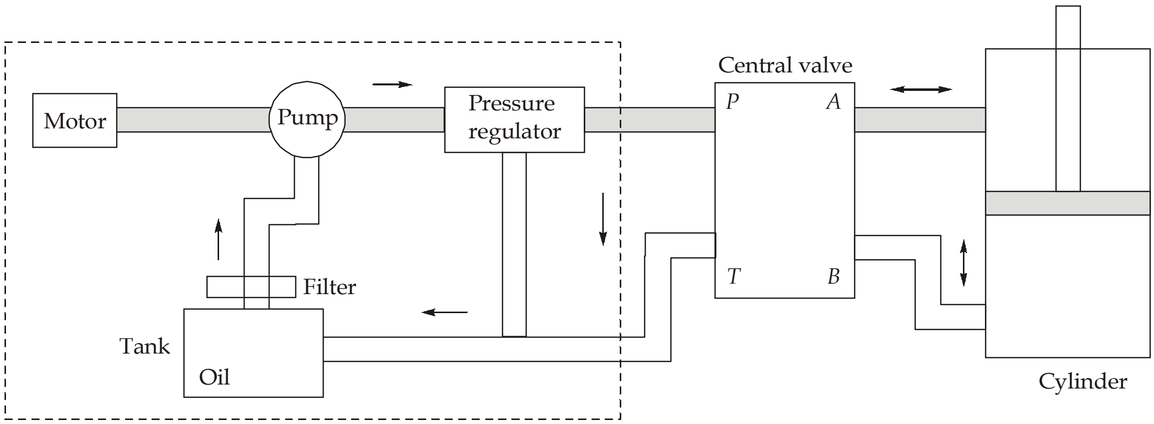
Advantages:

1. Easy to produce transmit, store, regulate and control, maintain and transform the hydraulic power.
2. Possible to generate high gain in force and power amplification.
3. Hydraulic systems are uniform and smooth, generate stepless motion and variable speed and force to a greater accuracy.
4. It is used where very high speed and large forces are required. It has higher load capacity.
5. Weight to power ratio of an hydraulic system is comparatively less than that for an electro mechanical system.

Disadvantages:

1. The manufacturing cost of the system is high since the hydraulic elements have to be machined to a high degree of precision.

2. Hydraulic elements have to be specially treated to protect them against rust, dirt, corrosion etc.
3. Petroleum based hydraulic oil may pose fire hazards thus limiting the upper level of working temperature.
4. Hydraulic power is not readily available compared to electric power.



Block diagram of hydraulic system

Pneumatic actuators: It uses pressurized air to transmit and control power. In this type, compressed air at high pressure is used which converts this energy into either linear or rotary motion. Pneumatic actuators enable large forces to be produced from relatively small pressure changes. The most common example is "Main engine pneumatic Actuator" used for changing of roller position over cam shaft for reversing.

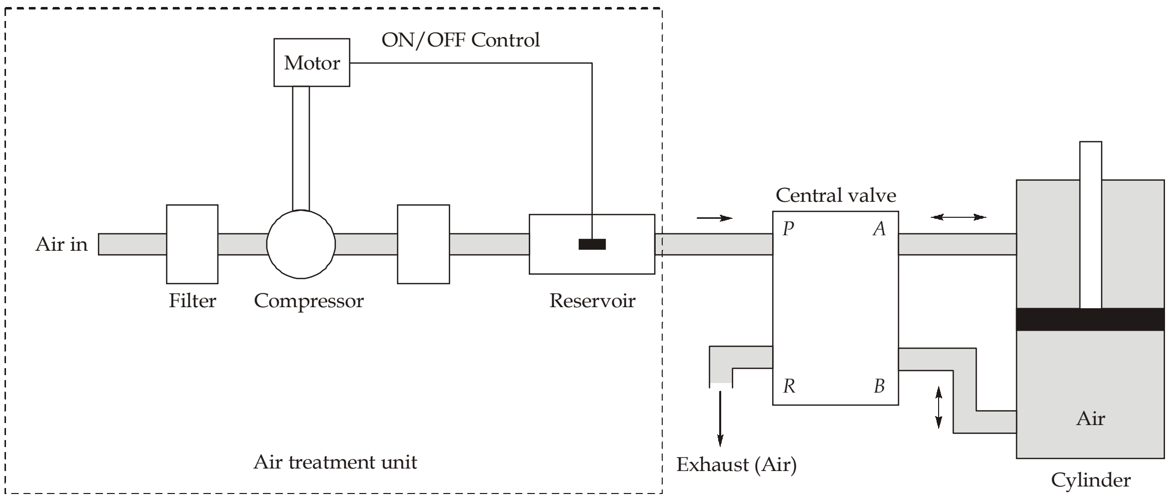
Advantages:

