MADE EASY

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Detailed Solutions

ESE-2019 Mains Test Series

Mechanical Engineering Test No : 11

Section A

Q.1 (a) Solution:

$$\text{Allowable stress} = \frac{\text{Ultimate stress}}{\text{FOS}} = \frac{650}{3}\,\text{MPa}$$

Now, considering vertical forces at joint C and D, we will get

$$F_{AC} \times \frac{5}{\sqrt{5^2 \times 10^2}} = 1\text{ kN}$$

$$\Rightarrow \qquad F_{AC} = \sqrt{5}\ \text{kN}$$

and

$$F_{AD} \times \frac{5}{\sqrt{5^2 \times 20^2}} = 2\text{ kN}$$

$$F_{AD} = 2\sqrt{17}\ \text{kN}$$

For rod AC,

$$\frac{F_{AC}}{A_{AC}} = \text{Allowable stress}$$

$$\Rightarrow \qquad \frac{\sqrt{5} \times 10^3}{\dfrac{\pi d_{AC}^2}{4}} = \frac{650}{3} \times 10^6$$

$$\Rightarrow \qquad d_{AC} = 3.625\text{ mm}$$

For rod *AD*,

$$\frac{F_{AD}}{A_{AD}} = \text{Allowable stress}$$

$$\Rightarrow \quad \frac{2\sqrt{17}\times 10^3}{\frac{\pi d_{AD}^2}{4}} = \frac{650}{3}\times 10^6$$

$$\Rightarrow \quad d_{AD} = 6.961 \text{ mm}$$

So, we have to take diameter of rod *AC* equal to 3.625 mm and that of rod *AD* to 6.961 mm.

Q.1 (b) Solution:

Considering the direction of moment applied, lower fibre will be in compression and upper fibre will be in tension.

Neutral axis:

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{(4\times 7)\times 2 - \left(\frac{\pi(2)^2}{2}\right)\times \frac{4\times 2}{3\times \pi}}{(4\times 7) - \left(\frac{\pi(2)^2}{2}\right)} = 2.33 \text{ cm}$$

Moment of inertia about neutral axis,

$$I = \left(I_{cg} + A\left(\bar{y} - y_{cg}\right)^2\right)_1 - \left(I_{cg} + A\left(\bar{y} - y_{cg}\right)^2\right)_2$$

$$= \left(I_{cg} + A\left(\bar{y} - y_{cg}\right)^2\right)_1 - \left[\left(I_{\text{along diameter}} - A\left(y_{cg}\right)^2\right) + A\left(\bar{y} - y_{cg}\right)^2\right]_2$$

$$= \frac{1}{12}(7)(4)^3 + (7\times 4)\times(0.33)^2 - \left[\frac{\pi(2)^4}{8} - \left(\frac{\pi(2)^2}{2}\right)\left(\frac{4\times 2}{3\times\pi}\right)^2 + \left(\frac{\pi(2)^2}{2}\right)\left(2.33 - \frac{4\times 2}{3\times \pi}\right)^2\right]$$

$$= 37.33 + 3.05 - [6.28 - 4.53 + 13.78] = 24.85 \text{ cm}^4$$

Considering, maximum tensile stress,

$$\frac{M\times(2-0.33)\times 10^{-2}}{24.85\times 10^{-8}} = 5\times 10^6$$

$$M = 74.40 \text{ Nm}$$

Considering, maximum compressive stress,

$$\frac{M \times (2.33) \times 10^{-2}}{24.85 \times 10^{-8}} = 7 \times 10^{6}$$

$$M = 74.66 \text{ Nm}$$

We can apply minimum of above values i.e. 74.40 Nm.

Q.1 (c) Solution:

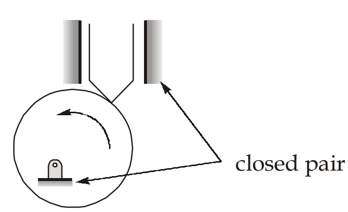
Kinematic pair: The connection between the two link is a joint or pair and this pair will be a kinematic pair if the relative motion between the links is a constrained motion (completely constrained motion or successfully constrained motion)

Kinematic pair can be classified according to

1. Nature of contact
2. Nature of mechanical constraint
3. Nature of relative motion

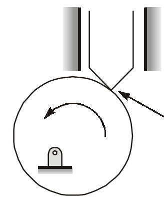
1. According to the nature of contact.
   - **(a) Lower pair :** When pair/joint having surface or area contact between the members. **Example:** Nut turning on a screw, shaft rotation in a bearing, all pairs of slider crank mechanism and universal joint.
   - **(b) Higher pair:** When pair/joint having line or point contact between members. **Example:** Wheel rolling on a surface, cam and follower pair, ball and roller bearings.
   - **(c) Wrapping pair:** When one link is wrapped over other link. **Example**: Belt and pulley, rop and pulley.

2. According to the nature of mechanical constraint.
   - **(a) Closed pair/self closed pair:** When the elements of pair are held mechanically. Generally all the lower pair and some of the higher pair are closed pairs.



   - **(b) Force closed pair/ unclosed pair/forcefully closed pair:** When the elements of a pair form contact either due to force of gravity or by some spring action, then it is known as force closed pair.

Example: cam and follower pair.



3. According to the nature of relative motion:
   (a) **Sliding pair:** If the two links having a translatory/sliding motion to each other.
   Example: A rectangular rod in a rectangular hole in a prism is a sliding pair.
   (b) **Turning pair**: If the two links having rotary/turning motion relative to each other then they form turning pair.
   Example: Circular shaft inside a bearing forms turning pair pin joint.
   (c) **Rolling pair**: When the links of pair have pure rolling motion.
   Example: A rolling wheel on a flat surface in a ball bearing, the ball and the shaft constitute one rolling pair.
   (d) **Screw pair (helical pair)**: When links of pair having a turning as well as sliding motion between them.
   Example: Lead screw and the nut of a lathe form screw pair.
   (e) **Spherical pair**: When one link in the form of sphere turns inside a fixed link then it forms spherical pair.
   Example: The ball inside the socket forms spherical pair.

Q.1 (d) Solution:

1. **Scoring**: Scoring is due to combination of two distinct activities. First, lubrication failure in the contact region and second, establishment of metal to metal contact. The scoring is classified into initial, moderate and destructive. Initial scoring occurs at high spots left by previous machining and once these high spots are removed, the stress comes down as shte load is distributed over a larger area. If the load, speed or oil temperature increases after initial scoring then moderate scoring at tolerable rate occurs and if the load, speed or oil temperature increases appreciably, then severe or destructive scoring takes place.
2. **Pitting of gears**: Pitting is a surface fatigue failure of the gear tooth. It occurs due to repeated loading of tooth surface and the contact stress exceeding the surface fatigue strength of the material. Material in the fatigue region gets removed and a pit is formed. The pit itself will cause stress concentration and soon the pitting spreads to adjacent region till the whole surface is covered. Subsequently, higher impact load resulting from pitting may cause fracture of already weakened tooth. However, the

failure process takes place over millions of cycles of running. There are two types of pitting, initial and progressive.

3. **Plastic flow/cold flow**: Plastic flow of tooth surface results when it is subjected to high contact stress under rolling cum sliding action. Surface deformation takes place due to yielding of surface or subsurface material. Normally it occurs in softer gear material but it can occur even in heavily loaded case hardened gears. Cold flow material over the tooth tip can be seen clearly in bevel gear.

Q.1 (e) Solution:

As per given data:

$$m = 220 \text{ kg}$$

$$I_w = 1.1 \text{ kg m}^2$$

$$I_e = 0.18 \text{ kg m}^2$$

$$r = 0.3 \text{ m}$$

$$G = \frac{\omega_e}{\omega_w} = 4.5$$

$$h = 0.62 \text{ m}$$

$$R = 35 \text{ m}$$

$$V = 45\times\frac{5}{18}\text{ m/s} = 12.5 \text{ m/s}$$

$$\text{Gyroscopic couple} = (2I_w + GI_e)\frac{V^2}{rR}\cos\theta$$

$$= (2\times1.1+4.5\times0.18)\frac{12.5^2}{0.3\times35}\cos\theta = 44.79\cos\theta$$

$$\text{Centrifugal couple, } C_c = m\frac{V^2}{R}h\cos\theta$$

$$= \frac{220\times12.5^2}{35}\times0.62\cos\theta = 608.928\cos\theta$$

$$\text{Total over turning couple} = (44.79\cos\theta + 608.928\cos\theta)$$

$$= 653.718\cos\theta \quad \text{...(1)}$$

$$\text{Rightening couple} = mgh\sin\theta$$

$$= 220 \times 9.81 \times 0.62\sin\theta$$

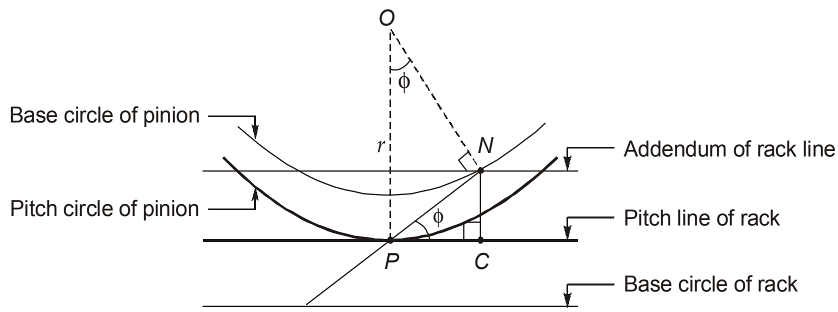
$$= 1338.084 \sin\theta \quad \text{...(2)}$$

$$\therefore \quad 1338.084\sin\theta = 653.718\cos\theta$$

$$\tan\theta = 0.48854$$

$$\Rightarrow \quad \theta = 26.037°$$

Q.2 (a) Solution:



From ΔPCN

$$\sin\phi = \frac{CN}{PN}$$

$$PN = \text{maximum path of approach}$$

$$CN = \text{maximum addendum of rack}$$

$$CN = PN\sin\phi$$

$$PN = r\sin\phi$$

$$CN = \frac{mt}{2}\sin^2\phi$$

$$t = \frac{2A_R}{\sin^2\phi} \qquad ...(1)$$

the equation (1) gives the relation between the maximum addendum of the rack, the number of teeth on the pinion or wheel and pressure angle.

