



# MADE EASY

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Detailed Solutions

**ESE-2018  
Mains Test Series**

**Civil Engineering  
Test No : 11**

## Section A

### Q.1 (a) Solution:

For a triangular channel section, if  $\theta$  is the angle of inclination of each of the sloping sides with the vertical and  $y$  is the depth of flow, then the wetted area 'A' and wetted perimeter 'P' can be respectively given by

$$\begin{aligned} & A = my^2 && [\because m = \tan\theta] \\ \Rightarrow & A = y^2 \tan\theta \\ \Rightarrow & y = \sqrt{A/\tan\theta} && \dots(i) \\ \text{and} & P = 2y \sec\theta && \dots(ii) \end{aligned}$$

Substituting value of  $y$  from (i) in (ii), we get

$$P = 2 \times \sqrt{\frac{A}{\tan\theta}} \times \sec\theta \quad \dots(iii)$$

Assuming wetted area,  $A$  to be constant, equation (iii) can be differentiated with respect to  $\theta$  and equated to zero for obtaining the condition for minimum wetted perimeter.

$$\begin{aligned} \therefore & \frac{dP}{d\theta} = 2\sqrt{A} \left[ \frac{\sec\theta \tan\theta}{\sqrt{\tan\theta}} - \frac{\sec^3\theta}{2(\tan\theta)^{3/2}} \right] = 0 \\ \Rightarrow & \sec\theta(2 \tan^2\theta - \sec^2\theta) = 0 && [\because \sec\theta \neq 0] \\ \therefore & 2 \tan^2\theta - \sec^2\theta = 0 \\ \Rightarrow & \sqrt{2} \tan\theta = \sec\theta \\ \Rightarrow & \theta = 45^\circ \end{aligned}$$

$$\Rightarrow m = \tan \theta = 1$$

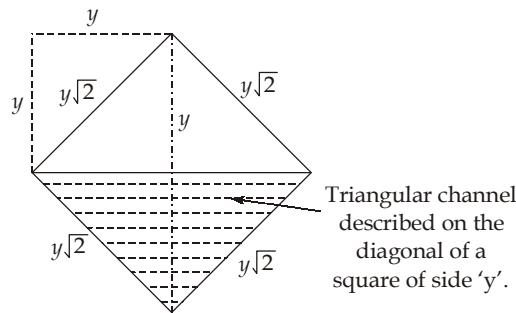
Hence a triangular channel will be most efficient when each of its sloping sides makes an angle of 45° with the vertical.

Hydraulic radius,  $R = \frac{A}{P} = \frac{y^2 \tan \theta}{2y \sec \theta} \quad (\theta = 45^\circ)$

$$\Rightarrow R = \frac{y^2 \tan 45^\circ}{2y} \times \cos 45^\circ$$

$$\Rightarrow R = \frac{y}{2\sqrt{2}}$$

Thus it can be seen that the most efficient triangular channel section will be half the square described on a diagonal and having equal sloping sides.



**Q.1 (b) Solution:**

$$\begin{aligned} \text{Runoff Depth} &= \frac{\text{Runoff volume}}{\text{Area}} \\ &= \frac{27.45 \times 10^6}{1830 \times 10^6} \times 100 \text{ cm} = 1.5 \text{ cm} = 15 \text{ mm} \end{aligned}$$

Neglecting other losses from precipitation such as interception, evaporation etc., we can write water budget equation as:

$$P \text{ (precipitation)} - I \text{ (Infiltration)} = R \text{ (Runoff)}$$

<b>Time (min)</b>	30	60	90	120	150	180	210	240
<b>Rainfall Intensity (mm/hr)</b>	5	9	20	13	6	8	16	3

Total storm duration 4 hr = 240 min

- Starting from lowest, let  $\phi < 3 \text{ mm/hr}$

$$(5 + 9 + 20 + 13 + 6 + 8 + 16 + 3) \times \frac{1}{2} - 4\phi = 15$$

$$\Rightarrow \phi = 6.25 \text{ mm/hr} \quad \therefore \text{Assumption is wrong}$$

- Let  $3 \text{ mm/hr} < \phi < 5 \text{ mm/hr}$ .

$$(5+9+20+13+6+8+16) \times \frac{1}{2} - 3.5\phi = 15$$

$$\Rightarrow \phi = 6.714 \text{ mm/hr}$$

∴ Assumption is wrong

3. Let  $5 \text{ mm/hr} < \phi < 6 \text{ mm/hr}$ .

$$(9 + 20 + 13 + 6 + 8 + 16) \times \frac{1}{2} - 3\phi = 15$$

$$\Rightarrow \phi = 7 \text{ mm/hr}$$

∴ Assumption is wrong

4. Let  $6 \text{ mm/hr} < \phi < 8 \text{ mm/hr}$ .

$$(9 + 20 + 13 + 8 + 16) \times \frac{1}{2} - 2.5\phi = 15$$

$$\Rightarrow \phi = 7.2 \text{ mm/hr}$$

(OK)

**Alternate solution:**

$$w\text{-index} = \frac{\left(\frac{5+9+20+13+6+8+16+3}{2}\right) - 15}{4} = 6.25 \text{ mm/hr}$$

As  $\phi\text{-index} > w\text{-index}$

Therefore, storm of intensities  $< 6.25 \text{ mm/hr}$  will not produce runoff.

$$\therefore \phi\text{-index} = \frac{\left(\frac{9+20+13+8+16}{2}\right) - 15}{2.5} = 7.2 \text{ mm/hr}$$

**Check :**

$$\text{Total runoff} = \left[ \frac{(9-7.2) + (20-7.2) + (13-7.2) + (8-7.2) + (16-7.2)}{1} \right] \times 0.5 = 15 \text{ mm}$$

which is same computed above.

**Q.1 (c) Solution:**

Rapid sand filters are typically designed as part of multi-stage treatment systems used by large municipalities. Various advantages and disadvantages of rapid sand filter with respect to slow sand filter are following:

**Advantages**

- Rapid sand filters have much higher flow rate than a slow sand filter; about 150 to 200 million gallons of water per acre per day.
- Rapid sand filter requires relatively small land area.
- These are less sensitive to changes in raw water quality, e.g. turbidity.

- Rapid sand filter requires less quantity of sand.

**Disadvantages:**

- Large pore size of rapid sand filter will not, without coagulant or flocculant, remove pathogens smaller than 20  $\mu\text{m}$ .
- Rapid sand filter requires more maintenance than a slow sand filter and is generally ineffective against taste and odour problems.
- It produces large volumes of sludge for disposal.
- Rapid sand filter, requires continuous investment in costly flocculation reagents.
- In rapid sand filter treatment of raw water with chemicals is essential.
- For rapid sand filter working skilled supervision is essential.
- Cost of maintenance is higher for rapid sand filter.

**Q.1 (d) Solution:**

For Reynolds number  $\leq 0.5$  (less than unity), a case of discrete settling can be assumed. When a discrete particle settles down in water, its downward settlement is opposed by the drag force offered by the water. The effective weight of the particle (actual weight - buoyancy) causes the particle to accelerate in the beginning, till it attains a sufficient velocity ( $v_s$ ) at which the drag force becomes equal to the effective weight of the particle. After attaining that velocity ( $v_s$ ), the particle falls down with that constant velocity.

Effective weight of the particle = Total weight - Buoyancy

$$= \frac{4}{3}\pi r^3 \times \gamma_s - \frac{4}{3}\pi r^3 \times \gamma_w \quad \dots(i)$$

(Where 'r' is radius of particle,  $\gamma_s$  is unit weight of particle and  $\gamma_w$  is unit weight of water)

Also, 
$$\text{Drag force} = C_D \times A \times \rho_w \times \frac{V^2}{2}$$

(Where  $C_D$  is coefficient of drag,  $A$  is area of particle,  $\rho_w$  is density of water and  $v$  is velocity of fall.) Now, when  $v$  becomes equal to  $v_s$ , the drag force becomes equal to the effective weight of the particle.

$$\begin{aligned} \therefore C_D \times A \times \rho_w \times \frac{V_s^2}{2} &= \frac{4}{3}\pi r^3 (\gamma_s - \gamma_w) && [\because A = \pi r^2] \\ \Rightarrow C_D \cdot \pi r^2 \cdot \rho_w \cdot \frac{V_s^2}{2} &= \frac{4}{3}\pi r^3 (\gamma_s - \gamma_w) \\ \Rightarrow V_s^2 &= \frac{4}{3} \times \frac{2r(\gamma_s - \gamma_w)}{\rho_w \cdot C_D} \end{aligned}$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times \frac{d(\gamma_s - \gamma_w)}{\rho_w \cdot C_D} \quad [ \because \gamma_s = \rho_s \times g \text{ and } \gamma_w = \rho_w \times g ]$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times \frac{d(\rho_s g - \rho_w g)}{\rho_w \times C_D}$$

$$\Rightarrow V_s^2 = \frac{4}{3} \times d \times \rho_w g \left( \frac{\rho_s}{\rho_w} - 1 \right) \times \frac{1}{\rho_w \cdot C_D}$$

$$V_s^2 = \frac{4}{3} \times g \times d \times (S_s - 1) \times \frac{1}{C_D} \quad \text{where } S_s = \frac{\rho_s}{\rho_w}$$

For  $Re = 0.5 (< 1)$

$$C_D = \frac{24}{Re} = \frac{24v}{V_s d}$$

[Where  $v$  is kinematic viscosity of water]

$$V_s^2 = \frac{4}{3} \times g \times d \times (S_s - 1) \times \frac{V_s d}{24v}$$

$$\Rightarrow V_s = \frac{g}{18} (S_s - 1) \times \frac{d^2}{v} \quad \text{(Hence proved)}$$

### Q.1 (e) Solution:

Design head,  $H_d = 4 \text{ m}$

Design head discharge coefficient,  $C_0 = 2.1$

$$H_e = H_d + H_a$$

where,  $H_a = \text{Head due to velocity of approach} = \frac{V^2}{2g}$

$$\text{Spillway discharge, } Q = C L_e H_e^{3/2}$$

where

$$L_e = L - 2[Nk_p + k_a]H_e$$

$$\therefore H_a = 0 \quad (\text{given})$$

$$\therefore H_e = H_d = 4 \text{ m}$$

$$\therefore L_e = 6 \times 8 - 2[5 \times 0.01 + 0.2] \times 4 = 46 \text{ m}$$

$$\text{Also } \frac{H}{H_d} = \frac{4.45}{4} = 1.11$$

$$\text{For } \frac{H}{H_d} = 1.11,$$

$$\frac{C}{C_0} = 1.01$$

$$\Rightarrow C = 1.01 \times 2.1 = 2.121$$

Discharge that will pass over the spillway at design head,

$$Q_1 = C_0 L_e H_e^{3/2}$$

$$\Rightarrow Q_1 = 2.1 \times 46 \times 4^{3/2}$$

$$\Rightarrow Q_1 = 772.8 \text{ m}^3/\text{s}$$

Discharge over spillway for head  $H (= 4.45 \text{ m})$  over the crest

$$Q_2 = C L_e H^{3/2}$$

$$Q_2 = 2.121 \times 46 \times 4.45^{3/2}$$

$$Q_2 = 915.88 \text{ m}^3/\text{s}$$

## Q.2 (a) Solution:

(i)

Analytical method for determination of lake evaporation can be broadly classified as

- **Water budget method:** This method is simplest in all three methods. In this by the hydrological continuity equation, we find out the evaporation from the lake.

