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Detailed Solutions

BPSC Main Exam 2019
ASSISTANT ENGINEER

CIVIL ENGINEERING
Subjective Paper-II

Test 6

Q.1 (a) Solution :

Design of super elevation : For mixed traffic conditions, the superelevation may be given as

$$e = \frac{V^2}{225R}$$

Given, V = 80 kmph and R = 500 m

$$= \frac{(80)^2}{225 \times 500} = 0.057 < 0.07$$

As the value is less than the maximum super elevation provided in plain and rolling terrain, then actual superelevation to be provided is 0.057.

Extra widening :

Width of pavement, w = 10.2 m

∴ number of lanes, n = 3

Extra widening, $$E_w = \frac{nl^2}{2R} + \frac{V}{9.5\sqrt{R}}$$

Assuming length of longest wheel base, l = 7.0 m

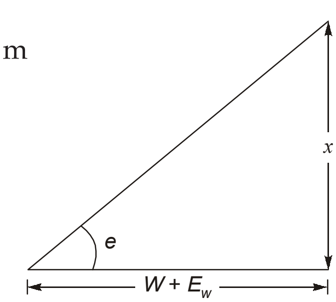
∴ $$E_w = \frac{3 \times 7^2}{2 \times 500} + \frac{80}{9.5\sqrt{500}} = 0.524 \text{ m}$$

Length of Transition curve :

Super elevation, e = 0.057

Assuming pavement is rotated about centre,

(1) Length of transition curve in flat open country is given by



$$L_s = eN(E + E_w)/2$$
$$= 0.057 \times 150\ (10.2 + 0.524)/2$$
$$= 45.85 \text{ m}$$

(2) Minimum length of transition curve as per *IRC*,

$$L_{min} = \frac{2.7V^2}{R} = \frac{2.7 \times 80^2}{500} = 34.56 \text{ m}$$

(3)
$$L_s = \frac{v^3}{R\left(\frac{80}{75+V}\right)} = \frac{\left(80 \times \frac{5}{18}\right)^3}{500 \times \left(\frac{80}{75+80}\right)} = 42.52 \text{ m}$$

Hence the length of transition curve should be 45.85 m. (Maximum of 1 , 2, 3)

Q.1 (b) Solution :

1. **Spot Speed:** Spot speed is the instantaneous speed of a vehicle at a specified section or location.
2. **Average Speed:** Average speed is the average of the spot speeds of all vehicles passing a given section on the highway.

$$\bar{V} = \frac{\sum_{i=1}^{n} V_i}{n}$$

where, $\bar{V}$ = Average speed in kmph

V_i = Observed instantaneous speed of i^{th} vehicle in kmph

n = Number of vehicle

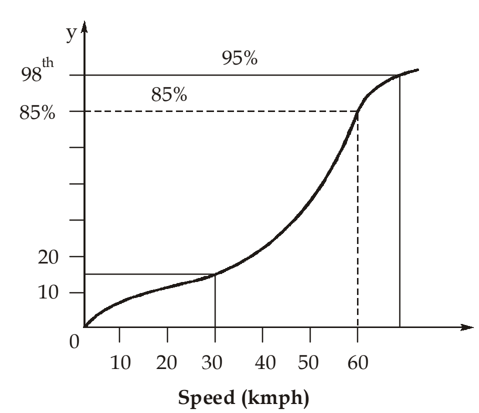
3. **Running Speed :** Running speed is average speed maintained by vehicle over a particular length of road, while vehicle is in motion,

$$V_{running} = \frac{d}{t} \text{ (kmph)}$$

where, d = length of road, t = time in which vehicle running (excluding delay time)

4. **Overall speed or Travel speed:** Effective speed achieved by vehicle on a particular way between two terminals. This speed include also delay time and stopping time during travel.

$$V_0 = \frac{\text{Total length of travel}}{\text{Total time of travel}}$$



Spot Speed Study: Spot speed may be useful in:

(a) Used in traffic planning, control and traffic regulation.
(b) Deciding design speed for new facilities.
(c) To use in accident studies.
(d) To study the traffic capacity.
(e) To decide the speed trends.

Presentation of spot speed data: A graph is plotted between average speed on x-axis and cumulative percentage of vehicle travelled at or below different speed on y-axis.

From above graph, we have

(i) Safe allowable speed : 85th percentile speed

(ii) Design speed : 98th percentile speed

Note: Safe speed and design speed are calculated by using spot speed studies.

