



MADE EASY

India's Best Institute for IES, GATE & PSUs

Detailed Solutions

**ESE-2019
Mains Test Series**

**Mechanical Engineering
Test No : 9**

Section A: Machine Design + Mechatronics and Robotics

Q.1 (a) Solution:

$$S_{ut} = 735 \text{ MPa}, S_e = 455 \text{ MPa}$$

Corrected endurance limit,

$$k_t = 1.18, q = 0.8$$

$$k_f = 1 + q(k_t - 1)$$

$$k_f = 1.144$$

$$k_d = \frac{1}{1.144} = 0.874$$

$$S_e' = 0.9 \times 1 \times 0.868 \times 0.874 \times 0.8 \times 455 = 248.527 \text{ MPa}$$

Maximum and minimum axial force,

$$F_{\max} = F_1^{\max} + F_2 = 3600 + 2250 = 5850 \text{ N}$$

$$F_{\min} = F_1^{\min} + F_2 = -900 + 2250 = 1350 \text{ N}$$

$$\text{Mean axial force, } F_m = \frac{F_{\max} + F_{\min}}{2} = 3600 \text{ N}$$

$$\text{Alternating axial force, } F_a = \frac{F_{\max} - F_{\min}}{2} = 2250 \text{ N}$$

Where, A = Area of the larger diameter(d) which experiences the fluctuating load.

$$\sigma_m = \frac{F_m}{A} = \frac{3600}{\frac{\pi}{4} \times d_1^2} = 198.9 \text{ MPa} \quad (\text{where, } d_1 = 0.48 \text{ cm})$$

$$\sigma_a = \frac{F_a}{A} = \frac{2250}{\frac{\pi}{4} \times d_1^2} = 124.3 \text{ MPa}$$

According to Goodman theory,

$$\frac{1}{N} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

$$\frac{1}{N} = \frac{124.3}{248.527} + \frac{198.9}{735}$$

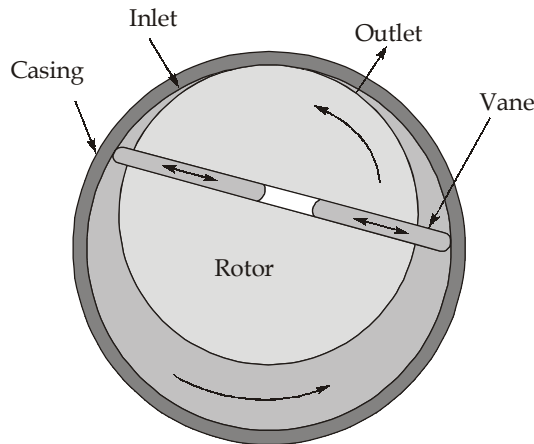
$$N = 1.2974$$

Q.1 (b) Solution:

Gear pumps have a disadvantages of small leakage due to gap between gear teeth and the pump housing. This limitation is overcome in vane pumps. The leakage is reduced by using spring or hydraulically loaded vanes placed in the slots of driven rotor. Capacity and pressure rating of vane pump are generally lower than the gear pumps but reduced leakage gives an improved volumetric efficiency of around 95%.

Vane pumps generate a pumping action by tracking of vanes along the casing wall. The vane pump generally consist of a rotor vanes, ring and ports with inlet and outlet port. Rotor is connected to the prime mover through a shaft. Vanes are located on slotted rotor. Rotor is eccentrically placed inside a cam ring. When prime rotor rotates the rotor the vanes are thrown outward due to centrifugal force. Vanes provide a tight hydraulic seal to the fluid which is more at the higher rotation speed due to higher centrifugal force. This produces a suction at the inlet and fluid is pushed into the pump through the inlet. The fluid is carried around to outlet by vanes whose retraction causes fluid to be expelled. Capacity of pump depends upon eccentricity, expansion of vanes, width of vanes and speed of rotor.

Application: Used in LPG cylinder filling, aerosol and propellants.



Schematic of working principle of vane pump

Q.1 (c) Solution:

$$S_y = 450 \text{ N/mm}^2, S_e = 270 \text{ N/mm}^2$$

Mean and amplitude stress,

$$\sigma_{xm} = \frac{1}{2}((\sigma_x)_{\max} + (\sigma_x)_{\min}) = \frac{1}{2}(100 + 50) = 75 \text{ N/mm}^2$$

$$\sigma_{xa} = \frac{1}{2}((\sigma_x)_{\max} - (\sigma_x)_{\min}) = \frac{1}{2}(100 - 50) = 25 \text{ N/mm}^2$$

$$\sigma_{ym} = \frac{1}{2}((\sigma_y)_{\max} + (\sigma_y)_{\min}) = \frac{1}{2}(80 + 10) = 45 \text{ N/mm}^2$$

$$\sigma_{ya} = \frac{1}{2}((\sigma_y)_{\max} - (\sigma_y)_{\min}) = \frac{1}{2}(80 - 10) = 35 \text{ N/mm}^2$$

$$(\tau_{xym}) = \frac{1}{2}((\tau_{xy} \max) + (\tau_{xy} \min)) = \frac{1}{2}(50 + 10) = 30 \text{ N/mm}^2$$

$$\tau_{xya} = \frac{1}{2}((\tau_{xy} \max) - (\tau_{xy} \min)) = \frac{1}{2}(50 - 10) = 20 \text{ N/mm}^2$$

Distortion energy theory:

$$\sigma^2 = \frac{1}{2} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$$

Substituting,

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

$$\sigma^2 = \frac{1}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y)^2 + (\sigma_x)^2 + 6\tau_{xy}^2 \right]$$

Equivalent mean stress,

$$\sigma_m^2 = \frac{1}{2} \left[(\sigma_{xm} - \sigma_{ym})^2 + (\sigma_{ym})^2 + \sigma_{xm}^2 + 6 \times \tau_{xym}^2 \right]$$

$$\sigma_m^2 = \frac{1}{2} \left[(75 - 45)^2 + (45)^2 + (75)^2 + 6 \times 30^2 \right]$$

$$\sigma_m = 83.5164 \text{ N/mm}^2$$

Equivalent amplitude stress,

$$\sigma_a^2 = \frac{1}{2} \left[(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya})^2 + (\sigma_{xa})^2 + 6 \times \tau_{xya}^2 \right]$$

$$\sigma_a^2 = \frac{1}{2} \left[(25 - 35)^2 + (35)^2 + (25)^2 + 6 \times 20^2 \right]$$

$$\sigma_a = 46.63 \text{ N/m}^2$$

Soderberg eq:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{N}$$

(N = factor of safety)

$$\frac{46.63}{270} + \frac{83.5164}{450} = \frac{1}{N}$$

$$N = 2.8$$

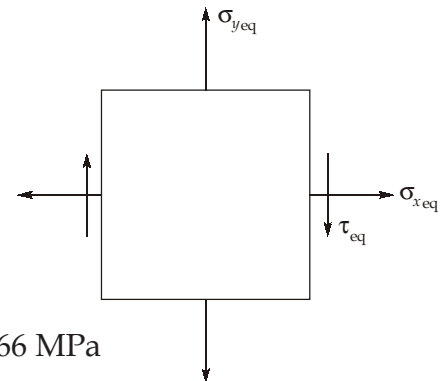
Alternative :

$$\sigma_{xeq} = \sigma_{xm} + \frac{\sigma_{xa} \times S_y}{S_e}$$

$$= 75 + \frac{25 \times 450}{270} = 116.66 \text{ MPa}$$

$$\sigma_{yeq} = \sigma_{ym} + \frac{\sigma_{ya} \times S_y}{S_e}$$

$$= 45 + \frac{35 \times 450}{270} = 103.33 \text{ MPa}$$



$$\tau_{eq} = \tau_{xym} + \frac{\tau_{xya} \times 0.5 \times S_y}{0.5 \times S_e} = 30 + \frac{20 \times 450}{270} = 63.33 \text{ MPa}$$

Principal stresses:

$$\begin{aligned} \sigma_{1,2} &= \frac{1}{2} \left[(\sigma_{xeq} + \sigma_{yeq}) \pm \sqrt{(\sigma_{xeq} - \sigma_{yeq})^2 + 4\tau_{eq}^2} \right] \\ &= \frac{1}{2} [219.99 \pm 127.36] \\ \sigma_1 &= 173.67 \text{ MPa} \\ \sigma_2 &= 46.315 \text{ MPa} \end{aligned}$$

Distortion energy theory

$$\begin{aligned} (\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2 &= 2 \left(\frac{S_y}{N} \right)^2 \\ N &= 2.888 \simeq 2.9 \end{aligned}$$

