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India's Best Institute for IES, GATE & PSUs

Detailed Solutions

ESE-2019 Mains Test Series

Mechanical Engineering Test No : 10

Section A

Q.1 (a) Solution:

Using steady flow energy equation across diffuser,

$$h_i + \frac{V_i^2}{2} + q = h_e + \frac{V_e^2}{2} + w \qquad (\because q = 0, w = 0) \qquad \text{... (i)}$$

Assume, air as an ideal gas,

From equation (i),

$$h_e - h_i = \frac{V_i^2 - V_e^2}{2}$$

$$c_P(T_e - T_i) = \frac{150^2 - 10^2}{2}$$

$$T_e = T_i + \frac{150^2 - 10^2}{2c_P} = 30 + \frac{150^2 - 10^2}{2 \times 1000} = 41.2°\text{C}$$

To estimate pressure,

$$\dot{m}_i = \dot{m}_e$$

$$\rho_i A_i V_i = \rho_e A_e V_e \quad \text{... (ii)}$$

$$\rho_e = \frac{\rho_i A_i V_i}{A_e V_e}$$

$$\text{Density at inlet, } \rho_i = \frac{P}{RT} = \frac{1.2\times10^5}{300\times303} = 1.32 \text{ kg/ m}^3$$

Now, $$\rho_e = \frac{1.32\times50\times150}{600\times10}$$

$$\rho_e = 1.65 \text{ kg/m}^3$$

Now, $$P_e = \rho_e R T_e$$

$$= 1.65 \times 300 \times (41.2 + 273)$$

$$= 155529 \text{ Pa}$$

$$P_e = 1.55 \text{ bar}$$

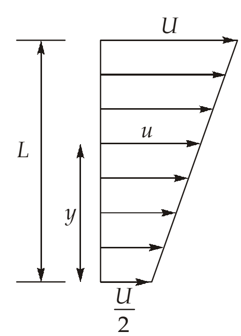
Q.1 (b) Solution:

As per given information.

From the steady flow continuity equation

$$\int_2 \rho u dA - \int_1 \rho u dA = 0 \quad \text{... (i)}$$

At section (2)



$$u = \frac{U}{2} + \frac{U}{2}\frac{y}{L}$$

$$u = \frac{U}{2}\left(1+\frac{y}{L}\right)$$

at section (1)

From equation (i)

$$2\int_0^L \rho \cdot \frac{U}{2}\left(1+\frac{y}{L}\right) b \cdot dy - \rho \cdot U \cdot 2 \cdot H \cdot b = 0$$

$$2\cdot\rho\cdot U\cdot h\cdot b = 2\cdot\rho\cdot b\cdot\frac{U}{2}\left[y+\frac{y^2}{2L}\right]_0^L$$

$$\rho\cdot U\cdot h\cdot b = \rho\cdot b\cdot\frac{U}{2}\left[L+\frac{L}{2}\right]$$

$$4\cdot H\cdot b = 3\cdot L\cdot b$$

$$H = \frac{3L}{4}$$

Now the linear momentum relation is used. Note that the drag force F is to the right (Force of the fluid on the body). Thus the force F of the body on fluid is to the left.

$$\Sigma F_x = 0$$

$$\int_2 u \cdot \rho u dA - \int_1 u \cdot \rho u dA + F_{\text{drag}} = 0$$

$$2\int_0^L \frac{U}{2}\left(1+\frac{y}{L}\right)\rho \cdot \frac{U}{2}\left(1+\frac{y}{L}\right) b \cdot dy - 2 \cdot H \cdot \rho \cdot U^2 \cdot b = -F_{\text{drag}}$$

$$-F_{\text{drag}} = 2\int_0^L b \cdot \rho \cdot \frac{U^2}{4}\left(1+\frac{y}{L}\right)^2 dy - 2 \cdot H \cdot \rho \cdot U^2 \cdot b$$

$$= 2 \cdot b \cdot \rho \cdot \frac{U^2}{4} \cdot L \left.\frac{\left(1+\frac{y}{L}\right)^3}{3}\right|_0^L - 2 \cdot H \cdot \rho \cdot U^2 \cdot b$$

$$= L \cdot 2 \cdot b \cdot \frac{U^2}{4}\left[-\frac{1}{3}+\frac{8}{3}\right] - 2 \cdot H \cdot \rho \cdot U^2 \cdot b$$

$$= \frac{7}{12} \cdot 2 \cdot L \cdot \rho \cdot U^2 \cdot b - 2 \cdot H \cdot \rho \cdot U^2 \cdot b$$

$$= \frac{14}{12} \cdot L \cdot \rho \cdot U^2 \cdot b - 2 \times \frac{3L}{4} \rho \cdot U^2 \cdot b \qquad \left[H = \frac{3L}{4}\right]$$

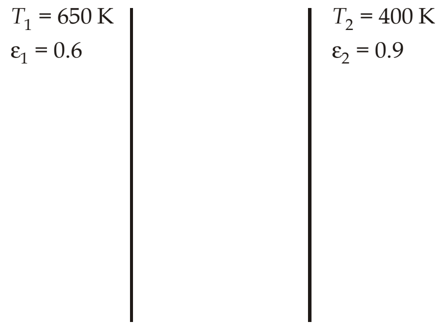
$$-F_{\text{drag}} = -\frac{1}{3} \cdot \rho \cdot U^2 \cdot L \cdot b$$

$$F_{\text{drag}} = \frac{1}{3}\rho U^2 L b$$

$$C_D = \frac{1}{3}$$

Q.1 (c) Solution:

Without shield,



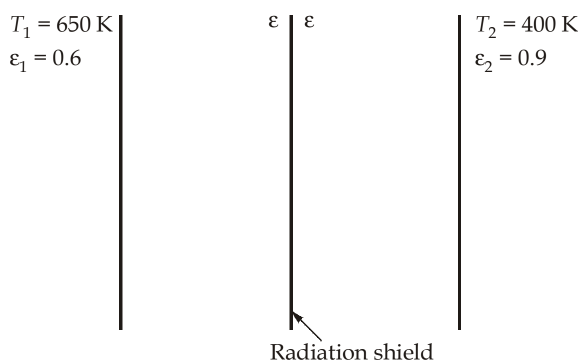
Net rate of radiation heat transfer between two large parallel plates per unit area,

$$\dot{q}_{12,\text{ no shield}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{5.67 \times 10^{-8} \times \left(650^4 - 400^4\right)}{\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1}$$

$$= 4876.754 \text{ W/m}^2$$

With Shield,

Let the emissivity of radiation shield be ε



Net rate of radiation heat transfer between two large parallel plates per unit area,

$$\dot{q}_{12,\text{ with shield}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{2}{\varepsilon} - 1\right)} = \frac{5.67 \times 10^{-8} \times \left(650^4 - 400^4\right)}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.9} - 1\right) + \left(\dfrac{2}{\varepsilon} - 1\right)}$$

$$= \frac{8669.784}{0.778 + \dfrac{2}{\varepsilon}}$$

As per the condition,

$$\dot{q}_{12,\text{ with shield}} = 0.15 \times \dot{q}_{12,\text{ no shield}}$$

$$\frac{8669.784}{0.778+\frac{2}{\varepsilon}} = 0.15 \times 4876.754$$

$$\Rightarrow \qquad 0.778+\frac{2}{\varepsilon} = 11.8518$$

$$\frac{2}{\varepsilon} = 11.8518 - 0.778$$

$$= 11.0738$$

$$\Rightarrow \qquad \varepsilon = 0.1806$$

Q.1 (d) Solution:

Comparison of Air and Water cooling systems: The following is a brief comparison of air and water cooling systems.

Advantages of Air cooling:

1. The direct transfer of heat from engine to air eliminates the use of water as a coolant. No water jacket, radiator and water pump are required. This may mean a reduction in weight by as much as 20%. The size of the engine is also small.
2. The engine design becomes much simpler.
3. The air-cooled engine is less sensitive to climatic conditions. No anti-freeze solution is needed. Due to greater temperature difference between the cooling air and cylinder, the cooling in hot weather does not deteriorate.
4. Due to high average cylinder temperature in the air-cooled engine, thermal losses are small. This results in lower specific fuel consumption.
5. The warm-up performance of the air-cooled engine is better. This results in low wear to cylinders.
6. Since the temperature difference between cooling air and cylinder is more, less amount of cooling air is required.
7. Higher mean cylinder temperature means reduced carbon deposits on combustion chamber wall. This gives sustained engine performance.
8. An air-cooled engine can take up some degree of damage. A broken fin does not affect much while a hole in the radiator may stop a water-cooled engine.
9. The control of cooling system is much easier than in water-cooled engines.

Disadvantages of Air cooling

1. Due to the absence of the water passage the combustion noise is not attenuated. Rather, the air fan is an additional source of noise.

2. The volumetric efficiency of an air-cooled engine is lower due to higher cylinder head temperatures.
3. High specific output engines cannot be air-cooled due to the complex nature of the fins that are required.

Advantages of Water-cooled engines

1. High specific output engines pose no problem with water cooling. The heat transfer coefficient of water is about 350 times that of air. This results in compact design.
2. Due to the high latent heat of water, the water-cooling system allows greater amount of heat from any local hot spot. This acts as a useful safety valve for overheating troubles.
3. The water-cooled engine can be installed anywhere in the vehicle.
4. The volumetric efficiency of water-cooled is higher than that of air-cooled engines.

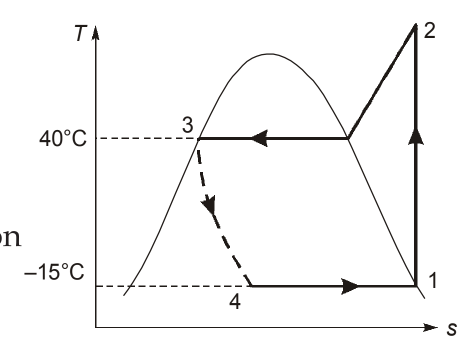
Disadvantages of Water-cooled engines

1. The need for a radiator and a pump increases the weight and the dimensions of the engine. Due to the presence of radiator the frontal area of the vehicle is increased resulting in greater air resistance.
2. Water-cooling system required more maintenance. A slight leakage of the radiator may result in breakdown of the engine.
3. The engine performance becomes more sensitive to climatic conditions. Cold weather starting requires use of anit-freeze solutions which may, sometimes, result in deposits on the water side of the cylinder and in reduced heat transfer.
4. The warm-up performance is poor. This results in greater cylinder wear.
5. The power absorbed by the pump is slightly higher than that necessary for air-cooled engines.

Q.1 (e) Solution:

Given: Ice produced, m = 10 tonnes/day, COP = 5

Consider ammonia samples saturation cycle as represented in T-s diagram here.



Cycle operates between –15°C and 40°C.