The continuity is written down as:

$$P + V_{iS} + V_{iG} = V_{oS} + V_{oG} + E_L + \Delta S + T_L$$

Where,  $P$  = precipitation,  $V_{iS}$  = surface inflow

$V_{iG}$  = ground water inflow,  $V_{oS}$  = surface outflow

$V_{oG}$  = Seepage outflow  $E_L$  = evaporation

$\Delta S$  = change in storage  $T_L$  = daily transpiration loss

$P$ ,  $V_{iS}$ ,  $V_{oS}$  and  $\Delta S$  can be measured, but it is not possible to measure  $V_{iG}$ ,  $V_{oG}$  and  $T_L$  and these quantities can only be estimated. Due to this reason, the result from this  $E_L$  is not accurate.

If time period is kept large for estimation of  $E_L$ , better accuracy is possible.

- **Energy budget method:** The energy budget method is an application of the *law of conservation of energy*. The energy available for evaporation is determined by considering the incoming energy, outgoing energy and energy stored in water body over a known time interval.

Consider the water body, the energy balance to the evaporating surface in a period of one day is given by

$$H_n = H_a + H_e + H_g + H_s + H_i$$

$r$  = reflection coefficient

$H_n$  = net heat energy received by the water surface

$$= H_c(1 - r) - H_b$$

$H_c(1 - r)$  = incoming solar radiation into a surface of reflection coefficient  $r$  (albedo)

$H_a$  = sensible heat transfer from water surface to air,

$H_e$  = heat energy used in evaporation

$$= \rho L E_L$$

$\rho$  = density,

$L$  = latent heat of evaporation

$E_L$  = evaporation in mm

$H_g$  = heat flux into the ground

$H_i$  = net heat conducted out of the system by water flow.

All the energy term are in calories/sq.mm/day

If the time period are short, the term  $H_s$  and  $H_i$  can be neglected as they are negligibly small. Then

$$E_L = \frac{H_n - H_g - H_s - H_i}{\rho L(1 + \beta)} \quad \text{where, } \beta = \text{Bowen's ratio}$$

Estimated of evaporation in a lake by the energy balance method has been found to give satisfactory result, when applied for a short period less than a week.

- **Mass-Transfer Method:** This method is based on theories of turbulent mass transfer in boundary layer to calculate the mass of water vapour transfer from the surface to the surrounding atmosphere.

The volume of water lost due to evaporation from a water body is calculated as

$$V_E = A E_{pm} C_p$$

$V_E$  = Volume of water lost in evaporation  $A$  = Average reservoir surface area

$E_{pm}$  = Pan evaporation

$C_p$  = relevant pan coefficient

Typically under Indian condition, evaporation loss from a water body is about 160 cm in a year. Value will increase in arid.

(ii)

**Duty:** It is usually defined as area of land in hectares which can be irrigated for growing any crop if one cumec of water is supplied continuously to land for entire base period

of crop.

**Delta:** It is the total depth of water applied over an irrigated land at different watering throughout the entire base period of crop.

Suppose  $B$  = Base period (in days)

$D$  = Duty (ha/cumec)

$\Delta$  = Delta (in cm)

Let say  $D$  hectare of field is irrigated for a particular crop during entire base period,  $B$  days with a discharge of 1 cumec resulting in  $\Delta$  cm depth of water due to irrigation.

$\therefore$  Total volume of water supplied,

$$\frac{\Delta}{100} \times D \times 10^4 = 1 \times B \times 86400$$

$$\{\therefore 1 \text{ day} = 24 \times 3600 \text{ sec} = 86400 \text{ sec.}\}$$

$$\Rightarrow D = \frac{864B}{\Delta}$$

**Given:**  $D = 432$  ha/cumec;  $B = 100$  Days;

$$\therefore D = \frac{864B}{\Delta}$$

$$\Rightarrow \Delta = \frac{864B}{D} = \frac{864 \times 100}{432} = 200 \text{ cm} = 2 \text{ m}$$

### Q.2 (b) Solution:

**For an agitator**

Power ( $P$ ) =  $f$ (speed ( $N$ ), Diameter ( $D$ ), Viscosity ( $\mu$ ), Density ( $\rho$ ))

Using Buckingham's  $\pi$  theorem

$$f(P, N, D, \mu, \rho) = c$$

$$\text{Number of } \pi \text{ terms} = 5 - 3 = 2$$

Using  $N, D, \rho$  as repeating variable

$$\phi(\pi_1, \pi_2) = c$$

$$\pi_1 = N^{a_1} D^{b_1} \rho^{c_1} P$$

$$\Rightarrow [M^0 L^0 T^0] = [T^{-1}]^{a_1} [L^1]^{b_1} [M^1 L^{-3}]^{c_1} [M^1 L^2 T^{-3}]$$

$$\therefore c_1 + 1 = 0$$

$$\Rightarrow c_1 = -1$$

$$b_1 + 2 - 3c_1 = 0$$

$$\Rightarrow b_1 = -5$$

$$-a_1 - 3 = 0$$

$$\Rightarrow a_1 = -3$$



$$\pi_1 = \frac{P}{\rho D^5 N^3}$$

$$\pi_2 = N^{a_2} D^{b_2} \rho^{c_2} \mu$$

$$[M^0 L^0 T^0] = [T^{-1}]^{a_2} [L^1]^{b_2} [M^1 L^{-3}]^{c_2} [M^1 L^{-1} T^{-1}]$$

$$c_2 + 1 = 0$$

$$\Rightarrow c_2 = -1$$

$$b_2 + (-3c_2) - 1 = 0$$

$$\Rightarrow b_2 = -2$$

$$-a_2 - 1 = 0$$

$$\Rightarrow a_2 = -1$$

$$\pi_2 = \frac{\mu}{ND^2 \rho}$$

$$\text{i.e., } \phi\left(\frac{P}{\rho D^5 N^3}, \frac{\mu}{ND^2 \rho}\right) = \text{Constant}$$

Here  $\pi_2$  looks similar to inverse of Reynold's number, so Reynold's model law follows.  
For similar pumps

$$\frac{\mu_{air}}{N_1 D_1^2 \rho_{air}} = \frac{\mu_{water}}{N_2 D_2^2 \rho_{water}}$$

$$\Rightarrow \frac{1.86 \times 10^{-5}}{N_1 \times 675^2 \times 1.23} = \frac{1.1 \times 10^{-3}}{23 \times (225)^2 \times 1000}$$

$$\Rightarrow N_1 = 35.13 \text{ rev/s} \quad \dots(i)$$

$$\text{Also } \frac{P_1}{\rho_{air} D_1^5 N_1^3} = \frac{P_2}{\rho_{water} D_2^5 N_2^3}$$

$$\Rightarrow \frac{P_1}{1.23 \times 675^5 \times (35.13)^3} = \frac{P_2}{1000 \times 225^5 \times 23^3}$$

$$\Rightarrow \frac{P_1}{P_2} = 1.065$$

$$\text{But } P = \frac{2\pi NT}{60}$$

$$\therefore N_1 T_1 = 1.065 N_2 T_2$$

$$\Rightarrow 35.13 T_1 = 1.065 \times 23 \times 1.1$$

$$\Rightarrow T_1 = 0.767 \text{ Nm}$$

**Q.2 (c) Solution:**

Vessel is empty.

Difference of mercury level

$$h_2 = 20 \text{ cm}$$

Let  $h_1 =$  Height of water above X-X

Sp. gr. of mercury,  $S_2 = 13.6$

Sp. gr. of water,  $S_1 = 1.0$

Density of mercury,  $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

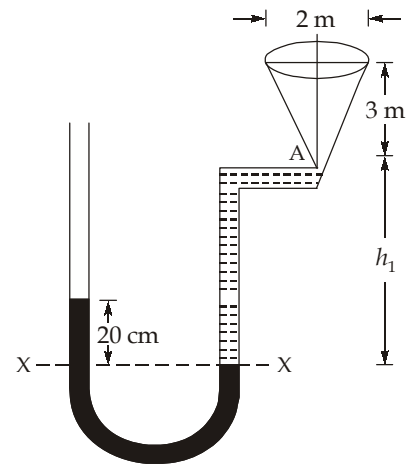
Density of water,  $\rho_1 = 1000 \text{ kg/m}^3$

Equating the pressure above datum line X-X, we have

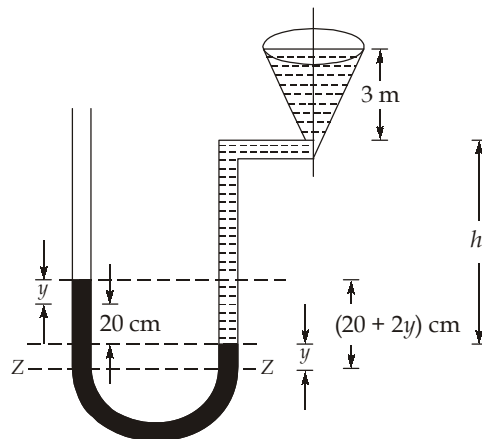
$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

$$\Rightarrow 13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$\Rightarrow h_1 = 2.72 \text{ m of water}$$



**Vessel is full of water:** When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be,  $y$  cm as shown in figure. The mercury will rise in the left by a distance of  $y$  cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z.