Q.1 (d) Solution:

$$S_e' = 0.5 S_{ut} = 0.5 \times 620 = 310 \text{ N/mm}^2$$

For 50% reliability, reliability factor $k_c = 1$

$$k_d = \frac{1}{k_f} = \frac{1}{3}$$

So, corrected endurance strength

$$S_e = k_c k_d S_e' = 103.33 \text{ N/mm}^2$$

$$k'_c = 2k'_b$$

k'_c = Combined stiffness of component

k'_b = Stiffness of bolt

So,

$$\Delta P = P \left(\frac{k'_b}{k'_c + k'_b} \right) = P \left(\frac{k'_b}{2k'_b + k'_b} \right) = \frac{P}{3}$$

$$C = \frac{(k'_b)}{(k'_c + k'_b)} = \left(\frac{1}{3} \right)$$

$$P_b = \text{load on bolt} = P_i + CP$$

$$P_i = \text{initial preload} = 5 \text{ kN}$$

$$P = \text{external load} = 0 \text{ to } 10 \text{ kN}$$

$$P_b = 5 + \frac{1}{3} \times P$$

$$(P_b)_{\min} = 5 \text{ kN} \quad (\text{at } P = 0)$$

$$(P_b)_{\max} = 5 + \frac{1}{3} \times 10 = 8.33 \text{ kN} \quad (\text{at } P = 10 \text{ kN})$$

$$(P_b)_m = \frac{1}{2} [(P_b)_{\max} + (P_b)_{\min}] = 6.67 \text{ kN}$$

$$P_a = \frac{1}{2} [(P_b)_{\max} - (P_b)_{\min}] = 1.66 \text{ kN}$$

$$S_a = \frac{S_{ut} - (P_i/A)}{1 + (S_{ut}/S_e)} \quad (\text{Area, } A = 36.6 \text{ mm}^2)$$

$$= 69.0538 \text{ N/mm}^2$$

$$\frac{S_a}{f(s)} = \sigma_a = \frac{(P_b)_a}{A}$$

$$\frac{69.0535}{f(s)} = \frac{1.67 \times 10^3}{36.6}$$

$$f(s) = 1.522$$

Q.1 (e) Solution:

In this case, motions alternate relative to the reference frame and current frame. Pre-multiplication will be done for transformation relative to fixed frame i.e., for those which are done with respect to fixed frame (reference frame) (xyz). Post multiplication will be done for transformation relative to rotating frame.

$${}^0T_1 = \text{Rot}(z, 90) \text{Rot}(x, 90) \text{Trans}(0, 0, 3) \text{Trans}(0, 5, 0)$$

$${}^1P = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

So, ${}^0P = {}^0T_1 \times {}^1P$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 4 \\ 1 \end{bmatrix}$$

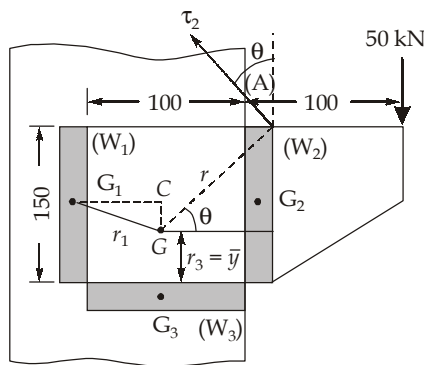
$$= \begin{bmatrix} 7 \\ 1 \\ 10 \\ 1 \end{bmatrix}$$

Q.2 (a) Solution:

Given: $P = 50 \text{ kN}$; $S_{ys} = 50 \text{ N/mm}^2$

There are two vertical welds (W_1 and W_2) and one horizontal weld (W_3).

Centre of Gravity (G) of the weld:



$$\bar{x} = 50 \text{ mm}$$

Take moment about W_3 weld and treating weld as a line,

$$(150 + 150 + 100)\bar{y} = 150 \times 75 + 150 \times 75 + 100 \times 0$$

$$\bar{y} = 56.25 \text{ mm (from } W_3 \text{ weld)}$$

$$A_1 = 150t \text{ mm}^2$$

$$A_2 = 150t \text{ mm}^2$$

$$A_3 = 100t \text{ mm}^2$$

$$\text{Total area, } A = A_1 + A_2 + A_3 = 400t \text{ mm}^2$$

Primary shear stress in the weld is given by,

$$\tau_1 = \frac{P}{A}$$

$$\Rightarrow \tau_1 = \frac{50 \times 10^3}{400t} = \frac{125}{t} \text{ N/mm}^2$$

$$G_1 G = \sqrt{(G_1 C)^2 + (GC)^2} = \sqrt{(50)^2 + (75 - 56.25)^2}$$

$$G_1G = 53.4 \text{ mm}$$

$$G_1G = r_1 = 53.4 \text{ mm}$$

We know that,

$$J_1 = J_2 = A_1 \left[\frac{l^2}{12} + r_1^2 \right]$$

where J_1 and J_2 are polar moment of inertia about G .

$$J_1 = J_2 = 150t \left[\frac{(150)^2}{12} + (53.4)^2 \right] = 708984t \text{ mm}^4$$

$$J_3 = A_3 \left[\frac{l^2}{12} + r_3^2 \right] = 100 \times t \left[\frac{(100)^2}{12} + (56.25)^2 \right]$$

$$= 399740t \text{ mm}^4$$

$$J = 2J_1 + J_3 = 2(708984t) + (399740t)$$

$$= 1817708 \text{ mm}^4$$

The secondary shear stress at point A is given by

$$\tau_2 = \frac{M \times r}{J}$$

(At A , radius (r) is maximum and secondary shear stress is maximum compared to all other extreme points).

$$\tau_2 = \frac{M \times r}{J}$$

$$r = \sqrt{(150 - \bar{y})^2 + (50)^2} = \sqrt{(150 - 56.25)^2 + (50)^2}$$

$$r = 106.25 \text{ mm}$$

$$M = P \times e$$

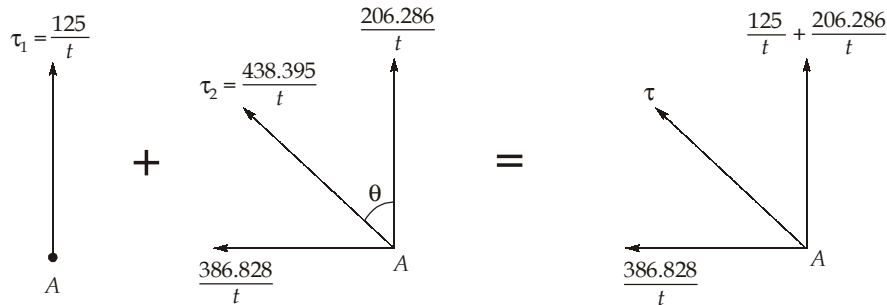
$$= 50 \times 10^3 \times 150 = 7500000 \text{ Nmm}$$

$$\tau_2 = \frac{7500000 \times 106.25}{1817708t} = \frac{438.395}{t} \text{ N/mm}^2$$

Resultant shear stress: The secondary shear stress is inclined at an angle of θ as shown in figure.

$$\tan\theta = \frac{150 - 56.25}{50}$$

$$\theta = 61.93^\circ$$



τ is the resultant shear stress,

$$\tau = \frac{509.299}{t} \text{ N/mm}^2$$

$$\Rightarrow \frac{S_{ys}}{FOS} = \tau$$

$$\frac{50}{2} = \frac{509.299}{t}$$

$$\Rightarrow t = 20.372 \text{ mm} \quad (\text{Throat of weld})$$

$$h = \frac{t}{0.707}$$

$$h = 28.81 \text{ mm} \quad (\text{Leg of weld})$$

Q.2 (b) Solution:

(a) Type '0' system with unit step input:

We know that, for unit step input $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s \rightarrow 0} \left(\frac{sR(s)}{1 + G(s)H(s)} \right)$$

$$= \lim_{s \rightarrow 0} \left(\frac{s \frac{1}{s}}{1 + G(s)H(s)} \right) = \lim_{s \rightarrow 0} \left(\frac{1}{1 + G(s)H(s)} \right)$$

Let
$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K(1+sT_a)(1+sT_b)\dots}{(1+sT_1)(1+sT_2)\dots}$$

$$K_p = K$$

So, steady state error is

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K}$$

(b) Type '0' system with unit ramp input:

We know that, for unit ramp input, $R(s) = \frac{1}{s^2}$

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \left(\frac{sR(s)}{1+G(s)H(s)} \right) \\ &= \lim_{s \rightarrow 0} \left(\frac{s \frac{1}{s^2}}{1+G(s)H(s)} \right) = \lim_{s \rightarrow 0} \left(\frac{1}{s(1+G(s)H(s))} \right) \\ &= \lim_{s \rightarrow 0} \left(\frac{1}{s+sG(s)H(s)} \right) = \lim_{s \rightarrow 0} \left(\frac{1}{sG(s)H(s)} \right) \end{aligned}$$

