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Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 12

Section-A

Q.1 (a) Solution:

The current at resonance is,

$$I_0 = \frac{V}{R} \quad \text{...(i)}$$

and the current at any other frequency is,

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{...(ii)}$$

For the given problem,

$$I = \frac{I_0}{2}$$

$$\Rightarrow \quad \frac{I_0}{I} = 2$$

$\therefore$ From equation (i) and (ii),

$$\frac{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}{R} = 2$$

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 4R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = 3R^2 \quad \Rightarrow \quad \omega L - \frac{1}{\omega C} = \pm\sqrt{3}R$$

Since power factor is lagging, circuit is inductive and $\omega L > \dfrac{1}{\omega C}$.

So,

$$\omega L - \frac{1}{\omega C} = \sqrt{3}R$$

$$\omega^2 LC - 1 = \sqrt{3}\,\omega RC$$

$$\omega^2 - \frac{\sqrt{3}R}{L} - \frac{1}{LC} = 0$$

or

$$\omega = \frac{\sqrt{3}R}{2L} \pm \sqrt{\left(\frac{\sqrt{3}R}{2L}\right)^2 + \frac{1}{LC}}$$

Taking positive roots of ω, we get,

$$\omega = \frac{\sqrt{3}R}{2L} + \sqrt{\left(\frac{\sqrt{3}R}{2L}\right)^2 + \frac{1}{LC}}$$

Q.1 (b) Solution:

For the given function F, the truth table is given below. As the input combination with $B = C$ can never occur, this condition can be taken as don't care.

Truth Table:

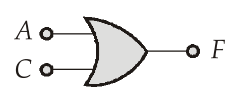
A	B	C	D	F
0	0	0	0	x
0	0	0	1	x
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	x
0	1	1	1	x
1	0	0	0	x
1	0	0	1	x
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	x
1	1	1	1	x

Minimization :

F: AB \ CD	00	01	11	10	
00	X	X	1	1	
01	0	0	X	X	C
11	1	1	X	X	
10	X	X	1	1	A

$$F = A + C$$

Logic circuit :



Q.1 (c) Solution:

(i)
$$R_{\text{series}} = \rho_n \frac{L(n)}{A(n)} + \rho_p \frac{L(p)}{A(p)}$$

$$= \frac{(0.2)\times 10^{-2}}{2\times 10^{-5}} + \frac{(0.1)(10^{-2})}{(2\times 10^{-5})} = 150\Omega$$

For $I_D = 1$ mA,
$$V = I_D R_{\text{series}} + V_T \ln\left(\frac{I_D}{I_0}\right)$$

$$= 1\times 10^{-3}\times 150 + 0.0259\ln\left(\frac{1\times 10^{-3}}{10^{-10}}\right) = 0.567 \text{ V}$$

For $I_D = 10$ mA,
$$V = (10\times 10^{-3})(150) + (0.0259)\ln\left(\frac{10\times 10^{-3}}{10^{-10}}\right) = 1.98 \text{ V}$$

(ii) When series resistance is present, the change in the value of applied voltage is equal to

$$\Delta V = 1.98 - 0.567 = 1.413 \text{ V}$$

thus, percentage change in voltage for change in current is equal to

$$\% \text{ change} = \frac{1.413}{0.567}\times 100 = 249.20\%$$

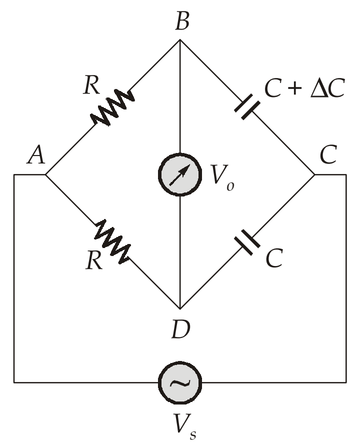
When series resistance is not considered, For $I = 1$ mA, $$V = V_T \ln\left(\frac{I_D}{I_O}\right) = 0.0259\ln\left(\frac{1\times10^{-3}}{10^{-10}}\right) = 0.417 \text{ V}$$

For $I = 10$ mA, $$V = V_T \ln\left(\frac{I_0}{I_0}\right) = 0.0259\ln\left(\frac{10\times10^{-3}}{10^{-10}}\right) = 0.477 \text{ V}$$

$\therefore$ $$\Delta V = 0.477 - 0.417 = 0.06\text{V}$$

$\therefore$ $$\%\text{change} = \frac{\Delta V}{V}\times100 = \frac{0.06}{0.417}\times100 = 14.39\%$$

Q.1 (d) Solution:



Let voltage across "AB" is V_{AB}

$$V_{AB} = \frac{R}{R+\dfrac{1}{j\omega(C+\Delta C)}}V_s = \frac{jR\omega(C+\Delta C)}{jR\omega(C+\Delta C)+1}V_s$$

Let voltage across "AD" is V_{AD}

$$V_{AD} = \frac{R}{R+\dfrac{1}{j\omega C}}V_s = \frac{jR\omega C}{jR\omega C+1}V_s$$

Output voltage, $$V_o = V_{AB} - V_{AD}$$

$$V_o = \frac{jR\omega(C+\Delta C)}{jR\omega(C+\Delta C)+1}V_s - \frac{jR\omega C}{jR\omega C+1}V_s$$

$$V_o = \left[\frac{jR\omega C + jR\omega\Delta C}{jR\omega C + jR\omega\Delta C+1} - \frac{jR\omega C}{jR\omega C+1}\right]V_s$$

$$\frac{V_o}{V_s} = \frac{1+R\omega\Delta C}{1-j+R\omega\Delta C}-\frac{1}{1-j} \qquad \text{(Given } \omega RC = 1\text{)}$$

$$\frac{V_o}{V_s} = \frac{1+R\omega\Delta C}{(1+R\omega\Delta C)-j}-\frac{1}{1-j}$$

$$= \frac{1+R\omega\Delta C-j-jR\omega\Delta C-1-R\omega\Delta C+j}{R\omega\Delta C-2j-jR\omega\Delta C}$$

$$= \frac{-jR\omega\Delta C}{R\omega\Delta C-j(2+R\omega\Delta C)} = \frac{-jR\omega C(\Delta C/C)}{R\omega C(\Delta C/C)-j\left[(2+R\omega C(\Delta C/C)\right]}$$

Since $R\omega C = 1$ and $\dfrac{\Delta C}{C} << 0.01,$

$$\frac{V_o}{V_s} = \frac{-j(\Delta C/C)}{(\Delta C/C)-2j} \simeq \frac{\Delta C}{2C}$$

Q.1 (e) Solution:

(i) The area of B-H curve = Hysteresis loss per cycle

= Area of parallelogram

= base × height

= 400×10^3 A/m × 2 Wb/m^2 = 8×10^5 J/m^3

(ii) $(BH)_{max}$ is derived from the demagnetizing portion (or) second quadrant of the B-H curve in the given figure.

The equation of demagnetizing curve is

$$B = \frac{-1}{200}H+1$$

$$BH = \frac{-1}{200}H^2+H$$

For $(BH)_{max}$

$$\frac{d}{dH}(BH) = -\frac{2}{200}H+1=0$$

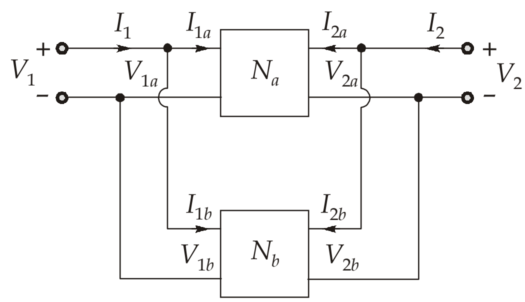
$$H_{max} = +100 \text{ kA/m}$$

$$B_{max} = -\frac{1}{200}(+100)+1 = 0.5 \text{ T}$$

$\therefore$ $(BH)_{max} = 100 \times 0.5 = 50$ kJ/m^3

Q.2 (a) Solution:

Let us consider the parallel connection of the two 2-port network shown below,



Here for network 'N_a' the Y-parameter equations are

$$I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$
$$I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}$$

Similarly for network 'N_b', the Y-parameter equations are

$$I_{1b} = Y_{11b}V_{1b} + Y_{12b}V_{2b}$$
$$I_{2b} = Y_{21b}V_{1b} + Y_{22b}V_{2b}$$

Assuming that the parallel connection can be made, which requires that,

$$V_1 = V_{1a} = V_{1b}$$
$$V_2 = V_{2a} = V_{2b}$$
$$I_1 = I_{1a} + I_{1b}$$
$$I_2 = I_{2a} + I_{2b} \qquad \text{(as shown in figure).}$$

By combining these equations, we get,

$$I_1 = I_{1a} + I_{1b} = (Y_{11a}V_{1a} + Y_{12a}V_{2a}) + (Y_{11b}V_{1b} + Y_{12b}V_{2b})$$
$$= (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

and
$$I_2 = I_{2a} + I_{2b} = (Y_{21a}V_{1a} + Y_{22a}V_{2a}) + (Y_{21b}V_{1b} + Y_{22b}V_{2b})$$
$$= (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

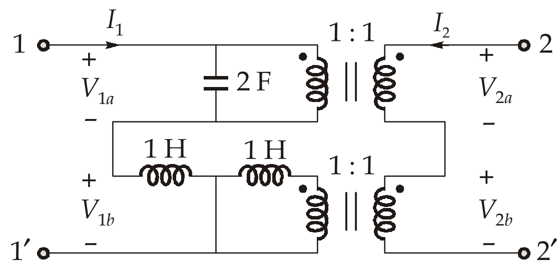
$\therefore$ The parallel connected combined network has Y-parameter matrix as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where,
$$Y_{11} = Y_{11a} + Y_{11b}$$
$$Y_{12} = Y_{12a} + Y_{12b}$$
$$Y_{21} = Y_{21a} + Y_{21b}$$
$$Y_{22} = Y_{22a} + Y_{22b}$$

Thus, for n numbers of two-port networks connected in parallel the overall Y-parameter matrix is simply the sum of Y-matrics of each individual two-port network connected in parallel.

The given circuit circuit can be modified as



For upper part of the circuit,

$$V_{1a} = \frac{1}{2s}I_1 + \frac{1}{2s}I_2 \quad \text{...(i)}$$

$$V_{2a} = \frac{1}{2s}I_1 + \frac{1}{2s}I_2 \quad \text{...(ii)}$$

For lower part of the circuit,

$$V_{1b} = sI_1 + 0I_2 \quad \text{...(iii)}$$

$$V_{2b} = 0I_1 + sI_2 \quad \text{...(iv)}$$

∵ The two networks are connected in series here,

∴ Over all Z parameter will be equal to the sum of individual Z-parameters.