1. Pneumatic systems are fire and explosion proof whereas hydraulic system are not, unless non flammable liquid is used.
2. In this no return pipes are used when air is used.
3. Ecological purity.
4. Low cost
5. High speed operation
6. Pneumatic system are insensitive to temperature changes in contrast to hydraulic system in which fluid friction due to viscosity depends greatly on temperature.
7. Ease in reversal of movements.

Disadvantages:

1. The normal operating pressure of pneumatic system is lower than that of hydraulic system.
2. Accuracy of pneumatic actuators is poor at low velocities.
3. Output power is less compared to hydraulic system.
4. Difficulties in performance at slow speed.

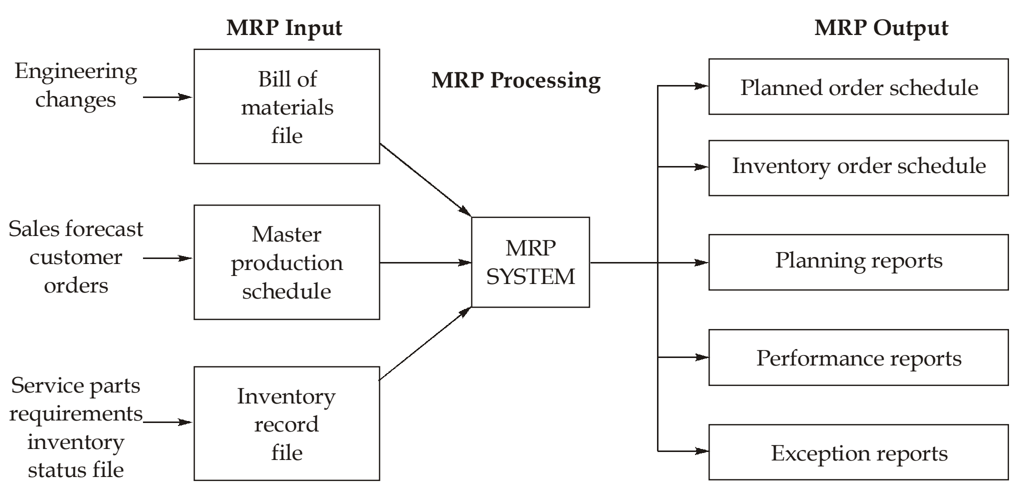
5. In pneumatic systems, external leakage is permissible to a certain extent, but internal leakage must be avoided because the effective pressure difference is rather small. In hydraulic system internal leakage is permissible to a certain extent, but external leakage must be avoided.



Block diagram of pneumatic system

Q.6 (b) Solution:

(i)



Primary outputs of MRP:

1. Planned order schedule.
2. Order release note.
3. Rescheduling notices.
4. Cancellation notices.
5. Reports on inventory status.

Secondary outputs of MRP:

1. Exception report
2. Performance report
3. Planning report

(ii)

The functions of inventory control are:

1. to minimize the capital investment in inventory by eliminating excessive stock,
2. to ensure availability of needed inventory for uninterrupted production and for meeting customer demand.
3. to provide a scientific basis for planning of inventory needs,
4. to minimize the risk of loss due to obsolescence, deterioration, etc. and
5. to tide over the demand fluctuations by maintaining reasonable safety stock.

In *ABC* analysis, the inventories of an organization are assumed to be of non equal value. Thus, the inventory is grouped into three categories (*A*, *B* and *C*) in order of their estimated importance.

'A' items are very important for an organization. These items are of high annual usage value and have a significant impact on overall inventory cost. Because of the high value of these '*A*' items, frequent value analysis is required. These items are held under very tight control and accurate records. In addition to that, an organization needs to choose an appropriate order pattern (e.g. just-in-time) to avoid excess capacity. 'A'-items have tight inventory control, more secured storage areas and better sales forecasts. Re-orders should be frequent and avoiding stock-outs on *A*-items is a priority.

Q.6 (c) Solution:

(i) Let, the true stress and true strain relationship is,

$$\sigma_T = k(\varepsilon_T^n)$$

where, σ_T is true stress, $\in_T$ true strain, k and n are constants.

