India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 14

Section-A

Q.1 (a) Solution:

The oriented graph of the network is shown in figure below. Since we have to find v, we take the branch (2) in the twig and a posible tree is selected.

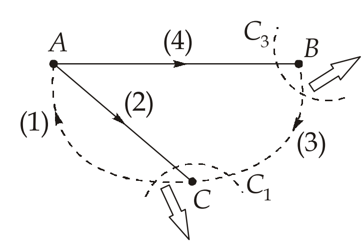
The fundamental cutsets are identified as

$$f\text{-cut-set-1} \;:\; [1, 2, 3]$$

$$f\text{-cut-set-2} \;:\; [3, 4]$$

The fundamental cut-set matrix is given as

$$Q_a = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline C_1 & -1 & 1 & 1 & 0 \\ C_2 & 0 & 0 & -1 & 1 \end{array}$$



The node equations are given as,

$$[Q]\,[Y_b]\,[Q^T]\,[V_t] = [Q]\,\{[Y_b]\,[V_s] - [I_s]\}$$

Here, $$[Q][Y_b][Q^T] = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$[Q]\times\{[Y_b][V_s]-[I_s]\} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \left\{ \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2v \end{bmatrix} \right\} = \begin{bmatrix} 1 \\ -2v \end{bmatrix}$$

Thus, the KCL equations are

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} V_{t2} \\ V_{t4} \end{bmatrix} = \begin{bmatrix} 1 \\ -2v \end{bmatrix}$$

Here, $V_{t2} = v$. Putting this in the KCL equations and solving we get, $v = \frac{4}{9}\text{ V}$.

Q.1 (b) Solution:

(i) We want the excess carrier concentration

$$\delta n = \delta p = 10^{15}\text{ cm}^{-3}$$

$$\therefore \quad g'\tau = 10^{15}$$

$$g' = \frac{10^{15}}{10^{-7}} = 10^{22}\text{ cm}^{-3}\text{ s}^{-1} = \text{generation rate}$$

now, we have, $h\nu = 1.9\text{ eV}$

$$\therefore \quad \lambda = \frac{1.24}{1.9} = 0.65\ \mu\text{m}$$

So that, $\alpha = 1.3 \times 10^4\text{ cm}^{-1}$

then $$g' = \frac{\alpha I(x)}{h\nu}$$

$$\therefore \quad I(x) = \frac{g'(h\nu)}{\alpha} = \frac{10^{22}\times1.6\times10^{-19}\times1.9}{1.3\times10^{4}}$$

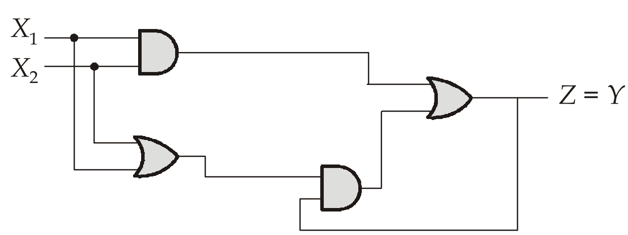
$$I(0) = 0.234 \text{ W/cm}^2$$

(ii) $$\frac{I(x)}{I_0} = 0.2 = \exp(-\alpha x)$$

$$x = \frac{-1}{1.3\times10^{4}}\ln(0.2) = 1.24\ \mu\text{m}$$

Q.1 (c) Solution:

The logic diagram of the circuit can be constructed by assuming X_1 and X_2 as the input and Y to be feedback path as its value is equivalent to the output value i.e., $Y \equiv Z$.



The state table can be constructed as

Present total state			Next total state			
Present state	Input		Present state	Input		Output
Y	X_1	X_2	Y	X_1	X_2	Z
0	0	0	0	0	0	0
0	0	1	0	0	1	0
0	1	1	1	1	1	1
0	1	0	0	1	0	0
1	0	0	0	0	0	0
1	0	1	1	0	1	1
1	1	1	1	1	1	1
1	1	0	1	1	0	1

Q.1 (d) Solution:

(i) We know that

$$\text{Gauge factor, } G_f = \frac{\Delta R/R}{\Delta L/L}$$

$$\therefore \quad \text{Change in length, } \Delta L = \frac{\frac{\Delta R}{R}\cdot L}{G_f}$$

$$\Delta L = \frac{\left(\frac{0.013}{240}\right)}{2.2}\times 0.1 = 2.46 \times 10^{-6} \text{ m}$$

$$\text{Stress } S = E\cdot\frac{\Delta L}{L} = \frac{207\times 10^9 \times 2.46\times 10^{-6}}{0.1}$$

$$\therefore \quad \text{stress } S = 5.092 \times 10^6 \text{ N/m}^2$$

$$\text{Force, } F = SA$$

$$= 5.092 \times 10^6 \times 4 \times 10^{-4}$$

$$F = 2.037 \times 10^3 \text{ N}$$

(ii) Advantages:

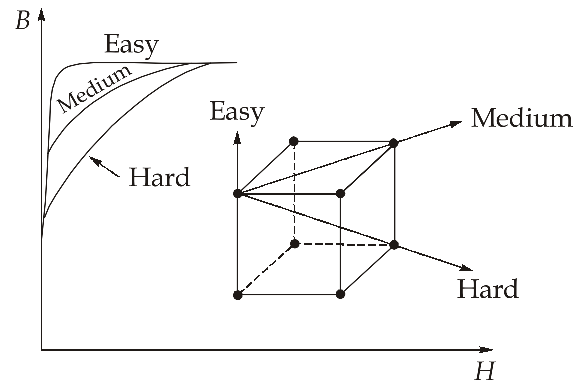
- Semiconductor strain gauges have a high gauge factor.
- Useful for measurement of very small strains of the order of 0.01 microstrain.
- They have excellent Hysterisis characteristics.
- Frequency of operation upto 10^{12} Hz.
- They have very small sizes ranging in length from 0.7 to 7 mm.

Disadvantages:

- These are very sensitive to changes in temperature.
- Linearity of the semiconductor strain gauge is poor.
- The gauge factor varies with strain.
- This gauge is non-linear at comparatively high strain levels.
- More expensive and difficult to attach to the object under study.

Q.1 (e) Solution:

(i) If in a magnetic material different magnetization is achieved by the application of magnetic field in different direction, this property is known as magnetic anisotropy. The magnetization curves in different directions of an iron single crystal are shown in figure.



In bulk materials, there are three basic methods by which uniaxial anisotropy can be induced:

(a) **Cold working:** Cold working such as cold rolling brings some regularities in the orientation of the unit cell and this help in achieving a large value of magnetization for the applied magnetic field.

Example: CRGO(Cold Rolled Grain Oriented)-Si steel is made from this process which is used in making core of the transformer.

(b) **Magnetic annealing:** The material is cooled down naturally in the presence of magnetic field. This also makes material anisotropic.

(c) **Magnetic quenching:** Material is fast cooled in the presence of a magnetic field through the Curie temperature, leading a uniaxial anisotropy, either parallel or perpendicular to the field direction.

(ii) 1. Meissner discovered that superconductors not only exhibit zero resistance but also spontaneously expell all magnetic flux when cooled through the superconducting transistion temperature that is they are also perfect diamagnets. This is the Meissner effect.

2. It has been observed that when a long superconductor is cooled in a longitudinal magnetic field from above the transition temperature, the lines of induction are pushed out. Then inside the specimen

$$B = 0$$

$$B = \mu_0(H + M)$$

For $B = 0$, we have

$$H + M = 0$$

$$\Rightarrow \quad H = -M$$

Since $\chi_m = M/H = -1$, we may state that magnetic susceptibility in a superconductor is negative. This is referred to as perfect diamagnetism. One of the Maxwell's equations gives

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

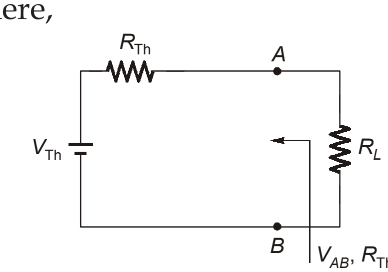
and from ohm's law

$J = \sigma E$ or $E = \rho J$ where ρ = resistivity with $\rho = 0$, $E = 0$. So $\partial B/\partial t$ should be zero, but this is not so because the flux exclusion from normal to superconducting state takes place. The perfect diamagnetism and zero resistivity are two independent properties in the superconducting state.

3. Meissner effect is not a consequence of zero resistance and Lenz's law.

Q.2 (a) Solution:

Let us consider a Thevenin's equivalent circuit as shown here,



$\therefore$ Voltage across terminal A-B is,

$$V_{AB} = \frac{R_L}{R_{Th} + R_L} . V_{Th} \quad \text{...(i)}$$

$\therefore$ Current through R_L is,

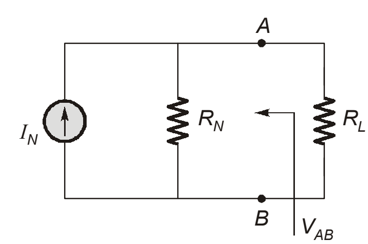
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad \text{...(ii)}$$

Now let us consider a Norton's equivalent circuit.

$$V_{AB} = \frac{R_N R_L}{R_N + R_L} . I_N \quad \text{...(iii)}$$

Current through R_L is

$$I_L = \frac{R_N}{R_N + R_L} \cdot I_N \quad \text{...(iv)}$$



If $R_N = R_{Th}$ and $I_N = \dfrac{V_{Th}}{R_{Th}}$, then equations (iii) becomes

$$\Rightarrow \quad V_{AB} = \frac{R_{Th} \cdot R_L}{R_{Th} + R_L} \cdot \frac{V_{Th}}{R_{Th}}$$

$$\Rightarrow \quad V_{AB} = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th} \quad \text{...(v)}$$