Pressure in left limb = Pressure in right limb

$$\Rightarrow 13.6 \times 1000 \times 9.81 \times \left( \frac{20 + 2y}{100} \right) = 1000 \times 9.81 \times \left( 3 + h_1 + \frac{y}{100} \right)$$

$$\Rightarrow 13.6 \times \left( \frac{20 + 2y}{100} \right) = \left( 3 + 2.72 + \frac{y}{100} \right)$$

$$\Rightarrow 272 + 27.2y = 5.72 \times 100 + y$$

$$\Rightarrow 26.2y = 300$$

$$\Rightarrow y = 11.45 \text{ cm}$$

The difference of mercury levels in two limbs

$$= (20 + 2y) \text{ cm of mercury}$$

$$= 20 + 2 \times 11.45 = 20 + 22.90$$

$$= 42.90 \text{ cm of mercury}$$

$\therefore$  Reading of manometer = 42.90 cm

### Q.3 (a) Solution:

(i) Suspended solids in waste water = 250 mg/l

Since 55% of these solids are removed in sedimentation, we have

The solids removed in sedimentation as sludge

$$= 55\% \times 250 \text{ mg/l} = 137.5 \text{ mg/l}$$

Let volume of waste water is 1 million litre, then solids removed as sludge

$$= \frac{137.5}{10^6} \times 10^6 \text{ kg} = 137.5 \text{ kg}$$

Sludge produced will, thus, have 137.5 kg solids and the rest will be water. Now, since the moisture content of sludge is 96%, we have,

4 kg of solids will produce 100 kg of wet sludge, by joining with 96 kg of water.

$\therefore$  Water contained in 4 kg of solids = 96 kg

$\therefore$  Water contained in 137.5 kg of solids =  $\frac{96}{4} \times 137.5 \text{ kg} = 3300 \text{ kg}$

Hence, volume of sludge produced per million litre of waste water

$$= \frac{\text{Mass of solids}}{\text{Density of solids}} + \frac{\text{Mass of water}}{\text{Density of water}}$$

$$= \left[ \frac{137.5}{1.2 \times 1000} + \frac{3300}{1000} \right] \text{cu-m}$$

[ $\therefore$  Density of solids = sp. gravity of solids  $\times$  Density of water =  $1.2 \times 1000 \text{ kg/m}^3$ ]

$$= 0.115 + 3.3 = 3.415 \text{ cu m.}$$

Hence, volume of sludge produced per million litre of waste water = 3.415 cu m.

(ii) Density of raw sludge =  $\frac{\text{Mass of solids} + \text{Mass of water}}{\text{Volume of sludge}}$

$$= \frac{137.5 + 3300}{3.415} \text{ kg/m}^3$$

$$= \frac{3437.5}{3.415} \text{ kg/m}^3 = 1006.59 \approx 1007 \text{ kg/m}^3$$

$$\begin{aligned} \therefore \text{Unit weight of raw sludge} &= \text{Density of raw sludge} \times g \\ &= 1007 \times 9.81 = 9878.7 \text{ N/m}^3 = 9.88 \text{ kN/m}^3 \end{aligned}$$

(iii) 45% of raw sludge is changed to liquid and gas, means that 45% of solids are consumed (i.e., digested).

$$\begin{aligned} \therefore \text{Mass of dry solids left in the digested sludge} &= (100 - 45)\% \text{ of total solid} \\ &= \frac{55}{100} \times 137.5 \text{ kg} = 75.625 \text{ kg} \end{aligned}$$

Since digested sludge contains 90% moisture content we have,

Total volume of digested sludge

$$\begin{aligned} &= \left[ \frac{\text{Mass of solids left in digested sludge}}{\text{Density of solids}} \right] + \left[ \frac{\text{Mass of water}}{\text{Density of water}} \right] \\ &= \left[ \frac{75.625}{1.2 \times 1000} + \left( \frac{75.625}{0.1 \times 1000} \right) \times 0.9 \right] \text{ m}^3 = 0.063 + 0.681 = 0.744 \text{ cu m} \end{aligned}$$

Hence, the volume of digested sludge per million litre of waste water = 0.744 cu-m

### Q.3 (b) Solution:

$$\begin{aligned} \text{Water required daily} &= \text{Population} \times \text{Per capita demand} \\ &= 275000 \times 200 \text{ l/c/d} \\ &= 55 \times 10^6 \text{ l/d} = 55 \text{ MLD} \end{aligned} \quad \dots(\text{i})$$

$$\begin{aligned} \text{Filtered water required for backwashing} &= 5\% \\ &= 5\% \text{ of } 55 \text{ MLD} = 2.75 \text{ MLD} \end{aligned} \quad \dots(\text{ii})$$

$$\begin{aligned} \therefore \text{Daily water demand of filtered water} &= (\text{i}) + (\text{ii}) \\ &= 55 + 2.75 = 57.75 \text{ MLD} \end{aligned}$$

Since 30 minutes (i.e., 0.5 hr) is lost daily in back washing the filters, the effective time left for working of filter units = 24 - 0.5 = 23.5 hr per day.

$$\therefore \text{Filtered water required per hour} = \frac{57.75}{23.5} = 2.457 \text{ ML/h}$$

$$\text{Now, filtration rate} = 15 \text{ m}^3/\text{m}^2.\text{h} \quad (\text{Given})$$

$$\therefore \text{Area of filter required} = \frac{\left[ \frac{2.457 \times 10^6}{10^3} \right] \text{m}^3/\text{h}}{15 \text{ m}^3/\text{m}^2 \cdot \text{h}} = 163.8 \text{ m}^2$$

Given, size of filter = 10 m × 4 m

$$\therefore \text{Area of one filter unit} = 10 \times 4 = 40 \text{ m}^2$$

$$\therefore \text{No. of filter units required} = \frac{163.8}{40} = 4.095 \approx 5 (\text{say})$$

Now using one filter unit as stand by unit, total number of filters required = 5 + 1 = 6.

### Q.3 (c) Solution:

Given:

$$D_1 = 0.4 \text{ m}; D_2 = 0.2 \text{ m}, L = 2 \text{ m}, Q = 20 \text{ lit/s} = 0.02 \text{ m}^3/\text{s}$$

#### (i) convective acceleration at middle when $Q = 20 \text{ lit/s}$

In this case, the rate of flow is constant and equal to  $0.02 \text{ m}^3/\text{s}$ . The velocity of flow is in  $x$ -direction only. Hence this is one-dimensional flow and velocity components in  $y$  and  $z$  directions are zero i.e.,  $v = 0, w = 0$ .

$$\therefore \text{Convective acceleration} = u \frac{\partial u}{\partial x} \quad \dots(1)$$

Let us find the value of  $u$  and  $\frac{\partial u}{\partial x}$  at a distance  $x$  from inlet.

The diameter ( $D_x$ ) at a distance  $x$  from inlet or at section X-X is given by,

$$D_x = 0.4 - \frac{0.4 - 0.2}{2} \times x = (0.4 - 0.1x) \text{ m}$$

The area of cross-section ( $A_x$ ) at section X-X is given by,

$$A_x = \frac{\pi}{4} D_x^2 = \frac{\pi}{4} (0.4 - 0.1x)^2$$

Velocity ( $u$ ) at the section X-X in terms of  $Q$  (i.e., in terms of rate of flow)

$$\begin{aligned} u &= \frac{Q}{\text{Area}} = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4} D_x^2} = \frac{4Q}{\pi(0.4 - 0.1x)^2} \\ &= \frac{1.273Q}{(0.4 - 0.1x)^2} = 1.273 Q(0.4 - 0.1x)^{-2} \text{ m/s} \quad \dots(2) \end{aligned}$$

To find  $\frac{\partial u}{\partial x}$ , we must differentiate equation (2) with respect to  $x$ .

$$\begin{aligned}\frac{\partial u}{\partial x} &= \frac{\partial}{\partial x} [1.273Q[0.4 - 0.1x]^{-2}] \\ &= 1.273 \times (-2)Q[0.4 - 0.1x]^{-3} \times (-0.1) \\ \frac{\partial u}{\partial x} &= 0.2546Q [0.4 - 0.1x]^{-3}\end{aligned}$$

So, convective acceleration at  $x = 1$  m

$$\begin{aligned}\frac{u\partial u}{\partial x} &= 1.273Q[0.4 - 0.1x]^{-2} \times 0.2546Q[0.4 - 0.1x]^{-3} \\ &= 0.05335 \text{ m/sec}^2\end{aligned}$$

**Case II:** Total acceleration at middle of pipe at 15th second.

Here  $Q$  changes from  $0.02 \text{ m}^3/\text{s}$  to  $0.04 \text{ m}^3/\text{s}$  in 30 seconds and we need to find the total acceleration at  $x = 1$  m and  $t = 15$  seconds.

