MADE EASY

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Detailed Solutions

ESE-2019 Mains Test Series

Mechanical Engineering Test No : 13

Section A

Q.1 (a) Solution:

1.

As the car starts moving from point 1 its speed will increase because of gravitational work. Potential energy will be converted into kinetic energy as there is no loss.

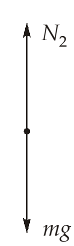
So, $$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

$\Rightarrow$ $$V_2^2 = 2gh_1 = 2 \times 9.81 \times 12 = 235.44\ (\text{m/s})^2$$

Drawing FBD at point 2:

As car is moving at radius of curvature equal to 6 m.

So, $$N_2 - mg = \frac{mv_2^2}{r_2}$$



$\Rightarrow$ $$N_2 = m\left(g + \frac{V_2^2}{r_2}\right)$$

$$= 900\left(9.81 + \frac{235.44}{6}\right) = 44.145 \text{ kN}$$

2.

At point 3, due to centrifugal force car will try to lift itself. The minimum safe radius of curvature will be when normal force approaches to zero.

Energy conservation

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_3 + \frac{1}{2}mv_3^2$$

$$\Rightarrow \quad v_3^2 = 2 \times g \times (h_1 - h_3)$$

$$= 2 \times 9.81 \times (12 - 4) = 156.96\ (\text{m/s})^2$$

For minimum safe radius of curvature

$$mg - N_3 = \frac{mv_3^2}{r_3}$$

$$r_3 = \frac{v_3^2}{g} = 16\ \text{m} \qquad (N_3 = 0)$$

Q.1 (b) Solution:

Due to the force at arm there will be shear stress, moment and torque at cross-section containing K.

$$V = 10\ \text{kN}$$

$$M = 10\ \text{kN} \times 0.15\ \text{m} = 1.5\ \text{kNm}$$

$$T = 10\ \text{kN} \times 0.2\ \text{m} = 2\ \text{kNm}$$

Shear stress at K due to V = 0

Normal stress at K due to M, $\sigma = -\frac{Mr_o}{I}$

Shear stress at K due to T, $\tau = \frac{Tr_o}{J}$

Given; $r_o = \frac{104}{2} = 52$ mm

$$r_i = r_o - 4\ \text{mm} = 48\ \text{mm}$$

$$I = \frac{\pi r_0^4}{4} - \frac{\pi r_i^4}{4} = \frac{\pi\left(52^4 - 48^4\right)}{4} = 1.5733 \times 10^{-6}\ \text{m}^4$$

$$J = \frac{\pi r_0^4}{2} - \frac{\pi r_i^4}{2} = 3.1466 \times 10^{-6}\ \text{m}^4$$

So, $\sigma = -\frac{Mr_o}{I} = \frac{-1.5 \times 10^3 \times 0.052}{1.5733 \times 10^{-6}} = -49.577$ MPa

$$\tau = \frac{Tr_o}{J} = \frac{2\times10^3\times0.052}{3.1466\times10^{-6}} = 33.051 \text{ MPa}$$

$$\text{Principal stress} = \frac{\sigma}{2}\pm\sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = -24.788 \pm 41.314$$

$$= -66.102 \text{ MPa}, 16.526 \text{ MPa}$$

(Negative sign is for compressive stress)

$$\text{Maximum shearing stress} = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = 41.314 \text{ MPa}$$

Please note that $\frac{D_i}{t} = \frac{96}{4} = 24 > 20$

So, we can take thin cylinder also.

In that case, $J = 2\pi r^3 t$ (here r is mean radius)

$$= 2\pi \times (50)^3 \times 4 \text{ mm}^4$$

$$= 3.1416 \times 10^{-6} \text{ m}^4$$

and $I = 1.5708 \times 10^{-6} \text{ m}^4$

Now $\sigma = -\frac{Mr_o}{I} = \frac{-1.5\times10^3\times0.052}{1.5708\times10^{-6}} = -49.656 \text{ MPa}$

$$\tau = \frac{Tr_o}{J} = \frac{2\times10^3\times0.052}{3.1416\times10^{-6}} = 33.104 \text{ MPa}$$

$$\text{Principal stress} = \frac{\sigma}{2}\pm\sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = -24.828 \pm 41.380$$

$$= -66.208 \text{ MPa}, 16.552 \text{ MPa}$$

(Negative sign is for compressive stress)

$$\text{Maximum shearing stress} = \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2} = 41.380 \text{ MPa}$$

Q.1 (c) Solution:

As per given information:

$$\omega_1 \Rightarrow \text{uniformly}$$

$$\frac{(N_2)_{max} - (N_2)_{min}}{(N_{mean})} = 0.08$$

$$(N_2)_{max} - (N_2)_{min} = 0.08(N_2)_{mean}$$

Similarly,

$$(\omega_2)_{max} - (\omega_2)_{min} = 0.08\,(\omega_2)_{mean}$$

$$\frac{\omega_1}{\cos\alpha} - \omega_1\cos\alpha = 0.08\omega_1$$

$$1 - \cos^2\alpha = 0.08\cos\alpha$$

$$\cos^2\alpha + 0.08\cos\alpha - 1 = 0$$

$$\cos\alpha = 0.96079$$

$$\cos\alpha = -1.0407 \text{ (Not possible)}$$

So, $\cos\alpha = 0.96079$

$$\alpha = \cos^{-1}(0.96079)$$

$$\alpha = 16.097°$$

Q.1 (d) Solution:

Given, $P = 12$ kW, $m_1 = 20$ kg, $R_1 = 80$ mm, $m_2 = 35$ kg, $R_2 = 120$ mm, $N_1 = 2000$ rpm

$$I_1 = m_1R_1^2 = 20 \times 0.08^2 = 0.128 \text{ kg-m}^2$$

$$I_2 = m_2R_2^2 = 35 \times 0.120^2 = 0.504 \text{ kg-m}^2$$

$$\omega_1 = \frac{2\pi N_1}{60} = 209.44 \text{ rad/s}$$

$$\text{Torque, } T = \frac{P\times 60}{2\pi N} = \frac{12\times 10^3\times 60}{2\pi\times 2000} = 57.295 \text{ Nm}$$

1. Time required to bring the output shaft to the rated speed from rest

$$t = \frac{(\omega_1 - \omega_2)I_1I_2}{(I_1 + I_2)T} \qquad \text{(where, } \omega_2 = 0)$$

$$t = \frac{(209.44 - 0)\times 0.128\times 0.504}{(0.128 + 0.504)\times 57.295}$$

$$t = 0.373 \text{ s}$$

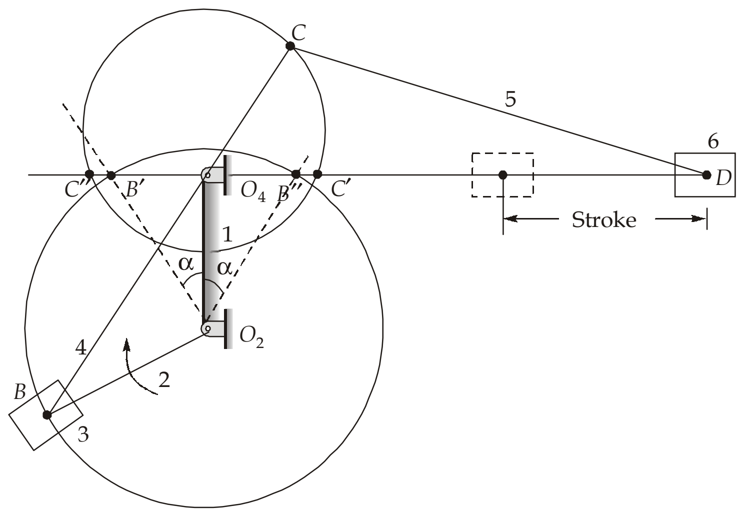
2. Heat generated during clutching:

$$E = \frac{1}{2}\frac{(\omega_1 - \omega_2)^2 I_1 I_2}{(I_1 + I_2)} = \frac{1}{2}\frac{(209.44 - 0)^2 \times 0.128 \times 0.504}{(0.128 + 0.504)}$$

$$E = 2238.786 \text{ J}$$

Q.1 (e) Solution:

As per given information:



$$\text{Time ratio (quick return ration)} = \frac{360 - 2\alpha}{2\alpha} = 2 \qquad \Rightarrow \alpha = 60°$$

$$\cos\alpha = \frac{O_2O_4}{O_2B'}$$

$$\cos 60° = \frac{76.2}{O_2B'}$$

$$O_2B' = \frac{76.2}{\cos 60°} = 152.4 \text{ mm} = O_2B$$

$$\text{Length of the driving crank} = 152.4 \text{ mm}$$

$$\text{Length the stroke} = 343 = 2 \times O_4C$$

$$O_4C = 171.5 \text{ mm}$$

as per given condition $CD = 3 \times O_4C$

$$CD = 3 \times 171.5 \text{ mm} = 514.5 \text{ mm}$$

Q.2 (a) Solution:

As per given information,

$$\text{total mass, } m = 90 \text{ kg}$$

$$\text{unbalanced mass, } m_o = 1.5 \text{ kg}$$

$$\text{Stroke length} = 8 \text{ cm} = 2r$$

$$r = 0.04 \text{ m}$$

$$\text{Crank shaft rotation, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 1200}{60} = 40\pi$$

maximum unbalanced periodic force, $F_o = m_o r\omega^2 = 1.5 \times 0.04 \times (40\pi)^2 = 947.482$ N

$$\text{equivalent stiffness} = 4\text{ K} = k_{eq}$$

$$\text{Transmissibility, } \varepsilon = \frac{1}{30}, \text{ when } \xi = 0$$

$$\varepsilon = \pm\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

First take +ve sign,

$$\varepsilon = +\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

$$\frac{1}{30} = \frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

$$1-\left(\frac{\omega}{\omega_n}\right)^2 = 30$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = -29 \qquad \text{(Not possible)}$$

