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Detailed Solutions

ESE-2019 Mains Test Series

Electrical Engineering Test No : 13

Section-A

Q.1 (a) Solution:

(i) Surge tank: The load on a generator keeps on fluctuating. Therefore the water intake to the turbine has to be regulated according to the load. A reduction in load on the alternator causes the governor to close the turbine gates. Sudden closure of turbine gates creates an increased pressure known as water hammer, in the penstock, when the governor opens the turbine gates suddenly to admit more water, there is a tendency to cause a vaccum in the penstock. The function of the surge tank is to absorb these sudden changes in water requirements so as to prevent water hammer and vaccum.

The surge tank helps in stablising the velocity and pressure in the penstock and reduces water hammer and negative pressure.

(ii) Penstock: A penstock carries water from the water storage system to the tubine. It may be a low pressure type or high pressure type. A low pressure penstock may consists of flume or a steel pipe. The high pressure penstock consists of thick steel pipes. The diameter may be upto a few meters for large units. Each turbine has a separate penstock. Small size plants have penstocks of concrete.

A penstock may be carried below earth surface or exposed.

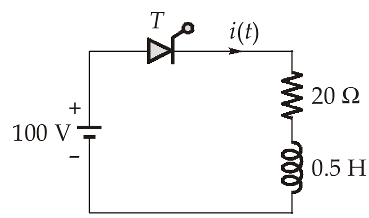
(iii) Spillway: Every dam is provided with an arrangement to discharge excess. water during floods. The arrangement may be a spillway or a by-pass tunnel or conduit. The spillway should be so designed as to discharge the major flood waters without damage to the dam but at the same time maintain a predetermined head. Spillways are classified as over flow spillway, chute spillway, side channel spillway, shaft spillway and siphone spillway.

The type of spillway for a particular location depends on type of dam and topographical, hydrological and geological factors.

(iv) Tailrace: A tailrace is required to discharge the water, leaving the turbine into the river. It is necessary that the draft tube must maintain water sealed all the time. Impulse turbines do not need a draft tube and discharges water directly. The design and size of the tailrace should be such that water has a free exit and the jet of water, after it leaves turbine, has unimpeded passage.

Q.1 (b) Solution:

Consider the SCR without resistance R. The circuit is shown below:



$$V_S = 100\text{ V},$$
$$R = 20\ \Omega,$$
$$L = 0.5\text{ H}$$
$$\text{Latching current} = 50\text{ mA}$$
$$\text{Pulse width} = 50\ \mu\text{s}$$

We have to check whether the SCR current rises above latching current in the firing pulse duration of 50 µs. The current in series R-L circuit is given by,

$$i(t) = \frac{V_S}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

Putting R, L and V_S

$$i(t) = \frac{100}{20}\left(1 - e^{-t20/0.5}\right)$$

$$i(t) = 5(1 - e^{-40t})$$

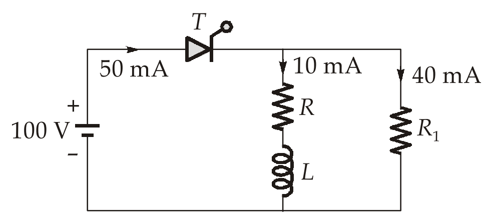
The current after t = 50 µs will be

$$i(t = 50\ \mu\text{s}) = 5\left(1 - e^{-40\times 50\times 10^{-6}}\right)$$

$$i(t = 50\ \mu\text{s}) = 9.99 \times 10^{-3} \approx 10\text{ mA} < 50\text{ mA}$$

Maximum current is less than the latching current. SCR fails to remain ON when firing pulse ends.

To determine of R_1 to ensure firing: The additional resistance connected in parallel with RL circuit increases the current through SCR. SCR takes 50 mA current to remain ON when R_1 is connected parallel to RL circuit. Hence on additional current of 40 mA can be passed through R_1 as shown below.

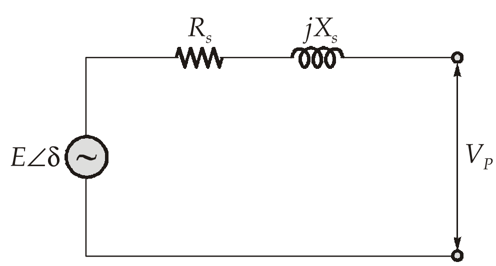


$$V_S = 100 = (40 \times 10^{-3})(R)$$

$$\therefore \quad R_1 = \frac{100}{40 \times 10^{-3}} = 2500\ \Omega$$

$$R_1 = 2.5\ \text{k}\Omega \text{ will ensure firing of SCR}$$

Q.1 (c) Solution:



Given: $R_s = 1.5\ \Omega$, $jX_s = j30\ \Omega$

$$V_P = \frac{13.5}{\sqrt{3}} = 7.794\ \text{kV}$$

$$\phi = \cos^{-1}0.8 = 36.87° \ \text{(lagging)}$$

Now,
$$3V_P|I|\cos\phi = 1280 \times 10^3$$

$$\text{Current, } |I| = \frac{1280 \times 10^3}{3 \times 7.794 \times 10^3 \times 0.8}$$

$$\therefore \quad I = 68.428\ \angle{-36.87°}\ \text{A}$$

Now,
$$E\angle\delta = (R_s + jX_s)I + V_P$$
$$= (1.5 + j30)(68.428\angle{-36.87°}) + V_P$$
$$= (1.5 + j30)(68.428\angle{-36.87°}) + 7794\ \angle 0°$$
$$= 9244\angle 9.845°\ \text{V}$$

Now, Voltage regulation $= \frac{|E|-|V_P|}{|V_P|} = \frac{9244-7794}{7794} = 18.6\%$

Q.1 (d) Solution:

(i) $$y(n) = -y(n-1) + x(n) + x(n-2)$$

Let an input $x_1(n)$ produce an output $y_1(n)$ and input $x_2(n)$ produce an output $y_2(n)$.

$$ay_1(n) + by_2(n) = -[ay_1(n-1) + by_2(n-1)] + [ax_1(n) + bx_2(n)] + [a\,x_1(n-2) + bx_2(n-2)]$$

Output due to weighted sum of input is

$$y_3(n) = -[ay_1(n-1) + by_2(n-1)] + [ax_1(n) + bx_2(n)] + [ax_1(n-2) + bx_2(n-2)]$$

$$y_3(n) = ay_1(n) + by_2(n)$$

So, the system is linear.

(ii) Output due to delay in input,

$$y(n,k) = -y(n-1-k) + x(n-k) + x(n-2-k)$$

The delayed output,

$$y(n-k) = -y(n-1-k) + x(n-\text{k}) + x(n-2-k)$$

$$y(n,k) = y(n-k)$$

∴ system is time invariant.

(iii) For $x(n) = \delta(n),\ y(n) = h(n)$

$$h(n) = -h(n-1) + \delta(n) + \delta(n-2)$$

$$h(0) = -h(-1) + \delta(0) + \delta(-2) = 1$$

$$h(1) = -h(0) + \delta(1) + \delta(-1) = -1$$

$$h(2) = -h(1) + \delta(2) + \delta(0) = 2$$

$$h(3) = -h(2) + \delta(2) + \delta(1) = -2$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = 1 + 1 + 2 + 2 + \ldots$$

$$= \infty$$

∴ Impulse response is not absolutely summable, so the system is unstable.

∴ Given system is non linear, casual, time invariant and unstable.

Q.1 (e) Solution:

(i) DMA (Direct Memory Access) technique is used for transferring data between disc storage and R/W memory. HOLD and HLDA (Hold Acknowledge) signals are pin no. 39 and pin no. 38 in 40 pin µp-8085.

Sequence of operations in DMA:

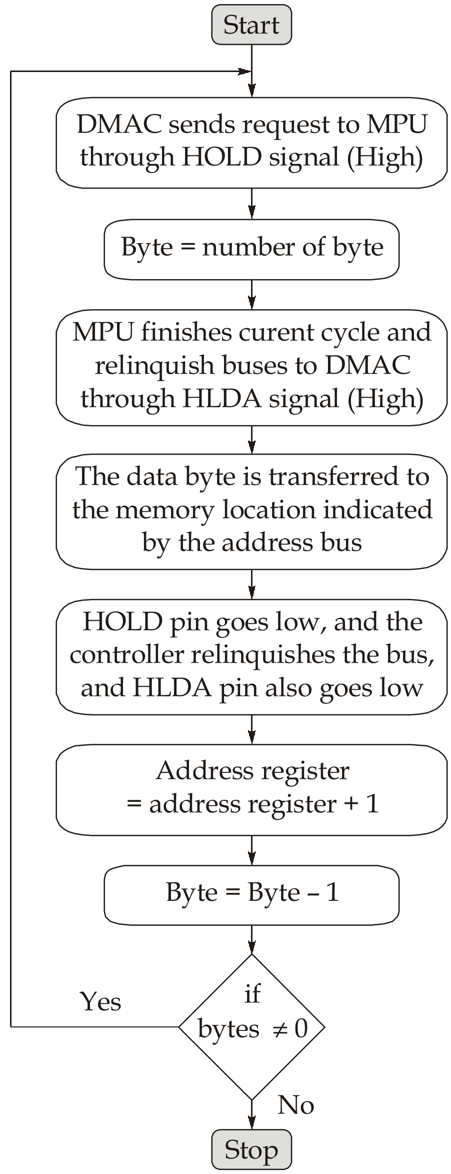
First the DMA will send active high input signal to HOLD pin of the µp to relinquish the address and data buses. After receiving the HOLD request, the µP relinquishes the buses in the following machine cycle. All buses are tri-stated and a hold acknowledge (HLDA) signal is sent out. The data is transferred from memory to external devices such as floppy disk.

