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ESE 2024 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 8

Section A : Power Systems [All Topics]

Section B : Electrical Machines-1 [Part Syllabus]

Section C : Control Systems-2 [Part Syllabus] + Engineering Mathematics-2 [Part Syllabus]

ANSWER KEY

1.	(c)	**16.**	(c)	**31.**	(a)	**46.**	(b)	**61.**	(c)
2.	(d)	**17.**	(d)	**32.**	(d)	**47.**	(a)	**62.**	(a)
3.	(b)	**18.**	(a)	**33.**	(d)	**48.**	(c)	**63.**	(c)
4.	(b)	**19.**	(b)	**34.**	(c)	**49.**	(b)	**64.**	(b)
5.	(c)	**20.**	(a)	**35.**	(c)	**50.**	(a)	**65.**	(b)
6.	(d)	**21.**	(c)	**36.**	(c)	**51.**	(a)	**66.**	(b)
7.	(c)	**22.**	(b)	**37.**	(b)	**52.**	(b)	**67.**	(d)
8.	(c)	**23.**	(b)	**38.**	(d)	**53.**	(a)	**68.**	(a)
9.	(b)	**24.**	(b)	**39.**	(d)	**54.**	(d)	**69.**	(c)
10.	(b)	**25.**	(c)	**40.**	(a)	**55.**	(d)	**70.**	(a)
11.	(d)	**26.**	(d)	**41.**	(a)	**56.**	(b)	**71.**	(d)
12.	(d)	**27.**	(c)	**42.**	(c)	**57.**	(c)	**72.**	(d)
13.	(a)	**28.**	(c)	**43.**	(a)	**58.**	(b)	**73.**	(a)
14.	(c)	**29.**	(b)	**44.**	(d)	**59.**	(c)	**74.**	(a)
15.	(c)	**30.**	(a)	**45.**	(c)	**60.**	(b)	**75.**	(b)

DETAILED EXPLANATIONS
Section A : Power Systems

1. (c)

$$\because \quad \text{Average load} = \frac{\text{Actual energy consumed}}{\text{Time duration}} = \frac{8\times50\times10+2\times1000\times4}{24} = 500 \text{ W}$$

$$\text{Load factor} = \frac{\text{Average load}}{\text{Maximum demand}} = \frac{500}{1200} = 0.417$$

2. (d)

- Renewable energy is energy produced from sources like the sun and wind that are naturally replenished and do not run out.
- Nuclear power plant is not an example of renewable energy.

3. (b)

Capacitance of a transmission line = $\dfrac{\pi\varepsilon_0}{\ln\left(\dfrac{d}{r}\right)}$ as mutual geometric mean distance increases, capacitance decreases. Capacitance depends on ground clearance only if ground is perfect conductor sheet.

4. (b)

100 km long line is a short transmission line,

so, $$Z = R + jX$$

For Y-connection, $$I_L = I_{ph} = \frac{S_{3\phi}}{\sqrt{3}\cdot V_L}$$

$$I_{ph} = \frac{150\times10^6}{\sqrt{3}\times110\times10^3} = \frac{15}{11\sqrt{3}}\times10^3$$

$$P_{loss} = 3I_{ph}^2\cdot R_{ph}$$

$$25\times10^6 = \frac{3\times15\times15}{121\times3}\times10^6\times R_{ph}$$

$$R_{ph} = 13.44\ \Omega/\text{phase}$$

5. (c)

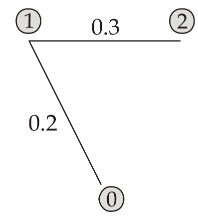
Series capacitor is not the primary method of voltage control in power system. In fact series capacitor is used to improve the steady state stability of system.

6. (d)

Fibre optic cables are used in power system applications mainly for

1. SCADA.
2. Communication between power station and load control centre.
3. Communication between power station and substation.