As we know,

The addendum of the rack cutter when the cutting the teeth of one wheel is also the addendum of the teeth on the mating gear.

in terms of module,

the maximum addendum of the rack cutter, is given by

$$A_R \times m = \frac{mt}{2}\sin^2\phi$$

maximum addendum of the wheel = maximum addendum of rack cutter when cutting the pinion

$$= \frac{mt}{2}\sin^2\phi$$

maximum addendum of pinion = maximum addendum of rack cutter when cutting the wheel

$$= \frac{mT}{2}\sin^2\phi$$

So maximum working depth = Sum of the maximum addenda

$$= \frac{m(t+T)}{2}(\sin\phi)^2 \qquad \text{...(ii)}$$

But the standard working depth = $2\,m$ ...(iii)

From (ii) and (iii)

$$\therefore \qquad 2m = \frac{m(t+T)}{2}(\sin\phi)^2$$

$$(t+T) = \frac{4}{\sin^2\phi} \qquad \text{...(iv)}$$

Condition-I:

If $\phi = 14\frac{1°}{2}$

Then, the sum of teeth $(t + \mathrm{T}) = \dfrac{4}{(\sin\phi)^2} = \dfrac{4}{\left(\sin\left(14\frac{1°}{2}\right)\right)^2} = 63.8 \simeq 64$ teeth

Condition-II:

If $\phi = 20°$

then, the sum of teeth $(t + T) = \dfrac{4}{(\sin\phi)^2} = \dfrac{4}{(\sin 20°)^2} = 34.194$

$$34.194 \simeq 35$$

Q.2 (b) Solution:

1. We know

$$\in = \left(\frac{\in_x + \in_y}{2}\right) + \left(\frac{\in_x - \in_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\in = \in_x \cos^2\theta + \in_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$

$$\theta_1 = 0°, \theta_2 = 60°, \theta_3 = 120°$$

For $\theta = \theta_1$

$$40\mu = \in_x \quad ...(1)$$

For $\theta = \theta_2$

$$980\mu = \frac{\in_x}{4} + \frac{3\in_y}{4} + \gamma_{xy}\left(\frac{\sqrt{3}}{4}\right) \quad ...(2)$$

for $\theta = \theta_3$

$$330\mu = \frac{\in_x}{4} + \frac{3\in_y}{4} - \gamma_{xy}\left(\frac{\sqrt{3}}{4}\right) \quad ...(3)$$

Solving eq. (1), eq. (2) and eq. (3) we get

$$\in_x = 40\mu, \in_y = 860\ \mu, \gamma_{xy} = 750\mu$$

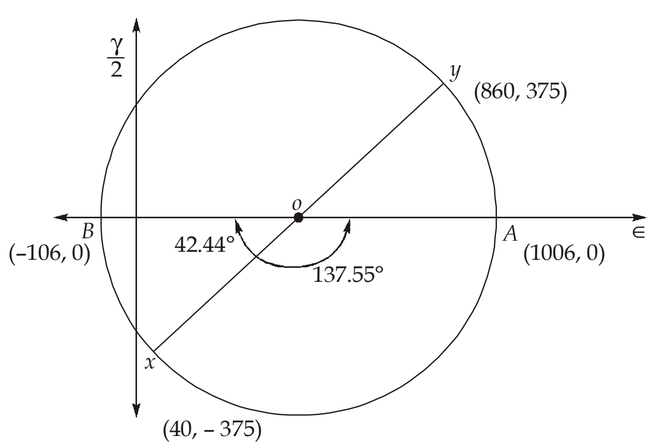
Principal strains: Let $\in_a$ and $\in_b$ be principal strains then

$$\in_a, \in_b = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 450\mu \pm 556\ \mu$$

$$= 1006\mu, -106\mu$$

Drawing mohr's circle,



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{750}{-820}$$

$$2\theta_p = -42.44, 137.55°$$

$$\theta_p = -21.22°, 68.77°$$

Since, $\sigma_z = 0$ (Q at surface)

$$\epsilon_c = -\frac{v}{E}(\sigma_a + \sigma_b) \quad ...(1)$$

We know $\epsilon_a = \frac{\sigma_a}{E} - \frac{\nu\sigma_b}{E}$

and $\epsilon_b = \frac{\sigma_b}{E} - \frac{\nu\sigma_a}{E}$

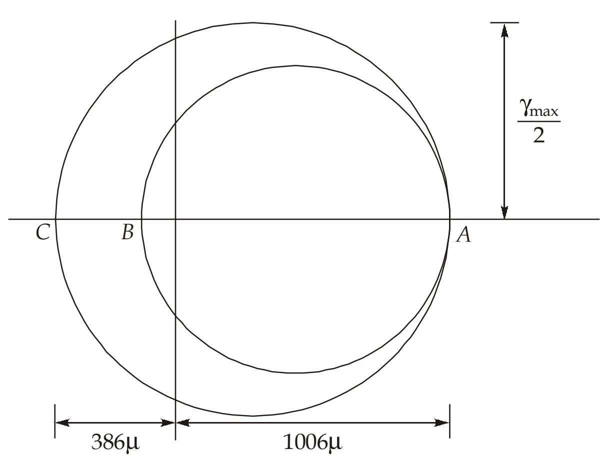
$$\Rightarrow \quad \epsilon_a + \epsilon_b = \frac{(1-\nu)}{E}(\sigma_a + \sigma_b) \quad ...(2)$$

From eq. (1) and (2)

$$\epsilon_c = -\frac{\nu}{E} \times \frac{(\epsilon_a + \epsilon_b)E}{(1-\nu)}$$

$$= -\frac{\nu}{(1-\nu)}(\epsilon_a + \epsilon_b) = -\frac{0.3}{0.7} \times 900\mu = -386\mu$$

So, now mohr's circle,

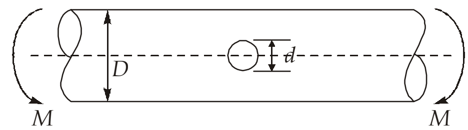


(c) Maximum shear strain, γ_{max},

$$\frac{\gamma_{max}}{2} = \frac{1}{2}(|\epsilon_a| + |\epsilon_c|)$$

$$\gamma_{max} = 1392\ \mu$$

Q.2 (c) Solution:



Considering only bending moment:

Mean and fluctuating bending stress:

$$\sigma_m = \frac{32 \times M_m}{\pi \times (0.06)^3}$$

$$M_m = \frac{400 + (-200)}{2} = 100 \text{ Nm}$$

So, $$\sigma_m = \frac{32 \times 100}{\pi \times (0.06)^3} = 4.72 \text{ MPa}$$

$$\sigma_v = \frac{32 \times M_v}{\pi \times (0.06)^3}$$

$$M_v = \frac{400 - (-200)}{2} = 300 \text{ Nm}$$

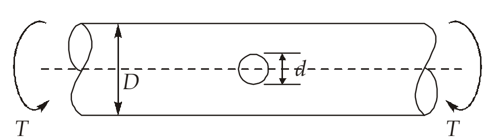
So, $$\sigma_v = \frac{32 \times 300}{\pi \times (0.06)^3} = 14.16 \text{ MPa}$$

For bending $$\sigma_{eb} = \sigma'_e \times 0.75 \times 0.78 \times 0.7 \times \frac{1}{2.25} = \sigma'_e \times 0.182$$

$$\sigma_{eb} = \sigma'_e \times 0.182 = 0.5 \times 600 \times 0.182 = 54.6 \text{ MPa}$$

So, $$\sigma_{eq} = \sigma_m + \frac{\sigma_v \times \sigma_y}{\sigma_{eb}} = 4.72 + \frac{14.16 \times 420}{54.6} = 113.64 \text{ MPa}$$