After completion of data transfer, the µP regains the control of buses after HOLD goes low. After that µP send HLDA-Hold acknowledge. This is an active high output signal indicating that the µP is relinquishing the control of the buses.

(ii) Cycle stealing:

A read or write signal is generated by the DMA controller (DMAC) and the I/O DMAC effectively steals cycles from the processor in order to transfer the byte, so single byte transfer is also known as "cycle stealing".

Burst-Mode: To achieve block transfers, some DMAC's incorporate an automatic sequencing of the value presented on the address bus. A register is used as a byte count, being decremental for each byte transfer, and upon the byte count reaching zero, the DMAC will release the bus. When the DMAC operates in burst mode, the CPU is halted for the duration of the data transfer.



Q.2 (a) Solution:

Given:

V = Supply voltage

I_a = Armature current

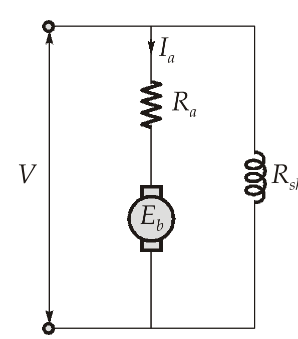
R_{sh} = Field resistance

R_a = Armature resistance

τ = Torque in Nm

E_b = Back emf

N = Speed in rpm



We know that,

For dc shunt motor, $E_b = k\omega$

and Torque, $\tau = kI_a$

At no load, $E_{bNL} = V_{NL} = 440$ V

$\therefore$ $440 = k(\omega_{NL})$

$$440 = k\left(\frac{2000 \times 2\pi}{60}\right)$$

$$k = \frac{440 \times 60}{2000 \times 2\pi} = 2.1 \text{ V/ rad/ sec}$$

At full load torque, let supply voltage, $V = V_1$

$$N_{FL} = 1000 \text{ rpm}$$

$$\omega_{FL} = 104.72 \text{ rad/sec}$$

$\therefore$ $E_{bFL} = k\omega_{FL} = 219.912$ V

We can write, $V_1 = I_{aFL} R_a + E_{bFL} = I_{aFL} R_a + 219.912$ ... (i)

At half the rated torque,

$$\text{Armature current} = I_{a2} = \frac{I_{aFL}}{2}$$

$$\text{Speed, } N_2 = 1050 \text{ rpm}$$

$$\omega_2 = \frac{1050}{60} \times 2\pi = 109.9557 \text{ rad/ sec}$$

$$E_{b2} = k\omega_2 = 2.1 \times 109.9557 = 230.907 \text{ V}$$

We can write, $V_2 = I_{a2}R_a + E_{b2}$

$$V_2 = \frac{I_{aFL}R_a}{2} + E_{b2} \qquad (\therefore\ V_2 = V_1 = V)$$

$$V = \frac{I_{aFL}R_a}{2} + 230.907 \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$0 = \frac{I_{aFL}R_a}{2} + 219.912 - 230.907$$

$$\frac{I_{aFL}R_a}{2} = 10.995$$

$$I_{aFL} R_a = 21.99 \text{ V}$$

The armature voltage drop at full load is 21.99 V

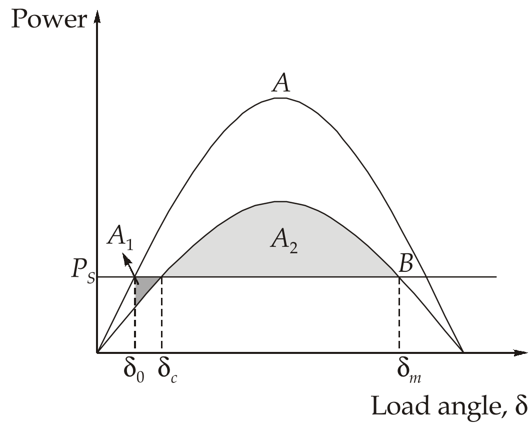
Q.2 (b) Solution:

The power angle curves are shown in figure below:

For curve A, i.e. when both lines are in operation,

$$\text{Transfer reactance,} \quad X = 0.25 + \frac{0.5 \times 0.4}{0.5 + 0.4} + 0.05$$

$$= 0.522 \text{ pu}$$



$$\text{Steady state power limit, } P_{max} = \frac{EV}{X} = \frac{1.2 \times 1}{0.522} = 2.298 \text{ pu}$$

$$\text{For initial condition, } P_0 = 1.0 \text{ pu}$$

$$\text{Initial load angle, } \delta_0 = \sin^{-1}\left(\frac{P_0}{P_{max}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2.298}\right) = 0.45 \text{ rad(elec) or } 25.797°$$

For curve B, when one line is in operation

$$\text{Transfer reactance, } X' = 0.25 + 0.5 + 0.05 = 0.8 \text{ pu}$$

$$\text{Steady state power limit, } P'_{max} = \frac{EV}{X'} = \frac{1.2 \times 1.0}{0.8} = 1.5 \text{ pu}$$

Electrical power develoed, $P_E' = P'_{max} \sin\delta = 1.5 \sin\delta$ and load angle,

$$\delta_m = 180° - \sin^{-1}\left(\frac{1}{1.5}\right) = 180° - 41.81°$$

$$= 138.19° \text{ or } 2.41 \text{ radian (elec)}$$

Applying equal area criterion for clearing angle δ_c.

$$A_1 = P_0(\delta_c - \delta_0) = 1.0\,(\delta_c - 0.45) = \delta_c - 0.45.$$

$$A_2 = \int_{\delta_c}^{\delta_m} (P_E' - P_0)\, d\delta = \int_{\delta_c}^{138.19} (1.5\sin\delta - 1)\, d\delta$$

$$= [-1.5\cos\delta - \delta]_{\delta_c}^{138.19°}$$

$$= -1.5(\cos 138.19° - \cos\delta_c) - (2.41 - \delta_c)$$

$$= 1.5\cos\delta_c + \delta_c - 1.292$$

$A_1 = A_2$, we have,

$$\delta_c - 0.45 = 1.5\cos\delta_c + \delta_c - 1.292$$

or

$$\cos\delta_c = \frac{1.292 - 0.45}{1.5} = 0.561$$

Critical clearing angle, $\delta_c = \cos^{-1}(0.5613) = 55.85°$ (electrical)

Q.2 (c) Solution:

Nodal equation at node 1,

$$\frac{V_1 - V_S}{R_3} = \frac{V_1 - V_0}{R_2} + \frac{V_1}{R_1} + V_1 sC_2 = 0 \quad \text{...(i)}$$

$$\frac{V_1}{R_1} = -V_0 sC_1$$

$$\Rightarrow \quad V_1 = -sR_1 C_1 V_0 \quad \text{...(ii)}$$

Putting the value of V_1 from equation (ii) in equation (i):

$$\frac{-sR_1C_1V_0 - V_S}{R_3} + \frac{(-sR_1C_1V_0 - V_0)}{R_2} - \frac{sR_1C_1V_0}{R_1} - sR_1C_1 \cdot sC_2V_0 = 0$$

$$V_0\left[\frac{sR_1C_1}{R_3} + \frac{sR_1C_1 + 1}{R_2} + sC_1 + s^2R_1C_1C_2\right] = \frac{-V_S}{R_3}$$

$$\Rightarrow \quad V_0\left[\frac{sR_1R_2C_1 + sR_1R_3C_1 + R_3 + sR_2R_3C_1 + s^2R_1R_2R_3C_1C_2}{R_2R_3}\right] = \frac{-V_S}{R_3}$$

$$\Rightarrow \quad \frac{V_0}{V_s} = \frac{-R_2}{s^2R_1R_2R_3C_1C_3 + sC_1(R_1R_2 + R_2R_3 + R_3R_1) + R_3}$$

The characteristic equation is:

$$s^2R_1R_2R_3C_1C_2 + sC_1(R_1R_2 + R_2R_3 + R_3R_1) + R_3 = 0$$

or

$$s^2 + \left(\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3C_2}\right)s + \frac{1}{R_1R_2C_1C_2} = 0$$

Natural frequency,

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

Damping factor,

$$\zeta = \frac{(R_1R_2 + R_2R_3 + R_3R_1)}{R_1R_2R_3C_2 \cdot 2\omega_n}$$

$$\zeta = \frac{\sqrt{R_1R_2C_1C_2}(R_1R_2 + R_2R_3 + R_3R_1)}{2R_1R_2R_3C_2}$$

$$\zeta = \frac{1}{2R_3} \cdot \frac{1}{\sqrt{R_1R_2}} \cdot \sqrt{\frac{C_1}{C_2}} \cdot (R_1R_2 + R_2R_3 + R_3R_1)$$

$$\zeta = \frac{1}{2} \cdot \sqrt{\frac{R_1R_2C_1}{C_2}} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Cut-off frequency,

$$\omega_c = \omega_n\left[(1-2\zeta^2) + \sqrt{(1-2\zeta^2)^2 + 1}\right]^{1/2}$$

where,

$$\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

and

$$\zeta = \frac{1}{2}\sqrt{\frac{R_1R_2C_1}{C_2}}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Q.3 (a) Solution:

$$x(t) = A\cos\omega t$$

$$= A\cos\frac{2\pi}{2\pi}t = A\cos t$$

$$t_0 = \frac{-\pi}{2}$$

$$t_0 + T = \frac{\pi}{2}$$

$$\therefore \quad T = \pi$$

fundamental frequency,

$$\omega_0 = \frac{2\pi}{\pi} = 2$$

Given function is even,

$$\therefore \quad b_n = 0$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt$$

$$a_0 = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} A\cos t\,dt = \frac{A}{\pi}[\sin t]_{-\pi/2}^{\pi/2} = \frac{2A}{\pi}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos\omega_0 t\,dt$$

$$a_n = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} A\cos t\cos 2nt\,dt = \frac{A}{\pi}\int_{-\pi/2}^{A/2}\left(\cos(1-2n)+\cos(1+2n)t\right)dt$$

$$= \frac{A}{\pi}\left[\frac{\sin(1-2n)t}{(1-2n)} + \frac{\sin(1+2n)t}{(1+2n)}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{\pi}\left[\frac{2\sin(1-2n)\frac{\pi}{2}}{(1-2n)} + \frac{2\sin(1+2n)\frac{\pi}{2}}{(1+2n)}\right]$$

$$= \frac{2A}{\pi}\left[\frac{\cos n\pi}{(1-2n)} + \frac{\cos n\pi}{(1+2n)}\right]$$

$$= \frac{2A}{\pi}\cos n\pi\left[\frac{2}{1-4n^2}\right]$$

$$a_n = \frac{4A(-1)^n}{\pi(1-4n^2)}$$

$$r(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t + b_n\sin\omega_0 t$$

$$x(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty}\frac{4A(-1)^n}{\pi(1-4n^2)}\cos 2nt$$

