DETAILED SOLUTIONS

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# ESE 2026 : Prelims Exam
## CLASSROOM TEST SERIES

## ELECTRICAL ENGINEERING

## Test 8

**Section A :** Power Systems [All Topics]

**Section B :** Electrical Machines-1 [Part Syllabus]

**Section C :** Control Systems-2 [Part Syllabus] + Engineering Mathematics-2 [Part Syllabus]

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (c) | **16.** | (b) | **31.** | (b) | **46.** | (a) | **61.** | (b) |
| **2.** | (a) | **17.** | (d) | **32.** | (c) | **47.** | (a) | **62.** | (c) |
| **3.** | (a) | **18.** | (c) | **33.** | (a) | **48.** | (a) | **63.** | (c) |
| **4.** | (c) | **19.** | (c) | **34.** | (b) | **49.** | (b) | **64.** | (a) |
| **5.** | (b) | **20.** | (d) | **35.** | (a) | **50.** | (d) | **65.** | (b) |
| **6.** | (b) | **21.** | (c) | **36.** | (c) | **51.** | (b) | **66.** | (c) |
| **7.** | (c) | **22.** | (b) | **37.** | (b) | **52.** | (b) | **67.** | (d) |
| **8.** | (a) | **23.** | (d) | **38.** | (d) | **53.** | (c) | **68.** | (a) |
| **9.** | (c) | **24.** | (c) | **39.** | (b) | **54.** | (b) | **69.** | (d) |
| **10.** | (c) | **25.** | (a) | **40.** | (d) | **55.** | (c) | **70.** | (c) |
| **11.** | (c) | **26.** | (d) | **41.** | (d) | **56.** | (a) | **71.** | (b) |
| **12.** | (c) | **27.** | (b) | **42.** | (c) | **57.** | (b) | **72.** | (c) |
| **13.** | (a) | **28.** | (b) | **43.** | (a) | **58.** | (c) | **73.** | (d) |
| **14.** | (c) | **29.** | (b) | **44.** | (d) | **59.** | (a) | **74.** | (c) |
| **15.** | (d) | **30.** | (a) | **45.** | (a) | **60.** | (d) | **75.** | (b) |

**DETAILED EXPLANATIONS**

**Section A : Power Systems**

**1. (c)**

Overall efficiency of the power station is

$$\eta_{overall} = \eta_{thermal} \times \eta_{electrical}$$
$$= 0.30 \times 0.92$$
$$= 0.276$$

Unit generated/hour $= (100 \times 10^3) = 10^5$ kWh

Heat produced per hour,

$$H = \frac{\text{Electrical output in heat unit}}{\eta \text{ overall}}$$

$$= \frac{10^5 \times 860}{0.276} = 311.6 \times 10^6 \text{ Kcal} \qquad (\because 1 \text{ kWh} = 860 \text{ Kcal})$$

$$\therefore \text{ Coal consumption/hour} = \frac{H}{\text{Calorific value}} = \frac{311.6 \times 10^6}{6400} = 48687 \text{ kg}$$

**2. (a)**

Nuclear power plants are used as base load plants.

**4. (c)**

$$Z_C = \sqrt{\frac{B}{C}} = \sqrt{\frac{57.97\angle 90°}{8.315 \times 10^{-4}\angle 90°}} = 250\ \Omega$$

Ideal power transfer capability

$$\text{SIL} = \frac{V^2}{Z_C} = \frac{(400)^2}{250} = 640 \text{ MW}$$

**5. (b)**

Diameter of conductor = 10 mm

Radius of conductor = 5 mm

Thickness of conductor = 5 m

$$R = t + r = 10 \text{ mm}$$

$$g_{max} = \frac{V}{r \ln\left(\frac{R}{r}\right)} = \frac{6.5 \times 10^3}{5 \times \ln\left(\frac{10}{5}\right)}$$