Considering only torsions:



$$\tau_m = 0$$

$$\tau_v = \frac{16 \times T}{\pi \times (0.06)^3} = \frac{16 \times 100}{\pi \times (0.06)^3} = 2.36 \text{ MPa}$$

For torsion, $$\sigma_{es} = \sigma_e{}' \times 0.75 \times 0.78 \times 0.78 \times \frac{0.7}{2.9} = 0.11\ \sigma'_e$$

$$\sigma_{es} = 0.11 \times 0.5 \times 600 = 33.042 \text{ MPa}$$

$$\tau_{eq} = \tau_m + \frac{\tau_y \times \tau_v}{\sigma_{es}} = 0 + \frac{0.577 \times \sigma_y \times \tau_v}{\sigma_{es}}$$

$$= \frac{0.577 \times 420 \times 2.36}{33.042}$$

$$\tau_{\text{eq}} = 17.308 \text{ MPa}$$

Equivalent principal stresses:

$$\sigma_{1,\,2} = \frac{1}{2}\left[\sigma_{eq} \pm \sqrt{(\sigma_{eq})^2 + 4 \times (\tau_{\text{eq}})^2}\right]$$

$$= \frac{1}{2}\left[113.64 \pm \sqrt{(113.64)^2 + 4 \times 17.308^2}\right]$$

$$= \frac{1}{2}[113.64 \pm 118.79]$$

$$\sigma_1 = 116.217 \text{ MPa}$$

$$\sigma_2 = -2.575 \text{ MPa}$$

Using Von mises:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2^2) + (\sigma_1)^2 = 2\left(\frac{\sigma_y}{N}\right)^2$$

$$(116.217 + 2.575)^2 + (116.217)^2 + (-2.575)^2 = 2\left(\frac{420}{N}\right)^2$$

$$N = 3.57$$

Q.3 (a) Solution:

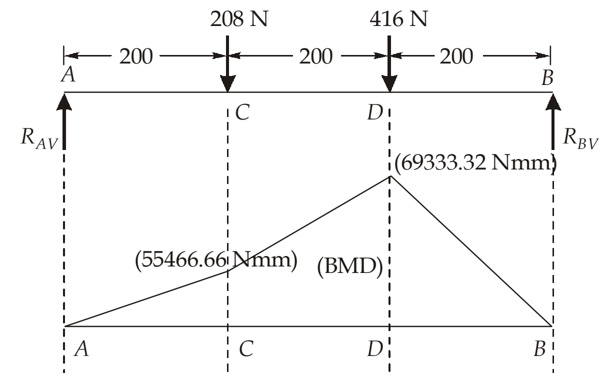
Permissible shear stress (Using maximum shear stress theory)

$$\tau_{\text{per}} = \frac{S_{sy}}{f(s)} = \frac{0.5 S_{yt}}{f(s)} = \frac{0.5 \times 400}{3} = 66.67 \text{ N/mm}^2$$

Vertical component in shaft:

Take moment about A,

$$R_{BV} \times 600 = 208 \times 200 + 416 \times 400$$



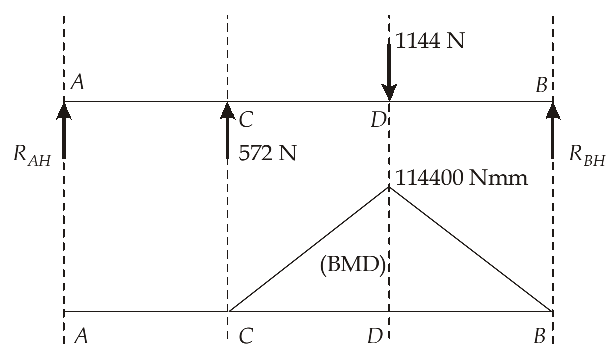
$$R_{BV} = 346.666 \text{ N}$$
$$R_{AV} = 277.333 \text{ N}$$

Bending moment: (BM_V)

$$(BM_V)_A = 0$$
$$(BM_V)_C = 55466.66 \text{ N-mm}$$
$$(BM_V)_D = 69333.32 \text{ N-mm}$$

Horizontal component in shaft:

Take moment about A,



$$R_{BH} \times 600 + 572 \times 200 = 1144 \times 400$$
$$R_{BH} = 572 \text{ N}$$

and $R_{AH} = 0$

Bending moment: (BM_H)

$$(BM_H)_A = 0$$
$$(BM_H)_C = 0$$
$$(BM_H)_D = 572 \times 200 = 114400 \text{ N-mm}$$

Torque in shaft, T:

$$T_C = 572 \times \frac{500}{2} = 143000 \text{ Nmm}$$

$$T_D = 1144 \times \frac{250}{2} = 143000 \text{ Nmm}$$

Net Bending moment:

$$(BM)_C = \sqrt{(BM_V)_C^2 + (BM_H)_C^2} = 55466.66 \text{ Nm}$$

$$(BM)_D = \sqrt{(BM_Y)_D^2 + (BM_H)_D^2}$$

$$= \sqrt{(69333.32)^2 + (114400)^2}$$

$$(BM)_D = (BM)_{max} = 133770.210 \text{ Nmm}$$

So it is clear that D is the critical point. Diameter of shaft is calculated with respect to critical point D.

We know that, for maximum shear stress theory,

$$d^3 = \frac{16}{\pi\tau_{per}}\sqrt{(2\times 133770.21)^2 + (1.5\times 143000)^2}$$

$$d^3 = 26195.180$$

$$d = 29.6989 \text{ mm} \simeq 30 \text{ mm}$$

So, diameter of shaft can be taken as 30 mm,

Net reaction at A and B is:

$$\text{at bearing } A\text{: } R_A = \sqrt{(R_{AV})^2 + (R_{AH})^2}$$

$$R_A = 277.33 \text{ N}$$

$$\text{at bearing } B\text{: } R_B = \sqrt{(R_{BV})^2 + (R_{BH})^2}$$

$$R_B = \sqrt{(346.66)^2 + (572)^2} = 668.847 \text{ N}$$

There is only radial load (no axial load)

$$L_{90} = \frac{60nL_{90h}}{10^6} = \frac{60\times 720\times 8000}{10^6} = 345.6 \text{ million rev.}$$

Dynamic load capacities:

$$\left(\frac{C}{P}\right) = (L_{10})^{1/3}$$

$$\left(\frac{C_A}{R_A}\right) = (345.6)^{1/3}$$

$$C_A = 277.33 \times (345.6)^{1/3}$$

$$C_A = 1946.20 \text{ N}$$

Similarly $C_B = (345.6)^{1/3} \times R_B = 4693.73$ N

From given table, at diameter of shaft 30 mm, bearing at A and B according to dynamic load values can be selected as:

Bearing at A = 61806

Bearing at B = 16006

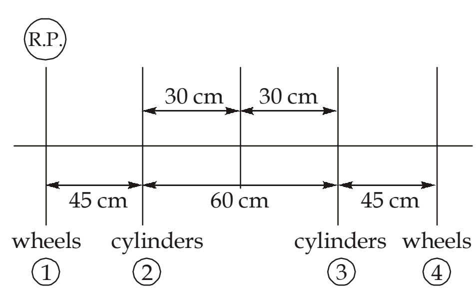
Q.3 (b) Solution:

As per given information:

It is the case of inside cylinder.

$$\text{Total mass is to be balanced} = \left(\text{Revolving mass} + \frac{2}{3}\text{Reciprocating mass}\right)$$

$$= \left(225 + \frac{2}{3} \times 270\right) = 405 \text{ kg}$$



$$m_2 = 405 \text{ kg}, m_3 = 405 \text{ kg},$$

$$r_2 = r_3 = \frac{64}{2} \text{ cm} = 32 \text{ cm} = 0.32 \text{ m},$$

$$r_1 = r_4 = 0.75 \text{ m}$$

$$N = 240 \text{ rpm}$$

From reference plane

$$l_2 = 0.45 \text{ m}, l_3 = 1.05 \text{ m}, l_4 = 1.5 \text{ m}$$

Cranks are placed at 90° to each other so crank(2) is at 0° and cranks(3) is at 90° from crank(2).

