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ESE 2024 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 6

Section A : Electrical Machines [All Topics]

Section B : Control Systems-1 + Engineering Mathematics-1 [Part Syllabus]

Section C : Electrical Circuits-2 + Digital Electronics-2 [Part Syllabus]

ANSWER KEY

1.	(b)	**16.**	(a)	**31.**	(b)	**46.**	(d)	**61.**	(b)
2.	(b)	**17.**	(b)	**32.**	(c)	**47.**	(c)	**62.**	(b)
3.	(d)	**18.**	(b)	**33.**	(a)	**48.**	(a)	**63.**	(c)
4.	(d)	**19.**	(a)	**34.**	(b)	**49.**	(b)	**64.**	(a)
5.	(d)	**20.**	(c)	**35.**	(b)	**50.**	(b)	**65.**	(b)
6.	(a)	**21.**	(d)	**36.**	(b)	**51.**	(a)	**66.**	(a)
7.	(d)	**22.**	(b)	**37.**	(c)	**52.**	(a)	**67.**	(d)
8.	(a)	**23.**	(b)	**38.**	(a)	**53.**	(d)	**68.**	(a)
9.	(a)	**24.**	(d)	**39.**	(d)	**54.**	(c)	**69.**	(b)
10.	(d)	**25.**	(c)	**40.**	(a)	**55.**	(d)	**70.**	(b)
11.	(d)	**26.**	(d)	**41.**	(a)	**56.**	(a)	**71.**	(d)
12.	(b)	**27.**	(a)	**42.**	(a)	**57.**	(a)	**72.**	(b)
13.	(d)	**28.**	(a)	**43.**	(a)	**58.**	(d)	**73.**	(d)
14.	(b)	**29.**	(c)	**44.**	(b)	**59.**	(c)	**74.**	(d)
15.	(d)	**30.**	(d)	**45.**	(d)	**60.**	(c)	**75.**	(a)

DETAILED EXPLANATIONS
Section A : Electrical Machines

1. (b)

$$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

Since, $\frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$

the flux density B_m remains constant,

Hence $\frac{P_i}{f} = a + bf$

$\therefore$ $\frac{2400}{50} = a + 50b$ ...(i)

and $\frac{800}{25} = a + 25b$ ...(ii)

Solving equation (i) and (ii),

$a = 16$

and $b = 0.64$

Therefore, at 50 Hz

Hysteresis losses, $P_h = af = 16 \times 50 = 800$ W

Eddy current losses, $P_e = bf^2 = 0.64 \times (50)^2 = 1600$ W

2. (b)

Iron losses for both transformers

= Reading of wattmeter, $W_1 = 5.0$ kW

Iron loss for one transformer,

$$P_i = \frac{5}{2} = 2.5 \text{ kW}$$

Full-load copper losses for both transformers

= Reading of primary series circuit wattmeter W_2

= 7.5 kW

Full-load copper loss for one transformer

$$P_{cfl} = \frac{7.5}{2} = 3.75 \text{ kW}$$

Copper loss of one transformer at 75% of full load

$= (0.75)^2 P_{cfl}$

$= (0.75)^2 \times 3.75 = 2.109$ kW

Output of each transformer at 75% of full load and 0.8 power factor

$$= 75\% \text{ of KVA at full load} \times \text{Power factor}$$

$$= \frac{75}{100} \times 250 \times 0.8 = 150 \text{ kW}$$

$$\text{Efficiency at 75\% full load} = \frac{\text{Output at 75\% full load}}{\text{Output at 75\% full load} + P_i + (0.75)^2 P_{cfl}}$$

$$= \frac{150}{150 + 2.5 + 2.109} = 97.02\%$$

3. (d)

To measure voltage and currents, these are known as instrument transformers.

4. (d)

- In an ordinary transformer the total electrical power is transferred from primary to secondary by induction.
- In an autotransformer electrical power is transferred from primary to secondary partly by the process of induction and partly by direct electrical connection.

5. (d)

All statements are correct.

6. (a)

Induced voltage in the primary per turn $\frac{E_{\text{ph}1}}{T_{p1}} = 12$

Since the hv side is delta connected, phase voltage = line voltage

$$E_{ph1} = E_{l1} = 11000$$

$$\therefore \quad \frac{11000}{T_{p1}} = 12$$

$$\frac{E_{\text{ph}2}}{T_{\text{ph}2}} = 4.44\, B_m\, A\, f$$

$$12 = 4.44 \times 1.2 \times A \times 50$$

$$A = \frac{12}{4.44 \times 1.2 \times 50} = 0.0450 \text{ m}^2$$

$$= 0.0450 \times 10^4 \text{ cm}^2 = 450 \text{ cm}^2$$

7. (d)

If the leakage flux in the core is zero, implies that all the flux in the core couples both winding.