$$= \frac{6.5 \times 10^3}{5 \times 0.69} = \frac{6.5 \times 10^3}{3.46}$$

$$= 1878.6 = 1.8 \text{ kV/mm}$$

**6. (b)**

$$Q_R = \frac{|V_R|}{X}(|V_S| - |V_R|)$$

$$1 = \frac{|V_R|}{0.2}(1.2 - |V_R|)$$

$$0.2 = 1.2|V_R| - |V_R|^2$$

$$|V_R|^2 - 1.2|V_R| + 0.2 = 0$$

$$(|V_R| - 1.0)(|V_R| - 0.2) = 0$$

$$\therefore \quad |V_R| = 1 \text{ p.u.}$$

$$\text{Voltage drop} = |V_S| - |V_R|$$

$$= 1.2 - 1 = 0.2 \text{ p.u.}$$

**7. (c)**

$$\text{Load factor} = \frac{\text{Average demand}}{\text{Maximum demand}}$$

$$0.60 = \frac{\text{Average demand}}{30}$$

$$\text{Average demand} = 18 \text{ MW}$$

$$\text{Plant capacity factor} = \frac{\text{Average demand}}{\text{Installed capacity}}$$

$$0.50 = \frac{18}{\text{Installed capacity}}$$

$$\text{Installed capacity} = \frac{18}{0.50} = 36 \text{ MW}$$

The energy corresponding to installed capacity per day

$$= 36 \times 24 = 864 \text{ MWh}$$

**8. (a)**

$$\text{Total load, } P_{G_1} + P_{G_2} = 250 \text{ MW}$$

For optimum load sharing,

$$\frac{dC_1}{dP_{G_1}} = \frac{dC_2}{dP_{G_2}}$$

$$0.20P_{G_1} + 40.0 = 0.25P_{G_2} + 30.0$$

$$5P_{G_2} - 4P_{G_1} = 200 \quad \text{...(i)}$$

$$P_{G_1} + P_{G_2} = 250 \quad \text{...(ii)}$$

On solving equations (i) and (ii), we obtain

$$P_{G_1} = \frac{350}{3} \text{ MW}$$

and $$P_{G_2} = \frac{400}{3} \text{ MW}$$

**10. (c)**

**Diversity factor:**

The ratio of the individual maximum demands of all the consumers supplied by it to the maximum demand of the power station.

$$\text{Diversity factor} = \frac{\text{Sum of individual maximum demands}}{\text{Maximum demand of power station}}$$

**11. (c)**

Corona effects can be minimized in a transmission line by using large diameter conductors which may be accomplished by using hollow conductors.

**12. (c)**

If we increase the spacing between the phase conductors, the line capacitance decreases.

**13. (a)**

Let $\Delta V$ = Voltage fluctuation

$\Delta Q$ = Reactive power variation (i.e. the size of the compensator)

$S_{s/c}$ = System short circuit capacity

Then, $$\Delta V = \frac{\Delta Q}{S_{s/c}}$$

or, $$\Delta Q = \Delta V \times S_{s/c}$$

$$= \pm (0.05 \times 5000)$$

$$= \pm 250 \text{ MVAR}$$

∴ The capacity of the static VAR compensator is 250 MVAR.

**14. (c)**

Steady state stability limit is enhanced by providing auxiliary stability controllers to damp low frequency oscillations. Hence, statement-5 is not correct.

**16. (b)**

Since the generators are in parallel, they will operate at same frequency at steady load.

Let,

load on generator 1 (200 MW) = $x$ MW

load on generator 2 (400 MW) = $(600 - x)$ MW

Reduction in frequency = $\Delta f$

Then, $$\frac{\Delta f}{x} = \frac{0.04 \times 50}{200}$$

$$\frac{\Delta f}{600 - x} = \frac{0.05 \times 50}{400}$$

Also, Equating $$\Delta f = \frac{0.04 \times 50}{200}(x) = \frac{0.05 \times 50}{400}(600 - x),$$

we have $x \approx 231$ MW (load on generator 1)

$\therefore$ $$\text{System frequency} = 50 - \left(\frac{0.04 \times 50}{200}\right) \times 231$$

$$= 47.69 \text{ Hz}$$

**17. (d)**

Turbine power will increase from 100 MW for a 0.03 p.u. droop or 1.5 Hz drop in frequency. Thus, we have regulation parameter

$$= R = \frac{-1.5}{100} = -0.015 \text{ Hz/MW}$$

For a frequency change of $\Delta f = -0.2$ Hz, the turbine power will experience a static change of

$$\Delta P_t = -\frac{1}{0.015} \times (-0.2) = 13.33 \text{ MW}$$

**18. (c)**

A sudden short circuit in an ac power system causes a rise in current in the short circuited phase.

