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Detailed Solutions

ESE-2019 Mains Test Series

Electrical Engineering Test No : 9

Section A : Electromagnetic Theory + Computer Fundamentals + Communication Systems

Q.1 (a) Solution:

The computer can be used to perform a specific task, only by specifying the necessary steps to complete the task. The collection of such ordered steps forms a 'program' of a computer. These ordered steps are the instruction. Computer instructions are stored in control memory locations and are executed sequentially one at a time.

A computer usually has a variety of instruction code formats. The most common fields in the instruction formats are:

- An operation code field that specifies the operation to be performed.
- An address fields that designates a memory address or a processor register.
- A mode field that specifies the way the operand or the effective address is determined.

Mode	Opcode	Address

Instruction format

Types of CPU organizations:

1. **Single accumulator organization:** All the operations are performed with an accumulator register. The instruction format in this type of computer uses one address field.

   Example: ADD X

2. **General register organization:** The instruction format in this type of computer needs three register address fields.

   Example: ADD R1, R2, R3

3. **Stack organization:** The instruction in a stack computer consists of an operation has the effect of popping the 2 top numbers from the stack, operation the number and pushing the sum into the stack.

   Example: ADD

Q.1 (b) Solution:

Pipelining is a process of arrangement of hardware elements of the CPU such that its overall performance is increased. Simultaneous execution of more than one instruction take place in a pipelines processor.

- Pipelined processor has multistage/segments such that output of one stage is connected to input of next stage and each stage performs a specific operation.
- Interface registers are used to hold the intermediate output between two stages. These interface registers are also called latch or buffer.
- All the stages in a pipeline along with the interface registers are controlled by a common clock.

Performance of a pipelined processor:

Consider a k segment pipeline with clock cycle time as 'T_P'. Let there be 'n' tasks to be completed in the pipelined processor.

Now, the first instruction will take 'k' cycles to come out of the pipeline but the other $n - 1$ instructions will take only one cycle each. i.e. a total of $(n - 1)$ cycles. So, time taken to execute 'n' instructions in a pipelined processor:

$$ET_{\text{pipeline}} = (k + n - 1) \text{ cycles}$$
$$= (k + n - 1)\, T_P$$

For a non-pipelined processor, execution time of 'n' instructions will be

$$ET_{\text{non-pipeline}} = n * k * T_P$$

So, speed up (S) of the pipelined processor over non-pipelined processor, when 'n' tasks are executed on the same processor is

$$S = \frac{ET_{\text{non-pipeline}}}{ET_{\text{pipeline}}}$$

$$S = \frac{(n \times k \times T_P)}{(k + n - 1)T_P} = \frac{n \times k}{(k + n - 1)}$$

When number of tasks 'n' are larger than k,

i.e. $$n >> k$$

$$S_{\max} = \frac{n \times k}{n} = k$$

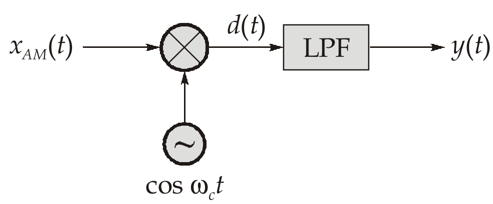
$$S_{\max} = k$$

$$\text{Efficiency} = \frac{\text{Given speed up}}{\text{Max. speed up}}$$

$$\eta = \frac{S}{S_{\max}} = \frac{S}{k}$$

Q.1 (c) Solution:

Given, $$x_{\text{AM}}(t) = [A + m(t)]\cos\omega_c t$$



$$d(t) = x_{\text{AM}}(t) \cos \omega_c t = [A + m(t)] \cos 2\, \omega_c t$$

$$= \frac{1}{2}[A + m(t)] + \frac{1}{2}[A + m(t)]\cos^2 \omega_c t$$

Hence, after low-pass filtering, we obtain

$$y(t) = \frac{1}{2}[A + m(t)] = \frac{1}{2}m(t) + \frac{1}{2}A$$

A blocking capacitor will suppress the direct-current (dc) term $\frac{1}{2}A$, yielding the output $\frac{1}{2}m(t)$.

Q.1 (d) Solution:

Given, $$D = D = \left(10xyz^2 + 4x\right)\hat{a}_x + 5x^2z^2\hat{a}_y + 10x^2yz\,\hat{a}_z \text{ nC/m}^2$$

(i) Total charge enclosed in a cube of volume 10^{-10} m^3 located at (2, 3, 4) is given by

$$Q = \int_{\text{vol}} \rho_v \cdot dV = \int_{\text{vol}} \nabla \cdot D dV$$

The volume charge density is,

$$\rho_v = \nabla \cdot D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(10xyz^2 + 4x\right) + \frac{\partial}{\partial y}(5x^2z^2) + \frac{\partial}{\partial z}\left(10x^2yz\right)$$

$$= 10\, yz^2 + 4 + 0 + 10\, x^2y$$

$$\text{Charge enclosed, } \Delta Q = \rho_v \cdot \Delta V$$

$$= \left(10yz^2 + 4 + 10x^2y\right) \times 10^{-10}\Big|_{x=2,y=3,z=4}$$

$$= 604 \times 10^{-9} \times 10^{-10}$$

$$= 6.04 \times 10^{-17}\ \text{C}$$

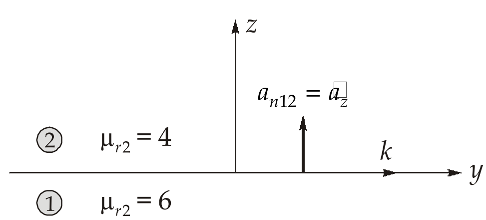
(ii) Flux leaving the volume is,

$$\Delta\psi = \Delta Q = 6.04 \times 10^{-17}\ \text{C}$$

(iii) The average volume charge density within the volume is

$$\rho_{av} = \frac{\Delta Q}{\Delta V} = \frac{6.04 \times 10^{-17}}{10^{-10}} = 6.04 \times 10^{-7}\ \text{C/m}^3.$$

Q.1 (e) Solution:



$$B_{1n} = B_{2n} = 8a_z \Rightarrow B_z = 8 \qquad \text{...(i)}$$

But,

$$H_2 = \frac{B_2}{\mu_2} = \frac{1}{4\mu_0}(5a_x + 8a_z)\ \text{mA/m} \qquad \text{...(ii)}$$

$$H_1 = \frac{B_1}{\mu_1} = \frac{1}{6\mu_0}(B_x a_x + B_y a_y + B_z a_z)\ \text{mA/m} \qquad \text{...(iii)}$$

Having found the normal components, we can find the tangential components by using $\left(H_1 - H_2\right) \times a_{n12} = K$.

$$H_1 \times a_{n12} = H_2 \times a_{n12} + K \qquad \text{...(iv)}$$

Substitute equation (ii) and (iii) in equation (iv), We get,

$$\frac{1}{6\mu_0}\left(B_x a_x + B_y a_y + B_z a_z\right)\times \hat{a}_z = \frac{1}{4\mu_0}(5a_x + 8a_z)\times \hat{a}_z + \frac{1}{\mu_0}\hat{a}_y$$