8. (c)



$$Z_{\text{bus (new)}} = \begin{bmatrix} Z_{\text{Bus col}} & Z_{1j} \\ & \vdots \\ Z_{j1} & Z_{j2} \ldots (Z_{j1} + Z_b) \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.5 \end{bmatrix}$$

9. (b)

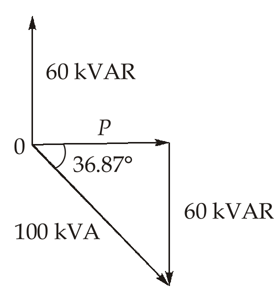
Bus type	Quantities specified	Quantities to be obtained
Load bus	P, Q	$\lvert V\rvert, \delta$
Generator bus	$P, \lvert V\rvert$	Q, δ
Slack bus	$\lvert V\rvert$, δ	P, Q

10. (b)

$$\text{K.E} = H \times S$$
$$= 15 \times 40 = 600 \text{ MJ}$$

11. (d)

According to power triangle,



Active power drawn from the supply = P

$$P \tan(36.87°) = 60 \text{ kVAR}$$
$$= 80 \text{ kW}$$

Now, the power factor is unity,

$$\text{Apparent power} = 80 \text{ kVA}$$

12. (d)

Current, $i = 2$ A/m

Resistance of distributor per meter,

$$r = 2 \times \frac{0.3}{1000} = 0.0006\ \Omega$$

Length of the distributor, $l = 200$ m

Voltage drop upto a distance of x meters from feeding point

$$= ir\left(lx - \frac{x^2}{2}\right)$$

For $x = 150$ m

$$\text{Desired voltage drop} = 2 \times 0.0006\left(200 \times 150 - \frac{150^2}{2}\right) = 22.5 \text{ V}$$

13. (a)

The first row of Y_{Bus} is $[Y_{11}\ Y_{12}\ Y_{13}]$

$$Y_{11} = y_{12} + y_{13} = 10 - j20 + 10 - j30$$
$$= (20 - j50)$$
$$Y_{12} = Y_{21} = -y_{12} = (-10 + j20)$$
$$Y_{13} = Y_{31} = -y_{13} = (-10 + j30)$$

The first row of Y_{Bus} is $[(20 - j50)\ (-10 + j20)\ (-10 + j30)]$

14. (c)

In line to line fault

$$I_1 = -I_2$$

and $$I_0 = 0$$

$$|I_f| = \sqrt{3}\, I_1$$
$$|I_f| = \sqrt{3}\, I_2 = \sqrt{3} \times 2$$
$$= 3.464 \text{ p.u.} \approx 3.46 \text{ p.u.}$$

15. (c)

The frequency of restriking voltage

$$= \frac{1}{2\pi\sqrt{LC}} \text{Hz}$$

$$\omega_n = \frac{1}{\sqrt{LC}} \text{ radians/sec}$$

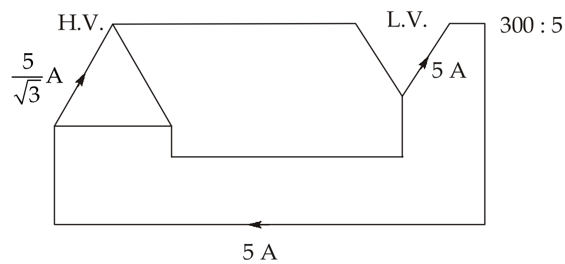
$$= \frac{1}{\sqrt{10 \times 10^{-3} \times 400 \times 10^{-12}}} = 500 \times 10^3 \text{ radians/sec}$$

16. (c)

C.T. connection on L.V. side will be ⅄ and H.V. side will be Δ.

$$i_1 = \frac{5}{\sqrt{3}} \text{ A}$$

and $$i_2 = 5 \text{ A}$$

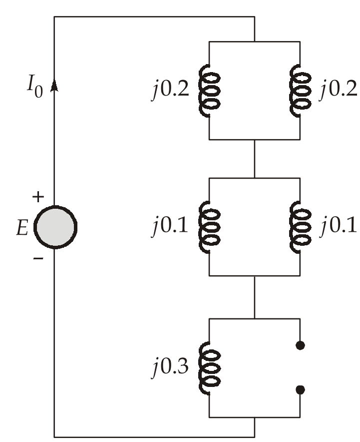


Now, $$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$I_1 = \frac{6.6}{33} \times 300 = 60 \text{ A}$$

$$\text{C.T. ratio on H.V. side} = \frac{I_1}{i_1} = \frac{60}{(5/\sqrt{3})} = 12\sqrt{3} : 1$$

17. (d)



$$X_0 = j\,0.15 + 3\,X_n$$
$$= j\,0.15 + j\,0.15 = j\,0.3 \text{ p.u.}$$

For LG fault the fault current is,

$$I_0 = \frac{E}{X_1 + X_2 + X_0 + 3(X_n + X_0)}$$

$$I_0 = \frac{E}{j0.1 + j0.05 + j0.3} = \frac{1}{j0.45}$$

$$I_f = 3I_0$$

$$I_f = \frac{20}{3}\angle -90° \text{ p.u.}$$

18. (a)

Total reactive power, $Q = \sqrt{3}|V_L||I_L|\sin\phi$

$$= \sqrt{3}\times 400\times\sqrt{3}\times 10\times\sin(36.87°)$$

($\because$ for Δ corrected load $I_L = \sqrt{3}I_{ph} = 10\sqrt{3}$ A)

$$= 7.2 \text{ kVAR}$$

19. (b)

- The dynamic system stability is improved due to increased damping provided.
- Load power factor is improved consequently line losses are reduced and the system efficiency is improved.

