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Detailed Solutions

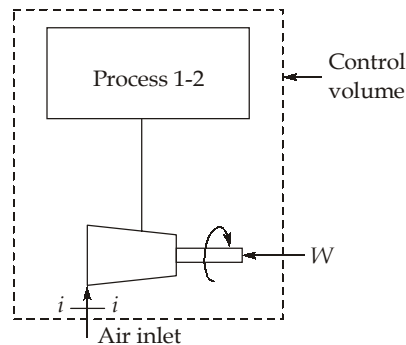
**ESE-2019
Mains Test Series**

**Mechanical Engineering
Test No : 14**

Section A

Q.1 (a) Solution:

Initial conditions:



$$v_1 = 2 \text{ m}^3$$

$$T_1 = 25^\circ\text{C} = (25 + 273)\text{K} = 298 \text{ K}$$

$$P_1 = 1 \text{ atm} = 101.325 \text{ kPa}$$

$$\text{Final mass: } m_2 = 5 m_1$$

Assumptions:

1. Adiabatic process: $Q = 0$
2. Change in kinetic and potential energy are negligible.

$$\text{Initial mass, } m_1 = \frac{P_1 V_1}{RT_1} = \frac{101.325 \times 2}{0.287 \times 298}$$

$$m_1 = 2.3694 \text{ kg}$$

$$m_2 = 5 m_1$$

$$m_2 = 11.847 \text{ kg}$$

For process 1 - 2 constant volume process,

$$\frac{P_1}{m_1 T_1} = \frac{P_2}{m_2 T_2} \quad [\text{Volume of tank} = \text{constant}]$$

$$\frac{101.325}{298} = \frac{P_2}{5T_2}$$

$$P_2 = 1.7 T_2$$

For adiabatic filling tank,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{298} = \left(\frac{1.7T_2}{101.325} \right)^{1.4-1/1.4}$$

$$T_2 = 298 \left(\frac{1.7T_2}{101.325} \right)^{1.4-1/1.4}$$

$$(T_2)^{1-0.4/1.4} = 298 \left(\frac{1.7}{101.325} \right)^{1.4}$$

$$T_2 = 567.28 \text{ K}$$

Now, $\left(\frac{dm}{dt} \right)_{cv} = \dot{m}_i - \dot{m}_e$

$$m_2 - m_1 = m_i \quad (\text{no exit mass, } \dot{m}_e = 0)$$

and, $U_2 - U_1 = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv}$

No heat transfer, $\dot{Q} = 0$, $\dot{m}_e = 0$

$$m_2 c_v T_2 - m_1 c_v T_1 = \dot{m}_i h_i - W_{cv}$$

$$(11.847 \times 567.28 - 2.3694 \times 298) \times 0.718 = (11.847 - 2.3694) \times 1.005 \times 298 - W_{cv}$$

$$W_{cv} = -1479.953 \text{ kW}$$

Negative sign indicates work supplied to the control volume.

Q.1 (b) Solution:

$$h_f = \frac{fLu^2}{2gD}, u = \frac{4Q}{\pi D^2}, u^2 = \frac{16Q^2}{\pi^2 D^2}, h_f = \frac{8fLQ^2}{\pi^2 gD^5}$$

Considering the two sections, (total drop)

$$60 = \frac{8 \times 0.024 \times 1500 \times Q_1^2}{\pi^2 g \times 0.2^5} + \frac{8 \times 0.024 \times 2500 \times Q_2^2}{\pi^2 g \times 0.2^5}$$

$$= 9295.5 Q_1^2 + 15492.54 Q_2^2$$

but

$$Q_2^2 = (Q_1 - 0.05)^2, \text{ Substituting and simplifying}$$

$$60 = 9295.52 Q_1^2 + 1549.525(Q_1^2 + 0.05^2 - 2Q_1 \times 0.05)$$

$$24788.05 Q_1^2 - 1549.25 Q_1 - 21.268 = 0$$

$$Q_1 = 0.074082 \text{ m}^3/\text{s}$$

$$Q_2 = Q_1 - 0.05$$

$$Q_2 = 0.024082 \text{ m}^3/\text{s}$$

The other solution is negative,

Check: For the first section

$$h_f = \frac{8 \times 0.024 \times 1500 \times 0.074082^2}{\pi^2 \times 9.81 \times 0.2^5} = 51.015 \text{ m}$$

For the second section

$$h_f = \frac{8 \times 0.024 \times 2500 \times 0.024082^2}{\pi^2 \times 9.81 \times 0.2^5} = 8.985 \text{ m,}$$

$$\text{Total head} = 60 \text{ m}$$

Q.1 (c) Solution:

Given: Wire diameter, ' d ' = 2 mm = 0.002 m, Radius, R = 0.001 m, h = 5000 W/m²K,

$$T_\infty = 100^\circ\text{C}$$

Centre temperature, $T_c = 150^\circ\text{C}$, Specific resistance, $\rho_s = 1.67 \mu\Omega \text{ cm}$

or

$$\rho_s = 1.67 \times 10^{-8} \Omega\text{-m}$$

$$k = 386 \text{ W/m-K}$$

As centre temperature of wire, $T_c = T_\infty + \frac{\dot{q}R}{2h} + \frac{\dot{q}R^2}{4k}$

Where,

\dot{q} = Heat generation rate per unit volume

$$150 = 100 + \frac{\dot{q} \times 0.001}{2 \times 5000} + \frac{\dot{q} \times (0.001)^2}{4 \times 386}$$

$$\dot{q} = 4.968 \times 10^8 \text{ W/m}^3$$

So heat generated in a wire of length L

$$Q = \dot{q} \times \pi R^2 \times L$$

If the resistance of wire be ' R_s '

Then,

$$Q = I^2 R_s = \dot{q} \times \pi R^2 L$$

or

$$I^2 \times \left(\rho_s \times \frac{L}{A} \right) = \dot{q} \times \pi R^2 L \quad \left[\text{As } R_s = \rho_s \frac{L}{A} \right]$$

$$I^2 \times 1.67 \times 10^{-8} \times \frac{L}{\frac{\pi}{4} \times (0.002)^2} = 4.968 \times 10^8 \times \pi \times 0.001^2 \times L$$

$$I^2 = 293606.642$$

$$I = 541.854 \text{ amp}$$

So current of 541.854 amp is being passing through the wire.

Q.1 (d) Solution:

(i)

Advantages:

1. It uses air as refrigerant, so there is no leakage problem.
2. Vortex tube is simple in design and it avoids control systems.
3. There are no moving parts in vortex tube.
4. It is light in weight and requires less space.
5. Initial cost is low and its working expenses are also less, where compressed air is readily available.
6. Maintenance is simple and no skilled labours are required.

Disadvantages:

1. It has low coefficient of performance.
2. Limited capacity and only small portion of the compressed air appearing as the cold air limits its wide use in practice.

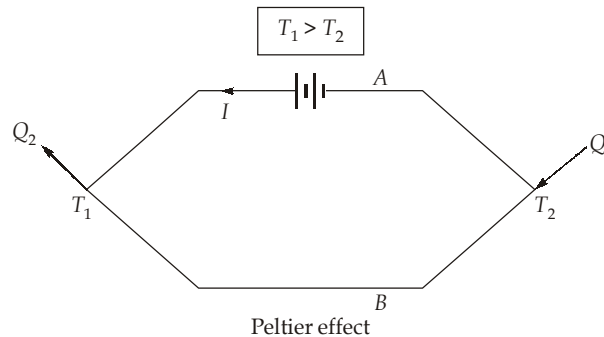
Applications:

1. Vortex tubes are extremely small and as it produces hot as well as cold air. It may be of use in industries where both are simultaneously required.
2. Temperature as low as -50°C can be obtained without any difficulty, so it is very much useful in industries for spot cooling of electronic components.
3. It is commonly used for body cooling of the worker in mines.

(ii)

Working principle: Thermo electric refrigeration works on the principle of "Peltier effect". When a battery is added in between the two conductors A and B whose junctions are initially at same temperature, and a current is made to flow through the circuit, the junction temperature will change, one junction becomes hot (T_1) and the other becomes cold (T_2). This effect is known as peltier effect. Refrigeration effect is obtained at the

cold junction and heat is rejected to the surroundings at the hot junction. This is the basis for thermoelectric refrigeration system.



The position of hot and cold junctions can be reversed by reversing the direction of current flow. The heat transfer rate at each junction is given by:

$$Q = \phi I$$

Where ϕ is the peltier coefficient in Volts and I is the current in Amperes.

Q.1 (e) Solution:

Supercharging: The power output of an engine depends upon the amount of air inducted per unit time, the degree of utilization of this air and the thermal efficiency of the engine. The amount of air inducted per unit time can be increased by increasing the density of air at intake. The method of increasing air density or pressure boosting is called supercharging.

Effects on performance:

- (i) Higher power output
- (ii) Greater induction of charge mass
- (iii) Better atomization of fuel
- (iv) Better mixing of fuel and air
- (v) Better scavenging of combustion products
- (vi) Better torque characteristics over the whole speed range
- (vii) Quicker acceleration of vehicle
- (viii) More complete and smoother combustion
- (ix) Inferior or poor ignition quality fuel can be used.

Q.2 (a) Solution:

- (i) Let a system be taken from an initial equilibrium state i to a final equilibrium state F by following the reversible path R_1 . The system is brought back from F to i by following another reversible path R_2 . Then the two paths R_1 and R_2 together constitute a reversible cycle.

