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ESE 2024 : Prelims Exam CLASSROOM TEST SERIES

CIVIL ENGINEERING

Test 14

Section A: Flow of Fluids, Hydraulic Machines and Hydro Power [All Topics]

Section B: Design of Concrete and Masonry Structures - I [Part Syllabus]

Section C: Structural Analysis - II [Part Syllabus]

1.	(b)	16.	(c)	31.	(a)	46.	(a)	61.	(c)
2.	(a)	17.	(c)	32.	(b)	47.	(a)	62.	(b)
3.	(d)	18.	(b)	33.	(c)	48.	(d)	63.	(c)
4.	(d)	19.	(b)	34.	(b)	49.	(b)	64.	(c)
5.	(c)	20.	(d)	35.	(d)	50.	(c)	65.	(b)
6.	(a)	21.	(a)	36.	(c)	51.	(d)	66.	(c)
7.	(c)	22.	(c)	37.	(a)	52.	(b)	667.	(d)
8.	(b)	23.	(a)	38.	(b)	53.	(b)	68.	(c)
9.	(c)	24.	(d)	39.	(c)	54.	(b)	69.	(b)
10.	(b)	25.	(b)	40.	(a)	55.	(b)	70.	(d)
11.	(b)	26.	(c)	41.	(b)	56.	(a)	71.	(c)
12.	(a)	27.	(d)	42.	(c)	57.	(c)	72.	(b)
13.	(c)	28.	(c)	43.	(c)	58.	(a)	73.	(b)
14.	(d)	29.	(a)	44.	(b)	59.	(c)	74.	(c)
15.	(a)	30.	(b)	45.	(a)	60.	(d)	75.	(b)

DETAILED EXPLANATIONS

1. (b)

Now,
$$\Delta p = \rho g h = \left[\frac{2\sigma}{R_1} - \frac{2\sigma}{R_2} \right]$$

$$\Rightarrow h \times 1000 \times 9.81 = 2 \times 0.073 \times \left[\frac{1}{3 \times 10^{-3}} - \frac{1}{8 \times 10^{-3}} \right]$$

$$\Rightarrow h = 3.1 \times 10^{-3} \text{ m} = 3.1 \text{ mm}$$

2. (a)

Centre of pressure,
$$\overline{h}_{cp} = \overline{h} + \frac{I_{gg}}{A \cdot \overline{h}}$$

$$\overline{h}_{cp} = \frac{D}{2} + \frac{\frac{\pi}{64} \times D^4}{\frac{\pi}{4} D^2 \times \frac{D}{2}} = \frac{5D}{8}$$

6. (a)

 \Rightarrow

$$F_{Dx} = \frac{1}{2}F_{DL}$$

$$F_{Dx} = C_{Dx}(B \times x) \times \frac{\rho u^2}{2}$$

$$F_{DL} = C_{DL}(B \times L) \times \frac{\rho u^2}{2}$$

$$\vdots$$

$$\frac{C_{Dfx}}{C_{DL}} \frac{x}{L} = \frac{F_{Dx}}{F_{DL}} = \frac{1}{2}$$
But
$$C_{Dx} = \frac{1.328}{\sqrt{\frac{ux}{v}}}$$

$$C_{DL} = \frac{1.328}{\sqrt{\frac{uL}{v}}}$$

Drag force,
$$F_{D} = C_{D'}A\frac{\rho V^{2}}{2}$$

Power, $P = F_{D} \times V$

$$= C_{D'}A \cdot \frac{\rho V^{3}}{2}$$

Now,

$$P = C_{D1'}A \times \rho \frac{V_{1}^{3}}{2} = C_{D2'}A \times \rho \frac{V_{2}^{3}}{2}$$

If $C_{D2} = 0.85C_{D1}$

then,

$$V_{2} = \left[\frac{1}{0.85}\right]^{1/3}V_{1} = 1.056V_{1}$$

So, percentage increase in velocity $= \frac{(V_{2} - V_{1}) \times 100}{V_{1}} = \frac{(1.056V_{1} - V_{1}) \times 100}{V_{1}}$

$$= 5.57\%$$

5.57% increase in the velocity.