Let

$$\begin{aligned} K_v &= \lim_{s \rightarrow 0} sG(s)H(s) \\ &= \lim_{s \rightarrow 0} \left(\frac{sK(1+sT_a)(1+sT_b)\dots}{(1+sT_1)(1+sT_2)\dots} \right) \end{aligned}$$

$$K_v = 0$$

So,

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

(c) Type '0' system with parabolic input:

We know that, for parabolic input, $R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s \rightarrow 0} \left(\frac{sR(s)}{1+G(s)H(s)} \right)$$

$$= \lim_{s \rightarrow 0} \left(\frac{s \frac{1}{s^3}}{1 + G(s)H(s)} \right) = \lim_{s \rightarrow 0} \left(\frac{1}{s^2 (1 + G(s)H(s))} \right)$$

$$= \lim_{s \rightarrow 0} \left(\frac{1}{s^2 (G(s) \times H(s))} \right)$$

Let

$$K_v = s^2(G(s)H(s))$$

$$K_a = \lim_{s \rightarrow 0} \left(\frac{s^2 K (1 + sT_a)(1 + sT_b) \dots}{(1 + sT_1)(1 + sT_2) \dots} \right)$$

$$K_a = 0$$

So,

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0}$$

$$e_{ss} = \infty$$

Q.2 (c) Solution:

$$l = d = 75 \text{ mm}$$

$$P = \frac{W}{ld} = \frac{15 \times 1000}{75 \times 75} = 2.667 \text{ N/mm}^2$$

$$c = 0.001 r = 0.001 \times \frac{75}{2} = 0.0375 \text{ mm}$$

$$h_o = 5(2 + 1) = 15 \text{ microns}$$

$$h_o = 0.015 \text{ mm}$$

So,

$$\frac{l}{d} = 1$$

$$\frac{h_o}{c} = \frac{0.015}{0.0375} = 0.4$$

from table,

$$S = 0.121, \frac{Q}{rcn_s l} = 4.33 = FV$$

$$\left(\frac{r}{c} \right) f = 3.22 = CFV$$

$$n_s = \frac{1440}{60} = 24 \text{ rev/s}$$

$$\frac{r}{c} = \frac{1}{0.001} = 1000$$

$$f = \frac{3.22}{\left(\frac{r}{c}\right)} = 3.22 \times 10^{-3}$$

$$S = \left(\frac{r}{c}\right)^2 \times \frac{\mu \times n_s}{P}$$

$$0.121 = (1000)^2 \times \mu \times \frac{24}{2.667}$$

$$\mu = 1.344 \times 10^{-8}$$

$$\mu = 13.44 \text{ N sec/mm}^2$$

$$\mu = 13.44 \text{ cP}$$

$$[1 \text{ poise} = 0.1 \text{ Pa.s}; 1 \text{ centipoise} = 0.001 \text{ N.s/m}^2]$$

$$Q = 4.33 \times n_s c r l$$

$$= 4.33 \times 24 \times 0.0375 \times \frac{75}{2} \times 75$$

$$Q = 10960.3125 \text{ mm}^3/\text{s} \quad \dots(1)$$

We know that

$$\Delta t = \frac{8.3P(CFV)}{FV} = \frac{8.3 \times 2.667 \times 3.22}{4.33}$$

$$\Delta t = 16.461^\circ\text{C}$$

OR

Heat dissipated by friction = Heat gained by lubricating oil

$$Hg = mc\Delta T$$

$$f \times W \times V = \rho \times Q \times C\Delta T$$

$$3.22 \times 10^{-3} \times 15 \times 10^3 \times \frac{\pi \times 0.075 \times 1440}{60} = 860 \times 10960.3125 \times 10^{-9} \times 1.76 \times 10^3 \times \Delta T$$

$$273.13 = 16.5895 \times \Delta T$$

$$\Delta T = 16.46^\circ\text{C}$$

So,

$$T_{\text{avg}} = T_i + \frac{\Delta t}{2}$$

⇒

$$T_{\text{avg}} = 41 + \frac{16.461}{2} = 49.2307^\circ\text{C} \quad \dots(2)$$

From eq. (1) and (2) it is observed that lubricating oil should have minimum viscosity of 13.44 cP at 49.2307°C. Suitable lubricating oil can be SAE 10 which will satisfy the above condition.

Q.3 (a) Solution:

Given, $S_{ut} = 660 \text{ N.mm}^2$, $S_e = 270 \text{ N/mm}^2$

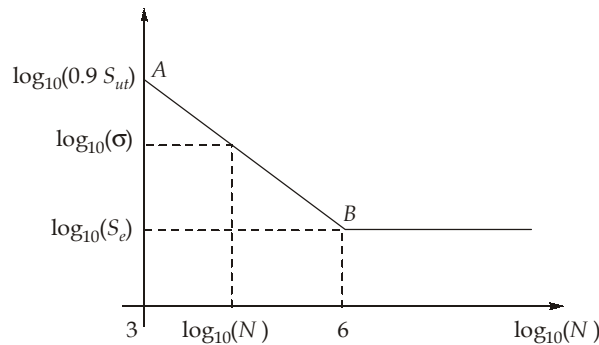
$$\log_{10}(0.9 S_{ut}) = \log_{10}(0.9 \times 660) = \log_{10}(594) = 2.7738$$

$$\log_{10}(S_e) = \log_{10}(270) = 2.43136$$

$$\log_{10}(\sigma_1) = \log_{10}(400) = 2.6021$$

$$\log_{10}(\sigma_2) = \log_{10}(300) = 2.4774$$

$$\log_{10}(\sigma_3) = \log_{10}(500) = 2.6989$$



Eq. of line A-B

$$(y - 2.7738) = \frac{(2.43136 - 2.7738)}{(6 - 3)}(x - 3)$$

$$(y - 2.7738) = -0.11414(x - 3)$$

For

$$\sigma_1 = 400 \text{ MPa}$$

$$(2.6021 - 2.7738) = -0.11414(x - 3)$$

$$x = 4.5043$$

$$N_1 = 31936.91 \text{ cycles}$$

Similarly for $\sigma_2 = 300 \text{ N/mm}^2$:

$$N_2 = 397423.06$$

and

$$\text{for } \sigma_3 = 500 \text{ N/mm}^2$$

$$N_3 = 4524.789 \text{ cycles}$$

Fatigue life of component is given by Miner's equation as follows:

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}$$

$$\frac{0.7}{31936.91} + \frac{0.15}{397423.06} + \frac{0.15}{4524.789} = \frac{1}{N}$$

$$\frac{0.7}{31936.91} + \frac{0.15}{397423.06} + \frac{0.15}{4524.789} = \frac{1}{N}$$

$$N = 18035 \text{ cycles}$$

Q.3 (b) (i) Solution:

$$\text{Gauge factor, } G_f = \frac{\Delta R/R}{\Delta L/L}$$

$$\text{Resistance of wire, } R = \frac{\rho L}{A}$$

$$A = \frac{\pi}{4} \times D^2 = kD^2 \quad (k \text{ is a constant})$$

$$R = \frac{\rho L}{kD^2}$$

$$\text{by differentiating, } dR = \frac{kD^2 (\rho dL + Ld\rho) - \rho L (2kD \times dD)}{(kD^2)^2}$$

$$dR = \frac{1}{kD^2} \left[(\rho dL + Ld\rho) - 2\rho L \frac{dD}{D} \right]$$

$$\frac{dR}{R} = \frac{1}{kD^2} \frac{\left[\rho dL + Ld\rho - 2\rho L \frac{dD}{D} \right]}{\frac{\rho L}{kD^2}}$$

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{2dD}{D}$$

$$\text{Now, Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-dD/D}{dL/L}$$

$$\frac{dD}{D} = -\mu \times \frac{dL}{L}$$

for small variations, the above relationship can be written as:

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\mu \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$$

$$G_f = \frac{\Delta R/R}{\Delta L/L}$$

$$\frac{\Delta R}{R} = G_f \frac{\Delta L}{L} = G_f \times e \quad \text{where, } e = \text{strain} = \frac{\Delta L}{L}$$

So,

$$G_f = 1 + 2\mu + \frac{\Delta \rho / \rho}{e}$$

G_f = Resistance change due to change of length + Resistance change due to change in area + Resistance due to piezo resistance effect.

Q.3 (b) (ii) Solution:

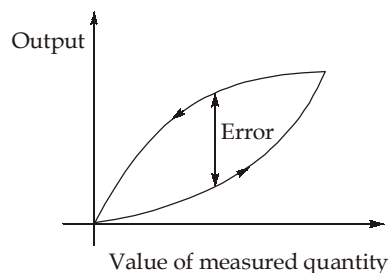
1. Range and span: Range: The range of a transducer defines the limits between which the input can vary.

Span: The span is the maximum value minus minimum value.

2. Sensitivity: The sensitivity is the relationship indicating how much output is there per unit input i.e. output/input.