We know that,
$$\sigma_T = \sigma(1 + \varepsilon)$$
$$(\sigma_T)_1 = 195\ (1 + 0.176) = 229.32 \text{ MPa}$$
$$(\sigma_T)_2 = 259\ (1 + 0.289) = 333.851 \text{ MPa}$$
$$\text{True strain, } \varepsilon_T = \ln(1 + \varepsilon)$$
$$(\varepsilon_T)_1 = \ln(1 + 0.176) = 0.16212$$
$$(\varepsilon_T)_2 = \ln(1 + 0.289) = 0.25387$$

Now,
$$\sigma_T = k\varepsilon_T^{\text{n}}$$
$$\ln\sigma_T = \ln k + n\ln\varepsilon_T$$

Putting values of (σ_T) and (ε_T) from above conditions,

$$\ln(229.32) = \ln k + n\ln(0.16212)$$
$$5.43512 = \ln k + n(-1.81942)$$
$$\ln k - (1.81942)n = 5.43512 \quad ...(1)$$

By second given condition,

$$\ln(333.851) = \ln k + n\ln(0.25387)$$
$$5.81069 = \ln k - (1.37093)\text{n}$$
$$\ln k - (1.37093)\text{n} = 5.81069 \quad ...(2)$$

On solving equation (1) and (2),

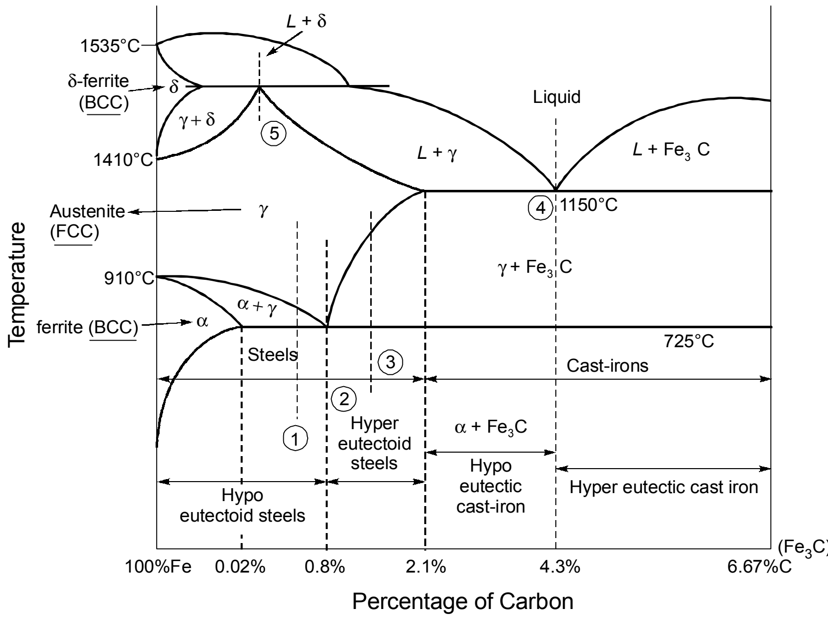
$$\ln k = 6.95875, \; n = 0.83741$$
$$k = 1052.28 \text{ MPa}$$
$$\sigma_T = 1052.28(\varepsilon_T^{0.83964}) \text{ MPa}$$

For engineering strain of 0.245,

$$\text{True strain, } \varepsilon_T = \ln(1+\varepsilon) = \ln(1.245) = 0.2191$$
$$\text{True stress, } \sigma_T = 1052.28 \times (0.2191)^{0.83964}$$
$$\sigma_T = 294.12 \text{ MPa}$$
$$\text{Engineering stress, } \sigma = \frac{\sigma_T}{(1+\varepsilon)} = \frac{294.12}{1+0.245}$$
$$\text{Engineering stress, } \sigma = 236.241 \text{ MPa}$$

Q.6 (c) Solution:

(ii)



Q.7 (a) Solution:

Step I: Converting the problem of maximization into equivalent minimization problem by subtracting each element of the matrix from the highest element of the matrix.

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

(Here, Highest element = 14)

Step II: Develop opportunity cost matrix

(a) Subtract the smallest element of each row from every element of the corresponding row.

(b) Subtract the smallest element of each column from every element of the corresponding column .