Equating components yields, $B_y = 0,\ \frac{-B_x}{6} = \frac{-5}{4} + 1$

$$B_x = \frac{6}{4} = 1.5 \quad \text{...(v)}$$

From equation (i) and (v), we get

$$B_1 = 1.5a_x + 8a_z \text{ mWb/m}^2$$

$$H_1 = \frac{B_1}{\mu_1} = \frac{1}{\mu_0}(0.25a_x + 1.33a_z) \text{ mA/m}$$

and

$$H_2 = \frac{B_2}{\mu_2} = \frac{1}{\mu_0}(1.25a_x + 2a_z) \text{ mA/m}$$

Q.2 (a) Solution:

```
# include <stdio.h>
# include <conio.h>
# include <math.h>
void main ( )
{
float x, sum = 0;
int i, j, limit;
printf ("Enter the value of x in degress:");
scanf ("%f", &x);
print ("Enter the number of terms required:");
scanf ("%d", & limit);
Q = x;
x = x * (3.1415/180);
for (i = 1, j = 1; i < = limit, i ++, j ++ = 2);
{
If (i% 2! = 0)
sum + = pow(x, j)/fac(j);
else
```

```
        sum = sum – pow (i, j)/fac (j);
    }
        printf ("sin (%0.1f): %f", Q, sum);
        getch ( );
    }
        int fac (int k)
    {   int p, foc = 1;
        for (p = 1; p < = k, p++)
        fac = fac * p
        return fac;
    }
```

Q.2 (b) Solution:

A single-tone AM signal can be expressed as,

$$x_{\text{AM}}(t) = A\cos\omega_c t + \mu A\cos\omega_m t\cos\omega_c t$$

$$= A\cos\omega_c t + \frac{1}{2}\mu A\cos(\omega_c - \omega_m)t + \frac{1}{2}\mu A\cos(\omega_c + \omega_m)t$$

$$P_c = \text{carrier power} = \frac{1}{2}A^2$$

$$P_s = \text{sideband power} = \frac{1}{2}\left[\left(\frac{1}{2}\mu A\right)^2 + \left(\frac{1}{2}\mu A\right)^2\right] = \frac{1}{4}\mu^2 A^2$$

The total power P_t is,

$$P_t = P_c + P_s = \frac{1}{2}A^2 + \frac{1}{4}\mu^2 A^2 = \frac{1}{2}\left(1 + \frac{1}{2}\mu^2\right)A^2$$

Thus,
$$\eta = \frac{P_s}{P_t}\times 100\% = \frac{\frac{1}{4}\mu^2 A^2}{\left(\frac{1}{2} + \frac{1}{4}\mu^2\right)A^2}\times 100\%$$

$$\eta = \frac{\mu^2}{2 + \mu^2}\times 100\%$$

With the condition that $\mu \le 1$

(i) For $\mu = 0.5$,

$$\eta = \frac{(0.5)^2}{2 + (0.5)^2}\times 100\% = 11.1\%$$

(ii) Since $\mu \le 1$, it can be seen that η_{max} occurs at $\mu = 1$ and is given by

$$\eta = \frac{1}{3} \times 100\% = 33.3\%$$

Q.2 (c) Solution:

(i) Given the total stored energy in the region-1 is three time that of the region-2 and $D_1 = 2D_2$. Let the region-1 has surface $2a_1$, the surface area of region-2 is $2(2 - a_1)$.

$$W_{E1} = 3W_{E2}$$

$$\frac{1}{2}\varepsilon_1 E^2 \times 2a_1 \times 2 = 3 \times \frac{1}{2}\varepsilon_2 E^2 \times 2(2 - a_1) \times 2$$

$$\varepsilon_1 a_1 = 3\varepsilon_2 (2 - a_1) \quad \text{...(i)}$$

Given that,

$$D_1 = 2D_2$$

$$\varepsilon_1 E = 2\varepsilon_2 E$$

$$\varepsilon_1 = 2\varepsilon_2 \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\frac{\varepsilon_1 a_1}{\varepsilon_1} = \frac{3\varepsilon_2(2 - a_1)}{2\varepsilon_2}$$

$$a_1 = \frac{3(2 - a_1)}{2}$$

$$2a_1 = 6 - 3a_1$$

$$5a_1 = 6$$

$$a_1 = \frac{6}{5} = 1.2 \text{ m}$$

$$\therefore \quad a_2 = 2 - 1.2 \text{ m} = 0.8 \text{ m}$$

$\therefore$ Dimension of first capacitor

$$= 2 \times 1.2 \times 2 \times 10^{-3} \text{ m}^3$$

Dimension of second capacitor

$$= 2 \times 0.8 \times 2 \times 10^{-3} \text{ m}^3$$

(ii) Let the total capacitance is $C_{total} = 50$ nF

Total capacitance is

$$C_{total} = 50 \times 10^{-9}$$

$$= \varepsilon_1 \times \frac{8.85 \times 10^{-12} \times 2 \times 1.2}{2 \times 10^{-3}} + \varepsilon_2 \times \frac{8.85 \times 10^{-12} \times 2 \times 0.8}{2 \times 10^{-3}}$$

$$12\varepsilon_1 + 8\varepsilon_2 = 56.5 \quad \text{...(iii)}$$

$\therefore$ From equation (ii) and (iii),

$$32\,\varepsilon_2 = 56.5$$

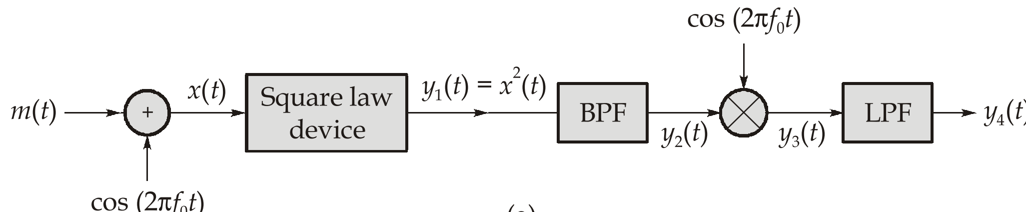
$$\varepsilon_2 = \frac{56.5}{32} = 1.765$$

$$\therefore \quad \varepsilon_1 = 3.53$$

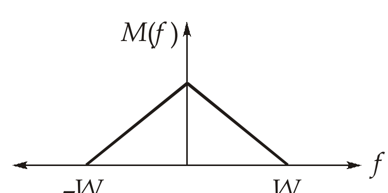
Q.3 (a) Solution:

The signal $x(t)$ is $m(t) + \cos(2\pi f_0 t)$

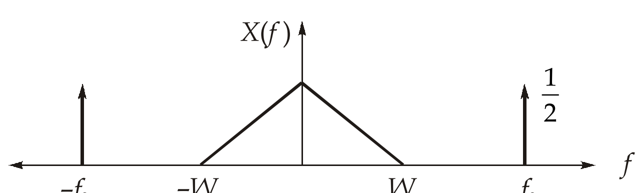
$$\text{Spectrum } X(f) = M(f) + \frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$$



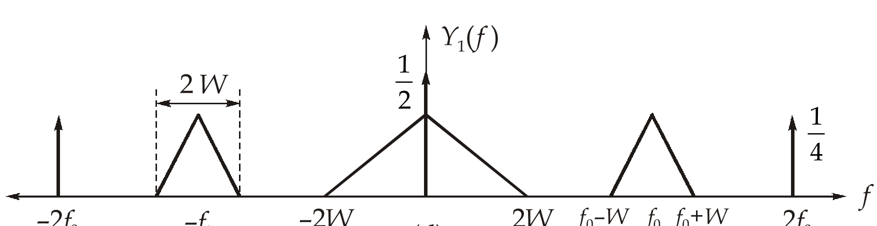
(a)