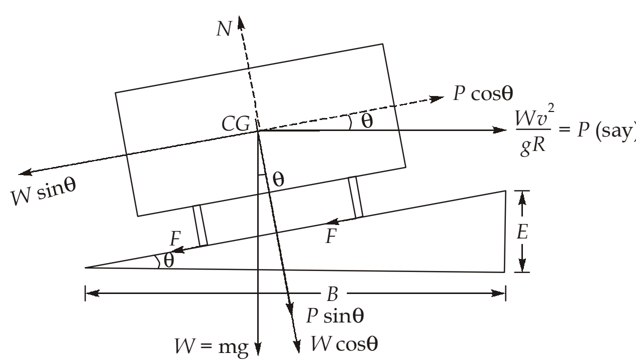
Speed regulation, Lower limit = 15% speed

Upper limit = 85% speed

OR

Q.1 (a) Solution :

Super-elevation: Super-elevation (or banking) is the raising of the outer edge of road with respect to the inner edge in order to counteract the centripetal force arising due to negotiation of a curve by a vehicle. Without super-elevation, if a vehicle negotiates a curve then it will be thrown out of the curve due to outward acting centripetal force.



Let W = Weight of vehicle

V = Velocity of vehicle

B = Width of road

E = Super-elevation where $\tan\theta = \dfrac{E}{B} = e$ (say)

F = Frictional force between the vehicle tyres and road = μN

P = Centrifugal force on horizontal curve

Thus resolving the various forces in two perpendicular directions,

we have, $F = \mu N = \mu\,(W\cos\theta + P\sin\theta)$ ...(i)

and $F + W\sin\theta = P\cos\theta$

$\Rightarrow \quad \mu\,(W\cos\theta + P\sin\theta) + W\sin\theta = P\cos\theta$ ...(ii)

$\Rightarrow \quad W(\sin\theta + \mu\cos\theta) = P(\cos\theta - \mu\sin\theta)$

$$\Rightarrow \quad \frac{P}{W} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$$\Rightarrow \quad \frac{P}{W} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta} \quad ...(iii)$$

Now $$P = \frac{Wv^2}{gR}$$

$$\Rightarrow \quad \frac{P}{W} = \frac{v^2}{gR} = \tan\theta$$

Substituting $\frac{P}{W}$ in (iii),

$$\frac{v^2}{gR} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta} = \frac{\mu + e}{1 - \mu e} \simeq \mu + e \quad (\because \mu e << 1)$$

$$\therefore \quad v = \sqrt{gR(\mu + e)}$$

Given radius of horizontal curve, $R = 500$ m

Speed of vehicle, $v = 100$ km/h $= 27.78$ m/s

Super-elevation is designed for 75% of maximum speed

$$v' = 0.75 \times 100 = 75 \text{ km/hr} = 20.83 \text{ m/s}$$

$$\therefore \quad e = \tan\theta = \frac{(v')^2}{gR} = \frac{(20.83)^2}{9.81 \times 500} = 0.088 > 0.07$$

Thus fix 'e' to 0.07

Now, $$\mu + e = \frac{v^2}{gR}$$

$$\Rightarrow \quad \mu = \frac{(27.78)^2}{9.81 \times 500} - 0.07 = 0.08 < 0.15 \quad \text{(OK)}$$

Thus provide a super-elevation of 0.07 or 7%.

Q.1 (b) Solution :

(i) **Ductility Test:** The ductility is expressed as the distance in centimeters to which a standard briquette of bitumen can be stretched before the thread breaks. A specimen of cross-section 10 mm × 10 mm is taken. The test is conducted at 27°C and a pull at a rate of 50 mm per minute is applied to specimens.

ISI has recommended a minimum ductility value of 75 cm for bitumens of grades of 45 and above. The minimum ductility value specified is 15 cm for the bitumen grade A-65 to 200 for use in certain regions.

(ii) Penetration Test: Penetration test determines the hardness or softness (consistency) of bitumen by measuring the depth in tenths of a millimeter to which a standard loaded needle will penetrate vertically in 5 seconds. The sample is maintained at a temperature of 25°C.

The penetrometer consists of a needle assembly of weight 100 gm. 80/100 reading means needle penetrate 1/10 of 80-100 of 1 mm i.e. 8 to 10 mm at 25°C.

The penetration values of various types of bitumen used in pavement construction range between 20 and 225, 30/40 and 80/100 grade bitumen are more commonly used in India.