Assuming reference plane (R.P.) across wheel 1.

$$\Sigma M_x = 0$$

$$-(m_4 r_4 l_4 \cos\theta_4) = m_2 r_2 l_2 \cos\theta_2 + m_3 r_3 l_3 \cos\theta_3$$

$$= 405 \times 0.32 \times 0.45 \cos 0° + 405 \times 0.32 \times 1.05 \cos 90°$$

$$m_4 r_4 l_4 \cos\theta_4 = -58.32 \quad \text{...(i)}$$

$$\Sigma M_y = 0$$

$$-(m_4 r_4 l_4 \sin\theta_4) = m_2 r_2 l_2 \sin\theta_2 + m_3 r_3 l_3 \sin\theta_3$$

$$-(m_4 r_4 l_4 \sin\theta_4) = 405 \times 0.32 \times 0.45 \sin 0° + 405 \times 0.32 \times 1.05 \sin 90°$$

$$m_4 r_4 l_4 \sin\theta_4 = -136.08 \quad \text{...(ii)}$$

From (i) and (ii)

$$m_4 r_4 l_4 = \sqrt{(-58.32)^2 + (-136.08)^2}$$

$$m_4 = \frac{\sqrt{(-58.32)^2 + (-136.08)^2}}{0.75 \times 1.5}$$

$$m_4 = 131.600 \text{ kg}$$

$$\theta_4 = \tan^{-1}\left(\frac{-136.08}{-58.32}\right)$$

$$\theta_4 = 180° + 66.801° \text{ (third quadrant)}$$

$$\theta_4 = 246.8014°$$

Similarly, it is symmetrical case $m_2 = m_3$

$$r_2 = r_3$$

$$m_1 = 131.600 \text{ kg}$$

$$\theta_1 = \tan^{-1}\left(\frac{-58.32}{-136.08}\right)$$

$$\Rightarrow \quad \theta_1 = 23.198° + 180°$$

$$\theta_1 = 203.198°$$

In order to balance 405 kg required balance mass is 131.600 kg

So in order to balance 1 kg, required balance mass = $\dfrac{131.600 \text{ kg}}{405 \text{ kg}}$

So for balancing 2/3rd or reciprocating mass i.e. 180 kg,

$$\text{required balance mass} = \frac{131.600\,\text{kg}}{405\,\text{kg}} \times 180 = 58.488 \text{ kg}$$

$$B = 58.488 \text{ kg}$$

$$b = 0.75 \text{ m}$$

$$\omega = \frac{2\pi \times 240}{60} = 25.132 \text{ rad/s}$$

$$\text{Hammer blow} = Bb\omega^2 = 58.488 \times 0.75 \times 25.132^2 = 27706.53 \text{ N}$$

maximum variation of tractive efforts:

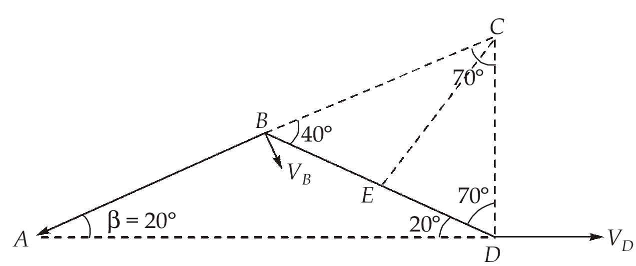
$$= \sqrt{2}(1-c)mr\omega^2$$

$$= \sqrt{2}\left(1-\frac{2}{3}\right) \times 270 \times 0.32 \times 25.132^2 = 25725.3675 \text{ N}$$

Q.3 (c) Solution:

As the system is released, due to gravity *AB* and *BD* rod will move. Rod *AB* will rotate about *A*. *D* will move horizontally.

Velocity of *B* will be perpendicular to *AB*. Instaneous center of rotation of rod *BD* can be found by making line perpendicular to $\vec{V}_B$ and $\vec{V}_D$.



Using sine rule

$$\frac{BD}{\sin 70°} = \frac{BC}{\sin 70°} = \frac{CD}{\sin 40°}$$

$$\Rightarrow \quad BC = BD = 0.75 \text{ m}$$

$$\Rightarrow \quad CD = \frac{BD \sin 40°}{\sin 70°} = 0.513 \text{ m}$$

Now, when, $\beta = 20°$

$$V_B = \omega_{BD} BC = 0.75\, \omega_{BD} \qquad \text{...(1)}$$

$$V_D = \omega_{BD} CD = 0.513\, \omega_{BD} \quad ...(2)$$

If E is midpoint of BD, using cosine rule we can find CE,

$$CE = \sqrt{BE^2 + BC^2 - 2(BE)(BC)\cos(\angle EBC)}$$

$$= \sqrt{(0.375)^2 + (0.75)^2 - 2 \times 0.375 \times 0.75 \times \cos(40°)}$$

$$= 0.522 \text{ m} \quad ...(3)$$

We can also write velocity of B w.r.t to A,

$$V_B = \omega_{AB} AB$$

$$= 0.75\, \omega_{AB} \quad ...(4)$$

Using (1) and (4)

$$\omega_{AB} = \omega_{BD}$$

(a) We can conserve energy between initial and final position as there is no loss.

$$E_1 = E_2$$

$$2mg\frac{l}{2}\sin\beta_1 = 2mg\frac{l}{2}\sin\beta_2 + \frac{1}{2}I_{AB}\omega_{AB}^2 + \frac{1}{2}I_{BD}\omega_{BD}^2 \quad ...(5)$$

where, I_{BD} is MOI of rod BD about I-centre at C.

$$\beta_1 = 60°$$

$$\beta_2 = 20°$$

$$m = 6 \text{ kg}$$

$$l = 0.75 \text{ m}$$

$$I_{AB} = \frac{ml^2}{3} = \frac{6(0.75)^2}{3} = 1.125 \text{ kg-m}^2$$

$$I_{BD} = \frac{ml^2}{12} + m(CE)^2 = \frac{6(0.75)^2}{12} + 6(0.522)^2$$

$$= 1.916 \text{ kg-m}^2$$

Putting these values in equation (5) we get

$$2 \times 6 \times 9.81 \times \left(\frac{0.75}{2}\right) \times (\sin 60 - \sin 20) = \frac{1}{2}(1.125)\omega_{AB}^2 + \frac{1}{2}(1.916)\omega_{BD}^2$$

as $\omega_{AB} = \omega_{BD}$

$$\omega_{AB} = 3.9 \text{ rad/sec}$$

(b) Using eq. (2),

$$V_D = \omega_{BD} CD = 2 \text{ m/s}$$

Q.4 (a) Solution:

Given; $k_c' = 2k_b'$, $P_i = 12$ kN, $P = 7$ kN, $\sigma_y = 400$ MPa, $A = 95$ mm^2

We know that, Net force on the bolt is

$$P_b = P_i + \Delta P$$

$$\Delta P = P\left(\frac{k_b'}{k_b'+k_c'}\right) = P\left(\frac{k_b'}{k_b'+2k_b'}\right) = \frac{P}{3}$$

$$\Delta P = \frac{7\times10^3}{3} = 2.333 \times 10^3 \text{ N}$$

So, $$P_b = P_i + \Delta P = 12 \times 10^3 + 2.333 \times 10^3 = 14.333 \times 10^3 \text{ N}$$

$$\text{Stress on the bolt, } \sigma = \frac{P_b}{A} = \frac{14.333\times10^3}{95}$$

$$\sigma = 150.877 \text{ MPa}$$

(i) So, factor of safety, (N)

$$N = \frac{\sigma_y}{\sigma} \qquad \text{(where, } \sigma_y = 400 \text{ MPa)}$$

$$N = \frac{400}{150.877} = 2.6511$$

(ii) Test for leak proof joint,

$$P_i > \text{P}(1 - K) \qquad \left(\text{Where, } K = \left(\frac{k_b'}{k_b'+k_c'}\right) = \frac{1}{3}\right)$$

$$12 > 7\left(1-\frac{1}{3}\right)$$

$$12 > 4.66$$

So as the condition is satisfied, the joint is leak proof.

(iii) Test for joint seperation,

(a) 1st condition $\quad P_i > KP$

$$12 > \frac{1}{3}\times7$$

$$12 > 2.333$$

Satisfied,

(b) 2nd condition $P_i < A \times \sigma_y$

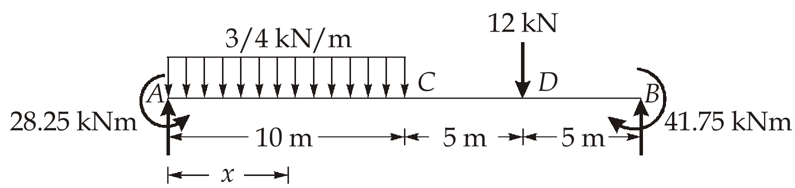
$$12 < \frac{95 \times 400}{1000}$$

$$12 < 38$$

Satisfied,

Hence as both the conditions are satisfied, the joint seperation does not takes place.