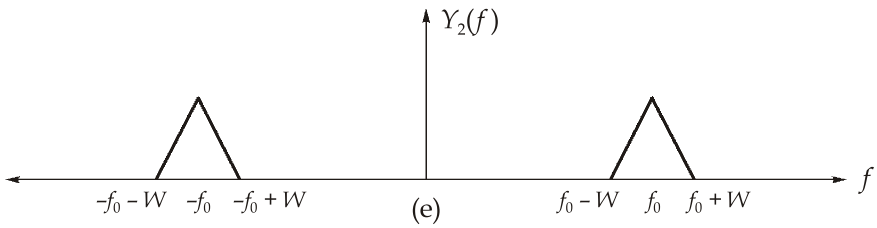
(b)



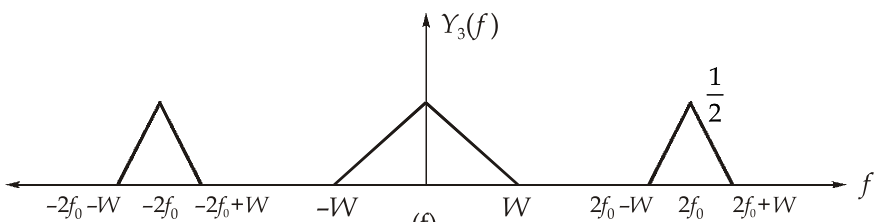
(c)



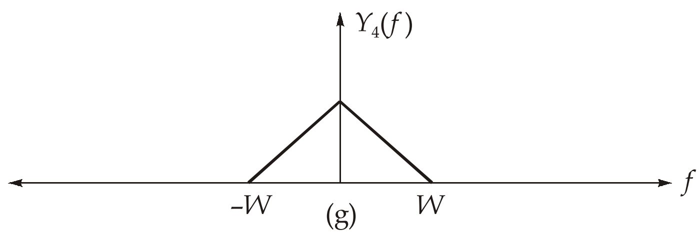
(d)



(e)



(f)



(g)

Bandwidth of $x(t)$ is f_0

The signal, $\quad y_1(t) = x^2(t) = [m(t) + \cos(2\pi f_0 t)]^2$

$$= m^2(t) + \cos^2(2\pi f_0 t) + 2\, m(t) \cos(2\pi f_0 t)$$

$$= m^2(t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi \cdot 2 f_0 t) + 2m(t)\cos(2\pi f_0 t)$$

Spectrum, $Y_1(f) = M(f) * M(f) + \frac{1}{2}\delta(f) + \frac{1}{4}[\delta(f - 2f_0) + \delta(f + 2f_0)] + M(f - f_0) + M(f + f_0)$

Bandwidth of $y_1(t)$ is $2f_0$

The BPF will cut off the low-frequency components $M(f) * M(f) + \frac{1}{2}\delta(f)$ and the terms with the double frequency components $\frac{1}{4}[\delta(f - 2f_0) + \delta(f + 2f_0)]$. Thus,

Spectrum, $Y_2(f) = M(f - f_0) + M(f + f_0)$

Bandwidth of $y_2(t)$ is 2 W

The signal $y_3(t)$ is $2\, m(t) \cos^2(2\pi f_0 t)$

$$y_3(t) = m(t) + m(t)\cos(2\pi \times 2 f_0 t)$$

$$Y_3(f) = M(f) + \frac{1}{2}[M(f - 2f_0) + M(f + 2f_0)]$$

Bandwidth,

The LPF will eliminate the spectral components $\frac{1}{2}\left[M(f-2f_0)+M(f+2f_0)\right]\cdot Y_4(t)=m(t)$

and its spectrum $Y_4(f) = M(f)$, and its bandwidth is W. The figures (c), (d), (e), (f) and (g) show the spectra of the signals $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$.

Q.3 (b) (i) Solution:

	Hardwired Control Unit	Micro programmed Control Unit
1.	It is a sequential circuit that generate control signals. It uses a fixed architecture.	It is a unit with micro instructions in the control memory to generate control signals.
2.	The speed of operation is fast.	The speed of operations is slow because it requires frequent memory access.
3.	To do modification, entire unit should be redesigned.	Modification can be implemented by changing the micro instructions in the control memory.
4.	It is more costly to implement.	It is less costly to implement.
5.	It is difficult to perform instruction decoding.	Instruction decoding is easier.
6.	It uses small instruction set.	It uses large instruction set.
7.	There is no control memory usage.	It uses control memory.
8.	It is used in processors that use a simple instruction set known as the reduced instruction set computer (RISC).	This control unit is used in processors based on complex instruction set known as complex instruction set computer (CISC)

Q.3 (b) (ii) Solution:

Array: An array is collection of items stored at contiguous memory locations. It stores the multiple items of same type together. The base index of an array can be freely chosen.

index of the element to be accessed

arr [0]

array variable

Arrays allow random access of elements. Arrays have better cache locality, which provides better performance.

Syntax : type arrayName [arraysize];

Structure: A structure is a user-defined data type available in *C* that allows to combining data items of different kinds. Structures are used to represent a record.

- The keyword struct is used to define a structure.
- When a variable is associated with a structure, the complier allocates the memory for each member. The size of the structure is greater than a equal to the sum of sizes of its members.
- Each member within a structure is assigned unique storage area of location.
- Altering the value of a member will not affect other members of the structure.
- Individual members can be accessed at a time.
- Several members of a structure can be initialized at once.

Syntax : struct struct_name
{ type element;
} variable;

Union: A union is a special data types available in *C* that allows storing different data types in the same memory location.

- The keyword union is used to define a union.
- When a variable is associated with a union, the complier allocates the memory by considering the size of the largest memory. So, size of union is equal to the size of largest member.
- Memory allocated is shared by individual members of union.
- Alter the value of any of the member, will alter other members value.
- Only one member can be accessed at a time.
- Only the first member of the union can be initialized.

Syntax : union union_name
{ type element;
} variable;

Q.3 (c) Solution:

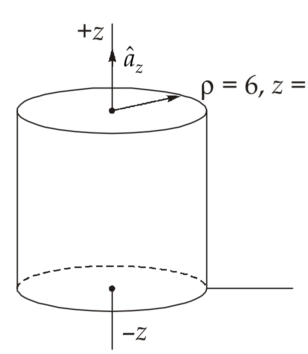
```
# include <conio.h>
# include <stdio.h>
void main ( )
{
int data [100], i, j, temp;
```

```
        printf ("Enter the number of element to be stored");
        scanf ("%d", & n);
        for (i = 0; i < n; i++)
        scanf ("%d", & data [ ]);
        for (i = 0; i < n; i++)
{
        for (j = i + 1; j < n; j ++)
{
        If (data [j] < data [i])
{
        temp = data [j];
        data [j] = data [i];
        data [i] = temp;
}}}
        printf ("In ascending order: ");
        for (i = 0; i < n; ++i)
        printf ("%d", data [i]);
        getch ( );
}
```

Q.4 (a) Solution:

The current density is $J = \frac{5}{\rho}\hat{a}_\rho + \frac{10}{\rho^2+1}\hat{a}_z \text{ A/m}^2$