Q.3 (b) Solution:

(i)

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

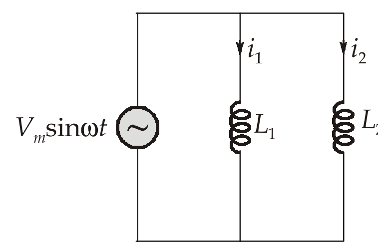
$$= \frac{1}{2}\left(\frac{\partial L_{11}}{\partial \theta}\right)i_1^2 + \left(\frac{\partial L_{12}}{\partial \theta}\right)i_1 i_2 + \frac{1}{2}\left(\frac{\partial L_{22}}{\partial \theta}\right)i_2^2$$

Substituting the value of inductance.

$$T_f = -(\sin\theta)\, i_1 i_2$$

Using KVL in given circuit

$$V_m \sin\omega t = 2\frac{di_1}{dt} + \cos\theta \frac{di_2}{dt}$$



and
$$V_m \sin\omega t = (\cos\theta)\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

Solving these we get,

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{V_m \sin\omega t}{(2 + \cos\theta)}$$

Integrating,
$$i_1 = i_2 = -\frac{V_m \cos\omega t}{\omega(2 + \cos\theta)}$$

Substituting in T_f,
$$T_f = -\frac{V_m^2 \cos^2 \omega t}{\omega^2 (2 + \cos\theta)^2}\sin\theta$$

$$T_f(av) = -\frac{V_m^2 \sin\theta}{2\omega^2 (2 + \cos\theta)^2}$$

Given: θ = 30°,
$$V = 100 \sin 314t$$

∴
$$T_f(av) = -\frac{(100)^2 \sin 30°}{2(2 + \cos 30°)^2 \times (314)^2}$$

$$= -3.086886 \times 10^{-3} \text{ Nm}$$

(ii) If coil 2 is shorted,

$$0 = \cos\theta\frac{di_1}{dt} + 2\frac{di_2}{dt}$$

$$\frac{di_2}{dt} = -\frac{1}{2}(\cos\theta)\frac{di_1}{dt}$$

or
$$i_2 = -\frac{1}{2}(\cos\theta)i_1$$

Given:
$$i_1 = I_m \sin\omega t$$

∴
$$i_2 = -\frac{1}{2}I_m(\cos\theta)\sin\omega t$$

Substituting in T_f,
$$T_f = (\sin\theta)\times\frac{1}{2}I_m^2\sin^2\omega t\cos\theta$$

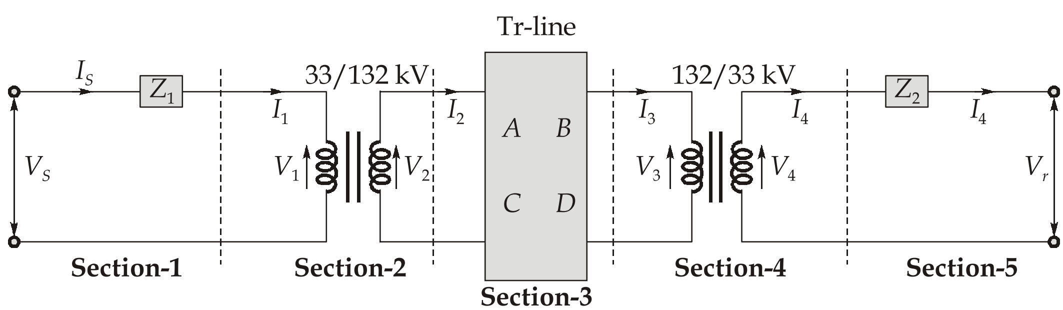
$$= \frac{1}{2}I_m^2(\sin\theta)\times(\cos\theta)\sin^2\omega t$$

$$T_f(av) = \frac{1}{8}I_m^2(\sin 2\theta) \qquad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

Given: $\theta = 45°$,
$$I_m = \sqrt{2}$$

∴
$$T_f(av) = \frac{1}{8}\times 2\sin 90° = 0.25 \text{ Nm}$$

Q.3 (c) Solution:



$$V_4 = V_r + Z_2 I_r; \qquad I_4 = I_r$$

$$\begin{bmatrix} V_4 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

$$V_3 = 4V_4; \quad I_3 = \frac{I_4}{4}$$

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} V_4 \\ I_4 \end{bmatrix}$$

$$V_2 = AV_3 + BI_3$$

$$I_2 = CV_3 + DI_3$$

or
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$V_1 = \frac{1}{4}V_2; \qquad I_1 = 4I_2$$

or,
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_S = V_1 + I_1 Z_1; \quad I_S = I_1$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

Combining all the equations,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1/4 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A + 16Z_1C & AZ_2 + \frac{1}{16}B + 16CZ_1Z_2 + DZ_1 \\ 16C & 16CZ_2 + D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \quad \text{... (i)}$$

If the overall constants for the whole system are, A_0, B_0, C_0, D_0.

$$V_S = A_0V_r + B_0I_r; \quad I_S = C_0V_r + D_0I_r$$

or
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \quad \text{... (ii)}$$

By comparing equations (i) and (ii) we get,

$$A_0 = A + 16Z_1C$$

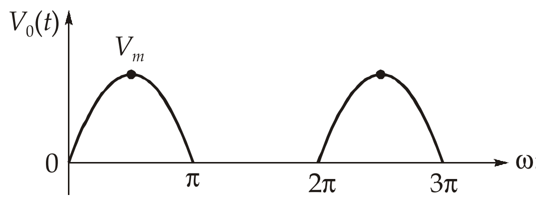
$$B_0 = AZ_2 + \frac{1}{16}B + 16CZ_1Z_2 + DZ_1$$

$$C_0 = 16C$$

$$D_0 = 16CZ_2 + D$$

Q.4 (a) Solution:

The general output voltage waveform of single phase halfwave rectifier is



The fourier series expansion of the waveform is,

$$V_0(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega t - \frac{2V_m}{3\pi}\cos 2\omega t - \frac{2V_m}{15\pi}\cos 4\omega t - \frac{2V_m}{35\pi}\cos 6\omega t$$

$$V_0 = \text{dc value} = \frac{V_m}{\pi} = \frac{100}{\pi} = 31.83 \text{ V}$$

$$\therefore \quad I_0 = \text{dc value} = \frac{V_0}{R} = \frac{31.83}{2} = 15.91 \text{ A}$$

$$\hat{I}_1 = \text{fundamental current component}$$

$$= \frac{\frac{V_m}{2}}{\sqrt{R^2 + (\omega L)^2}} = \frac{50}{\sqrt{4 + \left(2\pi \times 60 \times 2 \times 10^{-3}\right)^2}} = 23.39 \text{ A}$$

$$\hat{I}_2 = \text{first dominant harmonic}$$

$$= \frac{\frac{2V_m}{3\pi}}{\sqrt{R^2 + (2\omega L)^2}} = \frac{\frac{200}{3\pi}}{\sqrt{4 + (2 \times 0.7539)^2}} = 8.47 \text{ A}$$

$$\hat{I}_4 = \text{second dominant harmonic}$$

$$= \frac{\frac{2V_m}{15\pi}}{\sqrt{R^2 + (4 \times \omega L)^2}} = \frac{\frac{200}{15\pi}}{\sqrt{4 + (4 \times 0.7539)^2}} = 1.172 \text{ A}$$

$$\hat{I}_6 = \text{third dominant harmonic}$$

$$= \frac{\dfrac{2V_m}{35\pi}}{\sqrt{R^2+(6\times\omega L)^2}} = \frac{\dfrac{2V_m}{35\pi}}{\sqrt{R^2+(6\times 0.7539)^2}} = 0.368 \text{ A}$$

$$I_{\text{rms}} = \sqrt{(15.91)^2+\left(\frac{23.39}{\sqrt{2}}\right)^2+\left(\frac{8.47}{\sqrt{2}}\right)^2+\left(\frac{1.172}{\sqrt{2}}\right)^2+\left(\frac{0.368}{\sqrt{2}}\right)^2}$$

$$= \sqrt{253.1281+273.54+35.87+0.686+0.0677} = 23.73 \text{ A}$$

Power delivered to resistive load,

$$I_{\text{rms}}^2 R = (23.73)^2 \times 2 = 1126.2 \text{ W}$$

Q.4 (b) Solution:

- The open-loop transfer function of the uncompensated system is,

$$G_f(s) = \frac{10}{s(s+1)}$$

Finding the phase margin of uncompensated system :

