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Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 13

Section-A

Q.1 (a) Solution:

From the given response,

We have, $$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} = \frac{1.45 - 1}{1} = 0.45$$

and $$t_p = \frac{\pi}{\omega_d} = 2 \text{ sec}$$

$$\therefore \quad \omega_d = \frac{\pi}{2} = 1.57 \text{ rad/sec} \qquad \text{...(ii)}$$

From maximum peak overshoot,

$$M_p = 0.45 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

or $$\frac{\xi}{\sqrt{1-\xi^2}} = \frac{\ln(0.45)}{-\pi} = 0.254$$

Squaring both sides, we get,

$$\xi^2 = 0.0646\ (1 - \xi^2)$$

or $$\xi^2 = \frac{0.0646}{1.0646} = 0.06$$

or $$\xi = 0.246 \qquad \text{...(iii)}$$

From equation (ii) and (iii), we get,

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{1.57}{\sqrt{1-(0.246)^2}} = 1.619 \text{ rad/sec} \quad ...(iv)$$

From given system, the closed loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

On comparing it with standard transfer function.

$$2\xi\omega_n = \frac{1}{T}$$

$$\Rightarrow \quad T = \frac{1}{2\xi\omega_n} = \frac{1}{2\times 1.619\times 0.246} = 1.255$$

and

$$\omega_n^2 = \frac{K}{T}$$

$$\Rightarrow \quad K = \omega^2 T = (1.619)^2 \times 1.255 = 3.29$$

When the same system is subjected to ramp input.

The steady state error for unit ramp input is calculated as

$$e_{ss} = \frac{A}{K_v} \qquad \text{where } A = 5 \text{ (given) and,}$$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{sK}{s(1+sT)} = K$$

$$\therefore \quad e_{ss} = \frac{A}{K} = \frac{5}{3.29} = 1.52$$

Q.1 (b) Solution:

Directivity :

- The directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.
- The average radiation intensity is equal to the total power radiated by the antenna divided by 4π.

If the direction is not specified, the direction of maximum radiation intensity is implied.

Directivity can be written as

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

$$D_{max} = D_0 = \frac{U|_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

D_0 : Maximum directivity (dimensionless)
D : directivity (dimensionless)
U : radiation intensity (W/unit solid angle)
P_{rad} : total radiated power (W)
U_{max} :maximum radiation intensity (W/unit solid angle)
U_0 : radiation intensity of isotropic source (W/unit solid angle)

$\because$ $G_d = 18 \text{ dB} = 10\log_{10} G$

or $G_D = 10^{1.8} = 63.095$

Similarly, $G_r = 11 \text{ dB}$

$\Rightarrow$ $G_R = 10^{1.1} = 12.589$

Using Friis equation,

$$P_r = G_D G_R \left(\frac{\lambda}{4\pi r}\right)^2 \times P_t$$

$$\Rightarrow \quad P_t = P_r \left(\frac{4\pi r}{\lambda}\right)^2 \times \frac{1}{G_D G_R}$$

$$= 10 \times 10^{-3} \left(\frac{4\pi 120\lambda}{\lambda}\right)^2 \times \frac{1}{12.589 \times 63.095}$$

$$= 28.628 \text{ W}$$

Q.1 (c) Solution

For second extension of source:

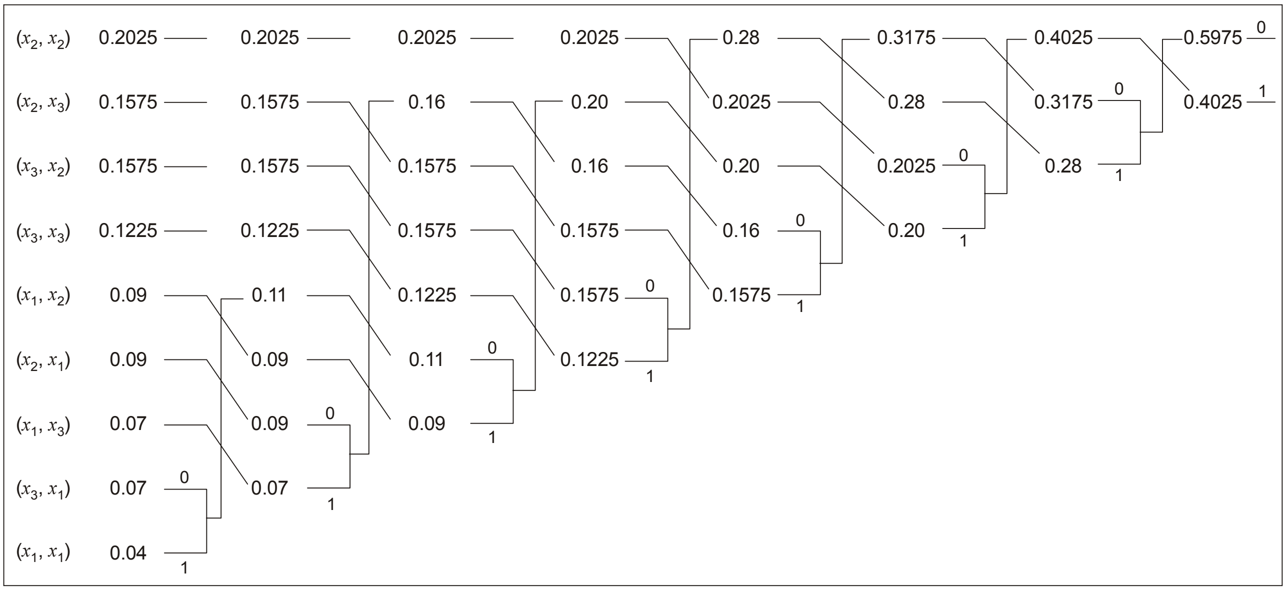
The possible symbols to be transmitted are:

$(x_1, x_1) \Rightarrow 0.2 \times 0.2 = 0.04$; $(x_1, x_2) \Rightarrow 0.2 \times 0.45 = 0.09$; $(x_1, x_3) \Rightarrow 0.2 \times 0.35 = 0.07$;

$(x_2, x_1) \Rightarrow 0.45 \times 0.2 = 0.09$; $(x_2, x_2) \Rightarrow 0.45 \times 0.45 = 0.2025$;$(x_2, x_3) \Rightarrow 0.45 \times 0.35 = 0.1575$;

$(x_3, x_1) \Rightarrow 0.35 \times 0.2 = 0.07$; $(x_3, x_2) \Rightarrow 0.35 \times 0.45 = 0.1575$; $(x_3, x_3) \Rightarrow 0.35 \times 0.35 = 0.1225$

Huffman coding:



The codewords resulted from the above tree diagram can be summarized as:

Block	(x_1, x_1)	(x_1, x_2)	(x_1, x_3)	(x_2, x_1)	(x_2, x_2)	(x_2, x_3)	(x_3, x_1)	(x_3, x_2)	(x_3, x_3)
Probability	0.04	0.09	0.07	0.09	0.2025	0.1575	0.07	0.1575	0.1225
Codeword	1101	111	0001	0000	10	001	1100	010	011
Length of codeword (l_i)	4	3	4	4	2	3	4	3	3

Coding efficiency:

$$\eta = \frac{H}{L}$$

where, $H(X)$ = entropy of source

L = average length of code

$$H(X) = -\sum_{i=1}^{3} P(x_i)\log_2(Px_i)$$

$$= -[0.2 \log_2(0.2) + 0.35 \log_2(0.35) + 0.45 \log_2(0.45)]$$

$$= 1.5129 \text{ bits/symbol}$$

For second extension:

$$H(X^2) = 2 \times H(X) = 3.026 \text{ bits/block}$$

Average length of code:

$$L = 2 \times (0.2025) + 3 \times (0.1575) + 3 \times (0.1575) + 3 \times (0.1225)$$
$$+ 3 \times (0.09) + 4(0.09) + 4 \times (0.07) + 4 \times 0.07 + 4 \times (0.04)$$
$$= 3.0675 \text{ bits/block}$$

$$\eta = \frac{H}{L} = \frac{3.026}{3.0675} = 0.9865 \text{ or } 98.65\%$$

Q.1 (d) Solution:

Given that,

$$x[n] = u[n] - u[n-4] \quad \text{...(i)}$$

So, even part of $x[n]$ is defined by,

$$x_e[n] = \frac{x[n] + x[-n]}{2} \quad \text{...(ii)}$$

$$x[-n] = u[-n] - u[-n-4]$$

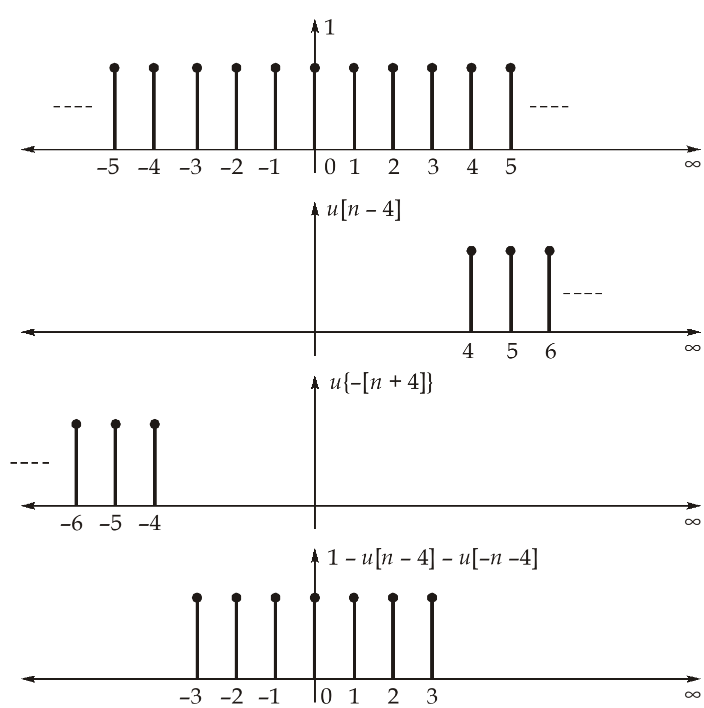
Since, $$x[-n] = u[-n] - u[-(n+4)] \quad \text{...(iii)}$$

