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Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 11

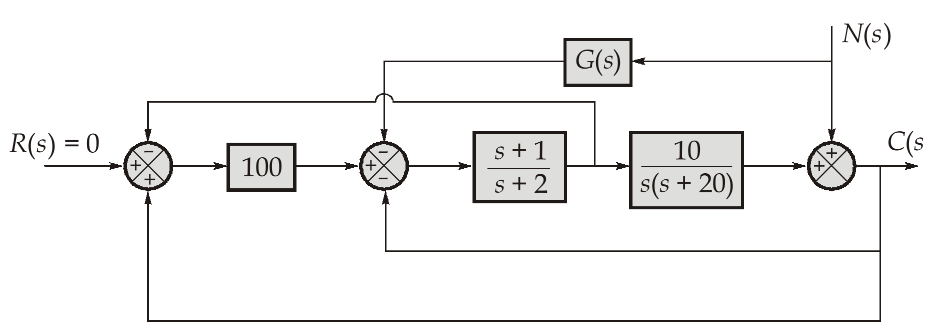
Section-A

Q.1 (a) Solution:

The output $C(s)$ will be totally independent of $N(s)$ if the response due to $N(s)$ is zero.

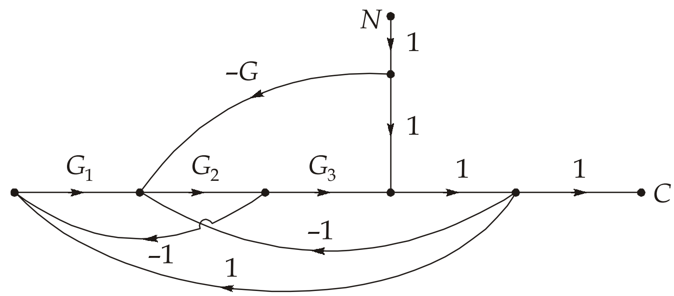
$$\therefore \qquad \frac{C(s)}{N(s)} = 0$$

To obtain the output response $C(s)$ for input $N(s)$. We assume $R(s) = 0$. So the block can be reduced as



In the block diagram, we have, $G_1 = 100$, $G_2 = \frac{s+1}{s+2}$ and $G_3 = \frac{10}{s(s+20)}$.

The signal flow graph of the above block diagram can be given by,



$\therefore$ Forward path becomes, $P_1 = 1$ and $P_2 = -GG_2G_3$

$\because$ All loops are touching to forward path P_2, so we have, $\Delta_2 = 1$

Also one loop is non-touching to forward path P_1, so we have, $\Delta_1 = 1 + G_1G_2$

Using Mason's gain formula,

The transfer function is,

$$\frac{C(s)}{N(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{1 + G_1G_2 - G_2G_3G}{\Delta}$$

$\therefore$ System is independent of $N(s)$.

$$\therefore \qquad \frac{C(s)}{N(s)} = 0$$

$$\therefore \qquad 1 + G_1G_2 - G_2G_3G = 0$$

or

$$G = \frac{1 + G_1G_2}{G_2G_3}$$

On putting the respective values in above equation, we get,

$$G = \frac{1 + 100\left(\frac{s+1}{s+2}\right)}{\left(\frac{s+1}{s+2}\right)\left(\frac{10}{s(s+20)}\right)} = \frac{[(s+2) + 100(s+1)]s(s+20)}{10(s+1)}$$

$$= \frac{s(101s^2 + 2122s + 2040)}{10(s+1)}$$

Q.1 (b) Solution

For a PM signal,

$$\Delta f_{max} = \frac{k_p}{2\pi}\left|\frac{dm(t)}{dt}\right|_{max}$$

$$m(t) = e^{-t^2}\ \text{V}$$

$$\frac{dm(t)}{dt} = -2te^{-t^2}$$

Let, $$x(t) = \frac{dm(t)}{dt} = -2te^{-t^2}$$

Finding the maximum value of $x(t)$:

$$\frac{dx(t)}{dt} = \frac{d}{dt}(-2te^{-t^2}) = -(2-4t^2)e^{-t^2} = 0$$

$$2 - 4t^2 = 0$$

$$t = \pm\frac{1}{\sqrt{2}}$$

So, at $t = \frac{1}{\sqrt{2}}$, $\left|\frac{dm(t)}{dt}\right|$ will attain its maximum value.

$$\left|\frac{dm(t)}{dt}\right|_{max} = (2te^{-t^2})\Big|_{t=\frac{1}{\sqrt{2}}} = \sqrt{2}\,e^{-0.5}$$

So, $$\Delta f_{max} = \frac{8000\pi}{2\pi}(\sqrt{2}e^{-0.5})\text{ Hz}$$

$$= 4\sqrt{2}(e^{-0.5})\text{ kHz} = 3.43\text{ kHz}$$

Q.1 (c) Solution:

(i) Given that, $$\vec{E} = 3\rho^2\sin\phi\,\hat{a}_\rho + 2\rho^2\cos\phi\,\hat{a}_\phi$$

In cylindrical coordinate system,

$$\nabla\cdot\vec{E} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + \frac{1}{\rho}\frac{\partial E_\phi}{\partial\phi} + \frac{\partial E_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(3\rho^3\sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(2\rho^2\cos\phi)$$

$$= \frac{1}{\rho}(9\rho^2\sin\phi) + \frac{1}{\rho}(2\rho^2(-\sin\phi))$$

$$= 9\rho\sin\phi - 2\rho\sin\phi$$

$$\nabla\cdot\vec{E} = 7\rho\sin\phi$$

Converting into Cartisian co-ordinates

$$\rho = \sqrt{x^2+y^2}$$

and

$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}}$$

$\therefore$

$$\nabla\cdot\vec{E} = 7\sqrt{x^2+y^2}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$= 7\sqrt{9^2+9^2}\cdot\frac{9}{\sqrt{9^2+9^2}} = 7\times 9 = 63$$

$$\nabla\cdot\vec{E} = 63$$

(ii) Let the vector quantity be

$$\vec{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z \qquad \text{...(i)}$$

We have to verify,

$$\nabla\cdot(\nabla\times\vec{A}) = 0 \qquad \text{...(ii)}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

by taking divergence of above equation, we get,

$$\nabla\cdot(\nabla\times\vec{A}) = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)(\nabla\times\vec{A})$$

$$= \frac{\partial^2 A_z}{\partial x\,\partial y} - \frac{\partial^2 A_y}{\partial x\,\partial z} - \frac{\partial^2 A_z}{\partial y\,\partial x} + \frac{\partial^2 A_x}{\partial y\,\partial z} + \frac{\partial^2 A_y}{\partial z\,\partial x} - \frac{\partial^2 A_x}{\partial z\,\partial y} = 0$$

Q.1 (d) Solution:

The volume of 1 mol of silicon is,

$$\frac{\text{Molecular weight of Si}}{\text{Density of Si}} = \frac{28.1 \text{ g/mol}}{2.33 \text{ g/cm}^3} = 12.06 \text{ cm}^3/\text{mol}$$

The volume of 1 mol of silicon dioxide is,

$$\frac{\text{Molecular weight of SiO}_2}{\text{Density of SiO}_2} = \frac{60.08 \text{ g/mol}}{2.21 \text{ g/cm}^3} = 27.18 \text{ cm}^3/\text{mol}$$

Since 1 mol of silicon is converted to 1 mol of silicon dioxide,

$$\frac{\text{Thickness of Si} \times \text{area}}{\text{Thickness of SiO}_2 \times \text{area}} = \frac{\text{Volume of 1 mol of Si}}{\text{Volume of 1 mol of SiO}_2}$$

$$\frac{\text{Thickness of Si}}{\text{Thickness of SiO}_2} = \frac{12.06}{27.18} = 0.44$$

$$\text{Thickness of Si} = (0.44)\,(\text{Thickness of SiO}_2)$$

To grow a SiO_2 layer of 100 mm, the thickness of Si consumed will be,

$$\text{Thickness of Si} = (0.44)\,(100) = 44 \text{ nm}$$

Q.1 (e) Solution:

(i) Derivation of the free space loss:

Let us assume a transmitter and a receiver are placed in a free-space with the following parameters.