Putting values in equation (iv),

$$I_L = \frac{R_{Th}}{R_{Th} + R_L} \cdot \frac{V_{Th}}{R_{Th}}$$

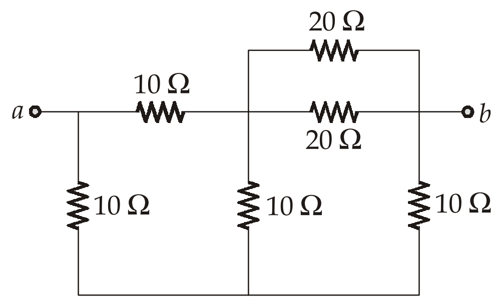
$$\Rightarrow \quad I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad \text{...(vi)}$$

Since equations (i) & (v) and (ii) & (vi) are same, it is clear that **Thevenin's and Norton's theorem is dual of each other and the values are related as,**

$$R_{Th} = R_N$$

and $$V_{Th} = I_N.R_{Th}, \quad \textbf{Proved}$$

In order to determine the value of R_{Th} the circuit can be redrawn as



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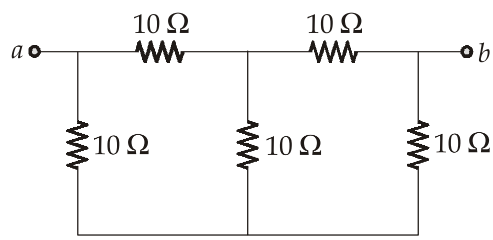
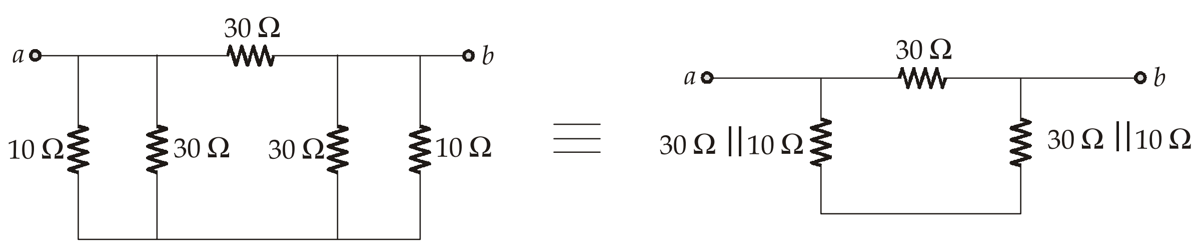
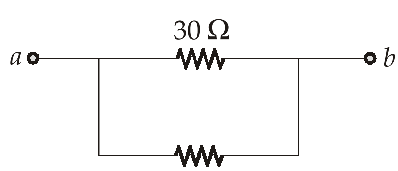


Using Y to Δ transformation, we get,

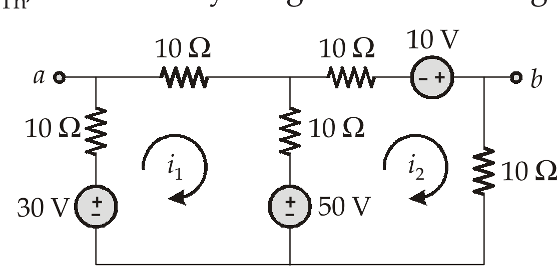




or $$R_{ab} = R_{\text{Th}} = 30\ \Omega\,||\,(7.5 + 7.5)\ \Omega$$

$$R_{\text{Th}} = 10\ \Omega$$

In order to determine V_{Th}, let us modify the given circuit using source transformation as



Using mesh analysis at loop (i), we get,

$$-30 + 50 + 30i_1 - 10i_2 = 0$$

or $$-2 = 3i_1 - i_2 \quad ...(i)$$

For loop (ii),

$$-50 - 10 + 30i_2 - 10i_1 = 0$$

or, $$6 = -i_1 + 3i_2 \quad ...(ii)$$

From equation (i) and (ii), we get,

$$i_1 = 0$$

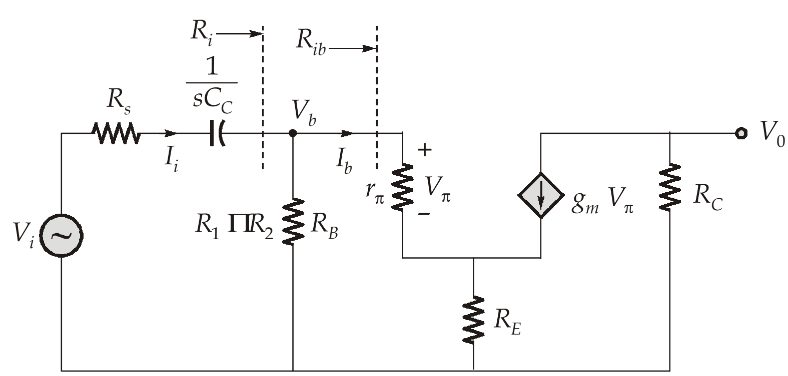
and $$i_2 = 2 \text{ A}$$

$\therefore$ $$V_{Th} = V_{ab} = 20 - 10 = 10 \text{ V}$$

$$V_{Th} = 10 \text{ V}$$

Q.2 (b) Solution:

The small signal equivalent circuit can be drawn as



Let the resistance $R_B = R_1 \parallel R_2$

the input current $$I_i = \frac{V_i}{R_s + \frac{1}{sC_c} + R_i}$$

now, resistance $$R_{ib} = \frac{V_b}{I_b}$$

$$V_b = r_\pi I_b + (\beta + 1)R_E I_b$$

$$R_{ib} = \frac{V_b}{I_b} = r_\pi + (\beta + 1)R_E$$

and $$R_i = R_B \parallel R_{ib}$$

Current, $$I_b = \frac{R_B}{R_B + R_{ib}} \cdot I_i \qquad \text{(current division)}$$

and then $$V_\pi = I_b r_\pi$$

and $$V_0 = -g_m R_C V_\pi$$

$$= -g_m R_C I_b \cdot r_\pi$$

$$= -g_m R_C r_\pi \cdot \left(\frac{R_B}{R_B + R_{ib}}\right) I_i$$

$$= -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right) \cdot \frac{V_i}{R_s + \frac{1}{sC_C} + R_i}$$

$$A_V(s) = \frac{V_0(s)}{V_i(s)} = -g_m r_\pi R_C \left(\frac{R_B}{R_B + R_{ib}}\right)\left[\frac{sC_C}{1 + s(R_S + R_i)C_C}\right]$$

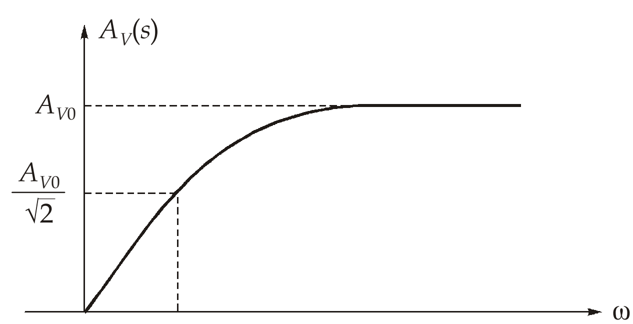
$$A_V(s) = \left(\frac{-g_m r_\pi R_C}{R_S + R_i}\right)\left(\frac{R_B}{R_B + R_{ib}}\right) \cdot \left(\frac{s\tau_s}{1 + s\tau_s}\right)$$

where $$\tau_s = (R_S + R_i)C_C$$

Now,

$$A_V(s) = \frac{-g_m r_\pi R_C}{R_S + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\left[\frac{R_1 \| R_2}{R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\right]\left[\frac{s(R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E])}{1 + s(R_s + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E])}\right]$$

$$A_{V0} = \frac{g_m r_\pi R_C}{R_S + R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\left[\frac{R_1 \| R_2}{R_1 \| R_2 \| [r_\pi + (\beta+1)R_E]}\right]$$

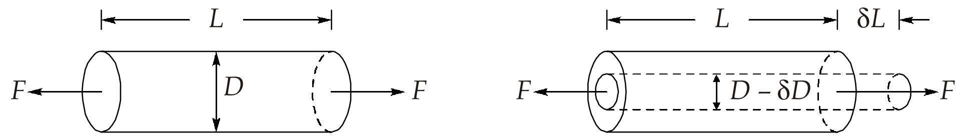


$$f_c = \frac{1}{2\pi C_C[R_s + R_1 \Pi R_2 \Pi(r_\pi + (\beta + 1)R_E)]}$$

Q.2 (c) Solution:

Let a wire be having a resistance, length, diameter and area of R, l, D and A respectively. Let a tensile stress S be applied on it.

Thus, the length of the wire increases and it's area reduces. Let charge in length be δl, change in area = δA, change in diameter be δD and change in resistance be δR.