8. (a)

We know,

$$\frac{T_{st}}{T_{\max}} = \frac{2s_m}{s_m^2 + 1}$$

$$0.5 = \frac{2s_m}{s_m^2 + 1}$$

$$\Rightarrow \quad s_m^2 - 4s_m + 1 = 0$$

$$s_m = \frac{4 \pm \sqrt{16 - 4 \times 1}}{2}$$

As $0 < s_m < 1$ we take (+) sign only

$$s_m = \frac{4 - \sqrt{12}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$= 2 - 1.732 = 0.268$$

9. (a)

In star-delta starting, phase voltage on starting

$$= \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9 \text{ V}$$

Since 400 V produce 100 A in phase winding, $\frac{400}{\sqrt{3}}$ V will produce $\frac{100}{\sqrt{3}}$ = 57.7 A

∴ Starting phase current = 57.7 A

In star connection,

Line current = Phase current

∴ starting line current = 57.7 A

10. (d)

- A double-cage rotor has low starting current and high starting torque.
- A double-cage induction motor has higher effective leakage reactance due to additional reactance of the inner cage.
- The pull-out torque a double-cage motor is smaller than that of a single-cage motor because the two cages produce the maximum torque at different speeds.

11. (d)

The main methods employed for speed control of induction motors are as follows:

- Pole changing
- Stator voltage control
- Supply frequency control
- Rotor resistance control
- Slip energy recovery.

12. (b)

At maximum efficiency,

$$\text{core loss} = \text{copper losses}$$
$$\text{core loss} = 80\text{ W}$$
$$\text{copper loss at rated current} = (25)^2 \times 0.5 = 312.5\text{ W}$$

When maximum efficiency occurred,

$$I_m^2R = 80$$
$$I_m^2 = \frac{80}{0.5} = 160$$
$$I_m = 12.649\text{ A}$$
$$xI_{\text{rated}} = I_m$$
$$x = \frac{I_m}{I_{\text{rated}}} = \frac{12.649}{25} = 0.5059$$
$$\%x = 50.59\%$$

13. (d)

In the case of a generator, the neutral plane shifts in the direction of rotation meaning that the conductors undergoing commutation have the same polarity of voltage as the pole they just left. To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the upcoming pole. In a motor, however the neutral plane shifts opposite to the direction of rotation, and the conductors undergoing commutation have the same flux as the pole they are approaching. In order to oppose this voltage, the interpoles must have the same polarity as the previous main pole.

14. (b)

There are two parallel paths through the rotor of this machine, each one consisting of $\frac{Z}{2} = 1440$ conductors, or 720 turns. Therefore, the resistance in each current path is

$$\text{Resistance/path} = (720\text{ turns})\ (0.011\ \Omega/\text{turn}) = 7.92\ \Omega$$

Since there are two parallel paths, the effective armature resistance is

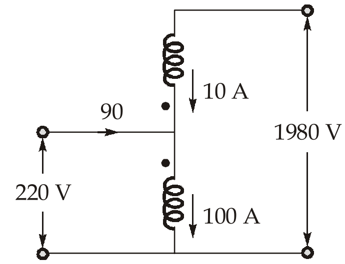
$$R_A = \frac{7.92\,\Omega}{2} = 3.96\ \Omega$$

15. (d)

The purpose of DC motor starters are:

- To protect the motor against damage due to short circuits in the equipment.
- To protect the motor against damage from long-term overloads.
- To protect the motor against damage from excessive starting currents.
- To provide a convenient manner in which to control the operating speed of the motor.

16. (a)



$$\text{Primary current, } I_P = \frac{22000}{220} = 100 \text{ A}$$

$$\text{Secondary current, } I_S = \frac{22000}{2200} = 10 \text{ A}$$

When connected as 220/1980 V auto transformer,

kVA rating of the transformer = 1980 × 10 = 19.8 kVA

17. (b)

In a induction machine, $E_2' \propto \phi$

The torque on full load, $T_{fl} \propto (E_2')^2 \propto \phi^2$

∴ If the flux density is reduced to half of its normal value then the torque will reduce to one fourth of its value.

18. (b)

To eliminate n^{th} harmonic, the chording angle,

$$\frac{n\alpha}{2} = \frac{\pi}{2}$$

$$\frac{5\alpha}{2} = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{5}$$

19. (a)

In a synchronous machine if the main field flux is ahead of armature field flux in the direction of rotation, the machine is acting like a synchronous generator.