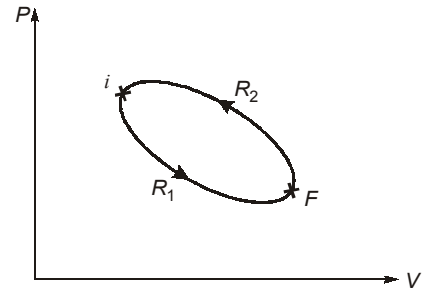
From Clausius theorem, $\oint_{R_1, R_2} \frac{\delta Q}{T} = 0$

As,
$$R_1 \int_i^F \frac{\delta Q}{T} + R_2 \int_F^i \frac{\delta Q}{T} = 0$$

or
$$R_1 \int_i^F \frac{\delta Q}{T} = - R_2 \int_F^i \frac{\delta Q}{T}$$

Since R_2 is reversible path

$$R_1 \int_i^F \frac{\delta Q}{T} = R_2 \int_i^F \frac{\delta Q}{T}$$



Since R_1 and R_2 represent any two reversible paths, $\left(\int_R \frac{\delta Q}{T} \right)$ is independent of the reversible path connecting i and F . Therefore, there exists a property of a system whose value at the final state F minus its value at the initial state i is equal to $\left(\int_i^F \frac{\delta Q}{T} \right)$.

This property is called entropy and is denoted by S . If S_i is the entropy at initial state i ,

and S_F is the entropy at the final state F . Then,

$$R \int_i^F \frac{\delta Q}{T} = S_F - S_i$$

When the two equilibrium states are infinitesimally near,

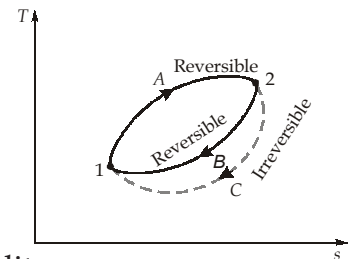
$$\frac{\delta Q_R}{T} = dS$$

or
$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_R$$

- (ii) Consider a cycle, where A and B are reversible processes and C is an irreversible process. For the reversible cycle consisting of A and B .

$$R \int \frac{\delta Q}{T} = A \int_1^2 \frac{\delta Q}{T} + B \int_2^1 \frac{\delta Q}{T} = 0$$

or
$$A \int_1^2 \frac{\delta Q}{T} = - B \int_2^1 \frac{\delta Q}{T} = B \int_1^2 \frac{\delta Q}{T} \dots (i)$$



For irreversible cycle consisting of A and C , by the inequality of clausius,

$$\oint \frac{\delta Q}{T} = \int_A^1 \frac{\delta Q}{T} + \int_C^2 \frac{\delta Q}{T} < 0 \quad \dots \text{(ii)}$$

From equations (i) and (ii)

$$-\int_B^1 \frac{\delta Q}{T} + \int_C^2 \frac{\delta Q}{T} < 0$$

$$\int_B^1 \frac{\delta Q}{T} > \int_C^2 \frac{\delta Q}{T}$$

Since the path B is reversible,

$$\int_B^1 \frac{\delta Q}{T} = \int_B^1 dS \quad \dots \text{(iii)}$$

And, $\int_B^1 dS = \int_C^2 dS$ [As entropy is a property] $\dots \text{(iv)}$

From equations (iii) and (iv)

$$\int_C^2 dS > \int_C^2 \frac{\delta Q}{T}$$

Thus for any irreversible process,

$$dS > \frac{\delta Q}{T}$$

For irreversible adiabatic process,

$$\delta Q = 0 \quad \text{and} \quad dS > \frac{\delta Q}{T}$$

$$dS > 0$$

\therefore Change in entropy is always positive. And for reversible adiabatic process,

$$dS = \left(\frac{\delta Q}{T} \right)_R$$

$$\delta Q = 0$$

$$dS = 0$$

\therefore Entropy is constant or reversible adiabatic process is isentropic.

Q.2 (b) Solution:

$$Q = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_m \left(\frac{\rho_m}{\rho} - 1 \right)}$$

$$A_1 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

$$\frac{A_1}{A_2} = 5$$

$$\therefore A_2 = \frac{0.0707}{5} = 0.0141 \text{ m}^2, \text{ Substituting,}$$

$$Q = \frac{0.96 \times 0.0141 \times 0.0707}{\sqrt{0.0707^2 - 0.0141^2}} \sqrt{2 \times 9.81 \times 0.04 \times \left(\frac{13.6}{0.8} - 1 \right)}$$

$$= 0.0486 \text{ m}^3/\text{s}$$

Point A and B level at A as datum

$$P_A + \rho g y + \rho g(0.04) = P_B + \rho g x + \rho g y + \rho_m g(0.04)$$

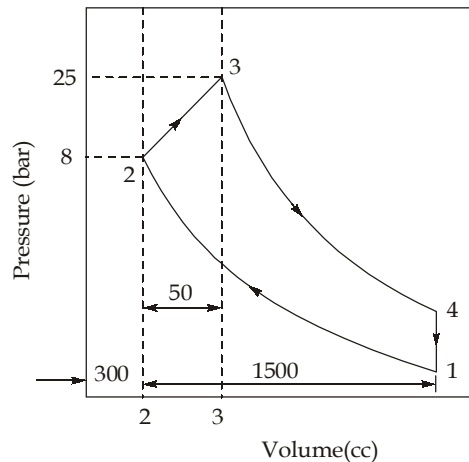
$$P_A - P_B = \rho g x + 0.04 \times g \times (\rho_m - \rho)$$

$$= \rho g(1 \times \sin 40^\circ) + 0.04 \times 9.81 \times (13600 - 800)$$

$$= 800 \times 9.81(1 \times \sin 40^\circ) + 0.04 \times 9.81 \times (13600 - 800)$$

$$= 10067.32 \text{ N/m}^2 \text{ or } 10.07 \text{ kN/m}^2$$

Q.2 (c) Solution:



$$V_1 - V_2 = 0.0015 \times 10^6 = 1500 \text{ cc}$$

$$V_2 = \frac{1500}{6-1} = 300 \text{ cc}$$

$$V_3 = \frac{1500}{30} + 300 = 350 \text{ cc}$$

$$T_2 = 350 + 273 = 623 \text{ K}$$

$$T_3 = T_2 \frac{p_3 V_3}{p_2 V_2} = 623 \times \frac{25}{8} \times \frac{350}{300} = 2271 \text{ K}$$

In order to estimate that heat added to the mixture during the process $2 \rightarrow 3$, it is required to calculate the work done and the increase in internal energy between 2 and 3.

$$W_{2-3} = \text{area under } 2-3 \text{ on } p\text{-}V \text{ curve.}$$

$$W_{2-3} = \left(\frac{25+8}{2} \right) \times 10^5 \times (350 - 300) \times 10^{-6} = 82.5 \text{ J}$$

$$\begin{aligned} \text{Mixture mass, } m &= \frac{pV}{RT} \\ &= \frac{8 \times 10^5 \times 300 \times 10^{-6}}{287 \times 623} = 1.342 \times 10^{-3} \text{ kg} \end{aligned}$$

$$\begin{aligned} \Delta E &= E_3 - E_2 \\ &= 1.342 \times 10^{-3} \times (1 - 0.287) \times (2271 - 623) = 1.577 \text{ kJ} \end{aligned}$$

$$Q = 0.0825 + 1.577 = 1.6595 \text{ kJ}$$

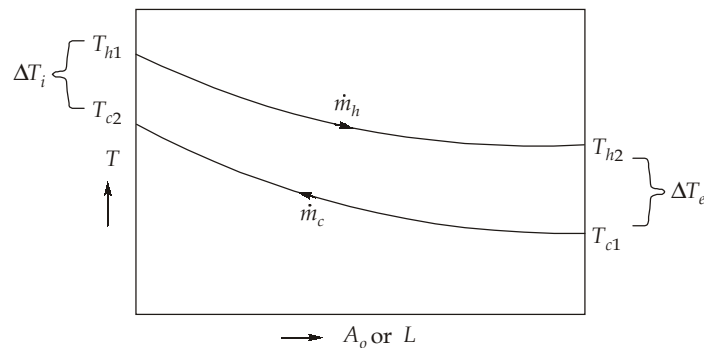
This is the quantity of heat actually given to the mixture in one cycle. But the heat liberated in one cycle must have been

$$\frac{1}{17} \times 1.342 \times 10^{-3} \times 42000 = 3.3155 \text{ kJ}$$

$$\text{heat lost during explosion} = 3.3155 - 1.6595 = 1.656 \text{ kJ}$$

$$\text{heat lost in kJ/kg} = \frac{1.656}{1.342 \times 10^{-3}} = 1234 \text{ kJ/kg of charge}$$

Q.3 (a) Solution:



Given:

$$T_{h1} = 110^\circ\text{C}, T_{h2} = 60^\circ\text{C}, T_{c1} = 40^\circ\text{C}, T_{c2} = 70^\circ\text{C}$$

$$\Delta T_i = 110 - 70 = 40^\circ\text{C}$$

$$\Delta T_c = 60 - 40 = 20^\circ\text{C}$$

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln \Delta T_i / \Delta T_e} = \frac{40 - 20}{\ln 40 / 20} = \frac{20}{\ln 2} = 28.85^\circ\text{C}$$

$$Q = \dot{m}_c c_c (T_{c2} - T_{c1}) = \dot{m}_h c_h (T_{h1} - T_{h2}) = U_o A_o \Delta T_m$$

$$A_o = \frac{1.4 \times 4.187 \times 30}{0.35 \times 28.85} = 17.42 \text{ m}^2$$

$$1.4 \times 4.187 \times 30 = \dot{m}_h \times 1.9 \times 50$$

$$\dot{m}_h = 1.851 \text{ kg/s}$$

$$\dot{m}_h c_h = 1.851 \times 1.9 = 3.52 \text{ kW/K}$$

$$\dot{m}_c c_c = 0.7 \times 4.187 = 2.93 \text{ kW/K} \quad (\text{with water flow halved})$$

$$\dot{m}_c c_c = C_{\min} \text{ and } \dot{m}_h c_h = C_{\max}$$

$$C = \frac{C_{\min}}{C_{\max}} = \frac{2.93}{3.52} = 0.832$$

$$NTU = \frac{U_o A_o}{C_{\min}} = \frac{0.35 \times 17.42}{2.93} = 2.08$$

$$\begin{aligned} \exp[-NTU(1-C)] &= \exp[-2.08(1-0.832)] \\ &= e^{-0.34944} = 0.705 \end{aligned}$$