8. (b)

Boundary shear stress, $\tau_0 = \rho u_*^2$ where u_* is shear velocity where, $u_* = V \sqrt{\frac{f}{8}}$ $= 0.098 \sqrt{\frac{0.032}{8}} = 0.098 \times 2 \times 10^{-1.5}$ $= \frac{0.098 \times 2}{31.6} = 6.2 \times 10^{-3} \, \text{m/s}$ \vdots $\tau_0 = 10^3 \times [6.2 \times 10^{-3}]^2$ $= 0.038 \, \text{Pg}$

10. (b)

Discharge varies inversely with the viscosity of the fluid.

11. (b)

Here
$$\left(\frac{\Delta V}{V}\right) = -0.11\% = -0.0011$$

$$\Delta P = 1500 \text{ kPa}$$
 Now,
$$\text{bulk modulus, } K = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{-1500}{-0.0011}$$

$$= 1.364 \times 10^6 \text{ kPa}$$

$$= 1.364 \times 10^9 \text{ Pa}$$

Velocity of sound,
$$C = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{1.364 \times 10^9}{0.87 \times 1000}}$$

= 1252.1 \simeq 1252 m/sec

12. (a)

Volume of spheres =
$$\frac{4}{3}\pi \times r^3$$

= $\frac{4}{3}\pi \times \left(\frac{1.5}{2}\right)^3 = 1.767 \,\text{m}^3$

For lower sphere, buoyant force, $F_b = 1.767 \times 10 = 17.67 \text{ kN}$

Tension,
$$T = W - F_b$$

= 20 - 17.67 = 2.33 kN

Buoyant force on upper sphere $F_b' = W + T$ = 4 + 2.33 = 6.33 kN

If the sphere is completely submerged, then the buoyant force would have been equal to 17.67 kN. Since only 6.33 kN of buoyant force is being exerted.

$$\therefore$$
 Percentage volume above water = $\frac{17.67 - 6.33}{17.67} \times 100 = 64.2 \simeq 64\%$

14. (d)

Here, Reynold's law is applicable

$$|R_{e}|_{m} = |R_{e}|_{p}$$

$$\frac{V_{m}D_{m}}{v_{m}} = \frac{V_{P}D_{P}}{v_{P}}$$

$$\Rightarrow V_{m} = V_{P}\left(\frac{D_{P}}{D_{m}}\right) \times \left(\frac{v_{m}}{v_{P}}\right)$$

$$= 12 \times 20 \times \frac{1}{2}$$

$$= 120 \text{ km/hr}$$

15. (a)

$$Q = A.V$$

$$= L_v L_H \times \sqrt{L_v}$$

$$= L_v^{3/2} \cdot L_H$$

$$= k^{5/2}$$

16. (c)

Head loss =
$$\frac{\Delta P}{\gamma} = \frac{100 \times 10^3}{900 \times 9.81}$$

For laminar flow,
$$h_f = \frac{32\mu VL}{\rho g D^2}$$

$$\frac{100 \times 10^3}{900 \times 9.81} = \frac{32 \times \mu \times 0.8 \times 800}{900 \times 9.81 \times (0.2)^2}$$

$$\mu = 0.195 \text{ Pa-sec}$$

17. (c)

:.

Momentum thickness,
$$\theta = \int_0^{\delta} \frac{u}{U_{\infty}} \left[1 - \left(\frac{u}{U_{\infty}} \right) \right] dy$$