(i) The total current crossing the surface $z = 3$, $\rho < 6$, in $\hat{a}_z$ direction, as shown in figure below,



$$I = \int_0^{2\pi}\int_0^{6}(J_z)_{z=3}\cdot\rho\, d\rho d\phi$$

$$= \int_0^{2\pi}\int_0^{6}\left(\frac{10}{\rho^2+1}\right)\cdot\rho\, d\rho d\phi = 20\pi\int_0^{6}\frac{\rho}{\rho^2+1}\cdot d\rho$$

$$= 20\pi\times\frac{1}{2}\left[\ln\left(\rho^2+1\right)\right]_0^6 = 10\,\pi\,\ln\,(37)$$

$$I = 113.44\text{ A}$$

(ii) Rate of change of volume charge density is

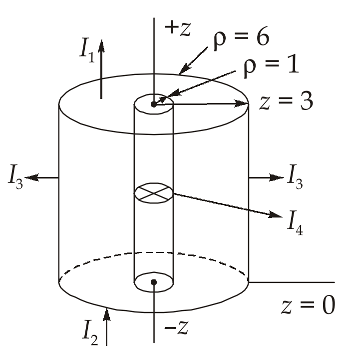
$$\frac{\partial\rho_v}{\partial t} = -\nabla\cdot J$$

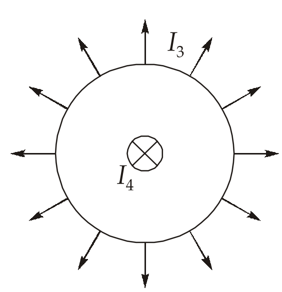
In cylindrical coordinate system we have,

$$\nabla\cdot J = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho J_\rho\right)+\frac{1}{\rho}\frac{\partial J_\phi}{\partial\phi}+\frac{\partial J_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(5)+\frac{\partial}{\partial z}\left[\frac{10}{\rho^2+1}\right] = 0$$

(iii)





The total current enclosed by the volume bounded by surfaces $z = 0$, $z = 3$, $\rho = 1$ and $\rho = 6$, shown in above figure

At $z = 3$, Current, $I_1 = \int_0^{2\pi}\int_1^{6}(J_z)_{z=3}\cdot\rho\, d\rho\, d\phi$

$$= \int_0^{2\pi}\int_1^{6}\frac{10}{\rho^2+1}\cdot\rho\, d\rho\, d\phi$$

$$= 20\pi\times\frac{1}{2}\left[\ln(\rho^2+1)\right]_1^6$$

$$= 10\pi \times [\ln 37 - \ln 2] = 91.66 \text{ A}$$

At $z = 0$, again current is $I_2 = -91.62$ A

At $\rho = 6$, Current is, $I_3 = \int_0^{2\pi}\int_0^3 (J_\rho)_{\rho=6} \cdot \rho\, dz\, d\phi$

$$= \int_0^{2\pi}\int_0^3 \frac{5}{\rho} \cdot \rho\, dz\, d\phi = 2\pi \times 5 \times 3 = 30\,\pi \text{ A}$$

At $\rho = 1$, Current is, $I_4 = -30\pi$ A

Thus the total current crossing the enclosed surface,

$$I = I_1 + I_2 + I_3 + I_4 = 0$$

Q.4 (b) Solution:

$$x_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$
$$= 10\cos(\omega_c t + 3\sin\omega_m t)$$

Thus, $m(t) = a_m \sin\omega_m t$

and $x_{PM}(t) = 10\cos(\omega_c t + k_p a_m \sin\omega_m t)$

For phase modulation, $\beta = k_p a_m$

or $x_c(t) = \text{A}\cos(\omega_c t + \beta\sin\omega_m t)$

$$\beta = k_p a_m = 3$$

We see that the value of β is independent of f_m.

When, $f_m = 1$ kHz

$$f_B = 2(\beta + 1)f_m = 8 \text{ kHz}$$

(i) When f_m is doubled, $\beta = 3$,

$$f_m = 2 \text{ kHz}$$

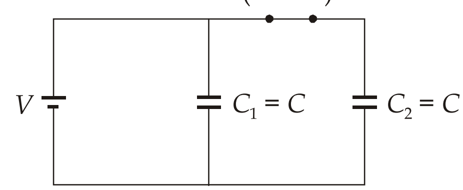
and $f_B = 2(3 + 1)2 = 16$ kHz

(ii) When f_m is decreased by one-half,

$$\beta = 3,$$
$$f_m = 0.5 \text{ kHz}$$

and $f_B = 2(3 + 1)(0.5) = 4$ kHz

Q.4 (c) Solution:



I-circuit condition with switch *S* closed.

We know that energy stored in a capacitor

$$= \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q = \text{Charge of capacitor}$$

$$C = \text{Capacitance}$$

$$V = \text{Applied voltage across capacitor}$$

$$\text{Charge on both capacitor} = CV$$

Let E_{T1} be total energy in (S closed) this condition

$$E_{T1} = \frac{1}{2}CV^2 + \frac{1}{2}CV^2$$

$$E_{T1} = CV^2$$

II-circuit condition with switch open and dielectric (K = 2) introduced between the capacitive plates. Both capacitance will increase by a factor of 2.

So, $$C_1 = C_2 = 2C$$

It is important to note that charge on C_2 is unchanged due to opening of switch and introducing dielectric as it is an isolated capacitor, so Q on $C_2 = CV$ only a voltage can alter charge on a capacitor. Now let E_{T2} is total energy in this arrangement with open switch and dielectric introduced.

$$E_{T2} = 7\frac{1}{2}C_1V^2 + \frac{Q^2}{2C_2} = \frac{1}{2}(2C)V^2 + \frac{(CV)^2}{2(2C)}$$

$$= CV^2 + \frac{CV^2}{4} = \frac{5}{4}CV^2$$

$$E_{T2} = \frac{5}{4}CV^2$$

$$\text{The ratio of } E_{T1} \text{ and } E_{T2} = \frac{CV^2}{\frac{5}{4}CV^2} = \frac{4}{5}$$

Section B : Systems & Signal Processing-2 + Electrical & Electronic Measurements-2

Q.5 (a) Solution:

$$X(z) = \log\left(\frac{1}{1-a^{-1}z}\right), \qquad |z| < |a|$$

$$= -\log(1 - a^{-1}z)$$

$$= +\left[a^{-1}z + \frac{(a^{-1}z)^2}{2} + \frac{(a^{-1}z)^3}{3} + \frac{(a^{-1}z)^4}{4} + ...\right]$$

$$= \sum_{n=1}^{\infty} \frac{(a^{-1}z)^n}{n} = \sum_{n=1}^{\infty} \frac{a^{-n}}{n} \cdot z^n$$