∴

$$V_1 = \left(s + \frac{1}{2s}\right)I_1 + \frac{1}{2s}I_2 \quad \text{...(v)}$$

$$V_2 = \frac{1}{2s}I_1 + \left(s + \frac{1}{2s}\right)I_2 \quad \text{...(vi)}$$

$$[Y] = [Z]^{-1}$$

where,

$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$

$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$

and

$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Here,

$$\Delta Z = \begin{vmatrix} s + \frac{1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & s + \frac{1}{2s} \end{vmatrix}$$

$$= \left(s + \frac{1}{2s}\right)^2 - \frac{1}{4s^2}$$

$$= s^2 + \frac{1}{4s^2} + 2 \times s \times \frac{1}{2s} - \frac{1}{4s^2}$$

$$\Delta Z = s^2 + 1$$

$$\therefore \quad Y_{11} = \frac{2s^2+1}{2s(s^2+1)}$$

$$Y_{21} = -\frac{1}{2s(s^2+1)}$$

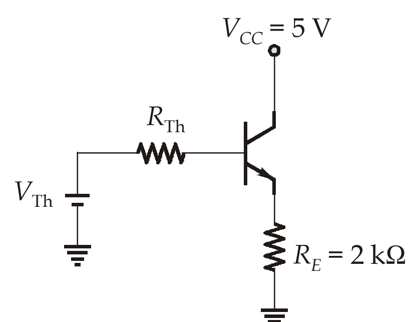
$$Y_{12} = -\frac{1}{2s(s^2+1)}$$

$$Y_{22} = \frac{2s^2+1}{2s(s^2+1)}$$

Q.2 (b) Solution:

To calculate the small signal voltage gain first we have to perform D.C. analysis.

Thus, open circuit the capacitor and taking Thevenins equivallent we get,



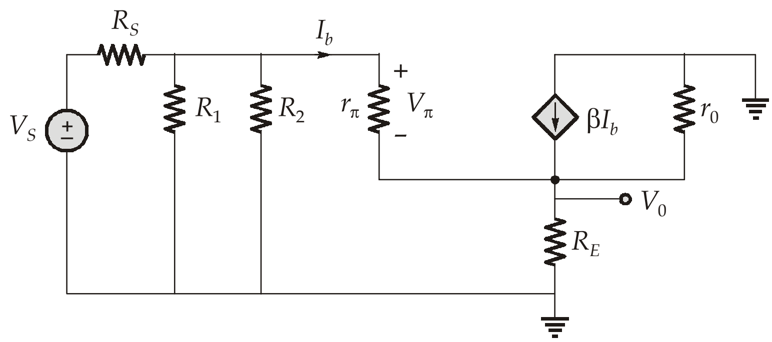
$$V_{Th} = \left(\frac{R_1}{R_1+R_2}\right) \times V_{CC} = \left(\frac{50\text{ k}\Omega}{50\text{ k}\Omega + 50\text{ k}\Omega}\right) \times 5\text{ V} = 2.5\text{ V}$$

and $\quad R_{\text{Th}} = R_1 \parallel R_2 = 50\text{ k}\Omega \parallel 50\text{ k}\Omega = 25\text{ k}\Omega$

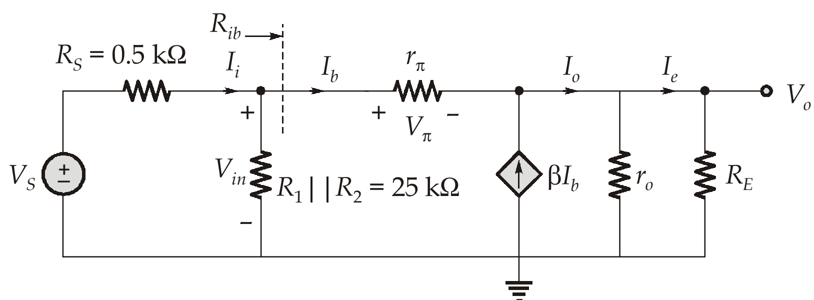
$$\therefore \quad I_B = \frac{V_{Th} - V_{BE(on)}}{R_{Th} + (\beta+1)R_E} = \frac{2.5 - 0.7}{25\text{ k}\Omega + 202\text{ k}\Omega} = 7.93\mu\text{A}$$

$$\therefore \quad I_{CQ} = \beta I_B = 100 \times 7.93 \times 10^{-6} = 0.793\text{ mA}$$

Now, drawing the small signal equivallent we get



the circuit can be redrawn as



now, $$I_0 = (1+\beta)I_b$$

and $$V_0 = I_b(1+\beta)(r_0 \parallel R_E) \quad \text{...(i)}$$

now, writing the KVL at base-emitter loop, we get

$$V_{\text{in}} = I_b[r_\pi + (1+\beta)(r_0 \parallel R_E)]$$

$$R_{ib} = \frac{V_{in}}{I_b} = r_\pi + (1+\beta)(r_0 \parallel R_E)$$

the value of V_{in} can be calculated as

$$V_{\text{in}} = \left(\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_s}\right)V_S \quad \text{...(ii)}$$

and $$I_b = \frac{V_{in}}{r_\pi + (1+\beta)(r_0 \parallel R_E)} \quad \text{...(iii)}$$

thus, combining equation (i), (ii) and (iii) we get,

$$A_V = \frac{V_0}{V_s} = \frac{(1+\beta)(r_0 \parallel R_E)}{r_\pi + (1+\beta)(r_0 \parallel R_E)}\left[\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_1 \parallel R_2 \parallel R_{ib} + R_S}\right]$$

now, $$r_\pi = \frac{V_T\beta}{I_{CQ}} = \frac{(0.026)\times 100}{0.793} = 3.28 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{80}{0.793} \approx 100\ \text{k}\Omega$$

thus, $$R_{ib} = 3.28\ \text{k}\Omega + (101) \times (100\ \text{k}\Omega \parallel 2\ \text{k}\Omega) \simeq 201\ \text{k}\Omega$$

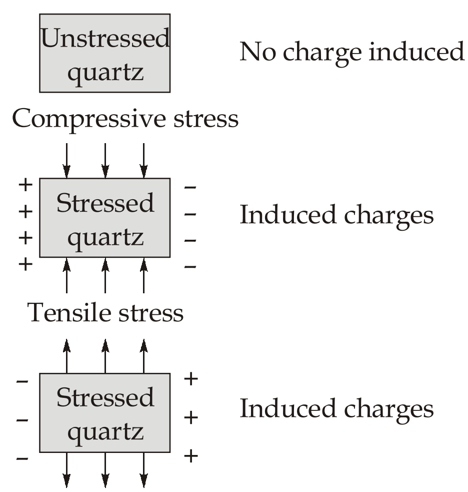
and $$R_1 \parallel R_2 \parallel R_{ib} = 50\ \text{k}\Omega \parallel 50\ \text{k}\Omega \parallel 201\ \text{k}\Omega \simeq 22.2\ \text{k}\Omega$$

$\therefore$ $$A_V = \frac{(101)(100\ \text{k}\Omega \parallel 2\ \text{k}\Omega)}{3.28\ \text{k}\Omega + (101)(100\ \text{k}\Omega \parallel 2\ \text{k}\Omega)} \times \left(\frac{22.2\ \text{k}\Omega}{22.2\ \text{k}\Omega + 0.5\ \text{k}\Omega}\right)$$

$$A_V = 0.962\ \text{V/V}$$

Q.2 (c) Solution:

Physics of piezo-electric material: When a crystal like quartz or tourmaline are stressed along any pair of opposite faces, electric charges of opposite polarity are induced in the opposite faces perpendicular to the stress. This is called piezo-electric effect.



Piezo-electric mechanism

Piezo-electric effects are only exhibited by certain crystals which lack centre of symmetry. In a piezo-electric crystal, the positive and negative electrical charges are separated, but symmetrically distributed, so that the crystal overall is electrically neutral. Each of these sides forms an electric dipole and dipoles near each other tend to be aligned in regions called Weiss domains. The domains are randomly oriented, but can be aligned during poling, a process by which a strong electric field is applied across the material at elevated temperature. When a mechanical stress is applied, this symmetry is distributed and the charge asymmetry generates a voltage across the material e.g. a 01 cm cube of quartz with 2 kN of force can produce a voltage of 12,500 V. Piezo-electric materials also show the opposite effect, where application of electric field creates mechanical deformation in the crystal.

Equivalent circuit and derivation of V_{out} with applied force F : A piezo-electric element used for converting mechanical motion to electrical signal may be thought as a charge generator and a capacitor. Mechanical deformation generates a charge and this charge appears as a voltage across electrode.

Charge induced ∝ Force,

$$Q \propto F$$

$$Q = dF \quad \text{...(i)}$$

d = Charge sensitivity of material in C/N

F = Force applied

For a material of thickness 't', area 'A' and 'E' as Young modulus.

$$F = \frac{AE}{t}\Delta t \quad \text{(in Newton)} \quad \text{...(ii)}$$

As, $$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{\Delta t}{t}$$

So, $$E = \frac{\frac{F}{A}}{\frac{\Delta t}{t}}$$

$\Rightarrow$ $$F = \frac{AE\Delta t}{t} \text{ Newton}$$

From (i) and (ii) we have,

$$Q = (d\,AE)\frac{\Delta t}{t} \quad \text{...(iii)}$$

Voltage is produced by charge across electrode

$$V_{out} = \frac{Q}{C_p} \quad \text{...(iv)}$$

C_p = Capacitance between electrode

$$C_p = \frac{\epsilon_r \epsilon_0 A}{t} \quad \text{...(v)}$$

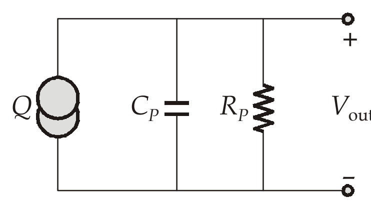
From (i), (iv) and (v), $V_{out} = \dfrac{Q}{C_p} = \dfrac{dF}{\epsilon_0 \epsilon_r A/t} = \dfrac{dt}{\epsilon_r \epsilon_0} \dfrac{F}{A}$

$$\therefore \qquad V_{out} = \frac{d}{\epsilon_r \epsilon_0} \cdot t \cdot \frac{F}{A} = \frac{gtF}{A}$$

'g' is the voltage sensitivity of crystal. This is constant for a given crystal cut. Its unit is V-m/N

$$V_{out} = \frac{gtF}{A}$$

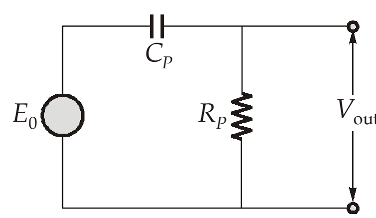
The basic equivalent circuit of a piezo-electric material transducer is as under:



The source is a charge generator. The value of the charge is

$$Q = dF$$

The charge generator can be replaced by an equivalent voltage source.



$$V_{out} = \frac{Q}{C_p} = \frac{dF}{C_p}$$

In series with C_p and R_p, R_p is the leakage resistance.