Now,
$$\theta = \int_0^{\delta} \left(\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) \left(1 - \frac{3}{2} \left(\frac{y}{\delta} \right) + \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right) dy$$

$$= \int_0^{\delta} \left(\frac{3}{2} \frac{y}{\delta} - \frac{9}{4} \left(\frac{y}{\delta} \right)^2 + \frac{3}{4} \left(\frac{y}{\delta} \right)^4 - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 + \frac{3}{4} \left(\frac{y}{\delta} \right)^4 - \frac{1}{4} \left(\frac{y}{\delta} \right)^6 \right) dy$$

$$= \left[\frac{3}{4} \frac{y^2}{\delta} - \frac{9}{12} \frac{y^3}{\delta^2} + \frac{3}{20} \frac{y^5}{\delta^4} - \frac{1}{8} \frac{y^4}{\delta^3} + \frac{3}{20} \frac{y^5}{\delta^4} - \frac{1}{28} \frac{y^7}{\delta^6} \right]_0^{\delta}$$

$$= \delta \left[\frac{3}{4} - \frac{3}{4} + \frac{3}{20} - \frac{1}{8} + \frac{3}{20} - \frac{1}{28} \right]$$

$$\theta = \frac{39}{280} \delta$$

18. (b)

$$h_{L} = \frac{fLV^{2}}{2gd}$$

$$\Rightarrow \frac{15V^{2}}{2g} = \frac{fLV^{2}}{2gd}$$

$$\Rightarrow \frac{15V^{2}}{2g} = \frac{0.02 \times L \times V^{2}}{2g \times (0.1)}$$

$$\Rightarrow L = 75 \text{ m}$$

19. (b)

For turbulent flow,
$$f = \frac{0.316}{(R_e)^{1/4}}$$

20. (d)

Boundary acts as limiting flow lines in a flownet.

21. (a)

For critical flow condition,

$$\frac{Q^2 T_c}{g A_c^3} = 1$$

Here,

$$A_c = \frac{\pi}{8}D^2 = \frac{\pi}{8} \times (1)^2 = 0.4 \,\mathrm{m}^2$$

$$T_c = D = 1 \text{ m}$$

$$\therefore \frac{Q^2}{g} = \frac{A_c^3}{T_C}$$

$$\Rightarrow$$

$$Q = \left[\left(\frac{0.4}{1} \right)^3 \times 9.81 \right]^{1/2} = \sqrt{0.63} = 0.79 \text{ m}^3/\text{s}$$

22. (c)

Sequent depth ratio,
$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8F_1^2} \right]$$

$$\Rightarrow \frac{2.0}{0.2} = \frac{1}{2} \left[-1 + \sqrt{1 + 8F_1^2} \right]$$

$$\Rightarrow F_1 = 7.42$$

Now,
$$F_1 = \frac{V_1}{\sqrt{gy_1}}$$

$$\Rightarrow \qquad 7.42 = \frac{V_1}{\sqrt{9.81 \times 0.2}}$$

$$\Rightarrow$$
 $V_1 = 10.39 \text{ m/sec}$

$$Q = V_1 \times y_1 \times B = 10.39 \times 0.2 \times 2$$
$$= 4.16 \text{ m}^3/\text{sec}$$

24. (d)

As per Manning's formula,

$$V = \frac{1}{n} (R)^{2/3} \sqrt{S_f}$$

where

$$S_f$$
 = water surface slope
= head loss per unit length

Hydraulic mean radius for full flow, $R_1 = \frac{A}{P}$

$$= \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$$

Hydraulic mean radius for half full sewer,

$$R_2 = \frac{A}{P}$$

$$= \frac{\frac{\pi}{8}D^2}{\frac{\pi}{2}D} = \frac{D}{4}$$

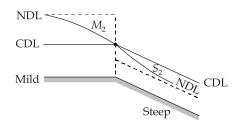
 $\begin{array}{rcl} R_1 &=& R_2 \\ V_1 &=& V_2 \end{array}$

 $n_1 = n_1$

 $S_{f1} = S_{f2}$

:.