Q.4 (b) Solution:



Taking moments about B, we get

$$\Sigma M_B = 0$$

$$\Rightarrow 28.25 + \left(10 \times \frac{3}{4} \times 15\right) + 12 \times 5 - R_A \times 20 - 41.75 = 0$$

$\Rightarrow$ $R_A = 7.95$ kN ($\uparrow$)

$\therefore$ $R_B = 7.5 + 12 - 7.95 = 11.55$ kN ($\uparrow$)

SFD : For AC, $F_x = 7.95 - 0.75x$ which is a linear variation

At $x = 0$, $F_A = 7.95$ kN

At $x = 10$ m, $F_C = 7.95 - 7.5 = 0.45$ kN

For CD, $F_x = 7.95 - 7.5 = 0.45$ kN which is constant from C to D

For DB, $F_x = 7.95 - 7.5 - 12 =$

$= -11.55$ kN which is constant from D to B

BMD

For AC, $M_x = -28.25 + 7.95x - \dfrac{0.75x^2}{2}$ which is a parabolic variation

At $x = 0$, $M_A = -28.25$ kNm

At $x = 10$, $M_C = -28.25 + 7.95\,(10) - \dfrac{0.75}{2}(10)^2 = 13.75$ kNm

$$M_x = 0 = -28.25 + 7.95x - \frac{0.75x^2}{2}$$

$$x = 4.51 \text{ m}$$

Hence, BM changes sign and is zero at $x = 4.51$ m

For CD, $M_x = -28.25 + 7.95x - 7.5(x - 5)$, which is linear

At $x = 10$ m, $M_C = -28.25 + 79.5 - 7.5 \times 5 = 13.75$ kNm as before

At $x = 15$ m, $M_D = -28.25 + 7.95 \times 15 - 7.5\ (15 - 5) = 16$ kNm

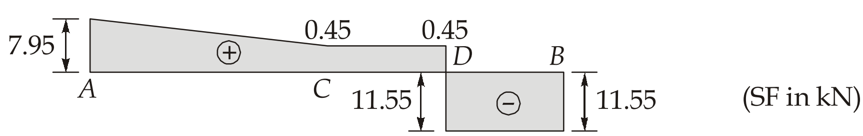
For DB: $M_x = -28.25 + 7.95x - 7.5(x - 5) - 12(x - 15)$ which is linear

At $x = 20$ m, $M_B = -25.25 + 7.95 \times 20 - 7.5\ (20 - 5) - 12\ (20 - 15)$

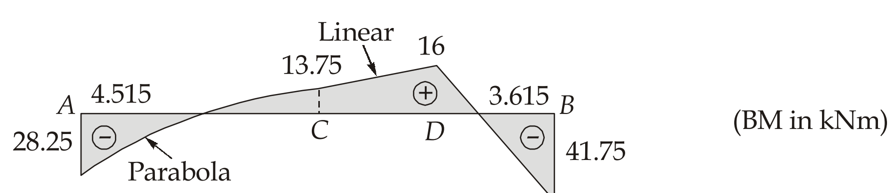
$= -41.75$ kNm which is same as given in question

Thus, the BM changes sign in DB and is zero at $x = 16.38$ m from A or at $\frac{41.75}{11.55} = 3.615$ m from B.

The SF and BM diagrams are shown below respectively.



SFD



BMD

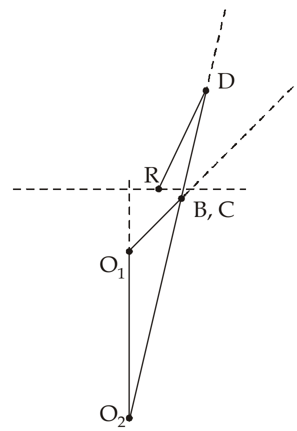
Q.4 (c) Solution:

First drawing configuration diagram according to the data given in the question:

Taking scale : 150 mm = 1 cm

After scaling $O_1O_2 = 4.67$ cm, $O_1B = 1.67$ cm

$O_2D = 8.33$ cm, $DR = 2.33$ cm

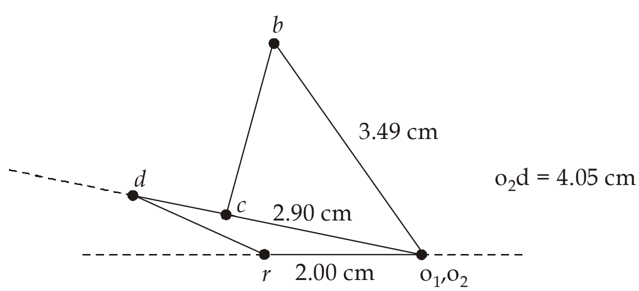


Now drawing velocity diagram,

$$V_B = \left(\frac{2\pi \times 40}{60}\right) \times 0.25 = 1.047 \text{ m/s}$$

Taking scale: 0.3 m/s = 1 cm

After scaling o_1b = 3.49 cm



(i) Since o_1 and o_2 are fixed points therefore these points are marked as one point in velocity diagram, draw vector o_1b perpendicular to O_1B such that

(ii) From point o_2 draw vector $\overrightarrow{o_2c}$ perpendicular to O_2C to represent the velocity of the coincident point C with respect to O_2 or simply velocity of C, and from b draw vector $\overrightarrow{bc}$ parallel to the path of motion of the sliding block (which is along line O_2D) to represent the velocity of C with respect to B (i.e. V_{cb}). The vectors $\overrightarrow{o_2c}$ and bc intersect at c.

(iii) Since the point d lies on o_2c produced, along line and the ratio $\dfrac{cd}{o_2d} = \dfrac{CD}{O_2D}$ will remain same.

(iv) Now from point d, draw vector dr perpendicular to DR to represent the velocity of R with respect to D(i.e. V_{rd}) and from point o_1 draw vector o, r parallel to the path of motion of R (which is horizontal) to represent velocity of r.
The vectors dr and $o_1 r$ intersect at r.

From velocity diagram

$$o_1 r = 2\text{ cm}$$

$$\Rightarrow \quad V_r = 2 \times 0.3 = 0.6\text{ m/s}$$

$$\text{Also as } o_1 d = 4.05\text{ cm}$$

$$\Rightarrow \quad V_d = 4.05 \times 0.3 = 1.215\text{ m/s}$$

$$\text{As,} \quad \omega_{O2D} = \frac{V_d}{O_2 D} = \frac{1.215\text{ m/s}}{1.25\text{ m}} = 0.972\text{ rad/s}$$

So, velocity of ram, $V_r = 0.6$ m/s

Angular velocity of slotted lever, $\omega_{O2D} = 0.972$ rad/s

Section B

Q.5 (a) Solution:

γ-iron has Face Centered Cubic structure and α-iron has Body Centered Cubic structure. Assume, Lattice parameter of FCC structure is a_1 and Lattice parameter of BCC structure is a_2. Radius of atom is 'r' which remain same.

(i) For FCC structure or for γ-iron

We know that,

$$\text{Lattice parameter, } a_1 = 2\sqrt{2}r$$

$$r = 0.35355 a_1$$

$$\text{or} \quad a_1 = 2.8284\, r$$

$$\text{Unit cell volume, } V_{C_1} = a_1^3 = (2.8284r)^3 = 22.6267r^3$$

In FCC structure, there are four atoms per unit cell.

Volume per atom in FCC structure,

$$V_1 = \frac{V_{C_1}}{4} = \frac{22.6267r^3}{4}$$

$$V_1 = 5.6566r^3$$

(ii) For BCC structure or for α-iron

We know that,

$$\text{Lattice parameter, } a_2 = \frac{4r}{\sqrt{3}} = 2.3094r$$

$$\text{Unit cell volume } V_{C_2} = a_2^3 = (2.3094r)^3 = 12.3168r^3$$

In BCC structure, there are two atoms per unit cell.

Volume per atom in BCC structure,

$$V_2 = \frac{V_{C_2}}{2} = \frac{12.3168r^3}{2}$$

$$V_2 = 6.1584\, r^3$$

$$\text{Percentage change in volume of atom} = \left(\frac{V_2 - V_1}{V_1}\right) \times 100\%$$

$$= \left(\frac{6.1584r^3 - 5.6566r^3}{5.6566r^3}\right) \times 100\%$$

Percentage change in volume of atom = 8.871%.

Q.5. (b) Solution:

We know that,

$$\text{Solidification time} \propto \left(\frac{\text{Volume}}{\text{Surface area}}\right)^2$$

As volume of all casting is same, assume volume as 1 unit.

$$\text{Solidification time} \propto \frac{1}{(\text{Surface area})^2}$$

I) For Sphere,

$$\text{Volume, } V = \frac{4}{3}\pi r^3$$

$$1 = \frac{4}{3}\pi r^3$$

$$r = \left(\frac{3}{4\pi}\right)^{1/3}$$

$$\text{Surface area, } A_1 = 4\pi r^2 = 4\pi \times \left(\frac{3}{4\pi}\right)^{2/3} = 4.836$$

II) For cube, Volume, $V = a^3$

$$1 = a^3$$

$$a = 1$$

Surface area, $A_2 = 6a^2 = 6 \times 1 = 6$

III) Cylinder, volume, $V = \pi r^2 h = \pi r^2 \times 2r = 2\pi r^3$

$$1 = 2\pi r^3$$

$$r = \left(\frac{1}{2\pi}\right)^{1/3}$$

Surface area, $A_3 = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 4\pi r^2 = 6\pi r^2$

$$A_3 = 6\pi\times\left(\frac{1}{2\pi}\right)^{2/3} = 5.536$$

IV) Cuboid: Volume, $V = a \times 2a \times 3a$

$$1 = 6a^3$$

$$a = \left(\frac{1}{6}\right)^{1/3}$$

Surface area, $A_4 = 2(a \times 2a + 3a \times 2a + 3a \times a)$

$$= 2a^2(2 + 6 + 3) = 22a^2 = 22\times\left(\frac{1}{6}\right)^{2/3} = 6.663$$

Solidification time for sphere, $t_1 = C\times\frac{1}{A_1^2} = \frac{C}{4.836^2} = 0.04276C$

Solidification time for cube, $t_2 = \frac{C}{A_2^2} = \frac{C}{6^2} = 0.0278C$

Solidification time for cylinder, $t_3 = \frac{C}{A_3^2} = \frac{C}{5.536^2} = 0.03263C$

Solidification time for cuboid, $t_4 = \frac{C}{A_4^2} = \frac{C}{6.663^2} = 0.02252C$

Minimum solidification time for cuboid and maximum for sphere.