20. (c)

Speed of induction motor is controlled by varying voltage and frequency,

Keeping V/f ratio constant

At maximum torque condition

$$R_2 = sX_2$$

or, $$s_{\max, T} = \frac{R_2}{X_2}$$

∴ Reactance $X_2 \propto$ frequency

So, $$s_{\max,T} \propto \frac{1}{f}$$

Hence, option (c) is correct.

21. (d)

The torque developed in induction motor, is proportional to V^2

$$T \propto V^2$$

$$\frac{T_1}{T_2} = \frac{V_1^2}{V_2^2}$$

$$T_2 = \left(\frac{250}{400}\right)^2 \times 150 = 58.59 \text{ N-m}$$

22. (b)

$$\text{Rotation speed} = 800 \text{ rpm}$$

$$N = \frac{800}{60} \text{ rev/sec}$$

Peripheral velocity of commutation

$$V_p = \pi \times D \times N$$

$$= \pi \times 20 \times \frac{800}{60} \text{cm/sec}$$

As we know,

V_p × time of commutation = Brush width

$$\therefore \text{ Time of commutation, } t_c = \frac{1 \times 60}{\pi \times 20 \times 800} = 1.194 \text{ msec}$$

23. (b)

Line current taken from the supply,

$$I_{st} = x^2 I_{sc} \quad \text{...(i)}$$

With auto-transformer starting,

$$I_{st} = \phi\, I_{fl} \quad \text{...(ii)}$$

Given short circuit current,

$$I_{SC} = 4I_{fl}$$

From equation (i) and (ii),

$$2I_{fl} = x^2 \times 4\, I_{fl}$$

$$x^2 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}} = 0.7071 \approx 70.7\%$$

24. (d)

At full load, $I_a = 40 - I_f$

$$= 40 - 1 = 39 \text{ A}$$

Under stalling conditions,

$$N = 0$$

So, $E_b = 0$

$$E_b = V - I_a(R_a + 2)$$

$$0 = 200 - 2.5I_a$$

$$I_a = \frac{200}{2.5} = 80 \text{ A}$$

$$T \propto I_a \quad (\because \phi \text{ is constant})$$

$$\frac{T_{stall}}{T_f} = \frac{80}{39}$$

$\therefore$ $$T_{stall} = 2.05T_f \approx 2T_f$$

25. (c)

$\because$ Constant loss = No load power input – No load copper loss

$$P_C = VI_0 - (I_0 - I_f)^2 R_a$$

$$= 400 \times 5 - (5 - 1)^2 \times 0.2$$

$$= 2000 - 3.2 = 1996.8 \text{ W}$$

For 50 A input current

Armature copper loss $= (I - I_f)^2 R_a$

$$= (50 - 1)^2 \times 0.2$$

$$= 480.2 \text{ W}$$

Hence, Total losses = Constant loss + Copper loss

= 1996.8 + 480.2

= 2477 W

26. (d)

Given, $V_{OC} = 1600$ V

$I_{SC} = 400$ A

Then synchronous impedance,

$$X_S = \frac{1600/\sqrt{3}}{400} = \frac{4}{\sqrt{3}}\Omega$$

Now internal voltage drop will be

$$= 100 \times \frac{4}{\sqrt{3}} = \frac{400}{\sqrt{3}} \text{V} = 230.94 \text{ V}$$

27. (a)

Given, $N = 970$ rpm

Synchronous speed, $$N_S = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\text{Slip, } s = \frac{1000 - 970}{1000} = 0.03$$

$$\text{Rotor copper loss} = \frac{s}{1-s} \times \text{Mechanical power developed}$$

$$P_{cu} = \left(\frac{0.03}{1-0.03}\right) \times P_{mech}$$

where, $P_{mech} = P_{shaft} + P_{mech.\ loss}$

$= 6000 + 900$

$= 6900 \text{ W}$

Rotor copper loss, $P_{cu} = \frac{0.03}{0.97} \times 6900 = 213.4 \text{ W}$

28. (a)

Stepping frequency or pulse rate

= Pulse per second (PPS)