From equation (i), (ii) and (iii) we get,

$$x_e[n] = \frac{u[n] - u[n-4] + u[-n] - u[-n-4]}{2}$$

$$u[n] + u[-n] = 1 + \delta[n]$$

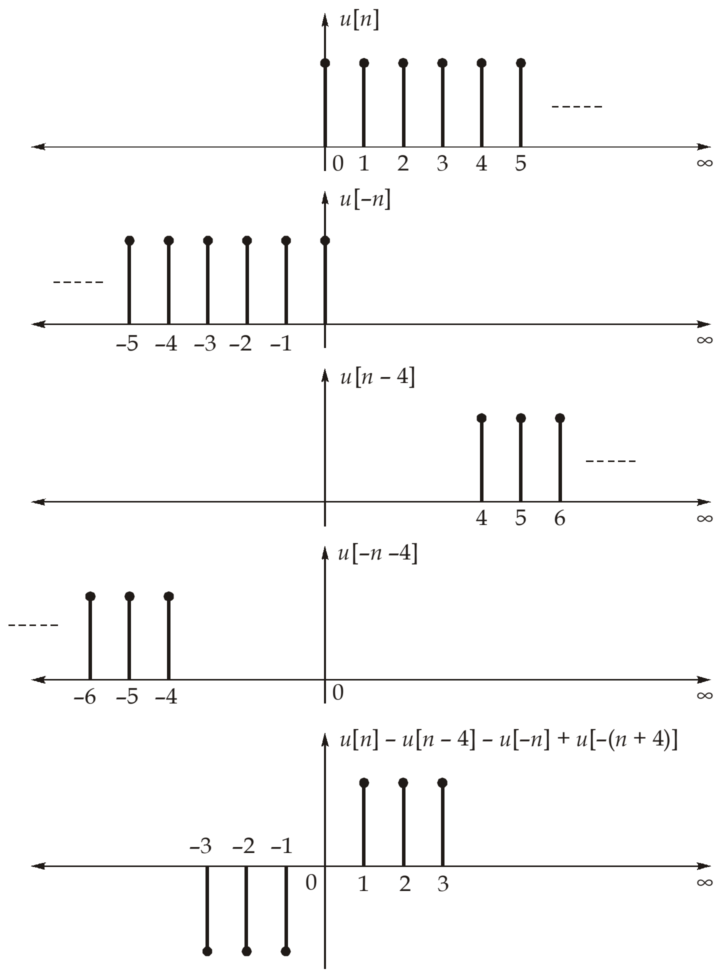
$$\therefore \quad x_e[n] = \frac{1 + \delta[n] - u[n-4] - u[-n-4]}{2}$$



$$x_e[n] = \frac{\delta[n] + u[n+3] - u[n-4]}{2}$$

Similarly, odd part of the $x[n]$ is given by

$$x_0[n] = \frac{x[n] - x[-n]}{2} = \frac{u[n] - u[n-4] - u[-n] + u[-(n+4)]}{2}$$



$$x_0[n] = \frac{u[n-1] - u[n-4] - u[-(n+1)] + u[-(n+4)]}{2}$$

or $$x_0[n] = \frac{\delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n+1] - \delta[n+2] - \delta[n+3]}{2}$$

Q.1 (e) Solution

The normalized frequency of the fiber can be given by,

$$V = \frac{2\pi a}{\lambda}(NA) = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$$

$$= \frac{\pi \times 62.5}{0.840} \times 1.48 \times \sqrt{2 \times 0.01} = 48.925$$

The total number of modes propagating is,

$$M = \frac{V^2}{2} = \frac{(48.925)^2}{2} \approx 1196$$

The percentage of optical power that propagates in the cladding is,

$$\frac{P_{\text{clad}}}{P} \times 100 \approx \frac{4}{3\sqrt{M}} \times 100\% = \frac{4}{3\sqrt{1196}} \times 100\% = 3.85\%$$

Q.2 (a) Solution:

The closed loop transfer function is given as

$$T(s) = \frac{K}{(s^2 + 10s + 100)(s + a)}$$

$$= \frac{K}{(s^3 + 10s^2 + 100s) + as^2 + 10as + 100a}$$

$$= \frac{K}{s^3 + (a + 10)s^2 + (100 + 10a)s + 100a} \quad \text{...(i)}$$

In order to find the error constant, the forward path transfer function of equivalent unity feedback system is required.

∵ The closed loop transfer function is defined as

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

For $H(s) = 1$

$$T(s) = \frac{G(s)}{1 + G(s)}$$

and
$$G(s) = \frac{T(s)}{1 - T(s)} \quad \text{...(ii)}$$

From equation (i) and (ii), we get,

$$G(s) = \frac{K}{s^3 + (10 + a)s^2 + (100 + 10a)s + 100a - K} \quad \text{...(iii)}$$

Now, for unit ramp input,

$$K_v = \lim_{s \to 0} sG(s) = 5 \qquad \text{(Given)}$$

$\therefore$ Equating the term,

$$100a - K = 0$$

we get, $$K = 100a \qquad \text{...(iv)}$$

$\therefore$ The error constant can be obtained as

$$K_v = \lim_{s \to 0} s \times \frac{K}{s(s^2 + (10 + a)s + (100 + 10a))}$$

$$5 = \frac{K}{100 + 10a} \qquad \text{...(v)}$$

$\therefore$ $$5 = \frac{100\,a}{100 + 10\,a}$$

or $$500 + 50a = 100a$$

or $$500 = 50a \quad \Rightarrow a = 10$$

and therefore, $$K = 1000$$

(ii) Now from the given transfer function by putting the value obtained for K and a. we get,

$$T(s) = \frac{1000}{(s^2 + 10s + 100)(s + 10)}$$

$$= \frac{1000}{s^3 + 10s^2 + 100s + 10s^2 + 100s + 1000}$$

$$= \frac{1000}{s^3 + 20s^2 + 200s + 1000}$$

and $$G(s) = \frac{1000}{s(s^2 + 20s + 200)} \qquad \text{...(vi)}$$

The open loop transfer function obtained from the given block diagram is

$$G(s) = \frac{G_c(s) \times 50}{s(s^2 + 25s + 50)} \qquad \text{...(vii)}$$

Equating equation (vi) and (vii), we get,

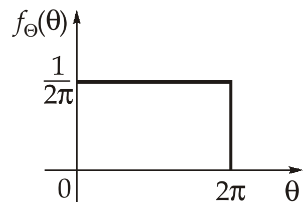
$$\frac{G_c(s) \times 50}{s(s^2 + 25s + 50)} = \frac{1000}{s(s^2 + 20s + 200)}$$

and $$G_c(s) = \frac{20(s^2 + 25s + 50)}{(s^2 + 20s + 200)}$$

2. (b) Solution

(i)

Given that, Θ is uniformly distributed in the range $[0, 2\pi]$



Mean value of $X, E[X] = \int_{-\infty}^{\infty} f_\Theta(\theta)\sin\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\sin\theta\, d\theta = 0$

Mean value of $Y, E[Y] = \int_{-\infty}^{\infty} f_\Theta(\theta)\cos\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos\theta\, d\theta = 0$

Correlation of X and Y,

$$E[XY] = \int_{-\infty}^{\infty} f_\Theta(\theta)\sin\theta\cos\theta\, d\theta$$

$$= \frac{1}{4\pi}\int_0^{2\pi}\sin 2\theta\, d\theta = 0 \quad \Rightarrow X \text{ and } Y \text{ are orthogonal} \qquad \text{...(i)}$$

Covariance of X and Y,

$$C_{XY} = E[XY] - \bar{X}\bar{Y}$$

$$= (0) - (0)(0) = 0 \quad \Rightarrow X \text{ and } Y \text{ are uncorrelated} \qquad \text{...(ii)}$$

Check for statistical independency:

- X and Y are said to be statistically independent, if $E[XY] = E[X]E[Y]$.
  But in this problem, $E[X] = E[Y] = E[XY] = 0$.
  So, we can't check for independency using the above mentioned relation.
- Now, we can use $E[X^2]$ and $E[Y^2]$ to check for independency.

$$E[X^2] = \int_{-\infty}^{\infty} f_\Theta(\theta)\sin^2\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta\, d\theta = \frac{1}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\, d\theta$$

$$= \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$E[Y^2] = \int_{-\infty}^{\infty} f_\Theta(\theta)\cos^2\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\, d\theta = \frac{1}{4\pi}\int_0^{2\pi}(1+\cos 2\theta)\, d\theta$$

$$= \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$E[X^2Y^2] = \int_{-\infty}^{\infty} f_\Theta(\theta)\sin^2\theta\cos^2\theta\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\sin^2\theta\, d\theta$$

$$= \frac{1}{8\pi}\int_0^{2\pi}\sin^2 2\theta\, d\theta = \frac{1}{16\pi}\int_0^{2\pi}(1-\cos 4\theta)\, d\theta = \frac{2\pi}{16\pi} = \frac{1}{8}$$

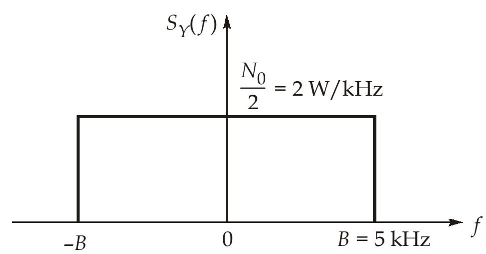
$$E[X^2Y^2] \neq E[X^2]\,E[Y^2] \Rightarrow X \text{ and } Y \text{ are not independent} \qquad \text{...(iii)}$$

From equations (i), (ii) and (iii), we can say that,

X and Y are orthogonal, uncorrelated but not independent.