The effectiveness of the heat exchanger is

$$\begin{aligned} \epsilon &= \frac{1 - \exp[-NTU(1-C)]}{1 - C \exp[-NTU(1-C)]} = \frac{1 - 0.705}{1 - 0.832 \times 0.705} \\ &= \frac{0.295}{0.413} = 0.714 \end{aligned}$$

Now,

$$\epsilon = \frac{(\Delta T)_1}{T_{h1} - T_{c1}} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = \frac{T_{c2} - 40}{110 - 40} = 0.714$$

$$T_{c2} = \text{exit water temperature} = 89.98^\circ\text{C}$$

$$\dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$$

$$1.851 \times 1.9(110 - T_{h2}) = 0.7 \times 4.187(90 - 40)$$

$$T_{h2} = 68.35^\circ\text{C} = \text{exit temperature of oil}$$

$$Q = 1.851 \times 1.9(110 - 68.35) = 146.48 \text{ kW}$$

Q.3 (b) Solution:

Initial exergy of the products,

$$\begin{aligned} \Psi_1 &= (h_1 - h_0) - T_0(s_1 - s_0) \\ &= c_{pg}(T_{g1} - T_0) - T_0 c_{pg} \ln\left(\frac{T_{g1}}{T_0}\right) \\ &= 1.1(563 - 300) - 300 \times 1.1 \ln\left(\frac{563}{300}\right) \\ \Psi_1 &= 81.566 \text{ kJ/kg} \end{aligned}$$

Final exergy of the products,

$$\begin{aligned}\Psi_2 &= (h_2 - h_0) - T_0(s_2 - s_0) \\ &= 1.1(463 - 300) - 300 \times 1.1 \ln\left(\frac{463}{300}\right) \\ \Psi_2 &= 36.098 \text{ kJ/kg}\end{aligned}$$

2. Decrease in exergy of the products

$$\begin{aligned}&= \Psi_1 - \Psi_2 \\ &= 81.566 - 36.098 = 45.468 \text{ kJ/kg}\end{aligned}$$

By making an energy balance for the air preheater,

$$\begin{aligned}\dot{m}_g c_{pg} (T_{g1} - T_{g2}) &= \dot{m}_a c_{pa} (T_{a2} - T_{a1}) \\ 10.5 \times 1.1(290 - 190) &= 9.5 \times 1.005(T_{a2} - 45) \\ T_{a2} &= 165.97^\circ\text{C} = (165.97 + 273) \\ T_{a2} &= 438.97 \text{ K}\end{aligned}$$

Now, increase in exergy of air,

$$\begin{aligned}&= \Psi_1 - \Psi_2 \\ &= (h_2 - h_1) - T_0(s_2 - s_1) \\ &= c_{pa} (T_{a2} - T_{a1}) - T_0 c_{pa} \ln\left(\frac{T_{a2}}{T_{a1}}\right) \\ &= 1.005(165.97 - 45) - 300 \times 1.005 \ln\left(\frac{438.97}{318}\right) \\ &= 24.377 \text{ kJ/kg}\end{aligned}$$

Irreversibility of the process,

$$\begin{aligned}I &= 10.5 \times 45.468 - 9.5 \times 24.377 \\ I &= 245.8325 \text{ kW}\end{aligned}$$

3. Let us assume that heat transfer from the products to air occurred through heat engines reversibly.

For reversible heat transfer,

$$\begin{aligned}\Delta \dot{S}_{\text{univ}} &= 0 \\ \Delta \dot{S}_{\text{sys}} + \Delta \dot{S}_{\text{surr}} &= 0 \\ \Delta \dot{S}_{\text{gas}} + \Delta \dot{S}_{\text{air}} &= 0 \\ \Delta \dot{S}_{\text{gas}} &= -\Delta \dot{S}_{\text{air}}\end{aligned}$$

$$\dot{m}_g c_{pg} \ln \frac{T_{g2}}{T_{g1}} = -\dot{m}_a c_{pa} \ln \frac{T_{a2}}{T_{a1}}$$

$$10.5 \times 1.1 \ln \left(\frac{463}{563} \right) = -9.5 \times 1.005 \ln \left(\frac{T_{a2}}{318} \right)$$

$$T_{a2} = 402.87 \text{ K}$$

Rate of heat supply from the gas to the working fluid in the heat engine,

$$\begin{aligned} \dot{Q}_1 &= \dot{m}_g c_{pg} (T_{g1} - T_{g2}) \\ &= 10.5 \times 1.1 (563 - 463) \\ &= 1155 \text{ kW} \end{aligned}$$

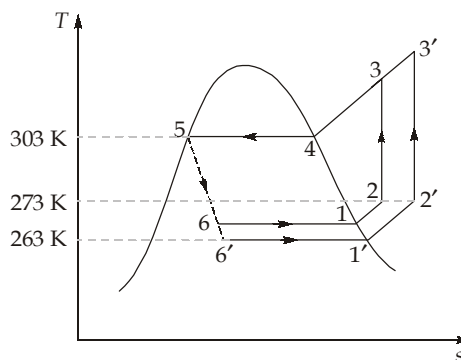
Rate of heat rejection from the working fluid in the heat engine to the air,

$$\begin{aligned} \dot{Q}_2 &= \dot{m}_a c_{pa} (T_{a2} - T_{a1}) \\ &= 9.5 \times 1.005 (402.87 - 318) \\ &= 810.296 \text{ kW} \end{aligned}$$

Total power developed by the heat engine,

$$\begin{aligned} \dot{W} &= \dot{Q}_1 - \dot{Q}_2 = 1155 - 810.296 \\ \dot{W} &= 344.704 \text{ kW} \end{aligned}$$

Q.3 (c) Solution:



$$\begin{aligned} 1. \quad h_2 &= h_{g@-5^\circ\text{C}} + c_{pv1}(T_2 - T_1) \\ h_2 &= 403.51 + 0.7642(0 + 5) \\ &= 407.33 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Now,} \quad h_3 &= h_{g@30^\circ\text{C}} + c_{pv4}(T_3 - T_4) \\ &= 414.18 + 0.9185(T_3 - T_4) \end{aligned} \quad \dots(i)$$

For, $T_{3'}$ (Process 2 - 3 isentropic)

$$s_2 = s_3$$

$$s_{g@-5^\circ} + c_{pv1} \ln\left(\frac{T_2}{T_1}\right) = s_{g@30^\circ} + c_{pv4} \ln\left(\frac{T_3}{T_4}\right)$$

$$1.7592 + 0.7642 \ln\left(\frac{0+273}{-5+273}\right) = 1.7120 + 0.9185 \ln\left(\frac{T_3}{303}\right)$$

$$T_3 = 323.92 \text{ K}$$

Now, from eq. (i),

$$h_3 = 414.18 + 0.9185(323.92 - 303)$$

$$h_3 = 433.39 \text{ kJ/kg}$$

(i) Mass flow rate, $\dot{m} = \frac{10 \times 3.5}{h_2 - h_6} = \frac{35}{407.33 - 236.31}$

$$\dot{m} = 0.2046 \text{ kg/s} \quad \dots(\text{ii})$$

(ii) Power, $P = \dot{m}(h_3 - h_2)$
 $= 0.2046(433.39 - 407.33)$

$$P = 5.3318 \text{ kW} \quad \dots(\text{iii})$$

(iii) $\text{COP} = \frac{RE}{P} = \frac{10 \times 3.5}{5.3318}$

$$\text{COP} = 6.564 \quad \dots(\text{iv})$$

2.

Mass flow rate, $\eta_v = 1 + c - c\left(\frac{P_3}{P_1}\right)^{1/n}$

$$0.88 = 1 + c \left[1 - \left(\frac{11.919}{4.215} \right)^{1/0.96} \right]$$

$$c = 0.0614$$

Now for condition (2) clearance ratio will be same.

$$\eta_v = 1 + c \left[1 - \left(\frac{P_3'}{P_1'} \right)^{1/n} \right]$$

$$\eta_v = 1 + 0.0614 \left[1 - \left(\frac{11.919}{3.543} \right)^{1/0.96} \right]$$

$$\eta_v = 0.8441$$

$$\eta'_v = \frac{\dot{m}' \times v_2'}{\dot{V}_s} \quad \dots(\text{vi})$$

$$\eta_v = \frac{\dot{m} \times v_2}{\dot{V}_s} \quad \dots(\text{vii})$$

Divide eq. (vi) and (vii),

$$\frac{\eta_v'}{\eta_v} = \frac{\dot{m}' \times v_2'}{\dot{m} \times v_2}$$

$$\frac{0.8441}{0.88} = \frac{\dot{m}' \times \left(\frac{T_2'}{T_1'} \times v_1' \right)}{\dot{m} \times \left(\frac{T_2}{T_1} \times v_1 \right)} \quad (v \propto T, \text{ for constant pressure process})$$

$$\frac{0.8441}{0.88} = \frac{\dot{m}' \times \left(\frac{273}{263} \times 0.0653 \right)}{0.2046 \times \left(\frac{273}{268} \times 0.0554 \right)}$$

Mass flow rate, $\dot{m}' = 0.1634 \text{ kg/s}$

Now, change in mass flow rate = $\dot{m}' - \dot{m}$
 $= 0.1634 - 0.2046 = -0.0412 \text{ kg/s}$

-ve sign indicates decrease in mass flow rate.

Q.4 (a) Solution:

$$\text{Swept volume, } V_s = \frac{\pi}{4} D^2 L$$

$$= \frac{\pi}{4} \times 0.32^2 \times 0.38 = 30.561 \times 10^{-3} \text{ m}^3$$

at atmospheric conditions, $\eta_{\text{vol}} = 0.71$

Actual volume of charge enters the cylinder per cycle,

$$V_{\text{act}} = V_s \times \eta_{\text{vol}} = 30.56 \times 10^{-3} \times 0.71 = 21.698 \times 10^{-3} \text{ m}^3$$

Calorific value of fuel is 18700 kJ/m^3 at 15°C and 1.033 bar .