Amount of heat removed to form ice in given condition

$$= m[c_{p,w}(T_f - 273) + h_{fusion} + c_{p,ice}(T_f - 273)]$$
$$= 10 \times 10^3 [4.187 \times 30 + 335 + 2 \times 5]$$
$$= 470.61 \times 10^4 \text{ kJ/day} = 4.7061 \times 10^6 \text{ kJ/day}$$

$$\text{Refrigeration capacity of plant} = \frac{4.7061 \times 10^6 \text{ kJ}}{24 \times 3600 \text{ s}} = 54.47 \text{ kW} = \frac{54.47}{3.5167} \text{ TR}$$
$$= 15.5 \text{ TR}$$

We have, $h_1 = 1675$ kJ/kg

$$h_4 = h_3 = h_{f_{40°C}} = 600 \text{ kJ/kg}$$

$\therefore$ Mass flow rate of refrigerant,

$$\dot{m} = \frac{Q_0}{h_1 - h_4} = \frac{54.47}{1675 - 600} = 0.05067 \text{ kg/s}$$

$$\text{COP of system} = \frac{Q_0}{W} = \frac{Q_0}{(h_2 - h_1)\dot{m}} = 5$$

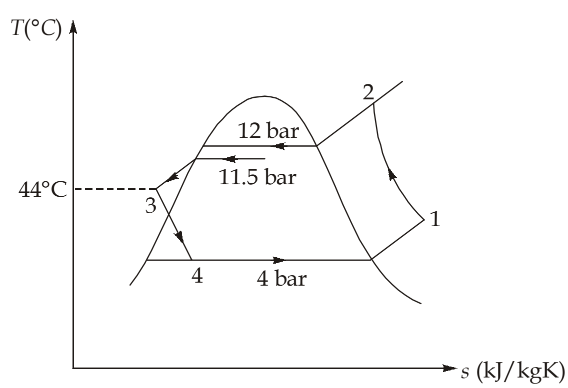
$$\Rightarrow \quad h_2 - h_1 = \frac{Q_0}{5\dot{m}} = \frac{54.47}{5 \times 0.05067}$$

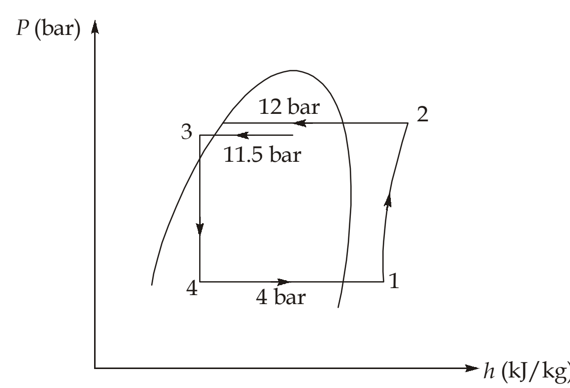
$$\Rightarrow \quad h_2 - 1675 = 215$$

$$\Rightarrow \quad h_2 = 1890 \text{ kJ/kg}$$

Hence, isentropic discharge enthalpy is 1890 kJ/kg.

Q.2 (a) Solution:





For power input,

$$w_c = \frac{n}{n-1}(P_1 v_1 - P_2 v_2) \quad \text{... (i)}$$

Now, for v_2

$$\frac{v_2}{v_1} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$v_2 = \left(\frac{4}{12}\right)^{1/1.01} \times 0.045 = 0.01516 \text{ m}^3/\text{kg}$$

Now from equation (i),

$$w_C = \frac{1.01}{1.01-1}(4 \times 100 \times 0.045 - 12 \times 100 \times 0.01516)$$

$$= -19.392 \text{ kJ/kg}$$

$$w_C = 19.392 \text{ kJ/kg (–ve sign indicates work input)}$$

Now, Refrigeration capacity $= \dot{m} \times RE$

$$10 \times 3.5 = \dot{m}(h_1 - h_4)$$

$$\dot{m} = \frac{10 \times 3.5}{359 - 241} = 0.2966 \text{ kg/ s}$$

$$\dot{m} = 17.796 \text{ kg/min}$$

Now, Power for compressor, $P = w_C \times \dot{m} = 19.392 \times 0.2966$

$$= 5.7516 \text{ kW}$$

Heat transfer rate for compressor,

Steady flow energy equation,

$$h_1 + q = h_2 + w \text{ (Neglecting KE and PE)}$$

$$359 + q = 378 - 19.392$$

$$q = -0.392 \text{ kJ/kg}$$

$$q = 0.392 \text{ kJ/kg} \quad \text{(Heat rejected)}$$

Or $Q = \dot{m} \times q$

$$Q_{rej} = 0.2966 \times 0.392$$

$\Rightarrow$ $Q_{rej} = 0.1163 \text{ kW}$

2. Coefficient of performance,

$$\text{COP} = \frac{RC}{W_{\text{comp}}} = \frac{10 \times 3.5}{5.7516} = 6.085$$

3. Irreversibility rate of condenser,

$$I = T_o\left[\dot{m}(s_3 - s_2) + \dot{m}_{cw}\left(s_{cw,out} - s_{cw,in}\right)\right] \quad \text{... (ii)}$$

Now,
$$\dot{m}_{cw}\left(h_{cw,out} - h_{cw,in}\right) = \dot{m}(h_2 - h_3)$$

$$\dot{m}_{cw}(125.66 - 83.86) = 0.2966\,(378 - 241)$$

$$\dot{m}_{cw} = 0.9721 \text{ kg/s}$$

Now, from equation (ii),

$$I = 293[0.2966(1.1444 - 1.5614) + 0.9721(0.4365 - 0.2963)]$$
$$= 3.693 \text{ kW}$$

Q.2 (b) Solution:

As per specified conditions:

$$\frac{\partial P}{\partial x} = 0, \qquad g_x = 0$$

$u \neq 0$, $v = 0$, $w = 0$

The Navier's stokes equation for the upper layer or lower layer can be written as in x-direction is:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad \text{... (i)}$$

From continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Since $v = 0$ and $w = 0$, therefore

$$\frac{\partial u}{\partial x} = 0, \; \frac{d^2 u}{dx^2} = 0$$

and in z-direction no motion. So,

$$\frac{\partial^2 u}{\partial z^2} = 0$$

So, equation (i) reduced to,

$$\frac{\partial^2 u}{\partial y^2} = 0$$

From integration,
$$\frac{\partial u}{\partial y} = A$$

Again integration

$$u = Ay + B \qquad \text{... (ii)}$$

Equation (ii) is the general equation in which A and B are the general constant because fluids are different.

Boundary condition for upper layer,

$$u = U \text{ at } y = 2h$$

Notation 1 refer for the upper layer, so

$$U = A_1 \times 2h + B_1$$

$$B_1 = U - 2hA_1$$

Boundary condition for lower layer,

$$y = 0, \qquad u = 0$$

From equation (ii) for layer lower,

$$u = A_2 y + B_2$$

$$\Rightarrow \quad 0 = A_2 \times 0 + B_2$$

$$\Rightarrow \quad B_2 = 0$$

For upper layer,

$$u_1 = A_1 y + B_1$$

$$= A_1 y + U - 2hA_1$$

$$u_1 = A_1(y - 2h) + U$$

For lower layer,

$$u_2 = A_2 y + B_2$$

$$u_2 = A_2 h$$

At interface, $y = h$ and $u_1 = u_2$

$$A_1(y - 2h) + U = A_2 h$$

$$A_2 = A_1\frac{y}{h} - \frac{A_1 \times 2h}{h} + \frac{U}{h} = -A_1 + \frac{U}{h} \quad \text{... (iii)}$$

Since, the velocity distribution is linear in each layer the shearing stress is,

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\frac{\partial u}{\partial y}$$

which is constant throughout each layer,

For the upper layer, $u_1 = A_1 y + B_1$

$$\Rightarrow \quad \frac{\partial u_1}{\partial y} = A_1$$

$$\tau_1 = \mu\frac{\partial u_1}{\partial y} \Rightarrow \tau_1 = \mu_1 A_1$$

For the lower layer, $u_2 = A_2 y$

$$\frac{\partial u_2}{\partial y} = A_2$$

$$\tau_2 = \mu_2\frac{\partial u_2}{\partial y} = \mu_2 A_2$$

At the interface, $\tau_1 = \tau_2$

$$\mu_1 A_1 = \mu_2 A_2$$

$$\frac{A_1}{A_2} = \frac{\mu_2}{\mu_1} \quad \text{... (iv)}$$

From equation (iii) + (iv)

$$A_2 = -A_1 + \frac{U}{h} = -\frac{\mu_2}{\mu_1}\cdot A_2 + \frac{U}{h}$$

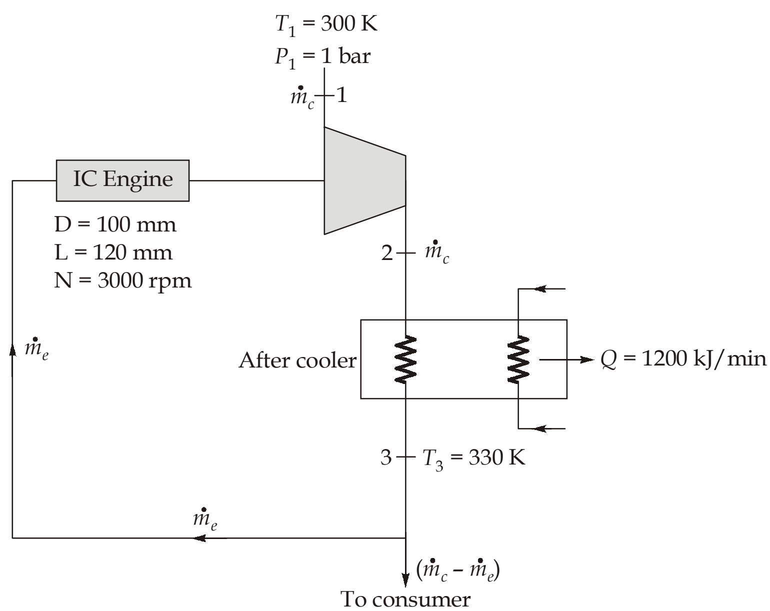
$$A_2 = \frac{U/h}{\left(1 + \frac{\mu_2}{\mu_1}\right)}$$

Thus velocity at the interface is

$$u_2\big|_{(y=h)} = A_2 h$$

$$u_2 = \frac{U \times h}{h\left(1 + \frac{\mu_2}{\mu_1}\right)} = \frac{U}{\left(1 + \frac{\mu_2}{\mu_1}\right)}$$

Q.2 (c) Solution:



(i) imep of supercharged engine

$$\text{BP} = \frac{2\pi NT}{60} = \frac{2\pi \times 3000 \times 140}{60} = 43.982 \text{ kW}$$

$$\text{IP} = \frac{BP}{\eta_{\text{mech}}} = \frac{43.982}{0.85} = \frac{P_m LAN}{2 \times 60} \times k$$

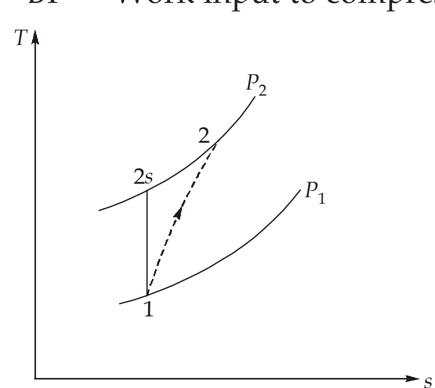
where k = 4 four cylinders.

$$P_m = \frac{43.982}{0.85} \times \frac{60 \times 2}{LAN\, k} = \frac{43.982 \times 120}{0.85 \times \frac{\pi}{4} \times (0.1)^2 \times 0.12 \times 3000 \times 4}$$

$$= 549 \text{ kPa} = 5.49 \text{ bar}$$

Since entire output of engine is used to drive the compressor.

$$\text{BP} = \text{Work input to compressor}$$



$$43.982 = \dot{m}_c c_P (T_2 - T_1)$$

$$\dot{m}_c = \frac{43.982}{c_P (T_2 - 300)} \quad \text{... (i)}$$

Engery balance across after-cooler (Neglecting KE, PE changes)

$$\dot{m}_c c_P (T_2 - T_3) = \frac{1200}{60}$$

$$\dot{m}_c = \frac{20}{c_P (T_2 - 330)} \quad \text{... (ii)}$$

From (i) and (ii)

$$\frac{43.982}{c_P (T_2 - 300)} = \frac{20}{c_P (T_2 - 330)}$$

or $$43.982(T_2 - 330) = 20(T_2 - 300)$$

or $$T_2 = 355.02 \text{ K}$$

From equation (i), $$\dot{m}_c = \frac{43.982}{1.005 \times (355.02 - 300)} = 0.7954 \text{ kg/ s}$$

or, $$\dot{m}_c = 2863.44 \text{ kg/h}$$

Isentropic efficiency of compressor,

$$\eta_{isen} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.85 = \frac{T_{2s} - 300}{355.02 - 300}$$

$\Rightarrow$ $$T_{2s} = 346.767 \text{ K}$$

Now, $$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma - 1/\gamma}$$

or, $$\frac{P_2}{P_1} = \left(\frac{346.767}{300}\right)^{1.4/0.4}$$

$\Rightarrow$ $$P_2 = 1.66038 \text{ bar}$$

Therefore, we get the intake condition of engine, i.e.