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	3	7
5	3	4	0	7

→

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

Step III: Make allocations in the opportunity cost matrix (i.e. put square on the zeros and cross all zeros (if any) of the corresponding row/column).

3	1	2	$\boxed{0}$	7
$\boxed{0}$	2	$\cancel{0}$	3	$\cancel{0}$
7	2	9	$\cancel{0}$	7
4	$\boxed{0}$	10	3	6
1	3	4	$\cancel{0}$	6

Since total number of allocations is less than the size of the matrix. So, current solution is not optimal. We have to perform optimally.

Step IV: Draw the minimum number of lines to cover all the zeros. Select the smallest element that do not have line through them. Subtract it from all the elements that do not have line through them. Add it to elements at the intersection of two lines and leave the remaining element of the matrix unchanged. Then again make allocations.

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

$\longrightarrow$

2	$\cancel{0}$	1	$\boxed{0}$	6
$\cancel{0}$	2	$\boxed{0}$	4	$\cancel{0}$
6	1	8	$\cancel{0}$	6
4	$\boxed{0}$	10	4	6
$\boxed{0}$	2	3	$\cancel{0}$	5

Since total number of allocations is less than the size of the matrix, so current solution is not optimal. Again we have to perform optimally.

Step V: Repeat step IV

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	4	6
0	2	3	0	5

$\longrightarrow$

2	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	5
1	3	$\cancel{0}$	5	$\boxed{0}$
6	1	7	$\boxed{0}$	5
4	$\boxed{0}$	9	4	5
$\boxed{0}$	2	2	$\cancel{0}$	4

Since total number of allocations is equal to the size of the matrix. So, current solution is optimal.

Optimal Assignment:

Machine	Job
1	3
2	5
3	4
4	2
5	1

Maximum profit = 10 + 5 + 14 + 14 + 7 = Rs. 50

Q.7 (b) Solution:

(i)

Load cells are elastic devices that can be used for measurement of force through indirect methods i.e., through use of secondary transducers.

Load cells utilize an elastic member as the primary transducer and strain gauges as secondary transducer. When the combination of the strain gauge-elastic member is used for weighing, it is called a “load cell”.

While designing load cells using strain gauges the following factors should be considered:

i. Stiffness of the elastic element.

ii. Optimum positioning of gauges on the element.

iii. Provision for compensation of the temperature.

When large loads are to be measured, the direct tensile -compressive member may be used, whereas, in case of small loads, strain amplification provided by bending may be used with advantage.

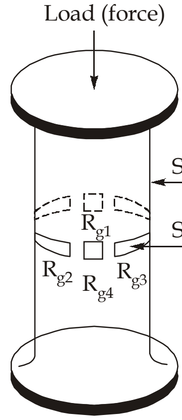
Strain-gauge load cells: These cells convert weight or force into electrical outputs which are provided by the strain gauges; these outputs can be connected to various measuring instruments for indicating, recording and controlling the weight or force.

Usually the strain gauges are directly applied to the force developing device and the device is calibrated against strain-gauge output.

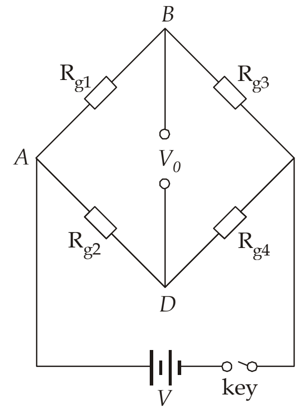
- These are excellent force measuring devices, particularly for transient and non-steady forces.
- These are used in conjuction with CRO (for display purposes) for measurement of rapidly changing loads.

Construction and working of the load cell:

It consists of a steel cylinder, on which four identical strain gauges are mounted. The gauges R_{g1} and R_{g4} are along the direction of applied load and the gauges R_{g2} and R_{g3} are attached circumferentially to gauges R_{g1} and R_{g4}. All the four gauges are connected electrically to the four limbs of a Wheatstone bridge circuit.



Load cell



Wheatstone bridge circuit

When there is no load on the cell, all the four gauges have the same resistance (i.e. $R_{g1} = R_{g2} = R_{g3} = R_{g4}$). Obviously the terminals B and D are at the same potential, the bridge is balanced and the output voltage is zero.

i.e., $$V_{AB} = V_{AD} = \frac{V}{2}$$

and, $$V_{AB} - V_{AD} = V_o = 0$$

On the application of a compressive load to the unit, the vertical gauges (R_{g1} and R_{g4}) undergo compression (i.e., negative strain) and, therefore, there is decrease in resistance. The circumferential gauges R_{g2} and R_{g3}, simultaneously, undergo tension (i.e., positive strain) leading to increase in resistance. The two strains are not equal; these are related to each other by a factor, μ, the Poisson's ratio.