Total acceleration = Convective acceleration + Local acceleration (at  $t = 15$  seconds)

The rate of flow at  $t = 15$  seconds is given by

$$\begin{aligned}Q &= Q_1 + \frac{Q_2 - Q_1}{30} \times 15 \text{ where } Q_2 = 0.04 \text{ m}^3/\text{s} \text{ and } Q_1 = 0.02 \text{ m}^3/\text{s} \\ &= 0.02 + \frac{(0.04 - 0.02)}{30} \times 15 = 0.03 \text{ m}^3/\text{s}\end{aligned}$$

The velocity ( $u$ ) and gradient  $\left(\frac{\partial u}{\partial x}\right)$  in terms of  $Q$  are given by equations (2) and (3) respectively.

$$\begin{aligned}\therefore \text{Convective acceleration} &= u \cdot \frac{\partial u}{\partial x} \\ &= [1.273Q (0.4 - 0.1x)^{-2}] \times [0.2546Q (0.4 - 0.1x)^{-3}] \\ &= 1.273 \times 0.2546Q^2 \times (0.4 - 0.1 \times 1)^{-5}\end{aligned}$$

$$\begin{aligned}\therefore \text{Convection acceleration (when } Q = 0.03 \text{ m}^3/\text{s} \text{ and } x = 1 \text{ m)} \\ &= 0.12 \text{ m/s}^2 \quad \dots(4)\end{aligned}$$

$$\begin{aligned}\text{Local acceleration} &= \frac{\partial u}{\partial t} = \frac{\partial}{\partial t} [1.273Q (0.4 - 0.1x)^{-2}] \\ & \quad [\because u \text{ from equation (2) is } u = 1.273Q (0.4 - 0.1x)^{-2}] \\ &= 1.273 \times (0.4 - 0.1x)^{-2} \times \frac{\partial Q}{\partial t}\end{aligned}$$

$$= 1.273 \times (0.3)^{-2} \times \frac{0.02}{30} = 0.00943 \text{ m/s}^2 \quad \dots(5)$$

$$\left[ \therefore \frac{\partial Q}{\partial t} = \frac{Q_2 - Q_1}{t} = \frac{0.04 - 0.02}{30} = \frac{0.02}{30} \right]$$

Hence adding equations (4) and (5), we get total acceleration.

$$\begin{aligned} \therefore \text{Total acceleration} &= \text{Convective acceleration} + \text{Local acceleration} \\ &= 0.12 + 0.00943 = 0.1294 \text{ m/s}^2 \end{aligned}$$

#### Q.4 (a) Solution:

##### (i) Sewer is in full flow condition,

Given,  $Q = 0.624 \text{ m}^3/\text{s}; \quad S = 10 \text{ in } 10,000; \quad n = 0.012$

From Manning's formula,  $Q = \frac{1}{n} \times A \times R^{2/3} \times S^{1/2}$

$$\Rightarrow 0.624 = \frac{1}{0.012} \times \left(\frac{\pi D^2}{4}\right) \times \left(\frac{D}{4}\right)^{2/3} \times \left(\frac{10}{10,000}\right)^{1/2}$$

$$\Rightarrow D = 0.902 \text{ m}$$

So, diameter of sewer = 0.902 m

##### (ii)

We know,  $\frac{d}{D} = \frac{1 - \cos(\alpha/2)}{2}$

$$\Rightarrow 0.4 = \frac{1 - \cos(\alpha/2)}{2}$$

$$\Rightarrow \alpha = 156.93^\circ$$

$$\text{Proportionate area} = \frac{a}{A} = \left[ \frac{\alpha}{360} - \frac{\sin \alpha}{2\pi} \right]$$

$$\frac{a}{A} = \left[ \frac{156.93}{360} - \frac{\sin 156.93}{2\pi} \right] = 0.374$$

$$\frac{p}{P} = \frac{\alpha}{360} = 0.436$$

So  $\frac{r}{R} = \frac{a \times P}{A \times p} = \frac{0.374}{0.436} = 0.858$

Now,  $\frac{q}{Q} = \frac{a}{A} \times \left(\frac{r}{R}\right)^{2/3} = 0.374 \times (0.858)^{2/3} = 0.338$

So,  $q = 0.338 \times 0.624 = 0.21 \text{ m}^3/\text{sec}$

$$\frac{v}{V} = \left(\frac{r}{R}\right)^{2/3} = 0.9029$$

$$v = 0.9029 \times \frac{0.624}{\frac{\pi}{4} \times (0.902)^2} = 0.882 \text{ m/s}$$

**Q.4 (b) Solution:**

(i)

A cross drainage work is a structure which is constructed at the crossing of a canal and a natural drain, so as to dispose of drainage water without interrupting the continuous canal supplies. In whatever way the canal is aligned, such cross drainage works generally become unavoidable. A cross-drainage work is generally a costly construction and must be avoided as far as possible. Since a watershed canal crosses minimum number of drains and thus such an alignment is preferred to a contour canal which crosses maximum number of drains.

**Types of cross drainage works are as follows:**

(i) *Aqueduct and syphon aqueduct:*

- In these works, the canal is taken over the natural drain, such that the drainage water runs below the canal either freely or under syphoning pressure.
- When the HFL of the drain is sufficiently below the bottom of the canal, so that the drainage water flows freely under gravity, the structure is known as an aqueduct.
- If the HFL of the drain is higher than the canal bed and the water passes through the aqueduct barrels under syphonic action, the structure is known as syphon aqueduct.

(ii) *Super Passage and Syphon:*

- In these works, the drain is taken over the canal such that the canal water runs below the drain either freely or under syphoning pressure.
- When FSL of the canal is sufficiently below the bottom of the drain trough, so that the canal water flows freely under gravity, the structure is known as a super passage.
- If the FSL of the canal is sufficiently above the bed level of the drainage trough, so that the canal flows under syphonic action under the trough, the structure is known as a canal syphon or a syphon.

(iii) *Level Crossing:*

- In this type of cross-drainage work, the canal water and drain water are allowed to intermingle with each other.



- A level crossing is provided generally when a large canal and a large drain approach each other practically at the same level.

(iv) *Inlets and Outlets:*

- An inlet is a structure constructed in order to allow the drainage water to enter the canal and get mixed with the canal water and thus to help in augmenting canal supplies.
- Such a structure is generally adopted when the drainage discharge is small and the drain crosses the canal with its bed level equal to or slightly higher than the canal FSL.
- When the drainage discharge is high or if the canal is small, so that the canal cannot take the entire drainage water, an outlet may sometimes be constructed to escape out the additional discharge at a suitable site, a little downstream along the canal.

(ii)

$$F_p = \int_0^t f_p dt \quad \text{where } f_p = 3.0 + e^{-2t}$$

(a) In the first 0.5 hour

$$\begin{aligned} F_{p1} &= \int_0^{0.5} (3.0 + e^{-2t}) dt = \left[ 3.0t - \frac{1}{2}e^{-2t} \right]_0^{0.5} \\ &= [(3.0 \times 0.5) - (1/2)(e^{-2 \times 0.5})] - [-(1/2)] \\ &= (1.5 - 0.184) + 0.5 = 1.816 \text{ cm} \end{aligned}$$

(b) In the second 0.5 hour

$$\begin{aligned} F_{p2} &= \int_{0.5}^{1.0} (3.0 + e^{-2t}) dt = \left[ 3.0t - \frac{1}{2}e^{-2t} \right]_{0.5}^{1.0} \\ &= [(3.0 \times 1.0) - (1/2)(e^{-2})] - [(3.0 \times 0.5) - (1/2)(e^{-2 \times 0.5})] \\ &= (3.0 - 0.0677) - (1.5 - 0.184) = 1.616 \text{ cm} \end{aligned}$$

**Q.4 (c) Solution:**

Given:

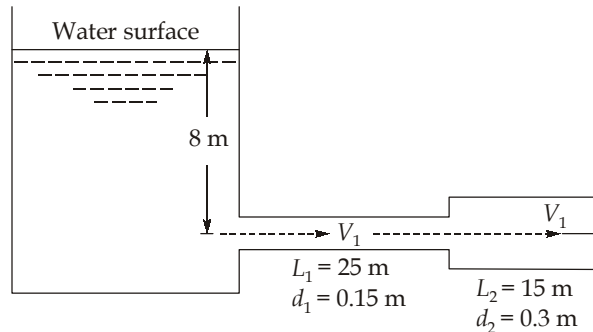
Total length of pipe,  $L = 40 \text{ m}$

Length of 1<sup>st</sup> pipe,  $L_1 = 25 \text{ m}$

Diameter of 1<sup>st</sup> pipe,  $d_1 = 150 \text{ mm} = 0.15 \text{ m}$

- Length of 2<sup>nd</sup> pipe,  $L_2 = 40 - 25 = 15 \text{ m}$
- Diameter of 2<sup>nd</sup> pipe,  $d_2 = 300 \text{ mm} = 0.3 \text{ m}$
- Height of water,  $H = 8 \text{ m}$
- Co-efficient of friction,  $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in figure and taking reference line passing through the centre of pipe,



$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

$$\Rightarrow 8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f1} + h_e + h_{f2} \quad \dots(i)$$

where  $h_i = \text{loss of head at entrance} = 0.5 \frac{V_1^2}{2g}$

$$h_{f1} = \text{head lost due to friction in pipe 1} = \frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$h_e = \text{head lost due to sudden enlargement} = \frac{(V_1 - V_2)^2}{2g}$$

$$h_{f2} = \text{head lost due to friction in pipe 2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$$

But from continuity equation, we have

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_1 = \frac{A_2 V_2}{A_1} = \frac{\frac{\pi}{4} d_2^2 \times V_2}{\frac{\pi}{4} d_1^2} = \left(\frac{d_2}{d_1}\right)^2 \times V_2 = \left(\frac{0.3}{0.15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of  $V_1$  in different head losses, we have

$$h_i = \frac{0.5V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f1} = \frac{4 \times 0.01 \times 25 \times (4V_2)^2}{0.15 \times 2 \times g} = \frac{4 \times 0.01 \times 25 \times 16}{0.15} \times \frac{V_2^2}{2g} = 106.67 \frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f2} = \frac{4 \times 0.01 \times 15 \times V_2^2}{0.3 \times 2g} = \frac{4 \times 0.01 \times 15}{0.3} \times \frac{V_2^2}{2g} = 2.0 \frac{V_2^2}{2g}$$

Substituting the values of these losses in equation (i), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67 \frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \frac{V_2^2}{2g}$$

$$\Rightarrow 8.0 = \frac{V_2^2}{2g} [1 + 8 + 106.67 + 9 + 2] = 126.67 \frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$$\therefore \text{Rate of flow, } Q = A_2 \times V_2 = \frac{\pi}{4} (0.3)^2 \times 1.113 = 0.07867 \text{ m}^3/\text{s} = 78.67 \text{ litres/s}$$

### Section B

#### Q.5 (a) Solution:

(i) Correction for pull:

$$C_p = \frac{(P - P_0)L}{AE} = \frac{(23 - 15) \times 1000}{0.0645 \times 2.11 \times 10^6} = 0.05878 \text{ m (+)ve}$$