Then take –ve sign,

$$\frac{1}{30} = \frac{-1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

$$-1+\left(\frac{\omega}{\omega_n}\right)^2 = 30$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 31 \qquad \text{(Possible)}$$

$$\frac{\omega}{\omega_n} = 5.567$$

$$\omega_n = \frac{40\pi}{5.567}$$

$$\omega_n = 22.572 \text{ rad/s}$$

$$\sqrt{\frac{k_{eq}}{m}} = 22.572$$

$$k_{eq} = 90 \times 509.538$$

$$k_{eq} = 45858.494 \text{ N/m}$$

$$4k = 45858.494$$

$$k = 11.464 \text{ kN/m}$$

After damping

$$x_1 = (1 - 0.2)x_o$$

$$\frac{x_o}{x_1} = \frac{1}{0.8}$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln 1.25$$

$$\xi = 0.0354$$

1. Now transmissibility, ε,

$$\varepsilon = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\varepsilon = \frac{\sqrt{1+(2\times 0.0354\times 5.567)^2}}{\sqrt{\left(1-(5.567)^2\right)^2+(2\times 0.0354\times 5.567)^2}}$$

$$\varepsilon = 0.03583$$

2. Amplitude of vibration at resonance ($\omega = \omega_n$)

$$A = \frac{F_0/k_{eq}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} = \frac{F_0/k_{eq}}{2\xi}$$

$$F_0 = mr\omega_n^2 = 1.5 \times 0.04 \times 22.572^2$$
$$= 30.569 \text{ N}$$

$$A = \frac{30.569/45858.494}{2\times 0.0354} = 9.415\times 10^{-3} \text{ m}$$

$$\varepsilon = \frac{\sqrt{1+(2\xi)^2}}{2\xi} = \frac{\sqrt{1+(2\times 0.0354)^2}}{2\times 0.0354} = 14.16 \text{ N}$$

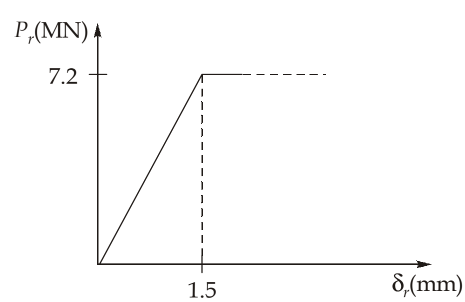
$$F_t = 14.16 \times 30.569 = 432.85 \text{ N}$$

Q.2 (b) Solution:

1. Load-deflection diagram for rod,

Load at yield point $= (\sigma_r)_y A_r = 360 \times 10^6 \times 200 \times 10^{-4} = 7.2$ MN

Deflection at yield point $= \dfrac{(\sigma_r)_y L}{E_r} = \dfrac{360\times 10^6}{120\times 10^9}\times 500\text{mm} = 1.5$ mm

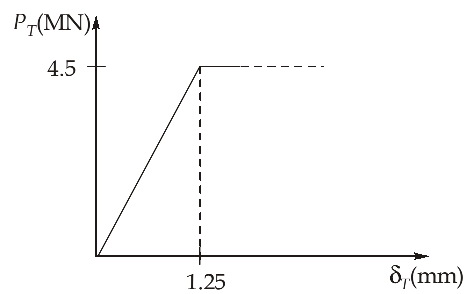


Load deflection diagram for tube,

Load at yield point $= (\sigma_t)_y A_t$

$= 450 \times 10^6 \times 100 \times 10^{-4} = 4.5$ MN

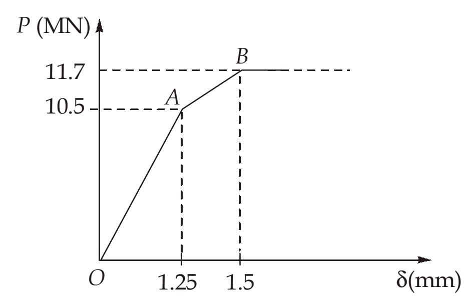
$$\text{Deflection at yield point} = \frac{(\sigma_t)_y L}{E_t} = \frac{450 \times 10^6}{180 \times 10^9} \times 500 \text{ mm} = 1.25 \text{ mm}$$



Load deflection diagram for rod tube assembly

$$\text{Load at 1.25 mm deflection} = 4.5 \text{ MN} + \frac{7.2}{1.5} \times 1.25 \text{ MN} = 10.5 \text{ MN}$$

$$\text{Load at 1.5 mm deflection} = 4.5 \text{ MN} + 7.2 \text{ MN} = 11.7 \text{ MN}$$



2. (i)

If 11 MN is applied, yield point of tube will be reached and rod will be in elastic region.

$$\text{Load on tube} = 4.5 \text{ MN}$$

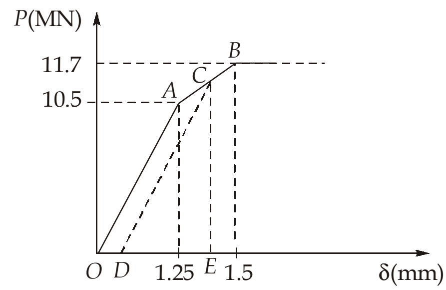
$$\text{Load on rod} = (11 - 4.5) \text{ MN} = 6.5 \text{ MN}$$

For P_r = 6.5 MN, elongation will be

$$\delta_{max} = \left(\frac{P_r}{A_r}\right)\frac{1}{E_r} \times L$$

$$= \frac{6.5 \times 10^6}{200 \times 10^{-4}} \times \frac{1}{120 \times 10^9} \times 500 \text{ mm} = 1.354167 \text{ mm}$$

ii.



During unloading, load will decrease parallel to the line OA.

$$\text{Slope of } OA = \frac{10.5}{1.25}\,(\text{MN/mm})$$

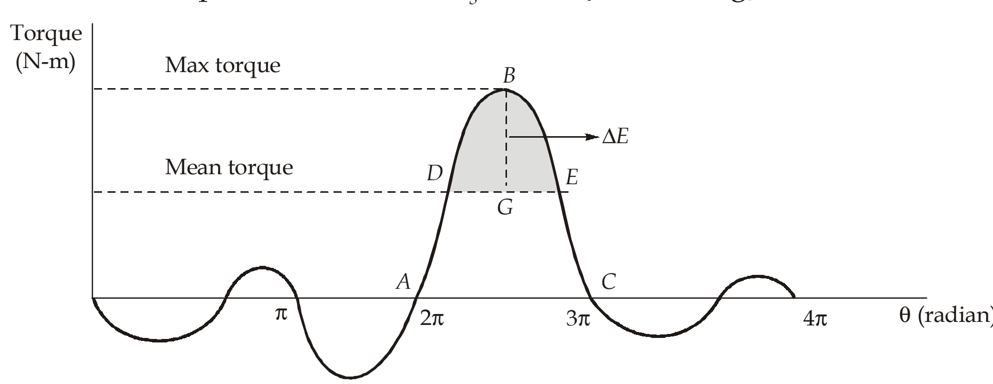
$$DE = \frac{CE}{\text{Slope of CD}} = \frac{11\text{ MN}}{\frac{10.5}{1.25}(\text{MN/mm})} = 1.309524\text{ mm}$$

$$\text{Permanent elongation} = OD = OE - DE = 1.354167 - 1.309524$$

$$= 0.044643\text{ mm}$$

Q.2 (c) Solution:

$P = 180$ kW, $N = 240$ rpm, $\sigma = 5.2$ MPa, $C_s = 0.03$, $\rho = 7220$ kg/m^3



$$P = \frac{2\pi N T_{mean}}{60}$$

$$T_{mean} = \frac{P\times 60}{2\pi N} = \frac{180\times 10^3\times 60}{2\pi\times 240} = 7161.97\text{ Nm}$$

$$\text{Work done per cycle} = T_{mean}\times 4\pi$$

$$= 7161.972 \times 4\pi = 90000\text{ J}$$

$$\text{Work done in power stroke} = 90000+\frac{1}{3}\times 90000 = 120000\text{ J}$$

$$\text{Work done in power stroke} = \frac{1}{2}\times T_{max}\times\pi$$

$$\frac{1}{2}\times T_{max}\times\pi = 120000$$

$$T_{max} = 76394.37\text{ Nm}$$

$$\frac{DE}{\pi} = \frac{T_{max}-T_{mean}}{T_{max}}$$

$$DE = \frac{69232.398\times\pi}{76394.37} = 2.847\text{ radian}$$

$$\text{Fluctuation of energy, }\Delta E = \frac{1}{2}\times BG\times DE = \frac{1}{2}\times 69232.398\times 2.847 = 98554.68\text{J}$$

We know that:

$$\Delta E = I\times\omega^2_{mean}\times c_s$$

$$\omega_{mean} = \frac{2\pi\times 240}{60} = 25.132\text{ rad/s}$$

$$\sigma = \rho v^2$$

$$\sigma = 7220\times R^2\times\omega^2_{mean}$$

$$\frac{5.2\times 10^6}{7220\times 25.132^2} = R^2$$

$$R = 1.0678\text{ m}$$

$$D = 2.135\text{ m}$$

So,
$$\Delta E = I\times\omega^2_{mean}\times c_s$$

$$98554.68 = m\times R^2\times(25.132)^2\times 0.03$$

(where, m is the mass in kg)

$$m = 4561.65\text{ kg}$$

Given,
$$\text{width} = 2\times\text{thickness}$$

$$b = 2\times t$$

$$\text{Cross-sectional area of rim, } A = b\times t$$

$$A = 2t\times t$$

$$A = 2t^2 \text{ m}^2$$

$$\text{mass} = \text{density} \times \text{volume}$$

$$4561.65 = 7220 \times A \times \pi \times D$$

$$4561.65 = 7220 \times 2t^2 \times \pi \times 2.135$$

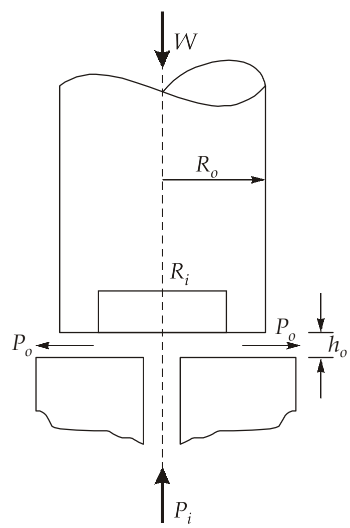
$$t = 0.21702 \text{ m}$$

$$t = 217.021 \text{ mm}$$

$$b = 2 \times t = 434.043 \text{ mm}$$

$$A = 2t^2 = 94196.22 \text{ mm}^2$$

Q.3 (a) Solution:



1. Load carrying capacity for hydrostatic step bearing is given by:

$$W = \frac{\pi P_i}{2} \frac{\left(R_0^2 - R_i^2\right)}{\log_e\left(\frac{R_o}{R_i}\right)}$$

$$P_i = 5 \text{ MPa}$$

$$R_o = 200 \text{ mm}$$

$$R_i = 125 \text{ mm}$$

So,

$$W = \frac{\pi \times 5}{2} \frac{(200^2 - 125^2)}{\log_e\left(\frac{200}{125}\right)} = 407317.71 \text{ N}$$

$$W = 407.32 \text{ kN}$$

Flow rate is given by

$$Q = \frac{\pi P_i h_0^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}$$

$$h_o = 0.15 \text{ mm}$$

$$\mu = 30 \times 10^{-9} \text{ N-s/mm}^2$$

$$Q = \frac{\pi \times 5 \times (0.15)^3}{6 \times (30 \times 10^{-9}) \log_e\left(\frac{200}{125}\right)}$$

$$Q = 626642.63 \text{ mm}^3/\text{s}$$

$$Q = 37.598 \text{ litre/min}$$

2. Pumping loss $(\text{kW})_P$

$$(\text{kW})_P = Q(P_i - P_o) \times 10^{-6} = (626642.63)(5-0) \times 10^{-6}$$

$$(\text{kW})_P = 3.13 \text{ kW}$$

$$\text{Frictional loss, } (\text{kW})_F = \frac{1}{58.05 \times 10^6} \frac{\mu n^2 (R_0^4 - R_i^4)}{h_o}$$

$$= \frac{1}{(58.05 \times 10^6)} \frac{(30 \times 10^{-9}) \times (720)^2 (200^4 - 125^4)}{0.15}$$

$$= 2.42 \text{ kW}$$

$$\text{Total energy loss} = (\text{kW})_t = (\text{kW})_P + (\text{kW})_F = 3.13 + 2.42$$

$$(\text{kW})_t = 5.55 \text{ kW}$$

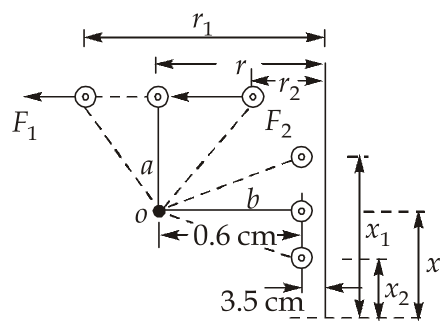
3. $$H = mc_p \Delta t$$

$$m = \rho Q = 0.86 \times 10^3 \times 626642.63 \times 10^{-9} = 0.5389 \text{ kg/s}$$

$$\Delta t = \frac{H}{mc_p} = \frac{5.55}{0.5389 \times 1.75} = 5.88°\text{C}$$

Q.3 (b) Solution:

As per given data:



m = 1.5 kg; a = 11 cm; b = 5 cm; r_1 = 12 cm; r_2 = 7 cm; N_2 = 300 rpm,

N_1 = 1.06 × 300 rpm = 318 rpm

$$F_1 = mr_1\omega_1^2 = 1.5\times 0.12\left(\frac{2\pi\times 318}{60}\right)^2 = 199.610 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 1.5\times 0.07\times\left(\frac{2\pi\times 300}{60}\right)^2 = 103.630 \text{ N}$$

$$\text{Spring constant, } k = 2\left(\frac{a}{b}\right)^2\left(\frac{F_1-F_2}{r_1-r_2}\right) = 2\left(\frac{0.11}{0.05}\right)^2\left(\frac{199.610-103.63}{0.12-0.07}\right)$$

$$= 18581.728 \text{ N/m} = 18.581 \text{ kN/m}$$

here sleeve weight is neglected.

Taking moment about point ‘o’.

$$F_1\cdot a = \frac{S_1}{2}\times b$$

$$S_1\cdot b = 2F_1\cdot a \quad \text{...(i)}$$

$$S\cdot b = 2F\cdot a \quad \text{...(ii)}$$

from equation (i) and (ii)

$$(S_1 - S)b = 2a(F_1 - F)$$

$$k(x_1 - x)b = 2a(F_1 - F)$$

from configuration, $$x_1 - x = (r_1 - r)\left(\frac{b}{a}\right)$$

$$\frac{k(r_1-r)}{2}\left(\frac{b}{a}\right)^2 = F_1 - F$$

$$F = F_1 - \frac{k(r_1-r)}{2}\left(\frac{b}{a}\right)^2$$

$$F = 199.610 - 18581.728\left(\frac{0.03}{2}\right)\left(\frac{5}{11}\right)^2$$

$$F = 142.022 \text{ N}$$

$$mr\omega^2 = 142.022$$

$$1.5 \times 0.09 \times \omega^2 = 142.022$$

$$\omega = 32.434$$

$\Rightarrow$ $$N = 309.729 \text{ rpm}$$

Q.3 (c) Solution:

$$EI\frac{d^4y}{dx^4} = -w = -w_0\left(1-\frac{4x}{L}+\frac{3x^2}{L^2}\right)$$

$$\Rightarrow \quad EI\frac{d^3y}{dx^3} = -w_0\left(x-\frac{2x^2}{L}+\frac{x^3}{L^2}\right)+C_1$$

at $x = L$, $\frac{d^3y}{dx^3}=0$ (as no shear force at free end)

$$\Rightarrow \quad 0 = -w_0(L - 2L + L) + C_1$$

$$\Rightarrow \quad C_1 = 0$$

$$\Rightarrow \quad EI\frac{d^3y}{dx^3} = -w_0\left(x-\frac{2x^2}{L}+\frac{x^3}{L^2}\right)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = -w_0\left(\frac{x^2}{2}-\frac{2x^3}{3L}+\frac{x^4}{4L^2}\right)+C_2$$

at $x = L$ $\quad \frac{d^2y}{dx^2} = 0$ (as no moment at free end)

$$\Rightarrow \quad 0 = -w_0\left(\frac{L^2}{2}-\frac{2L^2}{3}+\frac{L^2}{4}\right)+C_2$$

$$\Rightarrow \quad C_2 = w_0L^2\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right) = w_0L^2\left(\frac{6-8+3}{12}\right) = \frac{w_0L^2}{12}$$

$$\Rightarrow \quad \frac{EId^2y}{dx^2} = -w_0\left(\frac{x^2}{2}-\frac{2x^3}{3L}+\frac{x^4}{4L^2}\right)+\frac{w_0L^2}{12}$$

$$\Rightarrow \quad \frac{EIdy}{dx} = -w_0\left(\frac{x^3}{6}-\frac{x^4}{6L}+\frac{x^5}{20L^2}\right)+\frac{w_0L^2}{12}x+C_3$$

at $x = 0$, $\frac{dy}{dx} = 0 \Rightarrow C_3 = 0$

(a) $$y = \frac{w_0}{EI}\left(\frac{-x^4}{24}+\frac{x^5}{30L}-\frac{x^6}{120L^2}+\frac{L^2x^2}{24}\right)$$

(b) Deflection at free end

$$y_B = \frac{w_0 L^4}{EI}\left(\frac{-1}{24}+\frac{1}{30}-\frac{1}{120}+\frac{1}{24}\right)$$

$$= \frac{w_0 L^4}{EI}\left(\frac{-5+4-1+5}{120}\right) = \frac{w_0 L^4}{40EI}$$

Q.4 (a) Solution:

Given data; Spring index (C) = 6, τ_{perm} = 500 N/mm^2, G = 81370 N/mm^2, P = 500 N, δ = 20 mm

We know that,

$$\tau = K_w \frac{8PC}{\pi d^2}$$

$$K_w = \text{wahl factor} = \frac{4C-1}{4C-4}+\frac{0.615}{C}$$

$$K_w = 1.2525$$

$$\tau = \frac{1.2525\times500\times6\times8}{\pi d^2}$$

$$\tau < \tau_{perm}$$

$$\frac{1.2525\times500\times6\times8}{\pi d^2} = 500\times10^6$$

$$d = 4.3745\times10^{-3}$$

$$d = 4.37\text{ mm} \simeq 5\text{ mm}$$

(ii) Mean coil diameter, $D = C\times d = 5\times 6 = 30$ mm

(iii) Total number of coils:

As the spring has square and ground ends and it has 2 inactive coil so,

$$N_t = N + 2$$

(where N is the number of active coil)

We know that,

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$\delta = \frac{8\times500\times30^3\times N}{81370\times(5)^4} \quad \text{(where, } \delta = 20\text{ mm)}$$

$$N = 9.417 \simeq 10\text{ coils}$$

So,

$$N_t = 10 + 2 = 12\text{ coils}$$

(iv) Free length of the spring:

Actual deflection of spring for N = 10 coils

$$\delta' = \frac{8 \times 500 \times 30^3 \times 10}{81370 \times (5)^4} = 21.24 \text{ mm}$$