Power transmitted = P_t

Power received = P_r

Directive gain of the transmitted antenna = G_t

Directive gain of the receiving antenna = G_r

Effective aperture of the receiving antenna = A_{er}

Operating frequency = f

$$\text{Operating wavelength} = \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{f}$$

The power density produced by an isotropic antenna at a distance "d" can be given by,

$$p_{\text{iso}} = \frac{P_t}{4\pi d^2}$$

The power density produced by a practical antenna at a distance "d" can be given by,

$$p_{\text{practical}} = \frac{P_t G_t}{4\pi d^2} \quad \text{...(i)}$$

The power received by the receiving antenna placed at a distance "d" from the transmitter can be given by,

$$P_r = (p_{\text{practical}}) A_{er} \quad \text{...(ii)}$$

The effective aperture of a receiving antenna can be given by,

$$A_{er} = \frac{\lambda^2}{4\pi} G_r \quad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get,

$$P_r = \frac{P_t G_t G_r}{4\pi d^2}\left(\frac{\lambda^2}{4\pi}\right)$$

$$P_r = \frac{P_t G_t G_r}{(4\pi d / \lambda)^2} \quad \text{...(iv)}$$

In the right side of equation (iv), the component in the denominator causes the reduction in the received signal strength. This reduction is due to neither obstacles in the path nor absorption by the medium. But, this is due to the spread of the transmitted energy which is the free-space loss (FSL) or free space path loss.

So,
$$\text{FSL} = \left(\frac{4\pi d}{\lambda}\right)^2$$

Free-space loss in dB can be given by,

$$[\text{FSL}] = 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2 = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right)$$

$$= 20\log_{10}\left(\frac{4\pi df}{3\times 10^8}\right) \quad \because \lambda = \frac{c}{f}$$

If the frequency (f) is measured in "MHz" and distance (d) is measured in "km", then

$$[\text{FSL}] = 20\log_{10}\left(\frac{4\pi \times 10^9}{3\times 10^8}\right) + 20\log_{10}(fd)$$

$$[\text{FSL}] = 32.44 + 20\log_{10}(f) + 20\log_{10}(d)\ \text{dB}$$

(ii) Given data:

$$d = 42000 \text{ km}$$
$$f = 6 \text{ GHz} = 6000 \text{ MHz}$$

So, the free-space loss between the satellite and the ground station is,

$$[\text{FSL}] = 32.44 + 20\log_{10}(6000) + 20\log_{10}(42000) \text{ dB}$$
$$= 200.47 \text{ dB}$$

Q.2 (a) Solution:

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

$$M = \frac{K}{\omega\sqrt{1+\omega^2}\ \sqrt{4+\omega^2}}$$

$$\phi = -90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

When $\omega = 0,\quad M\angle\phi = \infty\angle -90°$

$\omega = 1,\quad M\angle\phi = \dfrac{K}{\sqrt{10}}\angle -161.56°$

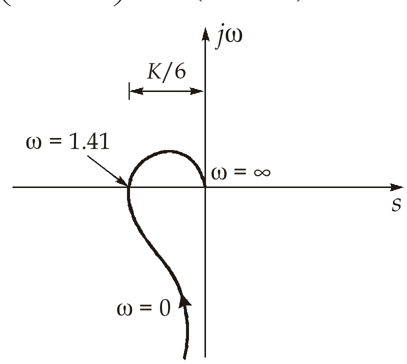
$\omega = 2,\quad M\angle\phi = \dfrac{K}{4\sqrt{10}}\angle -198.4°$

$\omega = \infty,\quad M\angle\phi = 0\angle -270°$

The polar plot is shown in figure below,

$$\omega_c = \frac{1}{\sqrt{T_1T_2}} = \frac{1}{\sqrt{1\times 0.5}} = \sqrt{2}$$

$$a = \frac{K}{2}\left(\frac{T_1T_2}{T_1+T_2}\right) = \frac{K}{2}\left(\frac{1\times 0.5}{1+0.5}\right) = 0.16667K = \frac{K}{6}$$



For stability, $N = Z - P$

Since $P = 0$, $Z = N$. But, for stability Z should be zero, i.e., N should be zero; which means that the point $(-1 + j0)$ should not be encircled. This will only happen, if

$$a < 1 \quad \text{or} \quad K/6 < 1 \quad \text{or} \quad K < 6$$

$$\text{Gain margin} = 20\log\frac{1}{a} = 3$$

i.e.,
$$20\log\frac{6}{K} = 3$$

$$\frac{6}{K} = 1.41$$

or
$$K = \frac{6}{1.41} = 4.25$$

Gain crossover frequency,

$$M = 1$$

$$\therefore \quad M = \frac{K}{\omega\sqrt{1+\omega^2}\,\sqrt{4+\omega^2}} = 1$$

or
$$(4.25)^2 = \omega^2(1 + \omega^2)(4 + \omega^2)$$

By hit and trial method

$$\omega = 1.183 \text{ is the value which satisfies the equation}$$

Phase margin

$$\text{P.M.} = \angle G(j\omega)H(j\omega)\big|_{\omega = 1.183} + 180°$$

$$= \left\{\left(-90° - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{2}\right)\Bigg|_{\omega = 1.183}\right\} + 180°$$

$$= -90° - \tan^{-1}\frac{1.183}{1} - \tan^{-1}\frac{1.183}{2} + 180° = 9.6°$$

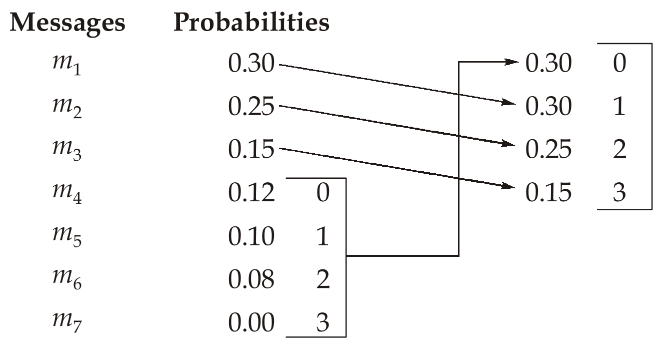
Q.2 (b) Solution:

For an r-ary code, we will have exactly r messages left in the last reduced set if, and only if, the total number of original messages is $r + k(r - 1)$, where k is an integer. This is because each reduction decreases the number of message by $(r - 1)$. Hence, if there is a total of k reductions, the total number of original messages must be $r + k(r - 1)$. In case the original messages do not satisfy this condition, we must add some dummy messages with zero probability of occurrence until this condition is fulfilled.

For the given problem, $r = 4$ and number of message = 6.

$$r + k(r - 1) = 7, 10, 13, \ldots\ldots \text{ for } k = 1, 2, 3, \ldots.$$

So, we need to add one dummy message to satisfy the required condition of $r + k(r - 1)$ messages and Huffman code can be obtained using the following table.