Q.2 (a) Solution :

(i) Weaving Length:

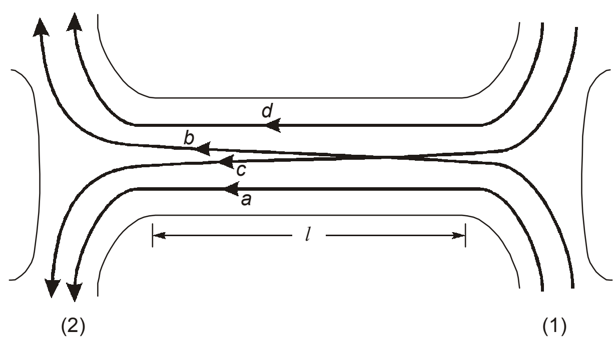
Assume,

a = Traffic generated from section under consideration and goes to left (say section (2))

b = Traffic generated from (1) and goes to other section excepting section (2)

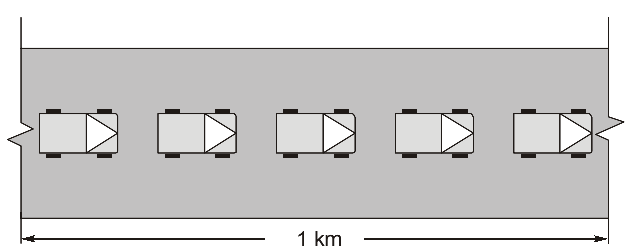
c = Traffic generated from section other than (1) and goes to section (2)

d = Traffic generated from section other than (1) and (2) and goes to section other than (1) and (2)



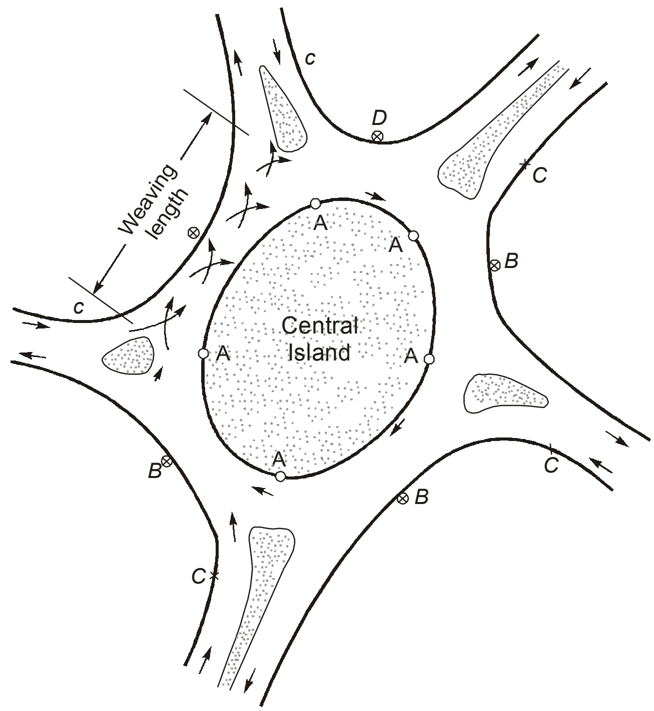
From above figure it is clear that the weaving of traffic occurs in segment of length 'l'. This length is called weaving length.

(ii) Traffic Density: It is the number of vehicles present in a unit length of roadways per lane at a particular instant. It is expressed as vehicles/km



The highest traffic density will occur when the vehicles are practically at a stand still on a given route and in this case traffic volume will approach to zero.

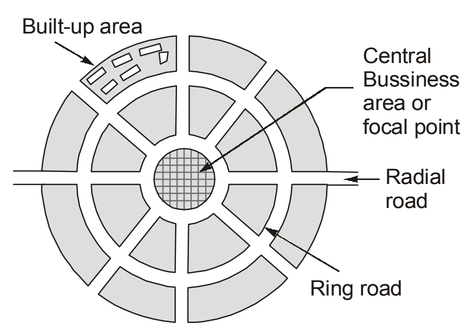
(iii) Traffic rotaries and round abouts: A rotary intersection or round about is an enlarged road intersection where all converging vehicles are forced to move around a large central island in one direction (clockwise direction) before they can weave out of traffic flow into their respective directions radiating from central island. The main objects of providing a rotary are to eliminate the necessity of stopping even for crossing streams of vehicles and to reduce the area of conflict.



Rotary Intersections

(iv) Radial and Circular Road Pattern:

In this system, the main radial roads radiating from central business area are connectioned together with concentric roads. In this areas, boundary by adjacent radial roads and corresponding circular roads, the built-up area is planned with a curved block system.



Radial and circular pattern

Advantages:

1. At traditional intersections with stop signs or traffic signals, some of the most common types of crashes are right angle, left-turn, and head-on collisions. These types of collisions can be severe because vehicles may be travelling through the intersection at high speeds with circular pattern, these types of potentially serious crashes essentially are eliminated because vehicles travel in the same direction.
2. Installing circular pattern in place of traffic signals can also reduce the likehood of rear-end crashes.