**20. (d)**

Three phase fault current is given as

$$(I_f)_{3-\phi} = \frac{V_3^0}{Z_{33}} = \frac{1}{j0.3} = -j\,3.33 \text{ p.u.}$$

**23. (d)**

The diagonal element $Y_{22}$ of $Y_{bus}$ is obtained by,

$$Y_{22} = y_{12} + y_{23} + y_{20}$$

$$= \frac{1}{Z_{12}} + \frac{1}{Z_{23}} + \frac{1}{Z_{20}} = \frac{1}{j0.1} + \frac{1}{j0.1} + \frac{1}{-j20} = -j19.95 \text{ p.u.}$$

**24. (c)**

$$\text{Stored energy} = G \times H = 800 \text{ MJ}$$

$$P_a = 80 - 50 = 30 \text{ MW}$$

$$= M\frac{d^2\delta}{dt^2}$$

$$M = \frac{GH}{180f} = \frac{800}{180\times 50} = \frac{4}{45} \text{ MJ-s/ele. deg}$$

Now, acceleration of rotor,

$$\alpha = \frac{d^2\delta}{dt^2} = \frac{30\times 45}{4}$$

$$= 337.5 \text{ ele. deg/s}^2$$

**26. (d)**

$$Z_1(\text{actual}) = Z_{pu.}\, Z_{base}$$

$$= 0.05\times\frac{400\times 400}{250} = 32\ \Omega$$

**27. (b)**

$$V_t = 100\times\frac{2\times 50}{450} = 22.22 \text{ kV}$$

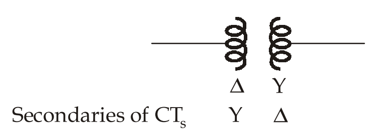
**28. (b)**

Synchronizing power coefficient,

$$S_p = \left.\frac{dP_e}{d\delta}\right|_{\delta=\delta_0} = P_{max}\cos\delta_0$$

**29. (b)**

They should be opposite in nature.




**30. (a)**

$$I_S = CV_r + DI_R$$

$$V_r = \frac{220}{\sqrt{3}},\ I_r = 0$$

$$\therefore \quad I_C = I_S = 0.5\times 10^{-4}\times\frac{220}{\sqrt{3}}\times 10^3$$

$$I_C = I_S = \frac{11}{\sqrt{3}}\text{A}$$

**31. (b)**

$$I_{PK} = 0.5\times 5 = 2.5$$

$$\text{PSM} = \frac{I_f}{I_{PK}\times CT \text{ ratio}} = \frac{3000}{2.5\times\frac{200}{5}} = 30$$

**32. (c)**

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a2} = \frac{1}{3}\left[I_a + \alpha^2 I_b + \alpha I_c\right]$$

$$I_{a2} = \frac{1}{3}\left[(0 + (1\angle 120°)^2 \times 6\angle 60° + (1\angle 120°) \times (6\angle -120°)\right]$$

$$= \frac{1}{3}\left(3 - 3\sqrt{3}j + 6\right) = 3 - j\sqrt{3}\text{ A}$$

**33. (a)**

Quantity of water per second,

$$Q = \frac{P}{\rho \times H \times 9.81 \times 10^{-3} \times \eta}$$

$$= \frac{30000}{1000 \times 50 \times 9.81 \times 10^{-3} \times 0.6} = 101.9\text{ m}^3/\text{s}$$

$$\text{Rate of fall of water level} = \frac{Q}{A} = \frac{101.9}{50 \times 10^6} \times 3600 = 7.33 \times 10^{-3}\text{ m/hr}$$

$$= 7.33\text{ mm/hr}$$

**34. (b)**

$$H = 5\text{ kW-sec/kVA}$$

$$= 5\text{ MJ/MVA}$$

$$\text{K.E.} = \text{H.S} = 5 \times 100 = 500\text{ MJ}$$

$$f_2 = f_1 \times \left(\frac{H.S + \Delta P_D \cdot t_d}{H.S}\right)^{1/2}$$

$$= 50 \times \left(\frac{500 + (100 - 50) \times 0.4}{500}\right)^{1/2}$$

$$= 50 \times \left(\frac{500 + 20}{500}\right)^{1/2} = 51\text{ Hz}$$

**35. (a)**

%winding unprotected,

$$\%\ W_u = \frac{I_n R_n}{V_{\text{ph}}} \times 100$$

$$I_n = \text{Out of balance } I \times \text{CT ratio}$$

$$= 1.8 \times \frac{1000}{5} = 360 \text{ A}$$

$$\%W_u = \frac{360 \times 10}{\frac{10 \times 10^3}{\sqrt{3}}} \times 100 = 62.36\%$$