Material with piezo-electric property:

1. **Quartz:** Its early application was used in phonograph pickups. It is also used as crystal oscillator, quartz crystal micro balance (used for measurement of small mass changes).
2. **Potassium sodium tartrate tetrahydrate (Rochelle salt):** It is used in transducer application such as ultrasonic generator, microphones and electromechanical resonators.
3. **Barium titanate:** Used for industrial cleansing apparatus, underwater detection system like sonar.

4. **Lithium Tantalate ($LiTaO_3$):** It finds application in passive infrared sensors such as motion detector, surface acoustic wave application.

Charge sensitivity, $d = 2$ pC/N

Gain of amplifier = 5 mV/pC

Sensitivity with amplifier, 1 pC = 5 mV

2 pC = 10 mV

So sensitivity = 10 mV/N ...(i)

Also,chart recorder sensitivity = 25 mm/V ...(ii)

From equation (i) and (ii), $1\text{ N} = 25 \times 10^{-5}\text{ m} = 250\ \mu\text{m}$

Overall sensitivity = 250 μm/N

For overall sensitivity, we need to calculate mm displacement when 1 N force is applied on crystal. 1 N force develops 10 mV after amplifier.

$$\text{10 mV would displace} = (10\text{ mV})\left(25\frac{\text{mV}}{\text{V}}\right) = 250\ \mu\text{m}$$

So, Overall sensitivity = 250 μm/N

Q.3 (a) Solution:

(i) Hysteresis loss, $H \propto f$ and Eddy current loss, $E \propto f^2$, where f is frequency.

$$H = K_1 f \text{ and } E = K_2 f^2$$

At 50 Hz iron losses = 2500 W

$$2500 = K_1 50 + K_2 50^2 = 50(K_1 + K_2 50)$$

$$K_1 + K_2 50 = 50 \qquad \text{...(i)}$$

At 25 Hz iron loss = 850 W

$$850 = K_1 25 + K_2 25^2 = 25(K_1 + K_2 25)$$

$$K_1 + K_2 25 = 34 \qquad \text{...(ii)}$$

By solving equations (i) and (ii)

$$K_1 = 18 \quad ; \quad K_2 = 0.64$$

At 50 Hz, Hysteresis loss = $K_1 50 = 18 \times 50 = 900$ W

Eddy current loss = $K_2 50^2 = 0.64 \times 50^2 = 1600$ W

(ii) Synchronous speed = $N_s = \dfrac{120 \times f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

Since the winding is full pitched so, $K_p = 1$

$$K_d = \text{Distribution factor} = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}}$$

where,
$$m = \frac{\frac{\text{Slots}}{\text{Pole}}}{\text{Phase}} = \frac{36}{4\times 3} = 3$$

$$\beta = \frac{180°}{\frac{\text{Slots}}{\text{Pole}}} = \frac{180°}{\frac{36}{4}} = 20°$$

$$K_d = \frac{\sin\frac{60°}{2}}{3\sin\frac{20°}{2}} = 0.96$$

$$\text{Number of turns per phase } (T) = \frac{Z}{2\times 3} = \frac{36\times 30}{2\times 3} = 180$$

$$\text{Induced phase voltage} = 4.44 \times K_p \times K_d \times \phi f T$$
$$= 4.44 \times 1 \times 0.96 \times 0.05 \times 50 \times 180 = 1918.08 \text{ V}$$

$$E_{\text{rms } L\text{-}L} = \sqrt{3}\times 1918.08 = 3322 \text{ V}$$

Q.3 (b) Solution:

(i) From the figure it is clear that accumulation occurs for a positive V_G and inversion occurs for a negative value of V_G, thus the given type of semiconductor is *n*-type.

(ii) Capacitance per unit area can be calculated as

$$C_{ox} = \frac{200\times 10^{-12}}{2\times 10^{-3}} = 1 \times 10^{-7} \text{ F/cm}^2,$$

now, the value of C_{ox} can also be calculated as

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad \text{(for oxide only)}$$

$$\therefore \quad t_{ox} = \frac{\varepsilon_{ox}}{C_{ox}} = \frac{3.9\times 8.85\times 10^{-14}}{10^{-7}}$$

$$\therefore \quad t_{ox} = 3.45 \times 10^{-6} \text{ cm}$$
$$= 345 \text{ A°}$$

(iii) The flot-band voltage can be given as

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

where Q'_{ss} is trapped charge in the oxide layer and ϕ_{ms} is the metal oxide work function difference.

$$\therefore \quad -0.8 = -0.5 - \frac{Q'_{ss}}{10^{-7}}$$

$$\therefore \quad Q'_{ss} = 3 \times 10^{-8}\ \text{C/cm}^2$$

(iv) The approximate value of flot-band capacitance per unit area can be calculated as

$$C'_{FB} = \frac{\varepsilon_{ox}}{t_{ox} + \frac{\varepsilon_{ox}}{\varepsilon_{si}}\sqrt{\left(\frac{KT}{q}\right)\left(\frac{\varepsilon_{si}}{qN_A}\right)}}$$

$$= \frac{(3.9)\times(8.85\times10^{-14})}{3.45\times10^{-6} + \left(\frac{3.9}{11.7}\right)\sqrt{(0.025)\left[\frac{(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(2\times10^{16})}\right]}}$$

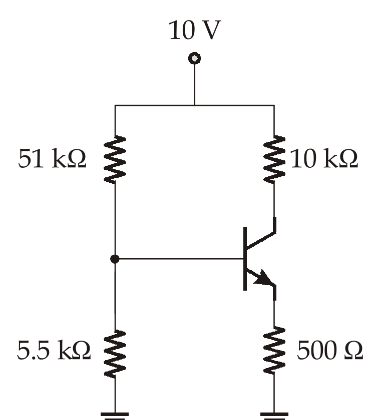
$$= 7.82 \times 10^{-8}\ \text{F/cm}$$

$\therefore$ Total flot-band capacitance $= A_{ox}\, C'_{FB}$

$$= 2 \times 10^{-3} \times 7.82 \times 10^{-8} = 156\ \text{pF}$$

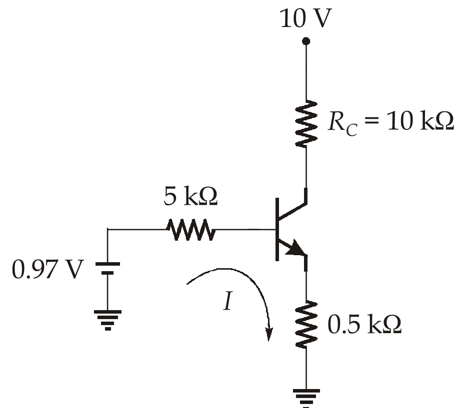
Q.3 (c) Solution:

To draw the small signal equivalent model, we need to calculate the D.C. parameters. Thus, the D.C. equivalent circuit can be drawn as



thus, $$V_{th} = 10\times\left(\frac{5.5}{5.5+51}\right) = 0.97\text{ V and } R_{th} = 5.5 \parallel 51 = 5\text{ k}\Omega$$

thus,



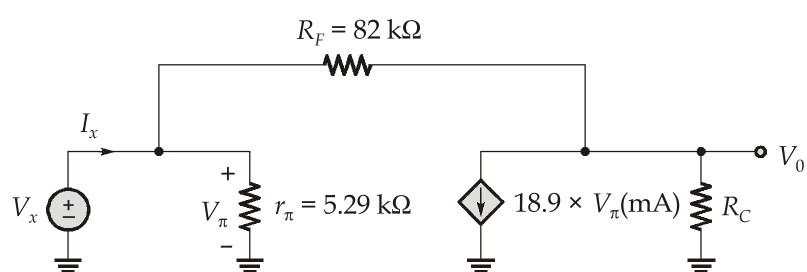
$$I_b = \frac{0.97-0.7}{5\text{ k}\Omega+50.5\text{ k}\Omega} = 4.9\ \mu\text{A}$$

$$I_c = 0.49\text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100\times0.026}{0.491} = 5.29\text{ k}\Omega$$

$$g_m = \frac{0.491}{0.026} \approx 18.9\text{ mA/V}$$

Input impedance



Writing a KCL equation of the input, we have

$$I_x = \frac{V_\pi}{r_\pi}+\frac{V_\pi-V_0}{R_F}$$

$$I_x = \frac{V_\pi}{5.29}+\frac{V_\pi-V_0}{82} \quad \text{...(i)}$$

Now, applying KCL at output node we get

$$\frac{V_0}{R_C}+g_mV_\pi+\frac{V_0-V_\pi}{R_F} = \frac{V_0}{10}+(18.9)V_\pi+\frac{V_0-V_\pi}{82} = 0$$

$$\frac{V_0}{10} + (18.9)V_\pi + \frac{V_0 - V_\pi}{82} = 0$$

$$(18.9)V_\pi - \frac{V_\pi}{82} = -\left(\frac{V_0}{10} + \frac{V_0}{82}\right)$$

$$\Rightarrow \quad 18.88\ V_\pi = -0.112\ V_0$$

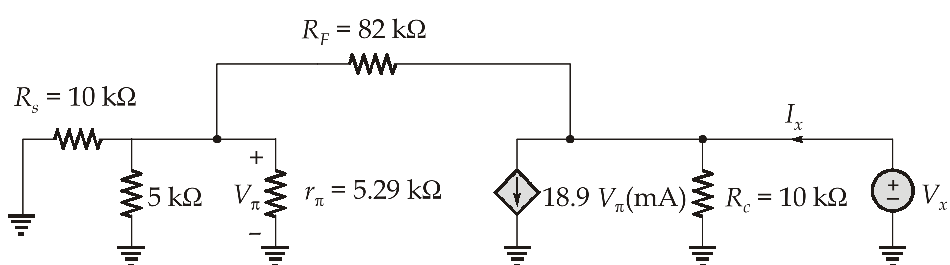
$$V_0 = -168.5\ V_\pi$$

Sub. in equation (1) we get

$$I_x = \frac{V_\pi}{5.29} + \frac{169.5}{82}V_\pi$$

$$R_{if} = \frac{V_x}{I_x} = 0.443\ \text{k}\Omega = 443\ \Omega \qquad (\because V_x = V_\pi)$$