Solid length of spring = $N_t \times d = 12 \times 5 = 60$ mm

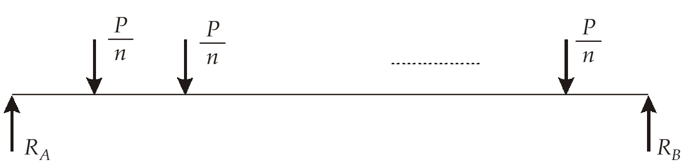
Total axial gap between coils = $(N_t - 1) \times$ axial gap $= 11 \times 1 = 11$ mm

free length = $21.24 + 60 + 11 = 92.24$ mm

(v) pitch of coil = $\dfrac{\text{Free length}}{(N_t - 1)} = \dfrac{92.24}{11} = 8.385$ mm

Q.4 (b) Solution:

FBD of given beam,



By symmetry, $R_A = R_B$

Equating forces in vertical direction,

$$R_A = R_B = \frac{n \times \left(\frac{P}{n}\right)}{2} = \frac{P}{2}$$

For n to be odd, at mid point there will be zero shear force and maximum moment will be there only.

$$M_{\max} = \frac{P}{2} \times \frac{L}{2} - \left(\sum_{i=1}^{\left(\frac{n-1}{2}\right)} \frac{P}{n} \left(\frac{L}{2} - \left(\frac{L}{n+1} \right) i \right) \right)$$

$$= \frac{PL}{4} - \frac{P}{n} \left(\frac{L}{2} \left(\frac{n-1}{2} \right) - \left(\frac{L}{n+1} \right) \frac{\left(\frac{n-1}{2} \right) \left(\frac{n+1}{2} \right)}{2} \right)$$

$$= \frac{PL}{4} - \frac{PL}{n} \left(\frac{n-1}{n} \right) \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{PL}{4} - \frac{PL}{8} \left(\frac{n-1}{n} \right)$$

$$= \frac{PL}{8} \left(2 - \frac{n-1}{n} \right) = \frac{PL}{8} \left(\frac{n+1}{n} \right)$$

For n to be even, there will zero shear force for $\left(\frac{L}{n+1}\right)$ section in the mid of the beam.

We will get maximum moment in that section.

Let's calculate moment at mid point of beam.

$$M_{max} = \frac{P}{2}\times\frac{L}{2}-\left(\sum_{i=1}^{n/2}\frac{P}{n}\left(\frac{L}{2}-\left(\frac{L}{n+1}\right)i\right)\right)$$

$$= \frac{PL}{4}-\frac{PL}{n}\left(\frac{1}{2}\times\frac{n}{2}-\frac{1}{n+1}\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right)$$

$$= \frac{PL}{4}-\frac{PL}{n}\left(\frac{n}{4}-\frac{n(n+2)}{8(n+1)}\right)$$

$$= \frac{PL}{4}-\frac{PL}{n}\left(\frac{2n^2+2n-n^2-2n}{8(n+1)}\right)$$

$$= \frac{PL}{4}-\frac{PL}{8}\left(\frac{n}{n+1}\right) = \frac{PL}{8}\left(\frac{n+2}{n+1}\right)$$

2. for $n = 1$

$$M_{max} = \frac{PL}{4}$$

for $n = 2$

$$M_{max} = \frac{PL}{8}\left(\frac{4}{3}\right) = \frac{PL}{6}$$

for $n = 3$

$$M_{max} = \frac{PL}{8}\left(\frac{4}{3}\right) = \frac{PL}{6}$$

for $n = 4$

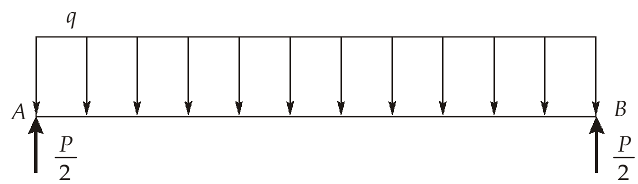
$$M_{max} = \frac{PL}{8}\left(\frac{6}{5}\right) = \frac{3PL}{20}$$

for $n = 5$

$$M_{max} = \frac{PL}{8}\left(\frac{6}{5}\right) = \frac{3PL}{20}$$

... and so on

3. For distributed load, $q = \dfrac{P}{L}$



$$M_{max} = \frac{P}{2}\times\frac{L}{2}-\left(q\times\frac{L}{2}\right)\frac{L}{4} = \frac{PL}{4}-\frac{qL^2}{8} = \frac{PL}{8}$$

$$\text{Also if } n \to \infty \text{ in } \quad \frac{PL}{8}\left(\frac{1+\frac{2}{n}}{1+\frac{1}{n}}\right) \text{ or } \frac{PL}{8}\left(\frac{1+\frac{1}{n}}{1}\right)$$

$$\text{we will get } M_{max} = \frac{PL}{8}$$

Q.4 (c) Solution:

As per given information,

module for gear A and B = $m = 2.5$ mm

module for gear C and D = $m' = 2.0$ mm

From configuration,

$$m\left(\frac{T_A}{2}+\frac{T_B}{2}\right) = m'\left(\frac{T_C}{2}+\frac{T_D}{2}\right)$$

$$2.5\left(\frac{T_A+T_B}{2}\right) = 2.0\left(\frac{T_C+T_D}{2}\right)$$

$$T_C + T_D = \frac{2.5}{2}(T_A+T_B) \quad \text{...(i)}$$

$$m\left(\frac{T_A+T_B}{2}\right) = 150 \text{ mm}$$

$$2.5(T_A+T_B) = 300$$

$$T_A + T_B = \frac{300}{2.5} = 120 \quad \text{...(ii)}$$

$$m'\left(\frac{T_C+T_D}{2}\right) = 150$$

$$T_C + T_D = 150 \quad \text{...(iii)}$$

$$\text{Velocity ratio} = \frac{N_A}{N_D} = 12$$

$$\frac{T_B \times T_D}{T_A \times T_C} = 12 \quad \text{...(iv)}$$

From eq. (ii) and (iii) and (iv)

$$(T_A + T_B)(T_C + T_D) = 120 \times 150 \qquad \{\because T_B \times T_D = 12\, T_A \times T_C\}$$

$$T_AT_C + T_AT_D + T_BT_C + 12\, T_AT_C = 120 \times 150$$

$$13T_AT_C + T_A(150 - T_C) + (120 - T_A)T_C = 120 \times 150$$

$$11T_AT_C = 120 \times 150 - T_A \times 150 - 120 \times T_C$$

$$T_A = \frac{18000 - 120T_C}{11T_C + 150} \quad \text{...(v)}$$

As per given condition, gear should not have less than 24 teeth.

$$T_C \geq 24 \quad \text{...(vi)}$$

So, from (v) and (vi)

when $T_C = 24 \Rightarrow T_A = 36.294$ (Teeth can not be in fraction)

$$T_C = 25 \Rightarrow T_A = 35.294$$

$$T_C = 26 \Rightarrow T_A = 34.128$$

$$T_C = 27 \Rightarrow T_A = 33.02$$

$$T_C = 28 \Rightarrow T_A = 31.965$$

$$T_C = 29 \Rightarrow T_A = 30.95$$

$$T_C = 30 \Rightarrow T_A = 30$$

So, teeth on gears,

$$T_A = 30$$

$$T_A + T_B = 120$$

$$T_B = 120 - T_A = 90$$

$$T_C = 30$$

$$T_C + T_D = 150$$

$$T_D = 150 - T_C = 150 - 30 = 120$$

Cross check

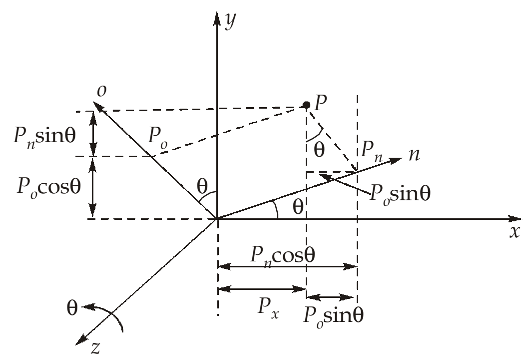
$$\text{centre distance} = m'\frac{(T_C + T_D)}{2} = 2.0\frac{(30+120)}{2} = 150 \text{ mm}$$

$$= \frac{m(T_A + T_B)}{2} = \frac{2.5(30+90)}{2} = 150 \text{ mm}$$

$$\text{Velocity ratio} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{90\times 120}{30\times 30} = 12$$

Section B

Q.5 (a) Solution:



So, $P_x = P_n\cos\theta - P_o\sin\theta$

Similarly $P_y = P_n\sin\theta + P_o\cos\theta$

$P_z = P_a$

In matrix form;
$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} P_n \\ P_o \\ P_a \end{bmatrix}$$

If $\theta = 60°$ and $P = [2, 3, 5]$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -1.598 \\ 3.232 \\ 5 \end{bmatrix}$$

Q.5 (b) Solution:

Most of the failures in rotating machines are due to

1. Damaged bearings, gears, vanes of impellers and shafts.
2. Assembly faults like unbalance, misalignment and bent shaft.
3. Oil whirl in hydrodynamic bearings.

All these factors affects the vibration emitted from the machines. Various mechanical faults can be detected by vibration measurement and analysis. Therefore, vibration is a good indicator of health of a machine.

In a new or just installed machine, high vibration indicate:

1. Improper installation/assembly.
2. Inadequate/improper lubrication.
3. Improper foundation condition.

As the time progresses, with the use of machinery, high vibration level means:

1. Deterioration due to wear which may be due to contaminants ingested/engrossed or corrosion induced by moisture oxidation or electric current leakage.
2. Material and manufacturing defects.