- The gain crossover frequency of uncompensated system is,

$$\frac{10}{\omega_{gc}\sqrt{1+\omega_{gc}^2}} = 1$$

$$\omega_{gc}^4+\omega_{gc}^2-100 = 0$$

$$\omega_{gc}^2 = 9.51 \quad \text{(taking only positive value)}$$

$$\omega_{gc} = 3.1 \text{ rad/sec}$$

- The phase of the system at $\omega = \omega_{gc}$ is,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) = -90° - \tan^{-1}(3.1) = -162°$$

- The phase margin of the uncompensated system is,

$$(\text{PM})_{\text{uncompensated}} = 180° - 162° = 18°$$

To design the required compensator:

$$(\text{PM})_{\text{uncompensated}} = 18°$$

$$(\text{PM})_{\text{overall}} \geq 45°$$

- So, the compensator to be designed should be a lead-compensator and the phase lead to be provided at new gain crossover frequency is,

$$\phi_m = 45° - 18° + \varepsilon$$

ε = margin of safety

So, $\phi_m = 45° - 18° + 5° = 32°$

- The general form of the transfer function of lead compensator to be designed can be given as,

$$G_c(s) = \frac{K_c(1+s\tau)}{(1+s\alpha\tau)} \quad ...(i)$$

- The value of K_c can be determined by using desired K_v as follows:

$$K_v = \lim_{s\to 0} sG_f(s)G_c(s) = \lim_{s\to 0} \frac{10K_c(1+s\tau)s}{s(s+1)(1+s\alpha\tau)}$$

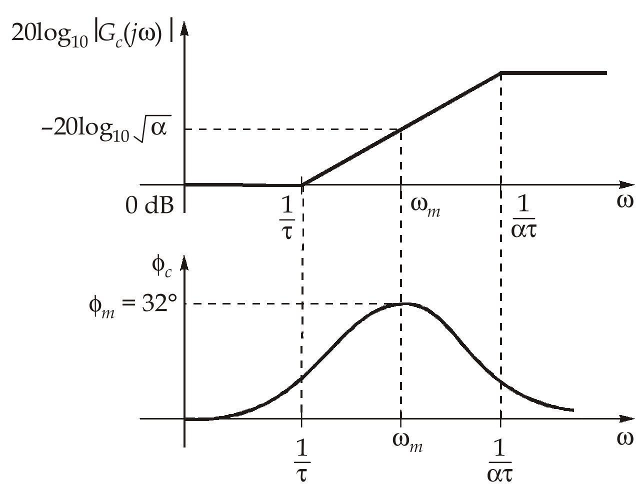
$$10K_c = 12 \Rightarrow K_c = 1.2$$

- The constant "α" can be given as,

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \frac{1-\sin 32°}{1+\sin 32°}$$

$$= \frac{1-0.5299}{1+0.5299} = \frac{0.4701}{1.5299} = 0.307$$

- The magnitude and phase bode plots of the lead compensator block can be given as follows:



$$\omega_m = \sqrt{\left(\frac{1}{\tau}\right)\left(\frac{1}{\alpha\tau}\right)} = \frac{1}{\sqrt{\alpha}\,\tau}$$

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}}$$

- The value of ω_m should be selected at the new gain crossover frequency. So, at $\omega = \omega_m$ the gain of overall open-loop transfer function should be unity and it can be determined as follows:

$$|G_f(j\omega)||G_c(j\omega)| = 1$$

$$|G_c(j\omega)|_{\omega=\omega_m} = \frac{1}{\sqrt{\alpha}}$$

So,

$$\big|G_f(j\omega)\big|\Big|_{\omega=\omega_m} = \sqrt{\alpha}$$

$$\frac{10}{\omega_m\sqrt{(1+\omega_m^2)}} = \sqrt{\alpha} = 0.554$$

Squaring both sides,

$$\frac{100}{\omega_m^2(1+\omega_m^2)} = 0.307$$

$$100 = 0.307\omega_m^2 + 0.307\omega_m^4$$

$$\omega_m^2 = 17.55 \qquad \text{(considering only positive value)}$$

$$\omega_m = 4.189 \text{ rad/sec}$$

$$\tau = \frac{1}{\omega_m\sqrt{\alpha}} = 0.4308$$

$$\alpha\tau = 0.132$$

- By substituting the value of τ, $\alpha\tau$ and K_c in equation (i), we get the transfer function of the desired compensator, which is as follows:

$$G_c(s) = \frac{1.2(1+0.431s)}{(1+0.132s)}$$

- The open-loop transfer function of the overall system (or) compensated system is,

$$G(s) = G_f(s)\,G_c(s) = \frac{12(1+0.431s)}{s(1+s)(1+0.132s)}$$

- The gain crossover frequency of the overall system is $\omega_{gc} = \omega_m = 4.19$ rad/sec
- The phase margin of the overall system can be calculated as,

$$\phi_{gc} = -90° - \tan^{-1}(\omega_{gc}) - \tan^{-1}(0.132\omega_{gc}) + \tan^{-1}(0.431\omega_{gc})$$

$$\phi_{gc} = -90° - 76.576° - 28.946° + 61.024°$$

$$= -134.498°$$

$$(\text{PM})_{\text{compensated}} = 180° - 134.498° = 45.502°$$

- The phase margin of uncompensated system is 18°. So, the designed compensator improved the phase margin of the system by 27.5°.

Q.4 (c) Solution:

Given:

$$\text{Input power } P_{\text{in}} = \sqrt{3} \times 460 \times 25 \times 0.85$$
$$= 16930.796 \text{ W}$$
$$\text{Stator copper loss} = P_{SC} = 1000 \text{ W}$$
$$\text{Rotor copper loss} = P_{RC} = 500 \text{ W}$$
$$\text{Rotational losses} = P_{WF} = 250 \text{ W}$$
$$\text{Core losses} = P_I = 800 \text{ W}$$
$$\text{Stray load} = P_{\text{stray}} = 200 \text{ W}$$

(i)

$$P_g \text{ (Air gap power)} = P_{\text{in}} - (P_{SC} + P_I)$$
$$= 16930.796 - 1800$$
$$= 15130.796 \text{ W}$$
$$\text{Slip, } s = \frac{\text{Rotor loss}}{P_g} = \frac{500}{15130.796} = 3.3\%$$

(ii) Developed mechanical power,

$$P_m = P_g - P_{RC}$$
$$= 15130.796 - 500$$
$$= 14630.796 \text{ W}$$

(iii)

$$\text{Output power} = P_m - P_{WF} - P_{\text{stray}}$$
$$= 14630.796 - 250 - 200$$
$$= 14180.796 \text{ W}$$

$\therefore$

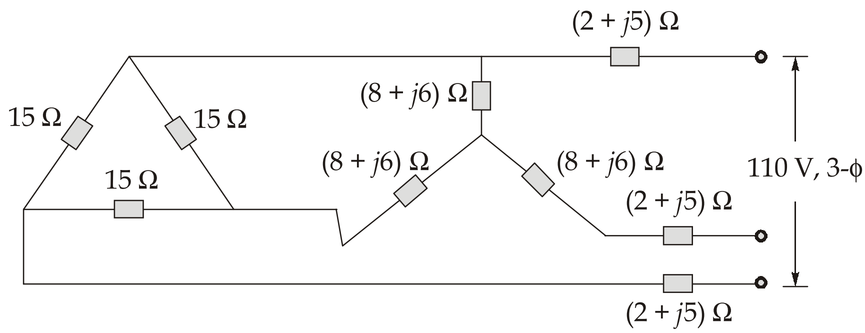
$$\text{Output horse power} = \frac{14180.796}{746} = 19 \text{ hp}$$

(iv)

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Input Power}} = \frac{14180.796}{16930.796} \times 100$$
$$= 83.757\%$$

Section-B

Q.5 (a) Solution:



Convert Δ to equivalent Y having $\frac{15}{3} = 5\Omega$ per phase

Equivalent impedance per phase

$$= \frac{5(8+j6)}{5+8+j6} = \frac{40+j30}{13+j6} = 3.49\angle 12.1°\Omega$$

$$Z_{total} = 3.49\angle 12.1° + 2 + j5 = 7.88\ \angle 46.64°\ \Omega$$

Current drawn from supply:

Supply current, $$|I| = \frac{110/\sqrt{3}}{7.88} = 8.06\text{ A}$$

Let V_t equals voltage at the load, line to neutral voltage:

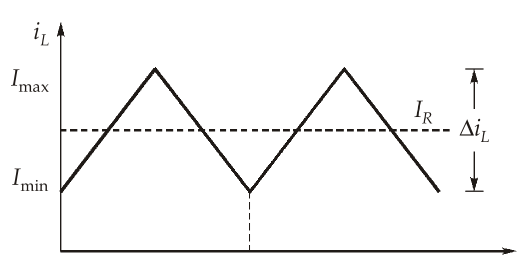
$$V_t = 8.06 \times 3.49 = 28.13\text{ V}$$

$$\text{Line to line } V_2 = \sqrt{3} \times 28.13 = 48.72\text{ V}$$

Q.5 (b) Solution:

(i)
$$V_0 = V_s \,.\, D$$
$$= (50)(0.4) = 20\text{ V}$$

(ii)



$$I_{max} = I_L + \frac{\Delta i_L}{2} = V_0\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{min} = V_0\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$I_{max} = 20\left(\frac{1}{20} + \frac{1-0.4}{(2)(400)\times 10^{-6}(20)(10^3)}\right)$$

$$= 1 + \frac{1.5}{2}$$

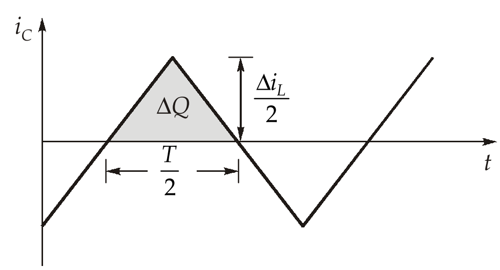
$$I_{max} = 1.75 \text{ A}$$

$$I_{min} = 20\left(\frac{1}{20} - \frac{(1-0.4)}{(2)(400)(10^{-6})(10^3)}\right)$$

$$= 1 - \frac{1.5}{2}$$

$$I_{min} = 0.25 \text{ A}$$

(iii)



$$\Delta Q = C\Delta V_0$$

$$\Delta V_0 = \frac{\Delta Q}{C}$$

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

$$V_0 = \frac{T\Delta i_L}{8C}$$

$$\Delta i_L = \frac{V_0}{L}(1-D)T$$

$$\Delta V_0 = \frac{T}{8C}\cdot\frac{V_0}{L}(1-D)T = \frac{V_0(1-D)}{8LC}T^2$$

$$\text{Ripple} = \Delta V_0 = \frac{(1-D)V_0}{8LCf^2}$$

$$\frac{\Delta V_0}{V_0} = \frac{1-0.4}{(8)(400)(10^{-6})(100)(10^{-6})(20\times 10^3)^2}$$

$$\frac{\Delta V_0}{V_0} = 0.00469$$

$$\Delta V_0 = (0.00469)\,(20) = 0.9375 \text{ V}$$

Q.5 (c) (i) Solution:

$(835)_{10} \longrightarrow (1000\ 0011\ 0101)_2$

$+(274)_{10} \longrightarrow +(0010\ 0111\ 0100)_2$

1109   1010 1010 1001

         + 0110 0110 0000

         1 0001 0000 1001 $\Rightarrow (1109)_{10}$

835 ⟶ 1000 0011 0101

− 274 ⟶ + 1101 1000 1011 (1's compliment)

561   1   0101 1100 0000

                         1

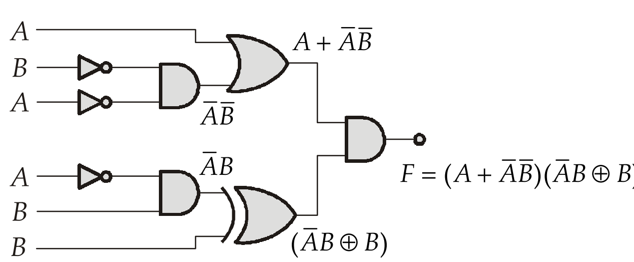
         0101 1100 0001

         0000 1010 0000

         0101 ① 0110 0001 $\Rightarrow (561)_{10}$

ignore this carry

Q.5 (c) (ii) Solution:



$$F = (A+\bar{A}\bar{B})(\bar{A}B \oplus B)$$

$$= \left(A+\bar{A}\bar{B})(\bar{A}B\cdot\bar{B} + \left(\overline{\bar{A}B}\right)B\right)$$

$$= \left(A+\bar{B}\right)\left[\left(A+\bar{B}\right)B\right]$$

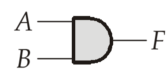
$$= \left(A+\bar{B}\right)\left(AB+\bar{B}B\right)$$

$$= (A+\overline{B})(AB)$$

$$= AB + AB\overline{B}$$

$$F = AB$$

∴ The simplest logic diagram is shown below,



Q.5 (d) Solution:

Given; $V = \dfrac{500}{\sqrt{3}} = 288.675\text{ V}$,

$$E = \frac{600}{\sqrt{3}} = 346.41\text{ V}$$

$$R = 0.4\ \Omega,\ jX = j3.6\ \Omega$$

$$Z\angle\theta = R + jX = 0.4 + j3.6 = 3.622 \angle 83.66°$$

Power supplied to the armature by each phase.

$$= \frac{62+1}{3} = 21\text{ kW}$$

$$\text{Line current, } I = \frac{V\angle\delta - E}{Z\angle\theta} = \frac{V}{Z}\angle\delta-\theta - \frac{E}{Z}\angle-\theta$$

$$EI^* = \text{Re}\left\{\frac{EV}{Z}\angle\theta-\delta - \frac{E^2}{Z}\angle\theta\right\}$$

$$21 \times 10^3 = \left\{\frac{(346.41)(288.675)}{3.622}\cos(\theta-\delta) - \frac{(346.41)^2}{3.622}\cos 83.66°\right\}$$

$$= \{27609 \cos(\theta - \delta) - 3658.578\}$$

$$\frac{21000 + 3658.578}{27609} = \cos(\theta - \delta)$$

$$\theta - \delta = 26.73°$$

$$\delta = \theta - 26.73° = 83.66° - 26.73° = 56.93°$$

$$\text{Line current, } I = \frac{V\angle\delta - E}{Z\angle\theta} = \frac{288.675\angle 56.93 - 346.41\angle 0}{3.622\angle 83.66°}$$

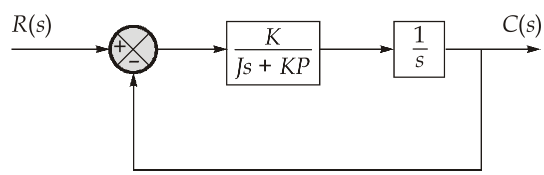
$$= 84.73 \angle 44.32°\text{ A}$$

Since line current and phase current are equal in a star system.

$$\text{Power factor} = \cos(56.93° - 44.32°)$$
$$= 0.97587 \text{ lagging}$$

Q.5 (e) Solution:

Simplifying the block diagram:



Solving we get,
$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + KPs + K}$$

Putting $J = 1$ kg-m^2

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + KPs + K}$$

Now comparing from second order system.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{K}, \quad 2\xi\omega_n = KP$$

$$M_P = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.25 \text{ (Given)}$$

Taking natural log on both sides in above equation

$$= \frac{\xi\pi}{\sqrt{1-\xi^2}} = 1.386$$

Solving we get $\xi = 0.404.$

The peak time $t_p = \frac{\pi}{\omega_d} = 2$ (Given)

$$\omega_d = 1.57 = \omega_n\sqrt{1-\xi^2}$$

Solving we get $\omega_n = \sqrt{K} = 1.7162 \approx 1.72$

$\Rightarrow$ $K = \omega_n^2 = 2.95$ N-m

$$P = \frac{2\xi\omega_n}{K} = \frac{(2)(0.404)(1.72)}{2.95} = 0.471 \text{ sec}$$

$K = 2.95$ N-m

$P = 0.471$ sec

Q.6 (a) Solution:

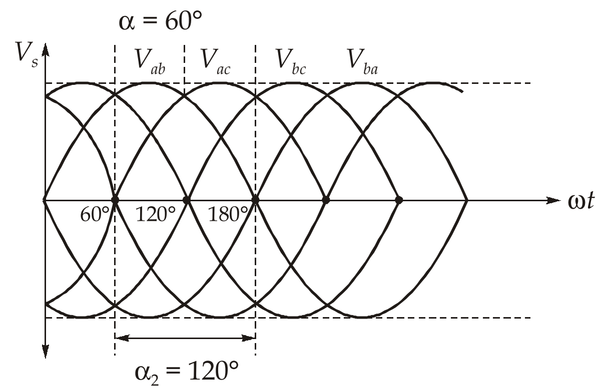
(i) For the perfect operation of dual conveter,

$$\alpha_1 + \alpha_2 = 180°$$

$$\alpha_2 = 180° - \alpha_1$$

Let, $$\alpha_1 = \frac{\pi}{3}$$

then, $$\alpha_2 = 180° - 60° = 120°$$



$$V_{ab} = V_{mL} \sin \omega t$$

$$V_{bc} = V_{mL} \sin (\omega t - 120°)$$

$$V_r = V_{ab} - V_{bc} = V_{mL}[\sin \omega t - \sin (\omega t - 120°)]$$

$$= V_{mL}[2 \cos(\omega t - 60°) \sin 60°]$$

$$V_r = \sqrt{3} V_{mL} \sin(\omega t + 30°)$$

Circulating current, $$i_c = \frac{1}{L}\int V_r \, dt$$

$$= \frac{1}{L} \int_{\left[\frac{\frac{\pi}{3}+\alpha_1}{\omega}\right]}^{t} \sqrt{3} V_{mL} \sin(\omega t + 30°) dt$$

$$= \frac{\sqrt{3} V_{mL}}{\omega L}\left(\cos(\omega t + 30° \Big|_{\frac{\frac{\pi}{3}+\alpha_1}{\omega}}^{t} \right)$$

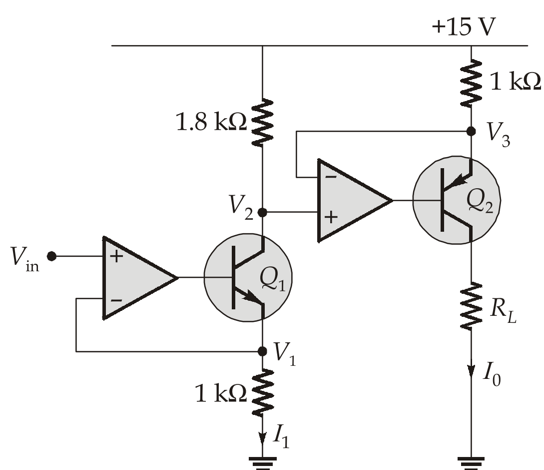
$$i_c(t) = \frac{\sqrt{3} V_{mL}}{\omega L}\left[-\sin \alpha_1 - \cos(\omega t + 30°)\right]$$