Tensile Force

Initial resistance, $R = \dfrac{\rho l}{A}$, Where ρ = resistivity of wire

$$\therefore \quad \frac{\delta R}{\delta S} = \frac{\delta}{\delta S}\left(\frac{\rho l}{A}\right) = \frac{l}{A}\cdot\frac{\delta\rho}{\delta S} + \frac{\rho}{A}\frac{\delta l}{\delta S} + \rho l\cdot\frac{\delta}{\delta S}\left(\frac{1}{A}\right)$$

$$\text{or,} \quad \frac{\delta R}{\delta S} = \frac{l}{A}\cdot\frac{\delta\rho}{\delta S} + \frac{\rho}{A}\cdot\frac{\delta l}{\delta S} - \frac{\rho l}{A^2}\cdot\left(\frac{\delta A}{\delta S}\right)$$

$$\text{or,} \quad \frac{1}{R}\cdot\frac{\delta R}{\delta S} = \frac{1}{\left(\frac{\rho l}{A}\right)}\cdot\left[\frac{l}{A}\cdot\frac{\delta\rho}{\delta S} + \frac{\rho}{A}\cdot\frac{\delta l}{\delta S} - \frac{\rho l}{A^2}\left(\frac{\delta A}{\delta S}\right)\right]$$

$$\text{or,} \quad \frac{1}{R}\cdot\frac{\delta R}{\delta S} = \left[\frac{1}{\rho}\cdot\frac{\delta\rho}{\delta S} + \frac{1}{l}\cdot\frac{\delta l}{\delta S} - \frac{1}{A}\cdot\frac{\delta A}{\delta S}\right] \qquad \text{...(1)}$$

$$\text{Now,} \quad A = \frac{\pi D^2}{4};\ \frac{\delta A}{\delta S} = \frac{\pi D}{2}\cdot\frac{\delta D}{\delta S};\ \frac{1}{A}\cdot\frac{\delta A}{\delta S} = \frac{1}{\left(\frac{\pi D^2}{4}\right)}\cdot\frac{\pi D}{2}\left(\frac{\delta D}{\delta S}\right)$$

$$\therefore \quad \frac{1}{A}\cdot\frac{\delta A}{\delta S} = \frac{2}{D}\cdot\frac{\delta D}{\delta S} \qquad \text{...(2)}$$

Substituting (2) in equation (1), we get:

$$\frac{1}{R}\cdot\frac{\delta R}{\delta S} = \frac{1}{\rho}\cdot\frac{\delta\rho}{\delta S} + \frac{1}{l}\cdot\frac{\delta l}{\delta S} - \frac{2}{D}\cdot\frac{\delta D}{\delta S}$$

$$\text{or,} \quad \frac{\delta R}{R} = \frac{\delta\rho}{\rho} + \frac{\delta l}{l} - 2\frac{\delta D}{D}$$

$$\text{or,} \quad \frac{\left(\frac{\delta R}{R}\right)}{\left(\frac{\delta l}{l}\right)} = \frac{\left(\frac{\delta\rho}{\rho}\right)}{\left(\frac{\delta l}{l}\right)} + 1 - \frac{2\left(\frac{\delta D}{D}\right)}{\left(\frac{\delta l}{l}\right)}$$

Neglecting $\dfrac{\left(\frac{\delta\rho}{\rho}\right)}{\left(\frac{\delta l}{l}\right)}$, we have

$$\left(\frac{\delta R/R}{\delta l/l}\right) = G_f = 1 + 2\gamma \text{ or } \boxed{G_f = \frac{\delta R/R}{\delta l/l} = 1+2\gamma}$$

where, $$\gamma = \frac{-\delta D/D}{\delta l/l} = \text{Poisson's ratio}$$

Q.3 (a) Solution:

At no load terminal voltage is $V_t = 2500 \text{ V} = E_f$

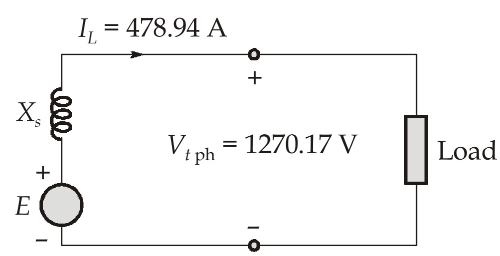
$$E_{f\,\text{ph}} = \frac{2500}{\sqrt{3}} = 1443.37 \text{ V}$$

At full load, $$V_t = 2200 \text{ V} \Rightarrow V_{tph} = \frac{2200}{\sqrt{3}} = 1270.17 \text{ V}$$

As we know that, $$P_{3\text{-}\phi} = \sqrt{3}\, V_L\, I_L \cos\theta$$

$$1460 \times 10^3 = \sqrt{3} \times 2200 \times I_L \times 0.8$$

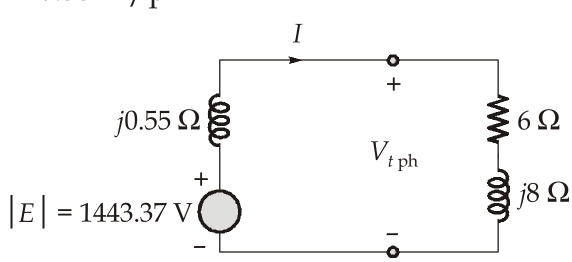
$$I_L = 478.94 \text{ A}$$



$$|E| = \sqrt{(V_{tph}\cos\theta + I_a R_a)^2 + (V_{tph}\sin\theta + I_a X_s)^2}$$

$$1443.37 = \sqrt{(1270.17\times 0.8)^2 + [(1270.17\times 0.6) + 478.94 X_s]^2}$$

$$X_s = 0.55\ \Omega/\text{ph}$$



$$I = \frac{1443.37}{6 + j8.55} = 138.18\angle -54.9^\circ \text{ A}$$

$$V_{ph\,t} = (138.18\ \angle -54.9^\circ) \times (6 + j8)$$

$$\left|V_{ph\,t}\right| = 1381.8 \text{ V}$$

Terminal voltage, $V_t = \sqrt{3} \times 1381.8 = 2393.34 \text{ V}$

Q.3 (b) Solution:

The built in voltage can be calculated as

$$V_j = V_T \ln\left[\frac{N_A N_D}{n_i^2}\right]$$

$$= 0.0259 \ln\left[\frac{10^{18} \times 5 \times 10^{15}}{(1.5 \times 10^{10})^2}\right] = 0.796 \text{ V}$$

Area, $A = \pi(5 \times 10^{-4})^2$

$$= 7.85 \times 10^{-7} \text{ cm}^2$$

and $W = \left[\frac{2\varepsilon_{Si} V_j}{q}\left[\frac{1}{N_A} + \frac{1}{N_D}\right]\right]^{1/2}$

$$= \left[\frac{2(11.8)(8.85 \times 10^{-14})(0.796)}{1.6 \times 10^{-19}}(10^{-18} + 10^{-16} \times 2)\right]^{1/2}$$

$$= 0.457\ \mu\text{m}$$

Now,

(i) $x_{no} = \frac{W}{1 + \frac{N_D}{N_A}} = \frac{0.457}{1 + 5 \times 10^{-3}} = 0.455\ \mu\text{m}$

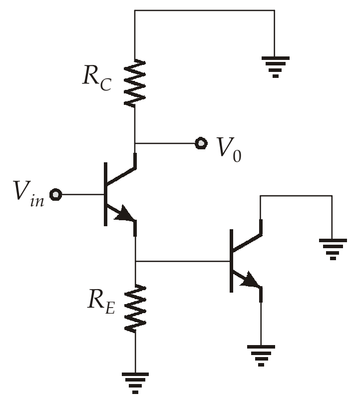
(ii) $x_{po} = \frac{0.457}{1 + \frac{N_D}{N_A}} = \frac{0.457}{1 + 200} = 2.27 \times 10^{-3}\ \mu\text{m}$

(iii) now, $|Q_n| = |Q_p| = |qAx_{no}N_D| = |qAx_{po}N_A|$

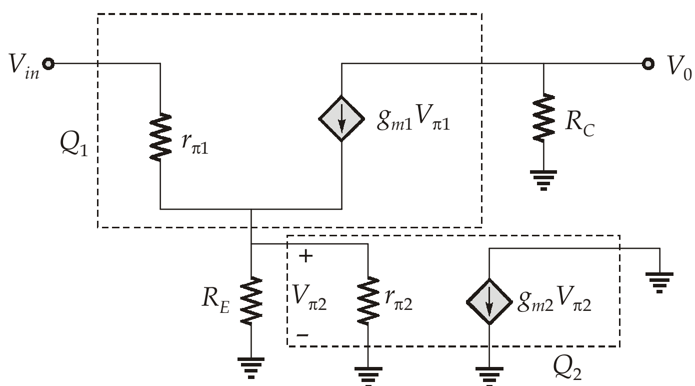
$$= |1.6 \times 10^{-19} \times (7.85 \times 10^{-7})(2.27 \times 10^{11})|$$

$$= 2.85 \times 10^{-14} \text{ C}$$

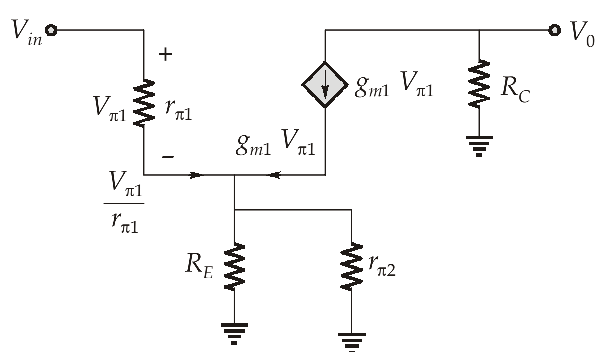
Q.3 (c) Solution:



Drawing the small-signal equivalent circuit will give



thus, the circuit can be redrawn as



$$V_{in} = V_{\pi 1} + \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)(R_E \parallel r_{\pi 2})$$

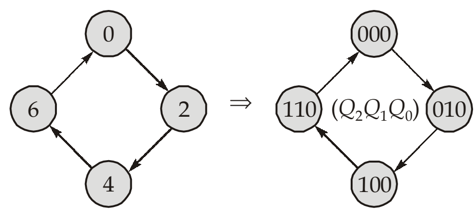
$$= \left[1 + \left(\frac{1}{r_{\pi 1}} + g_{m1}\right)(R_E \parallel r_{\pi 2})\right]V_{\pi 1}$$

Now, $V_0 = -g_{m1} R_C V_{\pi 1}$

$$\therefore \quad \frac{V_0}{V_{in}} = \frac{-g_{m1}R_C}{1+\left(\frac{1}{r_{\pi 1}}+g_{m1}\right)(R_E \parallel r_{\pi 2})}$$

4. (a) Solution

The sequence diagram of the counter to be designed is,



Excitation table:

	Present state			Next state			Excitations		
	Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
	0	0	0	0	1	0	0	1	0
*	0	0	1	×	×	×	×	×	×
	0	1	0	1	0	0	1	0	0
*	0	1	1	×	×	×	×	×	×
	1	0	0	1	1	0	1	1	0
*	1	0	1	×	×	×	×	×	×
	1	1	0	0	0	0	0	0	0
*	1	1	1	×	×	×	×	×	×

"*" indicates unused states of the counter

Minimization:

K-map for D_2

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0		×	×	1
1	1	×	×	

$D_2 = Q_2\overline{Q}_1 + \overline{Q}_2Q_1 = Q_2 \oplus Q_1$

K-map for D_1

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0	1	×	×	
1	1	×	×	

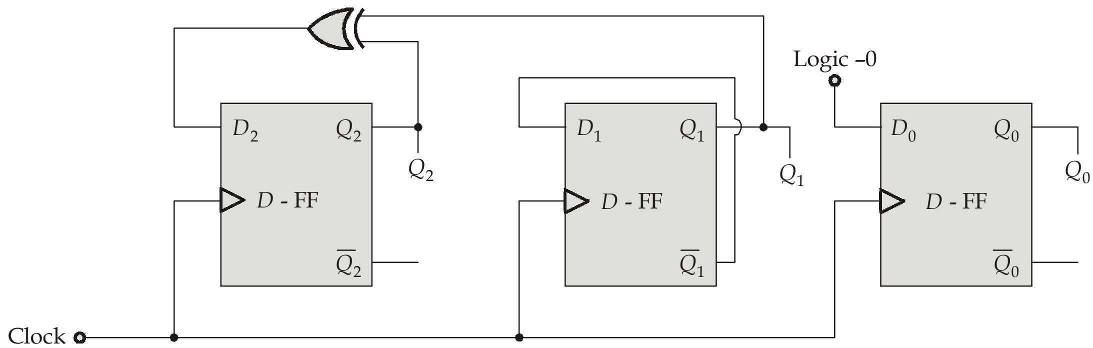
$D_1 = \overline{Q}_1$

K-map for D_0

$Q_2 \backslash Q_1Q_0$	00	01	11	10
0		×	×	
1		×	×	

$D_0 = 0$

Logic circuit:



Checking for self starting:

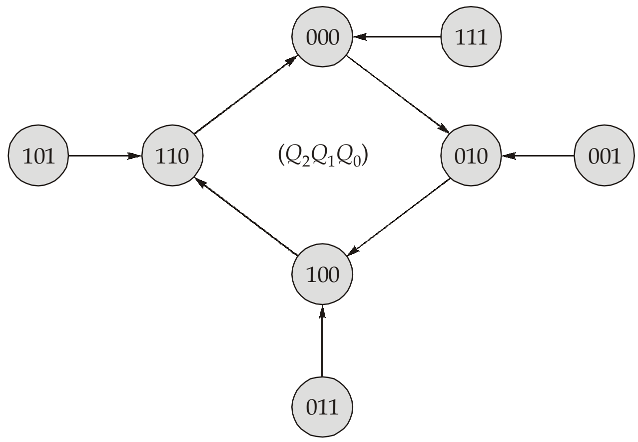
- A counter is said to be self starting when it enters into a used or valid state from an unused state within finite number of clock cycles.
- In the above designed counter, there are four unused states (1, 3, 5, 7). In order to determine the self starting capability of the counter, the next states of the unused states are to be examined, which can be done as shown below.

$$D_2 = Q_2 \oplus Q_1$$
$$D_1 = \bar{Q}_1$$
$$D_0 = 0$$

Present state			Excitations			Next state		
Q_2	Q_1	Q_0	D_2	D_1	D_0	Q_2^+	Q_1^+	Q_0^+
0	0	1	0	1	0	0	1	0
0	1	1	1	0	0	1	0	0
1	0	1	1	1	0	1	1	0
1	1	1	0	0	0	0	0	0

- From the above sequence table, it is clear that, from all the unused states, the counter will enter into a valid state within finite number of clock cycles. So, the designed counter is said to be self starting.

Complete sequence diagram:



Q.4 (b) Solution:

We need to consider the three time intervals $t \le 0$, $0 \le t \le 4$, and $t \ge 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0^+) = 0$$

For $0 \le t \le 4$, S_1 is closed so that the 4 Ω and 6 Ω resistors are in series. (Remember, at this time, S_2 is still open). Hence, assuming for now that S_1 is closed forever,

$$i(\infty) = \frac{40}{4+6} = 4 \text{ A},$$

$$R_{Th} = 4 + 6 = 10\ \Omega$$

$$\tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2} \text{ sec}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 4 + (0 - 4)e^{-2t} = 4(1 - e^{-2t}) \text{ A}, \quad 0 \le t \le 4$$

For $t \ge 4$, S_2 is closed; the 10 V voltage source is connected, and the circuit changes. This sudden change does not affect the inductor current because the current through the inductor cannot change abruptly. Thus, the initial current is

$$i(4) = i(4^-) = 4(1 - e^{-8}) \simeq 4 \text{ A}$$

To find $i(\infty)$, let v be the voltage at node P in figure. Using KCL,

$$\frac{40-v}{4} + \frac{10-v}{2} = \frac{v}{6} \quad \Rightarrow \quad v = \frac{180}{11} \text{ V}$$

$$i(\infty) = \frac{v}{6} = \frac{30}{11} = 2.727 \text{ A}$$

The Thevenin resistance at the inductor terminals is

$$R_{Th} = (4||2)+6 = \frac{4\times 2}{6}+6 = \frac{22}{3}\,\Omega$$

and $$\tau = \frac{L}{R_{Th}} = \frac{5}{22/3} = \frac{15}{22}\text{ sec}$$

Hence, $$i(t) = i(\infty) + [i(4) - i(\infty)]\, e^{-(t-4)/\tau}, \quad t \ge 4$$

We need $(t - 4)$ in the exponential because of the time delay. Thus,

$$i(t) = 2.727 + (4 - 2.727)\, e^{-(t-4)/\tau}, \quad \tau = \frac{15}{22}$$

$$= 2.272 + 1.273\, e^{-1.4667(t-4)}, \quad t \ge 4$$

Putting all this together,

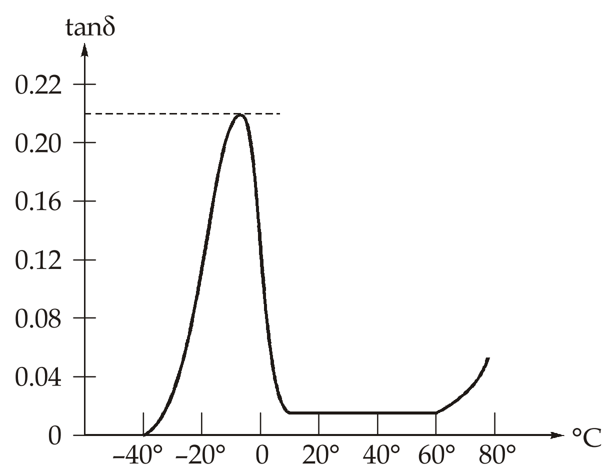
$$i(t) = \begin{cases} 0, & t \le 0 \\ 4(1-e^{-2t}), & 0 \le t \le 4 \\ 2.727 + 1.273\, e^{-1.4667(t-4)}, & t \ge 4 \end{cases}$$

At $t = 2$, $$i(2) = 4(1 - e^{-4}) = 3.93 \text{ A}$$

At $t = 5$, $$i(5) = 2.727 + 1.273\, e^{-1.4667} = 3.02 \text{ A}$$

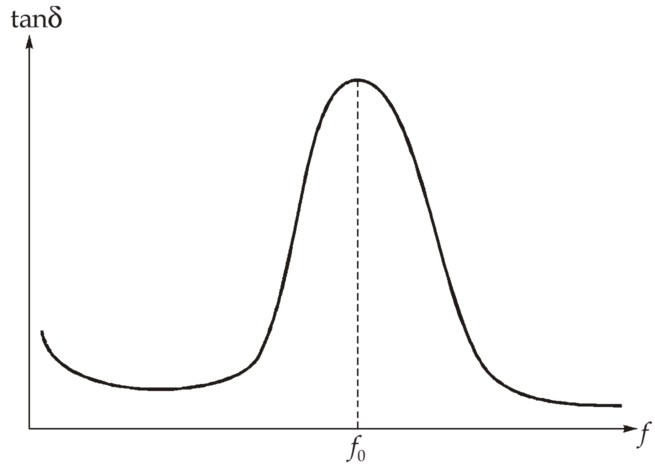
Q.4 (c) Solution:

(i) The dielectric loss of polar dielectrics consists of two components those due to leakage curent and those resulting from dipole polarisation. The dependence of tanδ on temperature is shown below:



At low temperatures, the loss due to dipole polarisation is greater than that due to the leakage current. At temperature much below 0°C due to less thermal motion orientation of dipoles is limited. With increase in temperature the dipoles acquire greater mobility hence it increase the dipole polarization which inturn will increase loss tangent tanδ. A further increase in temperature causes the loss tangent to drop off due to enhanced thermal agitation. After falling to a minimum, tanδ begins to increase, this time due to an increase in the leakage current.

The loss tangents of polar dielectrics depends on frequency. The variation of loss tangent tanδ with frequency at a constant temperature is shown below.



At low frequency the dipoles make less number of rotations per second resulting in a small amount of power loss. As the frequency is increased beyond a certain limit the dipolar polarisation ceases because the molecules will not be able to keep up with increased rate of field reversal. At same mid frequency depending on the temperature the loss tangent will be maximum as shown in above figure.

At zero frequency the loss is due to leakage current only and hence tanδ is minimum. At infinite frequency, the losses due to both polarisation and leakage current become zero.

(ii) $\varepsilon_r = 4.94 \quad n^2 = 2.69$

from clausius Mosotti relation

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\varepsilon_0} \qquad \text{...(i)}$$

If measurement are done in optical frequency range

$\varepsilon_r = n^2$ and $\alpha_i = 0$

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\varepsilon_0} \qquad \text{...(ii)}$$

from equation (i) and (ii)

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} \times \frac{n^2 + 2}{n^2 - 1} = \frac{\alpha_e + \alpha_i}{\alpha_e}$$

$$1+\frac{\alpha_i}{\alpha_e} = \frac{4.94-1}{4.94+2}\cdot\frac{2.69+2}{2.69-1} = 1.576$$

$$\frac{\alpha_i}{\alpha_e} = 0.576$$

$$\frac{\alpha_e}{\alpha_i} = 1.735$$

Section-B

Q.5 (a) Solution

Top-Down Technique: In top-down technique, generally a bulk material is taken and machined it to modify into the desired shape and product. Examples of this type of technique are the manufacturing of integrated circuits using a sequence of steps such as crystal growth, lithography, etching, ion implantation, etc. For nanomaterial synthesis, ball-milling is an important top-down approach, where macrocrystalline structures are broken down to nanocrystalline structures, but original integrity of the material is retained. Sometimes this method is used to prepare nanostructured metal oxides by chemical reaction between two constituents during crushing. The crystallites are allowed to react with each other by the supply of kinetic energy during milling process to form the required nanostructured oxide.