Messages (m_i)	m_1	m_2	m_3	m_4	m_5	m_6
Probabilities (P_i)	0.30	0.25	0.15	0.12	0.10	0.08
Codeword	1	2	3	00	01	02
Length of codeword (l_i)	1	1	1	2	2	2

Entropy,
$$H = -\sum_{i=1}^{6} P_i \log_2(P_i) = 2.42 \text{ bits/symbol}$$

$$= \frac{2.42}{\log_2(4)} = 1.21 \quad \text{4-ary units/symbol}$$

Average length of code word,

$$\bar{L} = \sum_{i=1}^{6} l_i P_i$$

$$= 1(0.30 + 0.25 + 0.15) + 2(0.12 + 0.10 + 0.08)$$

$$= 1.3 \quad \text{4-ary digits/symbol}$$

Code efficiency, $\eta = \dfrac{H}{\bar{L}} = \dfrac{1.21}{1.3} \approx 0.93 \text{ (or) } 93\%$

Code redundancy, $\gamma = 1 - \eta = 1 - 0.93 = 0.07 \text{ (or) } 7\%$

Q.2 (c) Solution:

(i) Given, difference equation,

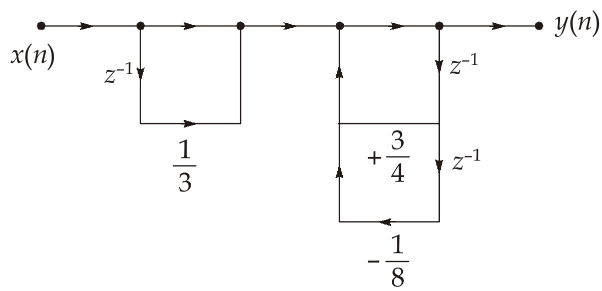
$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{3}x(n-1)$$

The Direct form I corresponds to first implementing the right-hand side of the difference equation (i.e., zeros) followed by the left-hand side (i.e., poles).

Thus, the Direct Form I for this difference equation is

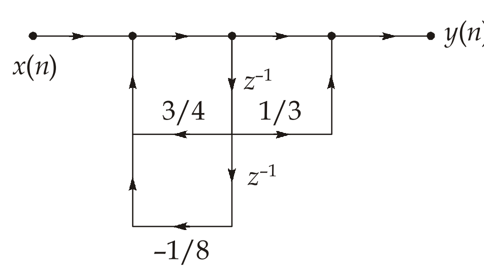
$$Y(z)-\frac{3}{4}z^{-1}Y(z)+\frac{1}{8}z^{-2}Y(z) = X(z)+\frac{1}{3}z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$



(ii) The direct form II corresponds to implementing the poles first followed by the zeros.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$



(iii) In the cascade form using first order sections, we must first factor the system function into a cascade of two first-order systems.

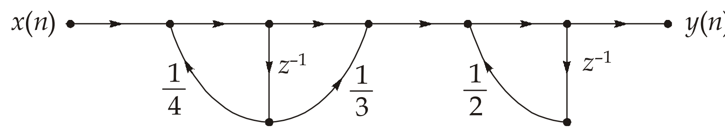
Applying the z-transform to both sides of the difference equation.

$$Y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = X(z)\left[1+\frac{1}{3}z^{-1}\right]$$

$$\therefore \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

In developing the cascade form, we can include the zero with either pole and arrange the cascade in either order.

i.e.,
$$H(z) = \left[\frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}\right]\left[\frac{1}{1-\frac{1}{2}z^{-1}}\right]$$



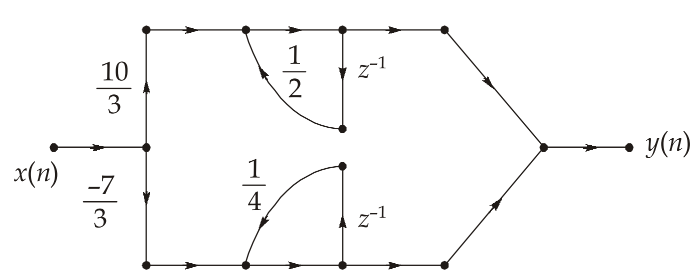
(iv) The parallel form corresponds to expanding H(z) in a partial fraction expansion.

$$\therefore \qquad H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

$$\frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} = \frac{A}{1-\frac{1}{4}z^{-1}}+\frac{B}{1-\frac{1}{2}z^{-1}}$$

$$A = \left.\frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}\right|_{z^{-1}=4} = \frac{1+\frac{4}{3}}{-1} = \frac{-7}{3}$$

$$B = \left.\frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}\right|_{z^{-1}=2} = \frac{1+\frac{2}{3}}{\frac{1}{2}} = \frac{10}{3}$$



Q.3 (a) Solution:

(i)

```
         LXI H, 3000H
         LXI D, 3001H
         MOV C, M
LOOP:    INX H
         MOV A, M
         ORI 00H
         JZ SKIP
         STAX D
         INX D
SKIP:    DCR C
         JNZ LOOP
         HLT
```

(ii)

```
LDA 5001H
LXI H, 5003H
ADD M
MOV C, A
LDA 5000H
DCX H
ADC M
MOV B, A
HLT
```

Q.3 (b) Solution:

(i) Six subnets are to be formed. So, three subnet bits ($2^3 = 8$) are needed.

Therefore the subnet patterns are 001 to 110.

(Note that broadcasts may occur on a subnet and, all-zero and all-one patterns may not be assigned for host bits.)

The address ranges of the 6 subnets will be,

Subnet 1:

HID range ⇒ 00100001 to 00111110 ⇒ 33 to 62

IP address range ⇒ (194.82.212.33) to (194.82.212.62)

Subnet 2:

HID range ⇒ 01000001 to 01011110 ⇒ 65 to 94

IP address range ⇒ (194.82.212.65) to (194.82.212.94)

Subnet 3:

IP address range ⇒ (194.82.212.97) to (194.82.212.126)

Subnet 4:

IP address range ⇒ (194.82.212.129) to (194.82.212.158)

Subnet 5:

IP address range ⇒ (194.82.212.161) to (194.82.212.190)

Subnet 6:

IP address range ⇒ (194.82.212.193) to (194.82.212.222)

(ii) **Simple Mail Transfer Protocol (SMTP):** It provides a basic electronic mail transport facility. It provides a mechanism for transferring messages among separate hosts. Features of SMTP include mailing lists, return receipts, and forwarding. The SMTP protocol does not specify the way in which messages are to be created; some local editing or native electronic mail facility is required. Once a message is created, SMTP accepts the message and makes use of TCP to send it to an SMTP module on another host. The target SMTP module will make use of a local electronic mail package to store the incoming message in a user's mailbox.

File Transfer Protocol (FTP): It is used to send files from one system to another under user command. Both text and binary files are accommodated, and the protocol provides features for controlling user access. When a user wishes to engage in file transfer, FTP sets up a TCP connection to the target system for the exchange of control messages. This connection allows user ID and password to be transmitted and allows the user to specify the file and file actions desired. Once a file transfer is approved, a second TCP connection is set up for the data transfer. The file is transferred over the data connection, without the overhead of any headers or control information at the application level. When the transfer is completed, the control connection is used to signal the completion and to accept new file transfer commands.

TELNET: It provides a remote logon capability, which enables a user at a terminal or personal computer to logon to a remote computer and function as if directly connected to that computer. The protocol was designed to work with simple scroll-mode terminals. TELNET is actually implemented in two modules: User TELNET interacts with the terminal I/O module to communicate with a local terminal. It converts the characteristics of real terminals to the network standard and vice versa.

Server TELNET interacts with an application, acting as a surrogate terminal handler so that remote terminals appear as local to the application. Terminal traffic between User and Server TELNET is carried on a TCP connection.