Vibration monitoring can effectively detect:

1. Unbalance
2. Misalignment
3. Eccentric rotor, bent shaft
4. Mechanical looseness, structural weakness,
5. Resonance, peak vibration
6. Mechanical rubbing
7. Problems of belt driven machine
8. Journal bearing defects.
9. Antifriction bearing defects
10. Problems of hydrodynamics and aerodynamic machines.
11. Gear problems (Tooth wear, tooth load, gear eccentricity, backlash, gear misalignment, cracked or broken tooth)
12. Electrical problem of AC and DC motor. (variable air gap, rotor bar defect).

Most of the mechanical failures and problems could be detected by vibration measurement and analysis.

Q.5 (c) Solution:

Interstitial Free Steels: The term 'Interstitial Free steel or IF steel' refers to the fact, that there are no interstitial solute atoms to strain the solid iron lattice, resulting in very soft steel. IF steels have interstitial free body centered cubic (bcc) ferrite matrix. These steels normally have low yield strength, high plastic strain ratio, high strain rate sensitivity and good formability.

Conventional IF steels which were developed commercially in Japan following the introduction of vacuum degassing technology. IF steel is termed as 'clean steel' as the total volume fraction of precipitates is very less. In spite of this, the precipitates appear to have a very significant effect on the properties of IF steels.

Liquid steel is processed to reduce C and N levels to low enough such that the remainder can be 'stabilized' by small additions of Ti and Nb. Ti and Nb are strong carbide/nitride formers, taking the remaining C and N out of solution in liquid iron, after which these latter two elements are no longer available to reside in the interstices between solidified iron atoms.

IF steel has ultra low carbon content, achieved by removing carbon monoxide, hydrogen, nitrogen, and other gasses during steelmaking through a vacuum degassing process. Interstitial elements like nitrogen or carbon are also in the form of nitrides and carbides due to the alloying elements such as Nb and/or Ti used for the stabilization of the residual interstitial. Therefore, IF steels posses typically non aging properties. Without free interstitial elements, these steels are very ductile and soft, don't age, and do not form strain lines during forming due to the absence of yield point elongation (YPE).

A typical IF steel composition is 0.002 % C, 0.01 % Si, 0.15 % Mn, 0.01 % P, 0.01 % S, 0.0025 % N, 0.04 % Al, 0.016 % Nb, and 0.025 % Ti.

The lack of interstitial atoms in the atomic structure enables IF steel to have extremely high ductility, ideal for deep-drawn products. In fact, IF steels are sometimes called extra deep drawing steels (EDDS). They have relatively low strength (although they are sometimes strengthened by the reintroduction of nitrogen or other elements), but high work hardening rates and excellent formability.

Advantages of IF steel include (i) superior stamping, forming, and drawing performance, (ii) the ability to make more complex parts, perhaps using a fewer numbers of dies, (iii) age hardening resistance (long shelf life for stored steel), and (iv) improved coating adhesion for galvanized products.

The main disadvantage of IF steel is that it can be very soft, resulting in shearing and punching difficulties, and its use may result in parts that are not as 'strong', i.e., dent resistant, compared to parts made from carbon steel.

Applications of IF steel: Applications for IF steel include elements of the body structure and closures. Nowadays, IF steel is widely known as the best affordable material for deep drawing operations. It has been utilized for broad applications ranging from automotive body to electronic components as well as from enamel wares to house hold appliances. IF steel has low yield strength but a poor dent resistance which are undesirable for certain automotive applications.

IF steel was initially developed as a highly formable material, and used extensively for deep drawn applications requiring high ductility and resistance to thinning. It also became the standard base for hot-dipped galvanized steels, as the stabilizing alloy elements in IF prevent aging behavior. A third type of IF steel, with nitrogen or other elements reintroduced, could be used to meet higher dent resistance and strength requirements.

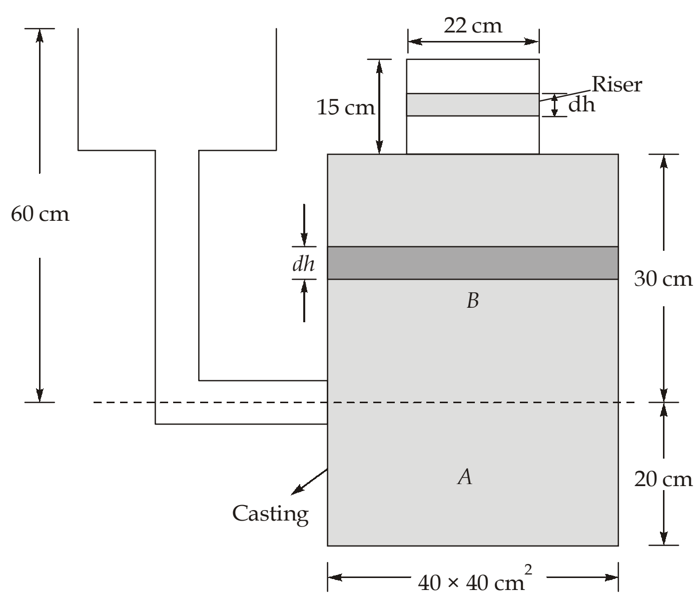
These steels have high strain hardening potential during forming, leading deep-drawn parts (trunks, tailgates, doors, linings, wheel arches, etc.) good dent resistance.

With their high mechanical strength guaranteeing good fatigue and impact resistance, these steels are intended for structural parts.

Q.5.(d) Solution:

Given: A_g = 6 cm^2; A_c = 40 × 40 cm^2; h_t = 60 cm; $(H_c)_1$ = 20 cm; $(H_c)_2$ = 30 cm

Section 'A' of the mould cavity will be filled like top gate.



$$(t_f)_A = \frac{A_c \times (H_c)_1}{A_g\sqrt{2gh_t}} = \frac{40\times40\times20\,\text{cm}^3}{6\times\sqrt{2\times9.81\times100\times60}} = 15.544 \text{ s}$$

Section 'B' of the mould cavity will be filled like bottom gate.

$$dt_f = \frac{A_c dH}{A_g \times \sqrt{2g(h_t - H)}}$$

On integrating,

$$(t_f)_B = \int_0^{30} \frac{A_c dH}{A_g\sqrt{2g(h_t - H)}} = \frac{A_c}{A_g\sqrt{2g}}\left[\frac{-(h_t - H)^{1/2}}{1/2}\right]_0^{30}$$

$$= \frac{40\times40\times2}{6\times\sqrt{2\times9.81\times100}}\left[-\sqrt{60-30}+\sqrt{60-0}\right]$$

$$= \frac{1600\times2}{6\times\sqrt{2\times981}}\left[\sqrt{60}-\sqrt{30}\right] = 27.317 \text{ s}$$

For riser, it will also be filled like bottom gate.

$$A_g \times V \times d(t_f) = A_R \times dH$$

$$6\times\left(\sqrt{2g(h_t - H)}\right)\times dt_f = \frac{\pi}{4}\times22^2\times dH$$

$$dt_f = \frac{\frac{\pi}{4}\times22^2}{6\sqrt{2g}}\frac{dH}{\sqrt{(h_t - H)}}$$

On integrating,

$$(t_f)_c = \int_{30}^{45}\frac{\frac{\pi}{4}\times22^2}{6\sqrt{2\times9.81\times100}}\frac{dH}{\sqrt{(h_t - H)}}$$

$$= \frac{\pi\times22^2}{24\times\sqrt{2\times981}}\left[\frac{-(h_t - H)^{1/2}}{1/2}\right]_{30}^{45}$$

$$(t_f)_c = \frac{\pi\times22^2\times2}{24\times\sqrt{2\times981}}\left[-\sqrt{(60-45)}+\sqrt{60-30}\right] = 4.589 \text{ s}$$

Total time required to fill the casting and cylindrical riser,

$$(t_f)_T = (t_f)_A + (t_f)_B + (t_f)_c = 15.544 + 27.317 + 4.589 = 47.450 \text{ s}$$

Q.5 (e) Solution:

PLC are specialized industrial devices for interfacing and controlling analog and digital devices. It is a digital electronic device that uses a programmable memory to store instructions and to implement functions such as logic sequencing, timing, counting and arithmetic in order to control machines and processes.

- They are designed with a small instruction set suitable for industrial control applications.
- They are usually programmed with "ladder logic", which is a graphical method of laying out the connectivity and logic between system inputs and outputs.

A PLC consist of following main components:

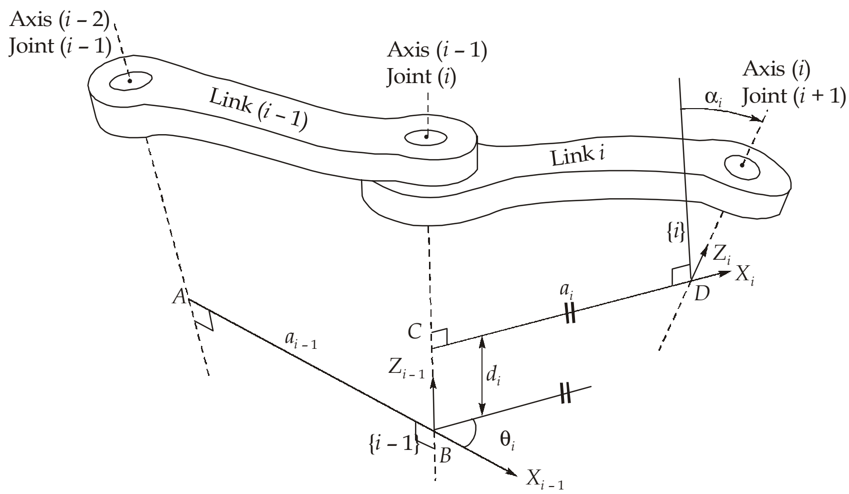
1. Central processing unit (CPU)
2. Memory
3. Input/Output circuitry

Advantages of a PLC:

1. Low cost
2. Ensure increased productivity
3. Easy to install
4. Faster operating speed
5. Provides high reliability and easier maintenance.
6. Can withstand harsh industrial environmental manufacturing environment.
7. Can input/output both analog and digital signal.
8. Reduced cost of scrap and rework.
9. Easier trouble shooting.