$$\% \; W_{dg} \text{ protected} = 100 - 62.36$$

$$= 37.64\%$$

**36. (c)**

$$E_{\max} = \frac{V}{r \ln\left(\frac{R}{r}\right)}$$

$$E_{\max 1} = \frac{V}{0.5R \ln\left(\frac{R}{0.5R}\right)} = \frac{2V}{R \ln 2}$$

$$E_{\max 2} = \frac{V}{0.25R \ln\left(\frac{R}{0.25R}\right)} = \frac{4V}{R \ln 4}$$

$$= \frac{2V}{R \ln 2}$$

$\because$ $E_{\max 2} = E_{\max 1}$

So no change in maximum voltage gradient.

**37. (b)**

$$\eta = \frac{V_{ph}}{2V_2} \times 100 = \frac{(V_1 + V_2)}{2V_2} \times 100$$

$$V_2 = V_1(1 + K)$$

Given, $V_2 = 12$ kV

$$K = \frac{C_m}{C_s} = \frac{C/3}{C} = \frac{1}{3}$$

$$V_1 = \frac{12}{\left(1 + \frac{1}{3}\right)} = \frac{12 \times 3}{4} = 9 \text{ kV}$$

$$\eta = \frac{9 + 12}{2 \times 12} \times 100 = \frac{21}{24} \times 100 = \frac{7}{8} \times 100 = 87.5\%$$

**38. (d)**

% winding unprotected = 100 – 85 = 15%

$$I_{fL} = \frac{100\times10^6}{\sqrt{3}\times1000} = 5249 \text{ A}$$

Output of balance current,

$$I_0 = \frac{20}{100}\times5249 = 1050 \text{ A}$$

$$V_{\text{ind}} = V_{\text{ph}}\times\frac{15}{100} = \frac{11000}{\sqrt{3}}\times\frac{15}{100} = 953 \text{ A}$$

$$R = \frac{V_{\text{ind}}}{I_0} = \frac{953}{1050} = 0.9\,\Omega$$

**39. (b)**

$$R = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{2}\sqrt{\frac{5}{0.02\times10^{-6}}} = \frac{1}{2}\times15.8\times10^3$$

$$= 7.9 \text{ k}\Omega$$

**40. (d)**

Steady state stability limit = $P_{SSSL} = \frac{EV}{X}$

So on increasing the value of $E$ and $V$ and on decreasing the value of $X$ stability limit increases.

**41. (d)**

Load flow study is generally carried out by using $Y_{\text{bus}}$ matrix.

**42. (c)**

For HVDC system, $f = 0$

So, $X_C = \frac{1}{2\pi fC}$

Hence, $I_C = 0$

and in EHVAC system, $f \neq 0$, hence $I_C$ value is present in EHVAC and it requires shunt and series compensation.

**45. (a)**

Reactance relay is more suitable for the protection of short lines because it is practically unaffected by arc resistance which may be large compared with the line impedance as a result of which more of the line can be protected at high speed using a reactance relay. Hence, both statement (I) and statement (II) are individually true and statement (II) is the correct explanation of statement (I).

## Section B : Electrical Machines-1

**46. (a)**

Net cross sectional area of core,

$$a = 55\ \text{cm}^2 = 0.0055\ \text{m}^2$$

Maximum value of flux,

$$\phi_{max} = \frac{E_1}{4.44 f N_1} = \frac{400}{4.44\times 50\times 350} = 5.148 \times 10^{-3}\ \text{Wb}$$

Peak value of flux density in the core,

$$B_{max} = \frac{5.148\times 10^{-3}}{0.0055} = 0.936\ \text{T}$$

**47. (a)**

- The core has infinite permeability so that zero magnetizing current is needed to establish the requisite amount of flux in the core.
- The core-loss (hysteresis as well as eddy-current loss) is considered zero.