25. (b)



26. (c) By weir formula,

$$Q = \frac{2}{3}C_d\sqrt{2g} \cdot L \cdot H^{3/2}$$

$$\Rightarrow \qquad 0.025 = \frac{2}{3}C_d \times \sqrt{2 \times 9.81} \times 0.4 \times (0.1)^{3/2}$$

$$\Rightarrow \qquad C_d = 0.669 \simeq 0.7$$

27. (d)

Celerity,
$$C = \sqrt{\frac{1}{2}} \cdot g \cdot \frac{y_2}{y_1} (y_1 + y_2)$$

Here,
 $y_1 = 2 \text{ m}$
 $y_2 = y_1 + \text{height of surge}$
 $= 2 + 0.5 = 2.5 \text{ m}$

$$\therefore \qquad C = \sqrt{\frac{9.81 \times 2.5}{2 \times 2} \times (2 + 2.5)}$$

$$= 5.25 \text{ m/sec.}$$

$$= 315 \text{ m/min}$$

29. (a)

> Cipolletti Weir is an exceptional case of trapezoidal weir with side slope value of 1H: 4V and is having a constant value of coefficient of discharge as 0.63.

31. (a)

$$Q = 10 \text{ m}^3/\text{sec.}$$

 $V = 1.25 \text{ m/sec.}$

:.

$$A = \frac{Q}{V} = \frac{10}{1.25} = 8 \,\mathrm{m}^2$$

For most efficient trapezoidal section,

$$A = \sqrt{3} y^2$$

 \Rightarrow

$$8 = \sqrt{3} \times y^2$$

 \Rightarrow

$$y = 2.149 \text{ m}$$

:.

Wetted perimeter,
$$P = 2\sqrt{3} \times y$$

= $2 \times \sqrt{3} \times 2.149$

$$= 7.44 \text{ m}$$

32. (b)

Unit power,
$$P_u = \frac{P}{H^{3/2}}$$

$$\therefore \frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

$$\Rightarrow$$

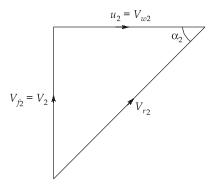
$$P_2 = P_1 \times \left[\frac{H_2}{H_1}\right]^{3/2}$$

=
$$8000 \times \left[\frac{16}{36} \right]^{3/2} = 8000 \times \frac{64}{216} \simeq 2370 \text{ kW}$$

34. (b

Specific speed is has dimensions of M° $L^{3/4}$ $T^{-3/2}$

35. (d)



From the outlet velocity triangle,

$$V_{f2} = V_2$$
 and $V_{r2} \cos \alpha_2 = V_{w2} = u_2$

But

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1450}{60} = 22.78 \text{ m/s}$$

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36.

39.

(c)

:.

Now,
$$\sigma_{c} = \frac{(NPSH)_{\min}}{H}$$

$$\Rightarrow \qquad 0.12 = \frac{(NPSH)_{\min}}{30}$$

$$\Rightarrow \qquad (NPSH)_{\min} = 3.6 \text{ m}$$
Also,
$$(NPSH)_{\min} = H_{\text{atm}} - H_{\text{vap.}} - Z_{s} - h_{L}$$

$$\Rightarrow \qquad Z_{s} = 9.8 - 0.3 - 0.3 - 3.6$$

$$= 5.6 \text{ m}$$

38. **(b)**

Power,
$$P = \rho Q(V_{w_1} \cdot u_1 - V_{w_2} \cdot u_2)$$
 $\Rightarrow 8000 \times 10^3 = 10^3 \times 30 \times (V_{w_1u_1} - 0)$

Here

 $u_1 = \frac{\pi D_1 N}{60}$
 $= \frac{\pi \times 3 \times 200}{60} = 31.42 \text{ m/s}$
 $\therefore 8000 \times 10^3 = 10^3 \times 30 \times 31.42 \times V_{w_1}$
 $\Rightarrow V_{w_1} = 8.49 \text{ m/s}$
 $\approx 8.5 \text{ m/s}$

Specific speed,
$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Here, $P = \eta_0 \cdot \gamma \cdot Q \cdot H$
 $\Rightarrow P = 0.8 \times 9.81 \times 2.45 \times 625$
 $= 12017.25 \text{ kW}$
 $\therefore N_s = \frac{400 \times \sqrt{12017.25}}{(625)^{5/4}} = 14$

If the flow is reversed then due to expansion, minor loss will occur which is given by,

$$h_L = \frac{\left(V_1 - V_2\right)^2}{2g}$$

43. (c)

Discharge is maximum when the friction loss is one-third of total head at the inlet.