3. Removing the reason for drivens to speed up as they approach green lights and by reducing abrupt stops at red lights.
4. Round abouts improve the efficiency of traffic flow, they also reduce vehicle emission and fuel consumption.
   Example: Connaught place in New Delhi is an example of radial and circular road pattern.

Q.2 (b) Solution :

Amount of useful water that can be extracted with the purpose of growth of crop (water content below permanent wilting point [P.W.P] cannot be extracted for growth of crop.)

So, Amount of useful water (maximum),

$$\text{To be stored in root zone} = \frac{\gamma_d (\text{F.C} - \text{P.W.P}) \times d}{100}$$

$$\text{FC} = 30\%$$

$$\text{P.W.P.} = 11\%$$

$$\gamma_d = \text{Dry unit wt.} = 1300 \text{ kg/m3}$$

$$\text{Depth of root zone, } d = 700 \text{ mm}$$

$$\text{So,Amount of water (In depth) (mm)} = \frac{1300}{1000} \times \frac{(30-11)\times 700}{100} = \mathbf{172.9 \text{ mm}}$$

This much of amount of water can be readily depleted.
For healthy growth, i.e, = 172.9 × 0.75 = **129.657 mm**
Assuming there is no rainfall,

$$\text{CIR} = C_u - R_e$$

$$\text{CIR} = 12 \text{ mm per day}$$

$$\text{Watering interval} = \frac{129.675}{12} = \mathbf{10.80625 \text{ Days}}$$

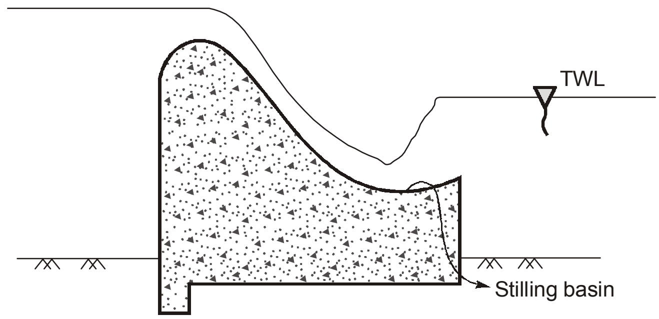
Being on safe side, watering interval = **10 days.**

OR

Q.2 Solution :

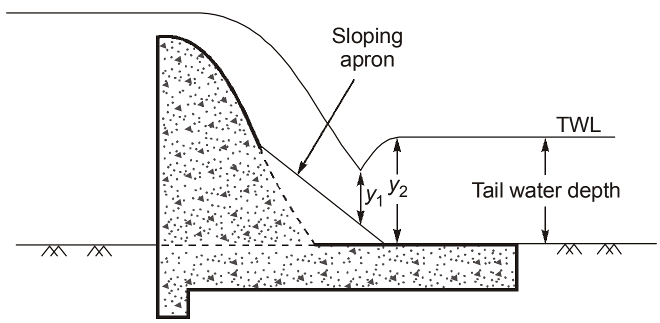
Methods for energy dissipation below spillway: The water flowing over the spillway acquires a lot of KE by the time it reaches near toe of the spillway. If arrangements are not made to dissipate this huge KE of water and if velocity of water is not reduced, large scale scour can take place on d/s side near toe of dam and away form it. These arrangements are known as energy dissipation arrangements or energy dissipatiors.

1. **Energy dissipators for case:** When TWC coincides with y_2 curve at all discharges:
   It is most ideal condition for jump formation. The hydraulic jump will from a toe of the spillway at all discharges:



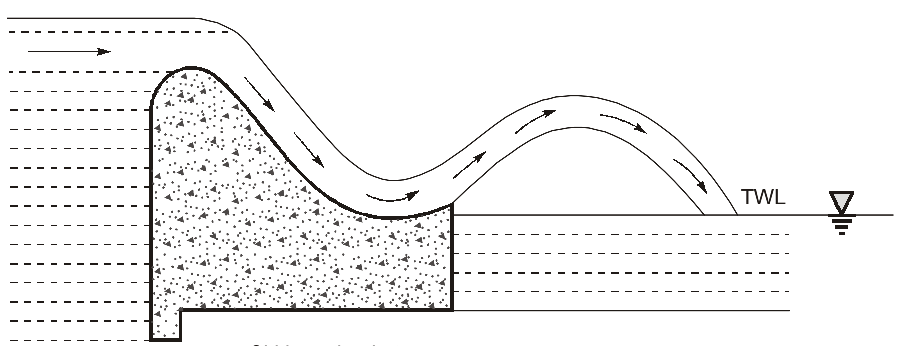
Simple horizontal

2. **Energy dissipators for case:** When TWC is lying above y_2 curve at all discharges:



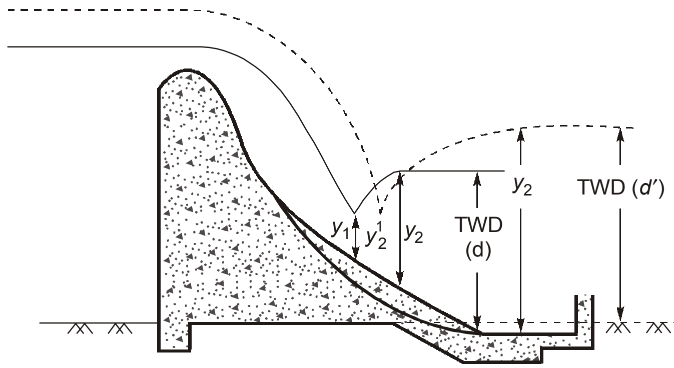
Sloping apron above river bed

3. **Energy dissipators for case:** When TWC lies below y_2 curve at all discharges:
   If the tail water is very low, the water may shoot up out of the above bucket and fall harmlessly into the river at some, distance downstream of the bucket.



Ski jump bucket

4. **Energy dissipators for case when TWC lies above the y_2 curve at low discharges and lies below y_2 curve at high discharges and when TWC lies above the y_2 curve at high discharge and lies below y_2 curve at low discharge:** In former case, at low discharges, the jump will be drowned and at high discharge, tail water depth is insufficient. While in the latter case, at high discharge, the jump will be drawned and at low discharge, fail water depth is insufficient. The solution to the problem lies in providing a sloping apron partly above and partly below the river bed. The horizontal apron and end silt should also be provided.



Q.3 (a) Solution :

There are 4 safety criteria for a gravity dam, which are as follows:

1. **Overturning :** The dam must not overturn about Toe.

$M_{clockwise}$ = Moments about Toe causing overturning

$M_{anticlockwise}$ = Moments about Toe which are balancing and restoring equation.

$$\text{F.O.S} = \frac{M_{anticlock}}{M_{clockwise}}$$

⇒ F.O.S. against overturning should very between 2 and 3.

2. **Sliding:** It is also called shear failure which calls for minimum value of B.

$$B \geq \frac{H}{\mu(S_c - C)}$$

which derives from, $\mu(W - U) \geq P$

$$\text{Shear friction factor} = \frac{\mu \Sigma V}{\Sigma H}$$

$$3 < \text{s.f.f} < 5$$

3. **Tension failure:** The dam is made of concreted earth materials which can withstand compression but not tension. If tension is developed there will be cracking which will consequently result into failure of whole dam.

$$|e| \leq \frac{B}{6} \quad \text{and} \quad B \geq \frac{H}{\sqrt{S_c - C}}$$

4. **Compression failure:** Also called as crushing failure. The principal compressive stress generated due to forces acting on body of dam must not exceed compressive strength of material.

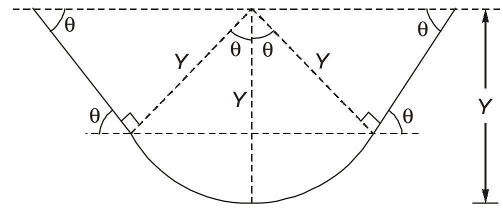
$$\sigma_{principal} = \gamma_w H(S_c - C + 1)$$

$$f_c = \text{comp. strength of dam material.}$$

$$\gamma_w H(S_c - C + 1) \leq f_c$$

$$H_{max} = \frac{f_c}{(S_c - C + 1)}$$

Q.3 (b) Solution :



Here, $P = 2y\theta + 2y \cot\theta$

$$A = \frac{1}{2}Y^2 2\theta + 2\left[\frac{1}{2} \times Y \times Y \cot\theta\right]$$

$$A = Y^2\theta + Y^2 \cot\theta$$

$$R = \frac{A}{P} = \frac{Y}{2}$$

$$S = \frac{1}{2000}$$

$$Q = 30 \text{ m}^3\text{s}^{-1}$$

$$\tan\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 26.56° = 0.464 \text{ radian}$$

$$A = 0.464\, y^2 + y^2 \times 2 = 2.464 y^2$$

$$Q = \frac{1}{n} \cdot A \cdot R^{2/3} \sqrt{S}$$

or, $$30 = \frac{1}{0.012} \times (2.464 y^2) \times \left(\frac{y}{2}\right)^{2/3} \sqrt{\frac{1}{2000}}$$