20. (a)

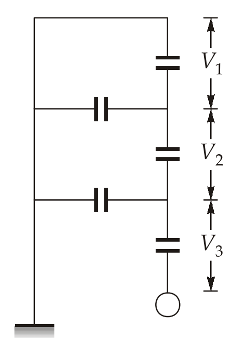
Gauss seidal method is sensitive to the choice of reference bus while NR method is not sensitive to the choice of slack bus/reference bus.

21. (c)

$\because$ Ratio of self capacitance to the pin-earth capacitance (k)

$$= 0.25$$

Voltage obtained across the string,



$$V = V_1 + V_2 + V_3$$

$$V_2 = (1+K)V_1$$

$$V_3 = (1+K)V_2 + KV_1$$

$$K = \frac{C_m}{C_s} = 0.25$$

$$V_2 = 1.25V_1$$

$$V_3 = [(1.25)^2 + 0.25]V_1$$

$$V_3 = 1.8125 \text{ V}$$

$$V = 1 + 1.25 + 1.8125 = 4.0625 \text{ V}$$

$\therefore$ % String efficiency $= \dfrac{4.0625\times 100}{3\times 1.8125}$

$$n = 74.71\%$$

22. (b)

Incremental cost of first generator is

$$\frac{dC_1}{dP_1} = 0.008P_1 + 2$$

Incremental cost of second generator is

$$\frac{dC_2}{dP_2} = 0.024P_2 + 2$$

For optimum load division,

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$0.008P_1 + 2 = 0.024P_2 + 2$$

$$P_1 = 3P_2 \quad \text{...(i)}$$

∴ Total load shared by the both generator

$$P_1 + P_2 = 400 \quad \text{...(ii)}$$

From (i) and (ii) equation,

$$\frac{4P_1}{3} = 400$$

$$P_1 = 300 \text{ MW}$$

23. (b)

The protection scheme used for detection of loss of excitation of a very large generating unit finding power into a grid employes offset mho relay.

24. (b)

We know that, $X_L = \frac{B}{1-A} = \frac{36.54}{1-0.99} = 3654\ \Omega$

$$L = \frac{X_L}{2\pi f} = \frac{3654}{2\times\pi\times 50}$$

⇒ $L = 11.63$ H

25. (c)

When switch S is open,

Transfer reactance, $X = X_{dg} + X_T + \frac{1}{2}X_l + \frac{1}{2}X_l$

$$= 0.8 + 0.1 + 0.3 + 0.3 = 1.5 \text{ p.u.}$$

Therefore, the steady-state power limit

$$P_{r\,\max} = \frac{EV}{X} = \frac{1.2\times 1.0}{1.5} = 0.8 \text{ p.u.}$$

26. (d)

Relay current setting = 5 × 0.5 = 2.5

$$\text{C.T. ratio} = \frac{400}{5} = 80$$

∴ Plug setting multiplier $= \frac{2000}{2.5\times 80} = 10$

27. (c)

$$Z_{Base} = \frac{(kV)^2_{Base}}{(MVA)_{Base}} = \frac{400^2}{250} = 640\ \Omega$$

$$Z_{Actual} = Zp.u. \times Z_{Base} = 0.05 \times 6400 = 32\ \Omega$$

28. (c)

The Jacobian matrix order for 'n' number of load buses and 'm' number of voltage control buses is

$$(2n - m - 2) \times (2n - m - 2) = (2 \times 12 - 3 - 2) \times (2 \times 12 - 3 - 2)$$

$$\text{Size of Jacobian matrix} = 19 \times 19$$

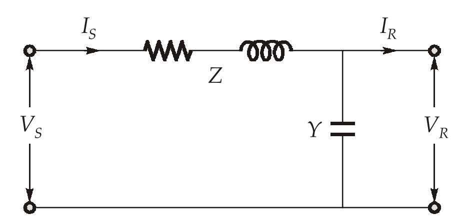
29. (b)

$$(SIL)_N = (SIL)_{old} \times \sqrt{1 + K_1} = (SIL)_{old} \times \sqrt{1 + 0.3}$$

$$\%\ \text{change} = \frac{\sqrt{1.3} - 1}{1} \times 100 = 14.02\%$$

31. (a)

End condenser network:



$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$= \begin{bmatrix} 1 + YZ & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Here $A \neq D$ therefore it is not symmetrical

$$AD - BC = 1 + YZ - YZ = 1$$

Therefore it is reciprocal.

