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Detailed Solutions

ESE-2019 Mains Test Series

Electrical Engineering Test No : 14

Section-A

Q.1 (a) Solution:

Given, $\omega = 300$ rad/s,

$R = 10\ \Omega,\ C = 30 \times 10^{-6}$ F

(i) Resonant frequency, $\omega_0 = 3 \times \omega = 900$ rad/s

$$\omega_0 = \frac{1}{\sqrt{LC}} = 900$$

$$L = \frac{1}{900 \times 900 \times 30 \times 10^{-6}} = 41.152 \text{ mH}$$

(ii) For 1st harmonic,

$\omega = 300$ rad/sec

$R = 10\ \Omega,$

$X_{L1} = 300 \times 41.152 \times 10^{-3} = 12.3456\ \Omega$

$$X_{C1} = \frac{-j}{300 \times 30 \times 10^{-6}} = -j111.11\ \Omega$$

$\therefore\quad Z_1 = [10 + j(12.3456 - 111.11)]\ \Omega$

$= (10 - j98.77)\ \Omega$

$$\therefore\quad I_1 = \frac{2000}{10 - j98.77} = 20.15\angle 84.21°\text{ A}$$

For 3rd harmonic (ω = 900 rad/s)

$$R = 10\ \Omega,$$

$$X_{L3} = 900 \times 41.152 \times 10^{-3}$$

$$= 37.037\ \Omega$$

$$X_{C3} = \frac{-j}{900 \times 30 \times 10^{-6}} = -j37.037\ \Omega$$

$$\therefore \quad Z_3 = 10 + j(37.037 - 37.037) = 10\ \Omega$$

$$\therefore \quad I_3 = \frac{400}{Z_3} = \frac{400}{10\angle 0^\circ} = 40\angle 0^\circ\ \text{A}$$

For 5th harmonic (ω = 1500 rad/s)

$$R = 10\ \Omega,$$

$$X_{L5} = j1500 \times 41.152 \times 10^{-3}$$

$$= j61.728\ \Omega$$

$$X_{C5} = \frac{-j}{1500 \times 30 \times 10^{-6}} = -j22.222\ \Omega$$

$$\therefore \quad Z_5 = 10 + j(61.728 - 22.222)$$

$$= (10 + j39.506)\ \Omega$$

$$\therefore \quad I_5 = \frac{100}{40.752\angle 75.795^\circ} = 2.454\angle -75.795^\circ\ \text{A}$$

The rms value of the current,

$$I_{\text{rms}} = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2}{2}}$$

$$= \sqrt{\frac{(20.15)^2 + (40)^2 + (2.454)^2}{2}} = 31.72\ \text{A}$$

The rms value of the voltage,

$$V_{\text{rms}} = \sqrt{\frac{V_1^2 + V_3^2 + V_5^2}{2}}$$

$$= \sqrt{\frac{(2000)^2 + (400)^2 + (100)^2}{2}} = 1443.95\ \text{V}$$

Q.1 (b) (i) Solution:

$$\text{Polarization,} \quad P = \varepsilon_0(\varepsilon_r - 1)E$$

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{4.3 \times 10^{-8}\, \text{C/m}^2}{(8.854 \times 10^{-12}\, \text{F/m})\,(1000\, \text{V/m})}$$

$$\varepsilon_r \approx 5.86$$

Q.1 (b) (ii) Solution:

- Carbon nanotube is polymer of element carbon generally formed by sheet of graphite rolled into a tube with both end capped with C_{60} fullerenes hemispheres. Fullerene C_{60} is itself polymorphic form of carbon with fixed number of atoms, generally arranged in hollow spherical cluster of sixty carbon atoms. Fullerenes are electrically insulating in nature.

Properties of carbon nanotubes:

- The length of carbon nanotube which behaves as a molecule has its length much greater compared to its diameter.
- Nanotubes are extremely strong and stiff and relatively ductile.
- Nanotubes have relatively low density.

Electrical properties:

- Electrical properties of carbon nanotubes are unique and dependent on their structure.
- Depending on orientation of the hexagonal units in the graphene plane (i.e.: tube wall) with tube axis, the nanotube can behave electrically as metal or a semiconductor.

Application:

- Flat panels and full color display in TVs and computers are fabricated with carbon nanotubes as field emitters.
- Single walled nanotubes tensile strengths extends from 50 - 200 GPa and it also inherits very high elastic modulus with low density. Therefore it has application in textile industry under name of ultimate fiber.

Q.1 (c) Solution:

$$A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$$

$$\Rightarrow \quad \bar{A} = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix} \quad ...(i)$$

$$(\bar{A})^T = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$$

$$A^\theta = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix} \quad ...(ii)$$

On adding (i) and (ii), we get

$$A + A^\theta = \begin{bmatrix} 2 & 2-2i & 4-6i \\ 2+2i & 4 & 2i \\ 4+6i & -2i & 14 \end{bmatrix}$$

Let, $$R = \frac{1}{2}(A + A^\theta) = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} \quad ...(iii)$$

On subtracting (ii) from (i), we get

$$A - A^\theta = \begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

let, $$S = \frac{1}{2}(A - A^\theta) = \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix} \quad ...(iv)$$

From (iii) and (iv), we have $$A = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} + \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

Hermitian matrix Skew-Hermitian matrix

Q.1 (d) Solution:

To obtain quiescent operating point, short circuit a.c. input

By applying KVL in base emitter loop:

$$-V_{BB} + I_B R_B + 0.7 = 0$$

$$I_B = \frac{3-0.7}{100\text{k}} = 2.3 \times 10^{-5} \text{ A}$$

$$I_C = \beta I_B = 100 \times 2.3 \times 10^{-5}$$

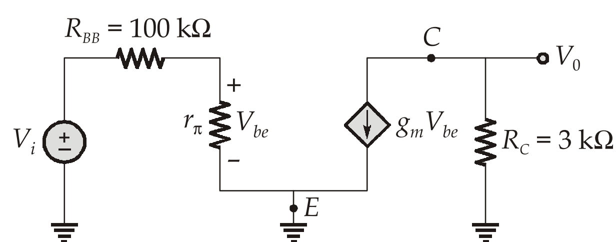
$$= 2.3 \times 10^{-3} \text{ A}$$

By applying KVL in collector emitter loop:

$$-V_{CC} + I_C R_C + V_{CE} = 0$$

$$V_{CE} = 10 - (2.3 \times 3) = 3.1 \text{ V}$$

The small signal a.c. equivalent circuit is,



$$\text{Transconductance, } g_m = \frac{I_c}{V_T} = \frac{2.3\times10^{-3}}{25\times10^{-3}} = 0.092 \text{ A/V}$$

Small signal input resistance,

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.092} = 1.086 \text{ k}\Omega \approx 1.09 \text{ k}\Omega$$

$$V_{be} = V_i \frac{r_\pi}{r_\pi + R_{BB}} = V_i \frac{1.09\text{k}}{(1.09+100)\text{k}} = 0.0108\ V_i$$

Output voltage,

$$V_0 = -g_m V_{be} \times R_C$$

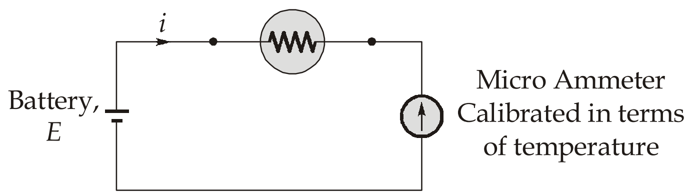
$$V_0 = -0.092 \times V_i \times 0.0108 \times 3 \times 10^3$$

$$A_V = \frac{V_0}{V_i} = -2.98 \approx -3$$

Q.1 (e) Solution:

A thermistor produces a large change of resistance with a small change in the temperature being measured. This large sensitivity of thermistor provides good accuracy and resolution. When this thermistor is connected in simple series circuit consisting of a battery and micro ammeter as shown in figure, any change in temperature causes a change in the resistance of thermistor and corresponding change in circuit current.

The micro ammeter may be directly calibrated in terms of temperature. The micro ammeter may be able to give a resolution of 0.1° C.



$$R_T = R_0 e^{\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)}$$

$$\ln\left(\frac{R_T}{R_0}\right) = \beta\left(\frac{1}{T}-\frac{1}{T_0}\right)$$

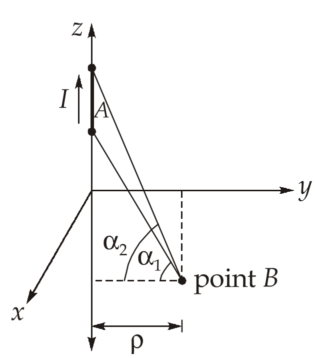
$$\ln\left(\frac{2330}{1050}\right) = 3140\left(\frac{1}{T}-\frac{1}{300}\right)$$

$$2.538 \times 10^{-4} = \frac{1}{T}-\frac{1}{300}$$

$$T = 278.77\ K = 5.77°$$

Q.2 (a) Solution:

A finite length current element on the z-axis is shown in below figure,



Let a differential element $IdL = I\,dz'$ lies at point A $(0, 0, z')$. Let there exist a point $B(0, y, z)$ at which the magnetic field intensity is desired. The vector from A to B is

$$R_{AB} = B - A = (y\hat{a}_y + z\hat{a}_z) - z'\hat{a}_z$$

$$= y\hat{a}_y + (z - z')\hat{a}_z$$

Its magnitude is, $$|R_{AB}| = \sqrt{y^2 + (z-z')^2}$$

According to Biot-Savart law,

$$\text{dH} = \frac{IdL \times R_{AB}}{4\pi |R_{AB}|^{3/2}} \text{A/m}$$

$$= \frac{I(\hat{a}_z dz') \times (y\hat{a}_y + (z-z')\hat{a}_z}{4\pi\left[y^2 + (z-z')^2\right]^{3/2}}$$

$$= \frac{I}{4\pi\left[y^2 + (z-z')^2\right]^{3/2}} \times \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & dz' \\ 0 & y & (z-z') \end{vmatrix}$$

$$\text{dH} = \frac{-Iydz'\hat{a}_x}{4\pi[y^2 + (z-z')^2]^{3/2}}$$

The direction of dH is $-\hat{a}_x$, which is $\hat{a}_\phi$ at point B. In the cylindrical coordinate system, using $y = \rho$ at point B,

$$dH_\phi = \frac{-I\rho dz'}{4\pi\left[\rho^2 + (z-z')^2\right]^{3/2}}$$

Assuming $$\tan\theta = \frac{z - z'}{\rho}$$

$$\sec^2\theta\, d\theta = \frac{-dz'}{\rho}$$

Thus, $$H_\phi = \frac{I\rho}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho\sec^2\theta\, d\theta}{\left[\rho^2 + \rho^2\tan^2\theta\right]^{3/2}}$$

$$= \frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \frac{\sec^2\theta}{\sec^3\theta}\cdot d\theta = \frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \cos\theta\cdot d\theta$$

$$H_\phi = \frac{I}{4\pi\rho}[\sin\theta]_{\alpha_1}^{\alpha_2}$$

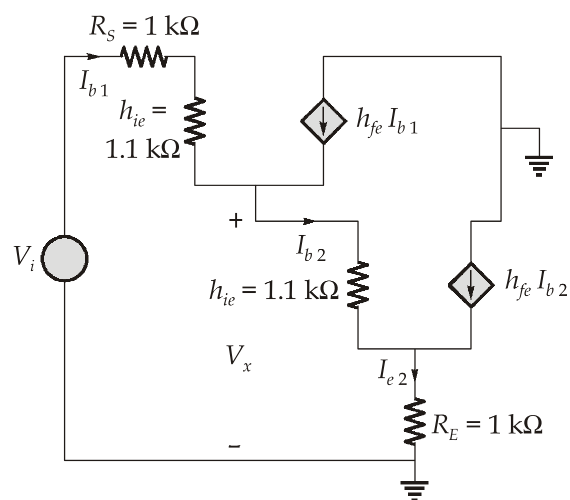
$$H_\phi = \frac{I}{4\pi\rho}[\sin\alpha_2 - \sin\alpha_1]a_\phi$$ Hence Proved.

Q.2 (b) Solution:

$$R_L h_{0e} = 1 \times 10^3 \times 25 \times 10^{-6} = 0.025 < 0.1$$

As $R_L\, h_{0e}$ is less than 0.1 we can use the approximate analysis.

The approximate a.c. equivalent circuit of Darlington emitter follower is shown below,



(i)

$$A_{I2} = \frac{I_0}{I_{i2}} = \frac{I_{e2}}{I_{b2}}$$

$$I_{e2} = I_{b2} + h_{fe} I_{b2}$$

$$I_{e2} = I_{b2}(1 + h_{fe})$$

$$\therefore \quad A_{I2} = 1 + h_{fe} = 1 + 50 = 51$$

(ii)

$$R_{I2} = \frac{V_x}{I_{b2}}$$

By applying KVL in the base emitter loop of transistor 2

$$-V_x + I_{b2}h_{ie} + I_{b2}(1+h_{fe})R_E = 0$$

$$V_x = I_{b2}\left[h_{ie} + R_E(1+h_{fe})\right]$$

$$\frac{V_x}{I_{b2}} = h_{ie} + (1+h_{fe})R_E$$

$$\therefore \quad R_{I2} = h_{ie} + (1+h_{fe})R_E$$

$$= (1.1 \times 10^3) + (51 \times 1 \times 10^3)$$

$$= 52.1 \text{ k}\Omega$$

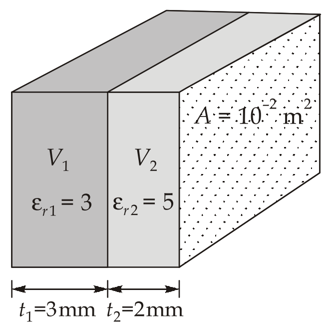
(iii)

$$A_{V2} = \frac{V_0}{V_2} = \frac{V_0}{V_x} = \frac{(I_{b2} + h_{fe} I_{b2}) R_E}{I_{b2}\left[h_{ie} + R_E(1+h_{fe})\right]}$$

$$= \frac{(1+h_{fe})R_E}{h_{ie} + (1+h_{fe})R_E} = \frac{A_{I2} R_E}{R_{I2}}$$

$$A_{V2} = \frac{51 \times 1 \times 10^3}{52.1 \times 10^3} = 0.978$$

Q.2 (c) (i) Solution:



Let the potential division between the two dielectrics be given by V_1 and V_2 and the respective field intensities, E_1 and E_2

Then, $$V_1 = E_1 t_1 = \frac{D_1}{\varepsilon_0}\frac{t_1}{\varepsilon_{r1}}$$

and $$V_2 = E_2 t_2 = \frac{D_2}{\varepsilon_0}\frac{t_2}{\varepsilon_{r2}}$$

At the interface, we have, from boundary relation:

$$\vec{D}_1 = \vec{D}_2$$

Consequently, $$\frac{V_1}{V_2} = \frac{t_1/\varepsilon_{r1}}{t_2/\varepsilon_{r2}} = \frac{3/3}{2/5} = \frac{5}{2}$$

As $V_1 + V_2 = 100$, we obtain

$$V_1 = (5/7)100 = 71.43 \text{ V}$$

and $$V_2 = (2/7)100 = 28.57 \text{ V}$$

Potential gradient in each dielectric is equal in magnitude to the electric field intensity. Accordingly, Potential gradient in dielectric 1

$$= \frac{V_1}{t_1} = \frac{71.43}{3} = 23.81 \text{ V/mm}$$

$$G_1 = 23.81 \text{ V/mm} = 23.81 \text{ kV/m}$$

Similarly, the gradient in dielectric 2

$$G_2 = 14.29 \text{ V/mm} = 14.29 \text{ kV/m}$$

To find the capacitance of the dielectric:

$$V = V_1 + V_2 = E_1t_1 + E_2t_2$$

and

$$D = \frac{Q}{A}$$

$$= \frac{Q}{A}\left[\frac{t_1}{\varepsilon_0\varepsilon_{r1}} + \frac{t_2}{\varepsilon_0\varepsilon_{r2}}\right] = \frac{Q}{A\varepsilon_0}\left[\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}}\right]$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{t_1/\varepsilon_{r1} + t_2/\varepsilon_{r2}} = \frac{\varepsilon_0 A\,\varepsilon_{r1}\varepsilon_{r2}}{t_1\varepsilon_{r2} + t_2\varepsilon_{r1}}$$

Substituting and simplifying, we get

$$C = 63.24 \text{ pF}$$

Let C_1 and C_2 be the capacitances of the parallel-plate capacitors formed by the dielectrics with relative primitivities ε_{r1} and ε_{r2} respectively.

Then we have,

$$C_1 = \varepsilon_0\varepsilon_{r1}\frac{A}{t_1} = \varepsilon_0\frac{3\times10^{-2}}{3\times10^{-3}} = 10\,\varepsilon_0 = 88.54 \text{ pF}$$

and

$$C_2 = \varepsilon_0\varepsilon_{r2}\frac{A}{t_2} = \varepsilon_0\frac{5\times10^{-2}}{2\times10^{-3}} = 25\varepsilon_0 = 221.35 \text{ pF}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{88.54\times221.35}{(88.54+221.35)} = 63.24 \text{ pF}$$

$$\text{Total energy stored} = \frac{1}{2}CV^2 = \frac{1}{2}(63.24\times10^{-12})100^2 \text{ J}$$

$$= 31.62 \times 10^{-8} \text{ J}$$

Q.2 (c) (ii) Solution:

- For eddy current loss per m^3 $= \dfrac{\pi^2 f^2 B_m^2 t^2}{6\rho} = \dfrac{\pi^2\times(60)^2\times(1)^2\times(0.4\times10^{-3})^2}{6\times(16\times10^{-8})}$

$$= 5921.76 \text{ W/m}^3$$

- Density of specimen in kg/m^3 = 7800 kg/m^3

$\therefore$ Eddy current loss per m^3 $= \dfrac{5921.76}{7800} = 0.759$ Watt/kg

$$\text{Hysteresis loss in watt per m}^3 = 240 \times 60 = 14400 \text{ W/m}^3$$

$$\text{Hysteresis loss in watt/kg} = \frac{14400}{7800} = 1.846 \text{ watt/kg}$$

- $$\text{Total core loss} = 1.846 + 0.759 = 2.605 \text{ watt/kg}$$

Q.3 (a) Solution:

For 1-cache

$$\text{No. of lines} = \frac{4\text{K}}{4} = 1\text{ K}$$

Tag	Lines	Offset
18	10	2

$$\#\text{ lines} = \log_2 2^{10} = 10$$

$$\#\text{ offset} = \log_2 4 = 2$$

$$\#\text{ Tag} = 30 - (10 + 2) = 18$$

$$\text{Capacity of tag memory} = \text{Tag bits} \times \text{line size} = 18 \text{ bits} \times 1\text{ K} = 18 \text{ Kbits}$$

For D-cache,

$$\text{No. of lines} = \frac{4\text{K}}{4} = 1\text{ K}$$

$$\text{No. of sets} = \frac{1\text{K}}{2} = 2^9$$

$$\#\text{ sets} = \log_2 2^9 = 9$$

Tag	Sets	Offset
19	9	2

$$\#\text{ tags} = 30 - (9 + 2) = 19$$

$$\text{Capacity of tag memory} = \text{No. of tag bits} \times \text{No. of sets} \times \text{No. of lines in each set}$$
$$= 19 \times 2^9 \times 2 = 19 \text{ Kbits}$$

For L2-cache

$$\text{No. of lines} = \frac{64\text{K}}{16} = 4\text{ K}$$

$$\text{No. of sets} = \frac{4\text{K}}{4} = 1\text{ K}$$

$$\#\text{ lines} = \log_2 2^{12} = 12$$

$$\#\text{ sets} = \log_2 2^{10} = 10$$

$$\#\text{ offset} = \log_2 2^4 = 4$$

Tag	Sets	Offset
16	10	4

$$\%\text{ Tag} = 30 - (10 + 4)$$

$$= 16$$

$$\text{Capacity of tag memory} = 16\text{ bits} \times 4 \times 1\text{ K}$$

$$= 64\text{ Kbits}$$

Q.3 (b) Solution:

(i) Peak to peak voltage at amplitude output = 0.2 V

Peak voltage at amplifier output = $\frac{0.2}{2}$ = 0.1 V

This is the voltage under loaded conditions,

We have, $$E_L = \frac{E_0}{1+\frac{Z_0}{Z_L}}$$

(or) peak open circuit voltage,

$$E_0 = E_L\left(\frac{1+Z_0}{Z_L}\right) = 0.1\left(1+\frac{250\times10^3}{2.5\times10^6}\right) = 0.11\text{ V}$$

The peak open circuit voltage at the amplifier output terminal is,

$$E_0 = Blv \times \text{gain}$$

$$\therefore \quad \text{Average flow rate} = \frac{0.11}{0.1\times50\times10^{-3}\times10^3} = 0.022\text{ m/s}$$

(ii) The effluent conductivity decreases by 20% and therefore it's impedance increases to

$$Z_0 = 1.2 \times 250 \times 10^3$$

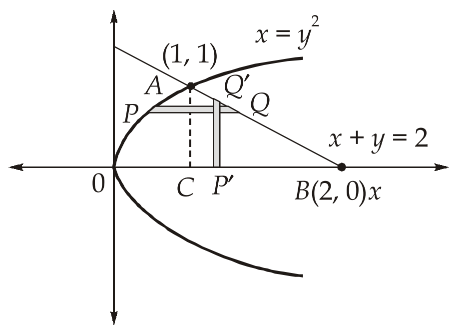
$$= 300\text{ k}\Omega$$

$\therefore$ for the same flow rate, the peak to peak voltage under loaded conditions

$$= \frac{2\times 0.11}{\left[1+\dfrac{300\times 10^3}{2.5\times 10^6}\right]} = 0.1964 \text{ V}$$

$$\text{Percent decrease in voltage} = \frac{0.2-0.1964}{0.2}\times 100 = 1.78\%$$

Q.3 (c) Solution:



Here the integration is first wrt x along horizontal strip PQ which extends from parabola $x = y^2$ to line $x = 2 - y$. Such a strip moves from $y = 0$ to $y = 1$. This gives region of integration curvilinear triangle as OAB.

Now on changing order to integration we first integrate wrt y-along vertical strip $P'Q'$.

This can be done by dividing OAB in two parts separated by line AC. i.e. (i) part ACB where strip moves from $x = 1$ to $x = 2$ with strip bounded by $y = 2 - x$ and $y = 0$.

Part OAC where strip moves from $x = 0$ to $x = 1$ with strip bounded by $y = \sqrt{x}$ and $y = 0$.

$$\Rightarrow \qquad I = \int_0^1 dx \int_0^{\sqrt{x}} (x+y)dy + \int_1^2 dx \int_0^{(2-x)} (x+y)dy$$

$$= \int_0^1 \left(xy + \frac{y^2}{2} \Big|_0^{\sqrt{x}} \right) dx + \int_1^2 xy + \frac{y^2}{2}\Big|_0^{2-x} dx$$

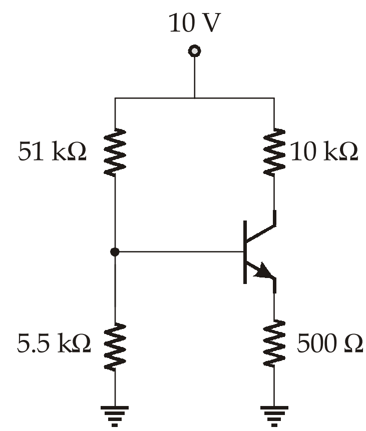
$$I = \int_0^1 \left(x^{3/2} + \frac{x}{2} \right) dx + \int_1^2 \left(x(2-x) + \frac{(2-x)^2}{2} \right) dx$$

$$= \left(\frac{2}{5}\cdot x^{5/2} + \frac{x^2}{4} \Big|_0^1 + \left(x^2 - \frac{x^3}{3} + \frac{(x-2)^3}{6} \Big|_1^2 \right.\right.$$

$$= \frac{2}{5} + \frac{1}{4} + \left[4 - \frac{8}{3} + 0 - \left(1 - \frac{1}{3} - \frac{1}{6} \right) \right] = \frac{89}{60}$$

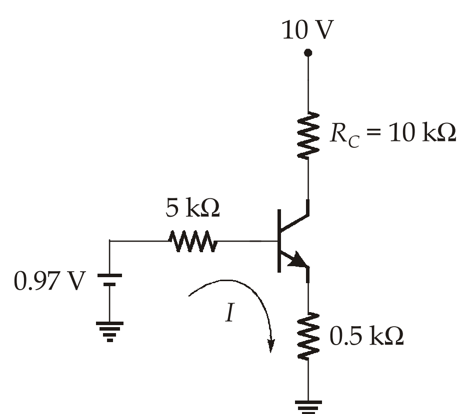
Q.4 (a) Solution:

To draw the small signal equivalent model, we need to calculate the D.C. parameters. Thus, the D.C. equivalent circuit can be drawn as



thus, $$V_{th} = 10\times\left(\frac{5.5}{5.5+51}\right) = 0.97\text{ V and } R_{th} = 5.5 \parallel 51 \approx 5\text{ k}\Omega$$

thus,



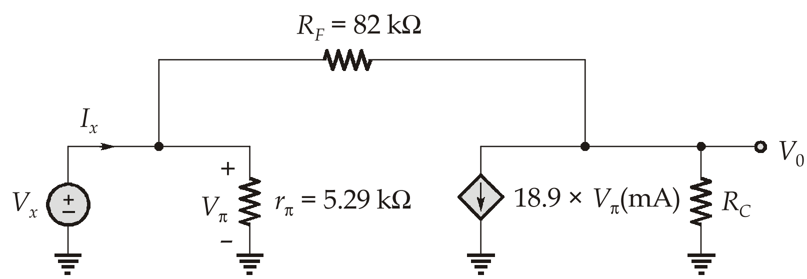
$$I_b = \frac{0.97-0.7}{5\text{ k}\Omega+50.5\text{ k}\Omega} \approx 4.9\ \mu\text{A}$$

$$I_c = 0.49\text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100\times0.026}{0.491} = 5.29\text{ k}\Omega$$

$$g_m = \frac{0.491}{0.026} \approx 18.9\text{ mA/V}$$

Input impedance



Writing a KCL equation of the input, we have

$$I_x = \frac{V_\pi}{r_\pi} + \frac{V_\pi - V_0}{R_F}$$

$$I_x = \frac{V_\pi}{5.29} + \frac{V_\pi - V_0}{82} \quad \text{...(i)}$$

Now, applying KCL at output node we get

$$\frac{V_0}{R_C} + g_m V_\pi + \frac{V_0 - V_\pi}{R_F} = \frac{V_0}{10} + (18.9)V_\pi + \frac{V_0 - V_\pi}{82} = 0$$

$$\frac{V_0}{10} + (18.9)V_\pi + \frac{V_0 - V_\pi}{82} = 0$$

$$(18.9)V_\pi - \frac{V_\pi}{82} = -\left(\frac{V_0}{10} + \frac{V_0}{82}\right)$$

$$\Rightarrow \quad 18.88\ V_\pi = -0.112\ V_0$$

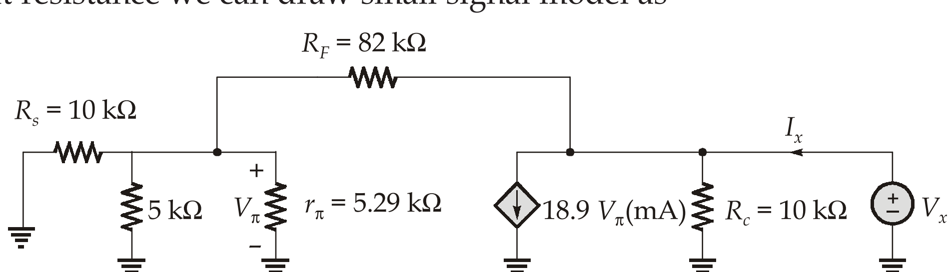
$$V_0 = -168.5\ V_\pi$$

Sub. in equation (1) we get

$$I_x = \frac{V_\pi}{5.29} + \frac{169.5}{82} V_\pi$$

$$R_{if} = \frac{V_x}{I_x} = 0.443\text{ k}\Omega = 443\ \Omega \quad (\because V_x = V_\pi)$$