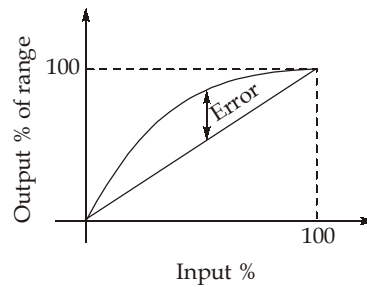
Stability: The stability of a transducer is its ability to give the same output when used to measure a constant input over a period of time, the term drift is often used to describe the change in output that occurs over time.

3. Hysteresis error: Transducer can give different outputs from the same value of quantity being measured according to whether that value has been reached by a continuously increasing change or a continuously decreasing change. This effect is called hysteresis.



4. Non-linearity error: For many transducers, a linear relationship between the input and output is assumed over the working range. i.e. a graph of output plotted against input is assumed to give a straight line. Few transducers, however, have a truly linear

relationship and thus error occurs as a result of the assumption of linearity. The error is defined as the maximum difference from the straight line.



- 5. Repeatability/Reproducibility:** The term repeatability and reproducibility of a transducer are used to describe its ability to give the same output for repeated applications of the same input value. The error resulting from the same output not being given with repeated applications is usually expressed as a percentage of the full range output.

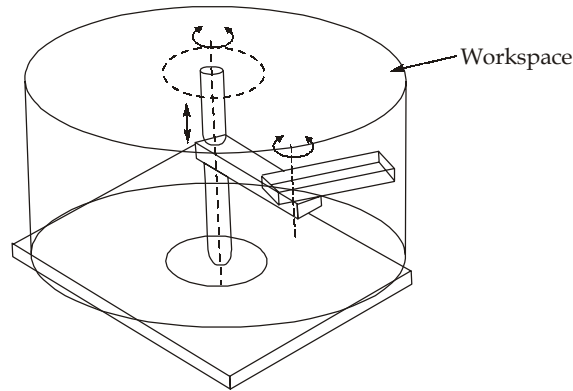
$$\text{Repeatability} = \frac{\text{Max - Min values given}}{\text{Full range}} \times 100$$

Q.3 (c) Solution:

(i)

New arm configurations can be obtained by assembling the links and joints differently, resulting in properties different from those of basic arm configurations outlined above. For instance, if the characteristics of articulated and cylindrical configurations are combined, the result will be another type of manipulator with revolute motions, confined to the horizontal plane. Such a configuration is called SCARA, which stands for selective compliance assembly robot arm.

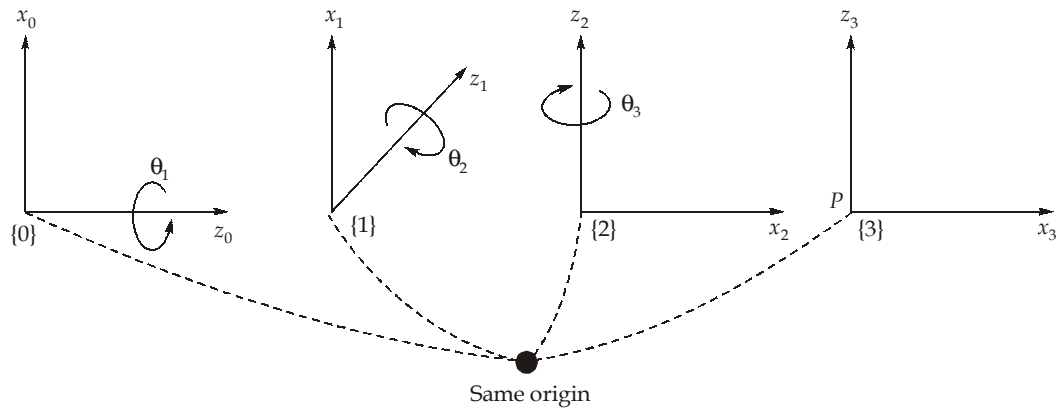
The SCARA configuration has vertical major axis rotations such that gravitational load, Coriolis, and centrifugal forces do not stress the structure as much as they would be if the axes were horizontal. This advantage is very important at high speed and high precision. This configuration provides high stiffness to the arm in the vertical direction, and high compliance in the horizontal plane, thus making SCARA congenial for many assembly tasks. The SCARA configuration and its workspace are presented pictorially in as shown below.



The SCARA configuration and its workspace

(ii)

Coordinate frames:



D-H parameter table:

Link i	a_i	α_i	d_i	θ_i	q_i	$C\theta_i$	$S\theta_i$	$C\alpha_i$	$S\alpha_i$
1	0	90°	0	θ_1	θ_1	C_1	S_1	0	1
2	0	90°	0	$\theta_2 + 90^\circ$	θ_2	$-S_2$	C_2	0	1
3	0	0	0	θ_3	θ_3	C_3	S_3	1	0

Q.4 (a) Solution:

$$\begin{aligned} \text{Work done by press} &= \text{Energy absorbed} \\ &= 25 \times 10^3 \times 0.2 \times 0.15 = 750 \text{ Nm} \end{aligned}$$

$$\begin{aligned} \text{Energy supplied in one cycle by motor to cam} &= \frac{\text{Power in kJ/s}}{\text{Revolution per second of Cam}} \\ &= \frac{3 \times 10^3 \times 60 \times 6}{950} = 1136.8421 \text{ Nm/cycle} \end{aligned}$$

$$\text{Energy supplied in one stroke} = \frac{1136.8421}{2} = 568.421 \text{ Nm/stroke}$$

$$\text{Energy supplied in 15\% of stroke} = 568.421 \times 0.15 = 85.2631 \text{ Nm}$$

$$\begin{aligned} \text{Fluctuation of energy, } (\Delta E) &= \text{Energy absorbed} - \text{Energy supplied} \\ &= 750 - 85.2631 = 664.736 \text{ Nm} \end{aligned}$$

We know that

$$\Delta E = I \times \omega^2 \times C_s$$

$$I(\text{for hollow disc}) = \frac{m}{2}(R_0^2 + R_i^2)$$

and

$$m = \rho \times V = \rho \times \pi(R_0^2 - R_i^2) \times t$$

So,

$$I = \rho \times \frac{\pi(R_0^2 - R_i^2) \times t}{2} \times (R_0^2 + R_i^2)$$

$$= \frac{\rho \times \pi}{2} (R_0^4 - R_i^4) \times t$$

$$I = \frac{\rho \times \pi}{2} R_0^4 \left(1 - \frac{R_i^4}{R_0^4}\right) \times t = 58.968 \times t$$

So,

$$\Delta E = 58.968 \times t \times \left(\frac{2\pi \times 950}{60}\right)^2 \times 0.02$$

$$t = 0.0569 \text{ m}$$

$$t = 56.95 \text{ mm}$$

$$\begin{aligned} \text{and mass of flywheel, } m &= \rho \times \pi(R_0^2 - R_i^2) \times t = 7850 \times \pi R_0^2 \left(1 - \frac{R_i^2}{R_0^2}\right) \times t \\ &= 45.505 \text{ kg} \end{aligned}$$

Q.4 (b) Solution:

The solution for joint displacement d_3 is directly obtained by equating r_{34}

$$\Rightarrow L_{12} + d_3 - L_4 = r_{34}$$

$$d_3 = r_{34} + L_4 - L_{12}$$

$$\Rightarrow L_2 C_{12} + L_{11} C_1 = r_{14} \quad \dots(1)$$

$$\Rightarrow L_2 S_{12} + L_{11} S_1 = r_{24} \quad \dots(2)$$

Squaring eq. (1) and (2), then adding,

$$L_{11}^2 + L_2^2 + 2L_{11}L_2C_2 = r_{14}^2 + r_{24}^2$$

$$C_2 = \frac{r_{14}^2 + r_{24}^2 - L_{11}^2 - L_2^2}{2L_{11}L_2}$$

$$S_2 = \pm\sqrt{1 - C_2^2}$$

So, $\theta_2 = \text{Atan2}(S_2, C_2)$

Now, θ_2 is used to compute, θ_1

$$\Rightarrow L_2(C_1C_2 - S_1S_2) + L_{11}C_1 = r_{14} \quad \text{(expanding } C_{12}, \text{ i.e. } \cos(\theta_1 + \theta_2))$$

$$L_2(S_1C_2 + C_1S_2) + L_{11}S_1 = r_{24} \quad \text{(expanding } S_{12}, \text{ i.e. } \sin(\theta_1 + \theta_2))$$

So, $(L_{11} + L_2C_2)C_1 - (L_2S_2)S_1 = r_{14} \quad \dots(3)$

$$(L_{11} + L_2C_2)S_1 + (L_2S_2)C_1 = r_{24} \quad \dots(4)$$

Let $L_{11} + L_2C_2 = r\cos\phi$ and $L_2S_2 = r\sin\phi$

So, $r = \sqrt{(L_{11} + L_2C_2)^2 + (L_2S_2)^2}$

and $\phi = \text{Atan2}\left(\frac{L_2S_2}{r}, \frac{L_{11} + L_2C_2}{r}\right)$