$$T = 330 \text{ K}$$

$$P = 1.66038 \text{ bar}$$

(b) The rate of air consumed by engine,

Volumetric efficiency of engine,

$$\eta_{vol} = \frac{\dot{V}_{act}}{\dot{V}_{swept}} = \frac{\dot{V}_{act}}{\frac{\pi}{4}D^2L\frac{Nk}{60\times 2}} \qquad (\because \text{ 4-stroke engine})$$

$$\dot{V}_{act} = \eta_{vol}\times\frac{\pi}{4}D^2L\frac{Nk}{120}$$

$$= 0.9\times\frac{\pi}{4}\times(0.1)^2\times 0.12\frac{3000\times 4}{120} = 0.084823\ \text{m}^3/s$$

Assuming air as ideal gas, $P\dot{V}_{act} = \dot{m}_e RT$

$$\dot{m}_e = \frac{P\dot{V}_{act}}{RT} = \frac{1.66038\times 100\times 0.084823}{0.287\times 330}$$

$$= 0.148705\ \text{kg/s}$$

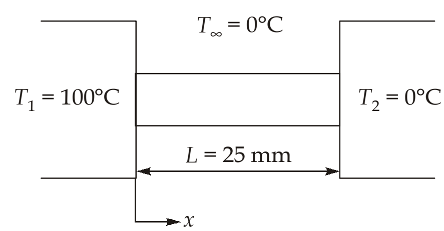
$$\dot{m}_e = 535.338\ \text{kg/h}$$

(c) The rate of air flow available to the consumer.

$$\dot{m}_{\text{consumer}} = \dot{m}_c - \dot{m}_e$$

$$= 2863.44 - 535.338 = 2328.102\ \text{kg/h}$$

Q.3 (a) Solution:



Thermal conductivity, k = 400 W/mK

Convection coefficient, h = 100 W/m²K

Temperature distribution is given by

$$\theta = C_1e^{mx} + C_2e^{-mx}$$

where,
$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{h\times\pi D}{k\times\frac{\pi}{4}D^2}} = \sqrt{\frac{4h}{kD}}$$

$$m = \sqrt{\frac{4\times100}{400\times\frac{1}{1000}}} = 31.6228 \text{ m}^{-1}$$

At $x = 0$, $\theta = \theta_1 = T_1 - T_\infty = 100 - 0 = 100°\text{C}$

$$100 = C_1 e^{(31.6228 \times 0)} + C_2 e^{-(31.3228 \times 0)}$$

$$\Rightarrow \quad 100 = C_1 + C_2 \qquad ...(1)$$

At $x = L = 0.025$ m, $\theta = \theta_2 = T_2 - T_\infty = 0 - 0 = 0°\text{C}$

$$0 = C_1 e^{(31.6228 \times 0.025)} + C_2 e^{-(31.6228 \times 0.025)}$$

$$\Rightarrow \quad 0 = 2.20465 C_1 + 0.45358 C_2 \qquad ...(2)$$

Solving (1) and (2)

$$C_1 = -25.903, \text{ and}$$

$$C_2 = 125.903$$

So,

$$\theta = -25.903 e^{31.6228x} + 125.903 e^{-31.6228x}$$

Differentiating this eq. w.r.t. x,

$$\frac{d\theta}{dx} = -(25.903 \times 31.6228) e^{31.6228x} - (125.903 \times 31.6228) e^{-31.6228x}$$

$$= -819.125 e^{31.6228x} - 3981.405 e^{-31.6228x}$$

Conduction heat transfer rate at $x = 0$,

$$q_{\text{cond},i} = -kA \left.\frac{d\theta}{dx}\right|_{x=0} \qquad \left[\because \frac{dT}{dx} = \frac{d\theta}{dx}\right]$$

$$= -400\times\frac{\pi}{4}\times0.001^2\times\left[-819.125 e^{(31.6228\times0)} - 3981.405 e^{-(31.6228\times0)}\right]$$

$$q_{\text{cond},i} = 1.5081 \text{ W}$$

Conduction heat transfer rate at $x = 0.025$ m,

$$q_{\text{cond,o}} = -kA \left.\frac{d\theta}{dx}\right|_{x=0.025\text{ m}}$$

$$= -400\times\frac{\pi}{4}\times0.001^2\times\left[-819.125 e^{(31.6228\times0.025)} - 3981.405 e^{-(31.6228\times0.025)}\right]$$

$$q_{\text{cond,o}} = 1.1347 \text{ W}$$

Rate of heat transfer by convection from single rod,

$$q_{conv} = q_{cond,\,i} - q_{cond,\,o}$$
$$= 1.5081 - 1.1347$$
$$q_{conv} = 0.3734 \text{ W}$$

(b) Total number of rods, $N = \dfrac{100 \times 100}{4 \times 4} = 625$

So, Area of unfined section, $A_{uf} = (100 \times 100) - 625 \times \left(\dfrac{\pi}{4} \times 1^2\right)$

$$A_{uf} = 9509.126 \text{ mm}^2$$

Total rate of heat transfer from surface at 100°C,

$$q = N\, q_{cond,\,i} + hA_{uf}(T_1 - T_\infty)$$
$$= 625 \times 1.5081 + 100 \times 9509.126 \times 10^{-6} \times (100 - 0)$$
$$q = 1037.654 \text{W}$$

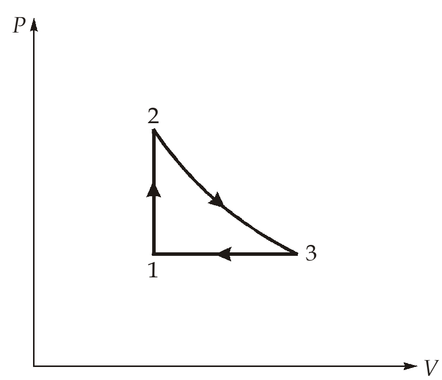
Q.3 (b) Solution:

Process: 1-2 $Q_{1\text{-}2} = (\Delta U)_{1\text{-}2}$

$$= m(u_2 - u_1)$$
$$Q_{1\text{-}2} = m\{h_2 - p_2 v_2 - (h_1 - p_1 v_1)\} \quad \text{... (i)}$$

Now, $h_1 = h_f + x h_{fg}$

$$= 762.51 + 0.4 \times (2014.6)$$
$$= 1568.35 \text{ kJ/kg}$$



and $v_1 = v_f + x v_{fg} = 0.001127 + 0.4(0.19436 - 0.001127)$

$$= 0.07842 \text{ m}^3/\text{kg}$$

$Q_{1\text{-}2} = 2(3129.364 - 35 \times 100 \times 0.07842 - 1568.35 + 10 \times 100 \times 0.07842)$ $\{\because v_1 = v_2\}$

$$= 2(1364.964)$$

$$Q_{1\text{-}2} = 2729.928 \text{ kJ}$$

Process: 2-3

$$Q_{2\text{-}3} = mT(s_3 - s_2)$$
$$= 2 \times 633.7(7.3362 - 6.6970)$$
$$= 2 \times 405.061$$
$$Q_{2\text{-}3} = 810.122 \text{ kJ}$$

Process: 3-1

$$Q_{3\text{-}1} = m(u_1 - u_3) + W_{3\text{-}1}$$
$$= 2[h_1 - p_1v_1 - (h_3 - p_3v_3)] + p(V_1 - V_3)$$

$$Q_{3\text{-}1} = 2\left[1568.35 - 10 \times 100 \times 0.07842 - 3180.5 + \frac{10 \times 100 \times 0.5754}{2}\right]$$
$$+10 \times 100 \times (0.07842 \times 2 - 0.5754)$$

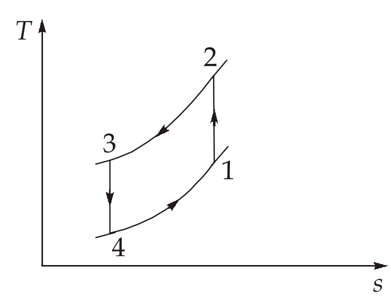
$$= -3224.3 \text{ kJ} \qquad \text{(–ve sign shows heat rejection)}$$

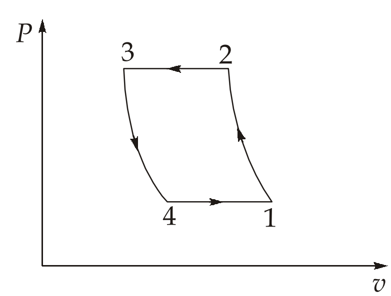
Thermal efficiency of cycle, $\eta = 1 - \dfrac{Q_{\text{rej}}}{Q_{\text{add}}} = 1 - \dfrac{3224.3}{2729.928 + 810.122}$

$$= 1 - \frac{3224.3}{3540.05} = 8.92\%$$

$$\eta = 8.92\%$$

Q.3 (c) Solution:





Now, COP of cycle $= \dfrac{RE}{W_{net}} = \dfrac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \dfrac{(T_1 - T_4)}{(T_2 - T_1) - (T_3 - T_4)}$

$$= \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{1}{\left(\frac{T_2 - T_3}{T_1 - T_4}\right) - 1} \quad \text{...(i)}$$

Now, from *T*-*s* diagram

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Assume pressure ratio, $\frac{P_2}{P_1} = r_p$

$$\frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{...(ii)}$$

and $$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{P_3}{P_4} = \frac{P_2}{P_1} = r_p$$

$$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} \quad \text{...(iii)}$$

From equations (ii) and (iii), we get

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\frac{T_2}{T_3} = \frac{T_1}{T_4}$$

$$\frac{T_2}{T_3} - 1 = \frac{T_1}{T_4} - 1$$

$$\frac{(T_2 - T_3)}{T_3} = \frac{(T_1 - T_4)}{T_4}$$

$\Rightarrow$ $$\frac{T_2 - T_3}{T_1 - T_4} = \frac{T_3}{T_4}$$

From equation (i),

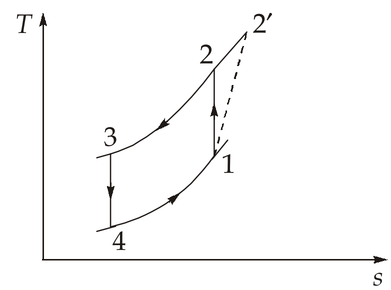
$$\text{COP} = \frac{1}{\frac{T_3}{T_4} - 1}$$

From equation (iii),

$$\text{COP} = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}$$

Numerical:

$$T_1 = 0°\text{C} = 0 + 273 = 273 \text{ K}$$
$$T_3 = 27°\text{C} = 27 + 273 = 300 \text{ K}$$



Now,
$$\frac{T_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 273 \times (3.5)^{\frac{1.4-1}{1.4}} = 390.49 \text{ K}$$

$$T_4 = T_3\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{1}{3.5}\right)^{\frac{0.4}{1.4}} = 209.74 \text{ K}$$

Now,
$$(\text{COP})_{\text{ideal}} = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{(3.5)^{\frac{0.4}{1.4}} - 1} = 2.324$$

Now,
$$(\text{COP})_{\text{actual}} = 0.9 \times (\text{COP})_{\text{ideal}}$$
$$= 0.9 \times 2.324 = 2.0916$$

$$(\text{COP})_{\text{actual}} = \frac{T_1 - T_4}{(T_2' - T_1) - (T_3 - T_4)}$$

$$2.0916 = \frac{273 - 209.74}{(T_2' - 273) - (300 - 209.74)}$$

$$T_2' = 393.504 \text{ K}$$

Now, isentropic efficiency of compressor,

$$\eta_{comp} = \frac{T_2 - T_1}{T_2' - T_1} = \frac{390.49 - 273}{393.504 - 273} = 0.9749$$

$$\eta_{comp} = 97.49\%$$

Q.4 (a) Solution:

Selection of refrigerant for a particular application is based on the following requirements:

1. Thermodynamic and thermo-physical properties.
2. Environmental and safety properties, and
3. Economics.

1. Thermodynamic and thermo-physical properties:

(a) **Suction pressure:** At a given evaporator temperature, the saturation pressure should be above atmospheric for prevention of air or moisture ingress into the system and ease of leak detection. Higher suction pressure is better as it better as it leads to smaller compressor displacement.