For output resistance we can draw small signal model as



Assume $\quad R_{eq} = r_\pi \parallel R_1 \parallel R_2 \parallel R_s$

$$R_{eq} = 2.04\ \text{k}\Omega$$

Applying KCL at output we get

$$I_x = \frac{V_x}{R_c} + g_m V_\pi + \frac{V_x}{R_F + R_{eq}}$$

and

$$V_\pi = \left(\frac{R_{eq}}{R_{eq} + R_F}\right)V_x$$

$\therefore$

$$I_x = \frac{V_x}{R_c} + g_m\left[\frac{R_{eq}}{R_{eq} + R_f}\right]V_x + \frac{V_x}{R_F + R_{eq}}$$

$$I_x = \frac{V_x}{10} + 18.9V_x \times \left[\frac{2.04}{2.04 + 82}\right] + \frac{V_x}{82 + 2.04}$$

$$R_{of} = \frac{V_x}{I_x} \approx 1.75\ \text{k}\Omega$$

Q.4 (a) Solution:

A practical dc voltage source can be drawn as

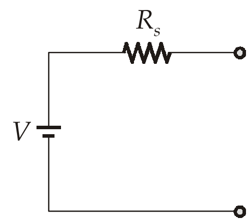


Fig. (i)

Where, R_s = internal resistance

At first, when it is momentarily short circuited, the circuit can be modified as

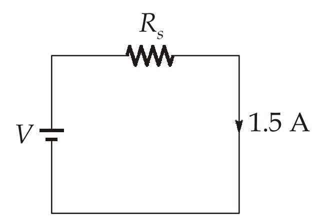


Fig. (ii)

and when connected to 5 Ω load, the circuit can be modified as

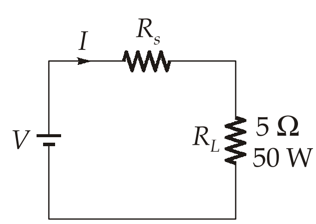


Fig. (iii)

From figure (ii),

$$\frac{V}{R_s} = 7.5 \text{ A} \quad \text{...(i)}$$

and from figure (iii),

$$I^2R_L = \left(\frac{V}{R_s+R_L}\right)^2 \times R_L \quad \text{...(ii)}$$

$$= \frac{V^2}{(R_s+R_L)^2} \times R_L = 50$$

or $$\frac{V^2}{(R_s+5)^2} \times 5 = 50$$

or $$\frac{V^2}{(R_s+5)^2} = 10$$

$\Rightarrow$ $$V^2 = 10(R_s + 5)^2 \quad \text{...(iii)}$$

On solving above equation, we get, (from (i) and (iii))

$$10(R_s + 5)^2 = R_s^2 \times (7.5)^2$$

$$10(R_s^2 + 25 + 10R_s) = 56.25\,R_s^2$$

$$10\,R_s^2 + 250 + 100R_s = 56.25\,R_s^2$$

or $-46.25\,R_s^2 + 100R_s + 250 = 0$

$$R_s = 3.645\ \Omega \text{ or } -1.483\ \Omega \quad \text{...(iv)}$$

Considering, $R_s = 3.645\ \Omega$

(i) The open circuit voltage becomes,

from equation (i),

$$V = 7.5 \times 3.645 = 27.337\text{ V}$$

(ii) The value of R_L to which it can deliver a maximum power is equal to that of R_s

$\therefore$ $$R_L = R_s = 3.645\ \Omega$$

and the maximum power delivered to this R_L is

$$P_{\max} = \left(\frac{V}{R_s + R_L}\right)^2 \times R_L$$

$$= \frac{V^2}{(2R_L)^2} \times R_L \qquad \because R_L = R_s$$

$$P_{\max} = \frac{V^2}{4R_L} = \frac{(27.337)^2}{4 \times 3.645} = \frac{747.31}{14.58} = 51.26\text{ W}$$

Q.4 (b) Solution:

$$\text{MOD} = 5$$

$\therefore$ $$2^n \geq N$$

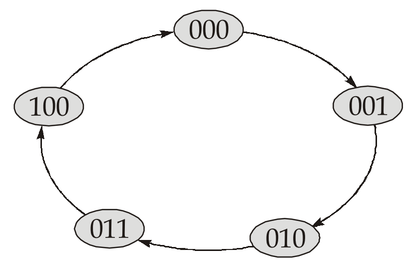
$\because$ $$2^n \geq 5, \quad \text{thus } n = 3$$

thus, three flip-flops are required to implement the counter.

Given Q_A = LSB

Q_C = MSB

Thus, the output state diagram of the counter will be



Excitation table of the JK flip-flop.

Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation table for JK flip-flop.

Present state			**Next state**			**Flip-flop-input**					
Q_C	Q_B	Q_A	Q_C^+	Q_B^+	Q_A^+	J_C	K_C	J_B	K_B	J_A	K_A
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	X	X	X	X	X	X	X	X	X
1	1	0	X	X	X	X	X	X	X	X	X
1	1	1	X	X	X	X	X	X	X	X	X

Map simplification

$Q_C \backslash Q_BQ_A$	00	01	11	10
0	0	0	1	0
1	X	X	X	X

$J_C = Q_BQ_A$

$Q_C \backslash Q_BQ_A$	00	01	11	10
0	X	X	X	X
1	1	X	X	X

$K_C = 1$

$Q_C \backslash Q_BQ_A$	00	01	11	10
0	0	1	X	X
1	0	X	X	X

$J_B = Q_A$

$Q_C \backslash Q_BQ_A$	00	01	11	10
0	X	X	1	0
1	X	X	X	X

$K_B = Q_A$

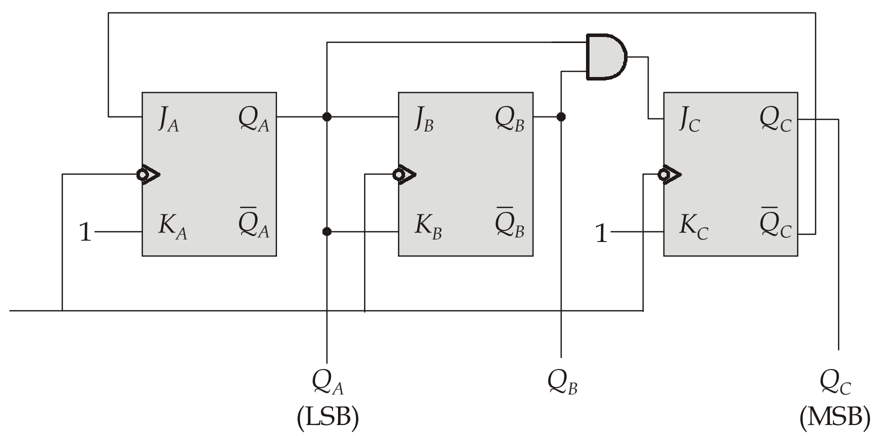
$Q_C \backslash Q_BQ_A$	00	01	11	10
0	1	X	X	1
1	0	X	X	X

$J_A = \bar{Q}_C$

$Q_C \backslash Q_BQ_A$	00	01	11	10
0	X	1	1	X
1	X	X	X	X

$K_A = 1$

Logic Diagram (Considering that the flip flops are negatively edge triggered).



Q.4 (c) Solution

(i)

If D is the density, M_{at} is the atomic mass and N_A is Avogadro's number, then the atomic concentration n_{at} is

$$n_{at} = \frac{DN_A}{M_{at}} = \frac{(971.2\,\text{kgm}^{-3})(6.022\times10^{23}\,\text{mol}^{-1})}{(22.99\times10^{-3}\,\text{kg mol}^{-1})}$$

$\therefore$ $$n_{at} = 2.544 \times 10^{28}\ \text{m}^{-3}$$

Which is also the electron concentration, given that each Na atom contributes 1 conduction electron. If d is the mean separation between the electrons then d and n_{at} are related as

$$d \approx \frac{1}{n_{at}^{1/3}} = \frac{1}{(2.544\times10^{28}\,\text{m}^{-3})^{1/3}}$$

$$d = 3.40 \times 10^{-10}\ \text{m or } 0.34\ \text{nm}$$

(ii)

Na is BCC with 2 atoms in the unit cell. So if 'a' is the lattice constant (side of the cubic unit cell), the density is

$$D = \frac{(\text{atoms in unit cell})(\text{mass of 1 atom})}{\text{Volume of unit cell}}$$

$$D = \frac{2\left(\frac{M_{at}}{N_A}\right)}{a^3}$$

$$a = \left[\frac{2M_{at}}{DN_A}\right]^{1/3}$$

$$a = \left[\frac{2\left(22.99\times10^{-3}\,\text{kg mol}^{-1}\right)}{\left(0.9712\times10^{3}\,\text{kgm}^{-3}\right)\left(6.022\times10^{23}\,\text{mol}^{-1}\right)}\right]^{1/3}$$

$$a = 4.284 \times 10^{-10} \text{ m or } 0.4284 \text{ nm}$$

For BCC structure, the radius of the metal ion R and the lattice parameter a are related by $(4R)^2 = 3a^2$

$\therefore$
$$R = \frac{1}{4}\sqrt{3a^2} = \frac{1}{4}\sqrt{3\times\left(0.4284\times10^{-9}\right)^2}$$

$$R = 0.1855 \text{ nm}$$

The mean separation of electron between two metal ions is equal to R.