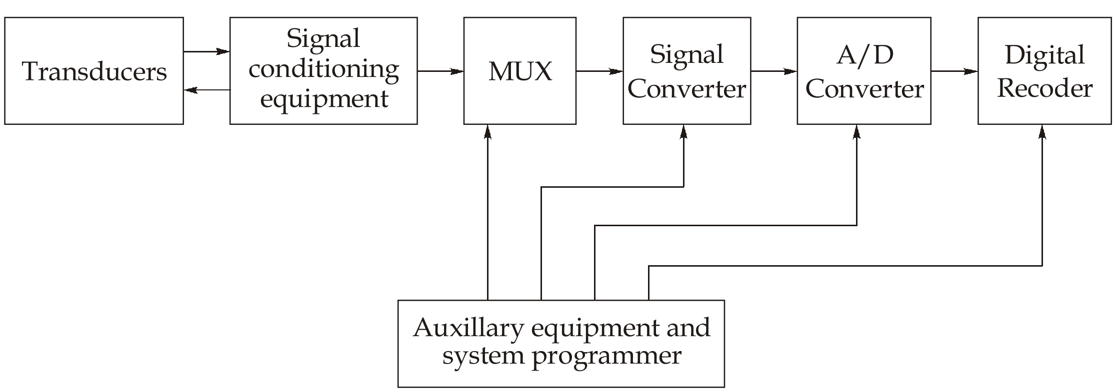
$$= \sum_{n=-1}^{-\infty} \frac{-1}{n} z^{-n} a^n$$

$$X(z) = \sum_{n=-1}^{-\infty} -\frac{a^n}{n} z^{-n}$$

$$\therefore \qquad x(n) = \begin{cases} 0 & n \geq 0 \\ -\frac{1}{n}a^n & n \leq -1 \end{cases}$$

Q.5 (b) Solution:

A generalized diagram of a digital data acquisition system is shown in below:



A digital data acquisition system may include some or all of the components shown in figure above. The essential functional operations of a digital data acquisition system are:

(a) handling of analog signals

(b) making the measurement

(c) converting the data to digital form and handling it

(d) internal programming and control

The various components and their functions are described below:

1. **Transducers:** They convert a physical quantity into an electrical signal which is acceptable by the data acquisition system.
2. **Multiplexer:** Multiplexing is the process of sharing a single channel with more than one input.
3. **Signal converter:** A signal converter translates the analog signal to form acceptable by the A/D converter.
4. **A/D converter:** An A/D converter converts the analog voltage to its equivalent digital form.
5. **Digital recorders:** Records of information in digital form may be load on punched cords, floppy discs, magnetic takes etc.

Q.5 (c) Solution:

Residue method:

In this method we obtain inverse z-transform by summing residues of $\left[X(z)z^{n-1}\right]$ at all poles.

Mathematically, this may be expressed as

$$x(n) = \sum_{\text{all poles of } X(z)} \text{Residue of} \left[X(z)\cdot z^{n-1}\right]$$

Here, residue of any pole of order m at $z = \beta$ is

$$\text{Residue} = \frac{1}{(m-1)!} \lim_{z\to\beta} \cdot \frac{d^{n-1}}{dz^{m-1}}\left[(z-\beta)^m X(z)\cdot z^{n-1}\right]$$

$$X(z) = \frac{z(z+1)}{(z-1)(z-2)}$$

Residue of $X(z)\ z^{n-1}$

At pole $z = 1$

$$R_1 = \left.\frac{(z-1)z^n(z+1)}{(z-1)(z-2)}\right|_{z=1} = (1)^n \cdot \frac{2}{-1} = -2$$

At pole $z = 2$,

$$R_2 = \left.\frac{(z-2)z^n(z+1)}{(z-1)(z-2)}\right|_{z=2}$$

$$= 2^n \cdot \frac{3}{1} = 3 \cdot 2^n$$

ROC : $1 < |z| < 2$

$z = 1$ is interior to c and $z = 2$ is exterior to c

$\therefore$ $$x(n) = -2\,u(n) - 3\,2^n\,u(-n-1)$$

Q.5 (d) Solution:

Let R_1 and L_1 be the effective resistance and inductance of the specimen respectively.

At balance $$(R_1 + j\omega L_1)\frac{1}{j\omega C_4} = R_3\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$L_1 = R_2R_3C_4 = 834 \times 100 \times 0.1 \times 10^{-6}\text{ H} = 8.34\text{ mH}$$

and $$R_1 = \frac{R_3C_4}{C_2} = \frac{100 \times 0.1}{0.12} = 83.33\,\Omega$$

Reactance of specimen at 2 kHz

$$X_1 = 2\pi \times 2 \times 1000 \times 8.34 \times 10^{-3} = 104.8\ \Omega$$

$\therefore$ Impedance of specimen, $$Z_1 = \sqrt{(83.33)^2 + (104.8)^2} = 133.89\,\Omega$$

Q.5 (e) Solution:

(i) $$\text{Resolution} = \frac{1}{10^n} = \frac{1}{10^4} = 0.0001$$

where the number of full digits is $n = 4$

(ii) There are 5 digit place is $4\frac{1}{2}$ digits, therefore 12.98 would be displayed as 12.980.

(iii) Resolution on 1 V range is 1 V × 0.0001 = 0.0001
Any reading up to the 4th decimal can be displayed.
Hence 0.6973 will be displayed as 0.6973.
Resolution on 10 V range = 10 V × 0.0001 = 0.001 V
Hence decimals up to the 3rd decimal place can be displayed.
Therefore on a 10 V range, the reading will be 0.697 instead of 0.6973.

Q.6 (a) Solution:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega}, & \frac{-\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j2\omega}\cdot e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-2)}d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-2)}}{(n-2)j}\right]_{-\pi/4}^{\pi/4} = \frac{1}{2\pi}\left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{j(n-2)}\right]$$

$$h_d(n) = \frac{e^{j\frac{n\pi}{4}}e^{\frac{-j\pi}{2}} - e^{-j\frac{n\pi}{4}}e^{\frac{j\pi}{2}}}{2\pi(n-2)}$$

$$= -\frac{\left[e^{j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{4}}\right]}{2\pi(n-2)}$$

$$= \frac{-1}{\pi(n-2)}\cdot\cos\left(\frac{n\pi}{4}\right);\ n \ne 2$$

$$h_d(2) = \lim_{n\to 2}\frac{\left(\sin\frac{n\pi}{4}\right)\times\frac{\pi}{4}}{\pi} = \frac{1}{4}$$

$$h_d(0) = \frac{1}{2\pi}$$

$$h_d(1) = \frac{1}{\pi\sqrt{2}}$$

$$h_d(2) = \frac{1}{4}$$

$$h_d(3) = \frac{1}{\pi\sqrt{2}};$$

$$h_d(4) = \frac{1}{2\pi}$$

Filter coefficients of the filter would be then,

$$h(n) = h_d(n) - w(n)$$

$$h(0) = \frac{1}{2\pi} = h(4)$$

$$h(1) = \frac{1}{\sqrt{2}\pi} = h(3)$$

$$h(2) = \frac{1}{4}$$

Frequency response,
$$H(e^{j\omega}) = \sum_{n=0}^{4} h(n)e^{-j\omega n}$$

$$= h(0) + h(1)e^{-j\omega} + h(2)^{-j2\omega} + h(3)e^{-j3\omega} + h(4)^{-j4\omega}$$

$$= \frac{1}{2\pi} + \frac{1}{\pi\sqrt{2}}e^{-j\omega} + \frac{1}{4}e^{-j2\omega} + \frac{1}{\sqrt{2}\pi}e^{-j3\omega} + \frac{1}{2\pi}e^{-j4\omega}$$

$$= \frac{e^{-j2\omega}}{\pi}\left[\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right] + \frac{2e^{-j2\omega}}{\pi\sqrt{2}}\left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right] + \frac{1}{4}e^{-j2\omega}$$

$$H(e^{j\omega}) = e^{-j2\omega}\left[\frac{\cos 2\omega}{\pi} + \frac{\sqrt{2}}{\pi}\cos\omega + \frac{1}{4}\right]$$

Q.6 (b) Solution:

Principle of operation: The signal wave form is converted to trigger pulses and is applied continuity to an AND gate, as shown in figure below. A pulse of 1s is applied to the other terminal and the number of pulses counted during this period indicates the frequency.