Equation 3 and 4 will be

$$r\cos(\theta_1 + \phi) = r_{14} \quad \dots(5)$$

$$r\sin(\theta_1 + \phi) = r_{24} \quad \dots(6)$$

From eq. 5 and 6

$$\theta_1 = \text{Atan2}\left(\frac{r_{24}}{r}, \frac{r_{14}}{r}\right) - \phi$$

Now, θ_1, θ_2 and θ_3 are known, θ_4 is computed as:

$$\Rightarrow C_{124} = r_{11}$$

$$S_{124} = r_{21}$$

So, $\theta_1 + \theta_2 - \theta_4 = \text{Atan2}(r_{21}, r_{11})$

$$\theta_4 = \theta_2 + \theta_1 - \text{Atan2}(r_{21}, r_{11})$$

Q.4 (c) Solution:

$$z_p = 18, N_p = 720 \text{ rpm}, N_G = 144 \text{ rpm}, \sigma_p = 120 \text{ MPa}, \sigma_G = 100 \text{ MPa}, C_s = 1.25$$

$$(i) \quad z_G N_G = z_p N_p$$

$$z_G = \frac{720}{144} \times 18 = 90$$

$$\text{Gear ratio, } G = \frac{720}{144} = 5$$

$$\text{For pinion, } y_p = 0.154 - \frac{0.912}{18} = 0.10333$$

$$\text{For gear, } y_G = 0.154 - \frac{0.912}{90} = 0.14386$$

$$\Rightarrow \quad \sigma_G \times y_G = 0.14386 \times 100 = 14.3866 \text{ N/mm}^2$$

$$\Rightarrow \quad \sigma_p \times y_p = 0.10333 \times 120 = 12.3996 \text{ N/mm}^2$$

$\sigma_p y_p$ is less than $\sigma_G y_G$, that means pinion is weaker and design is to be done for pinion.

$$\text{Torque, } M_t = \frac{5 \times 60 \times 10^3}{2\pi N_p} = 66.314 \text{ Nm}$$

$$M_t = 66314.566 \text{ Nmm}$$

$$M_t = P_t \times \frac{d_p}{2}$$

$$P_t = \frac{2M_t}{d_p} = \frac{2M_t}{m \times z_p} = \frac{2 \times M_t}{m \times 18}$$

$$P_t = \frac{7368.285}{m} \text{ N} \quad \dots(1)$$

$$\text{Effective load, } P_{\text{eff}} = \frac{P_t \times C_s}{C_v}$$

$$C_v = \frac{3}{3+v} \quad (v \text{ in m/s})$$

$$\Rightarrow C_v = \frac{3}{3 + \left(\frac{d_p}{2} \times \omega_p\right)} = \frac{3}{3 + \left(\frac{m \times z_p}{2 \times 1000} \times \omega_p\right)}$$

$$\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi \times 720}{60} = 75.398 \text{ rad/s}$$

$$C_v = \frac{3}{3 + 0.678m} \quad \dots(2)$$

$$P_{\text{eff}} = \frac{7368.285 \times 1.5(3 + 0.678m)}{m \times 3} \quad (\text{given, } C_s = 1.5)$$

$$P_{\text{eff}} = \frac{3684.1425(3 + 0.678m)}{m} = \frac{11052.427 + 2497.848m}{m}$$

$$P_{\text{eff}} \leq \frac{S_b}{f_s}$$

where S_b is the beam strength f_s is the factor of safety.

$$S_b = \sigma_p \times y_p \times \pi \times m \times b$$

$$S_b = 12.3996 \times \pi \times m \times 10 \text{ m} = 389.544 \text{ m}^2$$

$$P_{\text{eff}} \leq \frac{S_b}{f_s}$$

$$\frac{11052.427 + 2497.848m}{m} = \frac{389.544 \text{ m}^2}{1.5} \quad (\text{where, } f_s = 1.5)$$

$$11052.427 + 2497.848m = 259.6966 \text{ m}^3$$

$$m = 4.3937 \text{ m}$$

So, first preference value is 5 mm.

$$d_p = 5 \times 18 = 90 \text{ mm}$$

$$d_G = 5 \times 90 = 450 \text{ mm}$$

$$b = 10 \times m = 50 \text{ mm}$$

(ii) (F_s) factor of safety for wear strength = 2

$$S_w = \text{wear strength}$$

$$\text{For module, } m = 5 \text{ mm}$$

eq. (1) and eq. (2)

$$P_t = \frac{7368.285}{m} = 1473.657 \text{ (N)}$$

$$C_v = \frac{3}{3+0.678m} = 0.46948$$

$$P_{\text{eff}} = \frac{P_t \times C_s}{C_v} = 4708.369 \quad (C_s = 1.5)$$

So,

$$P_{\text{eff}} \leq \frac{S_w}{f_s}$$

$$S_w = d_p \times K \times Q \times b$$

$$Q = \frac{2G}{G+1} = \frac{2 \times 5}{5+1} = 1.667 \quad (\because \text{External gear})$$

So,

$$S_w = 90 \times 50 \times 1.667 \times 0.16 \left(\frac{BHN}{100} \right)^2$$

$$S_w = 1200.24 \left(\frac{BHN}{100} \right)^2$$

$$P_{\text{eff}} \leq \frac{S_w}{f_s}$$

$$4708.369 \leq \frac{1200.24}{2} \left(\frac{BHN}{100} \right)^2$$

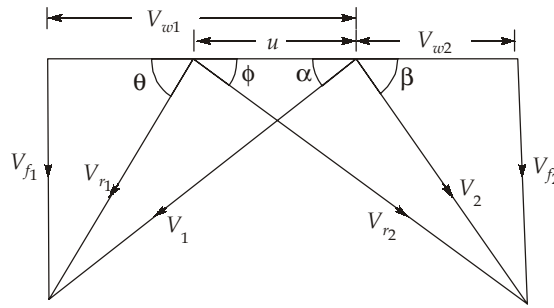
$$BHN = 280.10199$$

So, surface hardness is 281.

Q.5 (a) Solution:

It is the ratio of work done by the blade to the actual energy at the inlet of blade.

$$\eta_b = \frac{\dot{m}(V_{w1} + V_{w2}) \times u}{\frac{1}{2} \dot{m} V_1^2} = \frac{2(V_{w1} + V_{w2}) \times u}{V_1^2}$$



From velocity diagram,

$$V_{w1} + V_{w2} = V_{r1}\cos\theta + V_{r2}\cos\phi$$

$$= V_{r1} \cos\theta \left[1 + \left(\frac{V_{r2}}{V_{r1}} \right) \times \left(\frac{\cos\phi}{\cos\theta} \right) \right]$$

Let, $\frac{V_{r2}}{V_{r1}} = k$ and $\frac{\cos\phi}{\cos\theta} = x$

$$V_{w1} + V_{w2} = V_{r1}\cos\theta(1 + kx) \quad \dots(2)$$

Now, $V_{r1}\cos\theta = V_{w1} - u$

$$V_{r1}\cos\theta = (V_1\cos\alpha - u) \quad \dots(3)$$

Now, from eq. (2) and (3)

$$V_{w1} + V_{w2} = (V_1\cos\alpha - u)(1 + kx)$$

$$\eta_b = \frac{2(V_1\cos\alpha - u)(1 + kx) \times u}{V_1^2}$$

$$\eta_b = \frac{2(V_1 u \cos\alpha - u^2)(1 + kx)}{V_1^2} \quad \left(\text{Let, } \rho = \frac{u}{V_1} \right)$$

$$\eta_b = \frac{2V_1^2 \left(\frac{u}{V_1} \cos\alpha - \left(\frac{u}{V_1} \right)^2 \right) (1 + kx)}{V_1^2}$$

$$\eta_b = \frac{2(\rho \cos\alpha - \rho^2)(1 + kx)}{1}$$

Now for maximum blade efficiency,

$$\frac{d\eta_b}{d\rho} = 0$$

$$2(1 + kx)(\cos\alpha - 2\rho) = 0$$

$$\rho = \frac{\cos \alpha}{2}$$

$$\frac{d^2 \eta_b}{d\rho^2} = -4(1 + kx)$$

$$\left(\frac{d^2 \eta_b}{d\rho^2} < 0, \text{ so it is condition for maximization} \right)$$

Condition for maximum blade efficiency, $\rho = \frac{\cos \alpha}{2} = \frac{u}{V_1}$

Now,

$$\begin{aligned}
 (\eta_b)_{\max} &= 2 \left[\frac{\cos \alpha}{2} \times \cos \alpha - \frac{\cos^2 \alpha}{4} \right] (1 + kx) \\
 &= 2 \left[\frac{\cos^2 \alpha}{4} \right] (1 + kx) \\
 (\eta_b)_{\max} &= \frac{\cos^2 \alpha}{2} (1 + kx)
 \end{aligned}$$

If blades are frictionless, $V_{r1} = V_{r2}$

$$k = 1$$

If blades are symmetrical, $\theta = \phi$

$$\Rightarrow x = 1$$

$$(\eta_b)_{\max} = \frac{\cos^2 \alpha}{2} (1 + 1) = \cos^2 \alpha$$

Maximum blade efficiency of impulse turbine, $(\eta_b)_{\max} = \cos^2 \alpha$

Q.5 (b) Solution:

Certain control and supervisory instruments are provided for the safe and effective operation of a turbine given as follows.