32. (d)

Bundled conductor increase self GMD/GMR due to which inductance of the line is reduced in turn reducing the line reactance, increasing the transmission efficiency by reducing losses in line (Corona loss).

33. (d)

Shunt compensation in an EHV line is mainly used to overcome Ferranti effect i.e. for improving the voltage profile of the line.

34. (c)

Ring main system is preferred only due to its reliability and less voltage drop in the feeder.

35. (c)

- The voltage induced in a telephone line running parallel to a power line is reduced to zero if the power line is transposed and provided it carries balanced currents.
- Power line transposition is ineffective in reducing the induced telephone line voltage when power line currents are unbalanced or they contain third harmonics.
- Over the length of the one complete transposition cycle of power line, the net voltage induced in the line is zero, as it is the sum of three induced voltage which are displayed by 120° in time phase.

∴ Only 1 and 3 are correct.

36. (c)

Given, 50 Hz, 4-pole, 500 MVA, 22 kV generator,

p.f. = 0.8 lagging

Fault occurs which reduces output by 50%.

Pre-fault output power = 500 × 0.8 = 400 MW

Post-fault output power = 400 × 0.5 = 200 MW

P_a = 400 – 200 = 200 MW

∴ $P_a = T_a \times \omega$

Accelerating torque, $T_a = \dfrac{P_a}{\omega} = \dfrac{P_a \times 60}{2\pi N_s} = \dfrac{200 \times 60}{2\pi \times 1500} = 1.273 \times 10^6$ Nm

38. (d)

Corona loss is directly proportional to (f + 25). So corona loss in dc system is lesser than ac system for the same conductor diameter and operating voltage and in DC system frequency is zero.

∴ Statement-I is false and statement-II is true.

39. (d)

The best location for the power factor correction equipment to be installed is the receiving end (load end).

Hence statement I is false.

Section B : Electrical Machines-1

41. (a)

- The core has infinite permeability so that zero magnetizing current is needed to establish the requisite amount of flux in the core.
- The core-loss (hysteresis as well as eddy-current loss) is considered zero.

42. (c)

$$\eta = \frac{500 \times 1}{500 \times 1 + P_i + P_c} = 0.95 \quad \text{...(i)}$$

and
$$\frac{500 \times 0.6}{500 \times 0.6 + P_i + (0.6)^2 P_c} = 0.95 \quad \text{...(ii)}$$

Solving equation (i) and (ii) we get

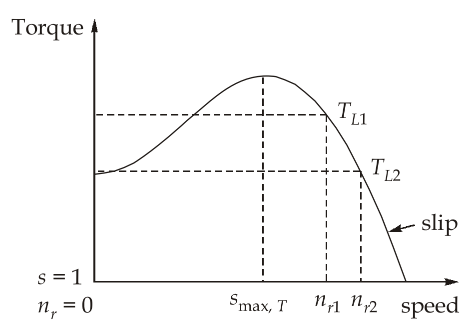
$$P_i = 9.87 \text{ kW}$$

and
$$P_c = 16.45 \text{ kW}$$

At 75% of full load,

$$\eta = \frac{500 \times 0.75}{500 \times 0.75 + 9.87 + (0.75)^2 \times 16.45} = 95.14\%$$

43. (a)



In stable region,

Slip varies from, $s = s_{max,\,T}$ to $s = 0$

In this region, if load torque increase ($T_{L1} > T_{L2}$)

The slip increase or speed decreases ($n_{r1} < n_{r2}$).

44. (d)

Stator input = 50 kW

Slip, s = 0.04 p.u.

Stator losses = 900 W

Stator output power = 50 – 0.9 = 49.1 kW

Rotor power input = 49.1 kW

Total rotor copper loss = s × rotor input

= 0.04 × 49.1 = 1.964 kW

$$\text{Per phase rotor copper losses} = \frac{1.964}{3} = 0.6546 \text{ kW}$$

45. (c)

Maximum torque by SRIM is

$$T_{max} = \frac{1}{2\omega_{sm}} \cdot \frac{V_1^2}{x_2'}$$

∴ Maximum torque is independent of rotor circuit resistance and proportional to the square of the supply voltage.