Q.6 (a) Solution:

The definition of a manipulator with four joint-link parameters for each link and systematic procedure for assigning right handed orthogonal co-ordinate frame, one to each link in an open kinematic chain is known as Denavit-Hartenberg (DH) rotation.



DH Convention for assigning frames to links
and identifying joint-link parameters

- A frame $\{i\}$ is rigidly attached to the distal end of link i and it move with link i.
- An n- DOF manipulator will have $(n + 1)$ frames with the frame $\{0\}$ or base frame acting as the reference inertial frame.

Figure shows a pair of adjacent links, link $(i – 1)$ and link i, their associated joints, joints $(i - 1)$, i and $(i + 1)$ and axes $(i – 2)$, $(i – 1)$ and (i) respectively.

A frame $\{i\}$ is assigned to link i as follows:

(i) The z_i- axis is aligned with axis i, its direction being arbitrary. The choice of direction defines the positive sense of joint variable θ_i.

(ii) The x_i-axis is perpendicular to axis $z_{i–1}$ and z_i points away from axis $z_{i – 1}$.

(iii) The origin of the i^{th} co-ordinate frame, frame $\{i\}$ is located at the intersection of axis of joint $i + 1$, i.e., axis i and the common normal between axes $(i - 1)$ and (i) (common normal is CD).

(iv) Finally y-axis completes the right hand orthonormal co-ordinate frame $\{i\}$.

Four-DH parameters- two link parameters $(a_i\ \alpha_i)$ and two joint parameters (d_i, θ_i) are:

Link length: For the two axes $(i – 1)$ and i there exist a mutual perpendicular, which gives the shortest distance between the two axes. This shortest distance along the common normal is defined as the link length (a_i). Distance measured x_i-axes with z_{i-1} axes to the origin of frame $\{i\}$.

Link-twist (α_i): The angle between the projection of axis $(i – 1)$ and axis i on a plane perpendicular to the common normal AB. It is angle between z_{i-1} and z_i-axes measured about x_i-axis in the right hand sense.

Joint distance (d_i): For two links connected by either a revolute or a prismatic joint the relative position of these links is measured by the displacement at the joint, which is either joint distance or joint angle. Joint distance is the distance measured along z_{i-1} axis from the origin of frame $(i-1)$ (point B) to the intersection of x_i-axis with z_{i-1} axis (point C) that is distance BC.

Joint angle (θ_i): The angle between x_{i-1} and x_i-axis measured about z_{i-1} axis in the right hand sense.

Q.6 (b) Solution:

In EDM process power source is a capacitor bank. During the major portion of the cycle capacitor bank charges and once the voltage reaches the discharge voltage. All the capacitor discharges simultaneously.

We know that,

$$\text{Discharge voltage, } V_d = V_o(1 - e^{-t/RC})$$

$$\frac{V_d}{V_o} = 1 - e^{-t/RC}$$

$$e^{-t/RC} = 1-\left(\frac{V_d}{V_o}\right)=\left(\frac{V_o - V_d}{V_o}\right)$$

where, V_o is open circuit voltage,

$$\frac{-t}{RC} = \ln\left(\frac{V_o - V_d}{V_o}\right)$$

$$t = RC\ln\left(\frac{V_o}{V_o - V_d}\right)$$

Where t is time of one charging and discharging.

$$\text{Energy released per spark, } E = \frac{1}{2}CV_d^2$$

$$\text{Average power, } P_{\text{avg}} = \frac{E}{t} = \frac{1}{2t}\times CV_d^2 = \frac{1}{2t}\times C\times V_0^2\left(1-e^{-t/RC}\right)^2$$

$$P_{\text{avg}} = \frac{RC}{2tR}\times V_0^2\left(1-e^{-t/RC}\right)^2 \qquad \left[\text{Let, } \frac{t}{RC} = N\right]$$

$$P_{\text{avg}} = \frac{1}{2NR}\times V_0^2\left(1-e^{-N}\right)^2$$

For maximum average power output,

$$\frac{dP_{avg}}{dN} = 0$$

$$\frac{2V_0^2}{2NR}\left(1-e^{-N}\right)\times e^{-N} - \frac{V_0^2}{2N^2R}\left(1-e^{-N}\right)^2 = 0$$

$$2e^{-N} = \frac{\left(1-e^{-N}\right)}{N}$$

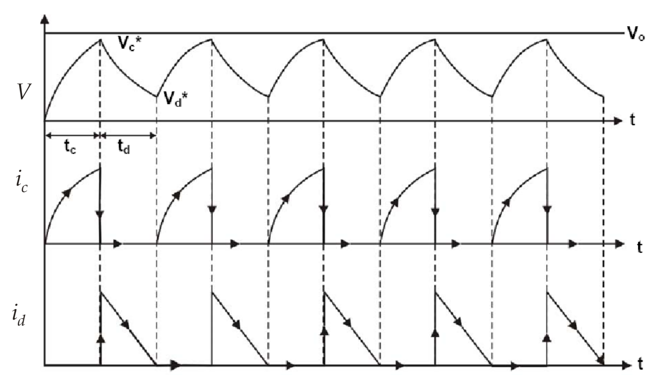
$$2N = \left(\frac{1}{e^{-N}}-1\right)$$

$$e^N - 2N - 1 = 0$$

On solving, $N = 1.2564$

Discharge voltage, $V_d = V_o(1 - e^{-N}) = V_o(1 - e^{-1.2564})$

$$V_d = 0.7153\ V_o$$



Schematic representation of the current pulse during charging and discharging in EDM process.

Q.6 (c) Solution:

For determining the resolved shear stress, first we need to calculate angle ϕ and λ. Where, ϕ is the angle between the normal to the (1 1 0) slip plane (i.e. the [1 1 0] direction) and the [0 1 0] direction. Where, λ represents the angle between [0 1 0] and [$\bar{1}$ 1 1] direction.

1.

For cubic unit cells, an angle θ between the directions 1 and 2, represented by [u_1 v_1 w_1] and [u_2 v_2 w_2], is given by

$$\theta = \cos^{-1}\left[\frac{u_1u_2+v_1v_2+w_1w_2}{\sqrt{\left(u_1^2+v_1^2+w_1^2\right)\left(u_2^2+v_2^2+w_2^2\right)}}\right]$$

For determining the value of ϕ, let $[u_1\ v_1\ w_1] = [0\ 1\ 0]$ and $[u_2\ v_2\ w_2] = [1\ 1\ 0]$

$$\phi = \cos^{-1}\left[\frac{0\times1+1\times1+0\times0}{\sqrt{(0^2+1^2+0^2)(1^2+1^2+0)}}\right] = \cos^{-1}\left[\frac{1}{\sqrt{2}}\right]$$

$$\phi = 45°$$

For λ, take $[u_1\ v_1\ w_1] = [0\ 1\ 0]$ and $[u_2\ v_2\ w_2] = [\bar{1}\ 1\ 1]$

Now,
$$\lambda = \cos^{-1}\left[\frac{0\times(-1)+1\times1+0\times1}{\sqrt{(0^2+1^2+0^2)((-1)^2+1^2+1^2)}}\right]$$

$$= \cos^{-1}\left[\frac{1}{\sqrt{3}}\right] = 54.73°$$

We know that,

Resolved shear stress, $\tau_R = \sigma\cos\phi\cdot\cos\lambda$

$$= 50\cos 45°\cdot\cos 54.73°$$

$$= 50\times\left(\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{3}}\right)$$

Resolved shear stress, $\tau_R = 20.41$ MPa

(2) The angle ϕ and λ will remain same as in part (1)

We know that, $\tau_R = \sigma\cos\phi\cos\lambda$

Yield strength can be given as,

$$\sigma_y = \frac{\tau_R}{\cos\phi\cos\lambda} = \frac{32}{\cos 45°\cos 54.73°} = 32\times(\sqrt{2})\times(\sqrt{3})$$

$$= 78.384 \text{ MPa}$$

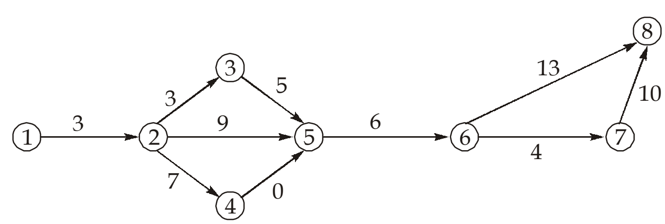
Q.7 (a) Solution:

Given that indirect cost is ₹ 50 per week.