**48. (a)**

Star/delta connection is the most commonly used connection for power systems. At transmission levels star connection is on the HV side i.e $\Delta/Y$ for step-up and $Y/\Delta$ for step-down.

**49. (b)**

Full load copper loss, $P_C = 300$ W

Iron loss, $P_i = 120$ W

kVA load corresponding to maximum efficiency

$$= x \times \text{Rated kVA}$$

$$= \sqrt{\frac{120}{300}}\times 20 = 12.649\ \text{kVA}$$

kVA load corresponding to maximum efficiency

$$= 12649 \times 0.8 = 10119\ \text{W} \qquad (\because P_C = P_i = 120\ \text{W})$$

Maximum efficiency of transformer

$$= \frac{\text{Output}}{\text{Input} + P_i + P_c}\times 100 = \frac{10119}{10119+120+120}\times 100 = 97.68\%$$

**50. (d)**

When the two windings are connected in series, the total voltage available is (11000 + 2200) ie., 13200 V. The other side can have the voltage of 11000 V or 2200 V. So the voltage rating can be either 13200/11000 V or 13200/2200 V

In the first case transformation ratio,

$$K_1 = \frac{11000}{13200} = 0.8333$$

$$\text{Rated output} = \frac{\text{Rating as a two-winding transformer}}{1-K}$$

$$= \frac{1000}{1-0.8333} = 6000 \text{ kVA}$$

**51. (b)**

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\Rightarrow \quad \frac{1000}{400} = \frac{I_2}{3}$$

$$\Rightarrow \quad I_2 = 7.5 \text{ A}$$

By changing supply from HV to LV or vice versa, losses in the system remains same as long as the system is operating under rated condition,

Power, $P = V_H I_H \cos\phi = 1000 \times 0.5 \times 3$

$= 1500$ W

$$1500 = V_L I_L \cos\phi$$

$$1500 = 400 \times 7.5 \times \cos\phi$$

$$\text{p.f.} = \cos\phi = 0.5 \text{ lag}$$

**52. (b)**

Synchronous speed, $N_s = \frac{120f}{P} = \frac{120\times 50}{20} = 300$ rpm

Full load speed, $N_f = 294$ rpm

Full-load slip, $S_f = \frac{N_s - N_f}{N_s} = \frac{300-294}{300} = 0.02$

Slip corresponding to maximum torque

$$S_{\text{max},\,T} = \frac{R_2}{X_2} = \frac{0.025}{0.28} = 0.0893$$

Ratio of maximum to full-load torque,

$$\frac{T_{\max}}{T_f} = \frac{\frac{S_{\max T}}{S_f} + \frac{S_f}{S_{\max T}}}{2} = \frac{\frac{0.0893}{0.02} + \frac{0.02}{0.0893}}{2} = 2.344$$

**53. (c)**

$$\text{TUF} = \frac{\sqrt{3}}{2} = 86.6\%$$

**54. (b)**

Autotransformer as step up transformer and to give maximum kVA, the connection should be additive polarity with HV common

So, $V_{HV} = 2000 + 200 = 2200$ V

$V_{LV} = 2000$ V

$$K = \frac{V_{LV}}{V_{HL}} = \frac{2000}{2200} = 0.909$$

$$\%(\text{voltage regulation})_{\text{auto}} = (1 - K)\,(V_R)_{\text{2-wdg}}$$

$$= (1 - 0.909) \times 3$$

$$= 0.273\%$$

**55. (c)**

Supply frequency, $f = 50$ Hz

Synchronous speed, $N_s = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm

Full-load slip, $s = \dfrac{\text{Frequency of rotor emf}}{\text{Supply frequency}}$

$$= \frac{f'}{f} = \frac{2}{50} = 0.04$$

Full load speed, $N = N_s(1 - s)$

$= 1000(1 - 0.04) = 960$ rpm

**56. (a)**

External resistance in the rotor circuit of a wound-rotor improves its starting power factor.