or, $$30 = \frac{1}{0.012} \times 0.03471 \times y^{8/3} = 2.8924 y^{8/3}$$

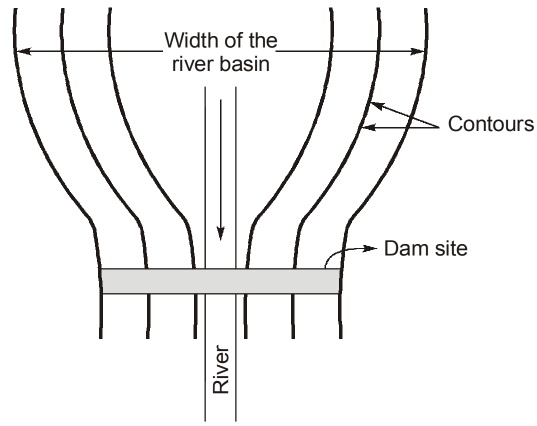
∴ y **= 2.40 m**

OR

Q.3 (a) Solution :

Selection of dam site: The selection of gravity dam site is governed by following factors:

1. Suitable foundations must be available.
2. For economy, the length of dam should be small as possible and for a given height, it should store the maximum volume of water. A general configuration of contours for suitable site is given below:



3. The general bed level at dam site should preferably be higher than that of the river basin. This will reduce the height of dam and will facilitated drainage.
4. A suitable site for the spillway should be available in the near vicinity. If the spillways is to be combined with the dam, the width of the gorge should be such as to accommodate both.
5. The reservoir basin should be reasonably water tight. The stored water should be as low as possible.
6. The dam site should be easily accessible, so that it can be economically connected to important towns and cities by rails, roads etc.

Q.3 (b) Solution :

Exit gradient can be defined as Hydraulic or pressure gradient of subsoil flow at downstream or exit end of floor.

It is given as,

$$G_E = \frac{H}{d} \times \frac{1}{\pi\sqrt{\lambda}}$$

$$\lambda = \frac{1+\sqrt{1+\alpha^2}}{2}; \ \alpha = b/d$$

Given: Floor width, b = 10 m; Depth of downstream pile, d = 1.5 m; H = 4 m

$$\alpha = \frac{10}{1.5} = \frac{20}{3}$$

$$\lambda = \frac{1+\sqrt{1+\left(\frac{20}{3}\right)^2}}{2} = 3.871$$

$$G_E = \frac{4}{1.5} \times \frac{1}{\pi \times \sqrt{3.871}} = \mathbf{0.43125}$$

Q.4 (a) Solution :

$$\text{Total water filtered in a day} = 1 \times (24 \times 60 \times 60) = 86400 \text{ m}^3/\text{day}$$

$$\text{Total surface area of filter required} = \frac{86400\,\text{m}^3/\text{day}}{140\,\text{m}^3/\text{day}/\text{m}^2} = 617.14 \text{ m}^2$$

$$\text{Area of one filters required} = 6 \times 8 = 48 \text{ m}^2$$

$$\text{Total number of filter} = \frac{617.14}{48} = 12.86 \simeq 13$$

As three filters are out of service, number of filters working = 13 – 3 = 10

Total surface area of ten filters = 48 × 10 = 480 m^2

$$\text{New loading rate} = \frac{86400 \text{ m}^3/\text{day}}{480 \text{ m}^2} = 180 \text{ m}^3/\text{day}/\text{m}^2$$

Q.4 (b) Solution :

$$\text{Population equivalent} = \frac{\text{Total 5 days oxygen demand}}{\text{Per capita BOD of sewage per day}}$$

$$\Rightarrow \quad \text{Total 5 days oxygen demand} = 3{,}17{,}000 \times \left(\frac{80}{1000}\right) = 25360 \text{ kg}$$

$$\text{Daily sewage flow} = 75 \times 10^6 \; l/\text{day}$$

$$\therefore \quad 25360 = (\text{Avg. 5 days BOD}) \times 75 \times 10^6$$

$$\Rightarrow \quad \text{Avg. 5 days BOD} = \frac{25360}{75 \times 10^6} \text{ kg}/l = \frac{25360 \times 10^6}{75 \times 10^6} \text{ mg}/l = 338.13 \text{ mg}/l$$

OR

Q.4 (a) Solution :

We know,

$$\text{Efficiency, } \eta = 1 - \exp\left(-\frac{AV}{Q}\right)$$

where,

V = Settling velocity of particle (in m/s)

Q = Air flow (in m^3/sec)