- 1. Pressure gauges:** Pressure gauges are provided to record the pressure of main steam at the stop valve, in the steam chest, at the first stage and the exhaust, the oil pressure to the bearings, the governor mechanism and the pressure of steam or water to the gland seals. For a condensing turbine, a vacuum gauge and a barometer are installed.
- 2. Thermometers:** Thermometers are provided to record steam temperature at the stop valve, in the steam chest, at the first stage, and at the gland. The oil temperatures entering and leaving the bearings are noted.

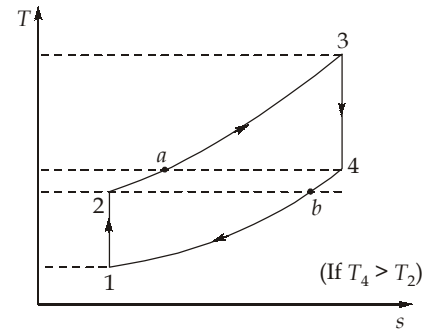
3. **Speed and cam-shaft position recorder:** A speed and cam-shaft position recorder is required to record the turbine speed in rpm. During operation, the turbine speed is obtained from the generator frequency recorder. Thus, the speed recorder is used to record the cam shaft position, which determines the opening of the valve and the load on the turbine.
4. **Eccentricity recorder:** An eccentricity recorder is provided to indicate and record the eccentricity of the shaft at the high pressure end of the turbine.
5. **Vibration amplitude recorder:** A vibration amplitude recorder is provided to record vibration of the rotor.
6. **Expansion Indicator:** An expansion indicator is provided on the turbine control board to show the axial expansion of the turbine casing.
7. **Noise meter:** A noise meter on the control board is used to pick up and amplify the noise made by the moving parts of the turbine.
8. **Flow meter:** Flow meters are mounted on the turbine control board to indicate, record and integrate the mass flow rate to the turbine, the steam bled at various points and the flow to the condenser.
9. **Wattmeters, Voltmeters and Ammeters:** Wattmeters, Voltmeters and Ammeters are also provided on the turbine control board, which along with the flowmeters are used to determine the steam and heat rates of the unit.
10. **Hand wheels:** Handwheels to operate the various drain valves are located at the turbine on the turbine board.
11. **Governor:** Governor controls are located at the turbine or on the turbine control board for proper regulation of valves.
12. **Trip lock lever:** A trip lock lever for testing the overspeed trip is usually mounted on the turbine control board.

Q.5 (c) Solution:

Regeneration: Regeneration is a process in which high temperature exhaust gases coming out from the turbine is utilised for heating cold air coming out from the compressor and before entering the combustion chamber. This preheating of cold air decreases the fuel requirement thus increasing the efficiency.

Effects of regeneration:

1. Compressor work → Remains same
2. Turbine work → Remains same
3. Net work output → Remains same
4. Heat supplied → Decreases
5. Heat rejected → Decreases
6. Mean temperature of heat addition → Increases
7. Mean temperature of heat rejection → Decreases
8. Specific fuel consumption → Decreases
9. Efficiency → Increases.



Ideal regenerative cycle: In an ideal regenerative cycle cold air is heated upto turbine exit temperature in a regenerator. It is possible for an infinitely large heat exchanger and at that point its effectiveness will be 100% and T_a will become equal to T_4 and T_b will become equal to T_2 .

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$\eta = 1 - \frac{(T_b - T_1)}{(T_3 - T_a)}$$

For ideal regenerative cycle, $T_a = T_4$, $T_b = T_2$

$$\begin{aligned} \eta &= 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)} \\ &= 1 - \frac{T_1 \left[\left(\frac{T_2}{T_1} \right) - 1 \right]}{T_3 \left[\left(1 - \frac{T_4}{T_3} \right) \right]} \\ &= 1 - \left(\frac{T_1}{T_3} \times \frac{T_2}{T_1} \right) \frac{[1 - T_1/T_2]}{[1 - T_4/T_3]} \end{aligned}$$

In Brayton cycle, $\frac{T_3}{T_4} = (r_p)^{\gamma-1/\gamma}$

$$\frac{T_2}{T_1} = (r_p)^{\gamma-1/\gamma}$$

Now, $\frac{T_3}{T_4} = \frac{T_2}{T_1}$ or $\frac{T_4}{T_3} = \frac{T_1}{T_2}$ also $1 - \frac{T_4}{T_3} = 1 - \frac{T_1}{T_2}$

$$\eta = 1 - \frac{T_1}{T_3} (r_p)^{\gamma-1/\gamma} \times 1$$

$$\eta_{\text{Ideal regen.}} = 1 - \frac{T_1}{T_3} (r_p)^{\gamma-1/\gamma}$$

Q.5 (d) Solution:

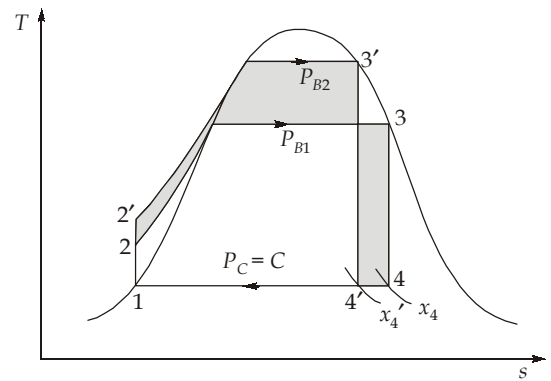
Where: W_p - Pump work, W_T - Turbine work, $W_{\text{net}} = (W_T - W_p)$

Q_S - Heat supplied, Q_R - Heat rejected, T_{ma} - Mean temperature at heat addition,

T_{mr} - Mean temperature at heat rejection, η - Efficiency of cycle, x - dryness fraction.

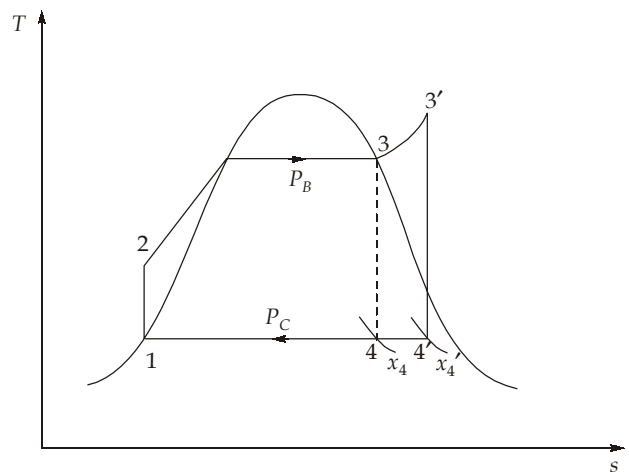
By increasing boiler pressure ($P_B \uparrow$):

- W_p - \uparrow
- W_T - $\uparrow\uparrow$
- W_{net} - \uparrow
- Q_S - cannot say (\downarrow normally)
- Q_R - \downarrow
- T_{ma} - \uparrow
- T_{mr} - Same
- η - \uparrow
- x - \downarrow
- Moisture content will increase.



By superheating :

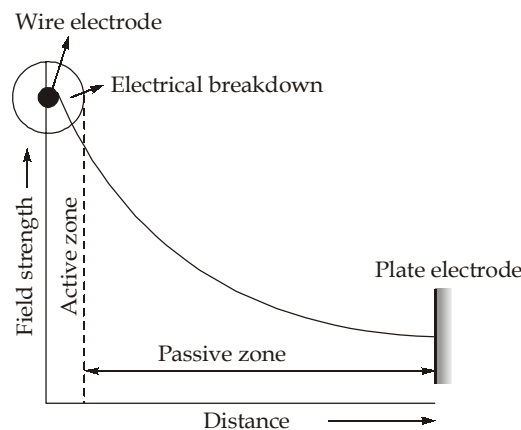
- W_p - Same
- W_T - \uparrow
- W_{net} - \uparrow
- Q_S - \uparrow
- Q_R - \uparrow
- T_{ma} - \uparrow
- T_{mr} - Same
- η - \uparrow
- x - \uparrow
- Moisture content will decrease



Q.5 (e) Solution:

Principle of ESP has four distinct phases as follows:

1. **Ionization or corona generation:** When the potential difference between the wire and electrode increases, a voltage is reached where an electrical breakdown of the gas occurs near the wire. This electrical break down or ion discharge is known as corona formation and thereby gas is transformed from insulating to conducting state. Two types of corona discharge can be generated which are:
 - a. **Negative corona:** In negative corona, discharge electrode is of negative polarity and the process of electron generation occurs at narrow region.
 - b. **Positive corona:** When positive voltage is applied to discharge electrodes in the same way as negative corona, large number of free electron and positive ions are generated. Or large number of positive ions produced move towards collecting electrode and thus transfer charge to dust particles upon collision.