**57. (b)**

Rotor emf, $f' = \dfrac{\text{Number of cycles completed per minute}}{60}$

$$= \frac{100/2}{60} = 0.833 \text{ Hz}$$

Slip, $s = \dfrac{f'}{f} = \dfrac{0.833}{50} = 0.0166$

Power input to rotor, $P_2 = 80$ kW

Total rotor copper losses $= sP_2$

$= 0.0166 \times 80 = 1.333$ kW

Mechanical power developed

$= 80 - 1.333$

$= 78.667$ kW

**58. (c)**

Starting current with normal voltage = $I_{SC}$

Starting current with 80% of normal supply voltage

$$= 0.8 I_{SC}$$

Starting torque with 80% of normal supply voltage applied to the stator

$$= T_f \left(\frac{0.8 I_{SC}}{I_f}\right)^2 S_f$$

= 0.64 times of starting torque with normal supply voltage

Reduction in starting torque = (1 – 0.64) × 100 = 36%

**59. (a)**

Synchronous speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000$ prm

Slip corresponding to speed of 960 rpm

$$s_1 = \frac{1000 - 960}{1000} = 0.04$$

Slip corresponding to speed of 750 rpm

$$s_2 = \frac{1000 - 750}{1000} = 0.25$$

For constant toque,

Slip ∝ Rotor resistance

So, $$\frac{R_2 + R}{R_2} = \frac{s_1}{s_2} = \frac{0.25}{0.04}$$

Where $R$ is additional resistance inserted in the rotor circuit per phase

$$\frac{0.2 + R}{0.2} = 6.25$$

$$R = 6.25 \times 0.2 - 0.2$$

$$= 1.05\ \Omega$$

**60. (d)**

The voltage regulation of an ideal transformer is always zero.

## Section C : Control Systems-2 + Engineering Mathematics-2

**61. (b)**

Critical gain = 50 dB

Operating gain = 25 dB

$(\text{Gain margin})_{dB}$ = Critical gain – Actual gain

= 50 – 25 = 25 dB

**62. (c)**

Lag compensator reduces steady state error and increases rise time.

**63. (c)**

$$\omega_n = \sqrt{16} = 4 \text{ rad/sec}$$

$$\xi = \frac{4}{2\times 4} = \frac{1}{2}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = 4\sqrt{1-2\times\left(\frac{1}{2}\right)^2}$$

$$= 4\times\frac{1}{\sqrt{2}} = 2.82 \text{ rad/sec}$$

**64. (a)**

$$\text{Initial slope} = 6 \text{ dB/oct} = 20 \text{ dB/dec}$$

$$\frac{a-0}{\log_{10}20-\log_{10}2} = 20$$

$$\frac{a}{\log_{10}\left(\frac{20}{2}\right)} = 20$$

$$\frac{a}{\log_{10}10} = 20$$

$$a = 20 \text{ dB}$$

**65. (b)**

Characteristic equation :

$$|sI - A| = 0$$

$$\begin{vmatrix} s & 1 \\ -1 & s+2 \end{vmatrix} = 0$$

$\Rightarrow$ $s(s + 2) + 1 = 0$

$\Rightarrow$ $s^2 + 2s + 1 = 0$

$\Rightarrow$ $(s + 1)^2 = 0$

$\Rightarrow$ $s = -1, -1$

Real and equal roots. So system is critically damped.

**66. (c)**

Given, $G(j\omega) = \dfrac{4}{3(1+s)^2}$

$\therefore$ $|G(j\omega)| = \dfrac{4}{3\left(\sqrt{1+\omega^2}\right)^2} = \dfrac{4}{3(1+\omega^2)}$

At $\omega = \omega_{gc}$, $|G(j\omega)| = 1$

$$\frac{4}{3(1+\omega^2)} = 1$$

or, $$(1+\omega^2) = \frac{4}{3}$$

or, $$\omega^2 = \frac{1}{3}$$

or, $$\omega = \omega_{gc} = \frac{1}{\sqrt{3}} \text{ rad/sec}$$

Phase angle, $$\phi = \angle G(j\omega) = -2\tan^{-1}\omega$$

At $$\omega = \omega_{gc} = \frac{1}{\sqrt{3}} \text{ rad/sec}$$

$$\phi = -2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{rad/sec}$$

$$= -2 \times 30° = -60°$$

Phase margin, $$\text{P.M.} = 180° + \phi = 180° - 60° = 120°$$

**67. (d)**

Here, $$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and $$C = [1 \quad 1]$$