If α is step angle,

$$\text{Motor speed, } n = \frac{\alpha f}{360} \text{rps}$$

Given, $N = 750 \text{ rpm} = \frac{750}{60} \text{rps}$

$f = 250 \text{ step/sec}$

So, $\frac{750}{60} = \frac{\alpha \times 250}{360}$

$\alpha = 18°$

29. (c)

$$T_L \propto N^2$$

$$\frac{T_1}{T_2} = \frac{N_1^2}{N_2^2}$$

As we know, $T \propto \phi I_a$

$T_1 \propto \phi_1 I_{a1}$

$\phi_1 \propto I_{a1}$

$$T_1 \propto I_{a1}^2 \quad \text{...(i)}$$

$T_2 \propto \phi_2 I_{a2}$

$$\phi_2 \propto \frac{I_a}{2}$$

$$T_2 \propto \frac{I_{a2}^2}{2} \quad \text{...(ii)}$$

From (i) and (ii),

$$\frac{T_1}{T_2} = \frac{2I_{a1}^2}{I_{a2}^2} = \frac{N_1^2}{N_2^2} \quad \text{...(iii)}$$

$$E_b \propto \phi N$$

As R_a and R_{se} is not given,

$$E_b \approx V \approx 240 \text{ Volt (for both case)}$$

$$\phi_1 N_1 = \phi_2 N_2$$

$$\frac{N_1}{N_2} = \frac{\phi_2}{\phi_1} = \frac{I_{a2}}{2I_{a1}} \quad \text{...(iv)}$$

Using (iii) and (iv),

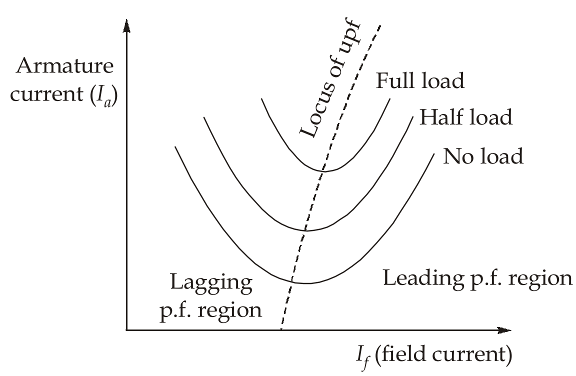
$$\frac{N_1}{N_2} = \frac{N_2\sqrt{2}}{N_1 \times 2}$$

$$N_2^2 = N_1^2\sqrt{2}$$

$$N_2^2 = 800 \times 800\sqrt{2}$$

$$N_2 = 951 \text{ rpm}$$

30. (d)



∴ As we go right of the unity power factor locus of V-curve we obtain over excitation and leading current input.

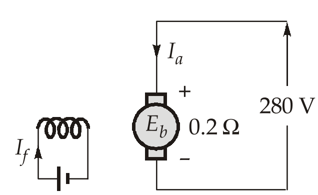
31. (b)

Back emf is given by,

$$E_b = \frac{NP\phi Z}{A \times 60}$$

$$= \frac{1000 \times 0.15 \times 100}{60} = 250 \text{ V}$$

$$I_a = \frac{280 - 250}{0.2} = 150 \text{ A}$$



32. (c)

$$\text{kVA shared} \propto \frac{1}{\text{leakage impedance}}$$

33. (a)

$$\text{Efficiency} = 80\%$$
$$\text{Power output at full load} = 28.8 \text{ kW}$$
$$\text{Input power} = \frac{28.8}{0.8} = 36 \text{ kW}$$
$$\text{Input power} = \sqrt{3} V_{LL} I_{LL} \cos\phi = 36 \text{ kW}$$
$$\cos\phi = \frac{36 \times 10^3}{\sqrt{3} \times 400 \times 50\sqrt{3}} = 0.6$$

34. (b)

$$N_s \text{ (stator field)} = \frac{120 \times 50}{4} = 1500 \text{ rpm;}$$
$$N_s \text{ (rotor field)} = \frac{120 \times 30}{4} = 900 \text{ rpm}$$
$$\therefore \quad N_r = 1500 \pm 900 = 2400 \text{ rpm, } 600 \text{ rpm}$$

35. (b)

Point R is corresponding to maximum voltage regulation.

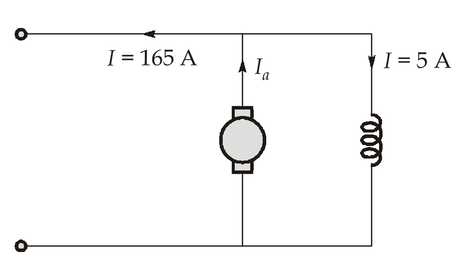
For maximum voltage regulation load power factor is equal to

$$\cos\phi = \frac{R}{Z}$$
$$Z = \sqrt{R^2 + X^2} = \sqrt{0.05^2 + 0.5^2} = 0.502$$

Power factor, $\cos\phi = \dfrac{0.05}{0.502} = 0.0995$ lagging

36. (b)

At full load,



Armature current, $I_a = 165 + 5 = 170$ A

$$\theta_e = \frac{P}{2} \cdot \theta_m = \frac{12}{2} \times 4 = 24°$$

$$\text{Cross magnetizing AT/Pole} = \left(\frac{180° - 2\theta_e}{180°}\right) \times \frac{Z/2}{P} \times \frac{I_a}{A}$$