(ii)

The output power spectral density of the filter can be given as,



$$S_y(f) = \frac{N_0}{2}\text{rect}\left(\frac{f}{2B}\right)$$

$$R_Y(\tau) \xleftrightarrow{\text{CTFT}} S_Y(f)$$

$$\text{sinc}(\tau) \xleftrightarrow{\text{CTFT}} \text{rect}(f)$$

$$2B\,\text{sinc}\,(2B\tau) \xleftrightarrow{\text{CTFT}} \text{rect}\left(\frac{f}{2B}\right)$$

$$N_0 B\,\text{sinc}(2B\tau) \xleftrightarrow{\text{CTFT}} \frac{N_0}{2}\text{rect}\left(\frac{f}{2B}\right)$$

So,
$$R_y(\tau) = N_0 B\,\text{sinc}(2B\tau)$$
$$= (4 \times 5)\,\text{sinc}(2 \times 5000\tau)$$
$$= 20\,\text{sinc}(10^4\tau)$$

Q.2 (c) Solution:

Main Program:

```
BACK:     MVI A, 40 H      ;
          SIM              ;  Send logic 0 on SOD pin
          CALL DELAY1      ;
          MVI A, C0H       ;
          SIM              ;  Send logic 1 on SOD pin
          CALL DELAY 2     ;
          JMP BACK         ;
```

Delay Calculations:

The following program can be used as the basic structure of the delay subroutines.

```
          MVI C, Count     ;  7T
LOOP:     DCR C            ;  4T
          JNZ LOOP         ;  10/7T
          RET              ;  10T
```

Total number of subroutine T-states = 7 + (4 + 10) Count – 3 + 10 = 14 + (14 × Count)

The number of main program T-states causing the delay (due to MVI, SIM, CALL) is equal to 7 + 4 + 18 = 29.

$$\text{Total delay} = [14 + (14\times\text{Count}) + 29]\frac{1}{f_{clk}}$$

$$= [43 + (14\times\text{Count})]\frac{1}{f_{clk}}$$

To get a delay of T_{OFF} = 89 μs,

$$43 + (14 \times \text{Count}) = 89 \times 3 = 267$$

$$\text{Count} = \frac{267-43}{14} = (16)_{10} = 10\text{H}$$

To get a delay of T_{ON} = 41 μs,

$$43 + (14 \times \text{Count}) + 10 = 41 \times 3 = 123$$ [Including 10 T-states for JMP Back]

$$\text{Count} = \frac{123-53}{14} = (05)_{10} = 05\text{H}$$

So, the delay subroutine programs can be given as follows:

```
DELAY1:   MVI C, 10H
LOOP1:    DCR C
          JNZ LOOP1
          RET
```

```
DELAY2:   MVI C, 05H
LOOP2:    DCR C
          JNZ LOOP2
          RET
```

Q.3 (a) Solution:

Hand-off Algorithms

At the cell-site, the received signal strength is always monitored on a reverse voice channel. When the received signal strength reaches the defined hand-off level (usually higher than the threshold level for the minimum required received signal level for desired voice quality) for a predefined period then the cell-site sends a request to the mobile telephone switching office (MTSO) for a hand-off on the call. Unnecessary hand-offs are avoided as a result of the direction and the speed of the vehicle.

There are two situations where hand-offs are necessary but cannot be made. Firstly, when the mobile subscriber is located at a signal-strength hole within a cell but not at the boundary -- the call must be kept in the old frequency channel until it is dropped as the result of an unacceptable received signal level. Secondly, when the mobile subscriber approaches a cell boundary but no channels in the new cell are available. The hand-off control algorithms tend to decentralise the decision making process, which help to reduce the hand-off delays. There are four basic type of hand-off algorithms:

- Network-controlled hand-off (NCHO)
- Mobile-assisted hand-off (MAHO)
- Soft hand-off (SHO)
- Mobile-controlled hand-off (MCHO)

(i) **Network-controlled hand-off (NCHO)** is a centralised hand-off protocol, in which the network makes hand-off decision based on measurements of the signal quality of a mobile subscriber unit at a number of adjacent cell-sites. Specifically, if the received signal level from a serving mobile subscriber is measured to have a weaker signal in its present serving cell, while a stronger signal in a neighbouring cell then a hand-off decision could be made by the cellular network to switch a cell-site from the present serving cell to the new target cell. Such a type of hand-off usually takes 100-200 milliseconds and produces a noticeable interruption in the conversation. The overall delay of this type of hand-off is generally in the range of 5-10 seconds. Thus, this type of hand-off is not suitable in a rapidly changing environment and a high density of subscribers due to the associated delay. In NCHO, the load of the network is high since the network handles the complete hand-off process itself. NCHO is used in the first-generation analog cellular systems such as Advanced Mobile Phone System (AMPS) where the mobile telephone switching office (MTSO) is responsible for overall hand-off decision. In NCHO, the network evaluates the necessary received signal strength indicator (RSSI) measurements and hand-off decision.

(ii) **Mobile-assisted hand-off (MAHO)** is a decentralised hand-off protocol and distributes the hand-off decision process. In order to reduce the load of the network, the mobile subscriber is responsible for making RSSI measurements and send them periodically to a cell-site in MAHO. Based on the received RSSI information data, the cell-site or the mobile switching centre (MSC) decides when to hand-off. MAHO is used in the second generation digital cellular system such as Global System for Mobile Communications (GSM). In a digital cellular system, the mobile receiver is capable of monitoring the signal strength of the control channels of the neighbouring cells during a call. When the received signal strength of its voice channel is weak, the mobile subscriber unit can request a hand-off with a candidate hand-off site. MSC then chooses the proper neighbouring cell for hand-off to take place based on the signal strengths of both forward and reverse control channels. Compared to NCHO, this mechanism has more distributed control, thereby helping to improve the overall hand-off delay, typically one second.

(iii) **Soft hand-off (SHO)** is often used in conjunction with MAHO. Rather than immediately terminating the connection between a mobile subscriber and a serving cell-site in the process of hand-off, a new connection is established first between the mobile subscriber and a new target cell-site, while maintaining the old communication link. Only after the new communication link can stably transmit data, the old link is released. Thus, SHO algorithm is a make-before-break type mechanism. This mechanism helps in ensuring the service continuity which is, however, at the expense of using more resource during the hand-off process as two connections are established simultaneously. The soft hand-off algorithm is generally applied to a CDMA digital cellular system which uses the same radio frequency in all the cells. There is no need to change from one frequency to another frequency but change from one code to another code during hand-off.

(iv) In the **Mobile-controlled hand-off (MCHO)** algorithm, the mobile subscriber unit controls and makes complete decisions on hand-off. A mobile subscriber keeps on measuring signal strength from all the neighbouring cell sites. If the mobile subscriber finds that there is a new cell-site with a stronger signal than that of an old serving cell-site, it may consider to hand-off from the old cell to the new cell, after reaching a defined signal threshold level. MCHO is the highest degree of hand-off decentralisation, thereby enabling it to have a very fast hand-off speed, typically of the order of 0.1 second only. MCHO extends the role of the mobile subscriber by giving overall control to it. The mobile subscriber and cell-site both make the necessary measurements and the cell-site sends the measurement data to the mobile subscriber. Then, the mobile subscriber decides when to hand-off based on the information gained from the cell-site and itself. MCHO algorithm is used in Digital European Cordless Telephone (DECT) system.

Q.3 (b) Solution

(i) Format of single precision floating point is

32 bits

Sign	Exponent (E)	Mantissa (M)
1 bit	8 bits	23 bits

0	1 0 0 0 0 1 1 1	101000000000...00

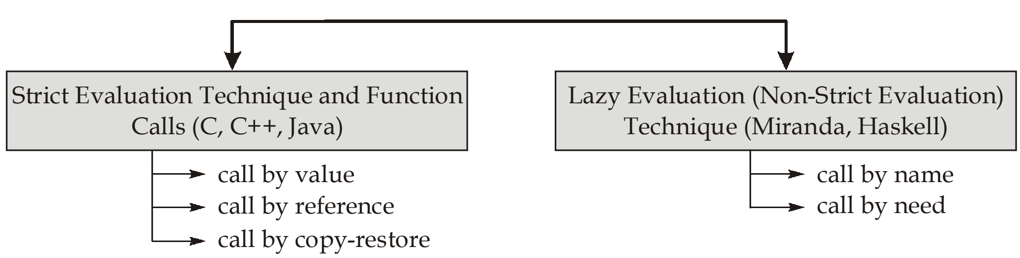
$$\begin{aligned}\text{Value} &= 1.M \times 2^{E-127} = 1.1010 \times 2^{135-127}\\ &= (1.1010)_2 \times 2^8 = 1.625 \times 2^8\\ &= (416)_{10}\end{aligned}$$

Octal representation

$$\begin{array}{r|rl} 8 & 416 & \\ \hline 8 & 52 & 0 \\ \hline & 6 & 4 \end{array}$$

Octal representation is (640).

(ii) Types of Evaluation of Function



Strict Evaluation Techniques:

Call by value

- In call by value method, the value of the variable is passed to the function as parameter.
- The value of the actual parameter can not be modified by formal parameter.

- Different Memory is allocated for both actual and formal parameters. Here, value of actual parameter is copied to formal parameter.

Call by reference

- In call by reference method, the address of the variable is passed to the function as parameter.
- The value of the actual parameter can be modified by formal parameter.
- Same memory is used for both actual and formal parameters since same address is used by both parameters.

Call by Copy-Restore: or (call by value-return):

- This technique is just like Call by Value in which, upon return, the modified values are copied back.
- The effect is similar to call by reference.
- This technique will yield different result corresponding to aliasing problems (i.e. different values referring to the same thing).

Lazy Evaluation Techniques:

Call by Name:

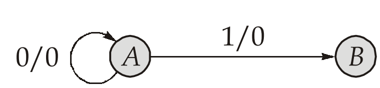
- The arguments passed to the function (which may be an expression) will be evaluated every time, if it is found inside the called function.

Call by Need:

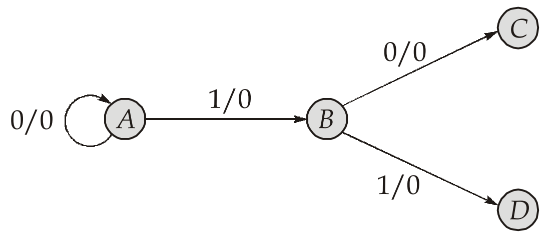
- It is same like call by name, except instead of re-evaluating the expression every time, the result of the evaluated expression is stored and re-used afterwards.
- The arguments passed to the function (which may be an expression) will be evaluated only for the first time, and for the next time it will be stored and re-used.