So the actual volume of charge at 15°C and 1.033 bar ,

$$\text{For } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \frac{288 \times 1.036 \times 21.698 \times 10^{-3}}{300 \times 1.033}$$

$$V_2 = 20.8905 \times 10^{-3} \text{ m}^3/\text{per cycle}$$

$20.8905 \times 10^{-3} \text{ m}^3$ contains the air and gas in proportion of 7.5 : 1.

$$\text{The gas taken in, per working stroke} = \frac{20.8905 \times 10^{-3}}{8.5} = 2.4577 \times 10^{-3} \text{ m}^3$$

From Brake thermal efficiency,

$$\eta_{\text{bth}} = \frac{B.P. \times 60}{(V_f) \times CV \times (N_w)}$$

$$N_w = \frac{B.P. \times 60}{\eta_{\text{bth}} \times (V_f) \times CV}$$

$$\text{Number of working cycles, } N_w = \frac{18.5 \times 60}{0.285 \times 2.4577 \times 10^{-3} \times 18700}$$

$$N_w = 84.74 \simeq 85 \text{ /min}$$

$$\text{Missed cycle} = \frac{N}{2} - N_w = \frac{280}{2} - 85 = 55 \text{ cycles}$$

Air consumption:

Air consumption per minute = Air during working (firing) cycle + air during non-firing cycles.

$$\begin{aligned} &= 20.8905 \times 10^{-3} \times \frac{7.5}{8.5} \times 85 + 20.8905 \times 10^{-3} \times 55 \\ &= 2.715 \text{ m}^3/\text{min} \end{aligned}$$

Mass of this air is given by,

$$\dot{m}_a = \frac{PV}{RT} = \frac{1.033 \times 100 \times 2.715}{0.287 \times 288} = 3.393 \text{ kg/min}$$

Gas consumption

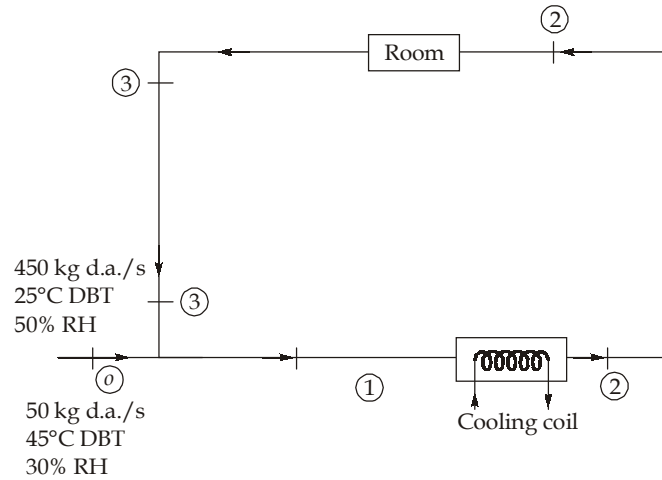
Gas consumption per minute = Gas used during firing cycles per minute

$$= 2.4577 \times 10^{-3} \times 85 = 0.2089 \text{ m}^3/\text{min at 1.033 bar and 15}^\circ\text{C}$$

Mass of this gas is given by,

$$\dot{m}_g = \frac{PV}{RT} = \frac{1.033 \times 100 \times 0.2089}{0.287 \times 288} = 0.26107 \text{ kg/min}$$

Q.4 (b) Solution:



1. At point o ,

$$\omega_o = 0.622 \times \frac{P_{v,o}}{P - P_{v,o}} \quad \dots(i)$$

Vapour pressure at o ,

$$RH = \frac{P_{v,o}}{P_{vs,o}}$$

$$0.3 = \frac{P_{v,o}}{0.09584}$$

$$P_{v,o} = 0.028752 \text{ bar}$$

From equation (i),

$$\omega_o = 0.622 \times \frac{0.028752}{1.01325 - 0.028752}$$

$$\omega_o = 0.01816 \text{ kg/kg d.a.}$$

At point 3,

$$RH = \frac{P_{v,3}}{P_{vs,3}}$$

$$0.5 = \frac{P_{v,3}}{0.03166}$$

$$P_{v,3} = 0.01583 \text{ bar}$$

$$\text{Specific humidity, } \omega_3 = 0.622 \times \frac{P_{v,3}}{P - P_{v,3}}$$

$$= 0.622 \times \frac{0.01583}{1.01325 - 0.01583} = 0.009872 \text{ kg/kg d.a.}$$

For mixing,

$$\dot{m}_{v,0} + \dot{m}_{v,3} = \dot{m}_{v,1}$$

$$w_0 \times \dot{m}_{a,3} + \omega_3 \times \dot{m}_{a,3} = \omega_1 \times \dot{m}_{a,1}$$

$$0.01816 \times 50 + 0.009872 \times 450 = \omega_1 \times 500$$

$$\omega_1 = 0.0107 \text{ kg/kg d.a.}$$

For temperature after mixing,

$$\dot{m}_{a,0} \times h_0 + \dot{m}_{a,3} \times h_3 = \dot{m}_{a,1} \times h_1$$

$$50 \times 45 + 450 \times 25 = 500 \times t_1$$

$$t_1 = 27^\circ\text{C}$$

Now,

$$BPF = \frac{t_2 - t_{ADP}}{t_1 - t_{ADP}}$$

$$0.15 = \frac{t_2 - 12}{27 - 12}$$

$$t_2 = 14.25^\circ\text{C}$$

After cooling coil,

$$BPF = \frac{\omega_2 - \omega_{ADP}}{\omega_1 - \omega_{ADP}}$$

$$0.15 = \frac{\omega_2 - 0.009}{0.0107 - 0.009}$$

$$\omega_2 = 0.009255 \text{ kg/kg d.a.}$$

(ii) Room sensible heat,

$$\begin{aligned} \text{RSH} &= m_{\text{d.a.}} \times c_p (\Delta T) \\ &= 500 \times 1.0216 \times (25 - 14.25) = 5491.1 \text{ kW} \end{aligned}$$

(iii) Room latent heat,

$$\begin{aligned} \text{RLH} &= 50 \text{ cmm} (\omega_3 - \omega_2) \\ &= 50 \text{ cmm} (0.009872 - 0.009255) && \dots(\text{ii}) \\ \text{RSH} &= 5491.1 = 0.0204 \text{ cmm} (25 - 14.25) && \dots(\text{iii}) \end{aligned}$$

Divide (ii) by (iii),

$$\frac{\text{RLH}}{5491.1} = \frac{50 \text{ cmm} (0.009872 - 0.009255)}{0.0204 \text{ cmm} (25 - 14.25)}$$

$$\text{RLH} = 772.459 \text{ kW}$$

Q.4 (c) Solution:

Given that,

Three thin walled infinitely long hollow cylinders, having radius

$$\begin{aligned} r_1 &= 5 \text{ cm} \\ r_2 &= 10 \text{ cm} \\ r_3 &= 15 \text{ cm} \end{aligned}$$

Temperature of innermost cylinders

$$T_1 = 1000 \text{ K}$$

Temperature of outermost cylinders

$$T_3 = 300 \text{ K}$$

Emissivity of each cylinder, $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.5$

Under steady condition,

$$Q_{12} = Q_{23}$$

As cylinders are infinitely long, hence

$$F_{12} = 1,$$

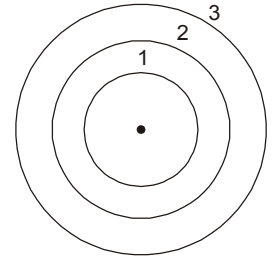
$$F_{23} = 1$$

$$Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}}$$

$$Q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)} \quad \dots(i)$$

$$Q_{23} = \frac{\sigma(T_2^4 - T_3^4)}{\frac{1 - \epsilon_2}{A_2 \epsilon_2} + \frac{1}{A_2 F_{23}} + \frac{1 - \epsilon_3}{A_3 \epsilon_3}}$$

$$Q_{23} = \frac{\sigma A_2 (T_2^4 - T_3^4)}{\frac{1}{\epsilon_2} + \frac{A_2}{A_3} \left(\frac{1}{\epsilon_3} - 1 \right)} \quad \dots(ii)$$



From equations (i) and (ii), we get

$$\frac{\sigma(T_1^4 - T_2^4)A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} = \frac{\sigma(T_2^4 - T_3^4)A_2}{\frac{1}{\epsilon_2} + \frac{A_2}{A_3}\left(\frac{1}{\epsilon_3} - 1\right)} \quad \dots(\text{iii})$$

$$A_1 = (2\pi r_1)L$$

$$A_2 = (2\pi r_2)L$$

$$A_3 = (2\pi r_3)L$$

$$\frac{A_1}{A_2} = \frac{r_1}{r_2} = \frac{5}{10} = 0.5$$

$$\frac{A_2}{A_3} = \frac{r_2}{r_3} = \frac{10}{15} = \frac{2}{3}$$

From equation (i),

$$\frac{A_1(T_1^4 - T_2^4)}{\frac{1}{0.5} + 0.5\left(\frac{1}{0.5} - 1\right)} = \frac{A_2(T_2^4 - T_3^4)}{\frac{1}{0.5} + \frac{2}{3}\left(\frac{1}{0.5} - 1\right)}$$

$$\frac{(2\pi r_1 L)(1000^4 - T_2^4)}{2.5} = \frac{(2\pi r_2 L)(T_2^4 - 300^4)}{\left(2 + \frac{2}{3}\right)} = \frac{(T_2^4 - 300^4)(2\pi r_2 L)}{\left(\frac{8}{3}\right)}$$

$$\Rightarrow 5 \times \frac{8}{3}(1000^4 - T_2^4) = 2.5(T_2^4 - 300^4) \times 10$$

$$\frac{8}{3}(1000)^4 + 5(300)^4 = \left(\frac{8}{3} + 5\right)T_2^4$$

$$T_2 = 770.86 \text{ K}$$

Heat flow per m^2 of the innermost cylinder,

$$Q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

$$= \frac{5.67 \times 10^{-8}(1000^4 - 770.86^4)}{\frac{1}{0.5} + 0.5\left(\frac{1}{0.5} - 1\right)}$$

$$Q_{12} = 1.467 \times 10^4 = 14.67 \text{ kW/m}^2$$

Section -B

Q.5 (a) Solution:

$$\text{Dry matter produced} = 4 \times 2 = 8 \text{ kg/day}$$

$$\text{Total cow dung produced} = \frac{8}{0.18} = 44.444 \text{ kg/day}$$

$$\text{Total mass of slurry} = 44.444 + 44.444 = 88.888 \text{ kg/day}$$

$$\text{Volume of slurry per day} = \frac{88.888}{1090} = 0.08155 \text{ m}^3/\text{day}$$

$$\begin{aligned} \text{Total volume of slurry} &= 0.08155 \times \text{Retention period} \\ &= 0.08155 \times 30 = 2.4465 \text{ m}^3 \end{aligned}$$

$$\text{Volume of biogas digester} = \frac{2.4465}{0.85} = 2.8782 \text{ m}^3$$

$$\text{Total volume of biogas} = 0.24 \times (2 \times 4) = 1.92 \text{ m}^3/\text{day}$$

$$\begin{aligned} \text{Thermal energy available per day} &= 1.92 \times 23 \times 0.6 \text{ MJ} \\ &= 26.496 \text{ MJ/day} \end{aligned}$$

$$= \frac{26.496 \times 10^6}{24 \times 3600} \text{ Watt}$$

$$\text{Thermal energy available, } P = 306.667 \text{ Watt}$$

Q.5 (b) Solution:

Given: Diameter of plunger, $D = 100 \text{ mm} = 0.1 \text{ m}$, Stroke length, $L = 200 \text{ mm} = 0.2 \text{ m}$

Crank radius, $r = \frac{L}{2} = 0.1 \text{ m}$, Suction head, $h_s = 4 \text{ m}$, Delivery head, $h_d = 14 \text{ m}$

Diameter of suction pipe, $d_s = 40 \text{ mm} = 0.04 \text{ m}$, Length of suction pipe, $l_s = 6 \text{ m}$

Diameter of delivery pipe, $d_d = 30 \text{ mm} = 0.03 \text{ m}$, Length of delivery pipe, $l_d = 18 \text{ m}$

$$\text{Separation pressure, } h_{\text{sep}} = \frac{7.848 \times 10^4}{1000 \times 9.81} = 8 \text{ m} \quad (\text{Below atmospheric Pressure})$$

$$\therefore h_{\text{sep}} = 10.3 - 8 = 2.3 \text{ m (abs)}$$

Case I: Speed of pump without separation during suction stroke:

Pressure head in the cylinder at the beginning of suction stroke

$$= (h_s + h_{as}) \text{ m below atmospheric}$$

$$h_{\text{sep}} = 10.3 - h_s - h_{as}$$

$$h_{as} = 10.3 - 4 - 2.3 = 4 \text{ m}$$

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r = \frac{6}{9.81} \times \left(\frac{D}{d_s} \right)^2 \omega^2 \times 0.1$$

or,

$$\omega^2 = \frac{9.81 \times 4 \times 0.04^2}{6 \times 0.1^2 \times 0.1} = 10.464$$

$$\omega = 3.2348 \text{ rad/s}$$

$$N = \frac{60 \times \omega}{2\pi} = \frac{60 \times 3.2348}{2 \times 3.14} = 30.9 \text{ rpm}$$

Case II: Speed of pump without separation during delivery stroke: During delivery stroke, the possibility of separation is only at the end of the delivery stroke. The pressure head in the cylinder at the end of delivery stroke = $H_{atm} + h_d - h_{ad}$

$$\Rightarrow h_{sep} = H_{atm} + h_d - h_{ad}$$

$$\Rightarrow h_{ad} = H_{atm} + h_d - h_{sep}$$

$$h_{ad} = 10.3 + 14 - 2.3 = 22 \text{ m}$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$$

or

$$22 = \frac{18}{9.81} \times \frac{D^2}{d_d^2} \times \omega^2 r$$

or

$$\omega^2 = \frac{22 \times 0.03^2 \times 9.81}{18 \times 0.1^2 \times 0.1} = 10.791$$

$$\omega = 3.28496 \text{ rad/s}$$

$$N = 31.3691 \text{ rpm}$$

Thus the maximum speed of the pump without separation during suction and delivery stroke is the minimum of these two speeds.

∴ Maximum speed = **30.9 rpm**

Q.5 (c) Solution:

$$\text{Nozzle efficiency, } \eta_n = \frac{\frac{1}{2}(\rho Q)V_1^2}{\rho QgH}$$

$$V_1 = \sqrt{2gH\eta_n} \quad \dots(1)$$

$$P = \eta_o(\rho QgH)$$

as

$$\eta_m = 100\%, \text{ so } P = \eta_n \eta_w(\rho QgH)$$

or,
$$P = \eta_n \eta_w \rho g H \times \frac{\pi}{4} d^2 V_1$$

or,
$$P = c_1 \times \eta_n \eta_w H d^2 \sqrt{2gH\eta_n} \quad \text{(from eq. (1))}$$

where,
$$c_1 = \rho g \times \frac{\pi}{4}$$

or,
$$P = c_2 \times \eta_w H^{3/2} d^2 \eta_n^{3/2}$$

...(2)

where,
$$c_2 = c_1 \times \sqrt{2g}$$

We know,
$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$N_s = \frac{N\sqrt{c_2 \eta_w H^{3/2} d^2 \eta_n^{3/2}}}{H^{5/4}} \quad \dots(3)$$

We know,
$$U = \phi \sqrt{2gH} \quad \text{(where, } \phi = \text{speed ratio)}$$

or
$$\frac{\pi DN}{60} = \phi \sqrt{2gH} \Rightarrow DN = C\sqrt{H}$$

$$\sqrt{H} = \frac{DN}{C} \quad \dots(4)$$

From eq. (3) and (4)

$$N_s = \frac{N \times \sqrt{C_2 \eta_w H^{3/2} d^2 \eta_n^{3/2}}}{H^{5/4}} = \frac{N \sqrt{C_2} d H^{3/4} \eta_n^{3/4} \eta_w^{1/2}}{H^{5/4}} = \frac{N \sqrt{C_2} d \eta_n^{3/4} \eta_w^{1/2}}{H^{1/2}}$$

from eq. (4),

$$N_s = \frac{k N d \eta_n^{3/4} \eta_w^{1/2}}{DN} = k \left(\frac{d}{D} \right) \eta_w^{0.5} \eta_n^{0.75}$$

Q.5 (d) Solution:

Advantages of floating drum type biogas plant:

1. It has less scum troubles because solids are constantly submerged.
2. No separate pressure equalizing device needed when fresh waste is added to the tank or digested slurry is withdrawn.
3. In it, the danger of mixing oxygen with the gas to form an explosive mixture is minimized.

4. Higher gas production per cubic meter of the digester volume is achieved.
5. Floating drum has welded braces, which help in breaking the scum (floating matter) by rotation.
6. No problem of gas leakage.
7. Constant gas pressure.

Disadvantages of floating drum type biogas plant:

1. It has higher cost, as cost is dependent on steel and cement.
2. Heat is lost through the metal gas holder, hence it troubles in colder regions and periods.
3. Gas holder requires painting once or twice a year, depending on the humidity of the location.
4. Flexible pipe joining the gas holder to the main gas pipe requires maintenance, as it is damaged by ultraviolet rays from the sun. It may be twisted also, with the rotation of the drum for mixing or scum removal.

Q.5 (e) Solution:

As compared to pressurized bubbling fluidized bed combustor (PBFB), pressurized circulating fluidized bed combustor (PCFB) offers the following advantages.

1. **Reduced space requirement:** PCFB has a higher velocity, higher heat release per unit area and fewer components inside the pressure vessel. So, the pressure vessel diameter is smaller.
2. **Less erosion and more accessibility:** PCFB has a low particle density fluidized bed with fine particles distributed throughout the furnace. In PBFB, submerged heat transfer surfaces are used. So in PCFB (i) the potential for erosion is greatly reduced with no immersed surfaces, (ii) plates for superheating are more accessible for inspection and maintenance.
3. PCFB has “more vigorous mixing” in the furnace which results in fewer feed points and results in a simpler, more reliable system.
4. PCFB “does not require a bed material handling system” to control the bed temperature.
5. The “exit gas temperature” of PCFB is higher since there is no free board where the flue gas may be cooled down. This is favourable to the gas turbine.
6. PCFB has “lower NO_x emission” due to enhanced staged combustion.
7. “Load following is more effectively accomplished by varying the fuel and air flow” and the proportion of primary and secondary air.
8. PCFB has “less bed inventory and less total metal and refractory”. This results in a shorter start-up and cool-down period and a quicker response to load changes.

9. At higher pressures, the partial pressure of CO_2 , in a bubbling bed is higher, which requires higher temperature for calcination of CaCO_3 . We have to use "dolomite instead of limestone" in PBF.

Q.6 (a) Solution:

$$Q = 545 \text{ L/s} = 0.545 \text{ m}^3/\text{s}, D_1 = 80 \text{ cm}; D_2 = 40 \text{ cm},$$

$$H = 14 \text{ cm}, \alpha_2 = 90^\circ \text{ (radial discharge)}$$

Peripheral velocity of wheel at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.8 \times 370}{60} = 15.5 \text{ m/s}$$

$$\text{Velocity of flow at the exit } V_{f2} = V_2 = 2.8 \text{ m/s}$$

$$\therefore \text{ Runner head} = \frac{V_{w1}U_1 - V_{w2}U_2}{g} \quad \{\because V_{w2} = 0, \text{ radial discharge}\}$$

$$RH = \frac{V_{w1}U_1}{g}$$

Assuming there is no draft tube and there is no frictional losses in casing, guide vane from energy equation across turbine,

$$H - \frac{V_{w1}U_1}{g} = \frac{V_2^2}{2g}$$

$$\text{So, runner head, } \frac{V_{w1}U_1}{g} = 14 - \left(\frac{2.8^2}{2 \times 9.81} \right) = 13.6 \text{ m}$$

$$\text{or, } V_{w1} = \frac{RH \times g}{U} = \frac{9.8 \times 13.6}{15.5} = 8.6075 \text{ m/s}$$

Since, $U_1 > V_{w1}$ so corresponding velocity Δ is as shown below.