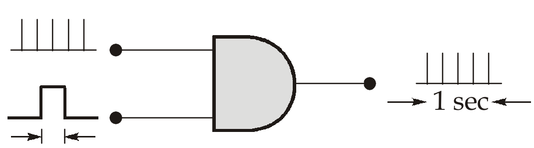
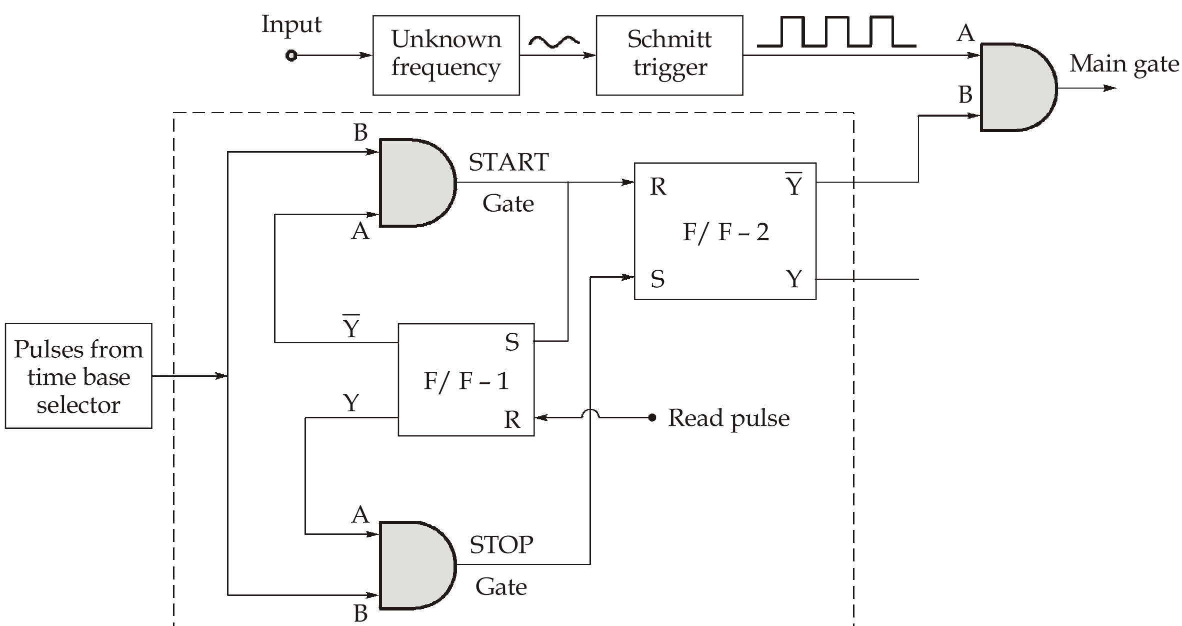


Fig: Principle of digital frequency measurement

The signal whose frequency is to be measured is converted into a train of pulses, one pulse for each cycle of the signal. The number of pulses occurring in adequate interval of time is then counted by an electronic counter. Since each pulse represents the cycle of the signal, the number of counter is a direct indication of the frequency of the signal.

Basic circuit for frequency measurement: The basic circuit or frequency measurement is as shown in figure below. The output of the unknown frequency is applied to a Schmitt trigger, producing positive pulses at the output. These pulses are called the counter signals and are present at point A of the main gate. Positive pulses from the time base selector are present at point B of the start gate and at point B of the stop gate.



Gate control F/F

Initially the flip-flop (FF – 1) is at its logic 1 state. The resulting voltage from output Y is applied to point A of the STOP gate and enables this gate. The logic 0 stage at the output $\bar{Y}$ of the F/F – 1 is applied to the input A of the START gate and disables the gate.

As the STOP gate is enabled, the positive pulses from the time base pass through the STOP gate to the set (S) input of the F/F – 2 thereby setting F/F – 2 to the 1 state and keeping it there.

The resulting 0 output level from $\bar{Y}$ of F/F – 2 is applied to terminal B of the main gate. Hence no pulses from the unknown frequency source can pass through the main gate.

In order to start the operation, a positive pulse is applied to (read input) reset input of F/F – 1, thereby causing its state to change. Hence $\bar{Y} = 1$, $Y = 0$ and as a result the STOP gate is disabled and the START gate is enabled.

The pulse from the unknown frequency source pass through the main gate to the counter and the counter starts counting. This same pulse from the START gate is applied to the set input of F/F – 1, changing its state from 0 to 1. This disables the START gate and enables this stop gate. However, till the main gate is enabled, pulses from the unknown frequency continue to pass through the main gate to the counter.

The counter counts the number of pulses occurring between two successive pulses from the time base selector. If the time interval between this two successive pulses from the time base selector is one second, then the number of pulses counted within this interval is the frequency of the unknown frequency source, in hertz.

Q.6 (c) (i) Solution:

$$\omega = 2\tan^{-1}\frac{\Omega T}{2}$$

$$(s + 0.1)^2 = -9 = (3j)^2$$

$$s = -0.1 \pm 3j$$

We know,

$$s = -\sigma + \Omega$$

$$\therefore \quad \Omega_C = 3 \text{ rad/sec}$$

$$\Omega_C = \frac{2}{T}\tan\frac{\omega_r}{2}$$

$$T = \frac{2}{3}\tan\frac{\pi}{8} = 0.276\,\text{sec}$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\frac{(z-1)}{(z+1)}} = H(s)\Big|_{s=7.24\left(\frac{z-1}{z+1}\right)}$$

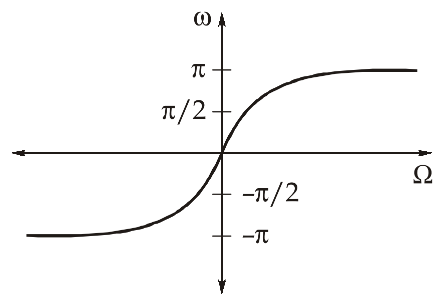
$$H(z) = \frac{7.24\left(\dfrac{z-1}{z+1}\right)+0.1}{\left(\dfrac{7.24z-7.24}{z+1}+0.1\right)^2+9}$$

$$= \frac{(7.24z-7.24+0.1z+0.1)(z+1)}{(7.24z-7.24+0.1z+0.1)^2+9(z+1)^2}$$

$$= \frac{(7.34z-7.14)(z+1)}{(7.34z-7.14)^2+9(z^2+2z+1)}$$

$$H(z) = \frac{(7.34z^2-0.2z-7.14)}{(62.88z^2-86.82z+59.97)}$$

Q.6 (c) (ii) Solution:



The IIR filter design is based on the established methods of analog filter design. After designing analog filter, it is converted into its corresponding digital filter by using bilinear transformation. But the use of bilinear transformation compare as. (–∞, ∞) frequency range of analog filter into finite range (–π, π) of digital filter. Due to this the designed digital filter does not have the same cut off frequency as it was given in the digital filter specification. This type of non linear distortion is known as "working".

To compensate frequency working digital filter specification is 1st of all converted into analog filter specification by applying pre working relationship.