(b) **Discharge pressure:** At a given condenser temperature, the discharge pressure should be as small as possible to allow light-weight construction of compressor, condenser etc.

(c) **Pressure ratio:** It should be as small as possible for high volumetric efficiency and low power consumption.

(d) **Latent heat of vaporization:** It should be as large as possible so that the required mass flow rate per unit cooling capacity will be small.

(e) **Isentropic index of compression:** It should be be as small as possible so that the temperature rise during compression will be small.

(f) **Liquid specific heat:** It should be small so that degree of subcooling will be large leading to smaller amount of flash gas at evaporator inlet.

(g) **Vapour specific heat:** It should be large so that the degree of superheating will be small.

(h) **Thermal conductivity:** Thermal conductivity in both liquid as well as vapour phase should be high for higher transfer coefficients.

(i) **Viscosity:** Viscosity should be small in both liquid and vapour phases for smaller frictional pressure drops.

2. Environmental and safety properties: The important environmental and safety properties are:

(a) **Ozone Depletion Potential (ODP):** The ODP of refrigerants should be zero, i.e., they should be non-ozone depleting substances. Refrigerants having non-zero ODP have either already been phased-out (e.g. R-11, R-12) or will be phased-out in near-future (e.g. R-22). Since ODP depends mainly on the presence of chlorine or bromine in the molecules, refrigerants having either chlorine (i.e. CFCs and HCFCs) or bromine cannot be used under the new regulations.

(b) **Global Warming Potential (GWP):** Refrigerants should have as low a GWP value as possible to minimize the problem of global warming. Refrigerants with zero ODP but a high value of GWP (e.g. R-134a) are likely to be regulated in future.

(c) **Total Equivalent Warming Index (TEWI):** The factor TEWI considers both direct (due to release into atmosphere) and indirect (through energy consumption) contributions of refrigerants to global warming. Naturally, refrigerants with low value of TEWI are preferable from global warming point of view.

(d) **Toxicity:** Ideally, refrigerants used in a refrigeration system should be non-toxic. However all fluids other than air can be called as toxic as they will cause suffocation when their concentration is large enough. Some refrigerants such as CFCsand HCFCs are non-toxic when mixed with air in normal condition. However, when they come in contact with an open flame or an electrical heating element, they decompose forming highly toxic elements (e.g. phosgene-$COCl_2$). In general the degree of hazard depends on:
- Amount of refrigerant used verses total space
- Type of occupancy
- Presence of open flames
- Odour of refrigerant, and
- Maintenance condition

(e) **Flammability:** The refrigerants should preferably be non-flammable and non-explosive. For flammable refrigerants special precautions should be taken to avoid accidents.

(f) **Chemical stability:** The refrigerants should be chemically stable as long as they are inside the refrigeration system.

(g) **Miscibility with lubricating oils:** Oil separators have to be used if the refrigerant is not miscible with lubricating oil (e.g. ammonia). Refrigerants that are completely miscible with oils are easier to handle (e.g. R-12). However, for refrigerants with limited solubility (e.g. R-22) special precautions should be taken while designing the system to ensure oil return to the compressor.

(h) **Ease of leak detection:** In the event of leakage of refrigerant from the system, it should be easy to detect the leaks.

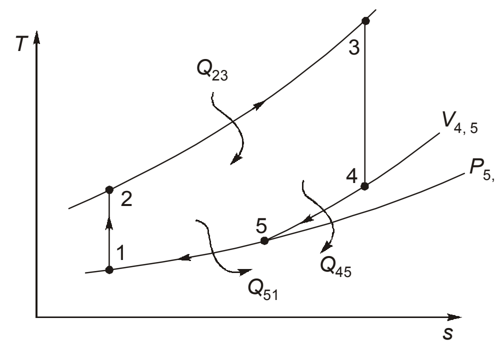
3. Economic properties: The refrigerant used should preferably be inexpensive and easily available.

Q.4 (b) Solution:

1 - 2: Isentropic compression 2 - 3: Constant volume heat input

3 - 4: Isentropic expansion4 - 5: Constant volume heat rejection

5 - 1: Isobaric heat rejection



Assumptions:

All the processes are internally reversible.

The working medium specific heat remains constant

Throughout the cycle.

Given, Compression ratio $= r_c$

Expansion ratio $= r_e$

Let, Clearance volume $= V_c$

$$\text{Efficiency of the cycle, } \eta = 1 - \frac{Q_R}{Q_S} = 1 - \left[\frac{Q_{45} + Q_{51}}{Q_{23}}\right]$$

We know that,

$$Q_{23} = mc_V (T_3 - T_2)$$

$$Q_{45} = mc_V (T_4 - T_5)$$

$$Q_{51} = mc_P (T_5 - T_1)$$

For process 1 - 2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r_c)^{\gamma-1}$$

$$T_2 = T_1(r_c)^{\gamma - 1}$$

For process 2 - 3:

Given,

$$T_3 - T_2 = \theta T_1$$

$$T_3 = T_2 + \theta T_1$$

$$= T_1(r_c)^{\gamma - 1} + \theta T_1$$

$$= T_1(r_c^{\gamma - 1} + \theta)$$

For constant pressure process 5 - 1,

$$\frac{T_5}{T_1} = \left(\frac{V_5}{V_1}\right)$$

$$T_5 = T_1\left[\frac{V_5}{V_2} \times \frac{V_2}{V_1}\right] = T_1\left(\frac{r_e}{r_c}\right)$$

For isentropic process 3 - 4,

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{1}{r_e}\right)^{\gamma-1}$$

$$T_4 = \frac{T_3}{(r_e)^{\gamma-1}} = \frac{T_1\left(r_c^{\gamma-1}+\theta\right)}{(r_e)^{\gamma-1}}$$

Now,

$$\eta = 1-\frac{m\left[c_V(T_4-T_5)+c_P(T_5-T_1)\right]}{mc_V(T_3-T_2)}$$

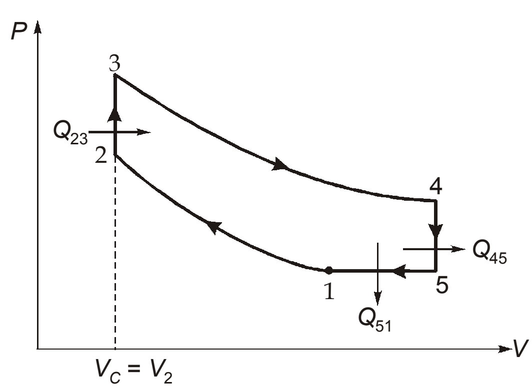
$$\eta = 1-\left[\frac{(T_4-T_5)+\gamma(T_5-T_1)}{(T_3-T_2)}\right] \quad \left\{\because \frac{c_P}{c_V}=\gamma\right\}$$

$$\eta = 1-\frac{\left[T_1\left(\frac{\theta+r_c^{\gamma-1}}{r_e^{\gamma-1}}\right)-T_1\left(\frac{r_e}{r_c}\right)+\gamma\left[T_1\left(\frac{r_e}{r_c}\right)-T_1\right]\right]}{T_1\left(r_c^{\gamma-1}+\theta\right)-\left(T_1 r_c^{\gamma-1}\right)}$$

$$\eta = 1-\frac{\left[\frac{\theta+r_c^{\gamma-1}}{r_e^{\gamma-1}}-\frac{r_e}{r_c}+\gamma\left(\frac{r_e}{r_c}\right)-\gamma\right]}{\theta} = 1-\frac{\left[\left(\frac{\theta+r_c^{\gamma-1}}{r_e^{\gamma-1}}-\gamma\right)+\frac{r_e}{r_c}(\gamma-1)\right]}{\theta}$$

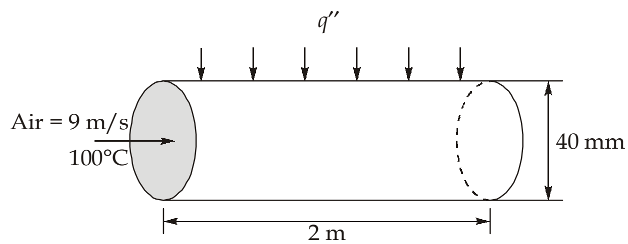
$$= 1-\frac{\left[\theta+r_c^{\gamma-1}-\gamma r_e^{\gamma-1}\right]r_c+r_e^{\gamma}(\gamma-1)}{\theta r_c r_e^{\gamma-1}}$$

$$= 1-\frac{(\theta-\gamma r_e^{\gamma-1})r_c+r_e^{\gamma}(\gamma-1)+r_c^{\gamma}}{\theta r_c r_e^{\gamma-1}}$$



P-V diagram for the given cycle

Q.4 (c) Solution:



At the inlet,

$$\text{Pressure of air, } p_i = 101.325 \text{ kPa}$$

$$\text{Temperature of air, } T_i = (100 + 273) \text{ K} = 373 \text{ K}$$

$$\text{Density of air, } \rho_i = \frac{p_i}{RT_i} = \frac{101.325}{0.287 \times 373} = 0.9465 \text{ kg/m}^3$$

$$\text{Velocity of air, } V_i = 9 \text{ m/s}$$

$$\text{Tube area, } A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.04^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

1. $$\text{Mass flow-rate of air, } \dot{m} = \rho_i A V_i = 0.9465 \times 1.2566 \times 10^{-3} \times 9$$

$$= 0.010704 \text{ kg/s}$$

$$= 0.6423 \text{ kg/min}$$

2. $$\text{Heat transfer rate, } q = 1000 \text{ W}$$

$$= \dot{m}c_P(T_e - T_i)$$

$\Rightarrow$ $$1000 = 0.010704 \times 1.0168 \times 10^3 (T_e - 100)$$

$$\text{Exit temperature of air, } T_e = 191.88°\text{C}$$

3. $$\text{Reynolds number, Re} = \frac{\rho VD}{\mu} = \frac{\dfrac{\rho Q}{\frac{\pi}{4}D^2} \times D}{\mu} = \frac{4\dot{m}}{\pi \mu D}$$

$$= \frac{4 \times 0.0107074}{\pi \times (28.2 \times 10^{-6} \times 0.84) \times 0.04} = 14383.608$$

Hence, the flow is turbulent.

$$\text{Prandtl number, Pr} = \frac{\mu c_P}{k} = \frac{(28.2 \times 10^{-6} \times 0.84) \times 1016.8}{0.035} = 0.688$$

Using Dittus-Boelter equation,

$$\overline{Nu} = 0.023(Re)^{0.8}\, Pr^{0.4} \text{ (as the fluid i.e. air is being heated)}$$

$$= 0.023(14383.608)^{0.8}(0.688)^{0.4}$$

$$= 41.98$$

$$= \frac{\bar{h} \times D}{k}$$

$$\Rightarrow \quad \frac{\bar{h} \times 0.04}{0.035} = 41.98$$

$$\bar{h} = 36.732 \text{ W/m}^2\text{-K}$$

At the outlet,

$$\text{Heat flux, } q'' = \frac{q}{\pi DL} = h(T_w - T_e)$$

$$\frac{1000}{\pi \times 0.04 \times 2} = 36.732 \times (T_w - 191.88)$$

$$\text{Exit wall temperature, } T_e = 300.2°\text{C}$$

Section B

Q.5 (a) Solution:

(i)

The major advantages of solar PV systems over conventional power systems are:

1. It converts solar energy directly into electrical energy without going through the thermal-mechanical link. It has no moving parts.
2. Solar PV systems are reliable, modular, durable and generally maintenance free.
3. These systems are quiet, compatible with almost all environments respond instantaneously to solar radiation and have an expected life span of 20 years or more.
4. A solar PV system can be located at the place of use and hence no or minimum distribution network is required, as it is universally available.

The major disadvantages of solar PV systems over conventional power systems are:

1. Currently the costs of solar cells are high, making them economically uncompetitive with other conventional power sources.
2. The efficiency of solar cell is low. As solar radiation density is also low, a large area of solar cell modules are required to generate sufficient useful power.

3. As solar energy is intermittent, some kind of electrical energy storage is required, which makes the whole system more expensive.