For output resistance we can draw small signal model as



Assume $$R_{eq} = r_\pi \parallel R_1 \parallel R_2 \parallel R_s$$
$$R_{eq} = 2.04\ \text{k}\Omega$$

Applying KCL at output we get

$$I_x = \frac{V_x}{R_c} + g_m V_\pi + \frac{V_x}{R_F + R_{eq}}$$

and $$V_\pi = \left(\frac{R_{eq}}{R_{eq} + R_F}\right) V_x$$

∴ $$I_x = \frac{V_x}{R_c} + g_m \left[\frac{R_{eq}}{R_{eq} + R_f}\right] V_x + \frac{V_x}{R_F + R_{eq}}$$

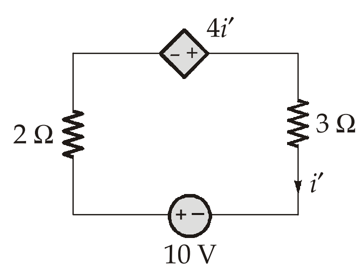
$$I_x = \frac{V_x}{10} + 18.9 V_x \times \left[\frac{2.04}{2.04 + 82}\right] + \frac{V_x}{82 + 2.04}$$

$$R_{of} = \frac{V_x}{I_x} \approx 1.75\ \text{k}\Omega$$

Q.4 (b) (i) Solution:

Case-I:

When 10 V source is acting alone

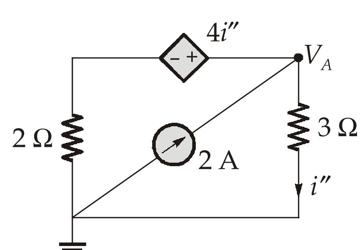


Apply KVL in above loop, we get

$$-4i' + 3i' - 10 + 2i' = 0$$
$$i' = 10\ \text{A}$$

Case-II:

When the 2 A source is acting alone



$$i'' = \frac{V_A}{3}$$

Apply KCL at node-A, we get

$$\frac{V_A - 4i''}{2} + (-2) + \frac{V_A}{3} = 0$$

$$\frac{V_A - \frac{4V_A}{3}}{2} + \frac{V_A}{3} = 2$$

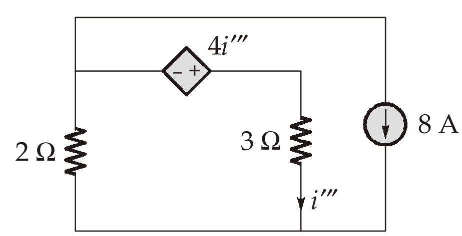
$$\frac{3V_A - 4V_A + 2V_A}{6} = 2$$

$$V_A = 12 \text{ V}$$

$$i'' = \frac{V_A}{3} = 4 \text{ A}$$

Case-3:

When the 8-A source is acting alone,



By apply KVL in above loop, we get

$$-4i''' + 3i''' + 2(i''' + 8) = 0$$

$$-4i''' + 3i''' + 2i''' + 16 = 0$$

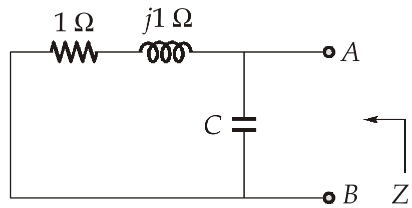
$$i''' = -16 \text{ A}$$

∴ Current when all the source are acting simultaneously is given by the superposition theorem as

$$i = (i' + i'' + i''')$$

$$= 10 + 4 - 16 = -2\text{A}$$

Q.4 (b) (ii) Solution:



$$Z = \frac{(1+j1)(-jX_c)}{1+j1-jX_c} = \frac{-jX_c(1+j1)}{1+j(1-X_c)}$$

$$|Z| = \frac{X_C\sqrt{1^2+1^2}}{\sqrt{1^2+(1-X_C)^2}}$$

$$\frac{d|z|}{dX_c} = 0$$

$$\frac{d|z|}{dX_c} = \left(\sqrt{1+(1-X_C)^2}\right)\sqrt{2} - \frac{1}{2}\left((1+(1-X_C)^2)\right)^{-1/2}(-2+2X_C)\sqrt{2}X_C$$

$$\sqrt{1-(1-X_C)^2} - \frac{1}{2}\frac{1}{\sqrt{1+(1-X_C)^2}}2(X_C-1)\cdot X_C = 0$$

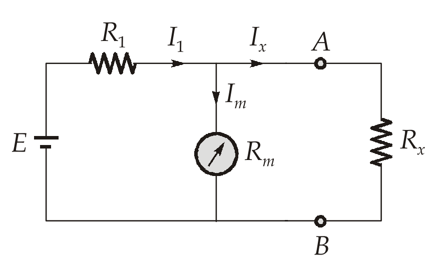
$$1 + (1 - X_C)^2 - X_C(X_C - 1) = 0$$

$$X_C = 2\ \Omega$$

$$\frac{1}{\omega C} = 2$$

$$C = \frac{1}{2} = 0.5\ \text{F}$$

Q.4 (c) Solution:



When $R_x = \infty$, full scale meter current is,

$$I_{fs} = \frac{E}{R_1 + R_m}$$

$$R_1 = \frac{E}{I_{fs}} - R_m$$

When resistance R_x to be measured,

$$\text{meter current, } I_m = \left(\frac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}}\right)\left(\frac{R_x}{R_m + R_x}\right) = \frac{ER_x}{R_1 R_m + R_x(R_1 + R_m)}$$

At half scale reading of meter,

$$i_m = 0.5\, I_{fs} \text{ and } R_x = R_h$$

$$0.5\, If_s = \frac{ER_h}{R_1 R_m + R_h(R_1 + R_m)}$$

$$\frac{I_m}{I_{fs}} = \frac{R_x(R_1 + R_m)}{R_1 R_m + R_x(R_1 + R_m)}$$

So,

$$\frac{I_m}{I_{fs}} = \frac{R_x}{R_m + R_x} \qquad \text{Since } [R_1 >> R_m]$$

$$2 = \frac{E}{R_1 + R_m} \cdot \frac{R_1 R_m + R_h(R_1 + R_m)}{ER_h}$$

$$2 = \frac{R_1 R_m + R_h R_1 + R_m R_h}{R_1 R_h + R_m R_h}$$

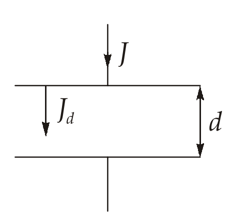
So,

$$R_h = \frac{R_1 R_m}{R_1 + R_m}$$

$$R_x = R_h = R_m \; (R_1 >> R_m \text{ so } R_1 + R_m \approx R_1)$$

Section-B

Q.5 (a) Solution:



The electric field inside the capacitor is,

$$E = -\frac{V}{d} = \frac{-20\cos 10^3 t}{0.1} = -200 \cos 10^3 t$$

The displacement current density is

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon_0 E)$$

$$= 2\times 10^5 \varepsilon_0 \sin 10^3 t \text{ A/m}^2$$

Since capacitor plate area, A is 2 m^2, the total displacement current carried by the capacitor is

$$I_d = J_d A = 4 \times 10^5\, \varepsilon_0 \sin 10^3 t \text{ A}$$

From Ampere's circuital law, (along a loop of radius r parallel to the capacitor plates)

$$2\pi r H = J_d\, \pi r^2$$

$$H = \frac{J_d r}{2} = 10^5\, r\varepsilon_0 \sin 10^3 t \text{ A/m}$$

Now the capacitor has a capacitance,

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 2}{0.1} = 20\,\varepsilon_0$$

Current in the capacitance terminal is

$$I = C\frac{dV}{dt} = 20\varepsilon_0 \frac{d}{dt}\left(20\cos 10^3 t\right)$$

$$= -4 \times 10^5\, \varepsilon_0 \cdot \sin 10^3\, t \text{ A}$$

But, since the direction of the current in the external circuit is opposite, we get $I_d = I$.

Q.5 (b) Solution:

(i) Drain current is given by,

$$I_D = \frac{1}{2}\mu_n C_{OX}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

Given, $I_{D2} = 0.2 \text{ mA}$

$$0.2 \times 10^{-3} = \frac{1}{2}\times(200\times10^{-6})\cdot\left(\frac{8\times10^{-6}}{0.8\times10^{-6}}\right)(V_{GS} - 0.6)^2$$

$$V_{GS} - 0.6 = \sqrt{0.2}$$

$$V_{GS} = 1.047 \text{ V} \approx 1.05 \text{ V}$$

$$V_{GS} = V_{DS}$$

$$\therefore \quad R_1 = \frac{V_{DD} - V_{GS}}{0.2\times10^{-3}} = \frac{3 - 1.05}{0.2\times10^{-3}} = 9.750 \text{ k}\Omega$$

(ii) Given, drain voltage = 1 V

For Q_2 to conduct 0.5 mA

$$R_2 = \frac{V_{DD} - V_D}{I_{D2}} = \frac{3-1}{0.5 \times 10^{-3}} = 4\ \text{k}\Omega$$

Since gates are connected together hence both transistors have same V_{GS},

$$\therefore \quad I_D \propto \left(\frac{W}{L}\right)$$

$$\frac{I_{D1}}{I_{D2}} = \frac{W_1}{W_2}$$

$$W_2 = W_1 \times \frac{I_{D2}}{I_{D1}} = 8 \times 10^{-6} \times \frac{0.5}{0.2} = 20\ \mu\text{m}$$

Q.5 (c) Solution:

Paging is a method of writing data to and reading it from, secondary storage for use in primary storage, also known as main memory. Paging plays a role in memory management for a computer's operating system.