**Check for controllability:**

$$Q_C = [B \quad AB]$$

$$[AB] = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$|Q_C| = 0 \quad \Rightarrow \text{Not controllable}$$

**Check for observability:**

$$Q_o = [C^T : A^T C^T]$$

$\Rightarrow$ $$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$|Q_o| = 0 \qquad \Rightarrow \text{Not observable}$$

**68. (a)**

It is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q(x)$$

Here, $$P = \frac{1}{x}$$

$$Q = \log x$$

then, $$\text{I.F.} = e^{\int P dx} = e^{\int \frac{dx}{x}} = e^{\log x} = x$$

Hence, the general solution is,

$$y(\text{I.F.}) = \int Q(IF)dx$$

i.e. $$yx = \int x \cdot (\log x)\, dx$$

$$yx = (\log x) \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} + c$$

$$yx = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

$$y = \frac{x \log x}{2} - \frac{x}{4} + \frac{c}{x}$$

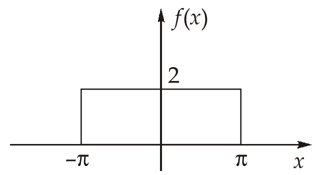
**69. (d)**

Here, $$I = \int_c \frac{\cos 2\pi z}{(2z-1)(z-3)} dz = \frac{1}{2}\int_c \frac{\cos(2\pi z)}{\left(z - \frac{1}{2}\right)(z-3)} dz$$

Both $z = \frac{1}{2}$ and $z = 3$ lies outside the closed curve $c$ and $\int_c f(z)dz = 2\pi i$ [Residue at these poles which are inside $c$]

$$2\pi\, i\, (0) = 0$$

**70. (c)**




Given function is even function

$\therefore$ value of coefficient $b_n = 0$

**71. (b)**

$$\ln f = x \ln y$$

$$\frac{1}{f}\frac{\partial f}{\partial x} = \ln y$$

$$\frac{\partial f}{\partial x} = y^x \ln y$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{y}\cdot y^x - x \ln y\ y^{x-1} = \frac{1}{1}(1)^2 - 2\cdot\ln(1)\cdot 1^1$$

$$= 1$$

**72. (c)**

Probability of item to be defective,

$$p = 0.1$$

Probability of item to be non-defective,

$$q = 1 - 0.1 = 0.9$$

$$P(x = 2) = {}^{10}C_2\, p^2 q^8 = \frac{10\times 9}{1\times 2}\times\left(\frac{1}{10}\right)^2\times\left(\frac{9}{10}\right)^8 = \frac{1}{2}\left(\frac{9}{10}\right)^9$$

**73. (d)**

$$\frac{1}{z\sin z} = \frac{1}{z\left[z - \frac{z^3}{3!} + \frac{z^5}{5!}\ldots\right]} = \frac{1}{z^2\left[1 - \frac{z^2}{3!} + \frac{z^4}{5!}\ldots\right]}$$

$$= \frac{1}{z^2}\left[1 - \left(\frac{z^2}{6} - \frac{z^4}{120}\ldots\right)\right]^{-1}$$

$$= \frac{1}{z^2}\left[1 + \frac{z^2}{6} + \frac{z^4}{120} + \ldots\right]$$

$$= \frac{1}{z^2} + \frac{1}{6} + \frac{z^2}{120}\ldots$$

$$\Rightarrow \qquad \text{Res}\ (z=0) = \left[\frac{1}{(2-1)!}\frac{d^{2-1}}{dz^{2-1}} z^2 f(z)\right]_{z=0}$$

$$= \frac{d}{dz}\left(1+\frac{z^2}{6}+\frac{z^4}{120}\right)_{z=0} = 0$$

$$\int_C \frac{dz}{z\sin z} = 2\pi i\ (0) = 0$$

**74. (c)**

Given, $$f(x) = \begin{cases} K(1-x^2) & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

We know that,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

i.e., $$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_0^1 K(1-x^2)\,dx + 0 = 1$$

$$K\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$K\left(1-\frac{1}{3}\right) = 1$$

$$\therefore \qquad K = \frac{3}{2}$$

**75. (b)**

Both statements are separately observed.

Hence, statement (II) is not the reason for statement (I).

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