Negative coronas are more commonly used in industrial application, while for cleaning air in inhabited space positive coronas are used. Due to ozone generation in negative corona its application for air cleaning in inhabited area is avoided.

2. **Charging of Particles:** Particle charging takes place in region between the boundary of corona glow and the collection electrode, where particles are subjected to the rain of negative ions from the corona process. Mainly two mechanisms are responsible for particle charging. Each mechanism becomes significant according to particle size ranges. For particles having diameter greater than $1\ \mu\text{m}$, field charging is dominant force and for particle size less than $0.2\ \mu\text{m}$ diffusion charging predominates.
3. **Migration and precipitation of particle.**
4. **Removal of deposited dust:** Once collected, particle can be removed by coalescing and draining, in the case of liquid aerosols and by periodic impact or rapping, in case of solid material.

In case of solid material, a sufficiently thick layer of dust must be collected so that it falls into the hopper or bin in coherent masses to prevent excessive re-entrainment of the material into the gas system.

Q.6 (a) Solution:

Atmospheric temperature,

$$T_a = 303 \text{ K}$$

Calorific value, C.V. = 42 MJ/kg

$$\text{Drought produced} = (\rho_a - \rho_g)gH$$

$$(20 \times 10^{-3}) \times g \times 10^3 = \left[\frac{P_a}{RT_a} - \frac{P_a}{RT_g} \right] gH$$

$$20 = \frac{1.01325 \times 100}{0.287} \left[\frac{1}{303} - \frac{1}{573} \right] \times H$$

$$H = \frac{20 \times 0.287 \times 303 \times 573}{101.325 \times 270}$$

Height of stack, $H = 36.4274 \text{ m}$

$$\therefore \text{Efficiency of boiler, } \eta = \frac{\dot{m}_s \times (h_1 - h_{feed})}{\dot{m}_f \times C.V.}$$

$$0.75 = \frac{8 \times 5000 \times (2920 - 127)}{\dot{m}_f \times 42 \times 10^3}$$

$$\dot{m}_f = 3546.67 \text{ kg/h}$$

Mass flow rate of fuel,

$$\dot{m}_f = 0.9852 \text{ kg/s}$$

Mass flow rate of gas, $\dot{m}_g = \dot{m}_a + \dot{m}_f$

$$= 16.5 \dot{m}_f + \dot{m}_f = 17.5 \dot{m}_f$$

$$\dot{m}_g = 17.5 \times 0.9852 = 17.241 \text{ kg/s}$$

We know that,

$$\text{Mass flow rate of gas, } \dot{m}_g = \rho_g \times A(V_g)$$

$$17.241 = \frac{P}{RT_{ge}} \times \frac{\pi}{4} D^2 \times (2 \times g \times H)^{1/2}$$

$$17.241 = \frac{101.325}{0.287 \times 593} \times \frac{\pi}{4} \times D^2 \times (2 \times 9.81 \times 36.4274)^{1/2}$$

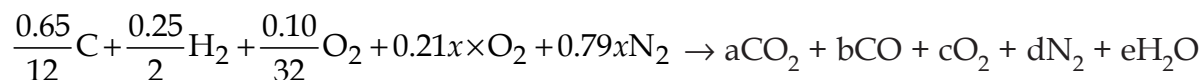
$$D^2 = \frac{4 \times 17.241 \times 0.287 \times 593}{\pi \times 101.325 \times 26.734} = 1.37920$$

Diameter at base, $D = 1.1744$ m

Q.6 (b) Solution:

Let us consider 1 kg fuel and let x k-mol of air is supplied.

Now,



$$\text{By carbon balance, } a + b = \frac{0.65}{12} = 0.054167 \quad \dots(1)$$

$$\text{By hydrogen balance, } e = \frac{0.25}{2} = 0.125$$

$$\text{By oxygen balance, } a + \frac{b}{2} + c + \frac{e}{2} = \frac{0.10}{32} + 0.21x \quad \dots(2)$$

$$\text{By nitrogen balance, } d = (0.79)x \quad \dots(3)$$

Given: $CO_2 = 6\%$, $CO = 1.5\%$

$$\text{Now, } \%CO_2 = \frac{a \times 100}{a + b + c + d} = 6$$

$$\%CO = \frac{b \times 100}{a + b + c + d} = 1.5$$

$$\frac{b}{a} = \frac{1.5}{6} = 0.25 \quad \dots(4)$$

By equation (1) and (4),

$$a + b = 0.054167$$

$$a + 0.25a = 0.054167$$

$$a = 0.0433336$$

$$b = 0.25 \times 0.0433336 = 0.0108334$$

$$\therefore \frac{a \times 100}{a+b+c+d} = 6$$

$$a+b+c+d = \frac{0.0433336 \times 100}{6} = 0.7222267$$

Now, we have, $a = 0.0433336$, $b = 0.0108334$, $e = 0.125$,

$$a+b+c+d = 0.7222267$$

Now, from equation (2),

$$a + \frac{b}{2} + c + \frac{e}{2} = \frac{0.10}{32} + 0.21x$$

Adding 'd' on both sides:

$$a + \frac{b}{2} + c + \frac{e}{2} + d = \frac{0.10}{32} + 0.21x + d$$

$$(a+b+c+d) - \frac{b}{2} + \frac{e}{2} = \frac{0.10}{32} + 0.21x + 0.79x$$

$$0.7222267 - \frac{0.108334}{2} + \frac{0.125}{2} = \frac{0.10}{32} + x$$

$$x = 0.776185 \text{ kilomole}$$

Hence,

$$\begin{aligned} \text{air supplied} &= 0.776185 \times 28.96 \\ &= 22.4783176 \text{ kg air per kg of fuel} \end{aligned}$$

Now for Brayton cycle:

$$T_1 = 27 + 273 = 300\text{K}, P_1 = 1 \text{ bar}, P_2 = 4 \text{ bar}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\gamma-1/\gamma} = (4)^{0.4/1.4}$$

$$T_2 = 445.798 \text{ K}$$

By energy balance on the combustion chamber:

$$m_a(c_p)_a [T_2] + m_f \times (\text{C.V.}) = (m_a + m_f) \times (c_p)_g \times T_3$$

$$22.4783176 \times 1.005 \times 445.798 + 1 \times 20000 = (22.4783176 + 1) \times 1.15 \times T_3$$

$$T_3 = 1113.734 \text{ K}$$

Maximum temperature in the cycle, $T_3 = 1113.734\text{K}$

Now,

$$\frac{T_3}{T_4} = (r_p)^{\gamma-1/\gamma}$$

$$\frac{1113.734}{T_4} = (4)^{1.33-1/1.33}$$

$$T_4 = \frac{1113.734}{(4)^{0.33/1.33}} = 789.584 \text{ K}$$

Regenerator effectiveness,

$$\epsilon = \frac{T_5 - T_2}{T_4 - T_2} = \frac{T_5 - 445.798}{789.584 - 445.798}$$

$$(0.78) [789.584 - 445.798] + 445.798 = T_5$$

$$T_5 = 713.951 \text{ K}$$

$$\begin{aligned} \text{Work input to compressor, } W_C &= m_a (c_p)_a [T_2 - T_1] \\ &= 22.4783176 \times 1.005 \times [445.798 - 300] \\ &= 3293.680 \text{ kJ/kg of fuel} \end{aligned}$$

$$\begin{aligned} \text{Work output from turbine, } W_T &= (m_a + m_f) \times (c_p)_g [T_3 - T_4] \\ &= (22.4783176 + 1) \times 1.15 \times [1113.734 - 789.584] \\ &= 8751.801 \text{ kJ/kg of fuel} \end{aligned}$$