Voltage due to applied load will be given as:

$$V_o = (1+\mu)\left(\frac{dR}{R}\times\frac{V}{2}\right)$$

Obviously, this voltage is a measure of the applied load.

The use of four identical strain gauges in each arm of the bridge provides full temperature compensation and also increases the sensitivity of the bridge 2(1 + μ)times.

Uses: The strain gauge load cells find extensive use in the following:

i. Road vehicle weighing devices.

ii. Draw bar and tool force dynamometers.

iii. Crane load monitoring etc.

(ii)

We know that, output voltage is:

$$V_o = (1+\mu)(G.F)\times \in \times \frac{V_i}{2}$$

G.F = Gauge factor

μ = Poission's ratio

$\in$ = Strain

V_i = Supply voltage

We know that
$$\in = \frac{\sigma}{E} = \frac{1\times 10^6}{200\times 10^9} = 5 \times 10^{-6}$$

So, output voltage,

$$V_o = (1+0.3)\times 2\times 5\times 10^{-6}\times \frac{6}{2} = 3.9 \times 10^{-5}$$

$$V_o = 39\ \mu V$$

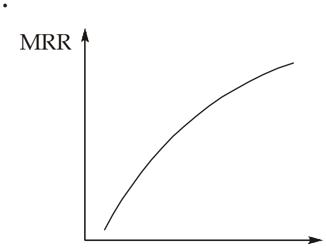
Sensitivity of load cell wheat stone bridge,

$$S = \frac{V_o}{P} = \frac{39\times 10^{-6}}{3\times 10^3} = 13 \times 10^{-9}\ V/N$$

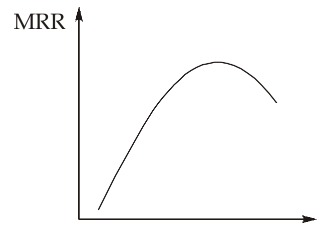
Q.7 (c) Solution:

(i)

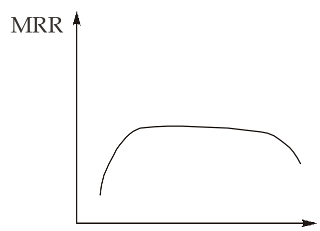
1. **Size of abrasive**: By increasing size of abrasive MRR increases because impact will be on more area, so MRR increases. Because standoff distance is large so MRR will never decrease.



2. **Percentage of grains**: As % of grains increases MRR increases because of increased concentration of abrasives. But after a limit grain will start colliding to each other this will decrease the MRR.



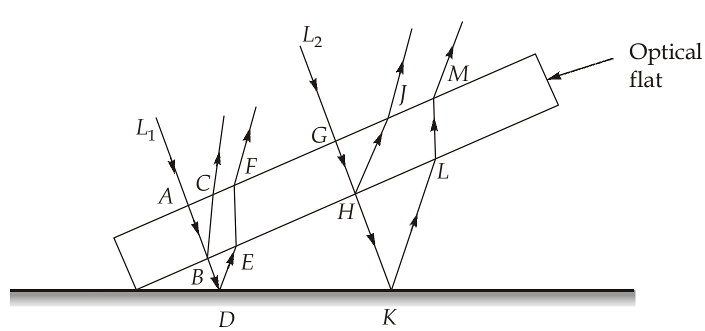
3. **Standoff distance**: Initially by increasing the standoff distance there will be more and more momentum due to increased acceleration time. These abrasives are thrown into atmosphere, so there will be drag between abrasives and atmospheric air, so in the intermediate portion of curve there will be a balance between acceleration and drag. So the curve will be flat. But after certain standoff distance drag will overcome the acceleration and hence MRR decreases.



(ii)

The flat, a glass or fused quartz disk with parallel flat surfaces, is placed on the surface of workpiece. When a monochromatic light beam is aimed at the surface at an angle, the optical flat splits it into two beams, appearing as light and dark bands to the naked eye. The number of fringes that appear is related to the distance between the surface of the part and the bottom surface of the optical flat.