Q.3 (c) Solution:

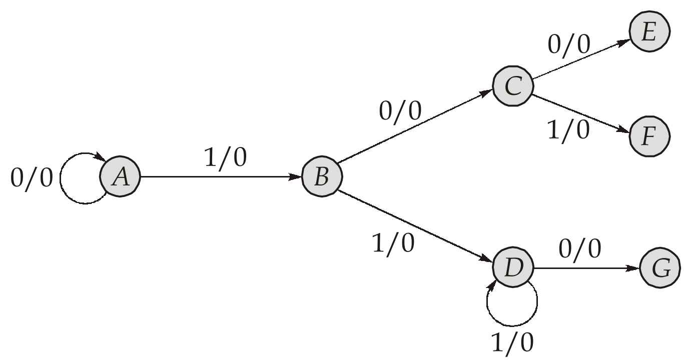
(i) If overlapping of the sequence is not allowed.
Let the initial state be 'A'.
Now, if a '0' is detected then the sequence will stay at 'A' and if '1' is detected then it will go to new state 'B'.



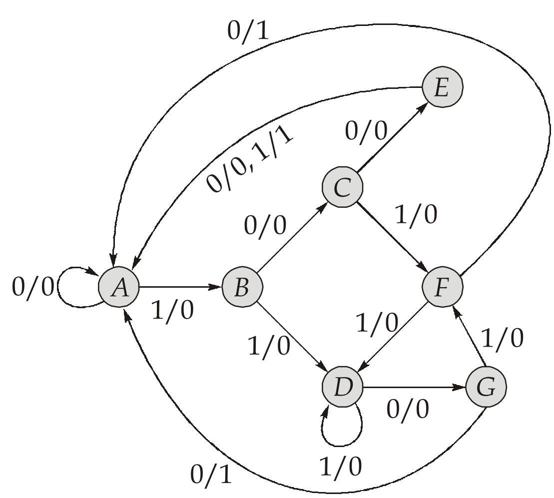
From state 'B' if '1' is received then it will go to next state 'D' and if '0' is received it will go to state 'C'.



From state '*C*' if a '0' is received it will move to a new state '*E*' and if '1' is received then it will move to stage '*F*'.
From state '*D*' if a '0' is received it will go to stage '*G*' and if '1' is received then it will stay at state '*D*'.

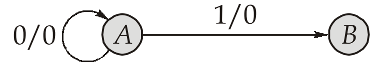


From state '*E*' if '1' is received then it will go to state *A* with output '1' and if '0' is received it will go to state '*A*' with output '0'.
From state '*G*' if '1' is received then it will go to state '*F*' with output '0' and if '0' is received it will go to state '*A*' with output equal to '1'.
From state '*F*' if '1' is received then it will go to state '*D*' with output '0' and if '0' is received it will go to state '*A*' with output '1'.

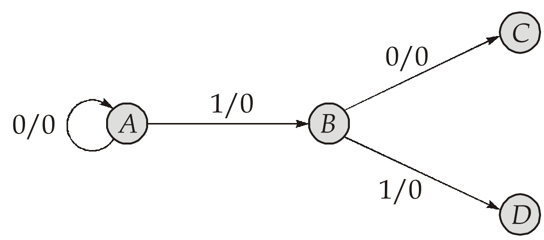


If overlapping sequences are not allowed

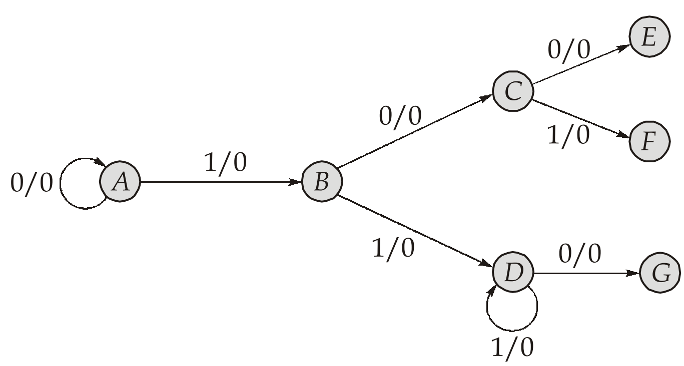
(ii) If overlapping of sequence is allowed.
Let 'A' be the initial state.
From state 'A' if '0' is received it will stay at 'A' and if '1' is received it will go to state 'B'.



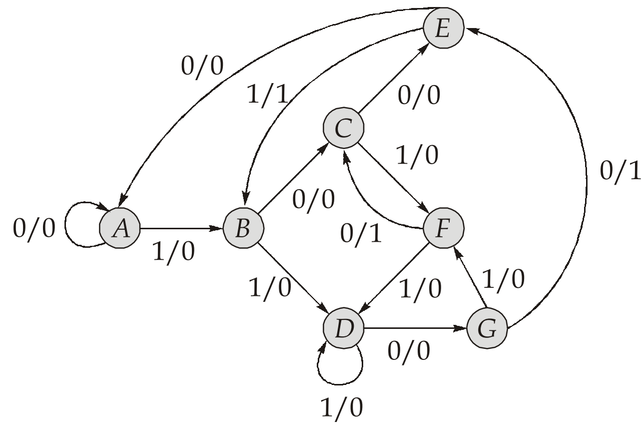
From state 'B' if '1' is received it will go to state 'D' and if '0' is received it will go to state 'C'.



From state 'C' if 1 is received it will go to 'F' and if '0' if received it will go to state 'E'.
From state 'D' if '1' is received it will stay at state 'D' and if '0' is received it will go to state 'G'.

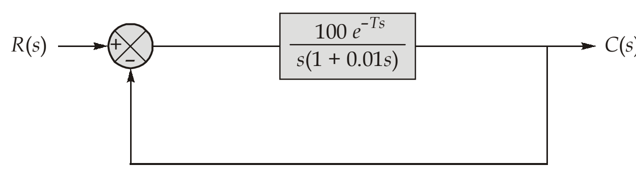


From state 'E' if '0' is received it will go to state 'A' with output '0' and if '1' is received it will go to state 'B' with output '1'.
From state 'F' if '1' is received it will go to state 'D' with output '0' and if '0' is received it will go to state 'C' with output '1'.
From state 'G' if '0' is received it will go to state 'E' with output '1' and if '1' is received it will go to state 'F' with output '0'.



If overlapping sequences are allowed

4. (a)



$$G(s) = \frac{e^{-Ts} \times 100}{s(1+0.01s)}$$

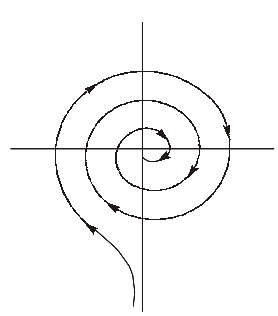
$$\therefore \qquad G(j\omega) = \frac{e^{-j\omega T} \times 100}{j\omega(1+0.01 \times j\omega)}$$

where, $\qquad e^{-j\omega T} = \cos(\omega T) - j\sin(\omega T)$

$$\therefore \qquad G(j\omega) = \frac{100[\cos(\omega T) - j\sin(\omega T)][1 - j\omega \times 0.01][-j]}{\omega\left(1+(0.01)^2\omega^2\right)}$$

$$= \frac{100[[-\cos(\omega T) \times 0.01 \times \omega - \sin\omega T]] - j[[\cos\omega T - \omega \times 0.01 \times \sin\omega T]]}{\omega\left[1+(0.01)^2\omega^2\right]}$$

For Nyquist plot,



ω	0	∞
$\lvert M \rvert$	∞	0
∠	–90	–∞

From, Nyquist plot, we can see that higher the values of ω the plot spirals round the origin.

The intersection of $G(j\omega)$ plot with real axis is obtained by equating imaginary part to zero.

$$\cos(\omega T) = \frac{\omega}{100} \times \sin \omega T \qquad \text{...(i)}$$

and magnitude $|G(j\omega)|$ at $(\omega = \omega_{pc})$ should be less than or equal to one for stability [Nyquist criteria]

$$|G(j\omega)|_{\omega_{pc}} = \frac{100[\sin \omega T + \omega \times 0.01 \times \cos \omega T]}{\omega\left(1+(0.01)^2 \omega^2\right)}$$

Putting value of equation (i),

then, $$\frac{100\left[\sin \omega T + \frac{\omega^2}{(100)^2} \sin \omega T\right]}{\omega\left(1+(0.01)^2 \omega^2\right)} \le 1$$

$$\therefore \qquad \frac{100(\sin \omega T)}{\omega} \le 1$$

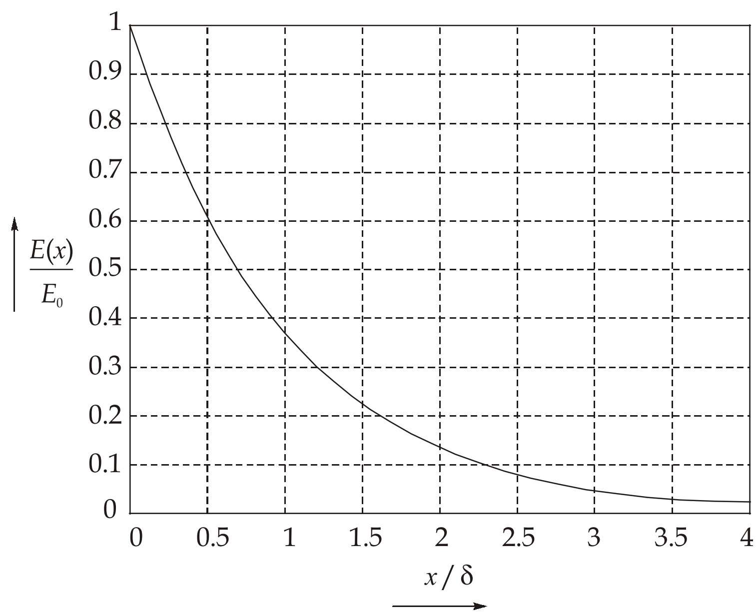
$$\therefore \qquad \sin \omega T \le \frac{\omega}{100}$$

$$\omega T \le \frac{\omega}{100} \qquad [\text{for smaller values } \sin \omega T \cong \omega T]$$

$$\therefore \qquad T \le \frac{1}{100}$$

Q.4 (b) Solution:

(i) Skin depth: Skin depth is defined as the distance over which a plane wave is attenuated by a factor of e^{-1} in a good conductor.