(For lap connected windings $A = P = 12$)

$$= \left(\frac{180 - 2 \times 24}{180°}\right) \times \frac{662/2}{12} \times \frac{170}{12}$$

$$= 286.56 \text{ AT/pole}$$

37. (c)

$$\frac{V_{Y(\text{ph})}}{V_{\Delta(\text{ph})}} = \frac{5}{1}$$

$$V_{\Delta(\text{ph})} = \frac{1}{5} V_{Y(\text{ph})} = \frac{1}{5} \times \frac{1000}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{V}$$

$$\sqrt{3} V_{\Delta L} I_{\Delta L} = \sqrt{3} V_{YL} I_{YL}$$

$$I_{\Delta L} = \frac{V_{YL} I_{YL}}{V_{\Delta L}} = \frac{1000}{\left(\frac{200}{\sqrt{3}}\right)} \times 100 = 500\sqrt{3} \text{ A}$$

38. (a)

$$T_{\text{start}} = x^2 \left(\frac{I_{sc}}{I_{fL}}\right)^2 s_{fL}$$

$\Rightarrow$ $0.75 = x^2 (6)^2 \times 0.035$

$\Rightarrow$ $x = 0.77$

39. (d)

The criteria of steady state stability of a synchronous machine is that the synchronzing power coefficient should remain positive.

40. (a)

The inertia of two phase servomotor is reduced by drag-cup rotor construction.

Section B : Control Systems-1 + Engineering Mathematics-1

41. (a)

Given, $$G(s)H(s) = \frac{Ks + b}{s[s + (a - k)]}$$

The velocity error constant,

$$K_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \frac{s(Ks + b)}{s(s + (a - k))}$$

$$= \frac{b}{a-k}$$

$\therefore$ the steady state error for a unit ramp input is

$$e_{ss}(t) = \frac{1}{K_v} = \frac{a-k}{b}$$

42. (a)

$$T = \frac{G}{1+GH}$$

also $$\frac{\partial G}{G} = 10\% = 0.1$$

$$S_G^T = \frac{\partial T}{T} \times \frac{G}{\partial G} = \frac{1}{1+GH}$$

or, $$\frac{\partial T}{T} = \frac{1}{21} \times 0.1 \times 100\% = 0.476\%$$

43. (a)

From the pole location shown in figure, the real part of complex poles is

$$-\xi\omega_n = -4$$

i.e. $$\xi\omega_n = 4$$

For 2% tolerance band,

$$\text{The settling time} = \frac{4}{\xi\omega_n} = \frac{4}{4} = 1 \text{ sec}$$

44. (b)

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

The Routh table is formulated as follows:

s^4	1	6	8
s^3	2	8	
s^2	2	8	
s^1	0		

All the elements in the s^1 row are zeros. That means, there are symmetrically located roots of the characteristics equation with respect to the origin of the s-plane.

$$A(s) = 2s^2 + 8 = 0$$

$$\frac{dA(s)}{ds} = 4s + 0$$

Replace the row of zeros with the coefficients of the first derivative of the auxiliary equation and complete the formation of the Routh table:

s^4	1	6	8
s^3	2	8	
s^2	2	8	
s^1	4	0	
s^0	8		

The auxiliary equation:

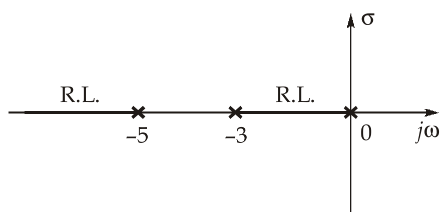
$$2s^2 + 8 = 0$$

$$\Rightarrow \quad s = \pm j2$$

Therefore the system oscillates at ω = 2 rad/sec

45. (d)

The segments of the real axis between 0 and –3, –5 and –∞, lie on the root locus.



46. (d)

It is desirable to remove the effect of disturbance on response. So $\frac{C(s)}{D(s)}$ ratio can be calculated as follows:

$$\frac{C(s)}{D(s)} = \frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

Response due to disturbance,

$$C(s) = D(s)\cdot\frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

$C(s)$ should be zero for $D(s) \neq 0$

$$\therefore \quad G_1H_2 = -1$$

47. (c)

Closed loop transfer function of system is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$T(s) = \frac{K}{s^2+2s+K}$$

Desired characteristic equation from given closed loop poles is,

$$1 + G(s)\,H(s) = \left(s+1-j\sqrt{5}\right)\left(s+1+j\sqrt{5}\right) = s^2 + 2s + 6$$

Comparing desired characteristic equation with given equation

$$K = 6$$

48. (a)

- Addition of pole to the transfer function shifts the root locus to RHS, therefore it reduces the stability margin.
- It increase the rise time and slows down the speed.

$$\text{Bandwidth} \propto \frac{1}{\text{Rise time}}$$

- Addition of pole at origin reduces steady state error.