A truly flat workpiece surface (that is when the angle between two surface is zero) will not split the light beam and no fringes will appear.



L_1 is a ray of monochromatic light, a portion of this ray will be reflected from B and a portion will be transmitted.

The transmitted ray will reflect back from the surface and come out of optical flat from F(as shown). For ray BC and EF there will be path difference equal to (BD + DE). If this path difference is equal to $\frac{\lambda}{2}$ there will be a dark band if this path difference is λ, a white fringe will appear.

So on the other end of optical flat the total path difference is (HK + KL).

Suppose there are 'n' fringes seen in optical flat (we count number of white fringes), then this path difference is equal to $n\lambda$.

$$HK \simeq KL \qquad 2HK = n\lambda$$

$$HK = \frac{n\lambda}{2}$$

Since angle is very small so HK can be approximated to a vertical line. If l is the length of surface(DK).

$$\frac{HK}{l} = \tan\theta \simeq \theta$$

Using this equation inclination angle of surface can be calculated.

Q.8 (a) Solution:

I^st case:

Given: $W_\alpha = 0.78$, $W_\beta = 0.22$, $C_0 = 30\%$ (wt% of Q)

We know that,

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$

$$0.78 = \frac{C_\beta - 0.30}{C_\beta - C_\alpha} \qquad \text{...(i)}$$

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha}$$

$$0.22 = \frac{0.30 - C_\alpha}{C_\beta - C_\alpha} \qquad \text{...(ii)}$$

Case II^nd case:

Given: $W_\alpha = 0.36$, $W_\beta = 0.64$, $C_0 = 65\%$ (wt% of Q)

We know that,

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$

$$0.36 = \frac{C_\beta - 0.65}{C_\beta - C_\alpha} \quad \text{...(iii)}$$

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha}$$

$$0.64 = \frac{0.65 - C_\alpha}{C_\beta - C_\alpha} \quad \text{...(iv)}$$

Equation (i) divided by equation (iii)

$$\frac{0.78}{0.36} = \frac{C_\beta - 0.30}{C_\beta - 0.65}$$

$$2.1667\ (C_\beta - 0.65) = C_\beta - 0.30$$
$$1.1667\ (C_\beta) = 1.408355 - 0.30$$
$$C_\beta = 0.94999$$
$$\simeq 0.95$$

Equation (ii) divided by equation (iv)

$$\frac{0.22}{0.64} = \frac{0.30 - C_\alpha}{0.65 - C_\alpha}$$

$$0.34375\ (0.65 - C_\alpha) = 0.30 - C_\alpha$$
$$0.65625\ (C_\alpha) = 0.30 - (0.34375 \times 0.65)$$
$$C_\alpha = \frac{0.0765625}{0.65625} = 0.1167$$

P-rich α-phase composition : 88.33 wt% of P and 11.67 wt% of Q

Q-rich β-phase composition : 5 wt% of P and 95 wt% of Q

Q.8 (b) Solution:

Volume of casting, $V_c = 25 \times 12.5 \times 5 = 1562.5 \text{ cm}^3$

Surface area of casting, $A_c = 2(25 \times 12.5 + 12.5 \times 5 + 5 \times 25) = 1000 \text{ cm}^2$

$$\text{For casting, } \left(\frac{V}{A}\right)_C = \frac{1562.5}{1000} = 1.5625$$

For side riser, Diameter of side riser = Height of side riser

$$d = h$$

$$\left(\frac{V}{A}\right)_r = \frac{\frac{\pi}{4}d^2h}{\frac{2\pi}{4}d^2 + \pi dh} = \frac{\left(\frac{1}{4}\right)d^3}{\frac{3}{2}d^2} = \frac{d}{6}$$

(i) Using Caine's method,

$$\text{Freezing ratio, } x = \frac{\left(\frac{V}{A}\right)_r}{\left(\frac{V}{A}\right)_c}$$

$$x = \frac{a}{y-b} + c \qquad \ldots(1)$$

where,
$$y = \frac{V_r}{V_c} = \frac{\frac{\pi}{4}d^2h}{1562.5} = \frac{\pi d^3}{6250}$$

Now,
$$x = \frac{\left(\frac{V}{A}\right)_r}{\left(\frac{V}{A}\right)_c} = \frac{\left(\frac{d}{6}\right)}{1.5625} = \frac{d}{9.375}$$