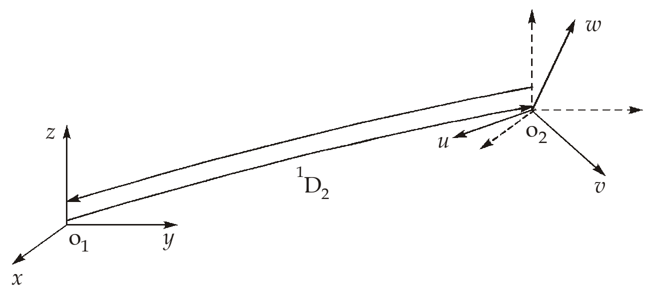
Ratio of solidification times $= t_1 : t_2 : t_3 : t_4$

$= 1.8987 : 1.2344 : 1.4489 : 1$

Comment: Cuboid shaped casting will solidify at fastest rate, and the sphere shaped casting will solidify at slowest rate.

Q.5 (c) Solution:

Let us consider a frame {1} having axis x, y and z. Another frame {2} having axis u, v and w is a translated and rotated frame with respect to frame {1}.



Homogenous transformation matrix, ${}^1T_2$.

$${}^1T_2 = \left[\begin{array}{c|c} {}^1R_2 & {}^1D_2 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Similarly,
$${}^2T_1 = \left[\begin{array}{c|c} {}^2R_1 & {}^2D_1 \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

we know that rotation submatrix R has the property.

$${}^2R_1 = {}^1R_2^{\,T}$$

The mapping of a point P from frame {2} to frame {1}

$${}^1P = {}^1D_2 + {}^1R_2{}^2P$$

Premultiplying both sides by ${}^2R_1$ gives

$${}^2R_1{}^1P = {}^2R_1\,{}^1D_2 + {}^2R_1\,{}^1R_2{}^2P$$

as $${}^2R_1\,{}^1R_2 = \mathrm{I}$$

So, $${}^2R_1\,{}^1P = {}^2R_1\,{}^1D_2 + {}^2P$$

$${}^2\mathrm{P} = {}^2R_1\,{}^1P - {}^2R_1\,{}^1D_2$$

$${}^2P = {}^2R_1\,{}^1\mathrm{P} + {}^2D_1$$

$\Rightarrow$ $${}^2D_1 = -{}^2R_1\,{}^1D_2$$

$${}^2D_1 = -{}^1R_2^{\,T}\,{}^1D_2$$

$$^{2}T_1 = [^{1}T_2]^{-1} = \left[\begin{array}{c|c} {}^{1}R_2^{T} & -{}^{1}R_2^{T}\,{}^{1}D_2 \\ \hline 0\ \ 0\ \ 0 & 1 \end{array}\right]$$

Q.5 (d) Solution:

(i) Volumetric efficiency, $\eta_v = \dfrac{\text{Theoretical flow rate of motor should consume}}{\text{Actual flow rate consumed by motor}}$

$$\eta_v = \frac{Q_T}{Q_A}$$

Now, $Q_A = 0.006\ \text{m}^3/\text{s}$ and

$$Q_T = V_D \times N = 160 \times \frac{2000}{60} \times 10^{-6} = 5.333 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$\eta_v = \frac{5.333 \times 10^{-3}}{0.006} = 0.8888 = 88.89\%$$

(ii) Mechanical efficiency $= \eta_m = \dfrac{\text{Actual torque delivered by motor}}{\text{Theoretical torque motor should deliver}}$

$$\eta_m = \frac{T_A}{T_t}$$

Theoretical torque $= T_t = \dfrac{P \times V_D}{2\pi} = \dfrac{75 \times 10^5 \times 160 \times 10^{-6}}{2\pi} = 190.986$ Nm

So, mechanical efficiency, $\eta_m = \dfrac{150}{190.986} = 0.7854 = 78.54\%$

(c) Overall efficiency, $\eta_o = \eta_m \times \eta_v = 0.8889 \times 0.7854$

$$\eta_o = 0.6981$$

$$\eta_o = 69.81\%$$

(d) Actual power:

$$\eta_o = \frac{\text{Actual power delivered by motor (Mechanical)}}{\text{Actual power delivered to motor (Hydraulic)}}$$

$$\eta_o = \frac{P_{act}}{Q_A \times P}$$

$$P_{act} = \eta_o \times Q_A \times P$$

$$= 0.6981 \times 0.006 \times 75 \times 10^5 \times 10^{-3}$$

$$P_{act} = 31.414 \text{ kW}$$

or $$P_{act} = T_A \times \frac{2\pi N}{60} = \frac{150 \times 2\pi \times 2000}{60 \times 1000} = 31.415 \text{ kW}$$

Q.5 (e) Solution:

Given, $D \times C_u$ = Rs. 75000 per year

Ordering cost, C_o = Rs. 45 per order

Carrying cost, C_h = 25% of C_u

$$C_h = 0.25\, C_u$$

At *EOQ* with no discount total cost per year,

$$= (D \times C_u) + \sqrt{2DC_oC_h}$$

$$= 75000 + \sqrt{2DC_0 \times C_u \times 0.25}$$

$$= 75000 + \sqrt{2 \times 75000 \times 45 \times 0.25}$$

$$(TAC)_{EOQ} = 75000 + 1299.04 = \text{Rs. } 76299.04 \text{ per year}$$

Let, x% discount is provided on unit cost.

$$(TAC)_x = (D \times C_u)_x + n \times C_o + C_h \times \left(\frac{D}{n} \times \frac{1}{2}\right)$$

$$(TAC)_x = (D \times C_u)_x + 4 \times 45 + \frac{(0.25 \times C_u \times D)_x}{8}$$

$$(TAC)_x = (1.03125)(D \times C_u)_x + 180$$

For minimum discount,

$$(TAC)_x = (TAC)_{EOQ}$$

$$1.03125\,(D \times C_u)_x + 180 = 76299.04$$

$$(D \times C_u)_x = \frac{76299.04 - 180}{1.03125}$$

$$(D \times C_u)_x = \text{Rs. } 73812.40$$

$$\%\text{discount} = \left[\frac{(D\times C_u)_{EOQ} - (D\times C_u)_x}{(D\times C_u)_{EOQ}}\right]\times 100\%$$

$$= \left[\frac{75000 - 73812.40}{75000}\right]\times 100\% = 1.5834\%$$

Q.6 (a) Solution:

- Ceramics are inorganic, non-metallic materials that consists of metallic and non-metallic elements for which the interatomic bonds are either totally ionic, or predominantly ionic but having some covalent character. The term "ceramics" comes from the greek word ***keramikos***, which means "burnt stuff", indicating that desirable properties of these materials are normally achieved through a high-temperature heat treatment process called firing.
- The properties of these materials also vary greatly due to differences in bonding. In general, these materials are typically hard and brittle with low toughness and ductility. Due to their desirable characteristics such as high hardness, wear resistance, chemical stability, high temperatures strength, and low coefficient of thermal expansion, advanced ceramics are being selected as the preferred material for many applications. These materials are usually good electrical and thermal insulators due to absence of conduction electrons.
- Ceramic materials normally have relatively high melting temperature and high chemical stability in many hostile environments, which are indispensable for many engineering designs.
- In general, we can divide ceramic materials used for engineering applications into two groups:
  (i) Traditional ceramic materials
  (ii) Engineering ceramic materials
- Typically, traditional ceramics are made from three basic components; clay, silica (flint), and feldspar. Bricks and tiles are familiar examples of traditional ceramics.

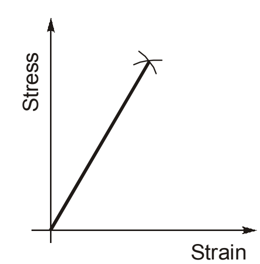
Electrical Properties of Ceramics

1. **Dielectric Constant :** Ceramics have high value of dielectric constant. Because of high dielectric constant they can be used for insulation purpose in transformers, alternators, electric motors etc.
2. **Dielectric Strength :** Dielectric strength can be defined as the voltage required per unit thickness to cause breakdown in the material. Ceramics have very high value of dielectric strength.

3. **Dielectric Loss :** For ceramics, value of dielectric losses is very low. Because of this property, they are highly suitable for insulation purpose.

Mechanical and Thermal Properties of Ceramics

1. **Strength:** Ceramics have very high strength in compression with respect to tension. Brick is used as a ceramic material in building because of its very high compressive strength.
2. **Corrosion Resistance:** Ceramics show very good corrosion resistance to prevent any action of chemicals and weathers. That is the reason of using ceramics as refractory material in nuclear power plants.
3. **Hardness and Abrasion Resistance:** Ceramic materials have very high hardness. Ceramic materials show excellent abrasion resistance.
4. **Water Absorptivity:** Ceramic materials have very low water absorptivity. So ceramics can be used in construction work.
5. **Brittleness, Ductility and Malleability:** Ceramics materials are highly brittle material. Ceramic materials show very less ductility and malleability (property of a material which permits the material to be extended in all directions without rupture).
6. **Melting Point:** As they have both ionic and covalent bond. Hence these materials have very high melting point. For example – Zirconia melts at 3200°C.
7. **Elasticity:** Ceramics show linear elastic behavior. This is reason because of which stress strain curve of ceramics is straight line upto fracture point.