Q.3 (c) Solution:

```
#include <stdio.h>
void main(void)
{
        int n, i = 0, j, base = 16, rem, num;
        char a [15];
        printf("Input a Decimal Number: ");
        scanf("%d", &n);
        num = n;
        do
        {
                rem = n % base'
                switch(rem)
                {
                        case10:
                                a[i++] = 'A';
                                break;
                        case11:
                                a[i++] = 'B';
                                break;
                        case12:
                                a[i++] = 'C';
                                break;
                        case13:
                                a[i++] = 'D';
                                break;
                        case14:
                                a[i++] = 'E';
                                break;
                        case15:
                                a[i++] = 'F';
                                break;
```

```
                default:
                        a[i++] = '0' + rem;
                }
                n/= base;
        }
        while(n != 0);
        a[i] = '\0';
        printf("\nHexa-Decimal Equivalent of Decimal Number %d = %d/n", num,
strrev(a));
}
Output:
        Input a Decimal Number: 123
        Hexa-Decimal Equivalent of Decimal Number 123 = 7B
```

Q.4 (a) Solution:

The overall transfer function is determined as:

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s(s+1)}}{1+\frac{1}{s(s+1)}\times(s\alpha+1)}$$

or
$$\frac{C(s)}{R(s)} = \frac{1}{s^2+(\alpha+1)s+1}$$

The characteristic equation is:

$$s^2 + (\alpha + 1)s + 1 = 0$$

which can be rearranged as:

$$s^2 + s + 1 + \alpha s = 0$$

or
$$1+\frac{\alpha s}{s^2+s+1} = 0$$

Therefore, the open-loop transfer function for sketching the root contour is given by:

$$G_1(s)H_1(s) = \frac{\alpha s}{s^2+s+1}$$

open-loop zero : $s = 0$

open-loop poles: $s = \frac{-1\pm\sqrt{1^2-4\times1}}{2} = -0.5\pm j0.866$

The number of root contour branches = 2

The starting point ($\alpha = 0$) of root contours is at

$$s = -0.5 \pm j0.866$$

The terminating point ($\alpha \to \infty$) of root contours is at $s = 0$ and $s \to \infty$.

The angle of asymptotes:

$$\angle\text{Asymptotes} = \frac{(2k+1)}{P-Z} \times 180° \;;\; k = 0$$

$$= \frac{(2\times 0+1)}{2-1} \times 180° = 180°$$

The root contour is present on entire negative real axis.

The characteristic equation is,

$$1 + \frac{\alpha s}{s^2+s+1} = 0$$

or $$\alpha = -\frac{s^2+s+1}{s}$$

$\therefore$ $$\frac{d\alpha}{ds} = -\left\{\frac{s(2s+1)-(s^2+s+1)\times 1}{s^2}\right\} = -\left\{\frac{s^2-1}{s^2}\right\}$$

The breakaway point is determined using $\dfrac{d\alpha}{ds} = 0.$

$\therefore$ $$s^2 - 1 = 0 \qquad s = \pm 1$$

The breakaway point is identified at $s = -1$ as it lies on root contour branch.

The angles of departure from complex poles $s = -0.5 \pm j0.866$ are determined below:

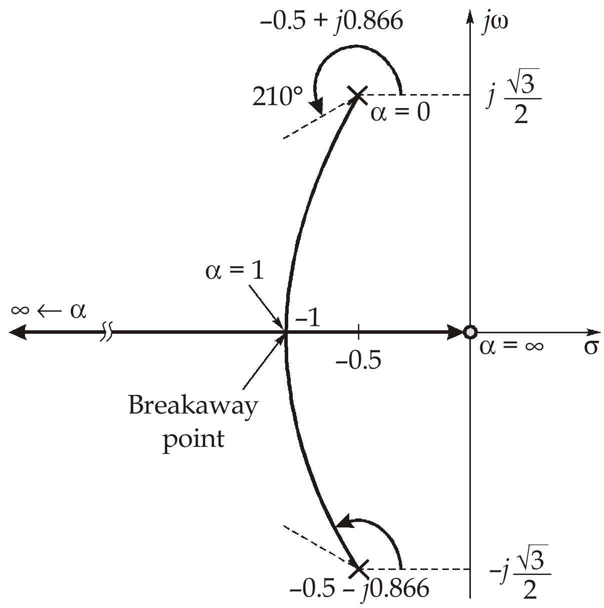
$$\theta_{d\,(-0.5+j0.866)} = 180° - \{(\theta_{p_2} - \phi_z)\} = 180° - \left\{90° - \left(90° + \tan^{-1}\frac{0.866}{0.5}\right)\right\}$$

$$= 180° - \{90° - (90° + 30°)\} = 210°$$

$$\theta_{d\,(-0.5-j0.866)} = 180° - \{(\theta_{p_1} - \phi_z)\} = 180° - \left\{-90° + \left(90° + \tan^{-1}\frac{0.866}{0.5}\right)\right\}$$

$$= 180° - \{-90° + (90° + 30°)\} = 150°$$

As per data calculated above the root contour plot is drawn and shown in figure below.



The critical damping occurs at breakaway point on real axis i.e., $s = -1$. As the point $s = -1$ lies on the root contour.

Therefore, $|G_1(-1)H_1(-1)| = 1$

$$\therefore \quad 1 = \left|\frac{\alpha(-1)}{(-1)^2 + (-1) + 1}\right|$$

$$\therefore \quad \alpha = 1$$

Q.4 (b) Solution:

Here $\sigma_1 = \sigma_2 = 0$ and $\mu_{R1} = \mu_{R2} = 1$.

(i) When 10% of energy is reflected then, the reflection coefficient will be square root of the 10%. In other words,

$$|\Gamma| = \sqrt{\frac{10}{100}} = \sqrt{0.1} = \pm 0.316$$

Using positive sign, we can write,

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\left(\frac{1}{\sqrt{\varepsilon_{R2}}}\right) - \left(\frac{1}{\sqrt{\varepsilon_{R1}}}\right)}{\left(\frac{1}{\sqrt{\varepsilon_{R2}}}\right) + \left(\frac{1}{\sqrt{\varepsilon_{R1}}}\right)} = 0.316$$

$$\frac{1-\left(\frac{\sqrt{\varepsilon_{R2}}}{\sqrt{\varepsilon_{R1}}}\right)}{1+\left(\frac{\sqrt{\varepsilon_{R2}}}{\sqrt{\varepsilon_{R1}}}\right)} = 0.316$$

$$\Rightarrow \qquad \sqrt{\frac{\varepsilon_{R2}}{\varepsilon_{R1}}} = \frac{1-0.316}{1+0.316} = 0.5197$$

$$\Rightarrow \qquad \frac{\varepsilon_{R2}}{\varepsilon_{R1}} = 0.27$$

Using negative sign, $\sqrt{\frac{\varepsilon_{R2}}{\varepsilon_{R1}}} = \frac{1+0.316}{1-0.316} = 1.923$

$$\therefore \qquad \frac{\varepsilon_{R2}}{\varepsilon_{R1}} = 3.70$$

(ii) When 10% energy is transmitted, then the reflected energy is 90%. In this case the reflection coefficient will be,

$$|\Gamma| = \sqrt{\frac{90}{100}} = \sqrt{0.9} = \pm 0.9486$$

Using a positive sign, we can write,

$$\frac{1-\left(\frac{\sqrt{\varepsilon_{R2}}}{\sqrt{\varepsilon_{R1}}}\right)}{1+\left(\frac{\sqrt{\varepsilon_{R2}}}{\sqrt{\varepsilon_{R1}}}\right)} = 0.9486$$

$$\Rightarrow \qquad \sqrt{\frac{\varepsilon_{R2}}{\varepsilon_{R1}}} = \frac{1-0.9486}{1+0.9486} = 0.02633$$

$$\Rightarrow \qquad \frac{\varepsilon_{R2}}{\varepsilon_{R1}} = 0.000694$$

Simultaneously using negative sign,

$$\sqrt{\frac{\varepsilon_{R2}}{\varepsilon_{R1}}} = \frac{1+0.9486}{1-0.9486} = 37.910$$

$$\therefore \qquad \frac{\varepsilon_{R2}}{\varepsilon_{R1}} = 1437.20$$

Q.4 (c) Solution:

(i)

- The condition required to eliminate the slope-overload distortion is,

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

$$m(t) = A\tanh(\beta t)$$

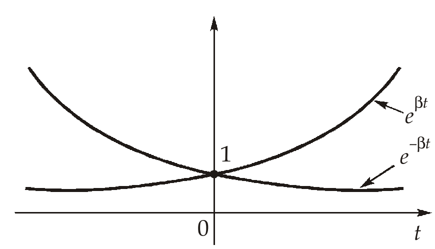
So,
$$\Delta f_s \geq \left|\frac{d}{dt}(A\tanh\beta t)\right|_{max}$$

$$\Delta f_s \geq \left|A\beta\,\text{sech}^2(\beta t)\right|_{max}$$

$$\text{sech}(\beta t) = \frac{1}{\cosh(\beta t)} = \frac{2}{e^{+\beta t} + e^{-\beta t}}$$

Minimum value of $(e^{+\beta t} + e^{-\beta t})$ occurs at $t = 0$ and that minimum value is 2.