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

This relationship compensated the non digital designed filter will have the same cut-off frequency given in the specification.

Q.7 (a) Solution:

(i)

$$X(s) = \int_0^\infty \sin\omega_0 t\, u(t+2)e^{-st}dt$$

$$= \int_0^\infty \sin\omega_0 t\, e^{-st}dt = \int_0^\infty \frac{\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)}{2j}e^{-st}dt$$

$$= \frac{1}{2j}\int_0^\infty \left[e^{-(s-j\omega_0)t} - e^{-(j\omega_0+s)t}\right]dt$$

$$= \frac{1}{2j}\left[\frac{e^{-(s-j\omega_0)t}}{-(s-j\omega_0)} - \frac{e^{-(j\omega_0+s)t}}{-(s+j\omega_0)}\right]_0^\infty$$

$$= \frac{1}{2j}\left[\frac{-1}{(s-j\omega_0)}(0-1) + \frac{1}{s+j\omega_0}(0-1)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega_0} - \frac{1}{s+j\omega_0}\right]$$

$$= \frac{1}{2j}\left[\frac{2j\omega_0}{s^2+\omega_0^2}\right] = \frac{\omega_0}{s^2+\omega_0^2}$$

(ii)

$$X(s) = \int_0^\infty x(t)e^{-st}dt$$

$$= \int_0^1 \sin \pi t\, e^{-st}dt = \int_0^1 \left(\frac{e^{j\pi t}-e^{-j\pi t}}{2j}\right)e^{-st}dt$$

$$= \frac{1}{2j}\int_0^1 \left[e^{-(s-j\pi)t} - e^{-(s+j\pi)}\right]dt$$

$$= \frac{1}{2j}\left[\frac{e^{-(s-j\pi)t}}{-(s-j\pi)} - \frac{e^{-(s+j\pi)t}}{-(s+j\pi)}\right]_0^1$$

$$= \frac{1}{2j}\left[\frac{-1}{(s-j\pi)}\left(e^{-(s-j\pi)}-1\right) + \frac{1}{(s+j\pi)}\left(e^{-(s+j\pi)}-1\right)\right]$$

$$= \frac{1}{2j}\left[\frac{e^{-s}}{s-j\pi} + \frac{1}{s-j\pi} - \frac{e^{-s}}{s+j\pi} - \frac{1}{s+j\pi}\right]$$

$$= \frac{1}{2j}\left[e^{-s}\left(\frac{2j\pi}{s^2+\pi^2}\right) + \frac{2j\pi}{s^2+\pi^2}\right]$$

$$= \frac{\pi}{s^2+\pi^2}(e^{-s}+1)$$

Q.7 (b) Solution:

(i) Total secondary circuit resistance

$$= 1.2 + 0.2 = 1.4\ \Omega$$

Total secondary circuit reactance

$$= 0.5 + 0.3 = 0.8\ \Omega,$$

Secondary circuit phase angle

$$\delta = \tan^{-1}\left(\frac{0.8}{1.4}\right) = 29.745°$$

$$\cos\delta = 0.8682 \text{ and } \sin\delta = 0.4961$$

Primary winding turns, $N_p = 1$

Secondary winding turns, $N_s = 200$

$\therefore$ Turns ratio, $n = 200$

$$\text{Magnetizing current, } I_m = \frac{\text{magnetizing mmf}}{\text{primary turns}} = \frac{100}{1} = 100 \text{ A}$$

$$\text{Loss component, } I_e = \frac{\text{mmf equivalent to iron loss}}{\text{primary winding turns}} = \frac{50}{1} = 50 \text{ A}$$

$$\therefore \quad \text{Actual ratio, } R = n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s}$$

$$= 200 + \frac{50 \times 0.8682 + 100 \times 0.4961}{5} = 218.6$$

Primary current, I_p = actual transformation ratio × secondary current

$= 218.6 \times 5 = 1093$ A

(ii) In order to eliminate the ratio error, we must reduce the secondary winding turns or in other words we must reduce the turns ratio.

The nominal ratio is 200 and therefore for zero ratio error the actual transformation ratio should be equal to the nominal ratio,

Nominal ratio, $K_n = 200$

$$\text{Actual ratio, } R = n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s}$$

$\therefore$ For zero ratio error, $K_n = R$

or
$$200 = n + \frac{50 \times 0.8682 + 100 \times 0.4961}{5}$$

$$= n + 18.6$$

Turns ratio, $n = 181.4$

Hence secondary winding turns

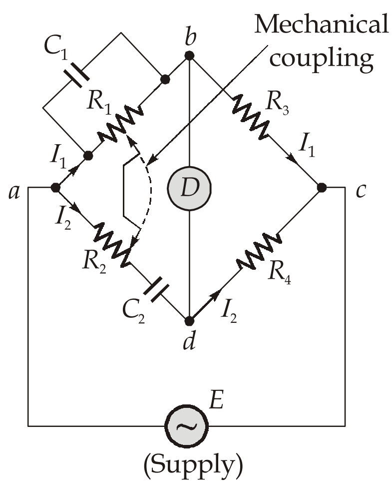
$$N_s = nN_p = 181.4 \times 1 = 181.4$$

Reduction in secondary winding turns

$$= 200 - 181.4 \approx 19$$

Q.7 (c) Solution:

Basic layout of Wein's bridge is shown below,



At balance condition,

$$Z_{ab} \cdot Z_{cd} = Z_{ad} \cdot Z_{bc}$$

$$\left(R_1 \,||\, \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$\Rightarrow \quad \frac{R_1 R_4}{1 + j\omega C_1 R_1} = R_3 \left[R_2 + \frac{1}{j\omega C_2}\right]$$

$$\Rightarrow \quad R_1 R_4 = R_3 \left[R_2 + \frac{C_1}{C_2} R_1\right] + jR_3 \left[\omega C_1 R_1 R_2 - \frac{1}{\omega C_2}\right]$$

Equating real part, we get

$$R_1 R_4 = R_2 R_3 + \frac{C_1}{C_2} R_1 R_3$$

$$\therefore \quad \frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

Equating imaginary part, we get

$$\omega C_1 R_1 R_2 - \frac{1}{\omega C_2} = 0$$

$$\therefore \quad \omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \text{ rad/sec}$$

also, $$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

Generally, in most of the Wein's bridges,

$$R_1 = R_2 = R$$

and $C_1 = C_2 = C$

then, equation becomes,

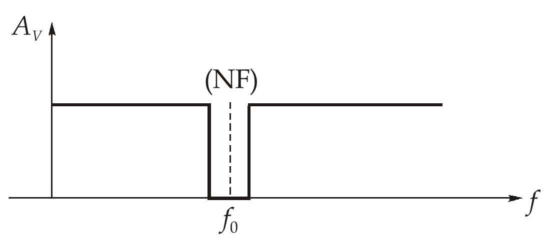
$$R_4 = 2R_3$$

and, $\omega = \dfrac{1}{RC}$ and $f = \dfrac{1}{2\pi RC}$

Wein's Bridge Applications:

- It may be employed in a "Harmonic distortion analyzer" where it is used as "Notch filter".
- It also finds applications in Audio and High frequency oscillators as the frequency determining device (100 Hz-100 KHz).
- The bridge may be used in "Frequency-determining device" balanced by a single control and this control may be calibrated directly in terms of frequency.
- It may also be used for the measurement of "Capacitance".
- Because of its "Frequency sensitivity", the Wein's bridge may be difficult of balance (unless the waveform of the supplied voltage is sinusoidal).
- It is possible to obtain an accuracy of 0.1 – 0.5%.