46. (b)

- Deeper and narrow slots increases the rotor resistance therefore $T_{starting}$ increase and the leakage reactance of the rotor T_{max} improves.
- Deeper and narrow slots improves power factor of induction motor.

47. (a)

Star/delta connection is the most commonly used connection for power systems. At transmission levels star connection is on the HV side i.e Δ/Y for step-up and Y/Δ for step-down.

48. (c)

Given, $$\frac{N_{\Delta(P)}}{N_{Y(P)}} = 4 = \frac{V_{\Delta(P)}}{V_{Y(P)}}$$

Also, $$V_{Y\,(P)} = \frac{500}{\sqrt{3}}\text{V}$$

∴ $$V_{\Delta(P)} = \frac{4\times 500}{\sqrt{3}} = \frac{2000}{\sqrt{3}} = 1154.70 \text{ V}$$

For delta connection,

$$V_{\Delta(L)} = V_{\Delta(P)} = 1154.70 \text{ V}$$

49. (b)

$$\text{Synchronous speed, } N_s = \frac{120\times 50}{6} = 1000 \text{ rpm}$$

$$\text{Stalling speed} = 900 \text{ rpm}$$

$$\text{Slip at stalling torque, } s = \frac{1000-900}{1000} = 0.1$$

Slip at maximum torque;

$$s_{mT} = \frac{R_2}{X_2} = \frac{0.01}{X_2}$$

∵ $$0.1 = \frac{0.01}{X_2}$$

$$X_2 = 0.1\ \Omega$$

To obtain maximum torque at starting,

Let rotor resistance = R_2'

at starting, $s = 1$

$$s_{mT} = \frac{R_2'}{X_2}$$

$$\Rightarrow \quad 1 = \frac{R_2'}{0.1}$$

$$\Rightarrow \quad R_2' = 0.1\ \Omega/\text{phase}$$

The external resistance to be added,

$$R_{ext} = 0.1 - 0.01$$
$$= 0.09\ \Omega/\text{phase}$$

50. (a)

$$N = 950 \text{ rpm},$$
$$P = 6,$$
$$f = 50 \text{ Hz}$$

$$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{1000 - 950}{1000} = 0.05$$

$$\text{Rotor copper loss} = \frac{s}{(1-s)} \times P_{\text{developed}}$$

$$P_{\text{developed}} = P_{\text{shaft}} + P_{\text{mech loss}} = 4000 + 500 = 4500 \text{ W}$$

$$\text{Rotor copper loss, } P_{cu} = \frac{0.05}{(1-0.05)} \times 4500 = 236.84 \text{ W}$$

51. (a)

$$\text{Synchronous speed, } N_S = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Slip, } s = \frac{E_{(\text{injected})}}{E_{(\text{rotor})}} = \frac{27}{54} = \frac{1}{2} = 0.5$$

$$\text{Rotor speed, } N_r = N_s(1 - s)$$
$$= 1500(1 - 0.5) = 750 \text{ rpm}$$

52. (b)

$$\text{The flux in the circuit, } \phi = \frac{MMF}{R} = \frac{N_1 i_1}{l/\mu A} = \frac{N_1 i_1 \mu A}{2\pi r}$$

According to Faraday's law, the emf induced in the second coil is

$$V_2 = -N_2 \frac{d\phi}{dt}$$

$$= \frac{-100 \times 200 \times 500 \times 4\pi \times 10^{-7} \times 10^{-3}}{2\pi \times 10 \times 10^{-2}} \frac{di(t)}{dt}$$

$$= -\frac{1}{50}\frac{d}{dt}(3\sin 100\pi t)$$

$$V_2 = -6\pi \cos 100\pi\, t \text{ V}$$

$$= \frac{6\pi}{\sqrt{2}}(\text{rms}) \text{ lagging the current by } 90°$$

53. (a)

Leakage flux in a transformer is reduced by using shell type construction.

54. (d)

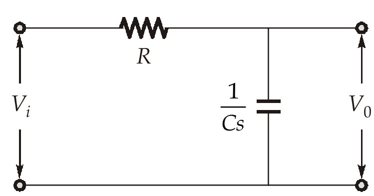
In distribution transformers to maintain the low voltage regulation, leakage reactance is kept low. Hence statement (I) is wrong.

55. (d)

Transformer having greater equivalent leakage impedance shares less kVA and that having lower leakage impedance shares greater kVA.