In a memory management system that takes advantage of paging, the OS read data from secondary storages in blocks called pages all of which have identical sizes. The physical memory containing a single page is called frame. When paging is used, a frame does not have to comprise a single physically contiguous region in secondary storage. It facilitates more efficient and faster use of storage.

Simple Paging	Virtual memory Paging
1. Main memory partitioned into small fixed size chunks called frames.	Main memory partitioned into small fixed size chunks called frames.
2. Program broken in the pages by the compiler or memory management system.	Program broken in the pages by the compiler or memory management system.
3. Operating system must maintain a page table for each process showing which frame each page occupies.	Operating system must maintain a page table for each process showing which frame each page occupies.
4. Processor use page number, offset to calculate absolute address.	Processor use page number, offset to calculate absolute address.
5. All the pages of a process must be in main memory for process to run, unless overlays are used.	Not all pages of a process need be in main memory frames for the process to run. Pages may be read in as needed.

Q.5 (d) Solution:

Here, $z = -1$ is a pole lies inside the circle

$z = 2$ is a pole lies out side the circle

$$\therefore \quad \int_c \frac{dz}{(z+1)^2(z-2)} = \int \frac{\frac{1}{z-2}}{(z+1)^2} dz$$

Here, $$f(z) = \frac{1}{z-2}$$

$$f'(z) = -\frac{1}{(z-2)^2}$$

Hence by Cauchy integral formula

$$\int_c \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^n(a)$$

$$\int_c \frac{dz}{(z+1)^2(z-2)} = \int_c \frac{\frac{1}{z-2}}{[z-(-1)]^2} dz = \frac{2\pi i}{1\,!} f'(-1)$$

$$= 2\pi i\left[\frac{-1}{(-1-2)^2}\right] \qquad \left(\therefore f'|z| = \frac{-1}{(z-2)^2}\right)$$

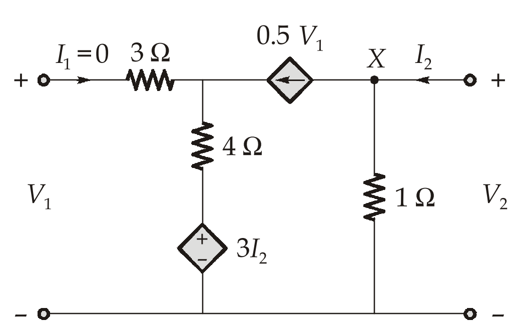
$$= 2\pi i\left[\frac{-1}{9}\right] = \frac{-2}{9}\pi i$$

Q.5 (e) Solution:

To find h-parameters, we consider two cases:

Case-I: When $I_1 = 0$

Here, no current will flow through the 3-Ω resistance



Apply KVL at the left mesh,

$$V_1 = 4 \times 0.5\, V_1 + 3I_2$$

$$= 2\,V_1 + 3I_2$$

$$V_1 = -3I_2$$

Apply KCL at node *X*, we get

$$I_2 = \frac{V_2}{1} + 0.5V_1 = V_2 + 0.5 \times (-3I_2)$$

$$2.5\,I_2 = V_2$$

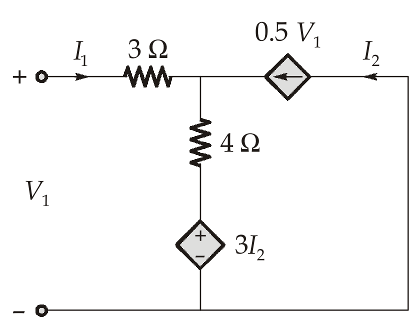
$$\therefore \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{2.5} = 0.4\ \mho$$

$$\therefore \quad V_1 = -3I_2 = -3 \times \frac{V_2}{2.5} = -1.2\,V_2$$

$$\therefore \quad h_{12} = \frac{V_1}{V_2} = -1.2$$

Case-II: when $V_2 = 0$

Here port-2 is short circuited. The 1 Ω resistance becomes redundant.



$$\therefore \quad I_2 = 0.5\,V_1 = 0.5 \times [3I_1 + 4I_1 + 4I_2 + 3I_2]$$

$$2.5\,I_2 = -3.5\,I_1$$

$$\therefore \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-3.5}{2.5} = -1.4$$

Also,

$$V_1 = 3I_1 + 4I_1 + 4I_2 + 3I_2$$

$$= 7I_1 + 7I_2$$

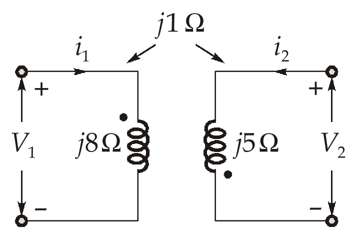
$$= -2.8\,I_1$$

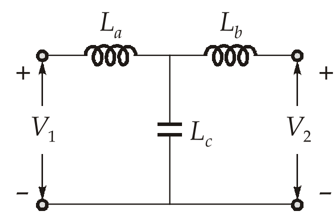
$$\therefore \quad h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = -2.8\ \Omega$$

Therefore the h-parameters of the network are given as

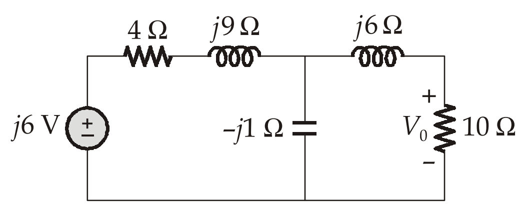
$$[h] = \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$$

Q.6 (a) (i) Solution:





T-equivalent circuit of given circuit,



An Equivalent T-circuit

$$L_a = L_1 - (-M) = 8 + 1 = 9 \text{ H}$$

$$L_b = L_2 - (-M) = 5 + 1 = 6 \text{ H}$$

$$L_c = -M = -1 \text{ H}$$

Applying mesh analysis, we obtain

$$j6 = I_1(4 + j9 - j1) + I_2(-j1) \quad \text{...(i)}$$

and
$$0 = I_1(-j1) + I_2(10 + j6 - j1) \quad \text{...(ii)}$$

$$I_1 = \left(\frac{10 + j5}{j}\right) I_2 = (5 - j10)I_2 \quad \text{...(iii)}$$

Substituting equation (iii) in equation (i), we get

$$j6 = (4 + j8)(5 - j10)I_2 - jI_2$$

$$= (20 + j40 - j40 + 80)I_2 - jI_2$$

$$= (100 - j)I_2$$

$$I_2 = \frac{j6}{100 - j} = 0.06\angle 90.573° \text{ A}$$

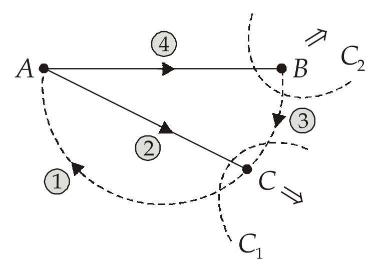
$$I_1 = (5 - j10)(0.06\angle 90.573°)$$

$$= 0.67\angle 27.14° \text{ A}$$

$$V_0 = -10I_2 = -0.6\angle 90.573° \text{ V}$$

Q.6 (a) (ii) Solution:

The oriented graph of the network shown in figure. Since we have to find voltage V_x, we take the node-2 in the twig and a possible tree is selected.



The fundamental cutsets are identified as,

f-cut - set - 1: [1, 2, 3]

f-cut - set - 2: [3, 4]

The fundamental cut-set matrix is given as,

Q_a =

	1	2	3	4
C_1	–1	1	1	0
C_2	0	0	–1	1

The node equations are given as,

$$[Q]\,[Y_b]\,[Q^T]\,[V_t] = [Q] \times \{[Y_b]\,[V_s] - [I_s]\}$$

$$[Q]\,[Y_b]\,[Q^T] = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix}$$

$$[Q] \times \{[Y_b]\,[V_s] - [I_s]\} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \left\{ \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2\,\text{V} \end{bmatrix} \right\} = \begin{bmatrix} -1 \\ -2V_x \end{bmatrix}$$

Thus the KCL equations are

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} \begin{bmatrix} V_{t2} \\ V_{t4} \end{bmatrix} = \begin{bmatrix} -1 \\ -2V_x \end{bmatrix};$$

Here, $V_{t2} = V_x$

Solving KCL equations we get,

$$V_x = \frac{-4}{9} \text{ V}$$

Q.6 (b) Solution:

Solids and liquids show almost same type of polarization phenomena because there is significant amount of interaction among the atoms of solids and liquids which affects the value of polarization. In case of gases as density of molecules is reasonably low hence internal field can be considered equal to applied field. In solids and gases internal field is also given by dipoles carried by surrounding particles. so internal field is not equal to applied electric field in general.