A = Surface area of electrostatic precipitator (in m^2)

$$0.99 = 1 - \exp\left(-\frac{0.12A}{50}\right)$$

$$-A\frac{0.12}{50} = \ln(1 - 0.99)$$

$$-A = \frac{50}{0.12}\ln(0.01)$$

$$A = -416.67 \ln(0.01)$$

$$\text{Surface area} = 1919 \text{ m}^2$$

Q.4 (b) Solution :

Negative head phenomenon is a problem observed in rapid sand filter. The water percolating through the filter moves downward under the force of gravity. This motion is opposed by the resistance offered by the sand grains and the impurities arrested by them. The percolating water, therefore looses some of its head. This loss of head is measured by inserting two piezometers, one in the water standing over the filter and the other in the outlet pipe. The difference in the readings of these two piezometers will give the loss of head.

When the filter is newly commissioned, the loss of head is generally very small, and is of the order of 15 to 30 cm. However, the loss of head goes on increasing as the time passes, and as more and more impurities get trapped into it. A stage is finally reached when the frictional resistance offered by the filter media exceeds the static head of water above the sand bed. Most of this resistance is caused by the top 10 to 15 cm sand layer. The bottom sand then acts like more or less as a vacuum, and water is sucked through the filter media rather than getting filtered through it. The fall of mercury level in the piezometer inserted in the outlet pipe below the centre line of the pipe, indicates the presence and extent of negative pressure. The negative pressure so developed, tends to release the air dissolved in water. It causes the formation of bubbles which stick to the sand grains, and there by seriously affecting the working of the filter. The phenomenon is known as air binding, as the air binds the filter and stops its functioning, thereby reducing the rate of filtration considerably.

In order to avoid such troubles, the filters are frequently cleaned particularly as soon as the loss of head exceeds the optimum allowable values. On an average, the loss of head is generally limited to 2.5 to 3.5 m, and the negative head to about 1.2 m. The depth of water during filter operation may also be increased by 15-20 cm as compared to the normal depth, as to increase the static head (to compensate lost head) and to reduce the negative head.

Q.5 (a) Solution :

(i) Infiltration index:

ϕ-index: This is the average rainfall above which, rainfall volume is equal to the runoff volume. ϕ-index basically represents infiltration rate during the period of rainfall excess. In the calculation of ϕ-index, initial losses are also considered as infiltration quantity.

W-index: This is the average infiltration rate during the whole period of storm. In the calculation of ω-index initial losses are separated from total infiltration quantity.

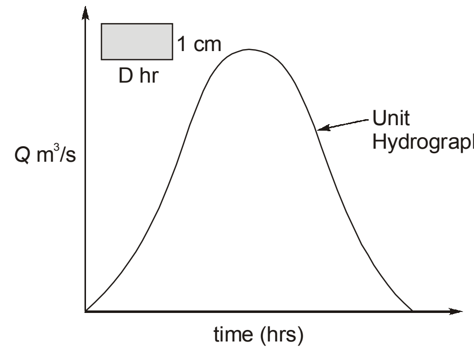
$$\text{W-index} = \frac{P - Q - \Delta I}{t}$$

Where, P = total rainfall, Q = total Runoff, ΔI = Initial losses

Note that, $\phi \geq W$

(ii) Unit Hydrograph: This is the direct runoff hydrograph which result in 1 cm (unit) runoff depth from a rainfall of some unit duration.

Shown D hours unit hydrograph indicates a direct run off hydrograph resulting in a run-off depth of 1 cm from a storm lasting for D hour. This D hour is called the unit duration.



(iii) Flood frequency: The time period after which a peak flood discharge is likely to be equal or exceeded is known as return period. Flood frequency represents annual exceedance probability (AEP), which is inverse of return period (T)

$$\therefore \quad f = \frac{1}{T} \times 100\%$$

Where, T = Return period

If Return period is 100 year, it means flood discharge can equal or exceed after 100 year.

$$\therefore \quad f = \frac{1}{100} \times 100\%$$

100 year flood have 1% flood frequency.

(iv) Benefit-cost ratio (of a project): Benefit-cost ratio is a ratio attempting to identity the relationship between the cost of project and benefits of a proposed project. Mathematically speaking, expenditure on a project is justified if the resultant annual benefits exceeds the annual cost (including interest on the capital expenditure) i.e. Benefit to cost ratio is more than one.

The benefit cost analysis is used to adopt most economical project as compare to competing project.