(ii) Correction for temperature:

$$\begin{aligned} C_t &= \alpha(T_m - T_0)L \\ &= 11.5 \times 10^{-6} \times (35 - 15) \times 1000 \\ &= 0.23 \text{ m (+ve)} \end{aligned}$$

(iii) Correction for slope:

$$C_{sl} = \frac{h^2}{2L} = \frac{2^2}{2 \times 1000} = 2 \times 10^{-3} \text{ m (-ve)}$$

(iv) Correction for mean sea level:

$$C_R = \frac{h}{R} L = \frac{1000 \times 1000}{6400 \times 1000} = 0.15625 \text{ m (-ve)}$$

$$\begin{aligned} \therefore \text{Total correction} &= 0.05878 + 0.23 - 2 \times 10^{-3} - 0.15625 \\ &= 0.13053 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Corrected length of the base line} &= 1000 + 0.13053 \\ &= 1000.13053 \simeq 1000.131 \text{ m} \end{aligned}$$

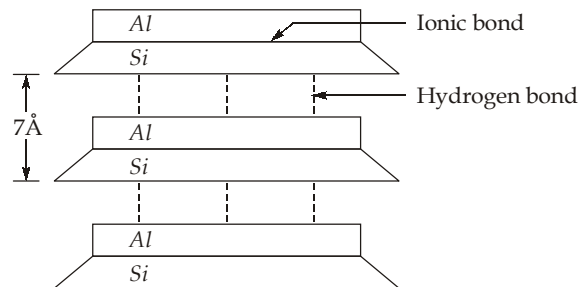
### Q.5 (b) Solution:

The clay minerals are basically composed of tiny crystalline substances of one or more members of a small group of minerals-commonly known as clay minerals. These clay minerals mainly evolved from the chemical weathering of certain rock forming minerals.

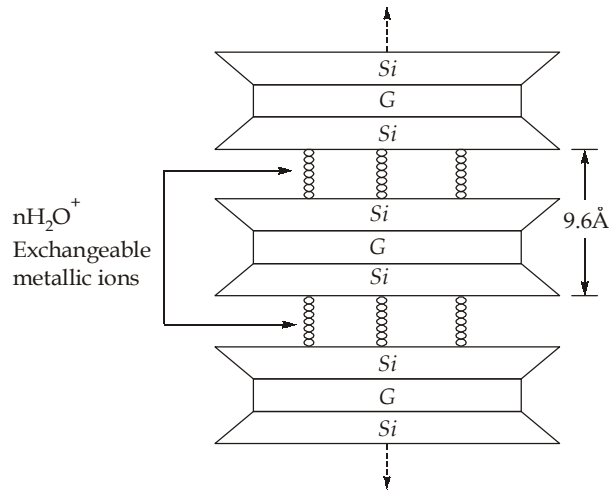
Important clay minerals are

1. Kaolinite
2. Montmorillonite
3. Illite

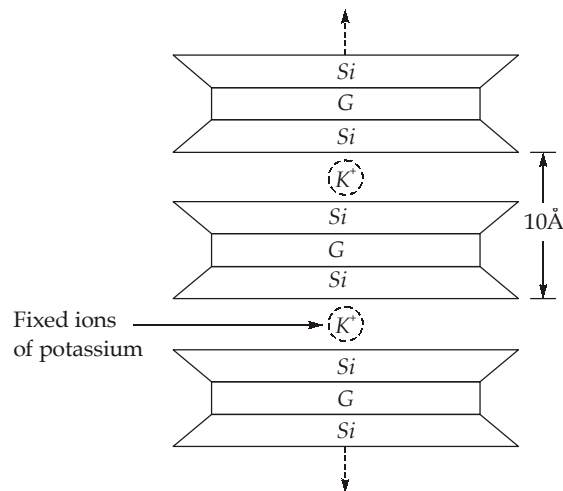
1. **Kaolinite** : The kaolinite structural unit consists of alternating layers of silica tetrahedral units with the tips embedded in alumina. The bonding is through hydrogen bond which results in considerable strength.



2. **Montmorillonite** : It is also called smectite. The structural unit of the mineral is composed of two silica sheets and one alumina sheet. The interlayer bonding between the top of silica sheets is mainly due to van der Waals forces which is very weak.



3. **Illite** : The Illite mineral is also a 2 : 1 mineral similar to montmorillonite. In Illite, potassium ion bonds are present and therefore Illite does not swell as much in the presence of water as montmorillonite.



Clay minerals are electrically charged with negative ions. They have large specific surface area and are very flaky in shape. Clay attracts dipolar water towards its surface by adsorption. This induces plasticity in clay. Therefore plasticity increases with specific surface area. Water in clays exhibits negative pressure due to which two particles are held close to each other due to this apparent cohesion developed in clays.

**Q.5 (c) Solution:**

Given:

$$N_{obs} = 25$$

$$\gamma = 20 \text{ kN/m}^3$$

$$\gamma_w = 9.81 \text{ kN/m}^3$$

(i) Overburden correction

$$\begin{aligned} \text{Effective stress at 12 m depth} &= 12 \times 20 - 10 \times 9.81 \\ &= 141.9 \text{ kN/m}^2 \end{aligned}$$

$$N' = C_N \cdot N_{\text{obs}}$$

$C_N$  from figure corresponding to  $\bar{\sigma} = 141.9 \text{ kN/m}^2$  is

$$C_N = 0.85$$

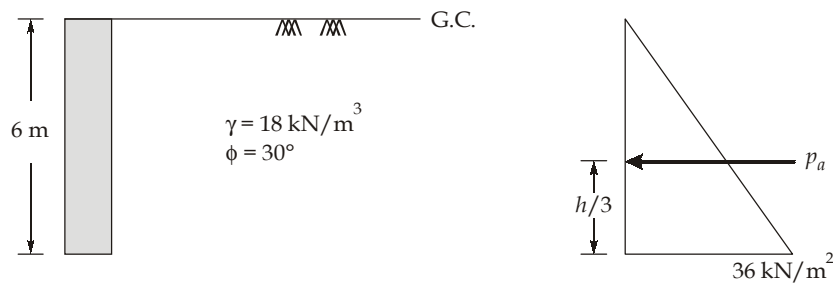
$$\therefore N' = 0.85 \times 25 = 21.25 > 15$$

(ii) Dilatency correction

$$N'' = 15 + \frac{(N' - 15)}{2} = 15 + \frac{(21.25 - 15)}{2} = 18.125$$

So corrected  $N$  value is 18.

**Q.5 (d) Solution:**



$$k_a = \tan^2 \left( 45 - \frac{30}{2} \right) = \frac{1}{3}$$

$$\text{At } z = 0, \quad p_a = 0$$

$$\text{At } z = 6 \text{ m} \quad p_a = \frac{1}{3} \times 18 \times 6 = 36 \text{ kN/m}^2$$

$$\Rightarrow P_a = \frac{1}{2} \times 36 \times 6 = 108 \text{ kN/m}$$

$$\text{Location of support from base} = \frac{h}{3} = \frac{6}{3} = 2 \text{ m}$$

$$\text{Force on support} = 108 \text{ kN/m}$$

**Q.5 (e) Solution:**

(i) Given: SSD (80 m) <  $L_C$  (250 m)

$$\therefore \text{Set back distance, } m = R - (R - d) \cos \frac{\alpha}{2}$$

$$\begin{aligned} \text{where,} \quad \frac{\alpha}{2} &= \frac{S}{2(R-d)} \times \frac{180}{\pi} \\ \Rightarrow \quad \frac{\alpha}{2} &= \frac{180 \times 80}{2\pi(350-2)} = 6.586^\circ \\ \therefore \quad m &= 350 - (350-2) \times \cos(6.586^\circ) \\ m &= 4.297 \text{ m} \end{aligned}$$

(ii) Given: SSD (300 m) >  $L_C$  (250 m)

$$\begin{aligned} \therefore \quad m &= R - (R-d) \cos \frac{\alpha}{2} + \left( \frac{S-L_C}{2} \right) \sin \frac{\alpha}{2} \\ \frac{\alpha}{2} &= \frac{L_C}{2(R-d)} \times \frac{180}{\pi} \\ \Rightarrow \quad \frac{\alpha}{2} &= \frac{250 \times 180}{2(350-2) \times \pi} = 20.58^\circ \\ \therefore \quad m &= 350 - (350-2) \cos(20.58) + \left[ \frac{300-250}{2} \right] \times \sin(20.58) \\ \Rightarrow \quad m &= 32.996 \text{ m} \simeq 33 \text{ m} \end{aligned}$$

### Q.6 (a) Solution:

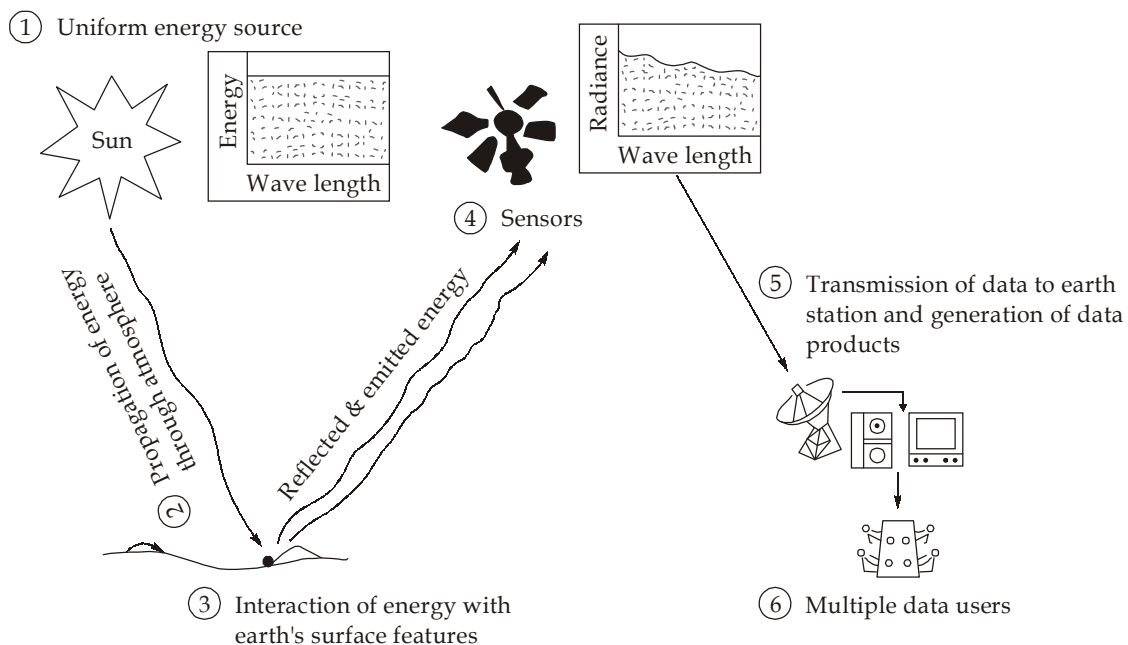
An idealised remote sensing system consists of the following stages as shown in figure below.