8. **Creep Strength:** Ceramics have very high melting point. Because of very high melting point these materials do not creep normally upto workable high temperature.
9. **Crack Propagation:** In ceramics materials crack is developed at the point of stress concentration. In ceramics crack can propagate under tensile load but crack cannot propagate under compressive load.
10. **Fracture of Ceramics:** In general, ceramic material fails under brittle mode because of very high degree of brittleness.
11. **Thermal conductivity, expansion and insulation:** Ceramic materials have very low value of thermal conductivity. These materials have very less value of thermal expansion with temperature. Ceramic materials are suitable for thermal insulation purpose.

Q.6 (b) Solution:

Given, Semidie angle, $\alpha = 12°$

Coefficient of friction, $\mu = 0.10$

$$B = \mu \cot\alpha = 0.1 \cot 12° = 0.47046$$

Initial diameter, $d_i = 12$ mm, final diameter, $d_f = 10$ mm

$$\sigma_f = (1000 \text{ MPa}) \in^{0.32}$$

$$\text{True strain, } \in = \ln\left(\frac{L_f}{L_i}\right) = \ln\left(\frac{A_i}{A_f}\right) = \ln\left(\frac{d_i}{d_f}\right)^2$$

$$\in = 2\ln\left(\frac{12}{10}\right) = 0.36464$$

$$\text{Mean flow stress, } \sigma_o = \left(\frac{k\in^n}{1+n}\right) = \frac{(1000)\times(0.36464)^{0.32}}{1+0.32}$$

$$\sigma_o = 548.558 \text{ MPa}$$

$$\text{Drawing stress, } \sigma_d = \sigma_o\left(\frac{1+B}{B}\right)\left[1-\left(\frac{d_f}{d_i}\right)^{2B}\right]$$

$$= (548.558)\times\left(\frac{1+0.47046}{0.47046}\right)\times\left[1-\left(\frac{10}{12}\right)^{2\times 0.47046}\right]$$

Drawing stress, $\sigma_d = 270.2866$ MPa

For minimum diameter or maximum possible reduction:

$$\sigma_d = \sigma_o\left(\frac{1+B}{B}\right)\left[1-\left(\frac{d_f}{d_i}\right)^{2B}\right]$$

For minimum possible diameter or maximum reduction, $\sigma_d = \sigma_o$

$$1 = \left(\frac{1+B}{B}\right)\left[1-\left(\frac{d_f}{d_i}\right)^{2B}\right]$$

$$1 = \left(\frac{1+0.47046}{0.47046}\right)\left[1-\left(\frac{d_f}{d_i}\right)^{2\times 0.47046}\right]$$

$$\left(\frac{0.47046}{1.47046}\right) = 1-\left(\frac{d_f}{d_i}\right)^{2\times0.47046}$$

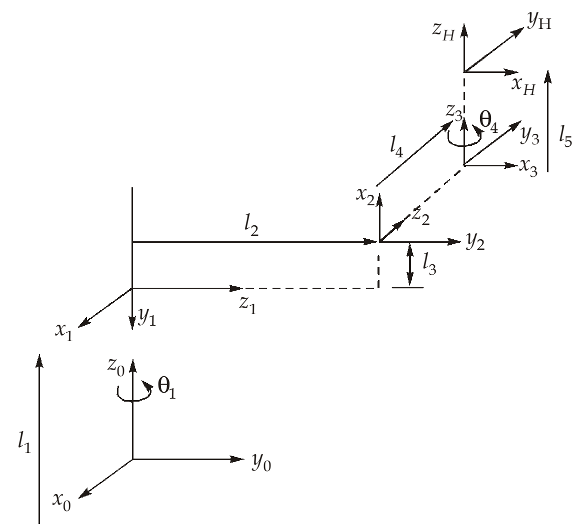
$$\left(\frac{d_f}{d_i}\right)^{2\times0.47046} = 1-\left(\frac{0.47046}{1.47046}\right)$$

$$\left(\frac{d_f}{d_i}\right)^{2\times0.47046} = 0.68006$$

$$d_f = (d_i) \times (0.68006)^{1/(2\times0.47046)}$$

Minimum possible diameter, d_f = 7.9655 mm

Q.6 (c) Solution:



Link	θ	d	a	α	$\sin\theta$	$\cos\theta$	$\sin\alpha$	$\cos\alpha$
1	$\theta_1 = 30°$	$l_1 = 20$ cm	0	–90°	0.5	0.866	–1	0
2	–90°	$l_2 = 50$ cm	5 cm	90°	–1	0	1	0
3	90°	$l_4 = 50$ cm	0	90°	1	0	1	0
4	$\theta_4 = 45°$	$l_5 = 10$ cm	0	0	0.707	0.707	0	1

Q.7 (a) Solution:

(i)

Controllability of a system: A state x_1, of a system is "controllable" if all initial conditions x_o at any previous time t can be transferred to x_1 in a finite time by some control function $u(t, x_0)$.

- If all the states are controllable then the system is completely controllable.
- If controllability is restricted to depend upon t_o, then the system is controllable at time t_o.

The controllability matrix for a system (A, B) is defined as:

$$Q_C = [B\ AB\ A^2B\ \dots\ A^{n-1}B]$$

where, state equation $\dot{x}(t) = Ax(t) + B\,u(t)$

$[A]_{n \times n} = n \times n$ system matrix, $[B]_{n \times m}$ = input matrix

The system is fully controllable if Rank $(Q_C) = n$

Observability of a system:

A state $x_1(t)$ at some given time is 'observable' if knowledge of the input $u(t)$ and output $y(t)$ over a finite segment of time completely determines $x_1(t)$.

The observability matrix for a system is defined as

$$O^T = [C^T\ C^TA^T\ \dots\ C^T(A^{n-1})^T]$$

The system is fully observable if Rank $O = n$

where output equation $y(t) = Cx(t) + Du(t)$

(ii)

Controllability:

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

$$Q_C = [B\ \ AB] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$|Q_C| = 0$$

As, determinant of Q_C is zero, so Rank is not equal to n, i.e. 2. So system is not controllable.

Observability:

$$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

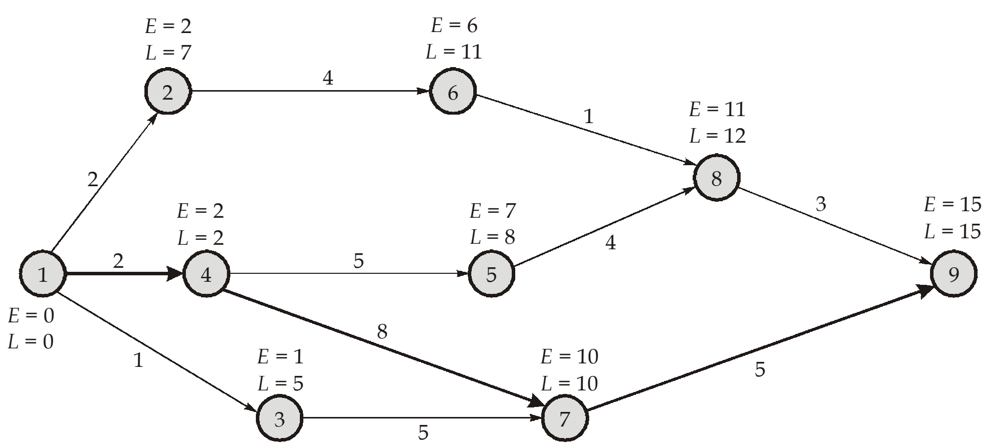
$$A^TC^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$Q_O = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$|Q_o| = 0$$

As determinant of Q_o is zero, hence rank is not equal to n i.e. 2, so system is not observable.

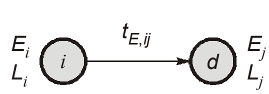
Q.7 (b) Solution:



Critical path: 1 - 4 - 7 - 9

Expected project completion time = 15 day

Activity	Duration	EST	EFT	LST	LFT	Total float	Free float	Independent float
1 - 2	2	0	2	5	7	5	0	0
1 - 4	2	0	2	0	2	0	0	0
1 - 3	1	0	1	4	5	4	0	0
2 - 6	4	2	6	7	11	5	0	–5
6 - 8	1	6	7	11	12	5	4	–1
4 - 5	5	2	7	3	8	1	0	0
4 - 7	8	2	10	2	10	0	0	0
5 - 8	4	7	11	8	12	1	0	–1
8 - 9	3	11	14	12	15	1	1	0
3 - 7	5	1	6	5	10	4	4	0
7 - 9	5	10	15	10	15	0	0	0



$$\text{TF} = L_j - (E_i + t_{E,ij})$$
$$\text{FF} = E_j - (E_i + t_{E,ij})$$

$$IF = E_j - (L_i + t_{E,ij})$$

Q.7.(c) Solution:

(i) Arc characteristics is given by, $I_a = 20(V - 15)$

Volt-ampere characteristic of power source,

$$I^2_t = -500(V - 45)$$

As current voltage variation is parabolic in nature, hence power source is constant current source.