Skin depth illustration

The equation for skin depth δ is found by

$$E(x) = E_0 e^{-\alpha x}$$

at $x = \delta$

$$E(\delta) = E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$\Rightarrow$ $$\alpha\delta = 1$$

or, $$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

For a good conducting medium,

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta$$

From the definition of skin depth for good conducting medium,

$$\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

where, $\omega = 2\pi f$ $\Rightarrow$ $\delta = \dfrac{1}{\sqrt{\pi f \mu\sigma}}$

$\because$ $$\delta = \frac{1}{\beta} = \frac{1}{2\pi/\lambda} \approx \frac{\lambda}{2\pi}$$

(ii) Given that, $\beta = 1.5$ rad/m

$f = 10^6$ Hz

From uniform plane wave,

$$E = E_0 e^{-\alpha z} \text{ V/m}$$

According to the question,

$$E = (1 - 0.45)E_0$$

$$= 0.65E_0 \text{ at } z = 1 \text{ m}$$

$\therefore$ $$0.65E_0 = E_0 e^{-\alpha}$$

or $$e^{-\alpha} = 0.65$$

Taking natural lag both the side, we get,

or $$\ln e^{-\alpha} = \ln 0.65$$

or $$\alpha = -\ln 0.65 = 0.431$$

$\therefore$ Skin depth, $$\delta = \frac{1}{\alpha} = \frac{1}{0.431} = 2.32 \text{ m}$$

and phase velocity,

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{1.5} = \frac{2\pi \times 10^6}{1.5} = 4.189 \times 10^6 \text{ m/s}$$

Q.4 (c) Solution:

Amplitude of $m(t)$, $A_m = 1$ V

Frequency of $m(t)$, $$f_m = \frac{2000\pi}{2\pi} = 1000 \text{ Hz} = 1 \text{ kHz}$$

(i) For FM signal, $$\beta = \frac{A_m K_f}{f_m} = \frac{1 \times 100}{1} = 100$$

Bandwidth of FM signal,

$$B_{FM} = (1 + \beta)2f_m = 101 \times 2 = 202 \text{ kHz}$$

For PM signal, $\beta = A_m K_p = 1 \times 10 = 10$

Bandwidth of PM signal,

$$B_{PM} = (1 + \beta)2f_m = 11 \times 2 = 22 \text{ kHz}$$

(ii) When the amplitude of $m(t)$ is doubled (i.e., $A_m = 2$ V):

For FM signal,

$$\beta = \frac{A_m K_f}{f_m} = \frac{2\times100}{1} = 200$$

$$B_{FM(1)} = (1 + 200)\,(2)\,(1) = 402 \text{ kHz}$$

For PM signal, $\beta = A_m K_p = 2 \times 10 = 20$

$$B_{PM(1)} = (1 + 20)\,(2)\,(1) = 42 \text{ kHz}$$

(iii) When the frequency of $m(t)$ is doubled (i.e., $f_m = 2$ kHz):

For FM signal, $\beta = \dfrac{A_m K_f}{f_m} = \dfrac{1\times100}{2} = 50$

$$B_{FM(2)} = (1 + 50)\,(2)\,(2) = 204 \text{ kHz}$$

For PM signal, $\beta = A_m K_p = 1 \times 10 = 10$

$$B_{PM(2)} = (1 + 10)\,(2)\,(2) = 44 \text{ kHz}$$

(iv)

- Doubling the amplitude of $m(t)$ roughly doubles the bandwidth of both FM and PM signals.
- Doubling the frequency of $m(t)$ [expanding the spectrum $M(f)$ by a factor 2] has hardly any effect on the FM bandwidth. However, it roughly doubles the PM bandwidth.
- It indicates that PM spectrum is sensitive to the shape of the baseband spectrum and FM spectrum is relatively insensitive to the nature of the spectrum $M(f)$.

Section-B

Q.5 (a) Solution:

Number of bits to be encrypted = $60 \times 10^6 \times 8 = 480 \times 10^6$ bits

Each pixel can hide 3 bits of data. Therefore, the number of pixels required in order to encrypt the entire file of 60 MB is $480 \times 10^6/3 = 160 \times 10^6$ pixels.

The aspect ratio of the image is 3 : 2.

So, $(3x)\,(2x) = 160 \times 10^6$

$$x = \sqrt{\frac{160\times10^6}{6}} \approx 5164$$

So,the required image size = 3(5164) × 2(5164) = 15492 × 10328 pixels

If the file were compressed to a third of its original size, then the number of bits to be encrypted would be $480 \times 10^6/3 = 160 \times 10^6$ bits, and the number of pixels needed would be $160 \times 10^6/3 = 53{,}333{,}333$ pixels.

$$x = \sqrt{\frac{53{,}333{,}333}{6}} = 2981.4 \approx 2982$$

So, the required image size = 3(2982) × 2(2982) = 8946 × 5964 pixels.

Q.5 (b) Solution:

Step 1: Read the three numbers into a, b, c

Step 2: If a > b and a > c

  Max = a;

  If b < c

    Max2 = b, Min = c

  Else

    Max2 = c, Min = b

    Goto Step 5

  Else

    Goto step 3

Step 3: If b > c then do the following else goto step 4

  Max = b;

  If (a > c)

    Max2 = a, Min = c

  Else

    Max2 = c, Min = a;

    Goto Step 5

Step 4: Max = c;

  If (a > b)

    Max2 = a, Min = b

  Else

    Max2 = b, Min = a

    Goto Step 5

Step 5: Print Min, Max2, Max;

Step 6: Stop.

Q.5 (c) Solution:

(i) The state model has the standard form

$$\dot{x} = Ax + Bu \quad \text{...(i)}$$

$$y = Cx + Du \quad \text{...(ii)}$$

Equation (i) can be written in s-domain as

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$(sI - A)X(s) = x(0) + BU(s)$$

or
$$X(s) = (sI - A)^{-1} x(0) + (sI - A)^{-1} BU(s) \quad \text{...(iii)}$$

Similarly from equation (ii), we get,

$$Y(s) = CX(s) + DU(s) \quad \text{...(iv)}$$

by substituting the values of $X(s)$ from equation (iii) to equation (iv), we get,

$$Y(s) = C(sI - A)^{-1} x(0) + C(sI - A)^{-1} BU(s) + DU(s)$$

Assuming zero initial conditions,

$$Y(s) = C(sI - A)^{-1} BU(s) + DU(s)$$

or
$$\frac{Y(s)}{U(s)} = T(s) = C(sI - A)^{-1} B + D \quad \text{...(v)}$$

(ii) Here,
$$A = \begin{bmatrix} 0 & 1 \\ -3 & -a \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = [1 \quad 1]$$

By using equation (v), we have,

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -a \end{bmatrix} = \begin{bmatrix} s & -1 \\ 3 & s+a \end{bmatrix}$$

$\therefore$
$$(sI - A)^{-1} = \frac{\text{adj}(sI - A)}{|sI - A|}$$

$$= \frac{1}{s(s+a)+3} \begin{bmatrix} s+a & 1 \\ -3 & s \end{bmatrix}$$

$$\therefore \qquad T(s) = C(sI - A)^{-1}B = [1 \quad 1]\frac{1}{s(s+a)+3}\begin{bmatrix} s+a & 1 \\ -3 & s \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= [1 \quad 1]\cdot\frac{1}{s(s+a)+3}\begin{bmatrix} s+a \\ -3 \end{bmatrix}$$

$$T(s) = \frac{s+a-3}{s^2+sa+3}$$

Here, the characteristic equation is

$$s^2 + sa + 3 = 0$$

On comparing it with standard second order characteristic equation, we get,

$$\omega_n^2 = 3 \quad \Rightarrow \omega_n = \sqrt{3}$$

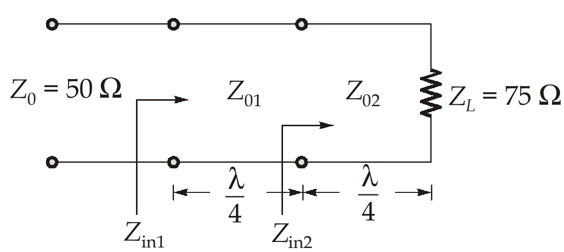
and $$2\xi\omega_n = a$$

or $$a = 2\times 0.5\times\sqrt{3}$$

or $$a = \sqrt{3}$$

Q.5 (d) Solution:

(i) The given figure can be redrawn as



For $\lambda/4$ transformer, we know that

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$\therefore \qquad Z_{in2} = \frac{Z_{02}^2}{Z_L}$$

and $$Z_{in1} = \frac{Z_{01}^2}{Z_L} = \frac{Z_{01}^2}{Z_{in2}} = Z_0 \qquad \text{(For perfect matching, } Z_{in1} = Z_0\text{)}$$

or $$Z_{in2} = \frac{Z_{01}^2}{Z_0} = \frac{Z_{02}^2}{Z_L}$$

$$\therefore \qquad Z_{01} = Z_{02}\sqrt{\frac{Z_0}{Z_L}} = 30\sqrt{\frac{50}{75}} = 24.5\,\Omega$$

$$\therefore \qquad Z_{01} = 24.5\ \Omega$$

(ii) Also,
$$\frac{Z_0}{Z_{01}} = \left(\frac{Z_{02}}{Z_L}\right) \Rightarrow Z_{02} = \frac{Z_0 Z_L}{Z_{01}} \qquad \text{...(i)}$$

and
$$\frac{Z_{01}}{Z_{02}} = \left(\frac{Z_{02}}{Z_L}\right)^2$$

$$(Z_{02})^3 = Z_{01}\, Z_L^2 \qquad \text{...(ii)}$$

From equation (i) and (ii),

$$(Z_{02})^3 = Z_{01}\, Z_L^2 = \frac{Z_0^3\, Z_L^3}{Z_{01}^3} \qquad \text{...(iii)}$$

or
$$Z_{01} = \sqrt[4]{Z_0^3\, Z_L} = \sqrt[4]{(50)^3\,(75)} = 55.33\,\Omega$$

From equation (iii),
$$Z_{02} = \sqrt[3]{Z_{01}\, Z_L^2} = \sqrt[3]{55.33 \times 75^2} = 67.76\,\Omega$$

Q.5 (e) Solution

(i) Area under a probability density function is unity.