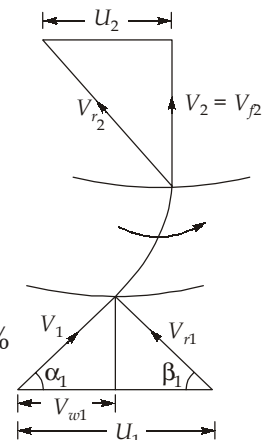
$$\text{Hydraulic efficiency, } \eta_h = \frac{13.6}{14} \times 100 = 97.14\%$$

Overall efficiency,

$$\eta_o = \frac{70 \times 10^3}{\rho Q g H} \times 100$$

$$\eta_o = \frac{70 \times 10^3}{10^3 \times 0.545 \times 9.81 \times 14} \times 100 = 93.52\%$$

Since width is constant throughout,



$$Q = \pi D_2 b_2 V_{f2} = \pi D_1 b_1 V_{f1}$$

$$V_{f1} = \frac{2.8 \times 0.4}{0.8} = 1.4 \text{ m/s}$$

From inlet velocity, Δ

$$\tan, \alpha_1 = \left(\frac{V_{f1}}{V_1} \right)$$

$$\alpha_1 = \tan^{-1} \left(\frac{1.4}{\sqrt{1.4^2 + 8.6075^2}} \right) = 9.12^\circ$$

$$\tan, \beta_1 = \left(\frac{V_{f1}}{U_1 - V_{w1}} \right) = \frac{1.4}{15.5 - 8.6075}$$

$$\beta_1 = 11.48^\circ$$

Q.6 (b) Solution:

At inlet, $\phi = 0.6$, $T_{a1} = 20^\circ\text{C}$, $P_{vs} = 0.0234 \text{ bar}$

$$\phi = \frac{P_v}{P_{vs}} \Rightarrow P_v = 0.6 \times 0.0234 = 0.01404 \text{ bar}$$

$$\omega_1 = \frac{0.622 P_v}{P_T - P_v} = \frac{0.622 \times 0.01404}{1.013 - 0.01404}$$

$$\omega_1 = 8.742 \times 10^{-3} \text{ kg/kg of dry air}$$

At outlet, $\phi = 1$, $T_{a2} = 26^\circ\text{C}$, $P_{vs} = 0.0336 \text{ bar}$

$$P_v = P_{vs} = 0.0336 \text{ bar}$$

$$\omega_2 = \frac{0.622 P_v}{P_T - P_v} = \frac{0.622 \times 0.0336}{1.013 - 0.0336} = 0.02133 \text{ kg/kg of dry air}$$

$$\text{Air pressure, } P_{a1} = 1.013 - 0.01404 = 0.99896 \text{ bar}$$

$$\text{Mass flow rate of air, } m_a = \frac{P_{a1} V_1}{R T_{a1}} = \frac{0.99896 \times 100 \times 10}{0.287 \times 293} = 11.88 \text{ kg/s}$$

Make-up water required,

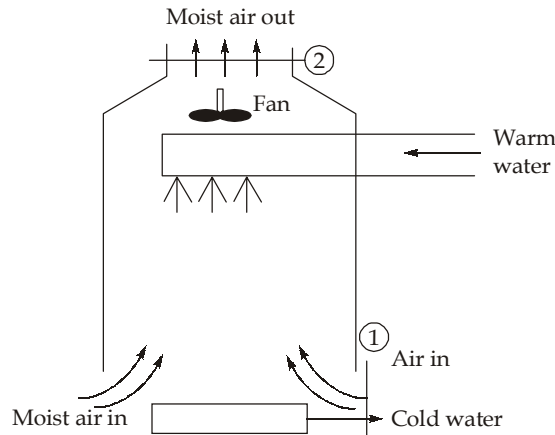
$$\Delta m_w = m_a \times (\omega_2 - \omega_1)$$

$$\Delta m_w = 11.88 \times (0.02133 - 8.742 \times 10^{-3})$$

$$\Delta m_w = 0.1495 \text{ kg/s}$$

Mass flow rate of water leaving the tower,

$$m_{w2} = 6 - 0.1495 = 5.8505 \text{ kg/s}$$



Applying SFEE across cooling tower

$$m_a h_1 + m_w h_{w1} + \dot{W}_{\text{fan}} = m_a h_2 + m_{w2} h_{w2} \quad \dots(1)$$

where h_1, h_2 are enthalpy of moist air.

$$\begin{aligned} h_1 &= 1.005 \times 20 + 8.742 \times 10^{-3}(2500 + 1.88 \times 20) \\ &= 42.28 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} h_2 &= 1.005 \times 26 + 0.02133(2500 + 1.88 \times 26) \\ &= 80.49 \text{ kJ/kg} \end{aligned}$$

From eq. (1)

$$11.88 \times 42.28 + 6 \times 4.187 \times 45 + 4.90 = 11.88 \times 80.49 + 5.8505 \times 4.187 \times T_{w2}$$

$$T_{w2} = 27.81^\circ\text{C} \text{ (Exit water temperature)}$$

Q.6 (c) Solution:

Given, $(r_p)_c = 4 : 1$, $T_1 = 288 \text{ K}$, $T_3 = 933 \text{ K}$, $T_5 = 898 \text{ K}$

$$T_{2s} = T_1 \times \left[(r_p)_c \right]^{\frac{\gamma-1}{\gamma}} = (273 + 15) \times (4)^{0.4/1.4} = 427.97 \text{ K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

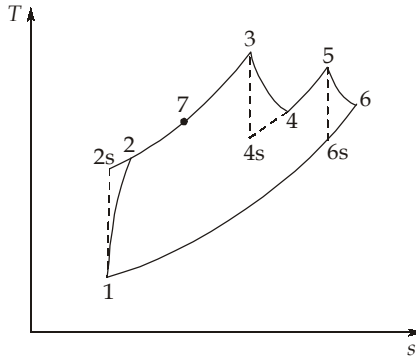
$$T_2 = T_1 + \left(\frac{427.97 - 288}{0.86} \right) = 288 + 162.76 = 450.756 \text{ K}$$

Given: Work required in compressor = $\eta_m \times$ (Work done in HP turbine)

$$c_p(T_2 - T_1) = 0.92 \times c_p(T_3 - T_4)$$

$$(450.756 - 288) = 0.92[(660 + 273) - T_4]$$

$$T_4 = 756.09 \text{ K}$$



Given:

$$(\eta_T)_{HP} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$T_{4s} = T_3 - \frac{(T_3 - T_4)}{(\eta_T)_{HP}}$$

$$T_{4s} = (660 + 273) - \frac{[(660 + 273) - 756.09]}{0.84}$$

$$T_{4s} = 722.393 \text{ K}$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^\gamma$$

$$\left(\frac{660 + 273}{722.393}\right) = \left(\frac{4}{P_4}\right)^{1.4}$$

$$\left(\frac{P_4}{4}\right)^{1.4} = \frac{722.393}{933}$$

$$P_4 = 1.634 \text{ bar}$$

In low pressure turbine,

$$P_4 = P_5 = 1.634 \text{ bar}, P_1 = P_6 = 1 \text{ bar},$$

$$T_5 = 273 + 625 = 898 \text{ K}$$

$$\frac{T_5}{T_{6s}} = 898 \times \left(\frac{1}{1.634}\right)^{1.4} = 780.45 \text{ K}$$

Now,

$$(\eta_T)_{LP} = \frac{T_5 - T_6}{T_5 - T_{6s}} = \frac{898 - T_6}{898 - 780.45}$$

$$T_6 = 898 - 0.8(898 - 780.45) = 803.96 \text{ K}$$

Given, effectiveness of heat exchanger = 75%

$$\epsilon = \frac{T_7 - T_2}{T_6 - T_2}$$

$$T_7 = 0.75 \times (803.96 - 450.756) + 450.756$$

$$T_7 = 715.66 \text{ K}$$

Hence, Total heat supplied, $Q = c_p[(T_3 - T_7) + (T_5 - T_4)]$

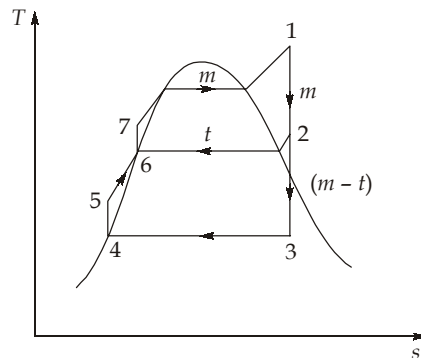
$$= 1.005[(933 - 715.66) + (898 - 756.09)] = 361.046 \text{ kJ/kg}$$

Net work output = Work output of LP turbine

$$= c_p(T_5 - T_6) = 1.005(898 - 803.96) = 94.5102 \text{ kJ/kg}$$

Overall efficiency = $\frac{W_{net}}{Q} = \frac{94.5102}{361.046} = 0.26177$ or 26.177%

Q.7 (a) Solution:



Given :

$$h_1 = 3625.3 \text{ kJ/kg}$$

$$s_1 = 6.9029 \text{ kJ/kgK}$$

$$h_2 = 3105.08 \text{ kJ/kg}$$

Now,

$$s_1 = s_3$$

$$6.9029 = 0.06493 + x \times 7.5009$$

$$x = 0.9116$$

$$h_3 = h_f + xh_{fg}$$

$$= 191.83 + (0.9116) \times 2392.8$$

$$h_3 = 2373.1065 \text{ kJ/kg}$$

$$h_4 = 191.83 \text{ kJ/kg}$$

$$h_5 = h_4 = 191.83 \text{ kJ/kg} \quad (\because \text{Pump work is neglected})$$

$$h_6 = 908.79 \text{ kJ/kg}$$

$$h_7 = h_6 = 908.79 \text{ kJ/kg} \quad (\because \text{Pump work is neglected})$$