45. (a)

Load combinations to be checked are:

$$1.5(DL + LL) = 1.5(20 + 30) = 75 \text{ kN-m}$$

 $1.5(DL + EQL) = 1.5(20 + 10) = 45 \text{ kN-m}$
 $1.2(DL + LL + EQL) = 1.2(20 + 30 + 10) = 72 \text{ kN-m}$

:. Design moment is 75 kN-m.

46. (a)

For Fe415

Maximum strain in tension reinforcement.

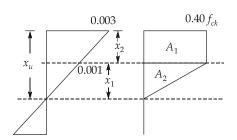
$$\in_{st} \not < \frac{f_y}{1.15E_s} + 0.002$$

$$\in_{st} \not < \frac{415}{1.15 \times 2 \times 10^5} + 0.002$$

$$\not < 0.0018 + 0.002$$

£ 0.0038

47. (a)



From similar triangles,

$$\frac{0.003}{x_u} = \frac{0.001}{x_1}$$

$$\Rightarrow x_1 = \frac{x_u}{3}$$

$$x_2 = \frac{2x}{3}$$

Total compressive force = Area of stress block \times *B* = $(A_1 + A_2) \times B$

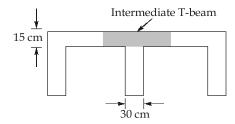
$$= \left[0.4f_{ck}x_2 + \frac{1}{2} \times 0.4f_{ck} \times x_1\right] \times B$$

$$= \left[0.4f_{ck}\left(\frac{2x_u}{3}\right) + \frac{1}{2} \times 0.4f_{ck} \times \frac{x_u}{3}\right] \times B$$

$$= \left[\frac{4f_{ck}x_u}{15} + \frac{f_{ck}x_u}{15}\right] \times B$$

$$= \frac{f_{ck}x_uB}{3}$$

48. (d)



Effective width of flange section = mininum of $\left\{ \frac{L_0}{6} + b_w + 6D_f \right\}$ c/c distance between beams

$$= mininum of \begin{cases} \frac{6000}{6} + 300 + 6(150) \\ 3000 \text{ mm} \end{cases}$$

$$= mininum of \begin{cases} 2200 mm \\ 3000 mm \end{cases}$$

 $= 2200 \, \text{mm}$

49. (b)

For minimum shear reinforcement,

$$S_v \leq \frac{0.87 f_y A_{sv}}{0.4b}$$

$$\Rightarrow$$

$$S_v \ \leq \ \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 8^2}{0.4 \times 250}$$

$$\Rightarrow$$

$$S_v \le 362.97 \, \text{mm}$$

Also,

Spacing
$$\neq$$
 (i) 0.75 d = 0.75 × 400 = 300 mm

- (ii) 300 mm
- (iii) $S_{77} = 362.96 \text{ mm}$
- Provide 8 mm diameter stirrups @ 300 mm c/c.

For bottom 1 m height of tank,

$$T = \frac{\gamma_w HD}{2}$$
$$= \frac{10 \times 5 \times 8}{2} = 200 \,\text{kN}$$

Now, total hoop tension is to be taken by steel only.

$$f_{st} = 130 \, \text{MPa}$$

$$A_{\rm st} = \frac{T}{f_{st}} = \frac{200 \times 10^3}{130}$$

= 1538.46 mm²/m
 $\simeq 1540$ mm²/m

51. (d)

A cantilever porch experiences tension at the top surface while compression at the bottom, hence tensile reinforcement is provided at top.

52. (b)

Check for deflection:

For cantilever, minimum effective depth required

$$= \frac{\text{Span}}{7} = \frac{7000}{7} = 1000 \text{ mm}$$

But effective depth available,

$$d = 650 - 50 = 600 \text{ mm}$$

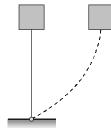
:. This cantilever beam fails in deflection.