So,
$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

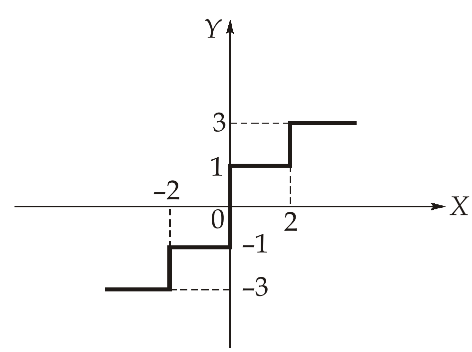
$$\int_{-4}^{4} k\,e^{-|x|}\,dx = 1$$

$$2k\int_{0}^{4} e^{-x}\,dx = 1$$

$$2k[-e^{-x}]_0^4 = 1$$

$$2k[1 - e^{-4}] = 1$$

$$k = \frac{1}{2(1-e^{-4})}$$



(ii)
$$\text{Step size } (s) = \frac{x_{\max} - x_{\min}}{L} = \frac{4-(-4)}{4} = \frac{8}{4} = 2$$

(iii) By assuming uniform mid-rise quantizer,

So, the quantizer levels are {–3, –1, +1, +3}

Q.6 (a) Solution

(i)

- Considering an elliptical orbit as shown in the figure below:

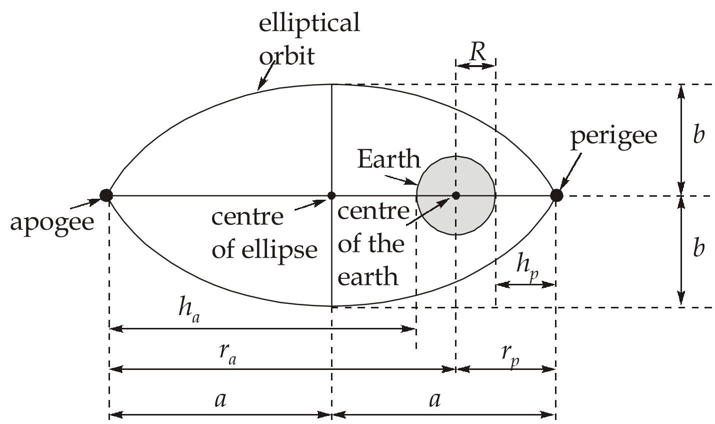


Figure-1

Here, R = mean radius of the earth

r_a = length of radius vector at apogee

r_p = length of radius vector at perigee

h_a = apogee height

h_p = perigee height

a = semimajor axis of the elliptical orbit

b = semiminor axis of the elliptical orbit

- The eccentricity (e) of the elliptical orbit can be defined as,

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

- **Given data :** e = 0.15

a = 9000 km

R = 6371 km

To determine the orbital period :

Kepler's third law states that, the square of the periodic time of orbit (orbital period) is proportional to the cube of the semi-major axis of the orbit.

i.e., $a^3 \propto T^2$

$$a^3 = \left(\frac{\mu}{4\pi^2}\right)T^2 \qquad ...(i)$$

Where, a = semimajor axis

T = orbital period

μ = Kepler's constant = $3.986004418 \times 10^{14}\ m^3/s^2$

• From equation (i), we can write,

$$T^2 = \left(\frac{4\pi^2}{\mu}\right)a^3$$

$$T = 2\pi\left(\frac{a^3}{\mu}\right)^{1/2} \quad \text{...(ii)}$$

• By substituting the values of "a" and "μ" in equation (ii), we get,

Orbital period, $$T = 2\pi\left[\frac{(9000\times10^3)^3}{3.986004418\times10^{14}}\right]^{1/2} \text{ seconds}$$

$= 2\pi(1352.368)$ seconds $\approx$ 8497.2 seconds

= 2 hours, 21 minutes, 37.2 seconds

To determine the apogee height:

• From the basic properties of ellipse, the radial length from the centre of the earth to the point apogee (r_a) can be written as,

$$r_a = a(1+e) \quad \text{...(iii)}$$

Where, a = semimajor axis of the elliptical orbit

e = eccentricity of the elliptical orbit

• By substituting the values of "a" and "e" in equation (iii), we get,

$r_a = a(1+e)$

= 9000(1 + 0.15) km

= 10350 km

• From the elliptical orbit shown in the figure-1, the apogee height can be written as,

$$h_a = r_a - R \quad \text{...(iv)}$$

Where, R = mean radius of the earth

= 6371 km (given)

• By substituting the values of "r_a" and "R" in equation (iv), we get,

$h_a = r_a - R$ = (10350 – 6371) km = 3979 km

To determine the perigee height:

• Similarly, from the basic properties of ellipse, the radial length from the centre of the earth to the point perigee (r_p) can be written as,

$$r_p = a(1-e) \quad \text{...(v)}$$

- By substituting the values of "a" and "e" in equation (v), we get,

$$r_p = a(1 - e)$$
$$= 9000(1 - 0.15) \text{ km} = 7650 \text{ km}$$

- From elliptical orbit shown in the figure-1, the perigee height can be written as,

$$h_p = r_p - R \qquad \text{...(vi)}$$

- By substituting the values of "r_p" and "R" in equation (vi), we get,

$$h_p = (7650 - 6371) \text{ km}$$
$$= 1279 \text{ km}$$

(ii)

Given data:

Free-space loss, [FSL]= 206 dB

Antenna pointing loss, [APL] = 1 dB

Atmospheric absorption, [AA] = 2 dB

Receiver feeder loss, [RFL] = 1 dB

Equivalent isotropic radiated power, [EIRP] = 48 dBW

Figure of merit of the receiver, [G/T] = 19.5 dB/K

Operating frequency, f = 12 GHz

Calculation of carrier-to-noise spectral density ratio:

$$\left[\frac{C}{N_0}\right] = [\text{EIRP}] + \left[\frac{G}{T}\right] - [\text{Losses}] - [k] \qquad \text{...(i)}$$

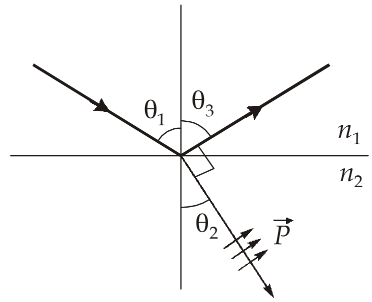
$$[k] = \text{Boltzmann constant in dB}$$
$$= 10\log_{10}(k) = -228.6 \text{ dB}$$
$$[\text{Losses}] = [\text{FSL}] + [\text{APL}] + [\text{AA}] + [\text{RFL}]$$
$$= 206 \text{ dB} + 1 \text{ dB} + 2 \text{ dB} + 1 \text{ dB}$$
$$= 210 \text{ dB}$$

- By substituting the values of [EIRP], [G/T], [Losses] and [k] in equation (i), we get,

$$\left[\frac{C}{N_0}\right] = (48 + 19.5 - 210 + 228.6) \text{ dBHz} = 86.10 \text{ dBHz}$$

Q.6 (b) Solution:

(i) **Brewster's angle:** For a plane elctromagnetic wave incident on a plane boundary between two dielectric media having different refractive indices, the angle of incidence at which transmittance from one medium to the other is unity, when the wave is linearly polarised with its electric vector parallel to the plane of incidence, is called Brewster's angle. This is demonstrated in Fig below.



Brewster's angle

For angle of incidence, θ_1 (called θ_B) equal to Brewster's angle or polarising angle, the reflectance for parallel polarization is zero and the reflected wave parallel polarisation only and is therefore, totally polarised.

Mathematical equation: We know, for parallel polarized wave, the reflection coefficient,

$$\frac{E_r}{E_i} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

So, it is zero, if $(\theta_1 + \theta_2) = \frac{\pi}{2}$ i.e., $(\theta_B + \theta_2) = \frac{\pi}{2}$

From Snell's law, $n_1 \sin\theta_B = n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos\theta_B$

$$\left[\begin{array}{l} n_1 = \sqrt{\dfrac{\mu_1\varepsilon_1}{\mu_0\varepsilon_0}}, \\ n_2 = \sqrt{\dfrac{\mu_2\varepsilon_2}{\mu_0\varepsilon_0}}, \text{ are refractive indices} \end{array}\right]$$

$$\therefore \qquad \tan\theta_B = \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

(since, $\mu_1 = \mu_2$)

At this angle, there is no reflected wave when the incident wave is parallel polarised.

(ii) From the definition of Poynting Theorem, we know,

$$P_{avg} = \frac{E^2}{2\eta}$$

$$\therefore \qquad P_{i\,avg} = \frac{E_{i0}^2}{2\eta_i} = \frac{E_{i0}^2}{2\eta_1}$$

$$P_{r\,\text{avg}} = \frac{E_{r0}^2}{2\eta_r} = \frac{E_{r0}^2}{2\eta_1}$$

and
$$P_{t\,\text{avg}} = \frac{E_{t0}^2}{2\eta_t} = \frac{E_{t0}^2}{2\eta_2}$$

It has been given that,

$$R = \frac{P_{r\,\text{avg}}}{P_{i\,\text{avg}}}$$

from above expression, we have

$$R = \frac{E_{ro}^2}{E_{io}^2} = \Gamma^2 = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)^2$$

where, Γ = Reflection coefficient

or
$$R = \left(\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right)^2$$

$$= \left(\frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_2}} + \sqrt{\frac{\mu_0}{\varepsilon_1}}}\right)^2 = \left(\frac{\sqrt{\mu_0\varepsilon_1} - \sqrt{\mu_0\varepsilon_2}}{\sqrt{\mu_0\varepsilon_1} + \sqrt{\mu_0\varepsilon_2}}\right)^2$$

Since, $\eta_1 = C\sqrt{\mu_1\varepsilon_1} = C\sqrt{\mu_0\varepsilon_1}$

and $\eta_2 = C\sqrt{\mu_2\varepsilon_2} = C\sqrt{\mu_0\varepsilon_2}$

$$\therefore \quad R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Hence proved.