Activity	T_n	T_c	Crash limit	ΔC/ΔT
1 - 2	3	2	1	100
2 - 3	3	3	0	–
2 - 4	7	5	2	80
2 - 5	9	7	2	45
3 - 5	5	4	1	50
4 - 5	0	0	0	–
5 - 6	6	4	2	45
6 - 7	4	3	1	70
6 - 8	13	10	3	40
7 - 8	10	9	1	200

Total direct cost, DC = ₹ 4220

IDC = ₹ 50/week



1 - 2 - 3 - 5 - 6 - 8 = 30 weeks

1 - 2 - 3 - 5 - 6 - 7 - 8 = 31 weeks

1 - 2 - 5 - 6 - 8 = 31 weeks

1 - 2 - 5 - 6 - 7 - 8 = 32 weeks

1 - 2 - 4 - 5 - 6 - 8 = 29 weeks

1 - 2 - 4 - 5 - 6 - 7 - 8 = 30 weeks

Critical path: 1 - 2 - 5 - 6 - 7 - 8

$$T_E = 32 \text{ weeks}$$

$$\text{Total cost} = 4220 + 50 \times 32 = \text{Rs. } 5820$$

Comparing the cost slope of activities along critical path:

1 - 2 = Rs. 100/week

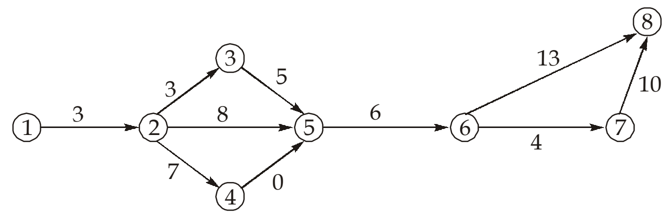
2 - 5 = Rs. 45/week

5 - 6 = Rs. 45/week

6 - 7 = Rs. 70/week

7 - 8 = Rs. 200/week

Crashing activity 2 - 5 by 1 week:



$$\begin{aligned}
1-2-3-5-6-8 &= 30 \text{ weeks}\\
1-2-3-5-6-7-8 &= 31 \text{ weeks}\\
1-2-5-6-8 &= 30 \text{ weeks}\\
1-2-5-6-7-8 &= 31 \text{ weeks}\\
1-2-4-5-6-8 &= 29 \text{ weeks}\\
1-2-4-5-6-7-8 &= 30 \text{ weeks}\\
T_E &= 31 \text{ weeks}\\
\text{Total cost} &= 5820 + 45 - 50 = \text{Rs. } 5815
\end{aligned}$$

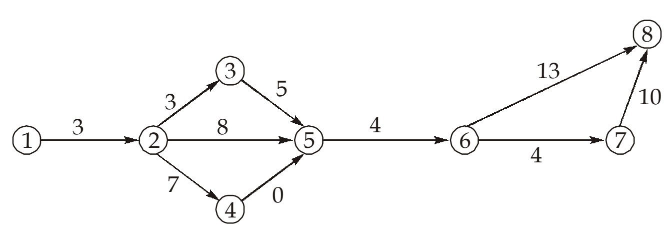
Now, comparing the cost slope and activities along critical path:

1 - 2, 3 - 5, 5 - 6, 6 - 7, 7 - 8

1 - 2, 2 - 5, 5 - 6, 6 - 7, 7 - 8

$$\begin{aligned}
1-2 &= \text{Rs. } 100/\text{week}\\
5-6 &= \text{Rs. } 45/\text{week}\\
6-7 &= \text{Rs. } 70/\text{week}\\
7-8 &= \text{Rs. } 200/\text{week}\\
3-5 \text{ and } 2-5 &= \text{Rs. } 95/\text{week}
\end{aligned}$$

Crashing activity 5 - 6 by 2 weeks



$$\begin{aligned}
1-2-3-5-6-8 &= 28 \text{ weeks}\\
1-2-3-5-6-7-8 &= 29 \text{ weeks}
\end{aligned}$$

$$1 - 2 - 5 - 6 - 8 = 28 \text{ weeks}$$
$$1 - 2 - 5 - 6 - 7 - 8 = 29 \text{ weeks}$$
$$1 - 2 - 4 - 5 - 6 - 8 = 27 \text{ weeks}$$
$$1 - 2 - 4 - 5 - 6 - 7 - 8 = 28 \text{ weeks}$$
$$T_E = 29 \text{ weeks}$$
$$\text{Total cost} = 5815 + 45 \times 2 - 50 \times 2 = \text{Rs. } 5805$$

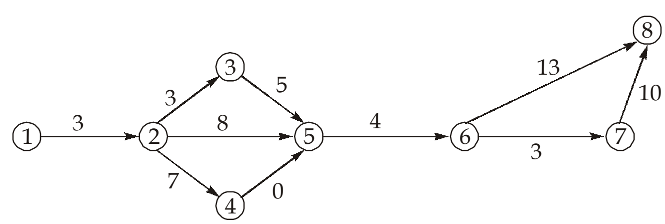
Now, comparing cost slope of activities along critical path:

1 - 2, 3 - 5, 6 - 7, 7 - 8

1 - 2, 2 - 5, 6 - 7, 7 - 8

$$1 - 2 = \text{Rs. } 100/\text{week}$$
$$6 - 7 = \text{Rs. } 70/\text{week}$$
$$7 - 8 = \text{Rs. } 200/\text{week}$$
$$3 - 5 \text{ and } 2 - 5 = \text{Rs. } 95/\text{week}$$

Crashing activity 6 - 7 by 1 week:



$$1 - 2 - 3 - 5 - 6 - 8 = 28 \text{ weeks}$$
$$1 - 2 - 3 - 5 - 6 - 7 - 8 = 28 \text{ weeks}$$
$$1 - 2 - 5 - 6 - 8 = 28 \text{ weeks}$$
$$1 - 2 - 5 - 6 - 7 - 8 = 28 \text{ weeks}$$
$$1 - 2 - 4 - 5 - 6 - 8 = 27 \text{ weeks}$$
$$1 - 2 - 4 - 5 - 6 - 7 - 8 = 27 \text{ weeks}$$
$$T_E = 28 \text{ weeks}$$
$$\text{Total cost} = 5805 + 70 - 50 = \text{Rs. } 5825$$

Since, total cost of project increases, so it is not advisable to crash activity 6 - 7.

$$\text{Optimum project time} = 29 \text{ weeks}$$
$$\text{and} \qquad \text{associated cost} = \text{Rs. } 5805$$
$$\text{Normal project time} = 32 \text{ weeks}$$
$$\text{and} \qquad \text{associated cost} = \text{Rs. } 5820$$

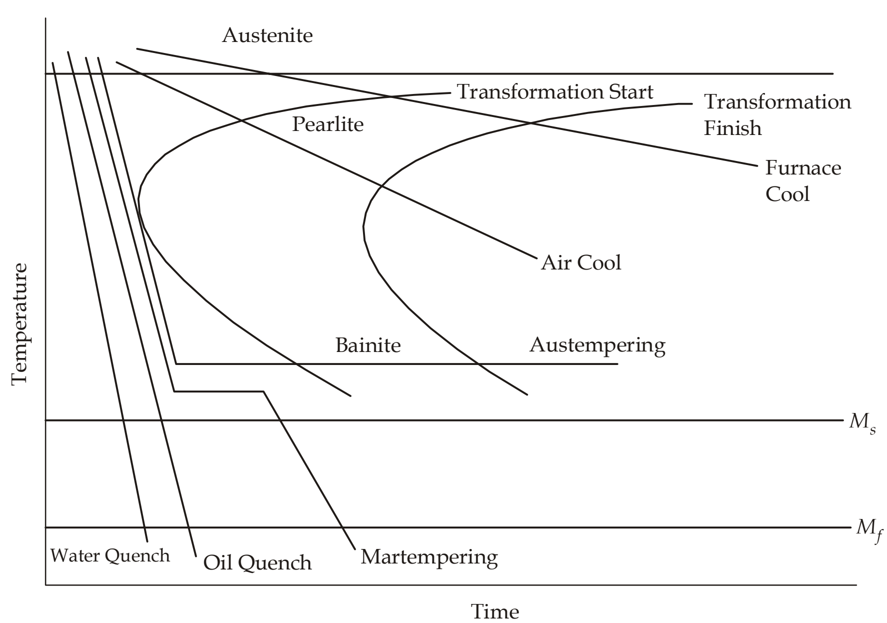
Q.7 (b) Solution:

Austempering: All lines on TTT diagram are that of decomposition of austenite into some other micro structure. Once the austenite converts into some other microstructure it never reconverts again. The slow cooling process produce coarse structure and fast cooling process produces fine structure. It can be observed that bainite cannot be produced by continuous cooling. To produce bainite, the samples has to quenched below the nose of TTT diagram but above martensite start line (220°C). The sample is then maintain at this temperature for substantial period of time till entire austenite converts into bainite. The process is called Austempering. Brittle and thick metals are suitable for austempering.

Advantage of Austempering:

- Quenching cracks between core and surface do not develop.
- Ductility is increased.
- Impact strength and toughness are increased.
- Only small sections are suitable for austempering because big sections can not be cooled rapidly to avoid formation of pearlite, steel sections less than 12 mm thick are suitable for austemparing.

  Upon quenching austenite specimen into water since surface is coming in contact with quenched medium. It will immediately convert into martensite at last there a core is still austenite.



Different cooling rate results in different microstructure

Martempering or Stepped Quenching:

(i) Articles are first quenched in water to a temperature of 300 - 400°C and then quickly transferred to a less intensive medium like oil or air where they are held until they are completely cooled.

(ii) This method is widely used in heat treatment of steel tools like taps, disc, milling cutter etc.

(iii) A larger holding time will cause, austenite decomposition. Austenite is transformed into martensite during the subsequent period of cooling to the room temperature.

Martempering has the following advantages over conventional quenching:

(i) Less volume change occurs due to the presence of a large amount of retained austenite and due to the possibility of self tempering of the austenite.

(ii) Less distortion since the transformation occurs almost simultaneously in the entire volume.

(iii) Less danger of quenching cracks, Martempering can be applied for thin sections from 5 to 8 mm thickness.