For the second part,

$$\text{considering, } \gamma = \frac{1}{3}$$

Lorentz internal field, $$E_i = E + E_s \quad \text{...(i)}$$

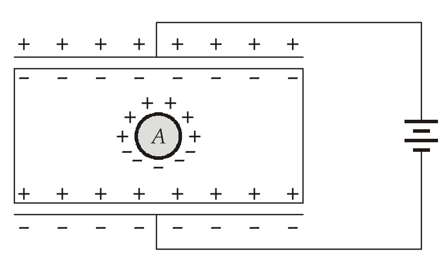
Where, $$E_s = \frac{P}{3 \in_0}$$

∴ $$E_i = E + \frac{P}{3 \in_0} \quad \text{...(ii)}$$

For providing above relation (ii) and determining self field,

Let as assume an imaginary spherical cavity around atom, when radius of cavity is large compared to radius of atom.

Now consider an atom in dielectric placed between plates of capacitor



The internal field at A is sum of four fields,

$$E_i = E_1 + E_2 + E_3 + E_4 \quad ...(iii)$$

E_1 is the field intensity due to charge on the plates of the capacitor,

$$E_1 = \frac{D}{\epsilon_0}$$

Also, $$D = \epsilon_0 E + P \quad ...(iv)$$

So, $$E_1 = E + \frac{P}{\epsilon_0} \quad ...(v)$$

E_2 is field intensity at A due to charge induced on the two sides of cubic dielectric opposite to the plates. As D is zero for this case

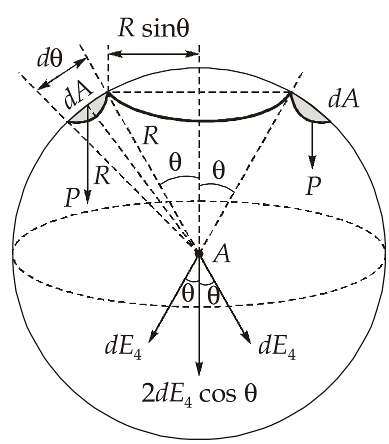
Putting, $D = 0$ in equation (iv),

$$E_2 = \frac{-P}{\epsilon_0}$$

E_3 is field due to other atoms in cavity. Assuming cubic structure and hence symmetry

$$E_3 = 0$$

and E_4 is field intensity due to polarization charges on surface of cavity (representing the self field) as shown below,



Assuming cavity to be empty of other molecules as charge density on spherical surface will be P per unit area,

Each unit area will contribute a radial field at A,

$$dE_4 = \frac{P\cos\theta dA}{4\pi\epsilon_0 R^2}$$

$\therefore$ All horizontal field due to all elements will cancel due to symmetry but vertical components will add to $dE_4 \cos\theta$

$$\therefore \quad \text{self field} = E_4 = E_s = \iint \frac{P\cos^2\theta\, dA}{4\pi \in_0 R^2}$$

dA is surface area of circular strip of radius $R \sin\theta$ with arc width of $R\, d\theta$

$$dA = 2\pi R \sin\theta .\, R\, d\theta$$

$$E_4 = \int_0^{\pi} \frac{P\cos^2\theta \cdot 2\pi R^2 \sin\theta\, d\theta}{4\pi \in_0 R^2}$$

$$E_4 = \int_0^{\pi} \frac{P}{2\in_0}\cos^2\theta \sin\theta\, d\theta$$

On solving $\quad E_4 = E_S = \dfrac{P}{3\in_0} \quad$ (Where E_s is the required self field).

$\therefore$ In cubic system, the internal field seen by atom is

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$= E + \frac{P}{\in_0} - \frac{P}{\in_0} + 0 + \frac{P}{3\in_0}$$

$$E_i = E + \frac{P}{3\in_0}$$

Where E is externally applied field.

Q.6 (c) Solution:

The A.E is $m^2 + 2m - 1 = 0$

$$m = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$$

$$\text{C.F.} = Ae^{\left(-1+\sqrt{2}\right)x} + Be^{\left(-1-\sqrt{2}\right)x}$$

$$P.I. = \frac{1}{(D^2+2D-1)}(x^2 + 2xe^x + e^{2x}) + \frac{1}{(D^2+2D-1)}\cos 2x \frac{(e^x + e^{-x})}{2}$$

$$\frac{1}{(D^2+2D-1)}x^2 = -\frac{1}{\left[1-(2D+D^2)\right]}x^2$$

$$= -\left[1+(2D+D^2)+(2D+D^2)^2+\ldots\right]x^2$$

$$= -[1 + 2D + D^2 + 4D^2]\; x^2$$

$$= -x^2 - 4x - 10$$

$$\frac{1}{D^2+2D-1}x^2 = -x^2 - 4x - 10$$

$$\frac{2}{D^2+2D-1}xe^x = \left(\frac{2e^x}{(D+1)^2+2(D+1)-1}\right)x$$

$$= 2e^x\frac{1}{D^2+2D+1+2D+2-1}x = \frac{2e^x}{D^2+4D+2}(x)$$

$$= \frac{2e^x}{2}\frac{1}{\left[1+\left(2D+\frac{D^2}{2}\right)\right]}x = e^x\left[1+\left(2D+\frac{D^2}{2}\right)\right]^{-1}x$$

$$= e^x\left[1-\left(2D+\frac{D^2}{2}\right)+\ldots\right]x = e^x(1-2D)x$$

$$\frac{2}{(D^2+2D-1)}xe^x = e^x[x-2] = (x-2)e^x$$

$$\frac{1}{D^2+2D-1}e^{2x} = \frac{1}{(4+4-1)}e^{2x} = \frac{e^{2x}}{7}$$

$$\frac{1}{D^2+2D-1}\cos 2x\left(\frac{e^x+e^{-x}}{2}\right) = \frac{1}{D^2+2D-1}\frac{e^x\cos 2x}{2} + \frac{1}{D^2+2D-1}\frac{e^{-x}\cos 2x}{2}$$

$$= \frac{e^x}{2}\frac{1}{(D+1)^2+2(D+1)-1}\cos 2x + \frac{e^{-x}}{2}\frac{1}{(D-1)^2+2(D-1)-1}\cos 2x$$

$$= \frac{e^x}{2}\frac{1}{D^2+2D+1+2D+2-1}\cos 2x + \frac{e^{-x}}{2}\frac{1}{-4-2}\cos 2x$$

$$= \frac{e^x}{2}\frac{(2D+1)\cos 2x}{2(2D-1)(2D+1)} - \frac{e^{-x}}{12}\cos 2x$$

$$= \frac{e^x}{2}\frac{1}{2(4D^2-1)}(-2.2\sin 2x + \cos 2x) - \frac{e^{-x}\cos 2x}{12}$$

$$= \frac{e^x}{4}\frac{(-4\sin 2x + \cos 2x)}{(-16-1)} - \frac{e^{-x}\cos 2x}{12}$$

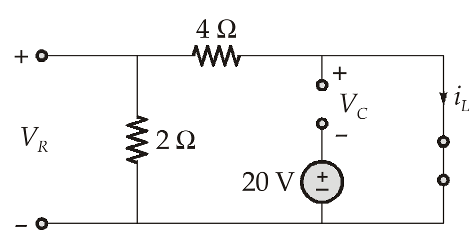
$$= -\frac{e^x(\cos 2x - 4\sin 2x)}{68} - \frac{e^{-x}\cos 2x}{12}$$

The general solution is

$$y = Ae^{\left(-1+\sqrt{2}\right)x} + Be^{-\left(1+\sqrt{2}\right)x} - 10 - 4x - x^2 + \frac{e^{2x}}{7} + (x-2)e^x - \frac{e^x}{68}(\cos 2x - 4\sin 2x) - \frac{e^{-x}}{12}\cos 2x$$

Q.7 (a) Solution:

(i) For $t < 2$, $3\,u(t) = 0$, At $t = 0^-$, since the circuit has reached steady state

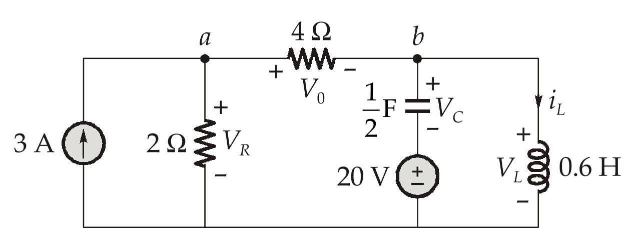


$$i_L(0^-) = 0,$$
$$V_R(0^-) = 0$$

and
$$V_C(0^-) = -20\text{ V}$$
$$i_L(0^+) = 0\text{ A},$$
$$V_C(0^+) = -20\text{ V}$$

For $t > 0$,

$$3\,u(t) = 3$$



Apply KCL at node (a)

$$3 = \frac{V_R(0^+)}{2} + \frac{V_0(0^+)}{4} \quad \text{...(i)}$$

Apply KVL to the middle mesh

$$-V_R(0^+) + V_0(0^+) + V_C(0^+) + 20 = 0$$

$\because$ $V_C(0^+) = -20$ V

$\therefore$ $V_R(0^+) = V_0(0^+)$ ...(ii)