1. Energy source.
2. Propagation of energy through atmosphere.
3. Energy interaction with earth's surface features.
4. Airborne/space borne sensors receiving the reflected and emitted energy.
5. Transmission of data to earth station and generation of data produce.
6. Multiple-data users.

1. **The energy source:** The uniform energy source provides energy over all wave lengths. The passive RS system relies on sun as the strongest source of EM energy and measures energy that is either reflected and/or emitted from the earth's surface features. However, active RS systems use their own source of EM energy.
2. **Propagation of energy through the atmosphere:** The EM energy, from the source passes through the atmosphere on its way to earth's surface. Also, after reflection from the earth's surface, it again passes through the atmosphere on its way to sensor. The atmosphere modifies the wave length and spectral distribution of energy to

some extent and this modification varies particularly with the wave length.

3. **Interaction of energy with surface feature of the earth:** The interaction of EM energy, with earth's surface features generates reflected and/or emitted signals (spectral response patterns or signatures). The spectral response patterns play a central role in detection, identification and analysis of earth's surface material.
4. **Air borne/space borne sensors:** Sensors are electromagnetic instruments designed to receive and record transmitted energy. They are mounted on satellites, aeroplanes or even on balloons. The sensors are highly sensitive to wave length, yielding data on the absolute brightness from the object as a function of wavelength.
5. **Transmission of data to earth station and data product generation:** The data from the sensing system is transmitted to the ground based earth station along with the telemetry data. The real-time (instantaneous) data handling system consists of high density data tapes for recording and visual devices (such as television) for quick look displays. The data products are mainly classified into two categories:
  - (i) Pictorial or photographic product (analogue)
  - (ii) Digital product.
6. **Multiple data users:** The multiple data users are those who have knowledge of great depth, both of their respective disciplines as well as of remote sensing data and analysis techniques. The same set of data becomes various forms of information for different users with the understanding of their field and interpretation skills.



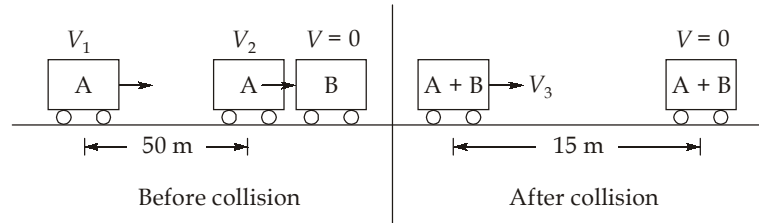


## Q.6 (b) Solution:

(i)

Assumptions:

1. Collision is perfectly plastic.
2. Brake efficiency = 100%



After collision for vehicle (A) + (B)

$$V_3^2 = 2gf \times S_2$$

$$V_3 = \sqrt{2 \times 9.81 \times 0.6 \times 15} = 13.288 \text{ m/s}$$

Applying momentum conservation equation,

$$m_A V_2 + m_B \times 0 = (m_A + m_B) V_3$$

$$\Rightarrow V_2 = \frac{(m_A + 0.6m_A)}{m_A} \times 13.288$$

$$\Rightarrow V_2 = 21.26 \text{ m/s}$$

Before collision for vehicle (A)

$$V_1^2 - V_2^2 = 2gfS_1$$

$$\Rightarrow V_1 = \sqrt{(21.26)^2 + (2 \times 9.81 \times 0.6 \times 50)}$$

$$\Rightarrow V_1 = 32.26 \text{ m/s}$$

(ii)

Gauge distance for BG track = 1.676 m

$$1. \quad \text{Curve lead (CL)} = 2 \text{ GN} = 2 \times 1.676 \times 12 = 40.224 \text{ m}$$

$$2. \quad \text{Switch lead (SL)} = \sqrt{2R_0 d}$$

where;  $R_0 = 1.5 \text{ G} + 2 \text{ GN}^2$  and  $d = \text{heel divergence} = 0.1 \text{ m}$  (Given)

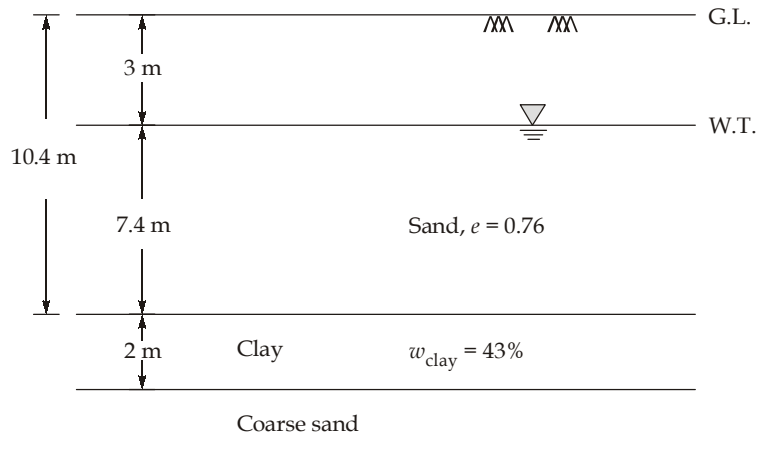
$$\therefore R_0 = 1.5 \times 1.676 + 2 \times 1.676 \times 12^2 = 485.202 \text{ m}$$

$$\therefore \text{SL} = \sqrt{2 \times 485.202 \times 0.10} = 9.85 \text{ m}$$

$$3. \quad \text{Lead} = \text{CL} - \text{SL} \\ = 40.224 - 9.85 = 30.37 \text{ m}$$

**Q.6 (c) Solution:**

(i)



$$\gamma_{sat} \text{ (sand)} = \frac{G_s + e}{1 + e} \gamma_w = \frac{2.7 + 0.76}{1 + 0.76} \times 9.81 = 19.2856 \approx 19.29 \text{ kN/m}^3$$

For clay,

$$S.e. = Gw$$

But

$$S = 1$$

$\therefore$

$$e_{clay} = G_s w = 2.7 \times 0.43 = 1.161$$

$\therefore$

$$\gamma_{sat} \text{ (clay)} = \frac{(G_s + S.e.)}{(1 + e)} \gamma_w = \frac{2.7 + 1.161}{1 + 1.161} \times 9.81 = 17.53 \text{ kN/m}^3$$

Effective stress at middle of clay layer

$$\begin{aligned} \sigma'_0 &= 3 \times 19.29 + (19.29 - 9.81) \times 7.4 + (17.53 - 9.81) \times 1 \\ &= 135.742 \text{ kN/m}^2 \end{aligned}$$

$\therefore$  Statement,

$$\Delta H = \frac{c_c H_0}{1 + e_0} \log_{10} \frac{\sigma'_0 + \Delta \sigma}{\sigma'_0}$$

$\Rightarrow$

$$\Delta H = \frac{0.3 \times 2000}{1 + 1.161} \log_{10} \frac{135.742 + 140}{135.742}$$

$\Rightarrow$

$$\Delta H = 85.5 \text{ mm}$$

(ii)

$$\text{OCR} = 1.5$$

Past maximum vertical effective stress,

$$\begin{aligned} \sigma'_p &= 1.5 \times 135.742 \text{ kN/m}^2 \\ &= 203.613 \text{ kN/m}^2 \end{aligned}$$

Also,  $\sigma'_0 + \Delta\sigma = 135.742 + 140 = 275.742 \text{ kN/m}^2$

$\therefore (\sigma'_0 + \Delta\sigma) > \sigma'_p$

$\therefore \Delta = \frac{H_0}{1 + e_0} \left[ c_r \log_{10} \frac{\sigma'_p}{\sigma'_0} + c_c \log_{10} \frac{\sigma'_0 + \Delta\sigma}{\sigma'_p} \right]$

$$\Delta = \frac{2000}{1 + 1.161} \left[ 0.05 \log_{10} \frac{203.613}{135.742} + 0.3 \log_{10} \frac{275.742}{203.613} \right]$$

$$\Delta = 44.71 \text{ mm}$$

**Q.7 (a) Solution:**

(i)

$\therefore \text{FOS} = \frac{C_u + \gamma Z \cos^2 i \tan \phi}{\gamma Z \sin i \cos i}$

$\therefore$  For clay  $\phi = 0$

$\therefore \text{FOS} = \frac{C_u}{\gamma Z \sin i \cos i}$

Since the slope has failed,

$\therefore \text{FOS} = 1$

$\Rightarrow C_u = 18.5 \times 2 \times \sin 38^\circ \cos 38^\circ$   
 $= 17.95 \text{ kPa}$

So undrained shear strength = 17.95 kPa

(ii)

Let  $\sigma_z$  at a point  $P$  vertically below  $A$  is required.