Wein's Bridge (WB) as "Notch Filter"



- "Wein's Bridge" rejects only one particular frequency "f_0" signal for which it is tuned, while it passes all other frequencies. Hence, Wein's Bridge is acting as a "Notch Filter".

Q.8 (a) Solution:

$$A_3(z) = 1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}$$

$$B_3(z) = \frac{1}{3} + \frac{5}{8}z^{-1} + \frac{13}{24}z^{-2} + z^{-3}$$

$$K_3 = \frac{1}{3}$$

$$A_2(z) = \frac{A_3(z) - K_3B_3(z)}{(1-K_3^2)} = \frac{A_3(z) - \frac{1}{3}B_3(z)}{\left(\frac{8}{9}\right)}$$

$$A_2(z) = \frac{9}{8}\left[\frac{8}{9}+\frac{1}{3}z^{-1}+\frac{4}{9}z^{-2}\right]$$

$$A_2(z) = \left(1+\frac{3}{8}z^{-1}+\frac{1}{2}z^{-2}\right)$$

$$B_2(z) = \frac{1}{2}+\frac{3}{8}z^{-1}+z^{-2}$$

$$K_2 = \frac{1}{2}$$

$$A_1(z) = \frac{A_2(z)-K_2B_2(z)}{1-K_2^2} = \frac{4}{3}\left(A_2(z)-\frac{1}{2}B_2(z)\right)$$

$$= \left(\frac{3}{4}+\frac{3z^{-1}}{16}\right)\times\frac{4}{3} = \left(1+\frac{z^{-1}}{4}\right)$$

$$B_1(z) = \frac{1}{4}+z^{-1}$$

$$K_1 = \frac{1}{4}$$

$$K_1 = \frac{1}{4};\qquad K_2 = \frac{1}{2},\qquad K_3 = \frac{1}{3}$$

Q.8 (b) (i) Solution:

Let, E_a = Voltage of pre-accelerating anode

m = Mass of electron (in kg)

E_d = Potential difference between deflecting plates (Volt)

d = Distance between deflecting plates (in m)

l_d = Length of deflecting plates (in m)

L = Distance between screen and the centre of the deflecting plates (in m)

D = Deflection of electron beam on the screen in y-direction

a_y = Acceleration of electron in y-direction

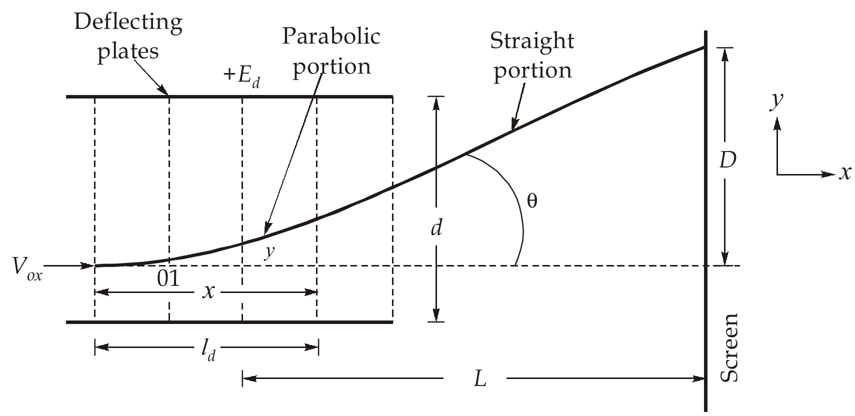
q = e = Charge on electron

V = Velocity of electron

The loss in potential energy of $e^-(P.E.) = q \times E_a$

Also, gain in kinetic energy, $(K.E.) = \frac{1}{2}\text{mV}^2$

Now, Loss in $P.E.$ = Gain in $K.E.$



$$\therefore \quad qE_a = \frac{1}{2}\mathrm{m}V^2$$

$$\Rightarrow \quad V = \sqrt{\frac{2qE_a}{m}}$$

or, $$V \propto \sqrt{E_a}$$

Velocity of electron along x-direction,

$$V_x = \frac{x}{t}$$

or, $$t = \frac{x}{V_x} \quad \text{...(i)}$$

$$\text{Velocity, } V_x = \sqrt{\frac{2qE_a}{m}}$$

Deflection is given by,

$$D = \frac{Ll_dE_d}{2dE_a}$$

$\therefore$ Voltage applied to deflecting plates

$$E_d = \frac{2dE_aD}{Ll_d} = \frac{2\times5\times10^{-3}\times2000\times3\times10^{-2}}{0.3\times2\times10^{-2}} = 100 \text{ V}$$

$\therefore$ Input voltage required for a deflection of 3 cm

$$V_i = \frac{E_d}{\text{gain}} = \frac{100}{100} = 1 \text{ V}$$

Q.8 (b) (ii) Solution:

Moment of inertia of beam,

$$I = \frac{1}{12}bd^3 = \frac{1}{12}\times 0.02\times(0.003)^3$$

$$= 45 \times 10^{-12} \text{ m}^4$$

$$\text{Deflection, } x = \frac{Fl^3}{3EI}$$

$$\therefore \quad \text{Force, } F = \frac{3EIx}{l^3}$$

$$= \frac{3\times 200\times 10^9 \times 45\times 10^{-12}\times 12.7\times 10^{-3}}{(0.25)^3} = 22 \text{ N}$$

Bending moment at 0.15 m from free end

$$M = Fx = 22 \times 0.15 = 3.3 \text{ Nm}$$

Stress at 0.15 m from free end

$$S = \frac{M}{I}\cdot\frac{t}{2} = \frac{3.3}{45\times 10^{-12}}\times\frac{0.003}{2} = 110 \text{ MN/m}^2$$

$$\text{Strain, } \varepsilon = \frac{\Delta L}{L} = \frac{S}{E} = \frac{110\times 10^6}{200\times 10^9} = 0.55 \times 10^{-3}$$

$$\therefore \quad \text{Gauge factor} = \frac{\Delta R / R}{\Delta L / L} = \frac{0.152/120}{0.55\times 10^{-3}} = 2.3$$

Q.8 (c) Solution:

$N = 4$

$$x(0) = \sin 0 = 0$$

$$x(1) = \sin\frac{\pi}{2} = 1$$

$$x(2) = \sin \pi = 0$$

$$x(3) = \sin\frac{3\pi}{2} = -1$$

$$x(n) = \{0, 1, 0, -1\}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$$

$$= \sum_{0}^{3} x(n)e^{-j2\pi nk/4}$$

$$= x(0) + x(1)e^{\frac{-j2\pi k}{4}} + x(2)e^{\frac{-j4\pi k}{4}} + x(3)e^{\frac{-j6\pi k}{4}}$$

$$= e^{\frac{-j\pi k}{2}} - e^{\frac{-j3\pi k}{2}}$$

$$X(0) = 1 - 1 = 0$$

$$X(1) = e^{-j\pi/2} - e^{-j3\pi/2} = -j - j = -j2$$

$$X(2) = e^{-j\pi} - e^{-j3\pi} = -1 + 1 = 0$$

$$X(3) = e^{-j3\pi/2} - e^{-j9\pi/2} = j + j = 2j$$

$$X(k) = \{0, -j2, 0, j2\}$$

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