So,
$$\left|\text{sech}^2(\beta t)\right|_{max} = \left|\left(\frac{2}{2}\right)^2\right| = 1$$



Hence,
$$\Delta f_s \geq A\beta$$

$$\Delta \geq \frac{A\beta}{f_s} = A\beta T_s$$

$$\Delta_{min} = \frac{A\beta}{f_s} = A\beta T_s$$

(ii)

- The sampling rate of each message signal is,

$$f_{s1} = 1.2 \times 2 \times f_{m\,(max)} = 24 \text{ kHz}$$

- The net sampling rate of the TDM multiplexed signal is,

$$f_s = 24 \times 5 = 120 \text{ kHz}$$

- The number of bits/sample can be determined as follows,

$$|q|_{max} \leq \left(\frac{0.50}{100}\right)m_p$$

$$\frac{\Delta}{2} \leq \left(\frac{0.50}{100}\right)m_p$$

$$\Delta = \frac{m_p - (-m_p)}{2^n} = \frac{2m_p}{2^n} \;;\; n = \text{number of bits/sample}$$

So, $$\frac{m_p}{2^n} \leq \frac{m_p}{200}$$

$$2^n \geq 200 \quad \Rightarrow n \geq \lceil \log_2(200) \rceil$$

$$n_{\min} = 8 \text{ bits/sample}$$

- The minimum information rate is,

$$r_{\min} = n_{\min} f_s = 8 \times 120 = 960 \text{ kbps}$$

- Including the framing and synchronization bits, the minimum transmission data rate is

$$R_{b(\min)} = \left(1 + \frac{0.50}{100}\right) r_{\min} = 964.8 \text{ kbps}$$

- The minimum channel bandwidth required to transmit the TDM multiplied signal is,

$$(\text{BW})_{\min} = \frac{R_b}{2} = \frac{964.8}{2} \text{ kHz} = 482.4 \text{ kHz}$$

Section-B

Q.5 (a) Solution:

(i) Number of cells in each cluster can be given by,

$$N = i^2 + ij + j^2$$

Given that, $i = 3$ and $j = 2$.

So, $$N = (3)^2 + (2)(3) + (2)^2 = 19$$

(ii) Area of a cell, $A_{\text{cell}} = 4 \text{ km}^2$

Area of a cluster, $A_{\text{cluster}} = NA_{\text{cell}} = 19 \times 4 = 76 \text{ km}^2$

Number of clusters in the system = $\frac{1520}{76} = 20$

(iii) Total number of cells in the system = $20 \times 19 = 380$

With frequency reuse,

Number of channels/cell = $\frac{1140}{19} = 60$

Without frequency reuse,

Number of channels/cell = $\frac{1140}{380} = 3$

Q.5 (b) Solution:

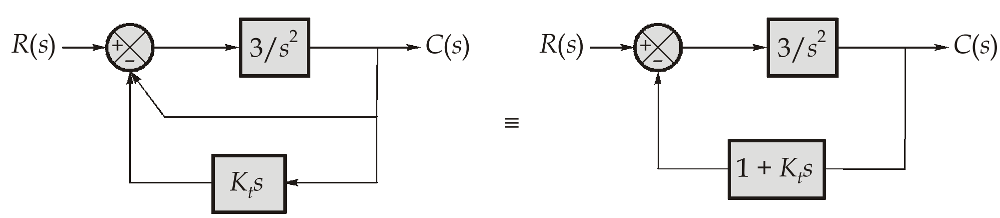
(i) **CPU utilization and response time:** CPU utilization is increased if the overheads associated with context switching are minimized. The context switching overheads could be lowered by performing context switches infrequently. This could however result in increasing the response time for processes.

(ii) **Average turnaround time and maximum waiting time:** Average turnaround time is minimized by executing the shortest tasks first. Such a scheduling policy could however starve long-running tasks and thereby increase their waiting time.

(iii) **I/O device utilization and CPU utilization:** CPU utilization is maximized by running long-running CPU-bound tasks without performing context switches. I/O device utilization is maximized by scheduling I/O-bound jobs as soon as they become ready to run, thereby incurring the overheads of context switches.

Q.5 (c) Solution:

With tachometer feedback, the system can be modified as



The closed loop transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{3/s^2}{1+\frac{3}{s^2}\times(1+K_t s)}$$

$$= \frac{3}{s^2+3+3K_t s} = \frac{3}{s^2+3K_t s+3} \quad \text{...(i)}$$

on comparing the above transfer function with standard second order transfer function, we get,

$$\omega_n^2 = 3$$

$$\Rightarrow \quad \omega_n = \sqrt{3} \text{ rad/sec}$$

and $2\xi\omega_n = 3K_t$

or $$\xi = \frac{3K_t}{2\omega_n} = \frac{\sqrt{3}}{2}K_t \quad \text{...(i)}$$

The maximum peak overshoot is given by,

$$M_{po} = e^{-\pi\xi/\sqrt{1-\xi^2}} = 40\%$$

$$\Rightarrow \quad 0.04 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\frac{\ln(0.04)}{\pi} = \frac{-\xi}{\sqrt{1-\xi^2}}$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 1.025$$

Squaring both the sides, we get,

$$\frac{\xi^2}{(1-\xi^2)} = 1.049$$

$$\text{or} \quad \xi^2 = \frac{1.049}{2.049} = 0.512$$

$$\Rightarrow \quad \xi = 0.715 \quad \text{...(iii)}$$

from equation (ii) and (iii)

$$K_t = \frac{2}{\sqrt{3}}\xi = \frac{2}{\sqrt{3}} \times 0.715 = 0.826$$

∴ The closed loop poles are located at

$$\text{CE} \Rightarrow \quad s^2 + 3K_t s + 3 = 0$$

$$\Rightarrow \quad s^2 + 2.48s + 3 = 0$$

$$s = (-1.24 \pm 1.21j)$$

Q.5 (d) Solution:

Dimensions of the waveguide, a = 2.5 cm and b = 3.5 cm

Operating frequency, f = 10 GHz = 10×10^9 Hz

Conductivity, $\sigma = 0$

Relative permittivity, $\varepsilon_r = 4$

Relative permeability, $\mu_r = 1$

For TE_{20} mode,

$$f_c = \frac{c}{2\pi\sqrt{\mu_r\varepsilon_r}}\left(\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}\right)$$