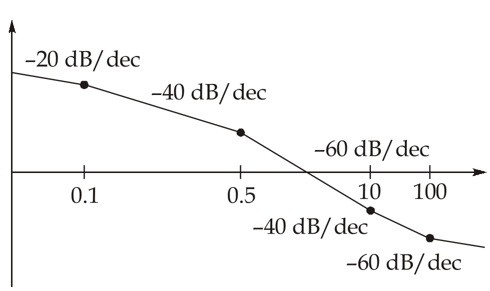
Section C : Control Systems-2 + Engineering Mathematics-2

56. (b)



$$\frac{V_0}{V_i} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

57. (c)



Given, $$P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$$

and $$C(s) = \frac{s+10}{s+0.1}$$

∴ $$P(s)C(s) = \frac{0.001(s+10)}{s(2s+1)(0.01s+1)(s+0.1)}$$

Slope at ω = 15 rad/sec is

$$= -20 - 20 - 20 + 20$$
$$= -40 \text{ dB/dec}$$

58. (b)

Phase cross-over frequency

$$-90° - \tan^{-1}\omega - \tan^{-1}4\omega = -180°$$
$$-\tan^{-1}\omega - \tan^{-1}4\omega = 90°$$
$$\tan^{-1}\left(\frac{\omega+4\omega}{1-\omega\times 4\omega}\right) = 90°$$
$$\tan^{-1}\left(\frac{5\omega}{1-4\omega^2}\right) = 90°$$
$$\frac{5\omega}{1-4\omega^2} = \infty$$
$$1 - 4\omega^2 = 0$$
$$\omega^2 = \frac{1}{4}$$
$$\omega = \frac{1}{2} = 0.50 \text{ rad/sec}$$

59. (c)

Transfer function $G(s)$ is gain by

$$G(s) = C(sI - A)^{-1}B$$
$$G_1(s) = [1 \quad 2]\begin{bmatrix} s & -1 \\ 2 & (s+3) \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$
$$= \frac{[1 \quad 2]\begin{bmatrix} (s+3) & 1 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}}{s(s+3)+2}$$

$$= \frac{[(s+1)(2s+1)]\begin{bmatrix}0\\2\end{bmatrix}}{s(s+3)+2}$$

$$G_1(s) = \frac{2(2s+1)}{(s+1)(s+2)}$$

System-2:

$$G_2(s) = [1 \quad 1]\begin{bmatrix}(s+3) & -1\\0 & (s+3)\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}$$

$$G_2(s) = \frac{[1 \quad 1]\begin{bmatrix}(s+3) & 1\\0 & (s+3)\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}}{(s+3)^2}$$

$$= \frac{[(s+3)(s+4)]\begin{bmatrix}0\\1\end{bmatrix}}{(s+3)^2} = \frac{(s+4)}{(s+3)^2}$$

$$\text{Overall T.F.} = G_1(s) \times G_2(s)$$

$$= \frac{2(2s+1)}{(s+1)(s+2)}\frac{(s+4)}{(s+3)^2}$$

60. (b)

The transfer function is calculated as follows:

$$[sI - F] = \begin{bmatrix}s & -1\\4 & s+2\end{bmatrix}$$

$$\det[sI - F] = s^2 + 2s + 4$$

$$\text{Adj}[sI - F] = \begin{bmatrix}s+2 & 1\\-4 & s\end{bmatrix}$$

$$[sI - F]^{-1} = \frac{1}{s^2+2s+4}\begin{bmatrix}s+2 & 1\\-4 & s\end{bmatrix}$$

$$G(s) = [1 \quad 0]\frac{1}{s^2+2s+4}\begin{bmatrix}s+2 & 1\\-4 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$$

$$= \frac{1}{s^2+2s+4}[1 \quad 0]\begin{bmatrix}1\\s\end{bmatrix} = \frac{1}{s^2+2s+4}$$

From the denominator of the transfer function, we get

$$\omega_n^2 = 4$$
$$\omega_n = 2$$
$$2\xi\,\omega_n = 2$$
$$\xi = \frac{2}{2\xi\omega_n} = 0.5$$

61. (c)

Given, $$G(s) = \frac{K}{s(1+0.4s)(1+0.05s)}$$

Putting $s = j\omega$

$$G(j\omega) = \frac{K}{(j\omega)(1+0.4j\omega)(1+0.05j\omega)}$$
$$G(j\omega) = \frac{K}{-0.45\omega^2 + j\omega(1-0.02\omega^2)}$$