Check for lateral stability: Span $\neq \min \begin{cases} 25b = 25 \times 300 = 7500 \text{ mm} \\ \frac{100 b^2}{d} = \frac{100 \times 300^2}{600} = 15000 \text{ mm} \end{cases}$

≯ 7500 mm

 \therefore It is safe in lateral stability.

53. (b)



$$(L_{\text{eff}})_{\text{theoretical}} = 2l$$

 $(L_{\text{eff}})_{\text{recommended}} = 2l$

54. (b)

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For axially loaded short column,

$$\begin{array}{rcl} P_u &=& 0.4 f_{ck} \, A_g + (0.67 f_y - 0.4 f_{ck}) \, A_{sc} \\ \Rightarrow & 1.5 \times 2000 \times 10^3 \, = \, 0.4 \times 20 \times (400 \times 600) + (0.67 \times 415 - 0.4(20)) A_{sc} \\ \Rightarrow & 3000000 \, = \, 1920000 + 270.05 \, A_{sc} \\ \Rightarrow & 1080000 \, = \, 270.05 \, A_{sc} \\ \Rightarrow & A_{sc} \, = \, 3999.25 \, \mathrm{mm}^2 \end{array}$$

55. (b)

Reduction coefficient is given as,

$$C_r = 1.25 - \frac{l_{eff}}{48B}$$
$$= 1.25 - \frac{4800}{48 \times 250}$$
$$= 1.25 - 0.4 = 0.85$$

56. (a)

Straight length of a lap shall not be less than 15\phi or 200 mm.

57.

Refer IS 456: 2000, Clause 26.2.3.2

58. (a)

$$\delta = \text{Minimum of} \begin{cases} 1 + \frac{3P_u}{A_g f_{ck}} \\ 1.5 \end{cases}$$
where,
$$P_u = 1000 \text{ kN}$$

$$A_g = 500 \times 500 \text{ mm}^2$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$\delta = \text{Minimum of} \begin{cases} 1 + \frac{3 \times 1000 \times 10^3}{500 \times 500 \times 25} \\ 1.5 \end{cases}$$

$$= \text{Minimum of} \begin{cases} 1.48 \\ 1.5 \end{cases}$$

$$= 1.48$$

60. (d)

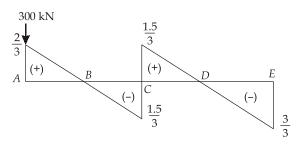
Minimum steel requirement in slab is based on shrinkage and temperature considerations because in slabs, there occurs a better distribution of load effects unlike in beams where it is based on strength considerations.

61.

Degree of static indeterminacy of truss is small. So, force methods are generally adopted for analysis of truss.

62. (b)

I.L.D. for shear force at 'C' is shown below.



For maximum positive shear force at *C*, load should be placed as shown in figure above.

So, maximum positive shear force at $C = \frac{2}{3} \times 300$

$$= 200 \text{ kN}$$

63. (c)

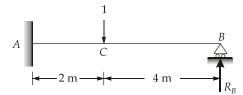
For underdamped system, damping ratio, $\xi < 1$ For overdamped system, damping ratio, $\xi > 1$ For critically damped system, damping ratio, $\xi = 1$

65. (b)

Force in members BF, CF, AI, DG, CG, CH, KN, OK will be zero because of the fact that if three members meet at a joint and there is no load at joint and out of 3 members, 2 are collinear and one is non-collinear, then there will be no force in non-collinear member.

66. (c)

To find ordinate of ILD of vertical reaction at 2 m from fixed end, apply a unit load at 2 m from fixed end.



Now, let the prop reaction be R_B .