Given :

$$\text{Total power output} = P_{\text{HPT}} + P_{\text{LPT}}$$

$$20 \times 10^3 = m(h_1 - h_2) + (m - t)(h_2 - h_3)$$

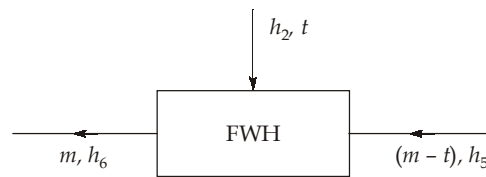
$$20,000 = m[3625.3 - 3105.08] + (m - t)[3105.08 - 2373.1065]$$

$$20,000 = (520.22)m + 731.9735(m - t)$$

$$(1252.1935)m - 731.9735t = 20000 \quad \dots(1)$$

For feed water heater,

By energy balance



$$m \times h_6 = t \times h_2 + (m - t)h_5$$

$$m \times 908.79 = t \times 3105.08 + (m - t) \times 191.83$$

$$716.96m - 2913.25t = 0$$

$$m = \frac{2913.25t}{716.96} = (4.0633t) \quad \dots(2)$$

Putting value of m in equation (1)

$$(1252.1935) \times (4.0633t) - (731.9735)t = 20,000$$

$$t = 4.5913 \text{ kg/s}$$

$$m = 4.0633 \times 4.5913 = 18.6558 \text{ kg/s}$$

Heat supplier in boiler,

$$Q_s = m(h_1 - h_7)$$

$$= 18.6558(3625.3 - 908.79)$$

$$= 50678.667 \text{ kJ/s}$$

$$Q_s = 50.678 \text{ MW}$$

Q.7 (b) Solution:

$$\frac{T_{02}}{T_{01}} = (r_p)^{\gamma-1/\gamma}$$

$$T_{02} = 293 \times (4)^{0.286} = 435.57 \text{ K}$$

$$T_{02'} - T_{01} = \frac{(T_{02} - T_{01})}{\eta_c}$$

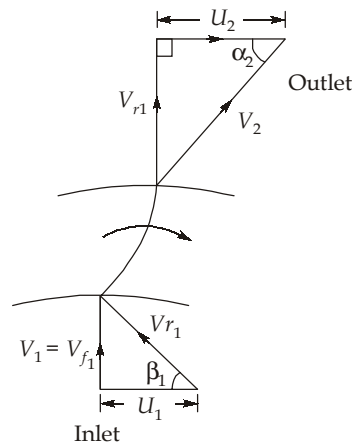
We know,
$$\dot{m} = \frac{P_1 V_1}{R T_1} = \frac{100 \times 600 / 60}{0.287 \times 293} = 11.892 \text{ kg/s}$$

Power input per kg to compressor,

$$P = c_p (T_{02'} - T_{01}) = \frac{c_p (T_{02} - T_{01})}{\eta_c}$$

$$P = 1.005 \times \frac{(435.57 - 293)}{0.82} = 174.735 \text{ kJ/kg}$$

or, Power input to compressor, $P = 11.892 \times 174.735 = 2077.95 \text{ kW}$



Also from velocity diagram,

$$P = (V_{w2} U_2 - V_{w1} U_1)$$

$$P = U_2^2$$

$$174.735 \times 10^3 = U_2^2$$

$$U_2 = 418.01 \text{ m/s}$$

$$\frac{\pi D_2 N}{60} = 418.01$$

$$\{V_{w2} = U_2 \text{ and } V_{w1} = 0\}$$

$$\Rightarrow D_2 = 0.7983 \text{ m}$$

$$\text{Also } D_1 = 0.5D_2 = 0.3991 \text{ m}$$

$$\text{Given, } Q = (\pi D_1 b_1) V_{f1}$$

But actual inlet to compressor eye is at different pressure and temperature.

$$\text{So } T_1 = T_{01} - \frac{V_1^2}{2c_p} = 293 - \frac{60^2}{2 \times 1005} = 291.2 \text{ K}$$

$$\left(\frac{P_1}{P_{01}} \right)^{\gamma-1/\gamma} = \left(\frac{T_1}{T_{01}} \right)$$

$$P_1 = \left(\frac{291.2}{293} \right)^{\frac{1.4}{0.4}} \times 1 = 0.97866 \text{ bar}$$

So, Q at $T_1 = 291.2 \text{ K}$ and $P_1 = 0.97866 \text{ bar}$ is

$$Q = \frac{600 \times 1}{0.97866} \times \frac{291.2}{293}$$

$$Q = 609.316 \text{ m}^3/\text{min}$$

$$\text{So, } \frac{609.316}{60} = \pi \times 0.3991 \times b_1 \times 60 \quad [V_f = 60 \text{ m/s}]$$

$$\text{Width at inlet, } b_1 = 0.135 \text{ m}$$

$$\text{Since, } U_1 = \frac{1}{2} U_2 = 209 \text{ m/s}$$

$$\therefore \beta_1 = \tan^{-1} \left(\frac{V_{f1}}{U_1} \right)$$

$$\text{Impeller angle, } \beta_1 = \tan^{-1} \left(\frac{V_{f2}}{U_2} \right) = 16.01^\circ$$

$$\text{Diffuser angle, } \alpha_2 = \tan^{-1} \left(\frac{V_{f2}}{U_2} \right)$$

$$\alpha_2 = \tan^{-1} \left(\frac{60}{418.01} \right) = 8.17^\circ$$

Q.7 (c) Solution:

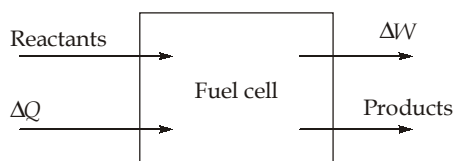
- (i) In a fuel cell, electrochemical reactions takes place, where reactants converted to products in a steady flow process.

From the first law of thermodynamics,

$$\Delta Q - \Delta W = \Delta H + \Delta KE + \Delta(PE)$$

Assuming change in kinetic and potential energy are negligible.

$$\Delta Q - \Delta W = \Delta H \quad \dots(1)$$



where, ΔQ = Heat transferred to the steady flow stream from the surrounding.

ΔW = Work done by the flow stream on the surrounding.

ΔH = Change in enthalpy of the flow stream from entrance to exit of the cell.

For ΔW to be maximum, the process must be reversible.

By second law of thermodynamics,

$$\Delta Q = T\Delta S \quad \dots(2)$$

From equation (1) and (2)

$$\Delta Q - \Delta W = \Delta H$$

Maximum work, $\Delta W_{\max} = T\Delta S - \Delta H$

$$\Delta W_{\max} = -(\Delta H - T\Delta S) \quad \dots(3)$$

Gibbs free energy

$$G = H - TS$$

$$\Delta G = \Delta H - (T\Delta S + S\Delta T)$$

As there is no change in temperature,

$$\Delta T = 0$$

$$\Delta G = (\Delta H - T\Delta S) \quad \dots(4)$$

From equation (3) and (4),

$$\Delta G = -\Delta W_{\max}$$

$$\Delta W_{\max} = -\Delta G \quad \dots(5)$$

The energy conversion efficiency of a fuel cell

$$\eta = \frac{\Delta W}{-\Delta H}$$

For maximum efficiency,

$$\eta_{\max} = \frac{\Delta W_{\max}}{-\Delta H} = \frac{-\Delta G}{-\Delta H} = \frac{\Delta G}{\Delta H}$$

(ii) Given, change in Gibbs free energy,

$$\Delta G = -39.45 \text{ kcal/mol}$$

$$= -39.45 \times 4.18$$

$$\Delta G = -164.901 \text{ kJ/mol}$$

Change in enthalpy,

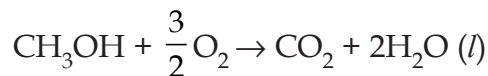
$$\Delta H = -56.80 \text{ kcal/mol}$$

$$= -56.8 \times 4.18$$

$$\Delta H = -237.424 \text{ kJ/mol}$$

The electrical work output per mole of methanol consumed,

$$\Delta W_{\max} = -\Delta G = 164.901 \text{ kJ}$$



\therefore 164.901 kJ of electrical work is produced from 1 mole (i.e., 32 gram) of methanol and 1.5 mole (i.e., $1.5 \times 32 = 48$ gram) of oxygen.

\therefore 164.901 kJ electrical work is produced by = 32 gram of methanol

1 kJ electrical work is produced by = $\left(\frac{1}{164.901}\right) \times 32$ gram of methanol

\therefore 250 kJ electrical work is produced by = $\left(\frac{250 \times 32}{164.901}\right) = 48.514$ gram of methanol.

For 250 kW electrical work output (i.e., 250 kJ/second), 48.514 gram of methanol per second is required.

For 250 kW electrical work output = 48.514 gm/second methanol
 $= 48.514 \times 3600$ gm/hour

Flow rate of methanol required = 174.6504 kg/hour

For 250 kW electrical work output oxygen required = $250 \times \left(\frac{48}{164.901} \right)$

$$= 72.770 \text{ gram/second}$$

Flow rate of oxygen required = 261.972 kg/hour

We know that,

Heat transferred, $\Delta Q = T\Delta S = \Delta H + \Delta W = \Delta H - \Delta G$

Now,
$$\Delta Q = -237.424 + 164.901$$

$$\Delta Q = -72.523 \text{ kJ/mol}$$

$$\Delta Q = -17.35 \text{ kcal/mol}$$

Here, negative sign represents heat rejection.

$$\text{Rate of heat rejection} = 17.35 \times \left(\frac{48.514}{32} \right)$$

$$= 26.304 \text{ kcal/second}$$

$$= 109.950 \text{ kJ/second}$$

Rate of heat rejection = 109.95 kW

Maximum conversion efficiency,

$$\eta_{\max} = \frac{\Delta G}{\Delta H} = \frac{(-39.45)}{(-56.80)}$$

$$\eta_{\max} = 0.6945 = 69.45\%$$

Q.8 (a) Solution:

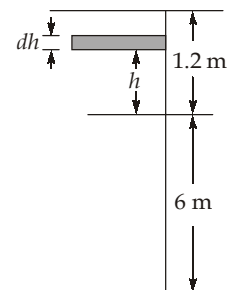
Work done by pump to raise the infinitesimal mass of water ' dm ' to height h .