$\therefore$ $d_{\text{electron-ion}} = R = 0.1855$ nm

(iii)

The electrical conductivity of Na is

$$\sigma = n_{at}e\mu$$

$$= (1.6 \times 10^{-19})\,(2.544 \times 10^{28})(53 \times 10^{-4})$$

$$\sigma = 2.16 \times 10^{7}(\Omega\,\text{m})^{-1}$$

Section-B

Q.5 (a) Solution

From the plot, intercept of the line for $(1/T) = 0$ is about 0.0008, and the slope of 0.6 per degree Kelvin.

so, $N(\alpha_e + \alpha_i) = (0.0008)\epsilon_0$

or Polarizability $= \alpha_e + \alpha_i$

$$= \frac{(0.0008)\times 8.854\times 10^{-12}}{2.5\times 10^{25}} = 2.83 \times 10^{-40}\ \text{Fm}^2$$

and $$\frac{Np_p^2}{3k} = (0.6)\in_0$$

$$p_p^2 = \frac{0.6\times 8.854\times 10^{-12}\times 3\times 1.38\times 10^{-23}}{2.5\times 10^{25}} = 8.810 \times 10^{-60}$$

Permanent dipole moment $p_p = 2.968 \times 10^{-30}$ coulomb meter

Q.5 (b) Solution:

(i) Total resistance of resistors connected in parallel and neglecting their errors is,

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{250}+\frac{1}{500}+\frac{1}{375}} = 115.38\ \Omega$$

(ii) The fractional error in $R_1 = 250\ \Omega$ is +0.025

$\therefore$ $\delta R_1 = (0.025 \times 250) = 6.25\ \Omega$

Hence, $R_1 = 250 + 6.25 = 256.25\ \Omega$

Similarly, $\delta R_2 = (-0.036 \times 500) = -18\ \Omega$

and $R_2 = 500 - 18 = 482\ \Omega$

$\therefore$ $\delta R_3 = (+0.014 \times 375) = 5.25\ \Omega$

and $R_3 = 375 + 5.25 = 380.25\ \Omega$

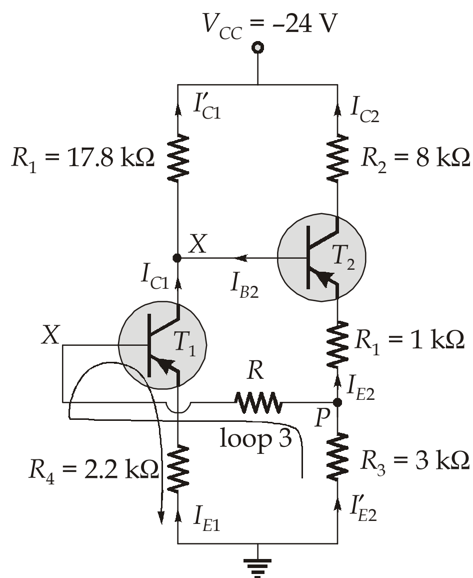
$\therefore$ The resultant resistance of three resistances in parallel is,

$$R'_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{256.25}+\frac{1}{482}+\frac{1}{380.25}} = 116.18\,\Omega$$

(iii) The fractional error of the parallel resistances based on the rated values is

$$\frac{R'_T - R_T}{R_T}\times 100 = \frac{116.18-115.38}{115.38}\times 100 = +6.93\%$$

Q.5 (c) Solution:



Applying KVL of collector-emitter loop for transistor T_1 we get,

$$(2.2\text{ k}\Omega)I_{E1} + V_{EC} + (17.8\text{ k}\Omega)\ I'_{C1} - 24 = 0$$

$$
\begin{aligned}
20 &= (2.2\text{ k}\Omega)\ I_{E1} + (17.8\text{ k}\Omega)\ I'_{C1} \\
&= (2.2\text{ k}\Omega)\ I_{E1} + (17.8\text{ k}\Omega)\ (I_{C1} + I_{B2}) \\
&= (2.2 \times 101 \times 10^3)\ I_{B1} + 17.8 \times 100 \times 10^3\ I_{B1} + 17.8 \times 10^3\ I_{B2} \\
&= 2002.2 \times 10^3\ I_{B1} + 17.8 \times 10^3\ I_{B2} \qquad \text{...(i)}
\end{aligned}
$$

Applying KVL at collector-emitter loop for transistor T_2 we get

$$(3\text{ k}\Omega)I'_{E2} + (1\text{ k}\Omega)I_{E2} + V_{EC} + (8\text{ k}\Omega)I_{C2} - 24 = 0$$

$$
\begin{aligned}
18 &= (3\text{ k}\Omega)\ (I_{E2} + I_{B1}) + (1\text{ k}\Omega)\ I_{E2} + 8\text{ k}\Omega\ I_{C2} \\
&= (3\text{ k}\Omega \times 101)\ I_{B2} + (3\text{ k}\Omega)\ I_{B1} + (101\text{ k}\Omega)\ I_{B2} + (800\text{ k}\Omega)\ I_{B2} \\
&= (1204\text{ k}\Omega)\ I_{B2} + (3\text{ k}\Omega)\ I_{B1} \qquad \text{...(ii)}
\end{aligned}
$$

$\therefore$

$$
\begin{aligned}
I_{B1} &= 9.85\ \mu\text{A} \\
I_{B2} &= 14.92\ \mu\text{A}
\end{aligned}
$$

Applying KVL in loose emitter loop (loop 3) we get

$$
\begin{aligned}
I_{B1}.\ R &= (3\times10^3)I'_{E2} - (2.2\times10^3)I_{E1} - 0.7\text{ V} \\
&= (3\text{ k}\Omega)\ [(\beta + 1)\ I_{B2} + I_{B1}] - 2.2\text{ k}\Omega \times (101)I_{B1} - 0.7 \\
&= (3 \times 10^3)[101 \times 14.9 \times 10^{-6} + 9.85 \times 10^{-6} - 2.2 \times 101 \times 9.85 \\
&\quad \times 10^{-6} \times 10^3 - 0.7]
\end{aligned}
$$

$$= 4.544 - 2.889 = 1.655$$

$$R = \frac{1.655}{9.85} \times 10^6 = 168 \text{ k}\Omega$$

Q.5 (d) Solution:

$$\text{Area} = 10^{-3} \text{ m}^2$$

$$l = 2\pi r = 2\pi \times 10 \times 10^{-2} = 0.6283 \text{ m}$$

$$i_1(t) = 3\sin 100\pi t \text{ A}$$

$$N_1 = 200 \text{ turns} ; N_2 = 100 \text{ turns}$$

$$\phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{NI}{\left[\dfrac{l}{\mu_o \mu_r A}\right]} = \frac{200 \times 3 \sin 100\pi t}{\left[\dfrac{0.6283}{4\pi \times 10^{-7} \times 500 \times 10^{-3}}\right]}$$

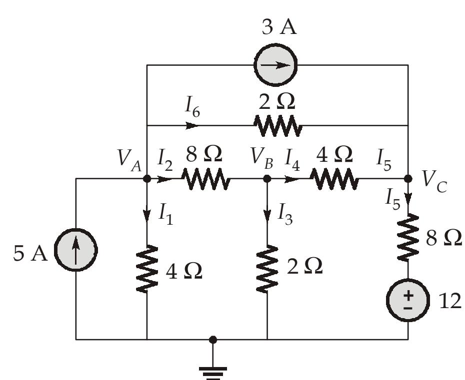
$$\phi = (6 \times 10^{-4}) \sin 100\pi t \text{ Wb}$$

$$\text{Induced emf} = -N_2 \frac{d\phi}{dt} = -100 \times \frac{d}{dt}[(6 \times 10^{-4}) \sin 100\pi t]$$

$$= -100 \times 6 \times 10^{-4} \times 100\pi \cos 100\pi t = -6\pi \cos 100\pi t \text{ V}$$

Q.5 (e) Solution:

Considering the given circuit as follows:



By applying KCL at node "A", we get,

$$I_1 + I_2 + I_6 + 3 \text{ A} = 5 \text{ A}$$

$$\frac{V_A}{4\,\Omega} + \frac{V_A - V_B}{8\,\Omega} + \frac{V_A - V_C}{2\,\Omega} = 2 \text{ A}$$

$$2V_A + V_A - V_B + 4V_A - 4V_C = 16 \text{ V}$$

$$7V_A - V_B - 4V_C = 16 \text{ V} \qquad \text{...(i)}$$

By applying KCL at node "B", we get,

$$I_2 = I_3 + I_4$$

$$\frac{V_A - V_B}{8\,\Omega} = \frac{V_B}{2\,\Omega} + \frac{V_B - V_C}{4\,\Omega}$$

$$V_A - V_B = 4V_B + 2V_B - 2V_C$$

$$V_A - 7V_B + 2V_C = 0 \qquad \text{...(ii)}$$

By applying KCL at node "C", we get,

$$I_5 = I_4 + I_6 + 3\text{ A}$$

$$\frac{V_C - 12\text{ V}}{8\,\Omega} = \frac{V_B - V_C}{4\,\Omega} + \frac{V_A - V_C}{2\,\Omega} + 3\text{ A}$$

$$V_C - 12\text{ V} = 2V_B - 2V_C + 4V_A - 4V_C + 24\text{ V}$$

$$-4V_A - 2V_B + 7V_C = 36\text{ V} \qquad \text{...(iii)}$$

By solving equations (i), (ii) and (iii), we get, $V_A = 10$ V, $V_B = 4.933$ V and $V_C = 12.267$ V.

Q.6 (a) Solution:

(i) Phase voltage $= \dfrac{1200}{\sqrt{3}} = 692.82\text{ V}$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = E_1 \times \frac{N_2}{N_1} = \frac{692.82 \times 1}{3.75} = 184.75\text{ V}$$

$$I_2 = \frac{E_{2\,\text{ph}}}{\sqrt{R_2^2 + X_2^2}} = \frac{184.75}{\sqrt{(0.016)^2 + (2\pi \times 50 \times 0.8 \times 10^{-3})^2}}$$

$$I_2 = 733.61\text{ A}$$

(ii) Rotor p.f. at starting, $\cos\phi = \dfrac{R_2}{\sqrt{R_2^2 + X_2^2}} = \dfrac{0.016}{\sqrt{(0.016)^2 + (0.251)^2}}$

$$\text{p.f.} = 0.064\text{ lagging}$$

(iii) $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1}$

$$I_2 = \frac{I_1 \times N_1}{N_2} = \frac{125 \times 3.75}{1} = 468.75\text{ A}$$

$$I_2 = 468.75 = \frac{E_2}{\sqrt{(R_2+r)^2 + X_2^2}}$$

$$468.75 = \frac{184.75}{\sqrt{(0.016+r)^2 + (0.251)^2}}$$

$$r = 0.288\ \Omega/\text{phase}$$

(iv)

$$I_2 = \frac{sE_2}{\sqrt{R^2 + (sX_2)^2}} = \frac{0.04 \times 184.75}{\sqrt{(0.016)^2 + (0.04 \times 0.251)^2}}$$