Now, putting value of x, y in equation (1),

$$x = \left(\frac{a}{y-b}\right) + c$$

$$\frac{d}{9.375} = \frac{0.1}{\frac{\pi d^3}{6250} - 0.03} + 1$$

$$\frac{d}{9.375} = \frac{\left[0.1 \times 6250 + \pi d^3 - 0.03 \times 6250\right]}{\pi d^3 - 0.03 \times 6250}$$

$$\pi d^4 - 187.5d = 9.375(\pi d^3 + 437.5)$$

$$\pi d^4 - 9.375\,\pi d^3 - 187.5d - 4101.5625 = 0$$

On solving, $d = 10.889$ cm

Hence dimensions of optimum cylindrical side riser,

$$\text{Diameter of side rise} = \text{Height of side riser} = 10.889 \text{ cm}$$

(ii) Using modified Caine's method:

$$\text{Shape factor, S.F.} = \frac{L+W}{T} = \frac{25+12.5}{5} = 7.5$$

From given table,

$$\left(\frac{V_r}{V_c}\right) = 0.5$$

(From table V_r/V_c value corresponding to shape factor)

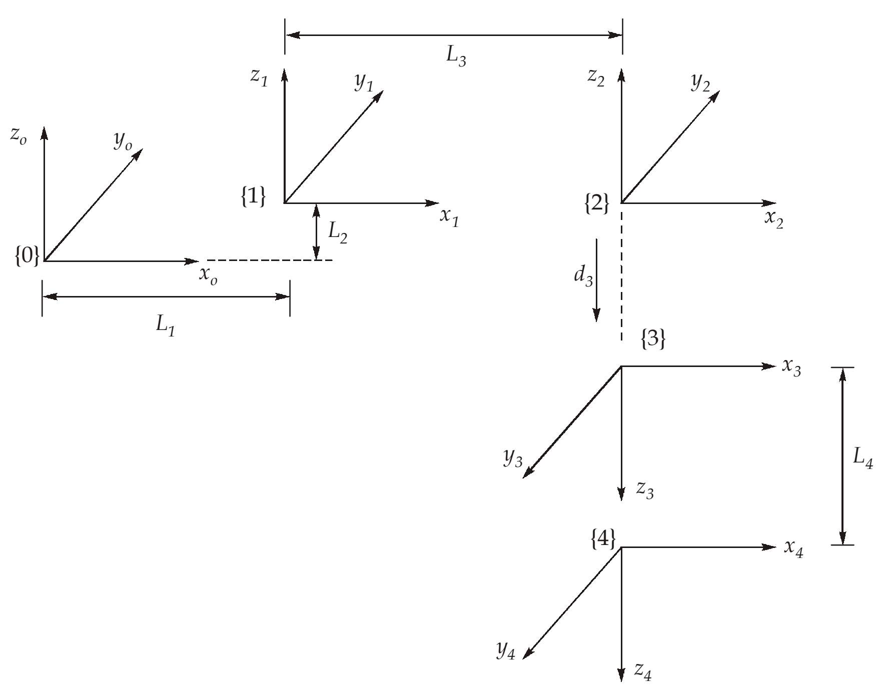
$$\frac{\frac{\pi}{4}d^2h}{1562.5} = 0.5$$

$$\pi d^3 = 0.5 \times 1562.5 \times 4$$

$$d^3 = 994.7184$$

$$\text{Diameter} = \text{Height} = d = 9.98 \text{ cm}$$

Q.8 (c) Solution:



Link i	a_i	α_i	d_i	θ_i	q_i	$c\theta_i$	$s\theta_i$	$c\alpha_i$	$s\alpha_i$
1	L_1	0	L_2	$\theta_1 = 30°$	θ_1	0.866	0.5	1	0
2	L_3	0	0	$\theta_2 = 45°$	θ_2	0.707	0.707	1	0
3	0	180°	d_3	0	d_3	1	0	–1	0
4	0	0	L_4	$\theta_4 = 30°$	θ_4	0.866	0.5	1	0

We know that,

$$^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1(\theta_1) = \begin{bmatrix} 0.866 & -0.5 & 0 & 50\times0.866 \\ 0.5 & 0.866 & 0 & 50\times0.5 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2(\theta_2) = \begin{bmatrix} 0.707 & -0.707 & 0 & 50\times0.707 \\ 0.707 & 0.707 & 0 & 50\times0.707 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_4(\theta_4) = \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall transformation matrix is $^{0}T_4 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4$

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