$$dW = \int_0^{1.2} \frac{\rho A g h dh}{\eta_{\text{pump}}}$$

$$\text{Pump work to raise water, } W = \frac{\rho A g \times 1.2^2}{2 \times 0.5} = \rho A g \times 1.2^2$$

Now, the height of water from datum is 7.2 m.

$$W = \eta_{\text{gen}} \frac{\rho A g \times 7.2^2}{2}$$



When water is not pumped,

$$\text{Work potential, } W_1 = \rho Ag \times \frac{6^2}{2} \times \eta_{gen}$$

$$\begin{aligned} \therefore \text{Increased in work potential} &= \eta_{gen} \frac{\rho Ag 7.2^2}{2} - \eta_{gen} \frac{\rho Ag \times 6^2}{2} - \rho Ag \times 1.2^2 \\ &= \rho Ag \left[\frac{0.5}{2} \times (7.2^2 - 6^2) - 1.2^2 \right] = 2.52 \rho Ag \end{aligned}$$

$$\text{Net energy gain as \% of input energy} = \frac{2.52 \rho Ag}{1.2^2 \rho Ag} \times 100 = 175\%$$

Q.8 (b) Solution:

The operation of a biogas plant or digestion process is affected by a number of parameters, which are to be optimized to obtain best result:

1. **Temperature:** Methane forming bacteria work best in temperature range of 20 – 55°C. Digestion rate is high at higher temperature than at lower temperature. The gas yield rates doubles at about every 5°C increase in temperature. In cold climate regions, the digester has to be heated to about 35°C, for heating some part of the biogas produced can be used or solar thermal collectors can be used. The gas production decreases sharply below 20°C and almost stops at 10°C.
2. **Pressure:** A minimum pressure of 6–10 cm of water column, i.e., 1.2 bar (absolute) is considered ideal for proper functioning of the plant, and it should never be allowed to exceed 40 – 50 cm of water column. Excess pressure inhibits release of gas from slurry. It also leads to leakage in masonry through micropores. Even normal gas taps and pipe joints start leaking due to excess pressure.
Addition of oxygen also results in an explosive mixture raising safety concern. Hence the plant should be completely leakproof.
3. **Solid to moisture ratio in the biomass:** Water is essential for survival and activity of micro organisms, hydrolysis process and activity of extra cellular enzymes. It helps in following ways:
 - (a) Better mixing of various constituents of the biomass.
 - (b) Movement of bacteria
 - (c) Faster digestion rate

If water content is too high, the mean slurry temperature and hence gas production drops. If water content is too low, acids accumulate and hinder fermentation process. So the optimum solid concentration is 7 to 9%. For various input materials, the optimum ratio of solid to moisture should be adjusted by mixing extra water for best results. Cow dung contains around 80 – 82% moisture (by weight). It is usually mixed with equal amount of water to reduce the solid content to 9 – 10%.

4. **pH value:** Micro-organisms are sensitive to pH of the digested slurry. For optimum biogas production, pH can be varied between 6.8 to 7.8. At pH of 6.2, acid conditions prevail which restrain the growth of methanogenic bacteria. Control on pH should be exercised by adding alkali when it drops below 6.6.
5. **Volumetric feeding rate:** If the digester is fed with too much raw material at a time, acids will accumulate and the digestion process may stop. At higher feeding rate, the retention period will be less and undigested slurry may come out. Therefore, a uniform feeding rate should be maintained. Over feeding and under feeding reduce the biogas production for a fixed retention period. For example, for desired retention period of 40 days, an amount equal to $1/40$ of the volume of the digester should be fed daily.
6. **Carbon to Nitrogen ratio:** Methanogenic bacteria needs carbon and nitrogen for its survival. Carbon is required for energy while nitrogen for building protein cell. The consumption of carbon is 30 to 35 times faster than that of nitrogen. A favorable ratio of C : N can be taken as 30 : 1. Any deviation from this ratio lowers the biogas production. A proper balance of C : N ratio is maintained either by adding saw dust having a high C : N ratio or by poultry waste having a low C : N ratio.
7. **Seeding of Biomass with Bacteria:** To start and accelerate the fermentation process, a small amount of digested slurry, containing methane-forming bacteria is added to the freshly charged plant. This is known as seeding. If the biomass is animal manure, it already contains acid and methane fermenters and seeding is not necessary. Seeding helps to accelerate the starting of the digestion process.
8. **Stirring digester contents:** Stirring the contents of the digester is necessary to mix the bacteria rich fluid in the slurry. It provides better contact between micro-organism and the substrate and uniformly distributes the volatile solids in the slurry. Gas production improves by 15% over the full cycle.
9. **Retention time:** Retention time is the time duration for which the slurry remains in the plant or the time that is available for biodigestion. It is determined by the volume of a digester divided by the volume of slurry added per day. Retention time is optimized to achieve a 70 – 80% complete digestion. The biodegradability period depends on the temperature and the type of biomass.

10. Effect of toxic substances: High concentration of ammonia, antibiotics, pesticides, detergents, heavy metals like chromium, copper, nickel and zinc are toxic to bacteria responsible for biodigestion. Low C : N ratio in slurry leads to high concentration of NH₃. Antibiotics given to animals in food or injected to them will have its trace in the manure produced. Synthetic materials are also toxic to bacteria. Heavy metals are mostly present in industrial water. Also, the digested slurry, if allowed to remain in the digester beyond a certain time, becomes toxic to micro-organisms.

Q.8 (c) Solution:

Work done per stage,

$$w = U(V_{w2} - V_{w1})$$

$$V_{w2} = U - V_f \tan \beta_2$$

$$V_{w1} = U - V_f \tan \beta_1$$

So,

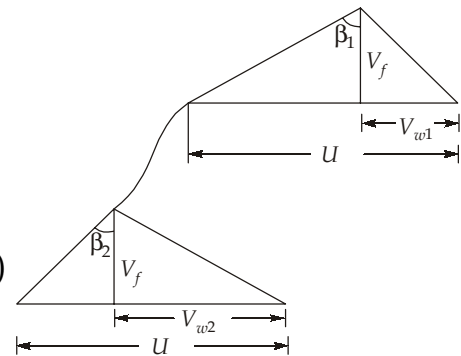
$$w = UV_f (\tan \beta_1 - \tan \beta_2)$$

$$c_p \times \Delta T_{\text{stage}} = UV_f (\tan \beta_1 - \tan \beta_2)$$

$$\Delta T_{\text{stage}} = \frac{UV_f}{c_p} (\tan \beta_1 - \tan \beta_2)$$

$$T_{01} \left((r_p)^{\gamma-1/\gamma} - 1 \right) = \frac{UV_f}{c_p} (\tan \beta_1 - \tan \beta_2)$$

$$r_p = \left[\frac{UV_f}{c_p T_{01}} (\tan \beta_1 - \tan \beta_2) + 1 \right]^{\gamma/\gamma-1}$$



From the expression of stage pressure ratio, it is clear that three factors mainly affect it in an axial flow compressor. They are

- a. tip speed
- b. axial velocity and
- c. fluid deflection in the rotor blades.

To obtain high temperature rise in a stage which is desirable to minimize number of stages for a given overall pressure ratio, the designers have to combine and compromise on the above 3 factors.

Tip speed: The centrifugal stress in the rotor blades depends on the rotational speed, the blade material and the length of the blade.

The maximum centrifugal tensile stress (σ_{ct}) which occurs at the blade root can be given by

$$(\sigma_{ct})_{\max} = \frac{\rho_b \omega^2}{a_r} \int_{r_r}^{r_t} a r dr$$

Where ρ_b = density of blade material, ω = angular velocity, a = Cross-section area of blade of any radius, r . Suffixes r and t represent root and tip of blades.

Since $\omega = 2\pi N$,

$$\text{So } (\sigma_{ct})_{\max} = \frac{\rho_b}{2} (2\pi N)^2 (r_1^2 - r_r^2) = 2\pi N^2 \rho_b A$$

Where, A = annulus area

$$\text{If } u_t \text{ is the tip peripheral of blade, then } (\sigma_{ct})_{\max} = \frac{\rho_b}{2} u_t^2 \left[1 - \left(\frac{r_r}{r_t} \right)^2 \right]$$

It is worth to note that the ratio, $\frac{r_r}{r_t}$ is known as hub-tip ratio. The above equation suggests that the maximum centrifugal stress is proportional to the square of the blade tip speed and the blade stress increases as the hub-tip ratio reduces. Hub-tip ratio also plays an important role in aerodynamics of bladings.

The blade stress obviously limits the blade speed. For tip speeds of around 350 m/s, stress problems are not usually critical in sizing the annulus. Tip speed of 450 m/s are common for turbofan engines.

Axial velocity: From the expression for stage pressure ratio it is obvious that it involves the stagnation stage temperature rise which involves axial velocity (V_f). It is desirable to have a high value of axial velocity for high stage pressure as well as to provide a high flow rate per unit frontal area, which is important for turbofan and turbojet engines. U and V_f are limited by the need to keep the relative gas velocity (relative Mach number) to an acceptable level at inlet. Generally, for subsonic compressor, relative Mach number at inlet should be slightly less than sonic. The value of V_f varies from 150 to 200 m/s.

Further, the hub-tip ratios below 0.4 are not used for aero-engines and significantly higher values are normal for industrial gas turbine compressor.

Fluid deflection in the rotor blades: The stage stagnation temperature rise is given by $\Delta T_{\text{stage}} = UV_f(\tan\beta_1 - \tan\beta_2)/c_p$ which means a higher value of fluid deflection will result in higher stage pressure ratio. Generally, β_1 is selected for shockless entry or negative incidence, so it is more or less fixed. To have a higher fluid deflection the angle β_2 has to be reduced which in turn will reduce the relative velocity C_{r2} . In other words, a high fluid deflection implies a high rate of diffusion. Generally, according

to de Haller criteria, $(C_{r2}/C_{r1}) < 0.72$. The lower value gives excessive losses. Due to its extreme simplicity, the de Haller number is still used in preliminary design work but for final design calculations criterion called the diffusion factor is preferred.