$$I_2 = 391.23\text{ A}$$

(v) Rotor p.f. at 4% slip,

$$\cos\phi = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} = \frac{0.016}{\sqrt{(0.016)^2 + (0.04 \times 0.251)^2}}$$

$$= 0.847\text{ lagging}$$

Q.6 (b) Solution:

$$V_{TN} = (|Q'_{SD}(\max)| - Q'_{SS})\left(\frac{t_{ox}}{\varepsilon_{ox}}\right) + 2\phi_{fp} + \phi_{MS}$$

we find that

$$\phi_{fP} = V_T \ln\left(\frac{N_a}{n_i}\right) = 0.0259 \ln\left[\frac{2 \times 10^{15}}{1.5 \times 10^{10}}\right]$$

$$= 0.306\text{ V}$$

and

$$x_d = \sqrt{\frac{4 \times 11.7 \times 8.854 \times 10^{-14} \times 0.306}{(1.6 \times 10^{-19})(2 \times 10^{15})}} \simeq 0.629\ \mu\text{m}$$

$$\therefore \quad Q'_{SD}(\max) = qN_D x_d$$

$$= (1.6 \times 10^{-19})\ (2 \times 10^{15})(0.629 \times 10^{-4})$$

$$|Q'_{SD}(\max)| = 2.01 \times 10^{-8}\text{ C/cm}^2$$

$$Q'_{SS} = (2 \times 10^{11})(1.6 \times 10^{-19})$$

$$Q'_{SS} = 3.2 \times 10^{-8}\text{ C/cm}^2$$

$$\therefore \quad V_{TN} = \frac{(2.01 \times 10^{-8} - 3.2 \times 10^{-8})(450 \times 10^{-8})}{(3.9)(8.854 \times 10^{-14})} + 2(0.306) + \phi_{ms}$$

$$V_{TN} = 0.457 + \phi_{ms}$$

(i) For aluminium gate ϕ_{ms} = –0.916 V (given)

$\therefore$ $$V_{TN} = 0.457 - 0.916$$

$\therefore$ $$V_{TN} = -0.459 \text{ V}$$

(ii) For n^+ polygate structure,

$$\phi_{ms} = \phi_{ss} = -0.866\text{V}$$

$\therefore$ $$V_{TN} = 0.457\text{V} - 0.866\text{V} = -0.409\text{V}$$

Q.6 (c) Solution:

(i) $$f(A, B, C, D) = AB\bar{C} + \bar{B}\bar{D} + C\bar{D} + \bar{B}C$$

$$f(A, B, C, D) = AB\bar{C}(D+\bar{D}) + (A+\bar{A})\bar{B}(C+\bar{C})\bar{D} + (A+\bar{A})(B+\bar{B})C\bar{D}$$
$$+ (A+\bar{A})\bar{B}C(D+\bar{D})$$

thus, $$f(A, B, C, D) = AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D}$$
$$+ A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$$
$$+ \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$$

$\therefore$ $$f(A, B, C, D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D}$$
$$+ A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D}$$

$$f(A, B, C, D) = m_0 + m_2 + m_3 + m_6 + m_8 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14}$$

but m_3, m_8 and m_{14} are don't care condition

$\therefore$ $$\sum_K m_K = \Sigma m(0, 2, 6, 10, 11, 12, 13)$$

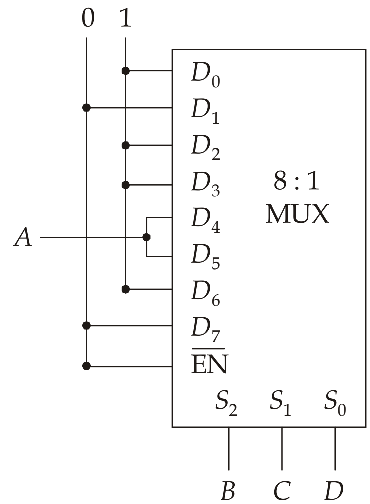
$\therefore$ $$f(A, B, C, D) = \Sigma m(0, 2, 6, 10, 11, 12, 13) + d(3, 8, 14)$$

(ii) Taking select lines as $S_2 = B; S_1 = C; S_0 = D$

We get

	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
$\bar{A}$	0	1	2	3	4	5	6	7
A	8	9	10	11	12	13	14	15
	1	0	1	1	A	A	1	0

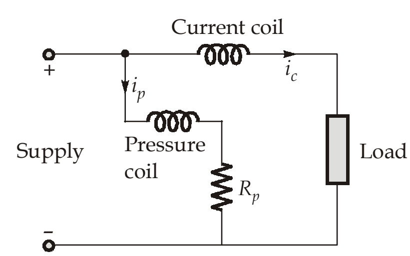
$\therefore$ Thus, the logic implementation can be expressed as



Q.7 (a) Solution:

Electrical power: Dynamometer type wattmeter consists of two coils i.e. fixed coil or field coils connected in series with load and pressure coil or voltage coil connected across supply voltage. The schematic is shown as under.

The instantaneous torque for dynamometer instrument is



$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

Let V and I are rms value of voltage and current being measured,

$$v = \sqrt{2}\, V \sin \omega t$$

If pressure coil (P.C.) is highly resistive then,

$$i_P = \frac{v}{R_P} = \left(\frac{\sqrt{2}V}{R_P}\right) \sin \omega t = i_1$$

$$I_P = \frac{V_P}{R_P} = \frac{V}{R_P}$$

$$R_P = \text{ Resistance of P.C.}$$

Similarly, $\quad i_C = \sqrt{2}\, I \sin(\omega t - \phi) = i_2 \quad$ (Assuming inductive load)

ϕ = Current in current coil (C.C.) lags voltage by angle ϕ

$$\therefore \quad T_i = \sqrt{2} I_P \sin \omega t \cdot \sqrt{2} I \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$= 2 I_P I \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$T_i = I_P I(\cos\phi - \cos(2\omega t - \phi))\frac{dM}{d\theta}$$

There is a component of power which varies twice the frequency of current and voltage.

Average deflection torque,

$$T_d = \frac{1}{T}\int_0^T T_i\, d(\omega t)$$

$$T_d = \frac{1}{T}\int_0^T I_P I[\cos\phi - \cos(2\omega t - \phi)]\frac{dM}{d\theta}d(\omega t) = I_P I\cos\phi\frac{dM}{d\theta}$$

$$T_d = \left(\frac{VI}{R_P}\right)\cos\phi\frac{dM}{d\theta}$$

Controlling torque, $T_C = K\theta$

K = Spring constant

θ = Final steady deflection

At balance, $T_C = T_d$

$$K\theta = \left(\frac{VI}{R_P}\right)\cos\phi\frac{dM}{d\theta}$$

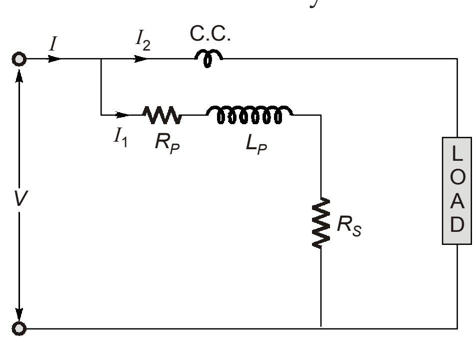
$$\theta = \left(\frac{VI\cos\phi}{R_P K}\right)\frac{dM}{d\theta} = \left(K_1\frac{dM}{d\theta}\right)P$$

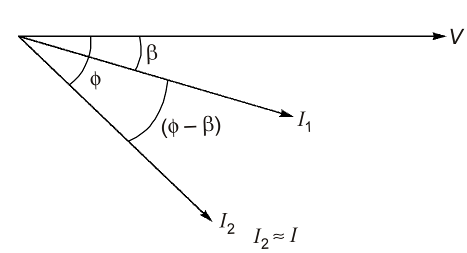
P = Power measured

$$K_1 = \frac{1}{R_P K}$$

So deflection, $\theta = \left(K_1\frac{dM}{d\theta}\right)P \Rightarrow \theta \propto P$ (Power)

Figure shows an electrodynamometer wattmeter with pressure coil inductance.





Wattmeter with Pressure Coil Inductance:

Let L = Pressure coil inductance

R_p = Pressure coil resistance

Due to L_p, I_1 lags V by angle β

where, $$\beta = \tan^{-1}\left(\frac{\omega L_p}{R_s}\right) \quad (R_p + R_s \approx R_s)$$

$$Z_p = \sqrt{(R_p + R_s)^2 + (\omega L_p)^2} \quad \text{(Impedance of p.c.)}$$

Now, deflecting torque,

$$T_d = I_1 I_2 \cos(\phi - \beta)\frac{dM}{d\theta}$$

Also, $$I_1 = \frac{V}{Z_p} \text{ and } I_2 \approx I$$

$$\cos\beta = \frac{R_s}{Z_p} \quad \text{or} \quad Z_p = \frac{R_s}{\cos\beta}$$

$\therefore$ $$T_d = \frac{VI\cos\beta}{R_s}\cos(\phi-\beta)\frac{dM}{d\theta}$$

Deflecting torque equation with p.c. inductance

Now, when there is no p.c. inductance then,

True value of deflecting torque is

$$T_d = \frac{VI\cos\phi}{R_s}\frac{dM}{d\theta}$$

At equilibrium, $T_c = T_d$

$\therefore$ $$\theta \propto \frac{VI\cos\phi}{R_s} \propto \text{Power}$$

$\therefore$ True power, $$W_T \propto \left(\frac{VI\cos\phi}{R_s}\right) \quad \ldots (i)$$

Also, when there is p.c. inductance then, measured value of deflecting torque,

$$Td_m = \frac{VI\cos\beta}{R_s}.\cos(\phi-\beta)\frac{dM}{d\theta}$$

At equilibrium, $T_c = Td_m$

$\therefore$ $$K\theta = \frac{VI\cos\beta}{R_s}.\cos(\phi-\beta)\frac{dM}{d\theta}$$

or $$\theta \propto \frac{VI\cos\beta}{R_s}\cos(\phi-\beta) \propto \text{Power}$$

$\therefore$ Measure Power, $W_m \propto \dfrac{VI\cos\beta}{R_s}\cos(\phi-\beta)$ ... (*ii*)

From equation (i) and (ii), we have

$$\frac{W_m}{W_T} = \frac{\cos\beta.\cos(\phi-\beta)}{\cos\phi}$$

or, $$\frac{W_T}{W_m} = \frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)} \quad \text{or} \quad \frac{W_T}{W_m} = \frac{\sec^2\beta}{(1+\tan\phi.\tan\beta)}$$

As β is small. so, $\tan^2\beta$ is very small

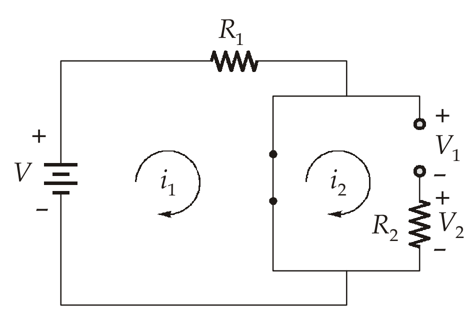
$$\therefore \quad \frac{W_T}{W_m} = \left[\frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)}\right] = \left[\frac{1}{1+\tan\phi.\tan\beta}\right] = \text{correction factor}$$

$$\therefore \quad \%\ \text{error} = \left(\frac{W_m - W_T}{W_T}\right)\times 100 = \tan\phi\ \tan\beta$$

Q.7 (b) Solution:

As the initial current through the inductor and the initial voltage through the capacitor is zero.