(ii) Power output required = 4 hp = 4 × 735 = 2940 Watt

$$\text{Actual power required, } P_{\text{required}} = \frac{P_{\text{motor}}}{\eta_{\text{motor}}} = \frac{2940}{0.78} = 3769.23 \text{ Watt}$$

Area of one module required, $A_m = 9 \times 4 \times 125 \times 125 \times 10^{-6}$

$= 0.5625 \text{ m}^2$

Assume, number of modules to be 'N'.

Solar radiation incident on panel, $I_g = 1.1 \text{ kW/m}^2 = 1100 \text{ Watt/m}^2$

Conversion efficiency, $\eta_{\text{conv.}} = 13\%$

Power output by a module of solar cell = 1100 × 0.5625 × 0.13

If there are N module, power output = 1100 × 0.5625 × 0.13 × N

Now, Total power required = Power output by N modules

$3769.23 = 1100 \times 0.5625 \times 0.13 \times N$

$$N = \frac{3769.23}{80.4375} = 46.859 \approx 47$$

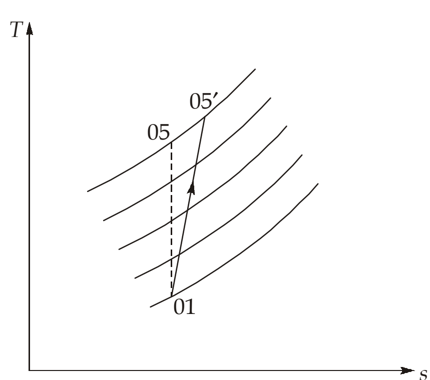
Hence, minimum 47 modules of photovoltaic cell are required.

Q.5 (b) Solution:

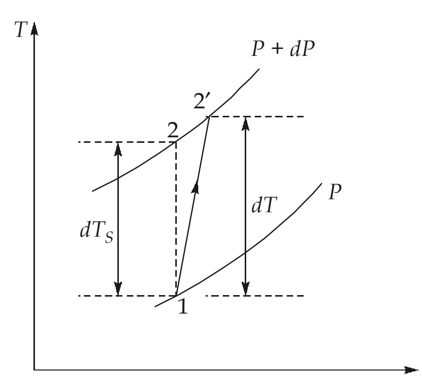
Assuming perfect gas is compressed. Polytropic law for a process,

$$PV^n = C \quad \text{... (i)}$$

Also, for perfect gas, $PV = RT$ ... (ii)



Multi-stage compression



Small stage compression

From (i) and (ii),

$$P^{1-n}\, T^n = C$$

Take log both sides,

$$(1 - n)\log P + n\log T = \log C$$

Differentiating this equation, we get

$$(1-n)\frac{dP}{P} + n\frac{dT}{T} = 0$$

Therefore, $$\frac{dT}{T} = -\frac{1-n}{n}\frac{dP}{P} = \frac{n-1}{n}\frac{dP}{P}$$

Thus the temperature rise in an elemental stage is given by

$$dT = \frac{n-1}{n}\frac{dP}{P}T$$

Had the process been isentropic, temperature rise in the elemental stage would have been

$$dT_S = \frac{\gamma-1}{\gamma}\frac{dP}{P}T$$

Hence, efficiency of small stage, which is defined as the ratio of the isentropic temperature rise to the actual temperature rise, i.e.

$$\eta_{pc} = \frac{dT_S}{dT} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right) \qquad \text{... (iii)}$$

Let r_c be the total head pressure ratio in compression,

$$\text{Isentropic efficiency, } \eta_{is} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

where, $(T_{02s} - T_{01})$ is total head temperature rise if compressed isentropically, and $(T_{02} - T_{01})$ is total head temperature rise if compression is polytropic. We know,

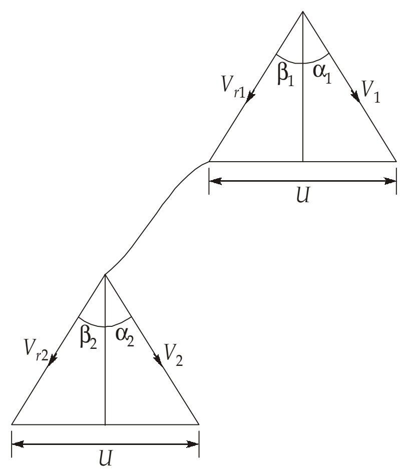
$$\frac{T_{02s}}{T_{01}} = (r_c)^{\gamma-1/\gamma}$$

and $$\frac{T_{02}}{T_{01}} = (r_c)^{n-1/n} = (r_c)^{\frac{\gamma-1}{\gamma}\times\frac{1}{\eta_{pc}}}$$ [$\because$ from equation 3]

$$\therefore \qquad \eta_{is} = \frac{\frac{T_{02s}}{T_{01}} - 1}{\frac{T_{02}}{T_{01}} - 1} = \frac{r_c^{\frac{\gamma-1}{\gamma}} - 1}{r_c^{\frac{\gamma-1}{\gamma}\frac{1}{\eta_{pc}}} - 1} \qquad \text{Hence proved.}$$

Q.5 (c) Solution:

Given: $T_{01} = 300$ K, $P_{01} = 1$ bar, $U = 200$ m/s, $V_f = 160$ m/s, $\Omega = 0.88$



From velocity Δ,

$$U = V_f(\tan\alpha_1 + \tan\beta_1)$$

$$200 = 160(\tan\alpha_1 + \tan\beta_1)$$

$$\tan\alpha_1 + \tan\beta_1 = \frac{200}{160} = 1.25 \qquad \text{... (i)}$$

We know,

Work done by compressor = Rise in enthalpy of air

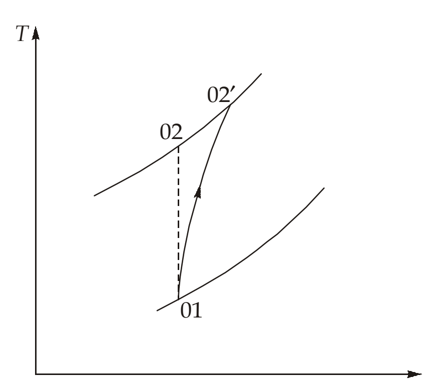
$$\left(\frac{T_{02}}{T_{01}}\right) = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02} = (20)^{2/7} \times 300$$

$$= 706.06 \text{ K}$$

$$\eta_{is} = \frac{T_{02} - T_{01}}{T_{02'} - T_{01}}$$

Actual rise temperature, $T_{02'} - T_{01} = \dfrac{T_{02} - T_{01}}{\eta_{is}}$



Given that equal temperature rise in each stage, so for single stage,

$$\frac{c_P(T_{02}-T_{01})}{(\eta_{is})\times 18} = \Omega\,(V_{w2} - V_{w1})U$$

$$\frac{1.005\times 10^3\times(706.06-300)}{0.87\times 18} = 0.88\times V_f U(\tan\alpha_2 - \tan\alpha_1)$$

$$26059.4 = 0.88\times 160\times 200\,(\tan\alpha_2 - \tan\alpha_1)$$

($\because$ Since 50% reaction stage so $\beta_1 = \alpha_2$)

$$\tan\beta_1 - \tan\alpha_1 = 0.9254 \quad \text{... (ii)}$$

From equation (i) and (ii)

$$\tan\beta_1 = 1.0877 \text{ and } \quad \tan\alpha_1 = 0.1623$$

$$\beta_1 = 47.4° \qquad \alpha_1 = 9.218°$$

and $$\alpha_2 = 47.4° \qquad \beta_2 = 9.218°$$

Q.5 (d) Solution:

Given: $h_1 = 3159.3$ kJ/kg, $s_1 = 6.9917$ kJ/kgK

We know that, $$s_1 = s_{2s}$$

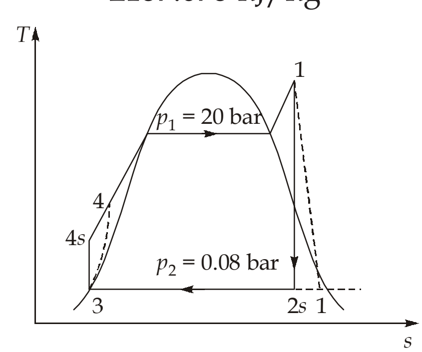
$$6.9917 = 0.5926 + x(7.6361)$$

$$x = \frac{6.9917-0.5926}{7.6361} = 0.838$$

$$h_{2s} = (h_f) + x\times(h_{fg})$$

$$= 173.88 + 0.838\times 2403.1$$

$$= 2187.678 \text{ kJ/kg}$$



(a) Ideal pump work, $$W_p = v(p_1 - p_2)$$

$$= 0.001008(20 - 0.08)\times 10^2 \text{ kJ/kg}$$

$$W_p = 2.008 \text{ kJ/kg}$$

Now, $h_{4s} = h_f + W_P$

$= 173.88 + 2.008 = 175.888$ kJ/kg

Now, Ideal turbine work, $W_T = (h_1 - h_{2s})$

$= 3159.3 - 2187.678$

$= 971.622$ kJ/kg

Net work, $W_{net} = W_T - W_P = 971.622 - 2.008$

$= 969.614$ kJ/kg

$$\text{Efficiency, } \eta_I = \frac{W_{net}}{\text{Heat supplied}} = \frac{969.614}{(h_1 - h_{4s})}$$

$$= \frac{969.614}{(3159.3 - 175.888)} = 0.325 = 32.5\%$$

(b) If $\eta_p = 80\%$, $\eta_T = 80\%$

$$\text{Now, Actual pump work, } (W_P)_a = \frac{2.008}{0.8} = 2.51 \text{ kJ/ kg}$$

Actual turbine work, $(W_T)_a = 0.8 \times (W_T)_{ideal}$

$= 0.8 \times 971.622$

$(W_T)_a = 777.2976$ kJ/kg

Net actual work output $= 777.2976 - 2.51$

$= 774.7876$ kJ/kg

$$\therefore \text{Reduction in work output} = \left(\frac{969.614 - 774.7876}{969.614}\right) \times 100\%$$

$= 20.093\%$

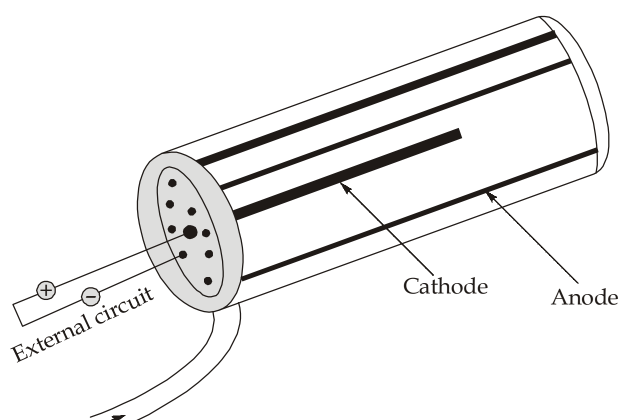
Q.5 (e) Solution:

Microbial fuel cells (MFCs) are devices that use bacteria as the catalysts to oxidize organic and inorganic matter and generate current (1-5 A). Electrons produced by the bacteria from these substrates are transferred to the anode (negative terminal) and flow to the cathode (positive terminal) linked by a conductive material containing a resistor, or operated under a load (i.e., producing electricity that runs a device).

Researchers from the US-based Pennsylvania state university have developed an electricity generator that is fuelled by human waste. The microbial fuel cell (MFC), is useful for countries where large-scale waste-processing plants are needed, but are prohibitively expensive, because of high power requirement. This cost can be offset by producing electricity, while treating waste, the programme is economically viable.

Operating process: Human waste contains undigested food comprising organic matter such as carbohydrates, proteins and lipids. Bacteria uses enzymes to oxidize the organic matter. In this process, electrons are released. Normally the electrons power the respiratory reactions of the bacterial cells, and combine with oxygen molecules. However by depriving the bacteria of oxygen on one side of MFC, the electrons are used to power an external circuit.