We know that, for stability of arc,

$$I_t = I_a$$

$$I^2_t = I^2_a$$

$$-50(V - 45) = [20(V - 15)]^2$$

$$-500V + 500 \times 45 = 400[V^2 - 30V + 225]$$

$$400V^2 - 12000V + 90000 + 500V - 22500 = 0$$

$$400V^2 - 11500V + 67500V = 0$$

$$V = 20.53 \text{ Volt}, 8.22 \text{ Volt}$$

Now, $I_a = 20(V - 15)$

If, $V = 20.53$ Volt,

$$I_a = 20(20.53 - 15) = 110.6 \text{ Ampere}$$

If, $V = 8.22$ Volt

$$I_a = 20(8.22 - 15) = -135.6 \text{ Ampere}$$

As current is negative for 8.22 Volt. Hence, voltage will be 20.53 Volt.

$$\text{Power of arc, } P = V \times I = 20.53 \times 110.6 = 2270.618 \text{ Watt}$$

$$P = 2.27 \text{ kW}$$

II) Arc length voltage relationship is given as,

$$V_a = 25 + 5l$$

Voltage-Ampere characteristics of power source is given by,

$$I^2_t = -500(V - 45)$$

$$I^2_t = -500(25 + 5l - 45)$$

$$I^2_t = -500 \times 5(l - 4)$$

$$I_t = \{-2500(l - 45)\}^{1/2}$$

We know that, Power is given by, $P = V \times I$

$$P = (25 + 5l) \times \{-2500(l - 4)\}^{1/2}$$

Now, for optimum arc length, $\frac{dP}{dl} = 0$

$$(25+5l)\times\frac{1}{2}\{-2500(l-4)\}^{-1/2}\times(-2500)+5\{-2500(l-4)\}^{1/2} = 0$$

$$2500\times5(5+l)\times\frac{1}{2}\{-2500(l-4)\}^{-1/2} = 5\{-2500(l-4)\}^{1/2}$$

$$2500 \times 5(5 + l) = \{-2500(l - 4)\}$$

$$1250 \times 5 + 1250l = -2500l + 10000$$

$$3750l = 10000 - 6250$$

$$\text{Optimum arc length, } l = 1 \text{ mm}$$

Q.8 (a) Solution:

(i) Risk priority number: Risk priority number (RPN) depends on three parameters.

1. O $\rightarrow$ Occurance
2. S $\rightarrow$ Severity
3. D $\rightarrow$ Detectability

All these (O, S and D) ranks are given on a scale from 1 to 10.

$$\text{RPN} = O \times S \times D$$

i. Low RPN is desirable.

ii. If the fault is occuring at a very high rate, we will give high number to 'O'. Where as if the fault is occuring less frequently (robust design), we will given low number to 'O'.

iii. If the fault is occuring at a very high rate, we will give lower number to 'S' otherwise we will give higher number to 'S'.

iv. If the fault can be easily detected, we will give lower number to 'D' otherwise higher number to 'D'.

v. RPN ranges from 1 to 1000.

vi. We apply condition based monitoring for systems having high RPN.

vii. RPN is a subjected evaluation put in number.

viii. The O, S and D rank depends on the application and FMECA standard that is used.

ix. The O, S and D and the RPN can have different meaning for each FMECA.

x. Sharing number between companies and group is very difficult.

(ii) Given, $\text{SPL}_1 = 100 \text{ dB}$

$$SPL_2 = 95 \text{ dB}$$
$$SPL_3 = 90 \text{ dB}$$
$$SPL_4 = 88 \text{ dB}$$

Total sound pressure level, $SPL_T = ?$

$$SPL_T = 10\log_{10}\left[10^{\frac{SPL_1}{10}} + 10^{\frac{SPL_2}{10}} + 10^{\frac{SPL_3}{10}} + ...\right]$$

$$= 10\log_{10}\left[10^{\frac{100}{10}} + 10^{\frac{95}{10}} + 10^{\frac{90}{10}} + 10^{\frac{88}{10}}\right]$$

$$= 10\log_{10}\left[10^{10} + 10^{9.5} + 10^{9} + 10^{8.8}\right]$$

Total sound pressure level, $SPL_T = 101.7$ dB

Q.8 (b) Solution:

(i)

Let 'm' be the number of holes,

Taylor's tool life equation,

$$VT^n = C$$

$$\Rightarrow \quad \left(\frac{\pi DN}{60}\right)\left(\frac{L}{fN}\times m\right)^n = C \qquad \text{(where } D, L, f \text{ are constant)}$$

$$\Rightarrow \quad N\left(\frac{m}{N}\right)^n = C_1$$

Case 1:

$$N_1 = 150 \text{ rpm}$$
$$m_1 = 300 \text{ holes}$$

$$N_1\left(\frac{m_1}{N_1}\right)^n = C_1$$

$$150\left(\frac{300}{150}\right)^n = 4 \qquad ...(1)$$

Case 2:

$$N_2 = 450 \text{ rpm}$$
$$m_2 = 200 \text{ holes}$$

$$450\left(\frac{200}{450}\right)^n = C_1 \qquad ...(2)$$

Equating (1) and (2):

$$150\left(\frac{300}{150}\right)^n = 450\left(\frac{200}{450}\right)^n$$

$$\left(\frac{2}{200}\times 450\right)^n = 3$$

$$n = \frac{\ln 3}{\ln 4.5} = 0.7304$$

Now, $N_3 = 200$ rpm

$$N_3\left(\frac{m_3}{N_3}\right)^n = C_1$$

$$\Rightarrow \qquad 200\left(\frac{m_3}{200}\right)^n = C_1 \qquad ...(3)$$

From (1) and (3)

$$150\left(\frac{300}{150}\right)^n = 200\left(\frac{m_3}{200}\right)^n$$

$$150(2)^{0.7304} = 200\left(\frac{m_3}{200}\right)^{0.7304}$$

$$m_3 = 269.776 \simeq 270$$

Maximum number of holes, $m_3 = 270$

(ii)

Approach angle, $\lambda = 75°$

Side cutting edge angle, $C_s = 90° - \lambda$

$= 90° - 75° = 15°$

Mean surface roughness, $R_a = 3\ \mu m$

$= 3 \times 10^{-3}$ mm

Maximum height of roughness, $H_{max} = 4\ R_a$

$= 4 \times 3 \times 10^{-3} = 12 \times 10^{-3}$ mm

As we know,

$$H_{max} = \frac{f}{\tan C_s + \cot C_e}$$

$$12 \times 10^{-3} = \frac{0.07}{\tan 15 + \cot C_e}$$

$$\tan 15 + \cot C_e = 5.8333$$

$$\cot C_e = 5.5654$$

$$\Rightarrow \quad C_e = 10.186°$$

So, end cutting edge angle, $C_e = 10.186°$

Q.8 (c) Solution:

To reach at desired, place 3 transformation will be done:

1. Translation along x-axis – Trans (r, 0, 0)
2. Rotation of α about z-axis – Rot (z, α)
3. Translation along z-axis – Trans (0, 0, l)

Since these transformation are all relative to universe frame (fixed frame), the total transformation caused by these three transformation is found by pre multiplying by each matrix.

$$^{R}T_P = T_{cyl}(r, \alpha, l) = \text{Trans}(0, 0, l)\,\text{Rot}(z, \alpha)\,\text{Trans}(r, 0, 0)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c\alpha & -s\alpha & 0 & 0 \\ s\alpha & c\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{R}T_P = T_{cyl}(r, \alpha, l) = \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After this transformation, moving frame will not be parallel to fixed frame due to α rotation about z-axis. To restore this orientation of moving frame and to make similar to fixed frame, rotate the (n, o, a) frame about the a-axis, through an angle of ($-\alpha$) and post multiply with the cylindrical co-ordiante matrix $T_{cyl}(r, \alpha, l)$. As a result frame will be at the same location but will be parallel to reference frame again as shown in given figure.

$$T'_{cyl} = T_{cyl}(r, \alpha, l) \times \text{Rot}(a, -\alpha)$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 & rc\alpha \\ s\alpha & c\alpha & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c(-\alpha) & -s(-\alpha) & 0 & 0 \\ s(-\alpha) & c(-\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & rc\alpha \\ 0 & 1 & 0 & rs\alpha \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As we are interested in the position vector, it will be same for both (i.e. before and after restoring to original fixed frame).

So, $l = 7$

$rc\alpha = 3$ ...(a)

and $rs\alpha = 4$ ...(b)

$$\tan\alpha = \frac{4}{3}$$

$\Rightarrow$ $\alpha = 53.1°$

By squaring and adding eq. *a* and equation *b*, we get

$$r = 5$$

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