$\therefore$  Stresses at  $P$  will be caused due to

(i) Point load of leg at  $A$ , for which  $r = r_A = 0$

(ii) Point load of leg at  $B$ , for which  $r = r_B = 6 \text{ m}$

(iii) Point load of leg at  $C$ , for which  $r = r_C = 6 \text{ m}$

$\therefore (\sigma_z)_{P,A} = \frac{3Q}{2\pi Z^2} \left[ \left\{ \frac{1}{1 + \left(\frac{r_A}{z}\right)^2} \right\}^{5/2} + 2 \left\{ \frac{1}{1 + \left(\frac{r_{B,C}}{z}\right)^2} \right\}^{5/2} \right]$

where,  $Q = \frac{600}{3} = 200 \text{ kN}$

$\therefore (\sigma_z)_{P,A} = \frac{3 \times 200}{2\pi(5)^2} \left[ \left\{ \frac{1}{1 + (0)^2} \right\}^{5/2} + 2 \left\{ \frac{1}{1 + (6/5)^2} \right\}^{5/2} \right]$

$$= 3.8197 (1 + 0.215)$$

$$(\sigma_z)_{P,A} = (\sigma_z)_{P,B} = (\sigma_z)_{P,C} = 4.64 \text{ kN/m}^2$$

**Q.7 (b) Solution:**

(i)

The F.S. at station 2 is determined from the rise given.

$$\text{F.S.} = 2.150 - 0.500 = 1.650 \text{ m}$$

$$\text{Fall at station 3} = 2.345 - 1.645 = 0.700 \text{ m}$$

$$\text{Rise at station 4} = 2.345 - 1.965 = 0.380 \text{ m}$$

$$\text{B.S. at station 4} = 1.825 - 0.400 = 1.425 \text{ m}$$

$$\text{Fall at station 8} = 2.100 - 1.690 = 3.790 \text{ m}$$

$$\text{Rise at station 9} = 449.100 - 448.060 = 1.040 \text{ m}$$

$$\text{I.S. at station 6} = 2.050 - 1.950 = 0.100 \text{ m}$$

$$\text{F.S. at station 7} = 0.100 - 0.120 = -0.020 \text{ m}$$

Station	B.S.(m)	I.S. (m)	F.S. (m)	Rise(m)	Fall (m)	R.L.(m)	Remarks
1	2.150	-				450.00	B.M.1
2	1.645	-	1.650	0.500		450.500	
3		2.345			0.700	449.800	
4	1.425	-	1.965	0.380		450.180	
5	2.050	-	1.825		0.400	449.780	
6		0.100		1.950		451.730	
7	-1.690	-	-0.020	0.120		451.850	B.M.2 staff
8	2.865	-	2.100		3.790	448.060	inverted
9		-	1.825	1.040		449.100	B.M.3
	8.445		9.345	3.990	4.890		

$$\sum \text{B.S.} - \sum \text{F.S.} = \sum \text{Rise} - \sum \text{Fall}$$

$$= \text{Last R.L.} - \text{First R.L.}$$

Thus,  $8.445 - 9.345 = 3.990 - 4.890 = 449.100 - 450.00 = -0.900 \quad (\text{OK})$

(ii)

(a) On verifying the observed bearing, it is found that the FB and BB of line DE differ by exactly  $180^\circ$ . So, the stations D and E are free from local attraction and the observed FB of EA and observed BB of CD are correct.

(b) The actual BB of EA should be

$$330^{\circ}15' - 180^{\circ}0' = 150^{\circ}15'$$

But the observed BB of EA  $147^{\circ}45'$ .

So, a correction of  $(150^{\circ}15' - 147^{\circ}45') = +2^{\circ}30'$  should be applied at A.

(c) Correct FB of AB =  $191^{\circ}45' + 2^{\circ}30' = 194^{\circ}15'$

Therefore, the actual correct BB of AB should be

$$194^{\circ}15' - 180^{\circ}00' = 14^{\circ}15'$$

But Observed BB of AB =  $13^{\circ}0'$

So, a correction of  $(14^{\circ}15' - 13^{\circ}0') = +1^{\circ}15'$  should be applied at B.

(d) Correct FB of BC =  $39^{\circ}30' + 1^{\circ}15' = 40^{\circ}45'$

$$\therefore \text{Correct BB of BC should be} = 40^{\circ}45' + 180^{\circ}0' = 220^{\circ}45'$$

But observed BB of BC =  $222^{\circ}30'$

So, a correction of  $(220^{\circ}45' - 222^{\circ}30') = -1^{\circ}45'$  should be applied at C.

(e) Correct FB of CD =  $22^{\circ}15' - 1^{\circ}45' = 20^{\circ}30'$

Therefore, the correct BB of CD should be

$$20^{\circ}30' + 180^{\circ}0' = 200^{\circ}30'$$

which is equal to the observed BB of CD.

So, D is free from local attraction, which also tallies with the remark made at the beginning.

The result is tabulated as follows:

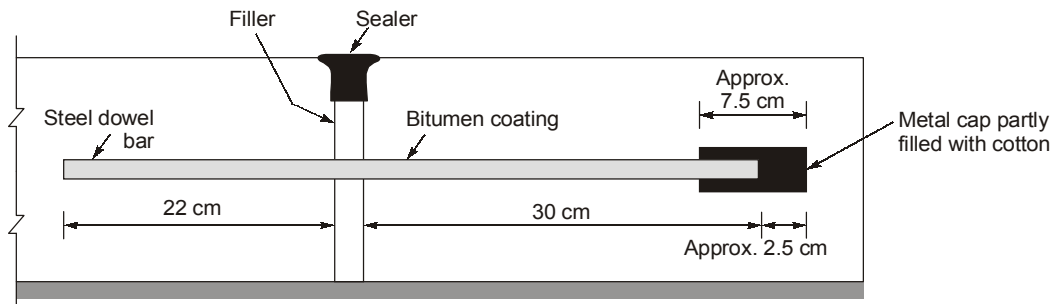
Line	Observed		Correction	Correct		Remarks
	FB	BB		FB	BB	
AB	$191^{\circ}45'$	$13^{\circ}00'$	$+2^{\circ}30'$ at A	$194^{\circ}15'$	$14^{\circ}15'$	
BC	$39^{\circ}30'$	$222^{\circ}30'$	$+1^{\circ}15'$ at B	$40^{\circ}45'$	$220^{\circ}45'$	
CD	$22^{\circ}15'$	$200^{\circ}30'$	$-1^{\circ}45'$ at C	$20^{\circ}30'$	$200^{\circ}30'$	
DE	$242^{\circ}45'$	$62^{\circ}45'$	$0^{\circ}$ at D	$242^{\circ}45'$	$62^{\circ}45'$	Station D is free from local attraction
EA	$330^{\circ}15'$	$147^{\circ}45'$	$0^{\circ}$ at E	$330^{\circ}15'$	$150^{\circ}15'$	Station E is also free from local attraction

### Q.7 (c) Solution:

(i)

Expansion joints and contraction joints are provided in transverse direction of traffic.

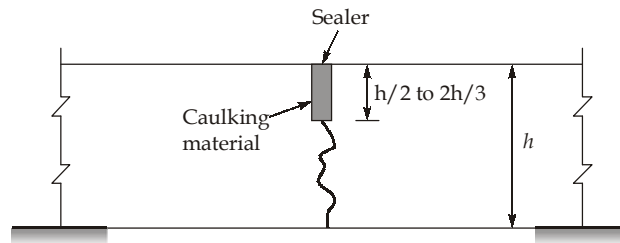
#### A. Expansion joints:



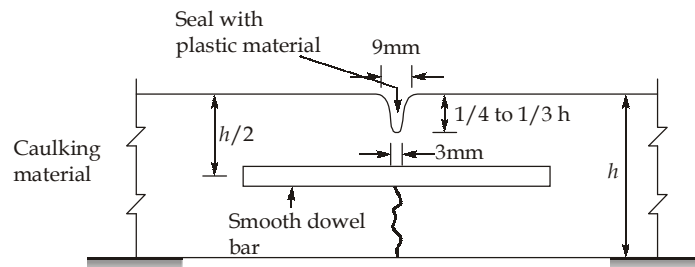
These joints are provided to allow for expansion of the slabs due to rise in slab temperature above the construction temperature of the cement concrete. Expansion joints in India are provided at an interval of 50 to 60 meter for smooth interface laid in winter and 90 to 120 meter for smooth interface laid in summer. However, for rough interface, the spacing between expansion joints may be 140 m. A typical expansion joint is shown in above figure. The approximate gap width for this type of joints is from 20 to 25 mm.

It may be stated here that the break in slab continuity forming a joint adds a weaker plane in the cement concrete pavement. The stresses include due to wheel loads at such joints are of very high order at the edge and corner regions. In order to strengthen these locations, following measures are adopted:

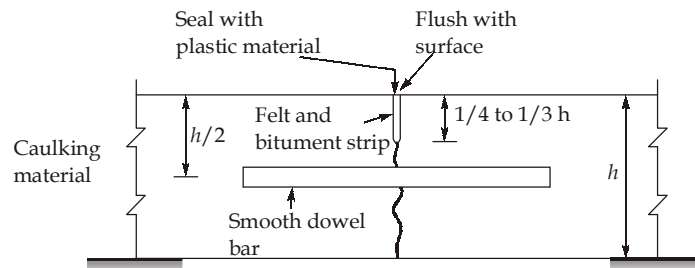
The load transfer across the transverse joint is carried out through a system of reinforcement provided at suitable intervals projecting in the concrete in longitudinal direction upto 60 cm length. Such a device is named as **Dowel Bar**. In the expansion joint, thus load transfer is affected through a system of dowel bars. Dowel bars are embedded and kept fixed in concrete at one end and other end is kept free to expand or contract by providing a thin coating of bitumen over it. Metal cap is provided at this end to offer a space of about 2.5 cm for movements during expansion. In the design, 40 percent of wheel load is expected to be taken up by the group of dowel bars and transferred to the adjoining slab. Spacing between the dowel bars is generally adopted as 30 cm. The size of the dowel varies with pavement thickness and it ranges between 20 to 30 mm. The total length of dowel bar varies between 40 cm to 73 cm depending upon the dowel diameter.

**B. Contraction joints:**

(a) Dummy Joint



(b) Contraction joint with Dowel bar



(c) Contraction joint with Dowel bar

Contraction joints are provided to permit the contraction of the slab. These joints are spaced closer than expansion joints. Load transference at the joints is provided through the physical interlocking by the aggregates projecting out at the joint faces. As per IRC specifications, the maximum spacing of contraction joints in unreinforced cement concrete slabs is 4.5 m and in reinforced slab of thickness 20 cm is about 14 meter. Since it is recommended to provide contraction joints at close spacing, there seems to be no need of providing any load transference, as mainly, this will be done by the aggregate interlocking. For added safety, some agencies recommended the use of dowel bars which are fully bounded in the concrete.