An MFC comprises a 15 cm long cylindrical metal container with a central cathode rod that is surrounded by a proton exchange membrane (PEM) as shown in figure. Eight anodes (long, slender graphite rods) are arranged around the cathode. Bacteria clustering around the anodes breakdown the organic waste as it is pumped, releasing electrons and protons. With no oxygen to help mop up the electrons, the bacteria enzymes transfer them to the anodes, while the protons migrate to the central cathode. Molecules on the PEM encourage the protons to pass through the cathode. There, they combine with oxygen from the air and electrons from the anodes to produce water. During the transfer of the electrons from the cathode, a voltage is created, enabling the MFC to power an external circuit. The equipment is designed for a laboratory test. It will be put in commercial uses and the system may produce about 51 kW of power from the waste of 100000 people.



Q.6 (a)(i) Solution:

The mean effective pressure is given by,

$$P_m = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

$$= \frac{\left(\frac{n}{n-1}\right)P_1V_1\left[\left(r_p\right)^{\frac{n-1}{n}}-1\right]}{V_S}$$

$$V_1 = V_S \text{ as clearance volume is neglected.}$$

$$P_m = \left(\frac{n}{n-1}\right)P_1\left[\left(r_p\right)^{\frac{n-1}{n}}-1\right]$$

$$= \frac{1.2}{0.2}\times 1\times\left[(7)^{\frac{0.2}{1.2}}-1\right] = 2.3 \text{ bar}$$

Average piston speed $= 2LN = 2.5$ m/s $= 150$ m/min

Indicated power, IP $= \frac{2P_mLAN}{60000}$ kW [∵ for double acting]

$$37 = \frac{2.3\times 10^5\times 150\times A}{60000}$$

$$A = 0.0643 \text{ m}^2$$

$$A = \frac{\pi}{4}D^2 = 0.0643 \text{ m}^2$$

$$D = 0.2861 \text{ m or } 28.61 \text{ cm}$$

$$2LN = 150 \text{ m/min}$$

$$L = \frac{150}{2N} = \frac{150}{2\times 200} = 00375 \text{ m}$$

$$L = 37.5 \text{ cm}$$

Q.6 (a)(ii) Solution:

The following methods are employed to achieve nearly isothermal compression for high speed reciprocating compressors.

1. **Spray injection:** This method assimilates the practice of injecting water into the compressor cylinder towards the compression stroke with the object of cooling the air. It entails the following demerits.
   (i) It necessitates the use of a special gear for injection;
   (ii) The injected water interferes with the cylinder lubrication and attacks cylinder walls and valves, and

(iii) The water mixed with air should be separated before using the air.

2. **Water injection:** It consists in circulating water around the cylinder through the water jacket which helps to cool the air during compression. This method is commonly used in all types of reciprocating air compressors.
3. **Inter-cooling:** When the speed of the compressor is high and pressure ratio required is also high with single-stage compression water jacketing proves to be less effective. The use of inter-cooling is restored to in addition to the water jacketing by dividing the compression into two or more stages. The air compressed in the first stage is cooled in an intercooler (heat exchanger) to its original temperature before passing it on to the following (second) stage.
4. **External fins:** The small capacity air compressors can be effectively cooled by using fins on their external surfaces.
5. **By a suitable choice of cylinder proportions:** By providing a short stroke and a large bore in conjunction with sleeve valves, a much greater surface is available for cooling, and the surface of the cylinder head is far more effective in this respect than the surface of the barrel, because the periodic motion of the piston does not allow the barrel to be exposed to the air for a sufficient time for heat to flow away. Moreover the air is compressed against the cylinder cover. Unfortunately clearance increases as the square of the bore, but in the Broom-Wade compressor this increase is compensated for by the mechanically operated valve.

Q.6 (b) Solution:

Given: Density, $\rho = 1025\ \text{kg/m}^3$, Rated power, $P = 60$ MW by each generator

For maximum power discharge is Q per turbine.

1. $$P_{ava} = \rho Q g h \qquad \left[\text{Power available } P_{ava.} = \frac{P}{\eta_{gen} \times \eta_t}\right]$$

$$\left(\frac{60 \times 10^6}{0.93 \times 0.93}\right) = 1025 \times Q \times 9.81 \times 10$$

$$Q = \frac{60 \times 10^6}{0.93 \times 0.93 \times 1025 \times 9.81 \times 10} = 689.91\ \text{m}^3/\text{s}$$

2. Total average power generated $= \dfrac{60 \times 140}{2} = \dfrac{8400}{2}$ MW = 4200 MW

$$\text{Total average power available} = \frac{4200}{0.93 \times 0.93} = 4856.05 \text{ MW}$$

We know that,
$$(P_{\text{available}})_{\text{avg}} = \frac{1}{2} \times \frac{\rho A g \left(R^2 - h^2\right)}{\text{Time}}$$

$$4856.05 \times 10^6 = \frac{1025 \times A \times 9.81 \times \left(10^2 - 2^2\right)}{2 \times 6 \times 3600}$$

$$A = 217.321 \times 10^6 \text{ m}^2$$

$$\text{Surface area of reservoir, } A = 217.321 \text{ km}^2$$

3. The wash behind the embankment of 16 km will be = $\dfrac{217.321}{16} = 13.582$ km

4. Energy produced only 12 hours of operation per day.

Output at 10 m head = 60 MW per generator

Output at the end of 6 hours = 0 MW

Average output during a cycle = 30 MW per generator

Energy output per day = 30 × 12 = 360 MWh per generator

Energy output per year = 360 × 365 = 131400 MWh per generator

Total energy output per year for 140 generator units

$= 131400 \times 140 = 18396 \times 10^3$ MWh

Total energy output per year for 140 generator units = 18.396 TWh

Results:

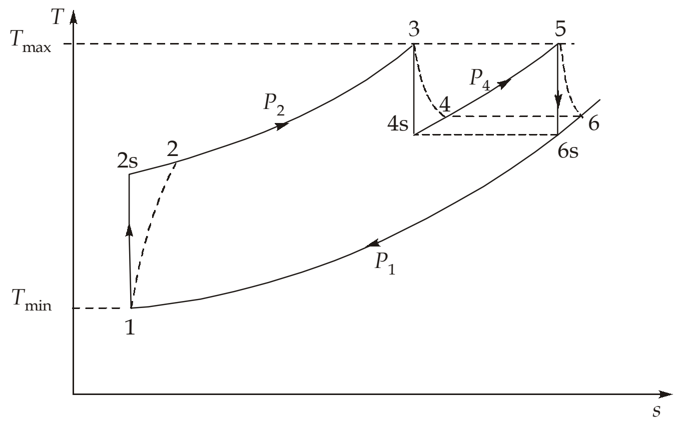
1. Maximum discharge per turbine = 689.91 m^3/s
2. Surface area of reservoir = 217.321 km^2
3. Wash behind the embankment = 13.582 km
4. Total energy produced = 18.396 TWh per year.

Q.6 (c) Solution:

For Ist case,

Let,
$$T_{min} = T_1$$
$$T_3 = T_5 = T_{max}$$

Given,
$$\frac{P_2}{P_4} = \frac{P_4}{P_1}$$



$$P_4 = \sqrt{P_2 P_1}$$

Let, $$\frac{P_2}{P_1} = r$$

$$P_4 = \sqrt{r} P_1$$

We know that, $$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r)^{\frac{\gamma-1}{\gamma}}$$

Let, $$\frac{\gamma-1}{\gamma} = x$$

$$\frac{T_{2s}}{T_1} = (r)^x$$

Now, $$T_{2s} = T_{min} \times (r)^x$$

Now, $$\eta_C = \frac{T_{2s} - T_1}{(T_2 - T_1)}$$

$$(T_2 - T_1) = \left(\frac{T_{2s} - T_1}{\eta_C}\right)$$

$$(\Delta T)_{comp} = \frac{T_{min} \times r^x - T_{min}}{\eta_C} = \frac{T_{min}}{\eta_C} \times \left(r^x - 1\right)$$

Now, $$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_4}\right)^x = \left(\frac{rP_1}{\sqrt{r}P_1}\right)^x = (r)^{x/2}$$

$$T_{4s} = T_3 \times r^{(-x/2)}$$

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$(\Delta T)_{\text{Turbine}} = \eta_T \times (T_3 - T_3 \times r^{-x/2})$$

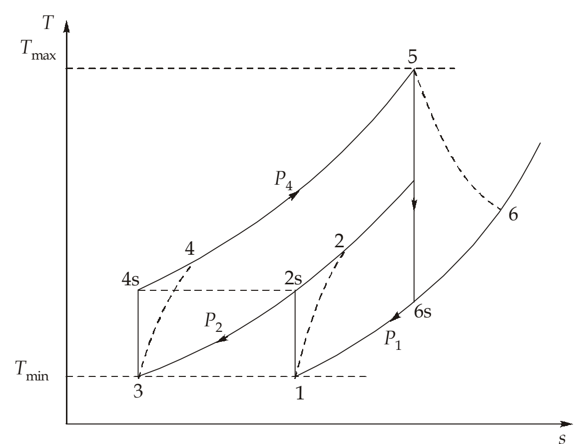
$$= \eta_T \times T_{\max} (1 - r^{-x/2})$$

Now, Net work output, $W_{\text{net}} = [W_{T1} + W_{T2} - W_C]$ $\quad (\because W_{T1} = W_{T2})$

$$= [2 \times c_p \times (\Delta T)_{\text{turbine}} - c_p \times (\Delta T)_{\text{comp.}}]$$

$$W_{\text{net}} = c_P\left[2\times\eta_T\times T_{\max}\left(1-r^{-x/2}\right)-\frac{T_{\min}}{\eta_C}\times\left(r^x-1\right)\right]$$

For second case:



$$T_1 = T_3 = T_{\min}$$

$$T_5 = T_{\max}$$

$$\frac{P_2}{P_1} = \frac{P_4}{P_2}$$

$$P_2 = \sqrt{P_1 P_4}$$

If overall pressure ratio is r, $\dfrac{P_4}{P_1} = r$

$$P_2 = \sqrt{r}P_1$$

Now, $$\frac{T_{2s}}{T_1} = \frac{T_{2s}}{T_{\min}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\sqrt{r}\right)^x$$

Now, $$T_{2s} = T_{\min} \times (r)^{x/2}$$

$$(\Delta T_s)_{\text{comp,1}} = (T_{2s} - T_1) = T_{\min} (r^{x/2} - 1)$$

$$(\Delta T)_{\text{comp,1}} = \frac{T_{\min}\times\left(r^{x/2}-1\right)}{\eta_C} = (\Delta T)_{comp,2}$$

Now, $$\frac{T_5}{T_{6s}} = \frac{T_{\max}}{T_{6s}} = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{6s} = \frac{T_{\max}}{(r)^x}$$

$$(\Delta T_s)_{\text{Turbine}} = (T_5 - T_{6s}) = T_{\max}(1 - r^{-x})$$

$$(\Delta T)_{\text{Turbine}} = \eta_T \times T_{\max} (1 - r^{-x})$$

$$W_{net} = [W_T - (W_{C1} + W_{C2})]$$
$$= [c_p(\Delta T)_{Turbine} - 2c_p(\Delta T)_{Comp.}]$$
$$W_{net} = c_P\left[\eta_T \times T_{max}\left(1-r^{-x}\right) - \frac{2\times T_{min}\times\left(r^{x/2}-1\right)}{\eta_C}\right]$$