Now, vertical deflection at B = 0

$$\Rightarrow \frac{1 \times 2^3}{3EI} + \frac{1 \times 2^2}{2EI} \times 4 - \frac{R_B \times 6^3}{3EI} = 0$$

$$\Rightarrow \frac{8}{3EI} + \frac{8}{EI} - \frac{R_B \times 72}{EI} = 0$$

$$\Rightarrow R_B = \frac{4}{27}$$

67. (d)

Natural frequency,
$$w_n = \frac{2\pi}{T} = \frac{2\pi}{0.5\pi} = 4 \text{ rad/s}$$

Maximum velocity, $y_{\text{max}} = Aw_n$

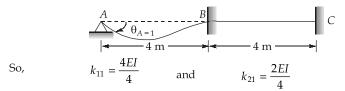
where 'A' is the amplitude.

$$\Rightarrow \qquad 0.136 = A \times 4$$

$$\Rightarrow$$
 $A = 0.034 \text{ m} = 34 \text{ mm}$

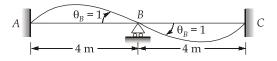
68. (c)

> To develop first column of stiffness matrix give unit displacement in direction of coordinate (1) i.e. give $\theta_A = 1$ and ensure $\theta_B = 0$.



69. (b)

> To develop second column of stiffness matrix, give unit displacement in direction of coordinate (2) i.e. give $\theta_B = 1$ and ensure $\theta_A = 0$.



Now,

$$k_{12} = k_{21} = \frac{2EI}{4} = \frac{EI}{2}$$
 and $k_{22} = \frac{4EI}{4} + \frac{4EI}{4} = 2EI$

70.

Let P_{1L} and P_{2L} are forces developed in coordinate directions due to given loads in locked structure.

$$\begin{array}{c|c}
 & 10 \text{ kN} \\
 & A & D & 6 \text{ kN/m} \\
 & D & & 4 \text{ m}
\end{array}$$

$$M_{FAB} = \frac{-10 \times 4}{8} = -5 \text{ kN-m}$$
 10×4

$$M_{FBA} = \frac{10 \times 4}{8} = 5 \text{ kN-m}$$

$$M_{FBC} = \frac{-6 \times 4^2}{12} = -8 \text{ kN-m}$$

$$M_{FCB} = \frac{6 \times 4^2}{12} = 8 \text{ kN-m}$$
 So,
$$P_{1L} = -5 \text{ kN-m}, P_{2L} = 5 - 8 = -3 \text{ kN-m}$$

$$\begin{bmatrix} FI & EI \end{bmatrix}$$

Stiffness matrix,
$$[k] = \begin{bmatrix} EI & \frac{EI}{2} \\ \frac{EI}{2} & 2EI \end{bmatrix}$$

$$= EI \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$$

$$[k]^{-1} = \frac{4}{7EI} \begin{bmatrix} 2 & \frac{-1}{2} \\ \frac{-1}{2} & 1 \end{bmatrix} = \frac{1}{7EI} \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix}$$

Now,
$$\begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \frac{1}{7EI} \begin{bmatrix} 8 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} - & (-5) \\ - & (-3) \end{bmatrix}$$

So,
$$\theta_A = \frac{1}{7EI}(40 - 2 \times 3) = \frac{34}{7EI}$$

As we have derived in previous question

$$\theta_B = \frac{1}{7EI}(-2 \times 5 + 4 \times 3) = \frac{2}{7EI}$$

72. (b)

$$M_{CB} = M_{FCB} + \frac{2EI}{L} \left(2\theta_C + \theta_B - \frac{3\Delta}{L} \right)$$

$$= 8 + \frac{2EI}{4} \left(\frac{2}{7EI} \right)$$

$$= 8 + \frac{1}{7} = \frac{57}{7} \text{kN-m}$$
[:: $\theta_C = \Delta = 0$]

73. (b)

Maximum moment at a point due to a UDL shorter than span occurs when the section divides the UDL in same ratio as it divides the span.

Consider joint C,

$$\Sigma F_y = 0$$

$$F_{BC} \sin 45^\circ - 60 = 0$$

$$\Rightarrow F_{BC} = \frac{60}{\sin 45^\circ} = 60\sqrt{2} \text{ kN}$$

$$\Sigma F_x = 0$$

$$\Rightarrow F_{DC} + F_{BC} \cos 45^\circ = 0$$

$$\Rightarrow F_{DC} = -F_{BC} \cos 45^\circ$$

$$= -60\sqrt{2} \times \frac{1}{\sqrt{2}} = -60 \text{ kN} = 60 \text{ kN (Compressive)}$$