(ii)

The requirements of highway drainage system are as under:

- The surface water from the carriageway and shoulder should effectively be drained off without allowing it to percolate to the subgrade.
- The surface water from the adjoining land should be prevented from entering the roadway.
- The side drain should have sufficient capacity and longitudinal slope to carry away all the surface water collected.
- Flow of surface water across the road and shoulders and along slopes should not cause formation of cross ruts or erosion.
- Seepage and other sources of under ground water should be drained off by the subsurface drainage system.
- Highest level of ground water table should be kept well below the level of subgrade, preferably by atleast 1.2 m.
- In waterlogged areas, special precautions should be taken, especially if detrimental salts are present or if flooding is likely to occur.

**Q.8 (a) Solution:**

$$C_u = \frac{q_u}{2} = \frac{70}{2} = 35 \text{ kN/m}^2$$

$$\text{Permissible, } C_u = \frac{35}{3} \text{ kN/m}^2$$

$$\text{Length of pile} = 10 \text{ m}$$

$$\text{Let the diameter of pile} = 0.5 \text{ m} = 50 \text{ cm}$$

$$\text{Let Spacing of piles} = 3d = 3 \times 50 = 150 \text{ cm}$$

$$\text{Let the number of piles} = n$$

Considering the piles to act individually, the load at failure is given by

$$Q_{\text{safe}} = \frac{nQ_{up}}{F} = \frac{n \left[ CN_c \frac{\pi}{4} d^2 + \alpha C \pi d L \right]}{3}$$

$$\Rightarrow 3000 = n \times \left[ \frac{35}{3} \times 9 \times \frac{\pi}{4} \times 0.5^2 + 0.8 \times \frac{35}{3} \times \pi \times 0.5 \times 10 \right]$$

$$\Rightarrow n = \frac{3000}{167.23} = 17.94$$



For square arrangement keep  $n = 16$

The modified length  $L$  will then have to be increased by the ratio  $\frac{17.94}{16}$

$$\therefore L' = 10 \times \frac{17.94}{16} = 11.21 \text{ m} \quad \text{Adopt, } L = 12 \text{ m}$$

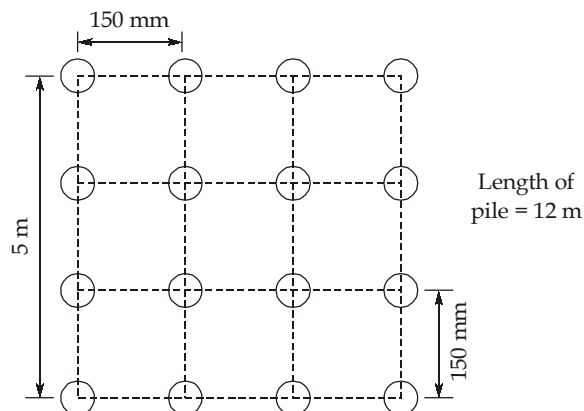
**Check for group action**

$$B = 3s + d = 3 \times 150 + 50 = 500 \text{ cm} = 5 \text{ m}$$

$$\therefore \text{Load taken by group action} = 4BL \cdot C + A_b \cdot CN_c$$

$$A_b = B^2 = (5)^2 = 25 \text{ m}^2, \quad N_c = 9 \text{ (for clay)}$$

$$\begin{aligned} \therefore Q_{vg} &= \left( 4 \times 5 \times 12 \times \frac{35}{3} \right) + 25 \times \frac{35}{3} \times 9 \\ &= 5425 \text{ kN} > 3000 \text{ kN} \quad \text{Hence safe} \end{aligned}$$



**Q.8 (b) Solution:**

(i)

Let  $s =$  Scale of the photograph for datum elevation

$$\therefore s = \frac{f}{H} = \frac{150}{600} = \frac{1}{4000}$$

For the datum elevation, we have,

$$\frac{B}{b} = \frac{H}{f}$$

$$\Rightarrow B = \frac{H}{f} b = \frac{b}{s} = 4000 \times \frac{6.375}{100} = 255 \text{ m}$$

The parallaxes for the top and the bottom of the chimney are calculated as,

$$p = \frac{Bf}{H-h}$$

For the bottom of the chimney,  $h = 0$  (since the bottom of the chimney is on the datum), and hence

$$p_1 = \frac{255 \times 150 (\text{mm})}{600} = 63.75 \text{ mm}$$

For the top of the chimney,  $h = 120 \text{ m}$

$$\therefore p_2 = \frac{255 \times 150 (\text{mm})}{(600 - 120)} = 79.69 \text{ m}$$

Hence difference of parallax is

$$\Delta p = (p_2 - p_1) = 79.69 - 63.75 = 15.94 \text{ mm}$$

**Check:**

$$\begin{aligned} \Delta h &= \frac{H \Delta p}{b + \Delta p} \\ &= \frac{600 \text{ m} \times 15.94 (\text{mm})}{63.75 (\text{mm}) + 15.94 (\text{mm})} = 120.015 \text{ m} \approx 120 \text{ m} \end{aligned}$$

which is the same as the given height of the chimney.

**(ii)**

Among various types of rocks, igneous rocks are inherently very competent and desirable for different civil engineering purposes. No other rock possesses all the desirable properties so as to make it ideally suitable for various types.

The requirement of good building stones are strength, durability (including frost resistance and fire resistance), colour, appearance, workability and its easy availability. These properties of rocks except the last one are, in turn, a function of their minerals, texture, structure, grain size, porosity and permeability.

**Minerals:** The type of minerals present in the rocks influence colour, durability and to some extent strength. The igneous rocks are composed mainly of silicate minerals. Among various minerals, silicate minerals are the most durable. Further, rocks rich in silica content are pleasingly light coloured as in the case of granite. All minerals of common igneous rocks are relatively very hard (quartz 7, feldspars 6, pyroxenes and amphiboles 5-6) and do not have cleavage to any harmful extent. These characters contribute to the strength of the rock and resistance to abrasion of rocks.

**Structure:** Since igneous rocks are formed out of solidification of a melt, they are

necessarily dense, compact and massive (except vesicular rocks). In other words, these rocks do not have any internal openings or hollow nature. This contributes to the strength and heaviness of these rocks. The same factor is also responsible for non-porous and impermeable character of these rocks which do not contribute to possible decrease in strength and durability. Impervious rocks are less prone to frost-attack. When rocks are porous, they naturally become permeable (with some exceptions). When rocks are saturated with water, it makes the interior of the rock accessible to percolating solutions, thereby making it susceptible to decay and decomposition. Both the factors, i.e., saturation of rocks with moisture and internal decay of rocks, substantially reduce the strength, toughness and durability of rocks. Further, igneous rocks do not have an inherent weakness due to the occurrence of bedding planes (as in many sedimentary rocks) or mineral alignment (as in many metamorphic rocks). From the workability point of view, of course, massive igneous rocks are difficult to use because of their toughness and hardness. However, suitably placed joints are not ruled out to enable these rocks to be worked out with less difficulty. This is so because joints are very common in all types of rocks. Mural joints in granites, columnar joints in basalts and sheet joints in all types of rocks are well known. Rift and grain are the microfractures occurring in association with mural joints in granites. These facilitate the dressing of quarried rocks into smaller blocks.

Igneous rocks by virtue of their texture and minerals present in them have the ability to take very good polish and thus have become increasingly popular for face work too. Above all, constructions made of these rocks have a majestic appearance.

#### Q.8 (c) Solution:

(i)

**The rational methods of foundation design which are being used now to reduce or prevent the effects of swelling can be grouped into three categories, namely,**

(a) isolating the structure from the swelling soil, (b) designing a structure to withstand the effects of swelling and (c) preventing the swelling.

Belled piers which are popular in USA for use in expansive soil conditions are isolated structures on swelling soils. The underreamed pile-beam construction is similar to that of belled piers and are designed for use under Indian conditions. The principal involved is to transfer the load of a building through the piles to a depth beyond the zone of seasonal variation in moisture content. The underreamed piles are bored cast in situ piles laid to a depth beyond the zone of seasonal variation in moisture content. The underreamed piles are bored cast in situ piles with their lower portions enlarged or reamed in the form of a bulb. The piles are connected at their top by plinth beams of reinforced concrete which support the super structure.

Designing a structure which is strong enough and rigid enough to withstand the effects of swelling may prove to be highly uneconomical except in the case of very small structures where even if the loads are supported by a central area or peripheral area much smaller than the plan area, the bearing pressures are within limits. Obviously, it is futile to attempt designing a structure that will remain undamaged in spite of swelling, because the cost of providing the structure with extra strength and rigidity would be much more as compared to the cost of underreamed pile or piers foundation.

Swelling can often be controlled, if not eliminated, by providing an impervious apron around the structure. By providing an apron, the moisture gradient between the centre of the structure and its edges is minimized and hence the differential swelling is controlled.

**Elimination of possible swelling can theoretically be brought about by** (a) pre-wetting the ground to a moisture content equal to the equilibrium moisture content, (b) making downward loads large enough to exceed swelling pressures and (c) chemical stabilization.

(ii)

Ballast is the granular material usually broken stones or bricks, shingle or kankar, gravel or sand placed and packed below and around the sleepers to transmit the load due to the wheels of the train from sleepers, to formation and at the same time allowing drainage of the track.

**Functions of Ballast:** Ballast performs the following functions:

- (i) It transfers the load from the sleeper to the subgrade and then distributes it uniformly over a larger area of the formation.
- (ii) It holds the sleepers in position and prevents the lateral and longitudinal movement due to dynamic loads and vibrations of moving trains.
- (iii) It imparts some degree of elasticity to the track.
- (iv) It provides easy means of maintaining the correct levels of the two lines of the track (i.e., level in straight portions and correct super-elevation on curves) and for correcting track alignment.
- (v) It provides good drained foundation immediately below the sleepers and helps to protect the top surface of the formation.