Bottom-Up Technique: Bottom-up technique is used to build something from basic materials, for example, assembling materials from the atoms/molecules up, and, therefore very important for nano-fabrication. Unlike lithographic technique of top-down approach, which is extensively used in silicon industry, this bottom-up nonlithographic approach of nanomaterial synthesis is not completely proven in manufacturing yet, but has great potential to become important alternative to lithographic process. Examples of bottom-up technique are self-assembly of nanomaterials, solgel technology, electrodeposition, physical and chemical vapour deposition (PVD, CVD), epitaxial growth, laser ablation, etc.

Q.5 (b) Solution:

Thermistor: It stands for thermal resistors. Thermistors are sensitive to temperature variation. Thermistors are available with positive and negative temperature coefficient of resistance. However, most of the application of thermistors are based on negative temperature coefficient of resistance.

The negative temperature coefficient of thermistors is very large. For some thermistors, resistance may decrease by 5% – 6% for 1°C rise in temperature. Thermistor are mainly used as temperature sensors, inrush current limiters, over current protection and self regulating heating element.

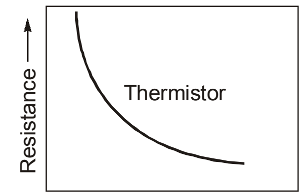
Resistance temperature characteristic:

For a thermistor, $$R = R_0 \exp\beta\left[\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]$$

$R_0 \rightarrow$ Resistance at T_0 (°K)

$R \rightarrow$ Resistance at T (°K)

$\beta \rightarrow$ A constant depending upon material



Thermistor resistance temperature characteristic

The characteristic of thermistor is non-linear and it has very high negative temperature coefficient of resistance.

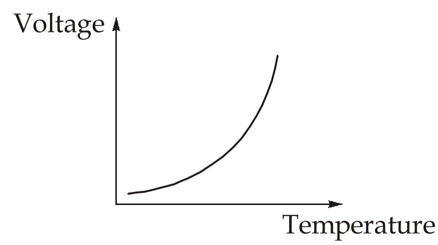
Thermocouple: A thermocouple is also used to measure temperature by measuring the emf induced in the thermocouple junction. Emf induced is a function of temperature.

$$E \; \alpha \; \Delta T$$

$\Delta T \rightarrow$ Temperature difference between two parts of junction

$E \rightarrow$ emf induced

Characteristics of thermocouple:



- Thermocouple has wide temperature ranges e.g.

  Type T = –200 to 350°C

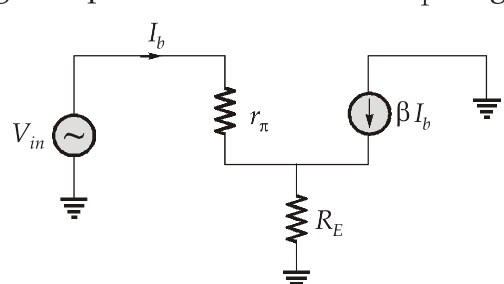
  Type J = 95 to 760°C

  Type K = 95 – 1260°K

  Thermistor has narrower sensing range –55°C to 150°C.
- Thermocouple are less non linear as compared to thermistor.
- In thermocouple, temperature is sensed by the voltage generated, whereas in thermistor temperature is sensed by the, change in resistance.

Q.5 (c) Solution:

(i) Drawing the small signal equivelent of transistor T_1 we get



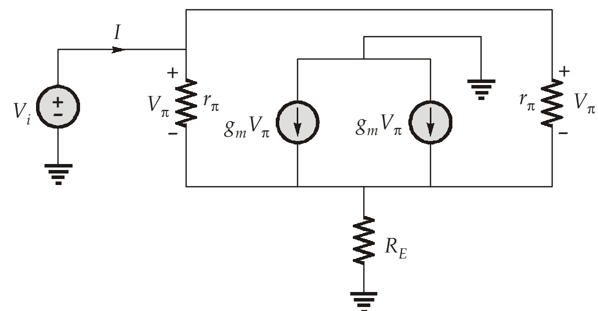
now, applying KVL in input loop we get

$$V_{in} = I_b r_\pi + (\beta + 1)I_b R_E$$

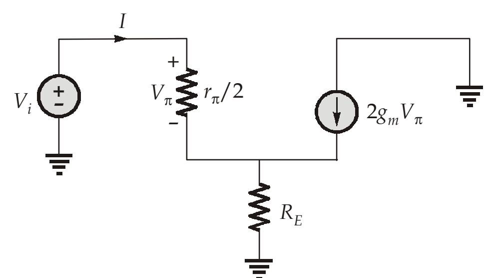
$$\therefore \qquad \frac{V_{in}}{I_b} = r_\pi + (\beta + 1)R_E$$

thus, Input resistance = $R_{in} = r_\pi + (\beta + 1) R_E$

(ii) Drawing the small signal equivalent model of the circuit, we get



Thus, the equivalent circuit can be drawn as



now, $$I = \frac{V_\pi}{r_\pi/2} = \frac{2V_\pi}{r_\pi} \qquad \text{...(i)}$$

and $$V_{in} = V_\pi + (I + 2g_m V_\pi) R_E$$

$$= V_\pi + \frac{R_E(2V_\pi)}{r_\pi} + 2g_m R_E V_\pi = \left(1 + \frac{2R_E}{r_\pi} + 2g_m R_E\right)V_\pi$$

$$V_{in} = (r_\pi + 2R_E + 2g_m r_\pi R_E)\frac{V_\pi}{r_\pi}$$

$$V_{in} = (r_\pi + 2(1+\beta)R_E)\frac{V_\pi}{r_\pi} \qquad (\because g_m r_\pi = \beta)$$

$$\therefore \qquad V_\pi = \frac{V_{in}\, r_\pi}{(r_\pi + 2(1+\beta)R_E)} \qquad \text{...(ii)}$$

now, substituting equation (ii) in (i), we get

$$I = \frac{2V_{in}}{(r_\pi + 2(1+\beta)R_E)}$$

$$\therefore \qquad R_{in} = \frac{V_{in}}{I} = \frac{r_\pi + 2(1+\beta)R_E}{2}$$

Q.5 (d) Solution:

$$\text{Length of airgap} = l_{ag} = 0.2 \times 10^{-2} \text{ m}$$
$$\text{Mean length} = l_m = 40 \times 10^{-2} \text{ m}$$

Without airgap, mean length of the core (l'_m) = 40 – 0.2 = 39.8 × 10^{-2} m.

$$\phi = \frac{\text{mmf}}{\text{Reluctance}}$$

$$\phi \propto \frac{1}{k}$$

$$k_1 = \frac{l}{\mu_o \mu_r A} = \frac{l_{ag}}{\mu_o A} + \frac{l'_m}{\mu_o \mu_r A} \qquad [\because \mu_r = \infty]$$

$$k_1 = \frac{l_{ag}}{\mu_o A} = \frac{0.2 \times 10^{-2}}{\mu_o A}$$

$$k_2 = \frac{l_{ag}}{\mu_o A} + \frac{l'_m}{\mu_o \mu_r A} = \frac{0.2 \times 10^{-2}}{\mu_o A} + \frac{39.8 \times 10^{-2}}{1000 \mu_o A} = \frac{0.2398 \times 10^{-2}}{\mu_o A}$$

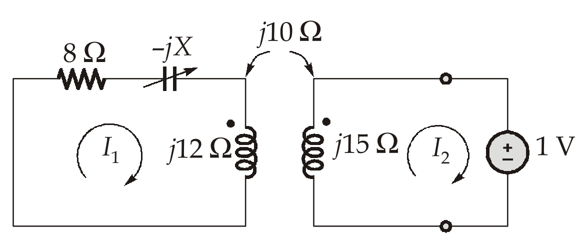
$$\phi \propto B \propto \frac{1}{k}$$

$$\therefore \qquad \frac{k_1}{k_2} = \frac{B_2}{B_1}$$

$$B_2 = \frac{B_1 \times k_1}{k_2} = 1 \times \frac{0.2 \times 10^{-2}}{0.2398 \times 10^{-2}} = 0.833 \text{ Tesla}$$

Q.5 (e) Solution

In order to find the maximum power transfer let us first calculate Z_{Th} across 20 Ω load as follows.