$\therefore$ Redrawing the circuit at $t = 0^+$, we get,



Let the clockwise mesh current be i_1 and i_2. At $t = 0^+$ the mesh equations are,

$$R_1 i_1 + \frac{1}{C}\int (i_1 - i_2)\,dt = V \qquad \text{...(i)}$$

$$\text{and } L\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C}\int (i_2 - i_1)\,dt = 0 \qquad \text{...(ii)}$$

(i) Since the capacitor behaves as a short circuit therefore,

$$\frac{1}{C}\int (i_1 - i_2)\,dt = 0$$

$\therefore$ from equation (i),

$$R_1 i_1(0^+) = V$$

or $$i_1(0^+) = \frac{V}{R_1}$$

and also since inductor behaves as an open circuit

$$i_2(0^+) = 0 \quad \text{...(iii)}$$

from equation (ii),

$\therefore$ $$V_1 = L\frac{di_2}{dt} = -R_2 i_2 - \frac{1}{C}\int (i_2 - i_1)dt$$

$$V_1(0^+) = L\frac{di_2(0^+)}{dt} = 0 \quad \text{...(iv)}$$

$$V_2 = R_2 i_2 = 0$$

(ii) Differentiating equation (ii), we get,

$$\frac{dV_1}{dt} + \frac{d}{dt}R_2 i_2 + \frac{i_2 - i_1}{C} = 0 \quad \text{...(v)}$$

from equation (iv),

$$\frac{di_2(0^+)}{dt} = 0 \quad \text{...(vi)}$$

$\therefore$ $$\frac{dV_2}{dt} = \frac{d}{dt}(R_2 i_2) = R_2\frac{di_2}{dt} \quad \text{...(vii)}$$

$\therefore$ $$\frac{dV_2(0^+)}{dt} = \frac{R_2 di_2(0^+)}{dt} = 0$$

From equation (v) and (vi),

$\therefore$ $$\frac{dV_1}{dt} = \frac{-R_2 d(i_2)}{dt} + \frac{i_1 - i_2}{C}$$

$$\frac{dV_1}{dt}(0^+) = \frac{-R_2 di_2}{dt}(0^+) + \frac{i_1(0^+) - i_2(0^+)}{C} = \frac{V}{R_1 C}$$

or $$\frac{dV_1}{dt}(0^+) = \frac{V}{R_1 C} \quad \text{...(viii)}$$

(iii) Differentiating equation (vii), we get,

$$\frac{d^2V_2}{dt^2} = R_2\frac{d^2i_2}{dt^2} \quad \text{...(ix)}$$

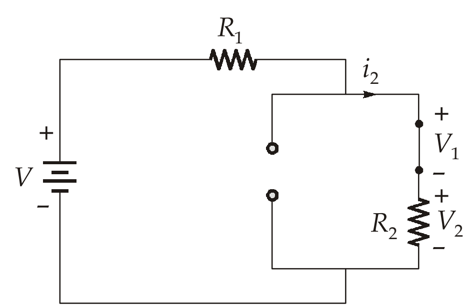
Now, $$\frac{dV_1}{dt} = \frac{d}{dt}\left(L\frac{di_2}{dt}\right) = L\frac{d^2i_2}{dt^2}$$

$$\frac{d^2i_2(0^+)}{dt^2} = \frac{1}{L}\left(\frac{dV_1}{dt}(0^+)\right) = \frac{V}{R_1LC}$$

From equation (ix),

$$\frac{d^2V_2(0^+)}{dt^2} = R_2\frac{d^2i_2}{dt^2}(0^+) = \frac{R_2V}{R_1LC}$$

(iv) At $t = \infty$, the circuit can be redrawn as



$\therefore$ $$V_1(\infty) = 0$$

Also since the capacitor acts an open circuit,

$$i_1(\infty) - i_2(\infty) = 0$$

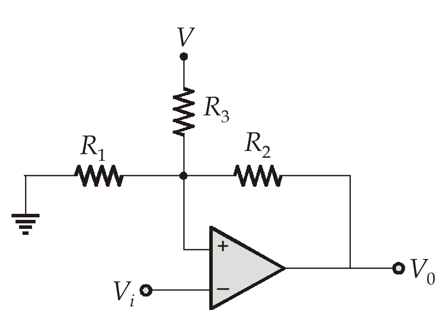
and the current through R_2 is

$$i_2 = \frac{V}{R_1 + R_2} \quad \text{...(viii)}$$

$\therefore$ $$V_2(\infty) = i_2(\infty)R_2 = \frac{VR_2}{R_1 + R_2}$$

Q.7 (c) Solution:

(i)



Applying KCL at (+ve) terminal we get

$$\frac{V_{TH}}{R_1} = \frac{V - V_{TH}}{R_3} + \frac{V^+ - V_{TH}}{R_2}$$

$$V_{TH}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_3} + \frac{V^+}{R_2}$$

$$V_{TH} = \left(\frac{V}{R_3} + \frac{V^+}{R_2}\right)(R_1 \parallel R_2 \parallel R_3)$$

for V_{TL}

$$\frac{V_{TL}}{R_1} = \frac{V - V_{TL}}{R_3} + \frac{V^- - V_{TL}}{R_2}$$

$$V_{TL}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_3} + \frac{V^-}{R_2}$$

$\therefore$
$$V_{TL} = \left(\frac{V}{R_3} + \frac{V^-}{R_3}\right)(R_1 \parallel R_2 \parallel R_3)$$

(ii) Now,
$$V_{TH} = 5.1 = \frac{\left[\frac{15}{R_3} + \frac{13}{R_2}\right]}{\left(\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

(Assume all resistance to be in kΩ range)

$$\frac{5.1}{10} + \frac{5.1}{R_2} + \frac{5.1}{R_3} = \frac{15}{R_3} + \frac{13}{R_2}$$

$$0.51 = \frac{7.9}{R_2} + \frac{9.9}{R_3} \quad \text{...(i)}$$

and
$$V_{TL} = 4.9 = \frac{\left(\frac{15}{R_3} - \frac{13}{R_2}\right)}{\frac{1}{10} + \frac{1}{R_2} + \frac{1}{R_3}}$$

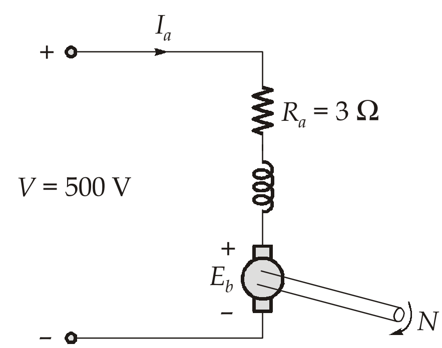
$$\frac{4.9}{10} + \frac{4.9}{R_2} + \frac{4.9}{R_3} = \frac{15}{R_3} - \frac{13}{R_2}$$

$$0.49 = \frac{-17.9}{R_2} + \frac{10.1}{R_3} \quad \text{...(ii)}$$

$\therefore$ Solving equation (i) and (ii) we get

$$R_2 = 856.8 \text{ k}\Omega$$
$$R_3 = 19.8 \text{ k}\Omega$$

Q.8 (a) Solution:



Mechanical power developed = $E_b\, I_a$ = 4000 W

$$I_a = \frac{4000}{E_b}$$

By applying KVL in motor circuit,

$$-V + I_a R_a + E_b = 0$$

$$E_b = V - I_a R_a = 500 - 3\left(\frac{4000}{E_b}\right)$$

$$E_b^2 = 500E_b - 12000$$

$$E_b^2 - 500E_b + 12000 = 0$$

Solving the above equation we obtain, E_b = 474.72 V and 25.28 V (Neglecting 25.28 V).

$$I_a = \frac{4000}{E_b} = \frac{4000}{474.72} = 8.426 \text{ A}$$

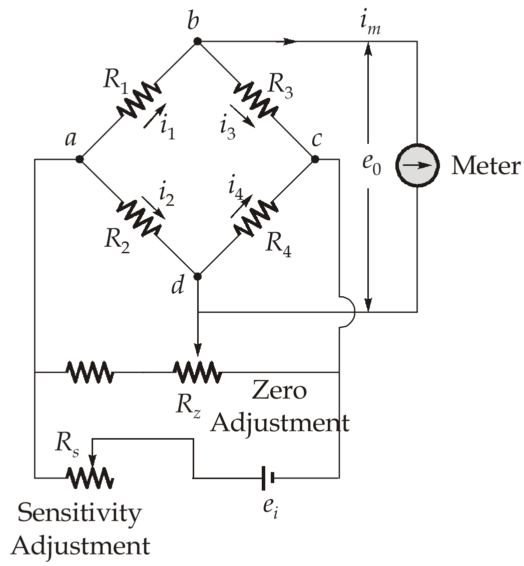
$$E_b = \frac{\phi ZNP}{60A}$$

$$474.72 = \frac{34.6 \times 10^{-3} \times 944 \times 4 \times N}{60 \times 2}$$

$\therefore$ N = 436 rpm

Q.8 (b) Solution:

(i)



For measurement of rapidly changing input signals, the deflection type bridge is used. When the input changes, the resistance R_1 changes producing an unbalance causing a voltage to appear across the meter. The deflection of meter is indicative of the value of resistance and the scale of the meter may be calibrated to read the value of resistance directly.

The deflection type bridge circuit is provided with a zero setting arrangement as shown in above figure. The series resistance R_s, is used to change the bridge sensitivity.