Given: $\eta_C = 0.85$, $\eta_T = 0.90$ and $\frac{T_{max}}{T_{min}} = 3.5$

$$r_{opt} = (0.85\times0.90\times3.5)^{(2\times1.4)/(3\times0.4)} = 9.955$$

Net work output in first case, $(W_{net})_1 = c_P\left[2\eta_T \times T_{max}\times\left(1-r^{-x/2}\right) - \frac{T_{min}}{\eta_C}\left(r^x - 1\right)\right]$

$$= c_P\left[2\times0.9\times3.5T_{min}\times\left(1-9.955^{-0.4/1.4\times2}\right) - \frac{T_{min}}{0.85}\left(9.955^{0.4/1.4}-1\right)\right]$$

$$(W_{net})_1 = 0.670\ c_p\ T_{min}$$

Net work output in second case, $(W_{net})_2 = c_P\left[\eta_T \times T_{max}\times\left(1-r^{-x}\right) - \frac{2T_{min}}{\eta_C}\left(r^{x/2} - 1\right)\right]$

$$= c_P\left[0.9\times3.5T_{min}\times\left(1-9.955^{-0.4/1.4}\right) - \frac{2\times T_{min}}{0.85}\left(9.955^{0.4/1.4\times2}-1\right)\right]$$

$$(W_{net})_2 = 0.602\ c_p\ T_{min}$$

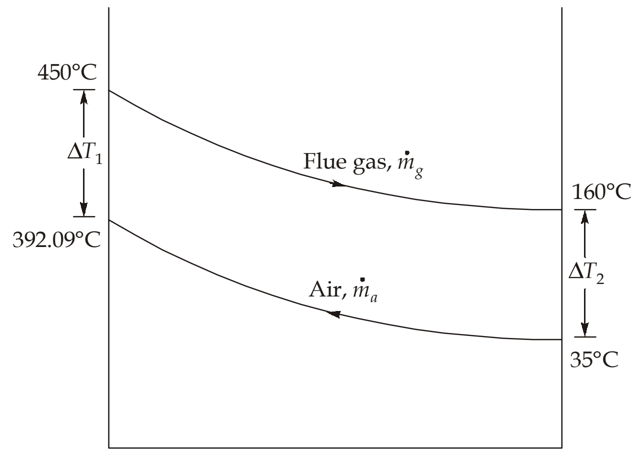
Q.7 (a) Solution:

Given, $\dot{m}_g = 1350\ \text{kg/s}$, $\dot{m}_a = 1200\ \text{kg/s}$

From the heat transfer analysis,

$$Q = \dot{m}_g(c_P)_g\times\left(T_{g1}-T_{g2}\right) = \dot{m}_a(c_P)_a\times\left(T_{a2}-T_{a1}\right)$$
$$= U_0\,A_0\,\Delta T_m$$

where ΔT_m is log mean temperature difference.



Now, $$Q = \dot{m}_g (c_P)_g \times (T_{g1} - T_{g2})$$
$$= 1350 \times 1.1 \times (450 - 160)$$
$$= 430650 \text{ kW}$$

Now, $$Q = \dot{m}_a (c_P)_a \times (T_{a2} - T_{a1})$$
$$430650 = 1200 \times 1.005 \times (T_{a2} - 35)$$
$$T_{a2} = 392.09°\text{C}$$
$$\Delta T_1 = 450 - 392.09 = 57.91°\text{C}$$
$$\Delta T_2 = 160 - 35 = 125°\text{C}$$

LMTD, $$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{57.91 - 125}{\ln\left(\frac{57.91}{125}\right)}$$
$$= 87.195°\text{C}$$

We know that, $$Q = 430650 \text{ kW}$$
$$U_0 A_0 \Delta T_m = 430650 \times 10^3$$
$$30 \times A_0 \times 87.195 = 430650 \times 10^3$$
$$A_0 = 164.631 \times 10^3 \text{ m}^2$$

Specific volume of flue gases at the inlet of air preheater,
$$v_g = \frac{RT_1}{P} = \frac{0.287 \times (450 + 273)}{1.01325 \times 100} = 2.04787 \text{ m}^3/\text{kg}$$

Let there are 'n' tubes,

We know that, $$\dot{m}_g = \left(n \times \frac{\pi}{4} D_i^2\right) \times \frac{V_g}{v_g}$$

$$1350 = n \times \frac{\pi}{4}\left(60 \times 10^{-3}\right)^2 \times \frac{12}{2.04787}$$

$$n = \frac{1350 \times 4 \times 2.04787}{\pi \times \left(60 \times 10^{-3}\right)^2 \times 12} = 81482.16$$

Hence, number of tubes is 81483.

We know, that, $$A_0 = (\pi D_0 l) \times n = 164.331 \times 10^3$$

$$l = \frac{164.331 \times 10^3}{81483 \times \pi \times 65 \times 10^{-3}} = 9.8762 \text{ m}$$

Results: Number of tubes = 81483

Length of tube = 9.8762 m

Q.7 (b) Solution:

Open circuit voltage, V_{OC} = 1 Volt

Short circuit current density, I_{SC} = 400 A/m^2

Temperature of cell during the trial = 35°C = 308 K

Boltzmann's constant, k = 1.381×10^{-23} J/K

Area of cell, A = 3 m^2

We know that, as per the photoelectric effect,

$$I = I_L - I_j$$

$$I = I_{SC} - I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

For open circuit voltage current will be zero.

$$0 = I_{SC} - I_0\left[\exp\left(\frac{eV_{OC}}{kT}\right) - 1\right]$$

$$I_0 = \frac{400}{\left[\exp\left(\frac{1.602 \times 10^{-19} \times 1}{1.381 \times 10^{-23} \times 308}\right) - 1\right]}$$

Dark current, I_0 = 1.75835×10^{-14} A/m^2

For maximum power condition,

$$P = V \times I$$

$$P = V \times \left[I_{SC} - I_0 \left\{ \exp\left(\frac{eV}{kT}\right) - 1 \right\} \right]$$

For maximum power, $\frac{dP}{dV} = 0$

$$\frac{dP}{dV} = I_{SC} - I_0 \left\{ \exp\left(\frac{eV}{kT}\right) - 1 \right\} - \left(\frac{eV}{kT}\right) \times I_0 \times \exp\left(\frac{eV}{kT}\right)$$

For maximum power, $\frac{dP}{dV} = 0$ and voltage, $V = V_m$

$$0 = I_{SC} + I_0 - I_0 \exp\left(\frac{eV_m}{kT}\right)\left(1 + \frac{eV_m}{kT}\right)$$

$$\frac{I_{SC} + I_0}{I_0} = \exp\left(\frac{eV_m}{kT}\right)\left(1 + \frac{eV_m}{kT}\right) \quad \text{... (i)}$$

$$\frac{400}{1.75835 \times 10^{-14}} + 1 = \exp\left(\frac{1.602 \times 10^{-19} \times V_m}{1.381 \times 10^{-23} \times 308}\right)\left(1 + \frac{1.602 \times 10^{-19} V_m}{1.381 \times 10^{-23} \times 308}\right)$$

$$2.274859954 \times 10^{16} = \exp\,(37.663278 V_m)(1 + 37.663278\ V_m)$$

On solving, voltage for maximum power condition,

$$V_m = 0.905522 \text{ Volt}$$

Now, for maximum power condition current value:

$$I = I_{SC} - I_0 \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

Current for maximum power condition is I_m and voltage is V_m.

$$I_m = (I_{SC} + I_0) - I_0 \left[\exp\left(\frac{eV_m}{kT}\right) \right] \quad \text{... (ii)}$$

From equation (i) and (ii)

$$I_m = (I_{SC} + I_0) - I_0 \left[\left(\frac{I_{SC} + I_0}{I_0} \right) \right] \times \frac{1}{\left(1 + \frac{eV_m}{kT}\right)}$$

$$I_m = \frac{(I_{SC}+I_0)\left(\frac{eV_m}{kT}\right)}{\left(1+\frac{eV_m}{kT}\right)}$$

Putting value of V_m, I_{SC} and I_0,

$$I_m = \frac{(400+1.75835\times10^{-14})\left(\frac{1.602\times10^{-19}\times1}{1.381\times10^{-23}\times308}\right)}{\left(1+\frac{1.602\times10^{-19}\times1}{1.381\times10^{-23}\times308}\right)}$$

$$= \frac{(400+1.75835\times10^{-14})\times(37.663278)}{(38.663278)}$$

$$I_m = 389.65426 \text{ A/m}^2$$

Maximum power density, $P_m = V_m \times I_m$

$= 0.905522 \times 389.65426$

$P_m = 352.8405 \text{ W/m}^2$

Total maximum power, $P_{\text{total}} = V_m \times I_m \times A_c$

$= 0.905522 \times 389.65426 \times 3$

$= 1058.5215$ Watt

$$\text{Fill factor} = \frac{V_m\times I_m}{V_{OC}\times I_{SC}} = \frac{352.8405}{1\times400} = 0.8821$$

$= 88.21\%$

Results:

1. Voltage for maximum power, $V_m = 0.905522$ Volt
2. Current density for maximum power, $I_m = 389.65426 \text{ A/m}^2$
3. Maximum power density, $P_m = 352.8405 \text{ W/m}^2$
4. Total maximum power, $P_t = 1058.5215$ Watt
5. Fill factor, FF = 0.8821
6. Voltage relation, $\exp\left(\frac{eV_m}{kT}\right)\left(1+\frac{eV_m}{kt}\right) = \frac{I_{SC}+I_0}{I_0}$
7. Current density, $I_m = \frac{(I_{SC}+I_0)\left(\frac{eV_m}{kT}\right)}{\left(1+\frac{eV_m}{kT}\right)}$

Q.7 (c) Solution:

Cavitation began, when, $\frac{p_1}{\rho g}+\frac{V_1^2}{2g} = 3.5 \text{ m}$

(where subscript 1 refers to the condition at inlet to the pump)

and at this condition, $p_1 = p_{vap}$

Therefore, $\frac{V_1^2}{2g} = 3.5-\frac{1.8\times10^3}{9.81\times10^3} = 3.32 \text{ m}$ (net positive suction head)

Hence, the cavitation parameter, $\sigma = \frac{V_1^2}{2gH} = \frac{3.32}{36} = 0.092$

This dimensionless parameter will remain same of the both the cases.

Applying Bernoulli's equation, between the liquid level at sump and the inlet to the pump (taking the sump level as datum),

We can write for the first case,

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{p_{atm}}{\rho g}-h_{f_1} \text{ (sum of head losses)}$$

or

$$(z_1+h_{f1}) = \frac{p_{atm}}{\rho g}-\sigma H-\frac{p_1}{\rho g} = (0.75\times13.6)-3.32-\frac{1.8}{9.81}$$

$$= 6.7 \text{ m}$$

For the second case,

$$\frac{p_1'}{\rho g}+\frac{V_1'^2}{2g}+z_1' = \frac{p_{atm}'}{\rho g}-h'_{f_1}$$

(Superscript (′) refer to the second case)

or

$$(z_1'+h'_{f1}) = \frac{p_{atm}'}{\rho g}-\sigma H-\frac{p_{vap}'}{\rho g} = (0.62\times13.6)-3.32-\frac{830}{9.81\times10^3}$$

$$= 5.03 \text{ m}$$

Since the flow rate is same, $h_{f1} = h'_{f1}$

Therefore, height of pump is to be reduced by some distance i.e. (6.7 – 5.03) m = 1.67 m

Q.8 (a) Solution:

Thermo-chemical conversion: Thermo-chemical conversion is a process to decompose biomass with various combinations of temperatures and pressures.

(a) Pyrolysis: Biomass is heated in absence of oxygen, or partially combusted in a limited oxygen supply, to produce a hydrocarbon, rich in gas mixture (H_2, CO_2, CO, CH_4 and lower hydrocarbons), an oil like liquid and a carbon rich solid residue (charcoal).

The pyrolytic or 'bio-oil' produced can easily be transported and refined into a series of products similar to refining crude oil. There is no waste product, the conversion efficiency is high (82%) depending upon the feed stock used, the process temperature in reactor and the fuel air ratio during combustion.

(b) Gasification: Gasification is conversion of a solid biomass, at a high temperature with controlled air, into a gaseous fuel. The output gas is known as producer gas, a mixture of H_2 (15-20%), CO (10-20%), CH_4(1-5%), CO_2(9-12%) and N_2(45-55%). The gas is more versatile than the solid biomass, it can be burnt to produce process heat and steam, or used in internal combustion engines or gas turbines to generate electricity. The gasification process renders the use of biomass which is relatively clean and acceptable in environmental terms.

(c) Liquefication: Liquefication of biomass can be processed through 'fast' or 'flash' pyrolysis, called 'pyrolytic oil' which is a dark brown liquid of low viscosity and a mixture of hydrocarbons. Pyrolysis liquid is a good substitute for heating oil.