Putting equation (ii) in equation (i), we get

$$3 = \frac{V_R(0^+)}{2} + \frac{V_R(0^+)}{4} = \frac{3V_R(0^+)}{4}$$

$$V_0(0^+) = V_R(0^+) = 4 \text{ V} \quad \text{...(iii)}$$

(ii)

$$V_L = L\frac{di_L}{dt}$$

$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

Applying KVL to the right mesh,

$$V_L(0^+) - V_C(0^+) - 20 = 0$$

$$V_L(0^+) = V_C(0^+) + 20 \quad \text{...(iv)}$$

$$= 0$$

Hence, $\frac{di_L(0^+)}{dt} = 0$

Similarly, $\frac{C\, dV_c}{dt} = i_C$

then, $\frac{dV_c}{dt} = \frac{i_C}{C}$

We apply KCL at node b to get i_C.

$$\frac{V_0(0^+)}{4} = i_C(0^+) + i_L(0^+)$$

Since, $V_0(0^+) = 4$ V and $i_L(0^+) = 0$

$\therefore$ $i_C(0^+) = 1$ A

Then $\frac{dV_C(0^+)}{dt} = \frac{I_C(0^+)}{C} = \frac{1}{0.5} = 2$ V/sec

To get $\frac{dV_R(0^+)}{dt}$, we apply KCL at node (a) and obtain

$$3 = \frac{V_R}{2} + \frac{V_0}{4}$$

Taking the derivative of each term and setting $t = 0$

gives,

$$0 = \frac{2dV_R(0^+)}{dt} + \frac{dV_0(0^+)}{dt} \quad \text{...(v)}$$

We also apply KVL to the middle mesh and obtain

$$-V_R + V_C + 20 + V_0 = 0$$

Again taking the derivative of each term and setting $t = 0^+$, yields

$$\frac{-dV_R(0^+)}{dt} + \frac{dV_C(0^+)}{dt} + \frac{dV_0(0^+)}{dt} = 0$$

Substituting for $\frac{dV_C(0^+)}{dt} = 2 \text{ V/sec}$

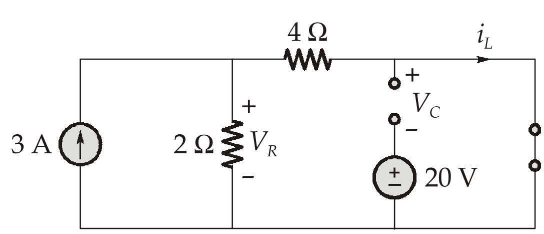
$$\frac{dV_R(0^+)}{dt} = 2 + \frac{dV_0(0^+)}{dt}$$

Putting equation (v),

$$\frac{dV_R(0^+)}{dt} = 2 - 2\frac{dV_R(0^+)}{dt}$$

$$\frac{dV_R(0^+)}{dt} = \frac{2}{3} \text{ V/ sec}$$

(iii) As $t \to \infty$,



$$i_L(\infty) = \frac{2}{6} \times 3 = 1 \text{ A}$$

$$V_C(\infty) = -20 \text{ V}$$

$$V_R(\infty) = 2 \times 2 = 4 \text{ V}$$

Q.7 (b) Solution:

(i) From Newton Raphsons iterative formula we have,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{...(i)}$$

Also given that,

$$x_n = \alpha + E_n \quad \text{...(ii)}$$

$$\Rightarrow \quad x_{n+1} = \alpha + E_{n+1} \quad \text{...(iii)}$$

Using equation (i), (ii) and (iii), we get

$$\alpha + E_{n+1} = \alpha + E_n - \frac{f(\alpha + E_n)}{f'(\alpha + E_n)}$$

$$\Rightarrow \quad E_{n+1} = E_n - \frac{f(\alpha + E_n)}{f'(\alpha + E_n)}$$

By Taylor's series expansion we have,

$$E_{n+1} = E_n - \frac{f(\alpha) + E_n f'(a) + \frac{1}{2!}E_n^2 f''(\alpha) + ...}{f'(\alpha) + E_n f''(\alpha) + ...}$$

Since, $f(\alpha) = 0$

$$\Rightarrow \quad E_{n+1} = E_n - \frac{E_n f'(\alpha) + \frac{1}{2!}E_n^2 f''(\alpha) + ...}{f'(\alpha) + E_n f''(\alpha) + ...}$$

Neglecting third and higher powers,

$$\Rightarrow \quad E_{n+1} = E_n - \frac{E_n f'(\alpha) + \frac{1}{2}E_n^2 f''(\alpha)}{f'(\alpha) + E_n f''(\alpha)}$$

$$\Rightarrow \quad E_{n+1} = \frac{E_n(f'(\alpha) + E_n f''(\alpha) - E_n f'(\alpha) - \frac{1}{2}E_n^2 f''(\alpha)}{f'(\alpha) + E_n f''(\alpha)}$$

$$\Rightarrow \quad E_{n+1} = \frac{E_n^2 f''(\alpha)}{2[f'(\alpha) + E_n f''(\alpha)]} = \frac{E_n^2 f''(\alpha)}{2f'(\alpha)}$$

(ii) Let, $f(x) = 3x - \cos x - 1$

$\Rightarrow$ $f'(x) = 3 + \sin x$

Now using Newton Raphson's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow \quad x_{n+1} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n} = \frac{x_n \sin x_n + \cos x_n + 1}{3 + \sin x_n}$$

$$\Rightarrow \quad x_1 = \frac{0.4 \sin 0.4 + \cos 0.4 + 1}{3 + \sin 0.4} = 0.6127$$

Now, $$x_2 = \frac{0.6127 \sin 0.6127 + \cos 0.6127 + 1}{3 + \sin 0.6127} = 0.6071$$

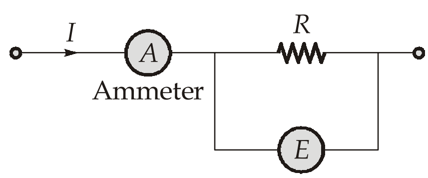
Now, $$x_3 = \frac{0.6071 \sin 0.6071 + \cos 0.6071 + 1}{3 + \sin 0.6071} = 0.6071$$

Since, $x_2 = x_3$

So desired root is 0.6071.

Q.7 (c) Solution:

Consider the circuit diagram as shown:



(i) $$P = \frac{E^2}{R}$$

$$\therefore \quad \frac{\partial P}{\partial E} = \frac{2E}{R}$$

and $$\frac{\partial P}{\partial R} = \frac{-E^2}{R^2}$$

Hence uncertainty in power measurement

$$W_p = \sqrt{\left(\frac{\partial P}{\partial E}\right)^2 W_E^2 + \left(\frac{\partial P}{\partial R}\right)^2 W_R^2}$$

$$= \sqrt{\left(\frac{2E}{R}\right)^2 W_E^2 + \left(\frac{-E^2}{R^2}\right)^2 W_R^2}$$

∴ Percentage uncertainly in measurement of power is calculated by putting,

$$P = \frac{E^2}{R}$$

$$\frac{W_P}{P} \times 100 = \sqrt{4\left(\frac{W_E}{E}\right)^2 + \left(\frac{W_R}{R}\right)^2} \times 100$$

$$= \sqrt{4(0.01)^2 + (0.01)^2} \times 100 = \pm 2.236\%$$

(ii) $$P = EI$$

∴ $$\frac{\partial P}{\partial E} = I \text{ (and) } \frac{\partial P}{\partial I} = E$$

Percentage uncertainty in power measurements

$$\frac{W_P}{P} \times 100 = \frac{1}{P}\sqrt{\left(\frac{\partial P}{\partial E}\right)^2 W_E^2 + \left(\frac{\partial P}{\partial I}\right)^2 W_I^2} \times 100$$

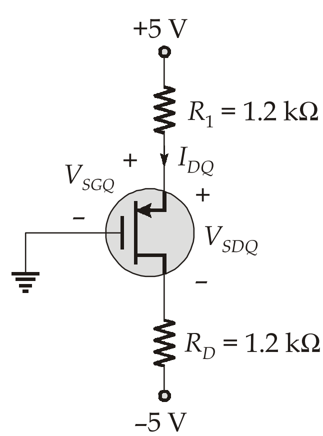
$$= \frac{1}{P}\sqrt{I^2 W_E^2 + E^2 W_I^2} \times 100$$

$$= \sqrt{\left(\frac{W_I}{E}\right)^2 + \left(\frac{W_I}{I}\right)^2} \times 100$$

$$= \sqrt{(0.01)^2 + (0.01)^2} \times 100 = \pm 1.414\%$$

Q.8 (a) Solution:

(i) For DC analysis of the given circuit, all the coupling capacitors can be open circuited and the resultant equivalent circuit will be as shown below:



By assuming that the transistor is in saturation mode and taking the numerical values of I_{DQ} in mA units, we get,

$$I_{DQ} = K_p(V_{SGQ} + V_{tp})^2$$

$$V_{SGQ} = 5 - 1.2\, I_{DQ}$$

So,
$$I_D = (1)\,(5 - 1.2\, I_{DQ} - 1.5)^2 = (3.5 - 1.2\, I_{DQ})^2$$

$$I_D = 1.44\, I_{DQ}^2 - 8.4 I_{DQ} + 12.25$$

$$1.44\, I_{DQ}^2 - 9.4 I_{DQ} + 12.25 = 0$$

By solving the above quadratic equation, we get,

$$I_{DQ} = 4.73 \text{ mA}, 1.8 \text{ mA}$$

For $I_{DQ} = 4.73$ mA, $V_{SGQ} = 5 - (1.2 \times 4.73) = -0.676 \text{ V} < |V_{tp}|$