$$= \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{2}{a}\right)^2}$$

$$= \frac{3\times10^{10}}{2\sqrt{4}}\sqrt{\left(\frac{2}{2.5}\right)^2}$$

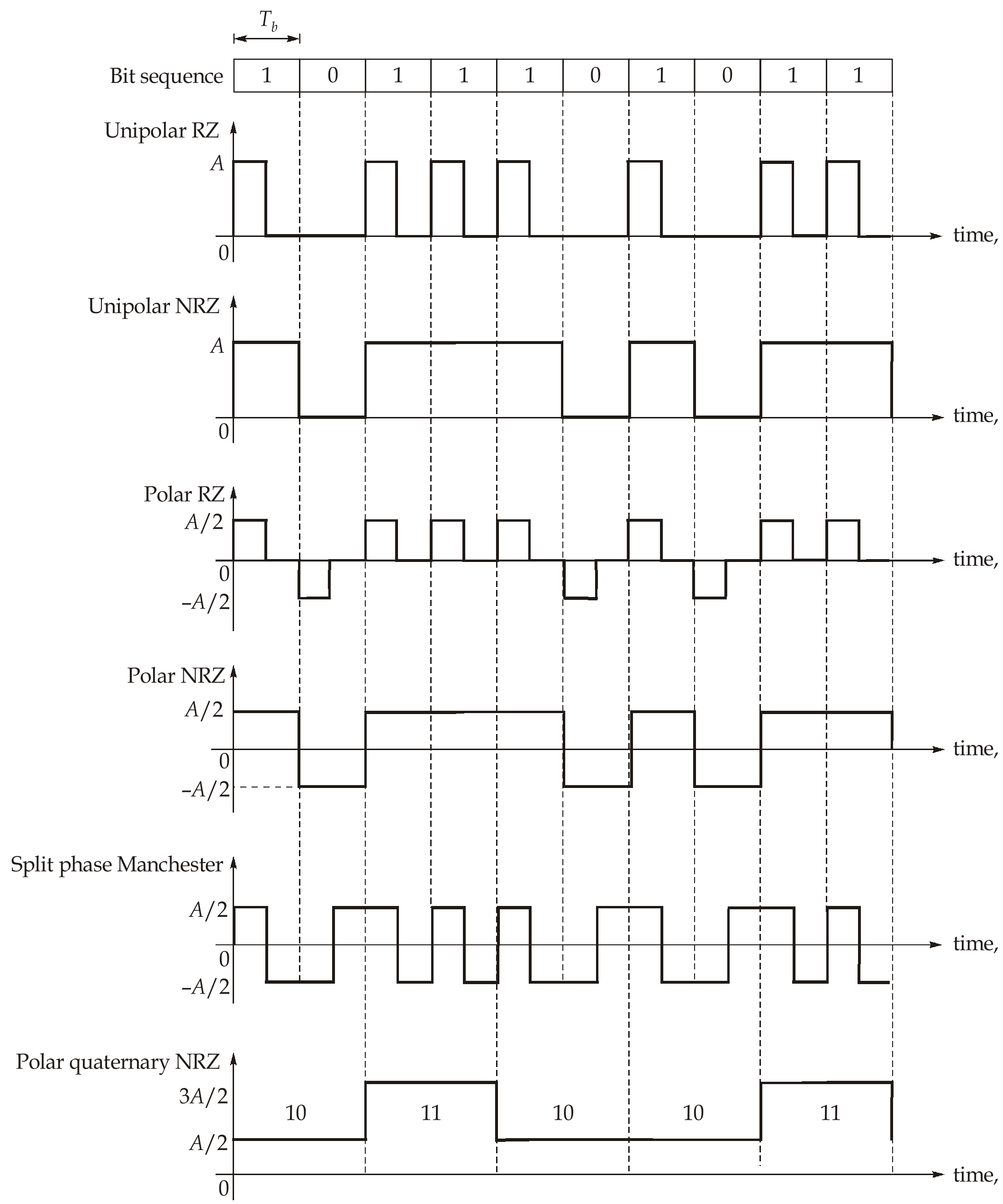
$$= \frac{3\times10^{10}}{4}\times\frac{2}{2.5} = 0.6\times10^{10} = 6\text{ GHz}$$

∴ Wave impedance for TE_{20} mode is given by

$$\eta_{TE20} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\eta_{TE} = \frac{120\pi/2}{\sqrt{1-\left(\frac{6}{10}\right)^2}} = \frac{60\pi}{\sqrt{1-0.36}} = \frac{60\,\pi}{0.8} = 75\,\pi\,\Omega = 235.62\ \Omega$$

Q.5 (e) Solution:



Q.6 (a) Solution:

(i) In FDMA, the output power of the transmitter is divided equally between the channels. For $P_{\text{transponder}}$ = 20 W and 500 channels,

$$\text{Power per channel} = \frac{20}{500} = 40 \text{ mW/channel}$$

(ii) Given that, for downlink,

Path loss, $L_p = 206$ dB at 11 GHz

Noise bandwidth, $B_N = 50$ kHz

Transmitter antenna gain, $G_t = 30$ dB

Receiver antenna gain, $G_r = 40$ dB

Noise equivalent temperature, $T_{eq} = 150$ K

So, the power received by the earth station can be given by,

In decilogs, $[P_r] = [P_t] + [G_t] + [G_r] - [L_p]$

From part (i), $[P_t] = 10\log_{10}(40 \times 10^{-3}) \approx -14$ dBW

So, $[P_r] = -14 + 30 + 40 - 206 = -150$ dBW

The noise power at the input of the earth station receiver is,

$$N = kT_{eq}B_N = (1.381 \times 10^{-23}) \times 150 \times 50 \times 10^3 \text{ W}$$

$$= 1.036 \times 10^{-16} \text{ W}$$

In decilogs, $[N] = 10\log_{10}(N) = -159.85$ dBW

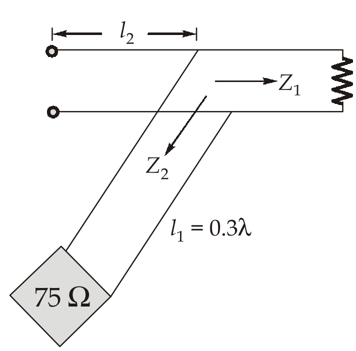
So, the C/N at input of receiver of an earth station can be given by,

In decilogs, $$\left[\frac{C}{N}\right] = [P_r] - [N] = (-150) - (-159.85) \text{ dB} = 9.85 \text{ dB}$$

(iii) $$\text{Margin} = \text{Receiver}\left[\frac{C}{N}\right] - \text{Threshold} = 9.85 - 6 \text{ dB} = 3.85 \text{ dB}$$

Q.6 (b) Solution:

In figure below at the junction we have two impedances Z_1 and Z_2 in parallel. Since the cable has been terminated in its characteristic impedance, Z_1 will be same as the characteristic impedance 50 Ω. Z_2 however will be transformed version of 75 Ω impedance. Hence, we have,



$$Z_2 = Z(l_1) = Z_0 \frac{75\cos\beta l_1 + j50\sin\beta l_1}{50\cos\beta l_1 + j75\sin\beta l_1}$$

and $\beta l_1 = \frac{2\pi}{\lambda} \times 0.3\lambda = 0.6\pi = 108°$, given

$$Z_2 = Z(l_1) = 50\frac{75\cos 108° + j50\sin 108°}{50\cos 108° + j75\sin 108°} = 35.2008 + 8.621j\ \Omega$$

Since, at the junction, the two impedances are connected in parallel, the impedance Z is

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = 20.9549 + 2.9389j\ \Omega$$

the impedane at a distance of l_2 from the junction is

$$Z(l_2) = Z_0 \frac{Z\cos\beta l_2 + j50\sin\beta l_2}{50\cos\beta l_2 + jZ\sin\beta l_2}$$

and $\beta l_2 = \frac{2\pi}{\lambda} \times 0.2\lambda = 0.4\pi = 72°$, we get

$$Z(l_2) = 50\frac{(20.9549 + 2.9389j)\cos 72° + j50\sin 72°}{50\cos 72° + j(20.9459 + 2.9389j)\sin 72°}$$

$$= 94 + j43.47\ \Omega$$

The magnitude of the reflection coefficient on the line is

$$|\Gamma| = \left|\frac{Z - Z_0}{Z + Z_0}\right| = 0.41$$

$\Rightarrow$ VSWR on the line, $\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.396$

Q.6 (c) Solution:

(i) For 4 KB page,

Page size = 4 × 1024 = 4096 bytes
20000 = (4 × 4096) + 3616
32768 = (8 × 4096) + 0
60,000 = (14 × 4096) + 2656

For 8 KB page,

Page size = 8 × 1024 = 8192 bytes
20000 = (2 × 8192) + 3616
32768 = (4 × 8192) + 0
60000 = (7 × 8192) + 2656

So, the virtual page number and the offset for the given virtual addresses can be tabulated as shown below:

Virtual address	With 4 KB page		With 8 KB page	
	Page number	Offset	Page number	Offset
20000	4	3616	2	3616
32768	8	0	4	0
60000	14	2656	7	2656

(ii) Total number of pages = $\dfrac{\text{Total address space}}{\text{Page space}} = \dfrac{2^{32}}{8 \times 2^{10}} = 2^{19} = 524288$

Loading page table needs to transfer 524288 addresses each of 32-bit, which can be done in one entry and each entry need 100 nsec.