Q.7 (c) Solution:

The position is achieved by Cartesian joints so, robot moves 4, 6 and 9 unit along the x, y and z axes. The orientation is achieved by Euler rotations. So,

$$\begin{bmatrix} n_x c\phi + x_y s\phi & o_x c\phi + o_y s\phi & a_x c\phi + a_y s\phi & 0 \\ -n_x s\phi + n_y c\phi & -o_x s\phi + o_y c\phi & -a_x s\phi + a_y c\phi & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta c\psi & -c\theta s\psi & s\theta & 0 \\ s\psi & c\psi & 0 & 0 \\ -s\theta c\psi & s\theta s\psi & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From element (2, 3)

$$-a_x s\phi + a_y c\phi = 0$$

$$\phi = \text{ATAN2}(a_y,\ a_x) \text{ or ATAN2}(-a_y, -a_x)$$

$$\phi = \text{ATAN2}(0.439, 0.628) = 35°$$

$$\phi = \text{ATAN2}(-0.439, -0.628) = 215°$$

From element (2, 1) and (2, 2)

$$s\psi = -n_x s\phi + n_y c\phi$$

$$c\psi = -o_x s\phi + o_y c\phi$$

$$\psi = \text{ATAN2}(s\psi, c\psi)$$

$$\psi(\text{at } \phi = 35°) = \text{ATAN2}(0, 1) = 0°$$

$$\psi(\text{at } \phi = 215°) = \text{ATAN2}(0, -1) = 180°$$

and finally from (1, 3) and (3, 3) element

$$s\theta = a_x c\phi + a_y s\phi$$

$$c\theta = a_z$$

$$\theta = \text{ATAN2}(s\theta, c\theta)$$

$$\theta(\text{at } \phi = 35°) = \text{ATAN2}[(0.628 \times 0.819 + 0.439 \times 0.573), 0.643] = 50°$$

$$\theta(\text{at } \phi = 215°) = -50°$$

So,
$$\phi = 35°, \quad \phi = 215°$$
$$\psi = 0, \quad \psi = 180°$$
$$\theta = 50°, \quad \theta = -50°$$

Q.8 (a) Solution:

(i)

Taylor's tool life equation

$$VT^n = C$$

Where V = cutting speed

T = Tool life (minutes)

n = exponent for conditions tested

C = Taylor's constant

= represents cutting speed for 1 minute as tool life.

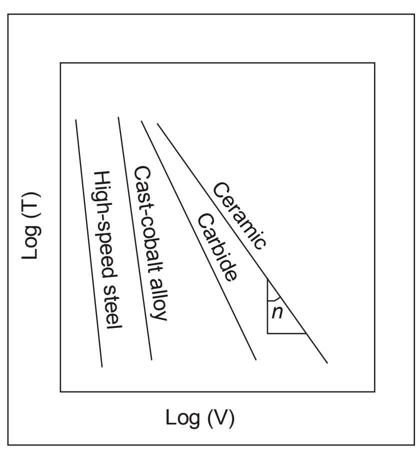


Figure: Tool-life curves for a variety of cutting-tool materials. The negative inverse of the slope of these curves is the exponent n in the Taylor tool-life equation and C is the cutting speed at $T = 1$ min, ranging from about 200 to 10,000 ft./min in this figure

(ii)

$$r = \frac{t_1}{t_2} = \frac{0.127}{0.228} = 0.557$$

$$\alpha = 10° \text{ (rake angle)}$$

$$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha} = \frac{0.557\cos 10°}{1-0.557\sin 10°} = \frac{0.5485}{0.9033}$$

$$\tan\phi = 0.607218$$

$$\phi = 31.27°$$

$$\mu = \frac{F}{N} = \frac{F_c\sin\alpha + F_T\cos\alpha}{F_c\cos\alpha - F_T\sin\alpha}$$

$$\mu = \frac{567\sin 10° + 227\cos 10°}{567\cos 10° - 227\sin 10°} = \frac{322}{519}$$

$$\tan\lambda = \mu \text{ or } \lambda = \tan^{-1}\frac{322}{519} = 31.82°$$

$$F_s = F_c\cos\phi - F_T\sin\phi$$

$$F_s = 567\cos 31.27° - 227\sin 31.27°$$

$$F_s = 484.632 - 117.83 = 366.8 \text{ N}$$

Shear stress along shear plane,

$$\tau_s = \frac{F_s}{\left(\frac{w\,t_1}{\sin\phi}\right)} = \frac{F_s\sin\phi}{w\,t_1}$$

$$\tau_s = \frac{366.8\times\sin 31.27}{6.35\times 0.127} = 236.1 \text{ MPa}$$

Power consumption, $W = F_c \times$ cutting speed

$= 567 \times 2 = 1134$ J/s

Chip velocity, $V_c = \dfrac{V\sin\phi}{\cos(\phi-\alpha)}$

$$= \frac{2\times\sin 31.27°}{\cos 21.27°} = 1.114 \text{ m/s}$$

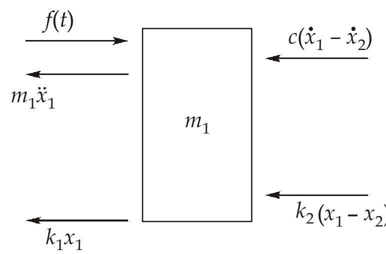
Shear strain, $\gamma = \cot\phi + \tan(\phi - \alpha)$

$$= \frac{1}{\tan 31.27°} + \tan 21.27°$$

$$= 1.6466 + 0.3893 = 2.036$$

Q.8 (b) Solution:

Free body diagram for body m_1 and m_2



So,

$$m_1\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2) = f(t)$$

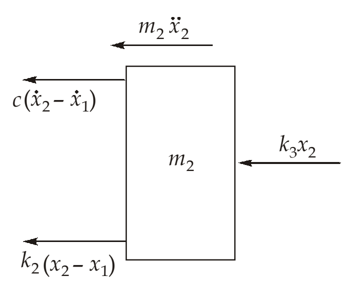
$$m_1\ddot{x}_1 + c\dot{x}_1 - c\dot{x}_2 + k_1x_1 + k_2x_1 - k_2x_2 = f(t)$$

Taking laplace on both side and assuming all initial condition is set to zero.

$m_1s^2x_1(s) + csx_1(s) - csx_2(s) + k_1x_1(s) + k_2x_1(s) - k_2x_2(s) = F(s)$

$(m_1s^2 + cs + k_1 + k_2)x_1(s) - (k_2 + cs)x_2(s) = F(s)$ ...(1)

For mass 2:



$$m_2\ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + k_3x_2 = 0$$

Taking laplace both side and assuming all initial condition is set to zero.

$m_2s^2\, x_2(s) + cs\, x_2(s) - csx_1(s) + k_2x_2(s) - k_2x_1(s) + k_3x_2(s) = 0$

$(m_2s^2 + cs + k_2 + k_3)x_2(s) - (cs + k_2)x_1(s) = 0$

$$\frac{x_2(s)}{x_1(s)} = \frac{(cs + k_2)}{(m_2s^2 + cs + k_2 + k_3)} \quad \text{...(2)}$$

Put value of $x_2(s)$ in equation (1)

$$(m_1s^2 + cs + k_1 + k_2)x_1(s) - \frac{(k_2 + cs)(k_2 + cs)x_1(s)}{(m_2s^2 + cs + k_2 + k_3)} = F(s)$$

$$\frac{\left(m_1s^2+cs+k_1+k_2\right)\left(m_2s^2+cs+k_2+k_3\right)-\left(k_2+cs\right)^2}{\left(m_2s^2+cs+k_2+k_3\right)} = \frac{F(s)}{x_1(s)}$$

$$\frac{x_1(s)}{F(s)} = \frac{\left(m_2s^2+cs+k_2+k_3\right)}{\left(m_1s^2+cs+k_1+k_2\right)\left(m_2s^2+cs+k_2+k_3\right)-\left(k_2+cs\right)^2} \quad ...(3)$$

Multiply equation 3 to equation 2

$$\frac{x_2(s)}{x_1(s)}\times\frac{x_1(s)}{F(s)} = \frac{\left(cs+k_2\right)}{\left(m_2s^2+cs+k_2+k_3\right)}\times\frac{\left(m_2s^2+cs+k_2+k_3\right)}{\left[\left(m_1s^2+cs+k_1+k_2\right)\left(m_2s^2+cs+k_2+k_3\right)-\left(k_2+cs\right)^2\right]}$$

$$\frac{x_2(s)}{F(s)} = \frac{\left(cs+k_2\right)}{\left(m_1s^2+cs+k_1+k_2\right)\left(m_2s^2+cs+k_2+k_3\right)-\left(k_2+cs\right)^2}$$

Q.8 (c) Solution:

(i) Here, we have:

D = 2500 units; C_0 = ₹15 per order

C_1 = ₹4 per unit per year.

$$\therefore \quad Q^\circ = \sqrt{\frac{2DC_0}{C_1}} = \sqrt{\frac{2\times 2500\times 15}{4}}$$

$= 136.93 \simeq 137$ Units

n° = Optimal number of orders

$$= \frac{D}{Q^\circ} = \frac{2500}{137} = 18.25$$

(ii) We are given: D = 2500 units

C_0 = ₹250 per production run

C_1 = ₹4 per unit per year

$$\therefore \quad Q^\circ = \sqrt{\frac{2DC_0}{C_1}}\times\sqrt{\frac{k}{k-r}}$$

$$= \sqrt{\frac{2\times 2500\times 250}{4}}\times\sqrt{\frac{4800}{4800-2500}} = 808 \text{ units}$$

t° = Average duration of the production run

$$= \frac{808}{4800} = 0.1683 \text{ year/ run}$$

(iii) When item is purchased from outside:

$$\text{Total cost} = D \times C + \sqrt{2DC_0C_1}$$
$$= 2500 \times 30 + \sqrt{2 \times 2500 \times 15 \times 4} = 75000 + 547.72$$
$$= ₹\ 75547.72 \text{ or } ₹\ 75548 \text{ approx.}$$

When item is produced internally:

$$\text{Cost per unit, } C = 80\% \text{ of } ₹30 = ₹\ 24$$

and C_1 ₹250 per production run,

$$\therefore \quad \text{Total cost} = D \times C + \frac{D}{Q^\circ} \times C_0 + \frac{1}{2} Q_1^\circ \times \frac{k-r}{k} \times C_1$$
$$= 2500 \times 24 + \frac{2500}{808} \times 250 + \frac{1}{2} \times 808 \times \frac{4800 - 2500}{4800} \times 4$$
$$= ₹60000 + ₹1547.84 = ₹\ 61547.84$$

From the above calculation, we observe that the company should manufacture the product internally.

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