For mesh (1)

$$0 = (8 - jX + j12)I_1 - j10I_2 \qquad \text{...(i)}$$

For mesh (2)

$$1 + j15I_2 - j10I_1 = 0 \qquad \Rightarrow I_1 = 1.5I_2 - 0.1j \qquad \text{...(ii)}$$

Substituting (ii) into (i) gives

$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

For maximum power transfer,

$$|Z_{Th}| = 20\ \Omega$$

$$\frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} = 20$$

$$400 = \frac{144 + 64 - 24X + 2.25X^2}{0.64 + 1.44 - 0.24X + 0.01X^2}$$

or $\quad 208 + 2.25X^2 - 24X = 832 - 96X + 4X^2$

or $\quad 1.75X^2 - 72X + 624 = 0$

On solving the above equation, we get,

$$X = 28.73, 12.40$$

Q.6 (a) Solution:

(i) Given data: $P = 4$, $N = 400$ rpm, $\phi = 0.05$ Wb/pole

Wave wound i.e. number of parallel paths = $A = 2$

Total number of turns = 220 × 10 = 2200

$\therefore$ Total number of conductors = Z = 2200 × 2 = 4400

$$\text{Emf induced} = E = \frac{\phi ZNP}{60\ A} = \frac{0.05 \times 4400 \times 400 \times 4}{60 \times 2} = 2933.33\ \text{V}$$

Number of parallel paths are 2, therefore conductors in each path

$$\frac{4400}{2} = 2200 \text{ conductors}$$

$$\text{Total number of turns in each path} = \frac{2200}{2} = 1100 \text{ turns}$$

Resistance of each turn = 0.02 Ω

$\therefore$ Resistance of each path = 1100 × 0.02 = 22 Ω

$$\therefore \quad \text{Total armature resistance} = \frac{22}{2} = 11\ \Omega$$

(ii)

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Output + Losses}}$$

At rated load:
$$0.97 = 1 - \frac{P_c + P_{cu}}{(10\times10^3\times1) + P_c + P_{cu}}$$

$$0.97 = \frac{(10\times10^3) + P_c + P_{cu} - P_c - P_{cu}}{(10\times10^3) + P_c + P_{cu}}$$

$$P_c + P_{cu} = \frac{300}{0.97} = 309.278 \qquad \text{...(i)}$$

At half rated load:
$$0.97 = 1 - \frac{P_c + (0.5)^2\ P_{cu}}{\left(10\times10^3\times\frac{1}{2}\right) + P_c + (0.5)^2\ P_{cu}}$$

$$0.97 = \frac{5\times10^3 + P_c + (0.5)^2 P_{cu} - P_c - (0.5)^2\ P_{cu}}{5\times10^3 + P_c + (0.5)^2 P_{cu}}$$

$$P_c + 0.25\ P_{cu} = 154.639 \qquad \text{...(ii)}$$

By solving equations (i) and (ii)

$$\text{Core loss} = P_{core} = 103.1\ \text{W}$$

$$\text{Ohmic loss} = \text{Copper loss} = P_{cu} = 206.185\ \text{W}$$

Q.6 (b) Solution:

(i) Truth table:

	A	B	C	X	Y	Z
m_0	0	0	0	0	0	1
m_1	0	0	1	0	1	0
m_2	0	1	0	0	1	1
m_3	0	1	1	1	0	0
m_4	1	0	0	0	1	0
m_5	1	0	1	0	1	1
m_6	1	1	0	1	0	0
m_7	1	1	1	1	0	1

Minimization:

K-map for X

$A \backslash BC$	00	01	11	10
0	(0)	(1)	(3) 1	(2)
1	(4)	(5)	(7) 1	(6) 1

$$X = AB + BC$$

K-map for Y

$A \backslash BC$	00	01	11	10
0		1		1
1	1	1		

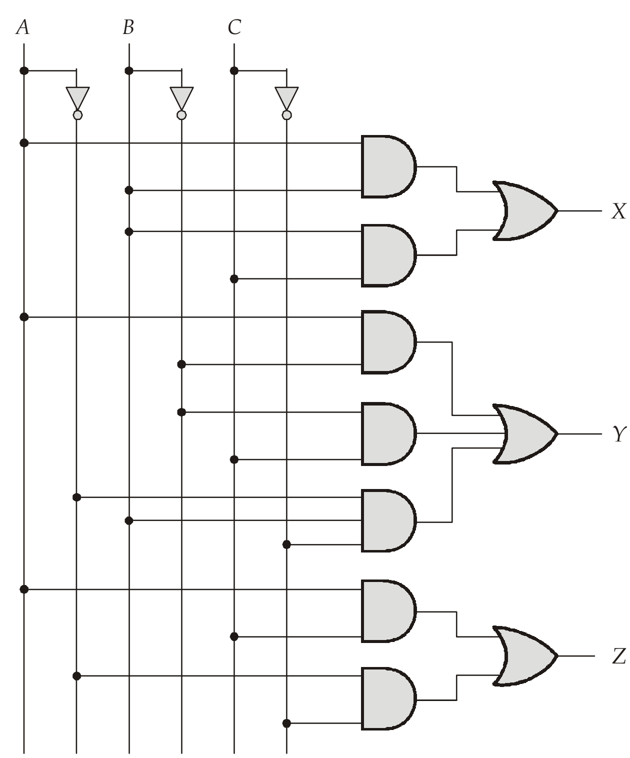
$$Y = A\bar{B} + \bar{B}C + \bar{A}B\bar{C}$$

K-map for Z

$A \backslash BC$	00	01	11	10
0	1			1
1		1	1	

$$Z = AC + \bar{A}\bar{C}$$

Logic circuit:



(ii) The truth table of the given circuit can be constructed as shown below.

A	B	C	D	$X = \bar{A}BC$	$Y = A + D$	$Z = \bar{Y}$	$F = XZ$
0	0	0	0	0	0	1	0
0	0	0	1	0	1	0	0
0	0	1	0	0	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	0	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	1
0	1	1	1	1	1	0	0
1	0	0	0	0	1	0	0
1	0	0	1	0	1	0	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	0	0
1	1	0	0	0	1	0	0
1	1	0	1	0	1	0	0
1	1	1	0	0	1	0	0
1	1	1	1	0	1	0	0

The given circuit can be used to detect the binary combination $(ABCD) = (0110)$.

Q.6 (c) Solution:

At $T = 300$ K, the junction voltage

$$V_{bi} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259 \ln\left[\frac{(5\times10^{17})(10^{17})}{(1.5\times10^{10})^2}\right]$$

$$V_{bi} = 0.8556 \text{ V}$$

For 1% change in V_{bi}, $n_i^2 \propto \exp\left(\frac{-Eg}{KT}\right)$

Now,
$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{\ln\left[\frac{N_A N_D}{n_i^2(T_2)}\right]}{\ln\left[\frac{N_A N_D}{n_i^2(T_1)}\right]} = \frac{\ln(N_A N_D) - \ln\left[n_i^2(T_2)\right]}{\ln(N_A N_D) - \ln\left[n_i^2(T_1)\right]}$$

$$= \frac{\ln(N_A N_D) - \ln(N_C N_V) - \left[\frac{-Eg}{kT_2}\right]}{\ln(N_A N_D) - \ln(N_c\, N_V) - \left[\frac{-Eg}{kT_1}\right]}$$

$$\ln(N_A N_D) = \ln[(5 \times 10^{17}) \times 10^{17}] = 79.897$$

$$\ln(N_C N_V) = \ln[(2.8 \times 10^{19})(1.04 \times 10^{19})] = 88.567$$

$$\therefore \quad \frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{-8.67 + \dfrac{Eg}{kT_2}}{-8.67 + \dfrac{Eg}{kT_1}} = \frac{-8.67 + \dfrac{Eg}{kT_2}}{-8.67 + \dfrac{1.12}{0.0259}} = \frac{-8.67 + \dfrac{Eg}{kT_2}}{34.57}$$

$$\text{For } 1\% \text{ change } \frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{0.8556 \times 0.99}{0.8556} = 0.99$$

$$\therefore \quad 0.99 = \frac{-8.67 + \dfrac{Eg}{KT_2}}{34.57}$$

$$\therefore \quad \frac{Eg}{KT_2} = 42.9 = \frac{1.12}{(0.0259)\left[\dfrac{T_2}{300}\right]}$$

$$T_2 = 302.4 \text{ K}$$

Q.7 (a) Solution:

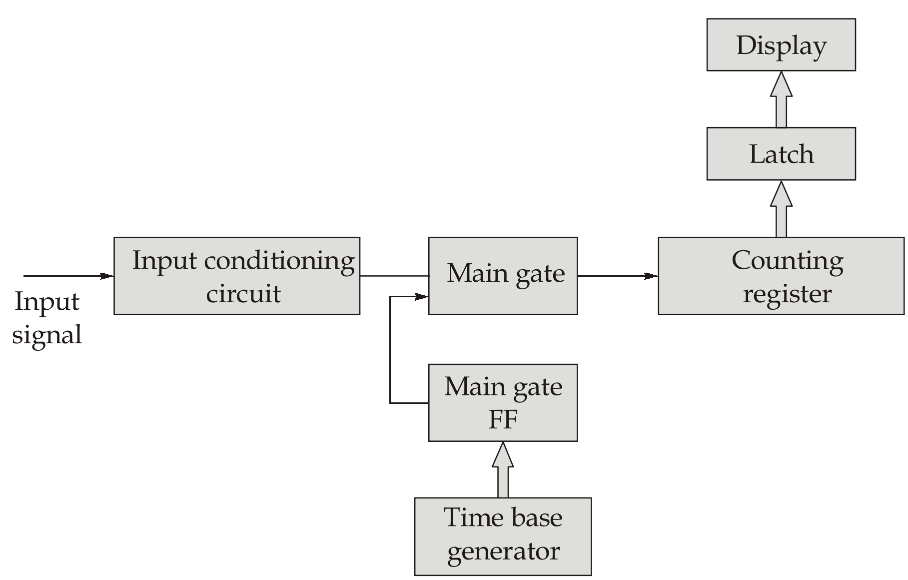


Fig. (a)

Fig. (b)

The frequency f, of repetitive signals is defined as the number of cycles of that signal per unit of time and is measured by the conventional counter by counting the number of cycles, N, and dividing it by the time interval t.

The input signal is first conditioned to form a signal compatible with the internal circuitary of the counting register. The block diagram of this conditioning circuit is given in figure (b). It consists of an attenuator, an amplifier and a wave shaping circuit in the form of schmitt trigger. Attenuator control, gain control and trigger controls are available on the instrument panel which control the attenuation level of the input, gain of the amplifier and threshold levels and hysterisis window of the schmitt trigger.

The output of the conditioning circuit produces a pulse train which is fed as one input of the main gate. The other input of the main gate is fed from main gate flip-flop (FF) which decides the opening time of the main gate. With the main gate open, pulses are allowed to pass through it and get totalized by the counting register and displayed through the latch. The main gate flip-flop (FF) is controlled by the time base.