Q.5 (b) Solution :

Assuming inlet as section 1 and throat at section 2

At inlet

Diameter, D = 200 mm

At throat

Diameter, D_2 = 10 cms

Using equation of continuity

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4} \times \left(\frac{200}{10}\right)^2 \times V_1 = \frac{\pi}{4} \times 10^2 \times V_2$$

$$\Rightarrow \quad V_2 = 4V_1$$

Differential head, $h = 0.30\left(\frac{13.6}{0.8}-1\right) = 4.8$ m

Using equation of energy conservation

$$\frac{V_2^2}{2g}-\frac{V_1^2}{2g} = 4.8$$

$$(4V_1)^2 - V_1^2 = 4.8 \times 2 \times 9.81$$

$$V_1 = 2.35 \text{ m/s}$$

Now, $A_1 = \frac{\pi}{4}\times 0.2^2 = 0.0314 \text{ m}^2$

$$A_2 = \frac{\pi}{4}\times 0.1^2 = 0.00785 \text{ m}^2$$

Theoretical discharge, $Q_{th} = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\times\sqrt{2gh} = 0.0786 \text{ m}^3/\text{s}$

Actual discharge, $Q_{act} = C_d Q_{th} = 0.6 \times 0.0736 = 0.0472 \text{ m}^3/\text{s}$

OR

Q.5 (a) Solution :

Def: A unit hydrograph is defined as the hydrograph of direct runoff resulting from unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate of a specified duration (D hours).

The unit hydrograph has 2 assumptions

(i) Time invariance: This assumption states that "DRH for an excess rainfall (ER) is same irrespective of the time when it occurs.

(ii) Linear response: The response of DRH to excess rainfall is linear.

Input $x_1(t) \rightarrow y_1(t)$ (output)

Input $x_2(t) \rightarrow y_2(t)$ (output)

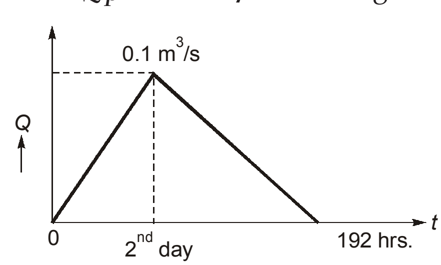
Input $x_1 + x_2 \rightarrow y_1 + y_2$ (output)

it is an important assumption as it enables usage of method of superimposition to derive DRH's.

UH: $D = 1$ **hr**

$t_B = 8$ days $= 192$ hr

$Q_P = 0.1 \text{ m}^3/\text{s}$ occurring on 2nd day.



We know rainfall depth = 1 cm

So, $$\text{depth} = \frac{\text{Volume}}{\text{Area}}$$

$$\text{Volume of water} = \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times 192\times 3600\times 0.1 = \mathbf{34560\ m^3}$$

$$1\text{ cm} = \frac{34560}{A\times 10^6}\times 100$$

$$\Rightarrow \quad A = \mathbf{3.456\ km^2} \quad \text{or} \quad A = \mathbf{345.6\ ha}$$

Q.5 (b) Solution :

(i) **Boundary layer thickness:** The velocity within the boundary layer increases from zero at the boundary surface to the velocity of the main stream asymptotic.

Therefore, the boundary layer thickness represented by δ is arbitrarily defined as that distance from the boundary surface in which the velocity reaches 99% of the velocity of the main stream. In other words, the boundary layer thickness δ may be considered equal to distance y from the boundary surface at which v = 0.99 V.

Displacement thickness: The displacement thickness δ* is defined as the distance by which the boundary surface would have to be displaced outwards so that total actual discharge would be same as that of an ideal (or frictionless) fluid past the displaced boundary.

Thus, expressed by the situation;

$$V\delta^* = \int_0^\infty (V - v)\,v\,dy$$

$$\Rightarrow \quad \delta^* = \int_0^\infty \left(1 - \frac{v}{V}\right)dy$$

Momentum thickness: The momentum thickness θ is defined as the distance from the actual boundary surface such that momentum flux corresponding to the main stream velocity V through the distance θ is equal to the deficiency or less in momentum due to boundary layer formation.

(ii) We know, the displacement thickness is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{V_\infty}\right)dy$$

Given, $$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

$$\therefore \quad \delta^* = \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right]dy = \frac{\delta}{8} \qquad \text{...(i)}$$

The momentum thickness is given by

$$\theta = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u}{V_\infty}\right)dy = \int_0^{\delta}\left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right]dy$$

$\therefore$ $$\theta = \frac{7}{72}\delta \quad \text{...(ii)}$$

Hence, $$\frac{\delta^*}{\theta} = \frac{\delta/8}{7\delta/72} = \frac{\text{Displacement thickness}}{\text{Momentum thickness}} = \frac{9}{7}$$

$\Rightarrow$ $$\frac{\delta^*}{\theta} = 1.2857$$

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