Similarly,
$$T = \frac{P_{t\,\text{avg}}}{P_{i\,\text{avg}}} = \frac{\eta_1}{\eta_2}\frac{E_{to}^2}{E_{io}^2} = \frac{\eta_1}{\eta_2}\tau^2$$

where, τ = transmission coefficient

$$\tau = 1 + \Gamma$$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\therefore \quad T = \frac{\eta_1}{\eta_2} \times \tau^2 = \frac{\eta_1}{\eta_2} \times \frac{4\eta_2^2}{(\eta_1 + \eta_2)^2} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2}$$

where, $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}}$

$$\therefore \quad T = \frac{4\sqrt{\frac{\mu_0}{\varepsilon_1}}\sqrt{\frac{\mu_0}{\varepsilon_2}}}{\left(\sqrt{\frac{\mu_0}{\varepsilon_1}}+\sqrt{\frac{\mu_0}{\varepsilon_2}}\right)^2} = \frac{4\sqrt{\frac{\mu_0}{\varepsilon_1}}\sqrt{\frac{\mu_0}{\varepsilon_2}}\times\varepsilon_1\varepsilon_2}{\left(\sqrt{\mu_0\varepsilon_1}+\sqrt{\mu_0\varepsilon_2}\right)^2} = \frac{4\sqrt{\mu_0\varepsilon_1}\times\sqrt{\mu_0\varepsilon_2}}{\left(\sqrt{\mu_0\varepsilon_1}+\sqrt{\mu_0\varepsilon_2}\right)^2}$$

$$T = \frac{4n_1n_2}{(n_1+n_2)^2}$$

Hence proved.

(iii) If $P_{r\,avg} = P_{t\,avg}$

$$RP_{i\,avg} = TP_{i\,avg}$$

or $R = T$

i.e., $$\frac{(n_1-n_2)^2}{(n_1+n_2)^2} = \frac{4n_1n_2}{(n_1+n_2)^2}$$

$$\Rightarrow \quad 4n_1n_2 = (n_1 - n_2)^2$$

or $$4n_1n_2 = n_1^2+n_2^2-2n_1n_2$$

or $$n_1^2-6n_1n_2+n_2^2 = 0 \Rightarrow \left(\frac{n_1}{n_2}\right)^2-6\left(\frac{n_1}{n_2}\right)+1=0$$

From above equation, we get,

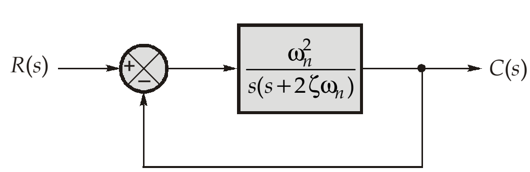
$$\frac{n_1}{n_2} = 5.82,\ 0.1715$$

Q.6 (c) Solution:

Technique	Description	Strengths	Weaknesses
Fixed Partitioning	Main memory is divided into a number of static operations at system generation time. A process may be loaded into a partition of equal or greater size.	Simple to implement; little operating system overhead.	Inefficient use of memory due to internal fragmentation; maximum number of active processes is fixed.
Dynamic Partitioning	Partitions are created dynamically, so that each process is loaded into a partition of exactly the same size as that process.	No internal fragmentation; more efficient use of main memory.	Inefficient use of processor due to need for compaction to counter external fragmentation.
Simple Paging	Main memory is divided into a number of equal-size frames. Each process is divided into a number of equal-size pages of the same length as frames. A process is loaded by loading all of its pages into available, not necessarily contiguous, frames.	No external fragmentation.	A small amount of internal fragmentation.
Simple Segmentation	Each process is divided into a number of segments. A process is loaded by loading all of its segments into dynamic partitions that need not be contiguous.	No internal fragmentation; improved memory utilization and reduced overhead compared to dynamic partitioning.	External fragmentation.
Virtual Memory Paging	As with simple paging, except that it is not necessary to load all of the pages of a process. Nonresident pages that are needed are brought in later automatically.	No external fragmentation; higher degree of multi-programming; large virtual address space.	Overhead of complex memory management.
Virtual Memory Segmentation	As with simple segmentation, except that it is not necessary to load all of the segments of a process. Nonresident segments that are needed are brought in later automatically.	No internal fragmentation, higher degree of multi-programming; large virtual address space; protection and sharing support.	Overhead of complex memory management.

Q.7 (a) Solution:

- The standard form of a second order system and its transfer function can be given by,



$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- For the purpose of frequency domain analysis, with sinusoidal excitation, the transfer function can be modified by replacing "s" with "$j\omega$" as follows:

$$T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n (j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j2\zeta\omega\omega_n}$$

- For the purpose of simplifying the calculations, let us consider the ratio $u = \dfrac{\omega}{\omega_n}$ as the normalized operating frequency and the transfer function can be written in terms of normalized frequency as follows:

$$T(j\omega) = \frac{1}{(1-u^2) + j2\zeta u}$$

- The magnitude and phase of the above transfer function can be given as,

$$|T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$$

$$\angle T(j\omega) = \phi = -\tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$$

- The resonant frequency (ω_r) of a system is the frequency at which the magnitude of $T(j\omega)$ attains its maximum and the maximum value of the magnitude is called as resonant peak (M_r).

So, $$M\big|_{\omega = \omega_r} = M_r$$

- Let us take the normalized resonant frequency as $u_r = \dfrac{\omega_r}{\omega_n}$.
- At $\omega = \omega_r$ or $u = u_r$, the slope of the magnitude curve will be zero.

i.e., $$\left.\frac{\partial M}{\partial u}\right|_{u = u_r} = 0$$

$$\left.\frac{\partial M}{\partial u}\right|_{u=u_r} = \left.\frac{4(1-u^2)u - 8\zeta^2 u}{2[(1-u^2)^2+(2\zeta u)^2]^{3/2}}\right|_{u=u_r} = 0$$

$$(1-u_r^2)u_r - 2\zeta^2 u_r = 0$$

$$1-u_r^2 = 2\zeta^2$$

$$u_r = \sqrt{1-2\zeta^2}$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

- The resonant peak (M_r) can be calculated as,

$$M_r = M|_{\omega=\omega_r} \text{ (or) } M|_{u=u_r}$$

$$= \left.\frac{1}{\sqrt{(1-u^2)^2+4\zeta^2u^2}}\right|_{u=u_r=\sqrt{1-2\zeta^2}}$$

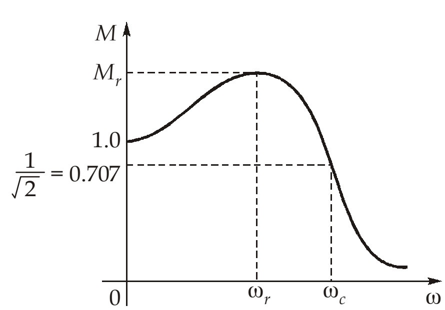
$$= \frac{1}{\sqrt{(1-1+2\zeta^2)^2+4\zeta^2(1-2\zeta^2)}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

- The phase of the system at $\omega = \omega_r$ can be given by,

$$\phi_r = -\tan^{-1}\left.\left(\frac{2\zeta u}{1-u^2}\right)\right|_{u=u_r=\sqrt{1-2\zeta^2}}$$

$$= -\tan^{-1}\left(\frac{\sqrt{1-2\zeta^2}}{\zeta}\right)$$

- A typical magnitude versus frequency characteristic of a feedback control system can be given as shown below.



- A standard second order control system has unity gain at $\omega = 0$ and the frequency at which the magnitude drops to $\frac{1}{\sqrt{2}}$ is called as cut-off frequency "ω_c". Generally for a control system the bandwidth is nothing but the cut-off frequency.

So, $$M\Big|_{\omega = \omega_c = \omega_b} = \frac{1}{\sqrt{2}}$$

- Let us take the normalized bandwidth as, $u_b = \frac{\omega_b}{\omega_n}$ and it can be calculated as follows:

$$\frac{1}{\sqrt{(1-u_b^2)^2 + 4\zeta^2 u_b^2}} = \frac{1}{\sqrt{2}}$$

$$1 - 2u_b^2 + u_b^4 + 4\zeta^2 u_b^2 = 2$$

$$u_b^4 - 2(1-2\zeta^2)u_b^2 - 1 = 0$$

$$u_b^2 = \frac{2(1-2\zeta^2) \pm \sqrt{4(1-2\zeta^2)^2 + 4}}{2}$$

$$= (1-2\zeta^2) \pm \sqrt{2 - 4\zeta^2 + 4\zeta^4}$$

u_b can be only a positive value.