49. (b)

1. Let open loop transfer function of system is

$$G(s) = \frac{K}{(s+a)}$$

And closed loop transfer function after applying unity negative feedback system is

$$T(s) = \frac{\frac{K}{(s+a)}}{1+\frac{K}{(s+a)}} = \frac{K}{(s+a+K)}$$

Time constant of open loop transfer function = $\frac{1}{a}$

Time constant of closed loop transfer function = $\frac{1}{(a+K)}$

$$\frac{1}{a} > \frac{1}{a+K}$$

2. Closed loop system may tend to instability if proper feedback is not applied.
3. Gain of closed loop system is affected by a factor of $(1 + GH)$

50. (b)

In Simpson's $\frac{1}{3}$rd rule the curve is replaced by a second degree polynomial i.e. parabola.

51. (a)

For the equation: $\frac{dy}{dx} + Py = Q$ the integrating factor is $e^{\int P dx}$.

For this given equation after dividing by $\cos^2 x$

$$P = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\text{Integrating factor} = e^{\int \sec^2 x\, dx} = e^{\tan x}$$

52. (a)

The auxilary equation is

$$2\lambda^2 + 8 = 0$$
$$\lambda = \pm 2\, i$$

Therefore the complimentary function,

$$y(t) = c_1 \cos 2t + c_2 \sin 2t$$
$$y'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t$$

But we have,

$$y(0) = 0,$$
$$y'(0) = 10$$

Therefore, we get

$$c_1 = 0 \text{ and } c_2 = 5$$

Therefore the solution is

$$y(t) = 5 \sin 2t$$

So, $$y(1) = 5 \sin 2$$

53. (d)

Orthogonal matrix, $A \cdot A^T = I$

Hermitian matrix, $A = A^*$

Skew Hermitian matrix, $A = -A^*$

54. (c)

Given matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

Applying $R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 6 & -11 \\ 0 & -7 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Applying $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 6 & -11 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$R_4 \to R_4 + \frac{1}{2}R_3$$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 6 & -11 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Above matrix is in row echelon form

Hence rank of matrix, $A = 3$

55. (d)

All statements are correct.

56. (a)

Let, $$y = \lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x}$$

$$\log y = \lim_{x\to\frac{\pi}{2}} \cos x \log \tan x = \lim_{x\to\frac{\pi}{2}} \frac{\log(\tan x)}{\sec x} = \lim_{x\to\frac{\pi}{2}} \frac{\sec x}{\tan x^2}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sec x \tan x}{2\tan x \sec^2 x} = \lim_{x\to\frac{\pi}{2}} \frac{1}{2\sec x}$$

$$\log y = 0$$

$$y = e^0 = 1$$

57. (a)

$$K_1 = h\, f(x_0, y_0)$$

$$K_1 = 0.2\,[x_0 + y_0]$$

$$K_1 = 0$$

$$K_2 = h\, f\,(x_0 + h, y_0 + K_1)$$

$$K_2 = 0.2[x_0 + h + y_0 + K_1]$$

$$= 0.2 \times 0.2 = 0.04$$

$$K = \frac{K_1 + K_2}{2} = 0.02$$

$$y_1 = y_0 + K = 0 + 0.02$$

$$y_1 = 0.02$$

58. (d)

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

$$f'(x) = 6x^2 - 18x + 12$$

$$f'(x) = 0$$

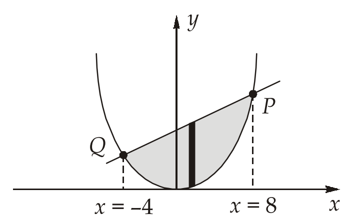
$$6(x^2 - 3x + 2) = 0$$

$$x = 1, 2$$

Further,
$$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5 = 0$$
$$f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 5 = -1$$
$$f(0) = -5$$
$$f(3) = 2(3)^3 - 9(3)^2 + 12(3) - 5 = 4$$

The absolute maximum value is 4.