At ωt = 150° the circulating current in the circuit is maximum, i.e. $$I_{cp} = \frac{\sqrt{3}V_{mL}}{\omega L}[1-\sin\alpha_1]$$

(ii) Given,

$$\alpha_1 = 60°$$
$$V_{ph} = 230\text{ V}$$
$$V_{m\,ph} = 230\times\sqrt{2}$$
$$V_{mL} = \sqrt{3}\times 230\times\sqrt{2}$$
$$V_{mL} = \sqrt{6}\times 230\text{V}$$
$$I_{cp} = \frac{\sqrt{3}\times\sqrt{6}\times 230}{2\pi\times 50\times 15\times 10^{-3}}[1-\sin 60°]$$
$$I_{cp} = 27.74\text{ A}$$

Q.6 (b) (i) Solution:



Since, $V_{in} = 5\text{ V}$

So, $V_1 = V_{in} = 1\cdot I_1$

⇒ $5 = I_1$

∴ $I_1 = 5\text{ mA}$

$V_2 = 15 - 1.8\,I_1$

⇒ $V_2 = 15 - 1.8\,I_1$

⇒ $V_2 = 15 - 9$

⇒ $V_2 = 6\text{ V}$

also, $V_3 = V_2 = 6\text{ V}$

$\because$ $$I_0 = \frac{15 - V_3}{1} = \frac{15-6}{1}$$

$\Rightarrow$ $$I_0 = 9\text{ mA}$$

Now, $$15 = (1 + R_L)I_0 + V_{CE}$$

$$15 = (1 + R_{L\max})\, I_0 + V_{CE,\, SAT}$$

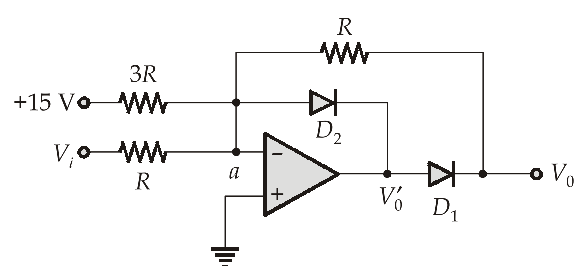
$\Rightarrow$ $$15 = (1 + R_{L\max})\, 9 + 0.3$$

$\Rightarrow$ $$1 + R_{L\max} = \frac{15 - 0.3}{9} = \frac{14.7}{9}$$

$\Rightarrow$ $$1 + R_{L\max} = 1.633$$

$\Rightarrow$ $$R_{L\max} = 0.633\text{ k}\Omega = 633.33\ \Omega$$

Q.6 (b) (ii) Solution:



During a portion of negative cycle V_o' can become positive. Then D_1 conducts and D_2 becomes off. Only then circuit provides a non-zero output V_o.

Using KCL at 'a'

$$\frac{V_i - 0}{R} + \frac{15 - 0}{3R} + \frac{V_o - 0}{R} = 0$$

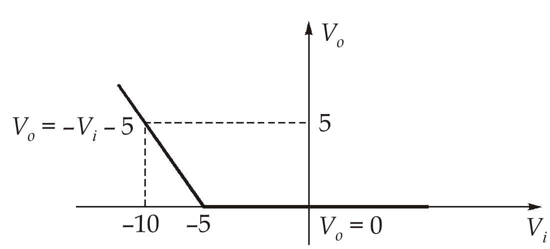
$$V_i + 5 + V_o = 0$$

$\Rightarrow$ $$V_o = -V_i - 5$$

From the above expression, it can be connected that positive output is generated when $V_i < -5$ V.

And for remaining values of V_i

i.e. $V_i > -5$ V, V_o remains zero



Q.6 (c) (i) Solution:

From the transfer function of open loop system following conclusion can be drawn there is one open loop pole in the right half of the s-plane. So the open loop system is unstable, $P = 1$; so for the close loop system to be stable, the Nyquist plot $G(s)\,H(s)$ must encircle $(-1 + j0)$ point of $q(s)$ plane once in counter-clockwise direction.

For sinusoidal form putting $s = j\omega$ in $G(s)H(s)$

$$G(j\omega)H(j\omega) = \frac{j\omega+4}{(j\omega+1)\,(j\omega-1)} = -\frac{4+j\omega}{(1+j\omega)\,(1-j\omega)}$$

$$= \frac{-4-j\omega}{1+\omega^2} = -\frac{4}{1+\omega^2} - j\frac{\omega}{1+\omega^2}$$

Along the segment (C_1) of the Nyquist contour on the $j\omega$-axis, s varies from $-j\omega$ to $+j\infty$.

At $\omega = -\infty$ $\quad G(j\omega)H(j\omega) = -0 + j0$

At $\omega = 0^-$ $\quad G(j\omega)H(j\omega) = \dfrac{-4}{[1+(-0)^2]} - \dfrac{j(-0)}{[1+(-0)^2]} = -4 + j0$

At $\omega = 0^+$ $\quad G(j\omega)H(j\omega) = -4 - j0$

At $\omega = +\infty$ $\quad G(j\omega)H(j\omega) = -0 - j0$

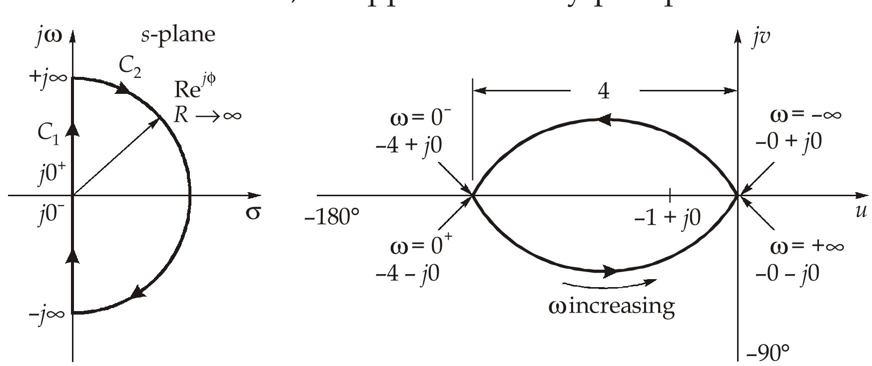
So, we got four points to draw an approximate Nyquist plot.

Along the semi-circular arc of the Nyquist contour $s = \lim_{R\to\infty} Re^{j\phi}$ (ϕ varying from +90° through 0 to –90°). It is mapped into

$$G(s)H(s) = \lim_{R\to\infty} \frac{(Re^{j\phi}+4)}{(Re^{j\phi}-1)\,(Re^{j\phi}+1)} = \lim_{R\to\infty} \frac{1}{Re^{j\phi}} = 0e^{-j\phi}$$

i.e. the origin of $G(s)H(s)$ plane. $G(s)H(s)$ locus thus turns at the origin with zero radius from –90° through 0° to +90°.

Based on the above information, an approximate Nyquist plot is shown below:



It indicates that the $(-1 + j0)$ point is encircled by this locus once in clockwise direction. So the closed loop system is stable.

Q.6 (c) (ii) Solution:

For a system to be completely observable rank of matrix $\left[C^T \quad A^T C^T \ldots (A^T)^{n-1} \quad C^T\right]$ should be n.

$$C^T = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix},\ A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -7 \\ -1 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T \cdot (A^T C^T) = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} -6 \\ -7 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -1 \end{bmatrix}$$

The rank of observability matrix

$$S_o = \left[C^T \quad A^T C^T \quad (A^T)^2 \quad C^T\right]$$

$$\det S_0 = \begin{vmatrix} 4 & -6 & 6 \\ 5 & -7 & 5 \\ 1 & -1 & -1 \end{vmatrix} \qquad (\because \det S_o = 0)$$

$$= 4(7 + 5) + 6(-5 - 5) + 6(-5 + 7) = 0$$

$$\rho(S_0) = \begin{bmatrix} 4 & -6 & 6 \\ 5 & -7 & 5 \\ 1 & -1 & -1 \end{bmatrix} \text{ is } 2\ (<3)$$

$\therefore$ System is not completely observable.

Q.7 (a) Solution:

$$\text{Secondary load kVA per phase} = \frac{150}{3} = 50 \text{ kVA}$$

$$\text{Secondary voltage per phase} = 1100 \text{ V (In } \Delta,\ V_p = V_l) = 1.1 \text{ kV}$$

$$\therefore \quad \text{Secondary current per phase} = \frac{\text{load kVA per phase}}{\text{voltage per phase in kV}} = \frac{50}{1.1}\text{A}$$

Phasor secondary current per phase referred to primary

$$I_2' = \left(\frac{50}{1.1} \times \frac{1100}{33000}\right) \angle -\cos^{-1} 0.8°$$

$$= 1.515\ (0.8 - j0.6)$$

$$= (1.212 - j0.909) \text{ A}$$

$$\text{Tertiary load kVA per phase} = \frac{50}{3} \text{kVA}$$

$$\text{Tertiary voltage per phase} = \frac{400}{\sqrt{3}} = 231 \text{ V} = 0.231 \text{ kV}$$

$$\text{Tertiary current per phase} = \frac{\text{load kVA per phase}}{\text{voltage per phase in kV}} = \frac{50/3}{0.231} \text{A}$$