So, $$u_b = \sqrt{(1-2\zeta^2) + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$$

- So, the bandwidth of a standard second order system is,

$$\omega_b = \omega_n \left[(1-2\zeta^2) + \sqrt{2 - 4\zeta^2 + 4\zeta^4} \right]^{1/2}$$

- The resonant frequency of a standard second order system is,

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

- The resonant peak of a standard second order system is,

$$M_r = \frac{1}{2\zeta\sqrt{1-2\zeta^2}}$$

Q.7 (b) Solution:

Here,
$$\beta l = 120°$$
$$\beta l = 120° \times \frac{\pi}{180°}$$
$\because$
$$l = 1.5 \text{ cm}$$
$\therefore$
$$\beta = \frac{120 \times \pi}{1.5 \times 180} = 1.396 \text{ rad/m}$$

Now, the cut-off frequency with dielectric,
$$f_{cd} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

where,
$$f_{cd} = \frac{f_{ca}}{\sqrt{\varepsilon_r}}$$

Also,
$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_{cd}}{f}\right)^2} = \frac{\omega}{c} \times \sqrt{\varepsilon_r}\sqrt{1 - \left(\frac{f_{ca}}{\sqrt{\varepsilon_r} \times f}\right)^2} \qquad \{\mu_r = 1\}$$
$$= \frac{\omega}{c} \times \sqrt{\varepsilon_r}\sqrt{1 - \left(\frac{f_{ca}}{\sqrt{\varepsilon_r} \times f}\right)^2}$$

or,
$$\varepsilon_r = \left(\frac{\beta c}{2\pi f}\right)^2 + \left(\frac{f_{ca}}{f}\right)^2$$
$$= \left(\frac{1.396 \times 3 \times 10^8}{2 \times \pi \times 3 \times 10^9}\right)^2 + \left(\frac{8 \times 10^9}{3 \times 10^9}\right)^2 = (0.0222)^2 + (2.67)^2$$
$$= 7.129$$

Q.7 (c) Solution:

(i) Given, analog filter
$$\frac{dy_a(t)}{dt} + 0.9y_a(t) = x_a(t)$$
applying the laplace transform to both sides of the differential equation we obtain,
$$s\,Y_a(s) + 0.9\,Y_a(s) = X_a(s)$$
$$\frac{Y_a(s)}{X_a(s)} = \frac{1}{s + 0.9} = H_a(s)$$

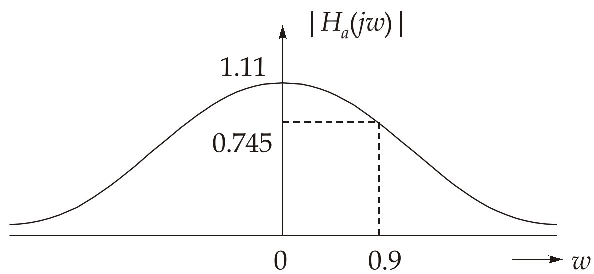
Put $$s = jw$$

$$H_a(jw) = \frac{1}{jw+0.9}$$

$$|H_a(jw)| = \frac{1}{\sqrt{w^2+0.9^2}}$$

at $w = 0$ $$|H_a(j0)| = \frac{1}{\sqrt{0.9^2}} = 1.11$$

$w = 0.9$ $$|H_a(j0.9)| = \frac{1}{\sqrt{2}\sqrt{0.9}} = 0.745$$



(ii) Given, digital filter,

$$\left[\frac{y(n+1)-y(n)}{T}\right]+0.9y(n) = x(n)$$

applying z-transform on both sides

$$\frac{zY(z)-Y(z)}{T}+0.9Y(z) = X(z)$$

$$Y(z)\left[\frac{z-1}{T}\right]+0.9Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{T}{z+(0.9T-1)}$$

with $$T = \frac{10}{9},$$

$$H(z) = \frac{\frac{10}{9}}{z+\left(0.9\times\frac{10}{9}-1\right)}$$

$$H(z) = \frac{\frac{10}{9}}{z}$$

put $$z = e^{jw}$$

$$H(e^{jw}) = \frac{10}{9}e^{-jw}$$

$$\left|H(e^{jw})\right| = \frac{10}{9}$$

for the particular choice for T (given), the frequency response is constant, independent of frequency in contrast to the analog filter which is a low pass filter.

(iii) For the system function determine in part (ii), the pole is at $z = 1 - 0.9T$. Assuming T to be positive, the pole is outside of the unit circle for $T > \frac{20}{9}$.

Q.8 (a) Solution:

Given that, $n_1 = 1.50$, $\Delta = 0.01$ and $L = 6$ km.

(i) The delay difference between the slowest and fastest modes can be given by,

$$\delta T_s \approx \frac{Ln_1\Delta}{c} = \frac{6000\times1.50\times0.01}{3\times10^8}\text{ s} = 300\text{ ns}$$

(ii) The rms pulse broadening due to intermodal dispersion can be given by,

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}\,c} = \frac{300}{2\sqrt{3}}\text{ ns} = 86.6\text{ ns}$$

(iii) The maximum bit rate can be estimated in two ways. Firstly, to get an idea of the maximum bit rate when assuming no pulse overlap, we get,

$$B_{T(\max)} = \frac{1}{2\tau} = \frac{1}{2\,\delta T_s} = \frac{1}{2\times300\times10^{-9}}\text{ bps} = 1.67\text{ Mbps}$$

Alternatively an improved estimate also can be used to get $B_{T(\max)}$ using rms pulse broadening, as

$$B_{T(\max)} = \frac{0.2}{\sigma_s} = \frac{0.2}{86.6\times10^{-9}} \approx 2.31\text{ Mbps}$$

(iv) By using the most accurate estimate of the maximum bit rate from part (iii), and assuming return to zero pulses, the bandwidth-length product can be given by,

$$B_{opt} \times L = 2.31 \times 10^6 \times 6 \times 10^3 = 13.86\text{ MHz-km}$$

Q.8 (b) Solution:

(i) One of the major problems in operating an instruction pipeline is the occurrence of branch instructions. A branch instruction can be conditional or unconditional. An unconditional branch always alters the sequential program flow by loading the program counter with the target address. In a conditional branch, the control selects the target instruction if the condition is satisfied or the next sequential instruction if the condition is not satisfied. The branch instruction breaks the normal sequence of the instruction stream, causing difficulties in the operation of the instruction pipeline. Pipelined computers employ various hardware techniques to minimize the performance degradation caused by instruction branching.

One way of handling a conditional branch is to pre-fetch the target instruction in addition to the instruction following the branch. Both are saved until the branch is executed. If the branch condition is successful, the pipeline continues from the branch target instruction. An extension of this procedure is to continue fetching instructions from both places until the branch decision is made. At that time control chooses the instruction stream of the correct program flow.

Another possibility is the use of a branch target buffer or BTB. The BTB is an associative memory included in the fetch segment of the pipeline. Each entry in the BTB consists of the address of a previously executed branch instruction and the target instruction for that branch. It also stores the next few instructions after the branch target instruction. When the pipeline decodes a branch instruction, it searches the associative memory BTB for the address of the instruction. If it is in the BTB, the instruction is available directly and pre-fetch continues from the new path. If the instruction is not in the BTB, the pipeline shifts to a new instruction stream and stores the target instruction in the BTB. The advantage of this scheme is that branch instructions that have occurred previously are readily available in the pipeline without interruption.

A variation of the BTB is the loop buffer. This is a small very high speed register file maintained by the instruction fetch segment of the pipeline. When a program loop is detected in the program, it is stored in the loop buffer in its entirety, including all branches. The program loop can be executed directly without having to access memory until the loop mode is removed by the final branching out.

Another procedure that some computers use is branch prediction. A pipeline with branch prediction uses some additional logic to guess the outcome of a conditional branch instruction before it is executed. The pipeline then begins pre-fetching the instruction stream from the predicted path. A correct prediction eliminates the wasted time caused by branch penalties.

A procedure employed in most RISC processors is the delayed branch. In this procedure, the compiler detects the branch instructions and rearranges the machine language code sequence by inserting useful instructions that keep the pipeline operating without interruptions. An example of delayed branch is the insertion of a no-operation instruction after a branch instruction. This causes the computer to fetch the target instruction during the execution of the no-operation instruction, allowing a continuous flow of the pipeline.

(ii) Three cases are possible here, which can be tabulated as shown below.

Location of referenced word	Probability	Total access time
In cache	0.9	20 ns
Not in cache, but in main memory	(0.1) (0.6) = 0.06	60 + 20 = 80 ns
Not in cache or memory	(0.1) (0.4) = 0.04	12000000 + 60 + 20 = 12000080 ns

So, the average access time can be given by,

Average access time = (0.9 × 20) + (0.06 × 80) + (0.04 × 12000080) = 480026 ns

Q.8 (c) Solutions

(i) The following two conditions should be satisfied by $\phi_1(t)$ and $\phi_2(t)$, to be qualified as orthonormal basis functions.

Condition for orthogonality:

$$\int_{-\infty}^{\infty} \phi_1(t)\phi_2(t)\,dt = 0$$

$$\int_0^{1/2} (A)(B)dt + \int_{1/2}^{1} (A)(-B)dt + \int_1^{3/2} (A)(B)dt + \int_{3/2}^{2} (A)(-B)dt = 0$$

$$\frac{AB}{2} - \frac{AB}{2} + \frac{AB}{2} - \frac{AB}{2} = 0$$

So, this condition is satisfied.

Unit Energy:

$$\int_{-\infty}^{\infty} \phi_1^2(t)\,dt = \int_{-\infty}^{\infty} \phi_2^2(t)\,dt = 1$$

$$\int_0^2 A^2 dt = \int_0^2 B^2 dt = 1$$

$$2A^2 = 2B^2 = 1$$

So, $$2A^2 = 1 \Rightarrow A = \frac{1}{\sqrt{2}}$$

$$2B^2 = 1 \Rightarrow B = \frac{1}{\sqrt{2}}$$

(ii) Given that, $Q = 120$, $IF = 465$ kHz

At f_s = 1 MHz,

Image frequency, $f_{si} = f_s + 2IF = [1000 + 2(465)]$ kHz = 1930 kHz

The image frequency rejection ratio,

$$\alpha = \sqrt{1+Q^2\rho^2}$$

where, $$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1930}{1000} - \frac{1000}{1930} = 1.412$$

$\therefore$ $$\alpha = \sqrt{1+(120)^2(1.412)^2} = 169.44$$

At f_s = 30 MHz,

Image frequency $f_{si} = f_s + 2IF = [30 \times 10^6 + 2(465 \times 10^3)]$ Hz = 30.93 MHz

$$\rho = \frac{30.93}{30} - \frac{30}{30.93} = 0.061$$

$\therefore$ The image frequency rejection ratio is,

$$\alpha = \sqrt{1+Q^2\rho^2} = \sqrt{1+(120)^2(0.061)^2} = 7.39$$

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