When a deflection type bridge is used, the bridge output on account of the unbalance may be connected either to a high input impedance device or to a low input impedence device. If the output of the bridge is connected directly to a low impedance device like a current galvanometer or a PMMC instrument, a large current flows through the meter. In this case the bridge is called a current sensitive bridge.

In most of the applications of deflection type bridge, the bridge output is fed to an amplifier which has a high input impedance and therefore the output current i_m is zero. This would also be the case if the bridge output is connected to a CRO or an Electronic Voltmeter. This bridge thus used as a voltage sensitive bridge.

Let us assume that the input impedance of the meter is infinite and therefore $i_m = 0$.

Hence, $i_1 = i_3$ and $i_2 = i_4$, output voltage e_0 = voltage across terminals b and d.

$\therefore$ $$e_0 = i_1R_1 - i_2R_2$$

But $$i_1 = \frac{e_i}{R_1 + R_3} \text{ and } i_2 = \frac{e_i}{R_2 + R_4}$$

$$e_0 = \left[\frac{R_1}{R_1+R_3} - \frac{R_2}{R_2+R_4}\right]e_i = \left[\frac{R_1R_4 - R_2R_3}{(R_1+R_3)(R_2+R_4)}\right]e_i$$

Suppose now R_1 changes by an amount ΔR_1. This causes a change Δe_0 in the output voltage.

Thus, $$e_0 + \Delta e_0 = \left[\frac{(R_1+\Delta R_1)R_4 - R_2R_3}{(R_1+\Delta R_1+R_3)(R_2+R_4)}\right]e_i$$

$$= \left[\frac{1+\frac{\Delta R_1}{R_1} - \frac{R_2R_3}{R_1R_4}}{\left\{1+\left(\frac{\Delta R_1}{R_1}\right)+\frac{R_3}{R_1}\right\}\left\{1+\frac{R_2}{R_4}\right\}}\right]e_i$$

In order to simplify the relationship, let us assume that initially all the resistances comprising the bridge are equal,

i.e., $$R_1 = R_2 = R_3 = R_4 = R$$

under these conditions, $e_0 = 0$

and $$\Delta e_0 = \left[\frac{\frac{\Delta R}{R}}{4+2\left(\frac{\Delta R}{R}\right)}\right]e_i$$

if the change in resistance is very-small as compared to initial resistance, then we have,

$$2\left(\frac{\Delta R}{R}\right) << 4$$

$\therefore$ $$\Delta e_0 = \frac{\frac{\Delta R}{R}}{4}e_i$$

Hence, bridge sensitivity

$$\frac{\Delta e_0}{\Delta R} = \frac{e_i}{4R}$$

The major disadvantage of a voltage sensitive deflection bridge is that calibration is dependent upon the value of supply voltage e_i.

(ii) Let the thermistor form resistance $R_T = R$ of the bridge given

Given, $$R_1 = R_2 = R_3 = R_4 = R = 10{,}000$$

Change in resistance, $$\frac{\Delta R}{R} = +5\% = 0.05$$

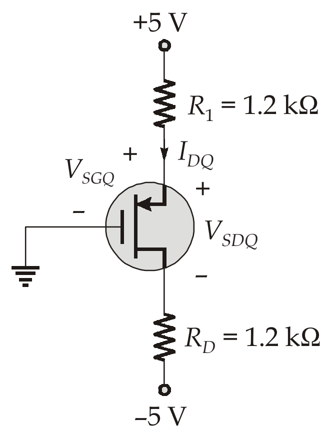
Output voltage of equal arm bridge

$$e_0 = \left[\frac{\left(\frac{\Delta R}{R}\right)}{4+2\left(\frac{\Delta R}{R}\right)}\right] e_i$$

$\therefore$
$$e_0 = \left[\frac{0.05}{4+2\times 0.05}\right]\times 6 = 0.0732 \text{ V} = 73.2 \text{ mV}$$

Q.8 (c) Solution:

(i) For DC analysis of the given circuit, all the coupling capacitors can be open circuited and the resultant equivalent circuit will be as shown below:



By assuming that the transistor is in saturation mode and taking the numerical values of I_{DQ} in mA units, we get,

$$I_{DQ} = K_p(V_{SGQ} + V_{tp})^2$$
$$V_{SGQ} = 5 - 1.2\, I_{DQ}$$

So,
$$I_D = (1)\,(5 - 1.2\, I_{DQ} - 1.5)^2 = (3.5 - 1.2\, I_{DQ})^2$$
$$I_D = 1.44\, I_{DQ}^2 - 8.4 I_{DQ} + 12.25$$

$$1.44\, I_{DQ}^2 - 9.4 I_{DQ} + 12.25 = 0$$

By solving the above quadratic equation, we get,

$$I_{DQ} = 4.73 \text{ mA}, 1.8 \text{ mA}$$

For $I_{DQ} = 4.73$ mA, $V_{SGQ} = 5 - (1.2 \times 4.73) = -0.676 \text{ V} < |V_{tp}|$

For $I_{DQ} = 1.8$ mA, $V_{SGQ} = 5 - (1.2 \times 1.8) = 2.84 \text{ V} > |V_{tp}|$

So, for the assumed case, the valid value of I_{DQ} is 1.8 mA.

$$V_{SDQ} = 10 - (2 \times 1.2 \times I_{DQ}) = 10 - (2.4 \times 1.8) = 5.68 \text{ V}$$

$$V_{SGQ} - |V_{tp}| = 2.84 - 1.5 = 1.34 \text{ V}$$

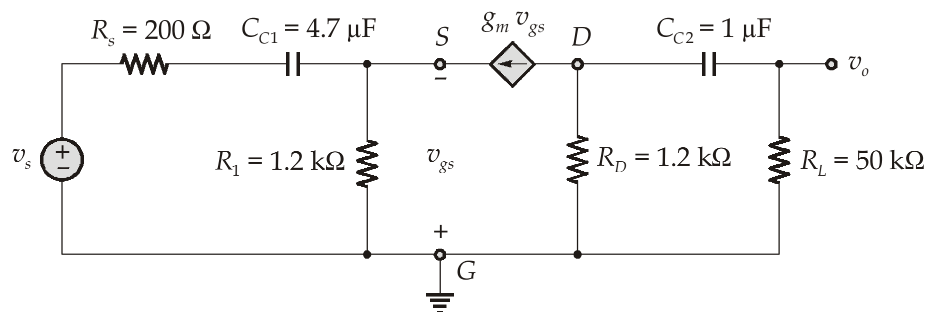
$V_{SDQ} > V_{SGQ} - |V_{tp}|$. So, the initial assumption is correct about the mode of operation of transistor.

The small-signal parameters of the transistor are,

$$g_m = 2K_p(V_{SGQ} + V_{tp}) = 2(1)\,(1.34) = 2.68 \text{ mA/V}$$

$$r_o = \frac{V_A}{I_{DQ}} = \frac{1}{\lambda I_{DQ}} = \infty$$

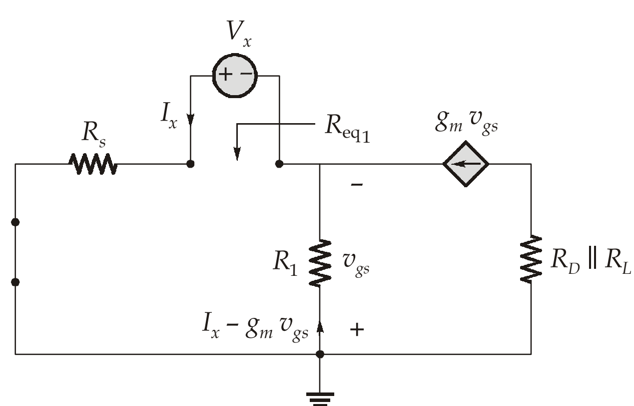
(ii) The small signal equivalent of the given amplifier will be,



Calculation of time constant (τ_1) associated with C_{C1} :

$$\tau_1 = R_{eq_1} C_{C1}$$

While calculating R_{eq2}, C_{C2} must be short circuited and the voltage source v_s should be deactivated as shown below.



$$V_x = R_s I_x + v_{gs}$$

$$v_{gs} = R_1(I_x - g_m v_{gs})$$

$$v_{gs} = \frac{R_1}{1 + g_m R_1} I_x$$

So,
$$V_x = \left(R_s + \frac{R_1}{1 + g_m R_1}\right) I_x$$

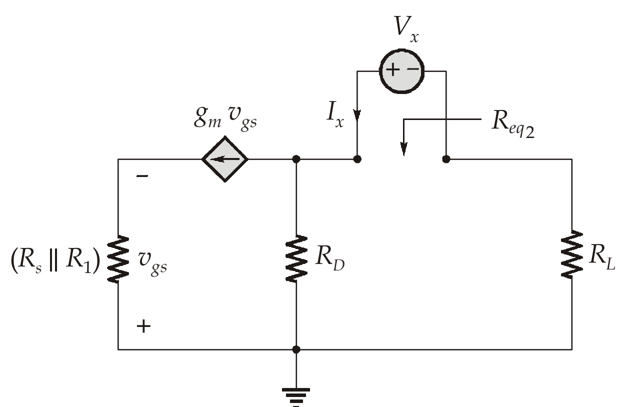
$$R_{eq_1} = \frac{V_x}{I_x} = R_s + \frac{R_1}{1+g_m R_1} = 484.63\ \Omega$$

$$\tau_1 = R_{eq_1} C_{C_1} = 484.63 \times 4.7\ \mu s = 2.28\ \text{ms}$$

Calculation of time constant (τ_2) associated with C_{C2}:

$$\tau_2 = R_{eq_2} C_{C2}$$

While calculating R_{eq_1}, C_{C1} must be short circuited and the voltage source must be deactivated as shown below.



$$v_{gs} = -g_m v_{gs} (R_s \| R_1)$$

So, $$v_{gs} = 0$$

$$R_{eq_2} = \frac{V_x}{I_x} = R_D + R_L = 51.2\ \text{k}\Omega$$

$$\tau_2 = R_{eq_2} C_{C_2} = 51.2\ \text{ms}$$

(iii) The corner frequency associated with C_{C1} is,

$$f_{C1} = \frac{1}{2\pi\tau_1} = 69.8\ \text{Hz}$$

The corner frequency associated with C_{C2} is,

$$f_{C2} = \frac{1}{2\pi\tau_2} = 3.12\ \text{Hz}$$

So, the corner frequency due to C_{C1} dominates that due to C_{C2}. Hence, the lower cut-off frequency of the amplifier can be given by,

$$f_L = f_{C1} = 69.8\ \text{Hz}$$

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