59. (c)



Let us find the point of intersection P and Q

$$x^2 = 8y \quad \text{...(i)}$$
$$x = 2y - 8 \quad \text{...(ii)}$$

Substituting the value of y from (ii) in (i),

$$x^2 = 4(x + 8)$$
$$\Rightarrow \quad x^2 - 4x - 32 = 0$$
$$\Rightarrow \quad x = 8, -4$$

$$\text{The required area} = \int_{-4}^{8} \int_{\frac{x^2}{8}}^{\left(\frac{x+8}{2}\right)} dy\, dx = \int_{-4}^{8} [y]_{x^2/8}^{(x+8)/2} dx$$

$$= \int_{-4}^{8} \left[\frac{x}{2} + 4 - \frac{x^2}{8}\right] dx = \left[\frac{x^2}{4} + 4x - \frac{x^3}{24}\right]_{-4}^{8} = 36$$

60. (c)

$$f(x) = x^3 - 12x^2 + 36x + 17$$
$$f'(x) = 3x^2 + (-12 \times 2)x + 36$$
$$= 3(x^2 - 8x + 12)$$
$$= 3(x - 2)(x - 6)$$

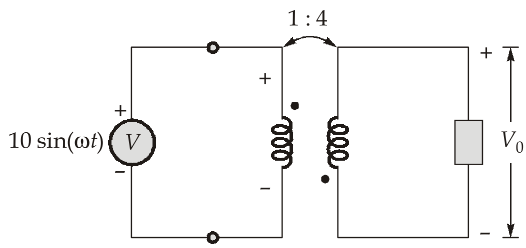
Now $f'(x) > 0$ when $x > 6$ or $x < 2$

for $x \in (-\infty, 2) \cup (6, \infty)$, $f(x)$ is strictly increasing

for $x \in (2, 6)$, $f(x)$ is strictly decreasing

Section C : Electrical Circuits-2 + Digital Electronics-2

61. (b)



$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$V_2 = V_i \sin\omega t\left(\frac{N}{1}\right) = NV_i \sin\omega t$$

$$V_0 = -V_2 = -4 \times 10 \sin \omega t$$
$$= -40 \sin \omega t$$

62. (b)

For this T-circuit, the z-parameters are given as

$$Z_{11} = Z_{22} = (30 + j20)\Omega$$
$$Z_{12} = Z_{21} = 30\ \Omega$$
$$\Delta Z = (Z_{11}Z_{22} - Z_{12}Z_{21})$$
$$= (30 + j20)^2 - 30^2$$
$$= (60 + j20)\ j20^2 = -400 + j1200$$

$\therefore$ $$A = \frac{Z_{11}}{\Delta Z} = \frac{30+j20}{(60+j20)j20} = 1 + j\frac{2}{3}$$

$\therefore$ $$B = \frac{\Delta Z}{Z_{21}} = \frac{(60+j20)j20}{30} = \left(-\frac{40}{3} + j40\right)\Omega$$

$\therefore$ $$C = \frac{1}{Z_{12}} = \frac{1}{30}\mho$$

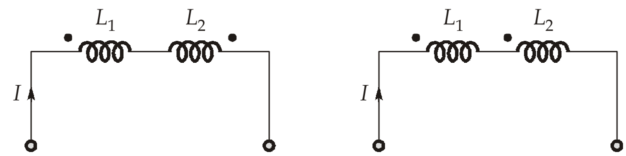
$\therefore$ $$D = \frac{Z_{22}}{Z_{12}} = \frac{30+j20}{30} = \left(1 + j\frac{2}{3}\right)$$

Alternate Solution:

$$[T] = \begin{bmatrix} 1 & j20 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{30} & 1 \end{bmatrix}\begin{bmatrix} 1 & j20 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & j20 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & j20 \\ \frac{1}{30} & j\frac{2}{3}+1 \end{bmatrix} = \begin{bmatrix} 1+j\frac{2}{3} & -\frac{40}{3}+j40 \\ \frac{1}{30} & 1+j\frac{2}{3} \end{bmatrix}$$

63. (c)



Opposing circuit Adding circuit

Let the self inductances of two coils be L_1 and L_2 respectively

Then, $20 \text{ H} = L_1 + L_2 - 2\text{ M}$ (for opposing)

$$40 \text{ H} = L_1 + L_2 + 2\text{ M} \text{ (for adding)}$$

$$20 = 4\text{ M}$$

$$\text{M} = \frac{20}{4} = 5 \text{ H}$$

64. (a)

$$L_1 + L_2 + 2M = 24$$

$$L_1 + L_2 - 2M = 8$$

Since, $L_1 = L_2$

$$2L_1 + 2M = 24 \quad \text{...(i)}$$

$$2L_1 - 2M = 8 \quad \text{...(ii)}$$

On solving equation (i) and (ii),

$$L_1 = 8 \text{ mH} = L_2$$

and $M = 4$ mH

To get maximum value in parallel connections,

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{L_1^2 - M^2}{2(L_1 - M)}$$

$$= \frac{64-16}{2(8-4)} = \frac{48}{8} = 6 \text{ mH}$$

65. (b)