Phasor tertiary current per phase referred to primary,

$$I_3' = \left(\frac{50/3}{0.231}\right)\left(\frac{231}{33000}\right)\angle - \cos^{-1} 0.9°$$

$$= 0.505\,(0.9 - j\,0.436)$$

$$= (0.4545 - j\,0.220) \text{ A}$$

$$\text{Magnetizing current, } I_\mu = 4\% \text{ of rated current}$$

$$= \frac{4}{100} \times \frac{(200 \times 10^3)/3}{33000} = 0.0808 \text{ A}$$

Core loss component of no load current,

$$I_w = \frac{(1000/3)}{33000} = 0.0101 \text{ A}$$

$$\therefore \quad \text{Primary no-load current, } I_0 = I_w - jI_\mu = (0.0101 - j\,0.0808) \text{ A}$$

$$\text{Total primary current, } I_1 = I_2' + I_3' + I_0$$

$$= 1.212 - j0.909 + 0.4545 - j0.220 + 0.0101 - j\,0.0808$$

$$= 1.6766 - j1.2098 = 2.0675\angle{-35.813°}$$

$$= 2.0675 \text{ A}$$

at a lagging power factor of cos 35.81° that is 0.8109

Q.7 (b) Solution:

Receiving end voltage is taken as reference,

$$V_R = \frac{132}{\sqrt{3}} = 76.21 \text{ kV}$$

$$A = 0.98\angle 3°, \quad B = 110\angle 75° \; \Omega/\text{phase}$$

$$\text{p.f.} = \cos\phi = 0.8 \text{ lagging}, \quad \sin\phi = 0.6$$

$$|I_R| = \frac{50 \times 10^6}{\sqrt{3} \times (132 \times 10^3)} = 218.69 \text{ A}$$

$\therefore$ $I_R = 218.69\angle -\cos^{-1} 0.8 = 218.69\angle -36.86° \text{ A}$

(i) $V_s = AV_R + BI_R$

$V_s = (0.98\angle 3°)\,(76.21 \times 10^3)\,(110\angle 75°)\,(218.69\angle -36.86°)$

$\therefore$ $V_s = 95.367\angle 11.35° \text{ kV}$

$|V_s| = 95.367 \text{ kV}$

or, $|V_s(\text{line})| = \sqrt{3}(95.367) = 165.18 \text{ kV}$

$= 165.18 \text{ kV}$

and power angle $(\delta) = 11.35°$

(ii) Since, $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$

or, $AD - BC = 1$

and $A = D$

$$C = \frac{A^2 - 1}{B} = \frac{(0.98)^2\angle 6° - 1}{110\angle 75°} = 9.996 \times 10^{-4}\angle 39.08°$$

$I_s = CV_R + DI_R$

$= (9.996 \times 10^{-4}\angle 39.08°)\,(76.21) \times 10^3$

$+ (0.98\angle 3°)\,(218.69\angle -36.86°)$

$\therefore$ $I_s = 247.617\angle -16.756° \text{ A}$

$\therefore$ $P_s = 3V_sI_s\cos(11.35° + 16.756°)$

$= 3(95.36 \times 10^3)\,(247.617)\cos 28.106°$

$= 62.484 \text{ MW}$

and $Q_s = 2V_sI_s\sin(11.35° + 16.756°)$

$= 3(95.36 \times 10^3)\,(247.617)\sin 28.106°$

$= 33.37 \text{ MVAR}$

(iii) Line loss $= P_s - P_R$

$= 62.484 - 50\,(0.8)$

$= 22.484 \text{ MW}$

VAR absorbed $= Q_s - Q_R$

$= 33.37 - 50(0.6) = 3.37 \text{ MVAR}$

(iv) For unit p.f. load,

$Q_R = 0$

and $V_s = V_R = 132 \times 10^3 \text{ kV}$ (line)

Since,
$$Q_R = \frac{|V_R||V_R|}{|B|}\sin(\beta-\delta) - \left|\frac{A}{B}\right||V_R|^2\sin(\beta-\alpha) = 0$$

$$\frac{(132\times10^3)(132\times10^3)}{110}\sin(75°-\delta) - \left|\frac{0.98}{110}\right|(132\times10^3)^2\sin(75°-3°) = 0$$

or, $\sin(75° - \delta) - 0.98 \sin 72° = 0$

$\therefore \quad \delta = 6.25°$

$$P_R = \frac{|V_s||V_R|}{|B|}\cos(\beta-\delta) - \left|\frac{A}{B}\right||V_R|^2\cos(\beta-\alpha)$$

$$= \frac{132\times10^3}{110}\cos(75°-6.25°) - \left(\frac{0.98}{110}\right)(132\times10^3)^2\cos 72°$$

$\therefore \quad P_R = 9.44$ MW at upf

Q.7 (c) Solution:

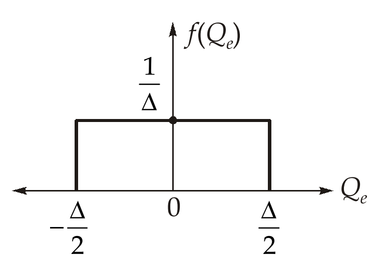
(i) Let message signal, $\quad m(t) = A\sin 2\pi f_0 t$

Signal power, $\quad S = \frac{A^2}{2}$

Quantization noise power,

$$N_Q = \text{Power}\,[Q_e] = E\left[Q_e^2\right] = \int Q_e^2 f(Q_e)\,dQ_e$$

Let Q_e having uniform density function,



$$N_Q = \int_{-\Delta/2}^{\Delta/2} Q_e^2\cdot\frac{1}{\Delta}\,dQ_e = \frac{1}{\Delta}\cdot\frac{Q_e^3}{3}\Bigg|_{-\Delta/2}^{\Delta/2}$$

$$N_Q = \frac{\Delta^2}{12},\quad \text{where, } \Delta = \frac{2A}{2^n}$$

$$= \frac{A^2}{3\times2^{2n}}$$

$$\frac{S}{N_Q} = \frac{3}{2} \times 2^{2n}$$

$$\left(\frac{S}{N_Q}\right)_{dB} = 10\log_{10}\left[\frac{3}{2} \times 2^{2n}\right]$$

$$= 10\log_{10}\frac{3}{2} + 10\log_{10} 2^{2n}$$

$$= 1.76 + 6.02\, n \cong (1.8 + 6n) \text{ dB}$$

(ii)

$$R_b = nf_s$$

$$50 \text{ m} = 7 \times f_s$$

$$f_s = \frac{50}{7}\text{MHz}$$

According to sampling theorem,

$$f_s \geq 2f_m$$

$$\frac{50}{7}\text{M} \geq 2f_m$$

$$f_m \leq \frac{50}{14}\text{M}$$

$$f_m \leq 3.57 \text{ MHz}$$

Maximum message bandwidth allowed is 3.57 MHz.

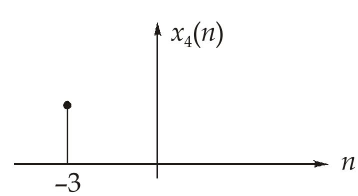
Q.8 (a) (i) Solution:

$$y[n] = x_1[n] * x_2[n] * x_3[n]$$

$$y[n] = 0.5^n \,.\, u[n] * [u(n + 3) * (\delta[n] - \delta(n - 1))]$$

$$= 0.5^n\, u(n) * [u[n + 3] - u[n + 2]]$$

$$x_4(n) = u[n + 3] - u[n + 2]$$



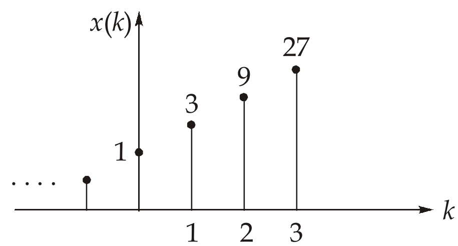
$$y[n] = 0.5^n\, u[n] * \delta[n + 3]$$

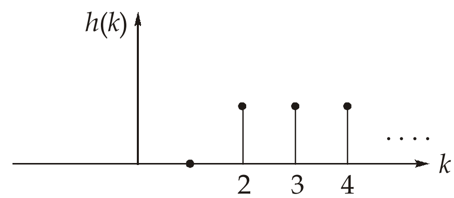
$$y[n] = 0.5^{n+3}\, u[n + 3]$$

Q.8 (a) (ii) Solution:

$$x(k) = 3^k\, u(-k+3)$$

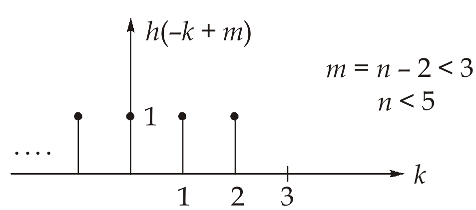
$$h(k) = u(k-2)$$

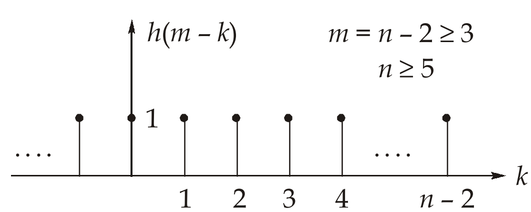




$$y(k) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k)$$

$$h(n-k) = u(n-k-2)$$





$n \geq 5$:

$$y(n) = \sum_{k=-\infty}^{3} 3^k = \sum_{k=-3}^{\infty} \left(\frac{1}{3}\right)^k$$

$$= 3^3 + 3^2 + 3^1 + 1 + \frac{1}{3} + \ldots.$$

$$= \frac{3^3}{1-\frac{1}{3}} = \left(\frac{81}{2}\right)$$

***n* < 5:**

$$y(n) = \sum_{k=-\infty}^{n-2} 3^k$$

$$= [3^{n-2} + 3^{n-3} + 3^{n-4} \ldots]$$

$$= \frac{3^{n-2}}{\left(1-\frac{1}{3}\right)} = \frac{1}{2}\cdot 3^{n-1}$$

$$y(n) = \begin{cases} \frac{81}{2} & n \geq 5 \\ \frac{1}{2}\cdot 3^{n-1} & n < 5 \end{cases}$$

Q.8 (b) (i) Solution:

- The Routh table for the given characteristic equation can be formed as follows:

s^4	1	2	10
s^3	K	$K+1$	0
s^2	$\frac{2K-(K+1)}{K} = \left(1-\frac{1}{K}\right)$	10	0
s^1	$(K+1) - \frac{10K^2}{(K-1)}$	0	0
s^0	10	0	0

- For the system to be stable, all the elements in the first column of the Routh table should have same sign. For this, the following conditions should be satisfied.