For $I_{DQ} = 1.8$ mA, $V_{SGQ} = 5 - (1.2 \times 1.8) = 2.84 \text{ V} > |V_{tp}|$

So, for the assumed case, the valid value of I_{DQ} is 1.8 mA.

$$V_{SDQ} = 10 - (2 \times 1.2 \times I_{DQ}) = 10 - (2.4 \times 1.8) = 5.68 \text{ V}$$

$$V_{SGQ} - |V_{tp}| = 2.84 - 1.5 = 1.34 \text{ V}$$

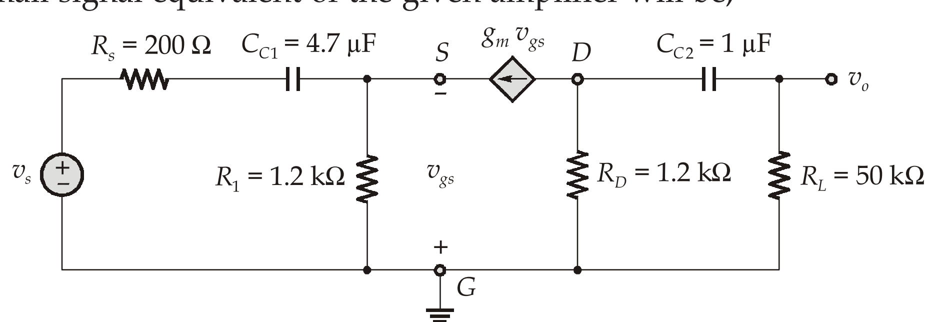
$V_{SDQ} > V_{SGQ} - |V_{tp}|$. So, the initial assumption is correct about the mode of operation of transistor.

The small-signal parameters of the transistor are,

$$g_m = 2K_p\,(V_{SGQ} + V_{tp}) = 2(1)\,(1.34) = 2.68 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_{DQ}} = \frac{1}{\lambda I_{DQ}} = \infty$$

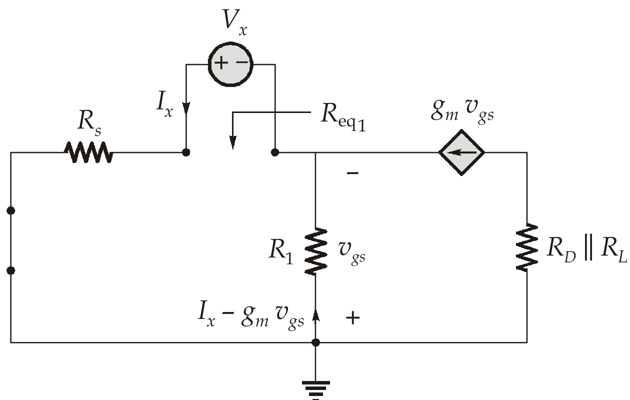
(ii) The small signal equivalent of the given amplifier will be,



Calculation of time constant (τ_1) associated with C_{C1} :

$$\tau_1 = R_{eq_1}\, C_{C1}$$

While calculating R_{eq2}, C_{C2} must be short circuited and the voltage source v_s should be deactivated as shown below.



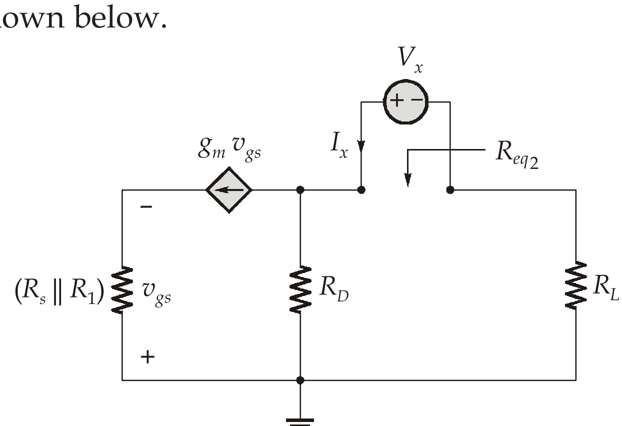
$$V_x = R_s I_x + v_{gs}$$
$$v_{gs} = R_1(I_x - g_m v_{gs})$$
$$v_{gs} = \frac{R_1}{1+g_m R_1} I_x$$

So,
$$V_x = \left(R_s + \frac{R_1}{1+g_m R_1}\right) I_x$$
$$R_{eq_1} = \frac{V_x}{I_x} = R_s + \frac{R_1}{1+g_m R_1} = 484.63\ \Omega$$
$$\tau_1 = R_{eq_1} C_{C_1} = 484.63 \times 4.7\ \mu s = 2.28\text{ ms}$$

Calculation of time constant (τ_2) associated with C_{C2}:

$$\tau_2 = R_{eq_2} C_{C2}$$

While calculating R_{eq_1}, C_{C1} must be short circuited and the voltage source must be deactivated as shown below.



$$v_{gs} = -g_m v_{gs} (R_s \| R_1)$$

So, $v_{gs} = 0$

$$R_{eq2} = \frac{V_x}{I_x} = R_D + R_L = 51.2\text{ k}\Omega$$

$$\tau_2 = R_{eq2} C_{C_2} = 51.2\text{ ms}$$

(iii) The corner frequency associated with C_{C1} is,

$$f_{C1} = \frac{1}{2\pi\tau_1} = 69.8\text{ Hz}$$

The corner frequency associated with C_{C2} is,

$$f_{C2} = \frac{1}{2\pi\tau_2} = 3.12\text{ Hz}$$

So, the corner frequency due to C_{C1} dominates that due to C_{C2}. Hence, the lower cut-off frequency of the amplifier can be given by,

$$f_L = f_{C1} = 69.8\text{ Hz}$$

Q.8 (b) Solution:

Given,
$$u + v = \frac{(\sin x + \cos y)\cdot(\sin 2x + \cos 2y)}{\sin x + \cos 2y}$$

$\Rightarrow$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\left[\cos x(\sin 2x + \cos 2y) + (\sin x + \cos y)(2\cos 2x)\right](\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)\cos x}{(\sin x + \cos 2y)^2}$$

Now,
$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\left[-\sin y(\sin 2x + \cos 2y) + (\sin x + \cos y)(-2\sin 2y)\right](\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)(-2\sin 2y)}{(\sin x + \cos 2y)^2}$$

$$2\frac{\partial u}{\partial x} = \frac{\begin{array}{c}\left[\cos x(\sin 2x + \cos 2y) + (\sin x + \cos y)(2\cos 2x)\right](\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)\cos x \\ +\left[-\sin y(\sin 2x + \cos 2y) + (\sin x + \cos y)(-2\sin 2y)\right](\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)(-2\sin 2y)\end{array}}{(\sin x + \cos 2y)^2}$$

...(iii)

Subtracting (ii) from (i),

$$2\frac{\partial v}{\partial x} = \frac{\begin{array}{c}\left[\cos x(\sin 2x + \cos 2y) + (\sin x + \cos y)(2\cos 2x)\right] \\ (\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)\cos x \\ +\left[-\sin y(\sin 2x + \cos 2y) + (\sin x + \cos y)(-2\sin 2y)\right] \\ (\sin x + \cos 2y) - (\sin x + \cos y)(\sin 2x + \cos 2y)(-2\sin 2y)\end{array}}{(\sin x + \cos 2y)^2} \quad \text{...(iv)}$$

Now since,

$$f'(z) = \frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z}$$

Putting, $x = z$ and $y = 0$ in (iii) and (iv),

$$f'(z) = \frac{\begin{array}{c}\left[\cos z(\sin 2z + 1) + (\sin z + 1)(\cos 2z)\right] \\ (\sin z + 1) - (\sin z + 1)(\sin 2z + 1)\cos z \\ +i\left[\cos z(\sin 2z + 1) + (\sin z + 1)(2\cos 2z)\right] \\ (\sin z + 1) - (\sin z + 1)(\sin 2z + 1)\cos z\end{array}}{2(\sin z)^2}$$

$$\Rightarrow \quad f'(0) = \frac{[1 + 1.2][1] - (1)(1)(1)}{2} + \frac{i(1+2) - 1}{2} = 1 + i$$

Q.8 (c) Solution:

```
    # include <conio.h>
    # include <stdio.h>
    # define max 10
    int a [10] {10, 14, 19, 26, 27, 18, 7, 5, 1, 9};
    int b [10]
void main ( )
{
    int i;
    printf (“list before sorting”);
    for (i = 0; i < = max; i ++)
    printf (“ %d”, a[i]);
    sort (0, max);
    print (“\n list after \n”);
```

```
        for (i = 0; i < = max; i++)
        printf (“%d, 0[i]”)
    }
        void sort (int low, int high)
    {
        int mid;
        if (low < high)
    {
        mid = (low + high)/2
        sort (low, mid);
        sort (mid + 1, high);
        merge (low, mid, high);
    }}
        void merge (int low, int mid, int high)
    {
        int l1, l2, i;
        for (l1 = low, l2 = mid + 1, i = row; l1 < = mid && l2 < = high; i ++)
    {
        if (a[l1] < = a[l2]
        b [i] = a[l1 ++]
    else
        b[i] = a[l2 ++];
    }
        while (l1 < = mid)
        b [i++] = a[l1 ++];
        while [l2 < = high]
        b[i ++] = a[l2++]
        for (i = low; i < = high; i++)
        a[i] = b[i];
    }
```

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