At phase cross over frequency (ω_p) the value of $G(j\omega)$ is purely real

∴ Equating imaginary part equal to zero

$$\omega_p\,(1 - 0.02\,\omega_p^{\,2}) = 0$$
$$\omega_p^2 = \frac{1}{0.02} = 50$$
$$\omega_p = 0 \text{ and } \omega_p = \sqrt{50}\text{ rad/sec}$$

(Important shortcut method for calculating ω_p in standard form $\frac{K}{s(1+sT_1)(1+sT_2)}$)

Phase cross over frequency,

$$\omega_p = \frac{1}{\sqrt{T_1T_2}}$$
$$= \frac{1}{\sqrt{0.4\times 0.05}} = \sqrt{50}\text{ rad/sec}$$
$$|G(j\omega)|_{\omega=\omega_p} = \frac{K}{0.45\omega_p^2} = \frac{K}{0.45\times 50} = \frac{K}{22.5}$$

Using, $$20\log(GM) = 40\text{ dB}$$
$$\log GM = 2$$
$$GM = 100$$

∴ $$a = \frac{1}{GM} = \frac{1}{100} = 0.01$$
$$\frac{K}{22.5} = 0.01$$
$$K = 0.225$$

62. (a)

For given state model,

$$A = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s+4 & 0 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{|sI - A|}\text{Adj}[sI - A] = \frac{1}{(s+3)(s+4)}\begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

So zero input response of the given system will be

$$x(t) = e^{At} \cdot x(0)$$

$$= \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 4e^{-4t} \\ -4e^{-3t} \end{bmatrix}$$

63. (c)

Here, $A = (1 + x^2)$,

$B = (5 + 2x^2)$,

and $C = (4 + x^2)$

$\therefore$ $B^2 - 4AC = (5 + 2x^2)^2 - 4(1 + x^2)(4 + x^2)$

$= 9 > 0$

So the equation is hyperbolic.

64. (b)

$$\arg(Z_1) = \theta_1 = \tan^{-1}\left(\frac{5\sqrt{3}}{5}\right)$$

$\Rightarrow$ $\theta_1 = 60°$

$$\arg(Z_2) = \theta_2 = \tan^{-1}\left(\frac{2\sqrt{3}}{6}\right)$$

$$\Rightarrow \qquad \theta_2 = 30°$$

$$\arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2)$$

$$= 60° - 30° = 30°$$

Hence, option (b) is correct.

65. (b)

$$y = 0.516x + 33.73 \qquad \text{...(i)}$$

$$x = 0.512y + 32.52 \qquad \text{...(ii)}$$

$$r\frac{\sigma_y}{\sigma_x} = 0.516 \qquad \text{...(iii)}$$

$$r\frac{\sigma_x}{\sigma_y} = 0.512 \qquad \text{...(iv)}$$

$$r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y} = (0.516)\ (0.512)$$

$$r^2 = (0.516)\ (0.512)$$

$$r = 0.514$$

Coefficient of correlation = 0.514

66. (b)

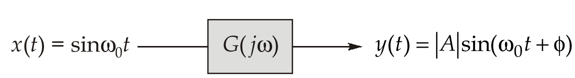
For given system gain crossover frequency can be calculated as,

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{100}{\omega_{gc}} = 1$$

$$\omega_{gc} = 100 \text{ r/sec}$$

67. (d)



$$|A| = |G(j\omega)|_{\omega=\omega_0 t}$$

$$G(j\omega) = \left.\frac{(16-\omega^2)(j\omega+2)}{(j\omega+1)(j\omega+9)}\right|_{\omega=\omega_0=4}$$

$$|G(j\omega)| = 0$$

68. (a)

A : Bolt is manufactured by machine A

B : Bolt is manufactured by machine B

C : Bolt is manufactured by machine C

$$P(A) = 0.25\ ,$$
$$P(B) = 0.35,$$
$$P(C) = 0.40$$

The probability of drawing a defective bolt manufactured by machine A is $P(D/A) = 0.05$

Similarly, $P(D/B) = 0.04$

and $P(D/C) = 0.02$

By Bay's theorem

$$P(B/D) = \frac{P(B)P(D/B)}{P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)}$$

$$= \frac{0.35\times 0.04}{0.25\times 0.05+0.35\times 0.04+0.40\times 0.02} = 0.41$$