From s^3 row, $K > 0$ ...(i)

From s^2 row, $1-\frac{1}{K} > 0$

$K > 1$ ...(ii)

From s^1 row, $(K+1) - \frac{10K^2}{(K-1)} > 0$

$K^2 - 1 - 10K^2 > 0$

$-9K^2 - 1 > 0$ ...(iii)

- No real value of "K" satisfies all the equations (i), (ii) and (iii) simultaneously. So, the given system is unstable for any real value of "K".

Q.8 (b) (ii) Solution:

The open loop transfer function is

$$= \frac{10}{(s+1+10k)s}$$

As k is not a multiplying factor, we modify the equation such that k appears as the multiplying factor. Since the characteristic equation is

$$s^2 + s + 10\,ks + 10 = 0$$

We rewrite this equation as follows:–

$$1 + \frac{10ks}{s^2+s+10} = 0$$

Let $10k = K$

The above equation becomes,

$$1 + \frac{Ks}{s^2+s+10} = 0$$

The system has a zero at $s = 0$, and two open loop poles

$$s = \frac{-1 \pm \sqrt{1-40}}{2} = -0.5 \pm j3.1225$$

Breakaway point is obtained by,

$$\frac{dK}{ds} = 0$$

From, C.E.,

$$\frac{dK}{ds} = \frac{d}{ds}\left[-\frac{(s^2+s+10)}{10s}\right] = 0$$

$$s = -3.16$$

As the system has two poles and one zero, there is a possibility of circular root loci. The angle equation is:

$$\angle \frac{Ks}{s^2+s+10} = \pm 180(2n+1)$$

$$\angle s - \angle s + 0.5 + j3.1225 - \angle s + 0.5 - j3.1225 = \pm 180(2n+1)$$

$$s = \sigma + j\omega$$

$$\angle \sigma + 0.5 + j(\omega + 3.1225) + \angle \sigma + 0.5 + j(w - 3.1225) = \pm 180\ (2n+1)$$

$$s = \sigma + j\omega$$

$$\tan^{-1}\frac{\omega + 3.1225}{\sigma + 0.5} + \tan^{-1}\frac{(\omega - 3.1225)}{\sigma + 0.5} = \tan^{-1}\left(\frac{\omega}{\sigma}\right) \pm 180°(2n+1)$$

Taking tangents on both side, we obtain

$$\frac{\dfrac{\omega+3.1225}{\sigma+0.5}+\dfrac{\omega-3.1225}{\sigma+0.5}}{1-\dfrac{(\omega+3.1225)(\omega-3.1225)}{(\sigma+0.5)^2}} = \frac{\omega}{\sigma}$$

$$\frac{2\omega(\sigma+0.5)}{(\sigma+0.5)^2-(\omega^2-3.1225^2)} = \frac{\omega}{\sigma}$$

$$2\omega\sigma(\sigma + 0.5) = \omega(\sigma + 0.5)^2 - \omega(\omega^2 - 10)$$

$$2\sigma^2 + \sigma = \sigma^2 + \sigma + 0.25 - \omega^2 + 10$$

which yields $\sigma^2 + \omega^2 = 10.25$

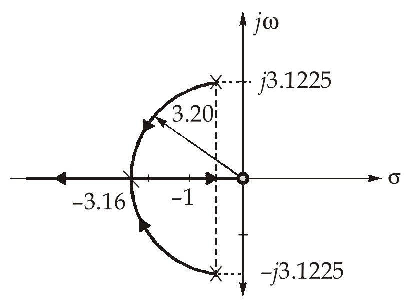
It is the equation of root loci of the system

The negative real axis corresponds to $K \geq 0$ and positive real axis corresponds to $K < 0$.

The equation

$$\sigma^2 + \omega^2 = 10.25$$

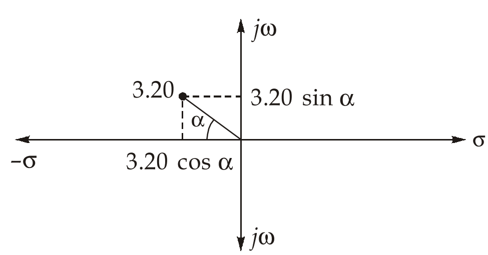
is the equation of circle with centre at $\sigma = 0$, $\omega = 0$ (i.e., origin) and radius = $\sqrt{10.25} = 3.20$.



As we require $\xi = 0.7$ for the closed loop poles, we find the intersection of the circular root locus and a line having angle

$$\cos^{-1}\xi = \cos^{-1}0.7 = 45.57°$$

This angle is with negative real axis as $\xi = 0.7$ is an underdamped system. The intersection can be found as:



$$\sigma = -3.20 \cos \alpha$$

$$\cos \alpha = \cos 45.57° = 0.7$$

$$\sigma = -\sqrt{10.25} \times 0.7 = -2.24$$

$$\omega = 3.20 \sin\alpha = 3.20 \cdot \sin 45.57° = 2.285$$

The intersection of $\xi = 0.7$ with root locus is at

$$s = -2.24 + j2.285$$

The gain K for this $s = -2.24 + j2.285$

$$K = -\left.\frac{(s^2 + s + 10)}{s}\right|_{s=(-2.24+j2.285)}$$

$$K = 3.427$$

Hence desired value of velocity feedback gain

$$k = \frac{K}{10} = 0.3427$$

Q.8 (c) Solution:

(i)

$$\text{slip, } s = \frac{-n}{m}\cos\alpha = -a\cos\alpha_m$$

For 25% speed range, $S_m = 0.25$ thus below the synchronous speed

$$0.25 = -a \cos 165°$$

$$a = \frac{-0.25}{\cos 165°} = 0.2588 \approx 0.259$$

$$\frac{n}{m} = a \text{ or } \frac{2}{m} = 0.259 \text{ or } m = 7.722$$

(ii) For a speed of 780 rpm

$$\text{slip, } s = \frac{1000 - 780}{1000} = 0.22$$

$$V_{d1} = \frac{3\sqrt{6}}{\pi} s \frac{V}{n} = \frac{3\sqrt{6}}{\pi} \times \frac{0.22 \times \frac{440}{\sqrt{3}}}{2} = 65.363 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi}\frac{V}{m}\cos\alpha$$

$$= \frac{3\sqrt{6}}{\pi}\frac{440}{\sqrt{3} \times 7.722} \times \cos 140° = -58.95 \text{ V}$$

$$R'_S = 0.1 \times (0.5)^2 = 0.025,$$

$$R_r = 0.08 \times 0.5 \times 0.5 = 0.02$$

$$I_d = \frac{V_{d1} + V_{d2}}{2(SR'_s + R_r) + R_d} = \frac{65.363 - 58.95}{2(0.22 \times 0.025 + 0.02) + 0.01}$$

$$= 105.11 \text{ A}$$

For 800 rpm $\quad$ slip, $s = \dfrac{1000-800}{1000} = 0.2$

$$\text{Torque, } T = \frac{|V_{d2}|I_d}{s\,\omega_{ms}} = \frac{58.95\times 105.11}{0.22\times 104.72} \approx 269 \text{ N-m}$$

(iii) $\quad$ Rated slip $= \dfrac{1000-970}{1000} = 0.03$

$$\text{Rated torue} = \frac{\dfrac{3}{104.72}\times\left(\dfrac{440}{\sqrt{3}}\right)^2\times\dfrac{0.08}{0.03}}{\left((0.1)^2+\dfrac{0.08}{0.03}\right)^2+(0.7)^2} = 605.32 \text{ N-m}$$

Half rated torque $= 302.66$ N-m

For 800 rpm, $\quad$ slip, $s = \dfrac{1000-800}{1000} = 0.2$

$$V_{d1} = \frac{3\sqrt{6}}{\pi}\times\frac{0.2\times\dfrac{440}{\sqrt{3}}}{2} = 59.42 \text{ V}$$

$$V_{d2} = \frac{3\sqrt{6}}{\pi}\times\frac{\dfrac{440}{\sqrt{3}}}{7.722}\cos\alpha = 76.95\cos\alpha$$

$$I_d = \frac{59.42+76.95\cos\alpha}{2(0.2\times 0.025+0.02)+0.01} = 990.33 + 1282.5\cos\alpha$$

$$\text{Torque, } T = \frac{|V_{d2}|I_d}{s\,\omega_{ms}} = \frac{76.95|\cos\alpha|\times(990.33+1282.5\cos\alpha)}{0.2\times 104.72}$$

$$T = (3.673\,|\cos\alpha|)\,(990.33 + 1282.5\cos\alpha)$$

Let, $\quad \cos\alpha = -X,$

$$\text{Torque, } T = (3.673X)\,(990.33 - 1282.5X)$$

This should be equal to half rated torque 302.66 N-m

$$(3.673X)\,(990.33 - 1282.5X) = 302.66$$

$$X^2 - 0.772\,X + 0.06425 = 0$$

$$X = 0.677 \text{ and } 0.0949$$

$$\alpha = 132.6° \text{ and } 95.45°$$

Later value of α corresponds to operation in unstable part of characteristics.

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