Write KVL equation in first loop,

$$V_1 = 2I_1 + V_2 \quad \text{...(i)}$$

Write KCL equation at node

$$I_1 = V_2 - I_2 \quad \text{...(ii)}$$

Substitute equation (ii) in equation (i),

$$V_1 = 2(V_2 - I_2) + V_2$$

$$V_1 = 3V_2 - 2I_2 \quad \text{...(iii)}$$

$$I_1 = V_2 - I_2 \quad \text{...(iv)}$$

From equation (ii) and (iii),

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\frac{C}{AD} = \frac{1}{3 \times 1} = \frac{1}{3}$$

66. (a)

The transfer function is,

$$H(s) = \frac{V_0}{V_i} = \frac{R \parallel \frac{1}{sC}}{sL + R \parallel \frac{1}{sC}}$$

$$R \parallel \frac{1}{sC} = \frac{\frac{R}{sC}}{R + \frac{1}{sC}} = \frac{R}{1 + sRC}$$

$$H(s) = \frac{R}{s^2 RLC + sL + R}$$

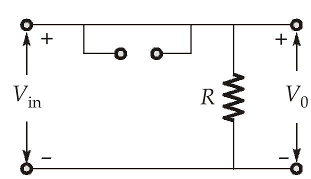
Put $s = j\omega$

$$H(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

Since $H(0) = 1$ and $H(\infty) = 0$, we conclude that the circuit is lowpass filter.

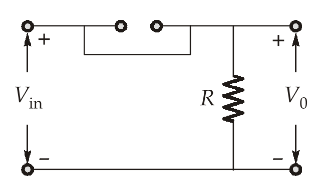
67. (d)

When $\omega = 0$



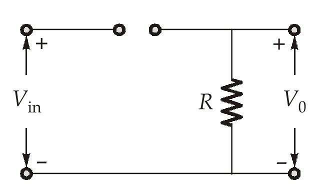
$V_0 = V_{in}$

When $\omega = \infty$



$V_0 = V_{in}$

at $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$



$V_0 = 0$

$\therefore$ Given circuit is band stop filter.

68. (a)

$$\text{Resolution} = \frac{V_r}{2^n - 1}$$

where, $V_r = 10$ V,

$$= \frac{10}{2^6 - 1}$$

$$\text{Resolution} = \frac{10}{63} = 0.1587 \text{ V}$$

69. (b)

$$\text{Decimal equivalent of binary data} \times \text{Resolution} > V_A + V_T$$

$$\text{Decimal equivalent of binary data} \times 50 \text{ mV} > 5000 \text{ mV} + 1 \text{ mV}$$

$$\text{Decimal equivalent of binary data} > \frac{5001}{50} = 100.02$$

Hence, average decimal equivalent will be 101.

70. (b)

$\because$ $$t = \frac{1}{f} = \frac{1}{100 \times 10^3} = 10 \text{ µsec}$$

For digital ramp ADC,

$$T_{max} = (2^N - 1) \times t$$

$$= (2^8 - 1) \times 10 \times 10^{-6}$$

$$= 2.55 \text{ msec}$$

71. (d)

For full scale output,

$$A_3A_2A_1A_0 = 1010$$

$$V_0 = \frac{-V_R}{(2^{N-1})}\left[\frac{R_f}{R}\right] \times [\text{Decimal equivalent of input}]$$

$$= \frac{-5}{(2^{4-1})} \cdot \left[\frac{2}{2}\right] \times 10 = \frac{-50}{8} = -6.25 \text{ V}$$

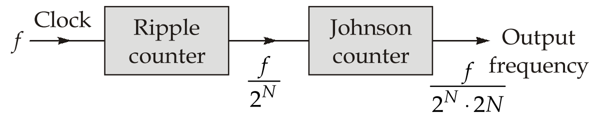
72. (b)

Output-Y will be

$$Y = \bar{A}B + A\bar{B}$$

Hence, it is a XOR gate.

73. (d)



$$100 = \frac{32 \times 10^3}{2N \times 2^N}$$

$$2N \times 2^N = 32 \times 10$$

$$= 2 \times 5 \times 2^5$$

$\therefore$ $N = 5$

Number of flip flop required = N for ripple counter = 5

74. (d)

- Noise margin is low.
- Propagation delay is high and speed of operation is low.

75. (a)

Step size, $$\Delta = \frac{V_{max} - V_{min}}{2^n} = \frac{10-0}{2^{10}} = \frac{10}{1024}$$

The maximum possible error = $\pm\frac{\Delta}{2}$

Therefore, $$\text{error} = \frac{10}{2 \times 1024} = 4.88 \text{ mV}$$

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