Another liquefaction method is through methanol synthesis. Gasification of biomass produces synthetic gas containing a mixture of H_2 and CO. The gas is purified by adjusting the hydrogen and carbon monoxide composition. Finally, the purified gas is subjected to liquefaction process, converted to methanol over a zinc chromium catalyst. Methanol can be used as liquid fuel.

Bio-chemical conversion: There are two types of biochemical conversions:

(a) Anaerobic digestion: This process converts the cattle dung, human wastes and other organic waste with high moisture content into biogas (gobar gas) through anaerobic fermentation in absence of air. Fermentation occurs in two stages by two different metabolic groups of bacteria. Initially the organic material is hydrolyzed into fatty acids, alcohol, sugars, H_2 and CO_2. Methane forming bacteria then converts the products of the first to CH_4 and CO_2, in the temperature range 30-55°C.

Biogas produced can be used for heating, or for operating engine driven generators to produce electricity. Fermentation occurs in a sealed tank called 'digester' where the sludge left behind is used as enriched fertilizer.

(b) Ethanol fermentation: Ethanol can be produced by decomposition of biomass containing sugar like sugarcane, cassava sweet sorghum, beet, potato, corn, grape etc, into sugar molecules such as glucose ($C_6H_{12}O_6$) and sucrose($C_{12}H_{22}O_{11}$).

Ethanol fermentation invoIes biological conversion of sugar into ethanol and CO_2.

$$C_{12}H_{22}O_{11} + H_2O \longrightarrow 2C_6H_{12}O_6$$

$$C_6H_{12}O_6 \xrightarrow{\text{Fermentation}} 2C_2H_5OH + 2CO_2$$

Ethanol has emerged as the major alcohol fuel and is blended with petrol.

Q.8 (b) Solution:

$$V_1 = C_V\sqrt{2gH_{pe}} = 0.97 \times \sqrt{2 \times g \times 760} = 118.45 \text{ m/s}$$

$$U = 0.48V_1 \qquad U = 56.856 \text{ m/s}$$

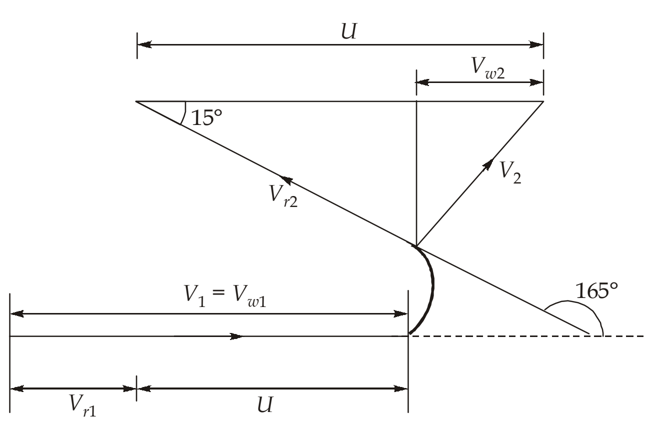
$$V_{r1} = V_1 - U = 118.45 - 56.856 = 61.594 \text{ m/s}$$

$$V_{r2} = 0.88V_{r1} = 0.88 \times 61.894 = 54.20 \text{ m/s}$$

$$V_{r2}\cos\beta_2 = 54.2\cos 15° = 52.35 \text{ m/s}$$

Since, $V_{r2}\cos\beta_2 < U$

The resulting velocity triangle is as follows:



$$V_{w2} = U - V_{r2}\cos\beta_2$$

$$= 56.856 - 52.35 = 4.506 \text{ m/s}$$

$$H_r = \frac{(V_{w1} - V_{w2})U}{g} = \frac{(118.45 - 4.506) \times (56.856)}{9.81} = 660.38 \text{ m}$$

$$\text{Runner power, RP} = \rho Q g H_r$$

Given, mechanical efficiency is 100%.

So, $$RP = SP$$

and $$SP = \frac{10000}{0.95} \text{ kW}$$

$$\frac{10000 \times 10^3}{0.95} = 10^3 \times Qg \times 660.38$$

$$Q = 1.6248 \text{ m}^3/\text{s}$$

$$Q = \frac{\pi}{4} d^2 V_1$$

$$1.6248 = \frac{\pi}{4} \times d^2 \times 118.45$$

$$d = 0.132 \text{ m or } d = 132 \text{ mm}$$

Force of jet on bucket.

$$\text{Runner power, RP} = F \times U$$

$$F = \frac{10000 \times 10^3}{0.95 \times 56.856} = 185.14 \text{ kN}$$

Power lost in nozzle due to friction,

$$P_N = \left(1 - C_V^2\right) \times H_{pe} \times \rho Q g$$

$$= (1 - 0.97^2) \times 760 \times 10^3 \times 1.6248 \times 9.81$$

$$= 715.93 \text{ kW}$$

Power lost in runner due to friction.

$$(h_L)_R = \frac{V_{r_1}^2 - V_{r_2}^2}{2g} = \frac{61.594^2 - 54.2^2}{2 \times g} = \frac{428.09}{g}$$

$$P_R = \rho Q g (h_L)_R$$

$$= 10^3 \times 1.6248 \times g \times \frac{428.09}{g} = 695.56 \text{ kW}$$

KE lost in outgoing stream,

$$V_{f2} = V_{r2} \sin\beta_2 = 54.2 \sin 15° = 14.02 \text{ m/s}$$

$$V_2 = \sqrt{V_{w_2}^2 + V_{f_2}^2} = \sqrt{4.356^2 + 14.02^2} = 14.68 \text{ m/s}$$

$$\text{KE of outgoing stream} = \frac{\rho Q V_2^2}{2} = 10^3 \times 1.6248 \times \frac{14.68^2}{2} = 175.07 \text{ kW}$$

$$\text{Hydraulic efficiency, } \eta_h = \frac{H_{\text{runner}}}{H_{pe}} \times 100 = \frac{660.38}{760} \times 100 = 86.89\%$$

$$\text{Bucket efficiency, } \eta_B = \frac{\text{Runner power}}{\text{Energy per sec at nozzle end}} \times 100$$

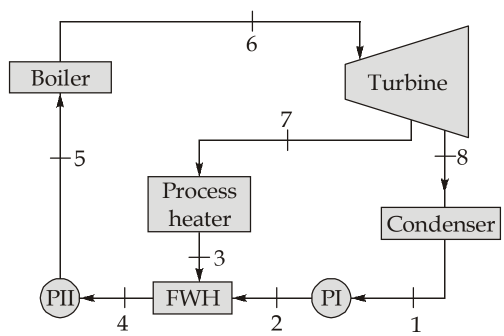
$$= \frac{\rho Q g \times 660.38}{\rho Q \times \dfrac{V_1^2}{2}} \times 100 = \frac{9.81 \times 2 \times 660.38}{(118.45)^2} = 92.34\%$$

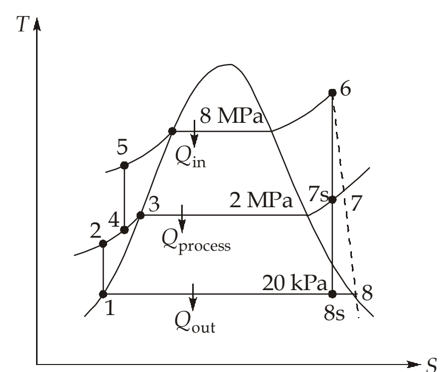
Q.8 (c) Solution:

Assumptions

1: Steady operating conditions exist.

2: Kinetic and potential energy changes are negligible.





Analysis from the steam tables

$$h_1 = h_{f\,@20\text{ kPa}} = 251.42 \text{ kJ/kg}$$

$$v_1 = v_{f\,@20\text{ kPa}} = 0.001017 \text{ m}^3/\text{kg}$$

$$W_{pI,\text{ in}} = \frac{v_1 (P_2 - P_1)}{\eta_P} = \frac{0.001017(2000 - 20)}{0.88}$$

$$= 2.29 \text{ kJ/kg}$$

$$h_2 = h_1 + W_{pI,\text{in}} = 251.42 + 2.29 = 253.71 \text{ kJ/kg}$$

$$h_3 = h_{f\,@2\text{ MPa}} = 908.47 \text{ kJ/kg}$$

Mixing chamber:

$$\dot{m}_3 h_3 + \dot{m}_2 h_2 = \dot{m}_4 h_4$$

(4 kg/s)(908.47 kJ/kg) + (11 – 4 kg/s)(253.71 kJ/kg) = (11 kg/s)h_4

$$h_4 = 491.81 \text{ kJ/kg}$$

$$v_4 = 0.001058 \text{ m}^3\text{/kg} \qquad \text{(given)}$$

$$W_{p\text{II,in}} = \frac{v_4(P_5 - P_4)}{\eta_P} = \frac{0.001058(8000 - 2000)}{0.88}$$

$$= 7.21 \text{ kJ/kg}$$

$$h_5 = h_4 + W_{\text{pII,in}} = 491.81 + 7.21 = 499.02 \text{ kJ/kg}$$

$$h_6 = 3399.5 \text{ kJ/kg}$$

$$\left.\begin{aligned} P_6 &= 8 \text{ MPa} \\ T_6 &= 500°\text{C} \end{aligned}\right\} s_6 = 6.7266 \text{ kJ/kg.K}$$

$$s_7 = s_6$$

By extrapolation for 2 MPa superheated steam,

$$\frac{s - s_1}{s_2 - s_1} = \frac{h - h_1}{h_2 - h_1}$$

$$\frac{6.7266 - 6.7684}{6.9583 - 6.7684} = \frac{h - 3024.2}{3137.7 - 3024.2}$$

$$h_{7s} = 3000.4 \text{ kJ/kg}$$

$$\eta_T = \frac{h_6 - h_7}{h_6 - h_{7s}} \Rightarrow h_7 = h_6 - \eta_T(h_6 - h_{7s})$$

$$= 3399.5 - (0.88)(3399.5 - 3000.4) = 3048.3 \text{ kJ/kg}$$

$$s_8 = s_6$$

$$6.7266 = 0.832 + x(7.9073 - 0.832)$$

$$x = 0.8331$$

$$h_{8s} = 251.42 + 0.8331(2608.9 - 251.42)$$

$$h_{8s} = 2215.5 \text{ kJ/kg}$$

$$\eta_T = \frac{h_6 - h_8}{h_6 - h_{8s}} \Rightarrow h_8 = h_6 - \eta_T(h_6 - h_{8s})$$

$$= 3399.5 - (0.88)(3399.5 - 2215.5) = 2357.6 \text{ kJ/kg}$$

Then,

$$\dot{Q}_{\text{process}} = \dot{m}_7(h_7 - h_3)$$

$$= (4 \text{ kg/s})(3048.3 - 908.47) \text{ kJ/kg} = 8559 \text{ kW}$$

(b) Cycle analysis:

$$(\dot{W}_T)_{\text{out}} = \dot{m}_7(h_6 - h_7) + \dot{m}_8(h_6 - h_8)$$

$$= (4\text{ kg/s})(3399.5 - 3048.3)\text{ kJ/kg} + (7\text{ kg/s})(3399.5 - 2357.6)\text{ kJ/kg}$$

$$= 8698\text{ kW}$$

$$\left(\dot{W}_P\right)_{\text{in}} = \dot{m}_1 W_{pI,in} + \dot{m}_4 W_{pII,in}$$

$$= (7\text{ kg/s})(2.29\text{ kJ/kg}) + (11\text{ kg/s})(7.21\text{ kJ/kg})$$

$$= 95\text{ kW}$$

$$\dot{W}_{net} = \dot{W}_{T,out} - \dot{W}_{P,in} = 8698 - 95 = 8603\text{ kW}$$

(c) Then,

$$\dot{Q}_{in} = \dot{m}_5\left(h_6 - h_5\right) = (11\text{ kg/s})(3395.5 - 499.02) = 31.905\text{ kW}$$

and

$$\varepsilon_u = \frac{\dot{W}_{net} + \dot{Q}_{\text{process}}}{\dot{Q}_{in}} = \frac{8603 + 8559}{31.905} = 0.538 = 53.8\%$$

■■■■