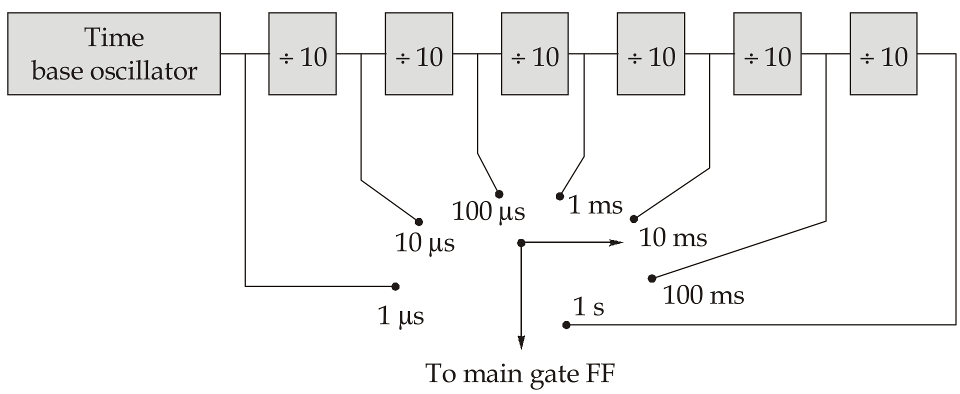


Fig. (c)

The time base consists of a number of decade (÷10) blocks which divide the frequency by 10. The input to the first decade block is provided by the time base oscillator.

By dividing this oscillator frequency a number of times by 10, a pulse train with different time is generated. A selector switch is then used to select proper time for the pulse output from the time base. Fig. (c) shows a typical time base circuit generating pulses with period starting from 1 μs to 1 s.

The number of pulses totaled by the counter for the selected gate time yields the frequency of the input signal.

$$\text{Frequency, } f_A = \frac{\text{Counts } (N)}{\text{gate time } (t)} \quad \text{or}$$

$$= \text{counts } (N) \times \text{Frequency of the time base } (f_c)$$

The total frequency measurement error is defined as the sum of its ± 1 count error and its total time base error.

Period measurement: The period of an input signal is the inverse of its frequency. i.e., it is the time taken for the signal to complete one cycle.

$$\text{Period} = \frac{1}{f_A}$$

If the time is measured over several input cycles, then the average period of the repetitive signal is determine. This is often referred to as multiple period averaging.

The basic block diagram for the conventional counter in its period measurement mode is shown in Fig. (d). In this mode of measurement, the duration over which the main gate is open is controlled by the frequency of the input signal rather than that of the time base. The counting register counts the output pulses from the time-base for one cycle of the input signal.

$$\text{Period} = \frac{1}{f_A} = \frac{\text{counts } (N)}{\text{frequency of the time base } (f_c)} = \text{Counts } (N) \times \text{Gate time } (t)$$

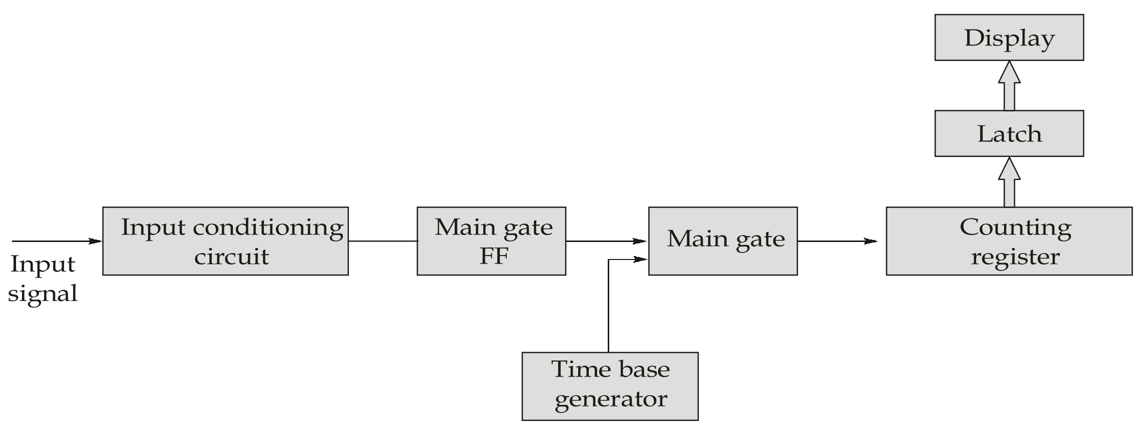


Fig. (d)

Gate can be started (triggered) at positive or negative edge of the input pulse and stopped at negative or positive edge or started at positive edge and stopped at the next positive edge.

The conditioned input signal may also be divided so that the gate is open in decade steps of the input signal period rather than for a single period. This is the basis of the multiple period averaging technique.

Further, lower the frequency of the input signal, the longer the time for which gate is opened and more accurate is the result. We can define the resolution as the smallest time interval that can be measured on the counter and is reciprocal of the time base frequency. Period measurement, therefore, allows more accurate measurement of unknown low-frequency signals because of increased resolution.

Q.7 (b) Solution:

From the governing equations of h-parameter network, we get,

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{...(i)}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{...(ii)}$$

From equation (ii), we get,

$$I_1 = \left(-\frac{h_{22}}{h_{21}}\right)V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2) \quad \text{...(iii)}$$

and from equation (i) and (iii), we get,

$$V_1 = h_{11}\left[\left(-\frac{h_{22}}{h_{21}}\right)V_2 + \left(-\frac{1}{h_{21}}\right)(-I_2)\right] + h_{12}V_2$$

or

$$V_1 = \left(\frac{-h_{11}h_{22} + h_{12}h_{21}}{h_{21}}\right)V_2 + \left(\frac{-h_{11}}{h_{21}}\right)(-I_2)$$

$$= \left(-\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}\right)V_2 - \left(\frac{-h_{11}}{h_{21}}\right)(I_2) \quad \text{...(iv)}$$

Comparing equation (iii) and (iv) with the original $ABCD$ (transmission) parameter equations, we get,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$\therefore$

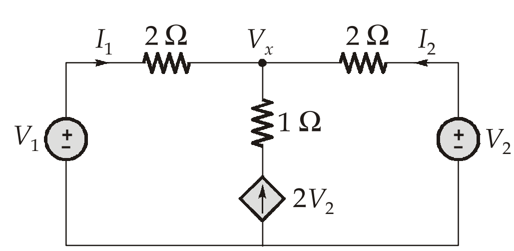
$$A = \left(-\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}\right)$$

$$B = -\left(\frac{h_{11}}{h_{21}}\right)$$

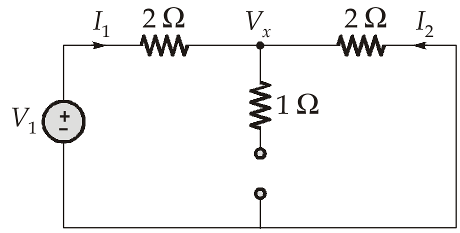
$$C = -\left(\frac{h_{22}}{h_{21}}\right)$$

$$D = -\left(\frac{1}{h_{21}}\right)$$

Now, the given circuit can be redrawn as



Considering $V_2 = 0$, the circuit can be redrawn as



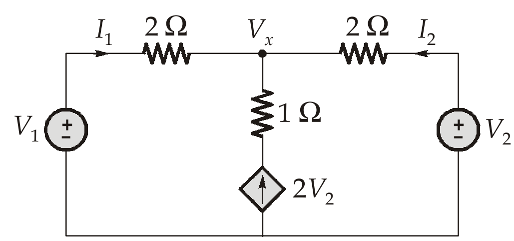
$$V_1 = 4I_1$$

and $$I_2 = -I_1$$

$\therefore$ $$h_{11} = 4\ \Omega$$

and $$h_{21} = -1$$

by considering $I_1 = 0$, the circuit can be redrawn as



$$\frac{V_x - V_1}{2} + \frac{V_x - V_2}{2} = 2V_2$$

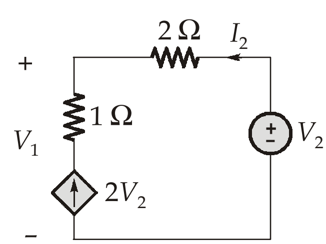
$\because$ $I_1 = 0$

$$\frac{V_x - V_2}{2} = 2V_2$$

or $$-I_2 = 2V_2$$

or $$h_{22} = \frac{I_2}{V_2} = -2\ \mho$$

Also as $I_1 = 0$, the circuit becomes



$\therefore$ $$\frac{V_1 - V_2}{2} = 2V_2$$

or $$V_1 - V_2 = 4V_2$$

or $\quad V_1 = 5V_2$

or $\quad h_{12} = 5$

From obtained value of h parameters, the $ABCD$ parameters can be calculated as,

$$A = -\left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{21}}\right) = -\left(\frac{4\times(-2) - 5\times -1}{-1}\right) = -8 + 5 = -3$$

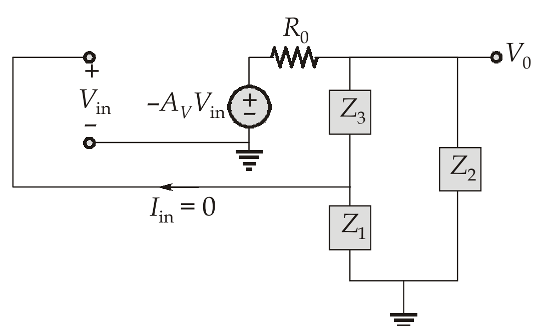
$$B = -\left(\frac{h_{11}}{h_{21}}\right) = \frac{-4}{-1} = 4\ \Omega$$

$$C = -\left(\frac{h_{22}}{h_{21}}\right) = -\left(\frac{-2}{-1}\right) = -2\ \mho$$

and $\quad D = \dfrac{-1}{h_{21}} = \dfrac{-1}{-1} = 1$

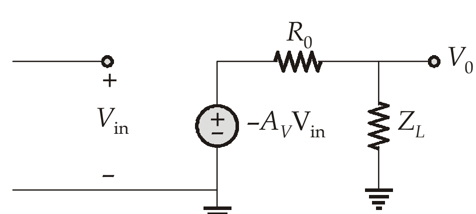
Q.7 (c) Solution:

Drawing the small signal model of the amplifier we have,



$\because \quad I_{in} = 0;$

The above circuit can be reduced as

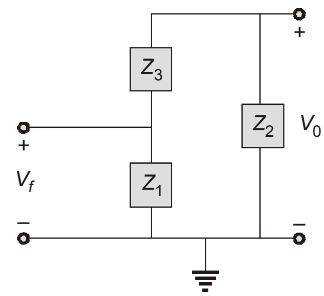


Thus, the overall gain of the amplifier,

$$A = \frac{V_0}{V_{in}} = \frac{-A_V Z_L}{Z_L + R_0}$$

where, $\quad Z_L = \dfrac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_2 + Z_3)}$

For the feedback circuit,



The feedback gain, $\beta = \dfrac{V_f}{V_0} = \dfrac{Z_1}{Z_1 + Z_3}$

$\because$ The phase shift of the feed back circuit is negative.

$$\therefore \quad A\beta = \frac{-A_V Z_1 Z_L}{(R_0 + Z_L)(Z_1 + Z_3)} = \frac{-A_V Z_1 \left[\dfrac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]}{\left[R_0 + \dfrac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right](Z_1+Z_3)}$$

$$= \frac{-A_V Z_1 Z_2}{R_0(Z_1+Z_2+Z_3)+Z_2(Z_1+Z_3)}$$

Now, $Z_1 = jX_1$, $Z_2 = jX_2$ and $Z_3 = jX_3$

$$\Rightarrow \quad A\beta = \frac{A_V(X_1X_2)}{jR_0(X_1+X_2+X_3)-X_2(X_1+X_3)}$$

To produce sustained oscillations the phase shift of the loop gain $A\beta$ should be 0°.

Thus, $R_0(X_1 + X_2 + X_3) = 0$

$\Rightarrow \quad X_1 + X_2 + X_3 = 0$

$(X_1 + X_3) = -X_2$

$$\therefore \quad A\beta = \frac{-A_V X_1}{(X_1 + X_3)}$$

$$\Rightarrow \quad A\beta = \frac{A_V X_1}{X_2}$$

Hence X_1 and X_2 should be of the same type of reactance.

Q.8 (a) Solution:

(i) Given data: Output = 10 kW ; Number of poles = 6 ; f = 50 Hz ; N = 960 rpm
η = 90% ; p.f. = 0.88 lagging.
Full load line current drawn by 3 phase, delta connected induction motor.

$$\eta = \frac{\text{Output power in watts}}{\sqrt{3}\, V_L\, I_L \cos\phi}$$

$$I_L = \frac{10\times 10^3}{\sqrt{3}\times 400\times 0.88\times 0.90} = 18.22 \text{ A}$$

$$I_{ph} = \frac{18.22}{\sqrt{3}} = 10.52 \text{ A}$$

On direct on line start, the starting current drawn by the motor per phase is given by,

$$I_{s,\,ph} = \frac{85}{\sqrt{3}} = 49.07 \text{ A}$$

$$\text{Synchronous speed} = N_s = \frac{120\times f}{P} = \frac{120\times 50}{6} = 1000 \text{ rpm}$$

$$\text{Slip}_{f1} = \frac{N_s - N_r}{N_s} = \frac{1000-960}{1000} = 0.04$$

$$\frac{T_s}{T_{f1}} = \frac{1}{3}\left(\frac{I_s}{I_{f1}}\right)^2 \cdot s_{f1} = \frac{1}{3}\left(\frac{49.07}{10.52}\right)^2 \times 0.04 = 0.29$$

(ii)

$$\text{Synchronous speed} = N_s = \frac{120\times f}{P} = \frac{120\times 50}{6} = 1000 \text{ rpm}$$

$$s_{max} = \frac{N_s - N_r}{N_s} = \frac{1000-900}{1000} = 0.1$$

$s_{max} = \frac{R_2}{X_2}$ $\Rightarrow$ $0.1 = \frac{0.25}{X_2}$

$\therefore$ $X_2 = 2.5\ \Omega$

Torque at any slip,

$$T = \frac{3}{\omega_s}\cdot\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$$

$$T = \frac{3E_1^2}{\omega_s}\cdot\frac{0.05\times 0.25}{(0.25)^2 + (0.05\times 2.5)^2}$$

$$T = \frac{3E_1^2}{\omega_s}\times 0.16 \quad \text{...(i)}$$

Given,

$$T_{max} = 200 \text{ N-m} = \frac{3}{\omega_s}\,\frac{E_1^2}{2X_2} = \frac{3E_1^2}{\omega_s}\times\frac{1}{5}$$

$$\frac{3E_1^2}{\omega_s} = 1000 \quad \text{...(ii)}$$

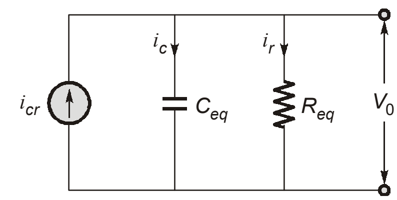
Substituting equation (ii) in equation (i), we get,

$$T = 1000 \times 0.16 = 160 \text{ N-m.}$$

Q.8 (b) Solution:

(i) Given charge, $$q = K_q x_i \quad ...(1)$$

Also, $$i_{cr} = \frac{dq}{dt} = K_q \frac{dx_i}{dt}$$



$$i_{cr} = i_c + i_r$$

$$V_0(t) = \frac{1}{C}\int i_c dt = \frac{1}{C}\int (i_{cr} - i_r)\,dt$$

$$\frac{dV_0}{dt} = \frac{1}{C}(i_{cr} - i_r)$$

$$C\frac{dV_0}{dt} = K_q \frac{dx_i}{dt} - \frac{V_0}{R}$$

$$RC\frac{dV_0}{dt} + V_0 = K_q \cdot R\frac{dx_i}{dt} \quad ...(2)$$

$\tau = RC$ is time constant

$$K = \frac{K_q}{C} = \text{Sensitivity in V/m}$$

$$\tau\frac{dV_0}{dt} + V_0 = K\tau\frac{dx_i}{dt}$$

Taking Laplace transform,

$$\tau s\, V_0(s) + V_0(s) = K\,\tau s\, x_i(s)$$

$$\frac{V_0(s)}{x_i(s)} = \frac{K\tau s}{(\tau s + 1)} = \left(\frac{K\tau s}{\tau s + 1}\right)$$

Transform function form.

(ii) Amplitude ratio, $$M = \frac{\omega\tau}{(1 + \omega^2\tau^2)^{1/2}}$$

Case I: Flat response within 5% is obtained for

$$M = 1 - 0.05 = 0.95$$

For 5% response, $\omega = 2\pi f$

$f = 5 \text{ kHz} = 5 \times 10^3 \text{ Hz}$,

τ = Time constant is required to be found out

$$\frac{\omega\tau}{(1 + \omega^2\tau^2)^{1/2}} = 0.95 \Rightarrow \frac{(\omega\tau)^2}{1 + \omega^2\tau^2} = (0.95)^2 \Rightarrow \omega\tau = 3.04$$

$$\tau = \frac{3.04}{2\pi f} = \frac{3.04}{(2)(\pi)(5\times10^3)} = 9.67 \times 10^{-5} \text{ sec}$$

Case II: 2% inaccuracy,

$$M = 1 - 0.02 = 0.98$$

$$\omega = ?$$

$$\tau = 9.676 \times 10^{-5} \text{ sec}$$

$$M = \frac{\omega\tau}{(1+\omega^2\tau^2)^{1/2}}$$

$$(0.98)^2 = \frac{\omega^2\tau^2}{(1+\omega^2\tau^2)}$$

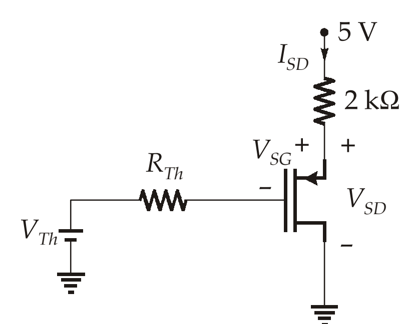
$$0.9604 + 0.9604\,\omega^2\tau^2 = \omega^2\tau^2$$

$$\omega^2\tau^2 = \frac{0.9604}{0.0396} = 24.25 \Rightarrow \omega\tau = 4.92$$

$$\omega = \frac{4.92}{9.6766\times10^{-5}} = 50.84 \text{ k-rad/s}$$

$$f = 8.095 \text{ kHz}$$

Q.8 (c) Solution:



$$V_{Th} = \frac{5\times59.4+(-5)\times20.6}{59.4+20.6} = 2.42 \text{ V}$$

$$V_{SG} = V_S - V_G$$

$$= 5 - 2\,I_{SD} - 2.42$$

$$= 2.58 - 2I_{SD}$$

$$\Rightarrow \quad I_{SD} = \frac{2.58 - V_{SG}}{2} \quad \text{...(i)}$$

Assume MOSFET in saturation,

$$I_{SD} = \frac{\mu_p C_{ox}}{2}\cdot\frac{W}{L}\left(V_{SG} - |1V_{TP}|\right)^2$$

$$\frac{2.58 - V_{SG}}{2} = 0.5\,(V_{SG} - 1.5)^2$$

After solving $V_{SG} = 2.15 \text{ V}, -0.153 \text{ V}$

But, $V_{SG} > |V_{TP}|$ (For MOS transistor to be ON)

(i) Therefore, $V_{SG} = 2.15\text{ V}$

(ii) $I_{SD} = \dfrac{2.58 - 2.15}{2} = 0.215\text{ mA}$ (From eqn. (i))

(iii) $V_{SD} = 5 - 2\,I_{SD} = 4.57\text{ V}$

$V_{SD} > V_{SG} - |V_{TP}| \Rightarrow$ MOSFET is in saturation.

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