69. (c)

$$\text{Probability of failure} = 20\% = \frac{20}{100} = \frac{1}{5}$$

$$\text{Probability of pass } (P) = 1-\frac{1}{5} = \frac{4}{5}$$

Probability of at least five pass = P(5 or 6)

$$= P(5) + P(6)$$

$$= 6\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^6$$

$$= \left(\frac{4}{5}\right)^5\left[\frac{6}{5}+\frac{4}{5}\right] = 2\left(\frac{4}{5}\right)^5 = 0.655$$

70. (a)

Let, $$f(x) = \frac{a_0}{2}+a_1\cos x+a_2\cos 2x+....+b_1\sin x+b_2\sin 2x+... \quad ...(i)$$

Hence $$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx = \frac{1}{\pi}\int_0^{2\pi} x\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{x\sin nx}{n} - 1\cdot\left(\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2} - \frac{1}{n^2}\right] = \frac{1}{n^2\pi}[1-1] = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - 1\cdot\left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\cos 2n\pi}{n}\right] = \frac{-2}{n}$$

Substituting the values of a_0 , a_n , b_n in (i) we get

$$x = \pi - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + ...\right]$$

71. (d)

$$n = 400,$$
$$p = 0.1$$
$$q = 1 - p$$

$$\text{Standard deviation} = \sqrt{npq}$$
$$= \sqrt{400\times 0.1\times(1-0.1)}$$
$$= \sqrt{400\times 0.1\times 0.9}$$
$$= 20 \times 0.3 = 6$$

72. (d)

Given, $$f(z) = \frac{e^{1/z}}{z^2}$$

$$= \frac{1}{z^2}\left(1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + ... + \frac{1}{n!z^n}\right)$$

$$= \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{2!z^4} + \frac{1}{3!z^5} + ... + \frac{1}{n!z^{n+2}}$$

Here, $f(z)$ has infinite number of terms in negative power of z.

Hence, $f(z)$ has essential singularity at $z = 0$.

73. (a)

$$P = 1\% = 0.01,$$
$$n = 100,$$
$$m = np = 100 \times 0.01 = 1$$
$$P(r) = \frac{e^{-m} \cdot (m)^r}{r!}$$

P(4 or more faulty condensers) $= P(4) + P(5) + ... + P(100)$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 1 - \left[1 - \frac{e^{-1}}{0!} + \frac{e^{-1}}{1!} + \frac{e^{-1}}{2!} + \frac{e^{-1}}{3!}\right]$$
$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right]$$
$$= 1 - \frac{8}{3e}$$

74. (a)

Here, $$f(z) = \frac{1}{\sinh z}$$

Poles are given by,

$$\sinh z = 0$$
$$\sin iz = 0$$
$$z = n\pi\, i \text{ where } n \text{ is an integer}$$

out of these, the poles $z = -\pi\, i$, 0 and $\pi\, i$ lie inside the circle $|z| = 4$.

The given function $\frac{1}{\sinh z}$ is of the form $\frac{\phi(z)}{\psi(z)}$

Its pole at $z = a$ is $\frac{\phi(a)}{\psi'(a)}$

$$\text{Residue (at } z = -\pi i) = \frac{1}{\cosh(-\pi i)} = \frac{1}{\cos i(-\pi i)} = \frac{1}{-1} = -1$$

$$\text{Residue (at } z = 0) = \frac{1}{\cosh 0} = \frac{1}{1} = 1$$

$$\text{Residue (at z} = \pi i) = \frac{1}{\cosh(\pi i)} = \frac{1}{\cos i(\pi i)} = \frac{1}{\cos(-\pi)}$$
$$= \frac{1}{\cos \pi} = \frac{1}{-1} = -1$$

Resiue at $-\pi i$, 0, πi are respectively –1, 1 and –1.
Hence the required integral $= 2\pi i\,(-1 + 1 - 1) = -2\pi i$

75. (b)

Given,

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

Above equation in symbolic form can be written as

$$(D^3 - 3D^2D' + 4D'^3)z = e^{x+2y}$$

Its A.E. is $\quad m^3 - 3m^2 + 4 = 0$

$$m = -1, 2, 2$$

$$\text{C.F.} = f_1(y - x) + f_2(y + 2x) + xf_3(y + 2x)$$

$$\text{P.I.} = \frac{1}{D^3 - 3D^2D' + 4D'^3}(e^{x+2y})$$

Put, $D = 1, D' = 2 = \dfrac{1}{1-6+32}e^{x+2y} = \dfrac{e^{x+2y}}{27}$

Hence complete solution is

$$z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x) + \frac{e^{x+2y}}{27}$$

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