So, the time required for loading the page table will be = 524288 × 100 nsec = 52.4288 msec.

Total run time of a process = 120 msec

Fraction of the CPU time devoted for loading the page table will be,

$$\frac{52.4288}{120} \simeq 0.437 \text{ (or) } 43.7\%$$

Q.7 (a) Solution:

Difficulties involved in Routh Hurwitz Criterion

(i) When the first term in any row of the Routh array is zero while rest of the row has atleast one non zero term. Because of this zero term, the terms in the next row become infinite and Routh's test breaks down.

Substitute a small positive number 'ξ' for the zero and proceed to evaluate the rest of the Routh array. Then examine the signs of the first column of Routh array by substituting $\xi \to 0$.

(ii) When all the elements in any one row of the Routh array are zero. This condition indicate that there are symmetrically located roots in the s-plane about the origin.

The polynomial whose coefficient are the element of the row just above the row of zero in the Routh array is called an auxiliary polynomial. This polynomial gives the number and location of root pairs of characteristic equation which are symmetrically located in s-plane. The order of auxiliary polynomial is always even.

Limitation of Routh Hurwitz Criterion

(i) Routh Hurwitz criterion can't be applied for an infinite series such as the polynomial containing sine, cosine and exponential terms.

(ii) The characteristic equation of a polynomial should have real coefficients.

(iii) The sign changes in first column determines the roots lies in the right half of s-plane but not their locations.

From the given characteristic equation, the Routh's array can be formed as

s^5	1	5	4
s^4	2	10	8
s^3	0(2)	0(5)	0
s^2	5/2	4	0
s^1	9/5	0	0
s^0	4		

Subsidiary equation for s^4 row

$$s^4 + 5s^2 + 4 = 0$$

derivative of subsidiary equation,

$$4s^3 + 10s = 0$$

or $$2s^3 + 5s = 0$$

Here s^3 row was a zero row, hence, it was formed by writing the subsidiary function of s^4 and then taking its derivative. There is no sign change in the first column of new formed Routh's array. However there is a zero row.

The subsidiary equation formed of s^4 row is one of the factors of the characteristic equation. The other factor is $(s + 2)$.

$\therefore s^5 + 2s^4 + 5s^3 + 10s^2 + 4s + 8 = 0$

$$(s^4 + 5s^2 + 4)(s + 2) = 0$$

$\therefore$ Roots of $s^4 + 5s^2 + 4 = 0 \Rightarrow s = \pm j2$ and $\pm j1$.

$s + 2 \Rightarrow s = -2$

Since non-repeated roots lie on imaginary axis and no roots are present in RHS of s-plane, the system is marginally stable.

Q.7 (b) Solution:

(i) Given, the maximum electric field.

$$E_{max} = 6 \text{ mV/m} = 6 \times 10^{-3} \text{ V/m}$$

Location of point of the field maxima

$$r = 20 \text{ km} = 20 \times 10^3 \text{ m}$$

Total radiated power,

$$P_{rad} = 100 \text{ kW} = 10^5 \text{ W}$$

Average radiated power of an antenna is defined as

$$P_{avg} = \frac{1}{2}\text{Re}\{E \times H*\}$$

So, the radiation intensity of the antenna is given as

$$U(\theta, \phi) = r^2 P_{avg}$$

$$= \frac{r^2}{2}\text{Re}\{E \times H*\}$$

Therefore, the maximum radiation intensity of the antenna is

$$U_{max} = \frac{r^2}{2}\text{Re}\left[E_s \times H_s^*\right] = \frac{r^2}{2}\left|\frac{E^2}{\eta}\right|$$

$$= \frac{r^2}{2\eta}(E_{max})^2$$

$$= \frac{(20 \times 10^3)^2}{2 \times 120\pi} \times (6 \times 10^{-3})^2 = 19.098 \text{ W}$$

∵ Directivity of an antenna is defined as,

$$D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \times 19.098}{100 \times 10^3} = 2.39 \times 10^{-3}$$

or $$D = 10\log(2.39 \times 10^{-3}) = -26.19 \text{ dB}$$

(ii) Consider the maximum power gain as G_p and directive gain as G_d, so the radiation efficiency is defined as

$$\eta_r = \frac{G_p}{G_d}$$

or $$G_p = \eta_r G_d = (0.95)G_d$$

$\because$ $$G_{d\max} = D$$

$\therefore$ $$G_p = 0.95\times(2.39\times10^{-3}) = 2.27\times10^{-3}$$

Q.7 (c) Solution:

(i)

- The state table of the given state diagram is,

PS	NS X = 0	NS X = 1	Output X = 0	Output X = 1
a	*b*	*d*	0	1
b	*e*	*f*	0	1
c	*b*	*d*	0	1
d	*e*	*b*	0	1
e	*e*	*f*	0	1
f	*f*	*c*	0	0

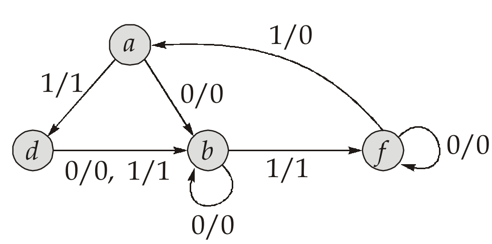
"*b*" and "*e*" are said to be equivalent states. So, remove "*e*" and replace it with "*b*".
"*a*" and "*c*" are said to be equivalent states. So, remove "*c*" and replace it with "*a*".

- So, the reduced state table is,

PS	NS X = 0	NS X = 1	Output X = 0	Output X = 1
a	*b*	*d*	0	1
b	*b*	*f*	0	1
d	*b*	*b*	0	1
f	*f*	*a*	0	0

No further equivalent states are found. So, it is not possible to reduce further.

- Now, the reduced state diagram can be drawn from the reduced state table as shown below:



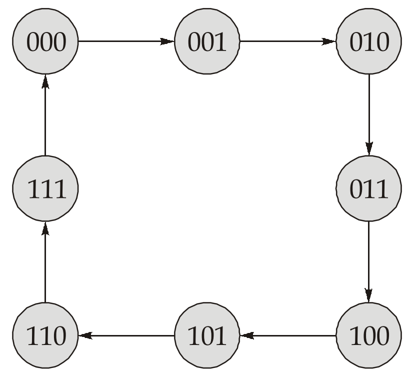
(ii)

$$\left.\begin{aligned} F_1 &\Rightarrow 10011000 \\ F_2 &\Rightarrow 11111100 \end{aligned}\right\} \text{Number of bits} = 8$$

So, minimum number of flip-flops required = 3

Truth table:

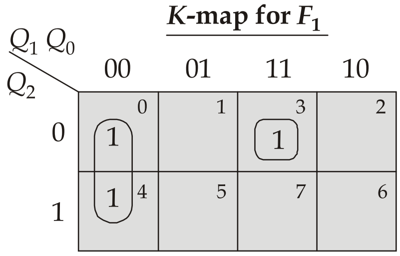
FF States				
Q_2	Q_1	Q_0	F_1	F_2
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	0	0



Sequence diagram of the counter

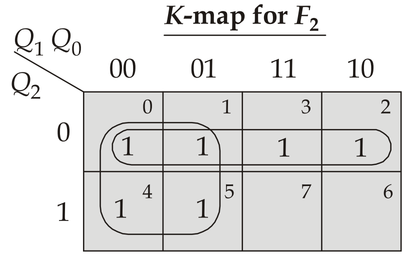
Minimization:

K-map for F_1



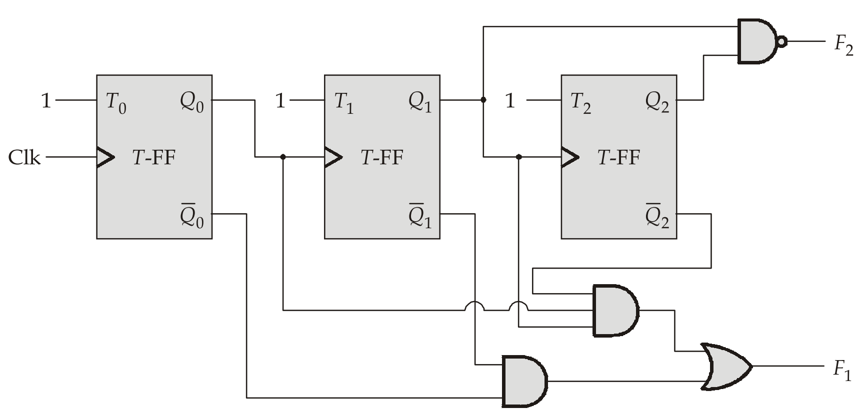
$F_1 = \bar{Q}_1 \bar{Q}_0 + \bar{Q}_2 Q_1 Q_0$

K-map for F_2



$F_2 = \bar{Q}_2 + \bar{Q}_1$

Logic circuit:



Q.8 (a) Solution:

(i) The mode spacing of the laser can be given by,

$$\Delta\lambda = \frac{\lambda^2}{2Ln} = \frac{(0.8)^2}{2(400)(3.6)}\ \mu\text{m} = 0.22\ \text{nm}$$

Therefore the number of modes in the range (750 to 850) nm will be,

$$\frac{850-750}{0.22} = 454.54$$

Taking only integer value, we get 454 laser modes in the range 750 nm < λ < 850 nm.

(ii) Bulk recombination life time,

$$\tau = \frac{\tau_r \tau_{nr}}{\tau_r + \tau_{nr}} = \frac{30\times 100}{30+100}\ \text{ns} \approx 23.1\ \text{ns}$$

Internal quantum efficiency,

$$\eta_{int} = \frac{\tau}{\tau_r} = \frac{23.1}{30} \approx 0.77\ \text{(or)}\ 77\%$$

Internal power level,

$$P_{int} = \eta_{int}\frac{hcI}{q\lambda} = 0.77\times\frac{6.626\times10^{-34}\times3\times10^{8}\times40}{1.6\times10^{-19}\times1.31\times10^{-6}}\ \text{mW}$$

$$= 29.21\ \text{mW}$$

Q.8 (b) Solution:

(i)

$$\text{Rotational latency} = \frac{1}{600}\ \text{minutes} = \frac{60}{600}\ \text{sec} = 100\ \text{msec}$$

$$\text{Average rotational delay} = \frac{\text{Rotatonal latency}}{2} = 50\ \text{msec}$$

Capacity of track = 100 × 500 = 5 × 10^4 bytes

In one rotation, we can transfer the whole track (i.e., 5 × 10^4 bytes). So, the average data transfer time for 250 bytes will be,

$$\text{Average data transfer time} = \frac{250}{5\times10^4}\times100\ \text{msec} = 0.5\ \text{msec}$$

$$\text{Average seek time} = \frac{0+1+2+......+499}{500}\ \text{msec}$$

$$= \frac{499\times500}{2\times500} = 249.5\ \text{msec}$$

Average transfer time = Average seek time + Average rotational delay + Average data transfer time.

$$= 249.5 + 50 + 0.5 = 300 \text{ msec}$$

(ii) In direct mapped cache, location of the memory block in the cache can be decided by using the formula,

Cache location = (main memory block number) % (number of cache blocks)

0	~~8~~ ~~0~~ ~~16~~ 24	8 % 8 = 0	0 % 8 = 0	16 % 8 = 0	24 % 8 = 0
1	~~9~~ ~~1~~ ~~25~~ 17	9 % 8 = 1	17 % 8 = 1	25 % 8 = 1	17 % 8 = 1
2	~~2~~ ~~18~~ ~~2~~ 82	2 % 8 = 2	18 % 8 = 2	2 % 8 = 2	82 % 8 = 2
3	3	3 % 8 = 3			
4	20	20 % 8 = 4			
5	5	5 % 8 = 5			
6	30	30 % 8 = 6			
7	63	63 % 8 = 7			

So, the memory blocks present in the cache, at the end of the given sequence, are 24, 17, 82, 3, 20, 5, 30, 63 respectively in the cache blocks 0 to 7.

Q.8 (c) Solution:

(i)

$$m_1(t) \xleftrightarrow{CTFT} M_1(f)$$

$$m_2(t) \xleftrightarrow{CTFT} M_2(f)$$

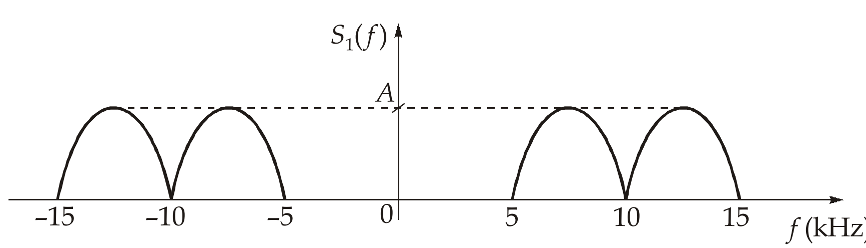
$$2\cos(2\pi f_c t) \xleftrightarrow{CTFT} [\delta(f + f_c) + \delta(f - f_c)]$$

$$m_2(t)[2\cos(2\pi f_c t)] \xleftrightarrow{CTFT} M_2(f) * [\delta(f + f_c) + \delta(f - f_c)]$$

So, the spectrum of the signal at point (1) can be given

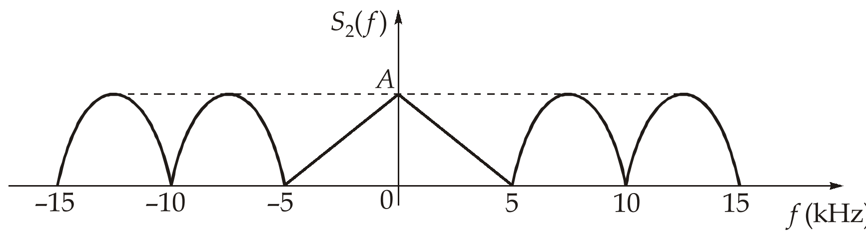
$$S_1(f) = M_2(f) * [\delta(f + 10 \text{ kHz}) + \delta(f - 10 \text{ kHz})]$$

$$= M_2(f + 10 \text{ kHz}) + M_2(f - 10 \text{ kHz})$$



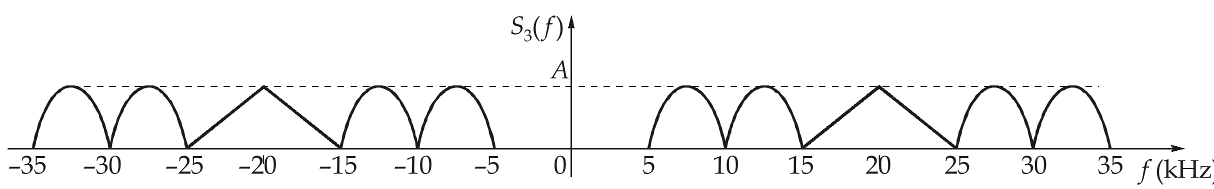
The spectrum of the signal at point (2) can be given by,

$$S_2(f) = M_1(f) + S_1(f)$$



The spectrum of the signal at point (3) can be given by,

$$S_3(f) = S_2(f) * [\delta(f + 20 \text{ kHz}) + \delta(f - 20 \text{ kHz})]$$
$$= S_2(f + 20 \text{ kHz}) + S_2(f - 20 \text{ kHz})$$



The minimum channel bandwidth required by the channel to transmit the output signal is equal to (35 – 5) kHz = 30 kHz.

(ii) The signals $m_1(t)$ and $m_2(t)$ can be recovered from the modulated signal at point (3) by using the receiver shown below.