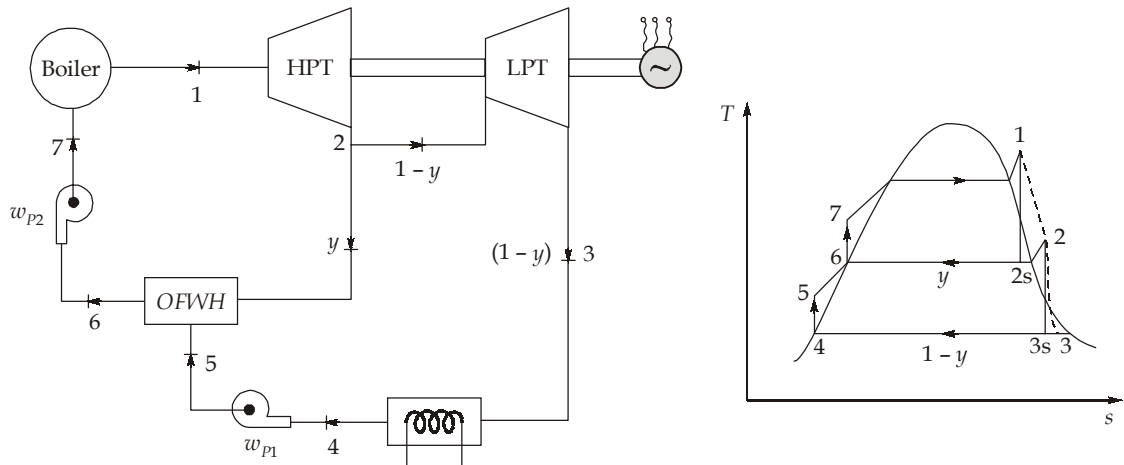
$$\begin{aligned} \text{Heat supplied, } Q_s &= (m_a + m_f) + (c_p)_g \times [T_3 - T_5] \\ &= 23.4783176 \times 1.15 \times [1113.734 - 713.951] \\ &= 10794.167 \text{ kJ/kg of fuel} \end{aligned}$$

$$\text{Thermal efficiency of cycle, } \eta_{\text{cycle}} = \frac{W_T - W_C}{Q_s} = \frac{8751.801 - 3293.680}{10794.167}$$

$$\eta_{\text{cycle}} = 0.50565 = 50.565\%$$

$$\begin{aligned} \text{Specific fuel consumption} &= \frac{3600}{W_{\text{net}}} = \frac{3600}{8751.801 - 3293.680} \\ &= 0.659567 \text{ kg/kW-h} \end{aligned}$$

Q.6 (c)



At 8 MPa and 480°C (from table),

$$h_1 = 3348.4 \text{ kJ/kg}$$

$$s_1 = 6.6586 \text{ kJ/kgK}$$

Since 1 - 2s is isentropic process,

$$s_1 = s_{2s}$$

$$\Rightarrow 6.6586 = s_f + x_{2s} s_{fg} = 1.9922 + x_{2s} \times (6.708 - 1.9922)$$

$$x_{2s} = 0.9895$$

$$h_{2s} = 2741.824 \text{ kJ/kg}$$

$$\text{Turbine efficiency, } \eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$0.85 = \frac{3348.4 - h_2}{3348.4 - 2741.824}$$

$$h_2 = 2832.81 \text{ kJ/kg}$$

At $p = 0.7 \text{ MPa}$ and $h = 2832.81 \text{ kJ/kg}$ (from table),

$$s_2 = 6.8606 \text{ kJ/kgK}$$

Since 2 - 3s is isentropic process,

$$s_2 = s_{3s}$$

$$\Rightarrow 6.8606 = 0.5926 + x_{3s} \times (8.2287 - 0.5926)$$

$$x_{3s} = 0.8208$$

$$\begin{aligned} h_{3s} &= h_f + x_{3s} h_{fg} \\ &= 173.88 + 0.8208 \times (2576.98 - 173.88) \\ &= 2146.34 \text{ kJ/kg} \end{aligned}$$

$$\text{Turbine efficiency, } \eta_T = \frac{h_2 - h_3}{h_2 - h_{3s}}$$

$$0.85 = \frac{2832.81 - h_3}{2832.81 - 2146.34}$$

$$h_3 = 2249.31 \text{ kJ/kg}$$

$$\begin{aligned} w_{p1} &= v_{f@0.7} \Delta p = 1.0084 \times 10^{-3} \times (0.7 - 0.008) \times 10^3 \\ &= 0.6978 \text{ kJ/kg} \end{aligned}$$

$$h_5 = h_4 + w_{p1} = 173.88 + 0.6978 = 174.578 \text{ kJ/kg}$$

$$w_{p2} = v_{f@0.008} \Delta p = 1.1080 \times 10^{-3} \times (8 - 0.7) \times 10^3$$

$$= 8.088 \text{ kJ/kg}$$

$$h_7 = h_6 + w_{p2} = 697.22 + 8.088 = 705.308 \text{ kJ/kg}$$

OFWH:

Energy balance:

$$yh_2 + (1 - y)h_5 = h_6$$

$$y \times 2832.81 + (1 - y) \times 174.578 = 697.22$$

$$2832.81y + 174.578 - 174.578y = 697.22$$

$$y = 0.1966$$

$$\text{Pump work, } w_p = (1 - y)w_{p1} + w_{p2}$$

$$= (1 - 0.1966) \times (0.6978) + (8.088)$$

$$w_p = 8.6486 \text{ kJ/kg}$$

$$\text{Turbine work, } w_T = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$= (3348.4 - 2832.81) + (1 - 0.1966)(2832.81 - 2249.31)$$

$$w_T = 984.3739 \text{ kJ/kg}$$

$$\text{Heat supplied, } q_s = h_1 - h_7 = 3348.4 - 705.308 = 2643.092 \text{ kJ/kg}$$

$$\text{Thermal efficiency, } \eta_{th} = \frac{w_T - w_p}{q_s} = \frac{984.3739 - 8.6486}{2643.092} = 0.3692 = 36.92\%$$

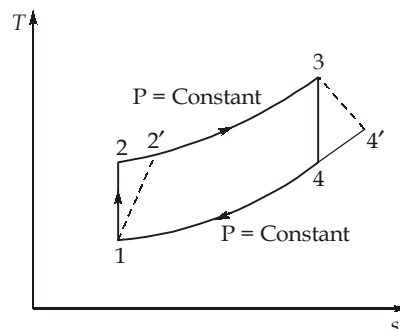
$$\text{Power output of cycle, } P = 100 \text{ MW} = 100 \times 10^3 \text{ kW} = \dot{m}_s (w_T - w_p)$$

$$100 \times 10^3 = \dot{m}_s (984.3739 - 8.6486)$$

$$\Rightarrow \dot{m}_s = 102.488 \text{ kg/s}$$

Q.7 (a) Solution:

Given: Pressure ratio, $r_p = 6$



$$T_1 = 273 + 20 = 293 \text{ K}$$

$$\frac{T_2}{T_1} = (r_p)^{(\gamma-1)/\gamma} = (6)^{(1.4-1)/1.4}$$

$$T_2 = 488.87 \text{ K}$$

$$\eta_C = \frac{T_2 - T_1}{T_2' - T_1}$$

$$T_2' = T_1 + \left(\frac{T_2 - T_1}{\eta_C} \right) = 293 + \left(\frac{488.87 - 293}{0.80} \right)$$

$$T_2' = 537.837 \text{ K}$$

$$T_3 = 800 + 273 = 1073 \text{ K}$$

$$\frac{T_3}{T_4} = (r_p)^{(\gamma-1)/\gamma} = (6)^{(1.34-1)/1.34}$$

$$T_4 = \frac{1073}{(6)^{0.34/1.34}} = 681.02 \text{ K}$$

$$\eta_T = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$\begin{aligned} T_4' &= T_3 - \eta_T(T_3 - T_4) \\ &= 1073 - 0.86(1073 - 681.02) \end{aligned}$$

$$T_4' = 735.897 \text{ K}$$

$$\begin{aligned} \text{Power consumed by compressor, } W_C &= C_p(T_2' - T_1) \\ &= 1.005(537.837 - 293) \\ &= 246.06 \text{ kJ/kg of air} \end{aligned}$$

$$\begin{aligned} \text{Work done by turbine, } W_T &= (c_p)_g(T_3 - T_4') \\ &= 1.128(1073 - 735.897) \\ &= 380.25 \text{ kJ/kg of gas} \end{aligned}$$

(i) Gas required per kg of air compressed in compressor drive turbine to supply power of compressor.

By energy balance:

$$(m_a + m_f)h_3 - m_a h_2' = m_f \times \text{C.V.}$$

$$(1 + m_f)h_3 - h_2' = m_f \times \text{C.V.}$$

{If $m_a = 1 \text{ kg of air}$ }

$$(1 + m_f) \times 1.128 \times 1073 - 1.005 \times 537.837 = m_f \times 42000$$

$$1.128 \times 1073 - 1.005 \times 537.837 = m_f(42000 - 1.128 \times 1073)$$

$$m_f = 0.01642 \text{ kg of fuel per kg of air}$$

(iii) Total amount of gases per kg of air supplied in turbine

$$= 1 + 0.01642 = 1.01642 \text{ kg}$$

Gases required per kg of air in compressor drive turbine = 0.6471 kg

Hence, gases supplied to output turbine = 1.01642 - 0.6471

$$= 0.36932 \text{ kg/kg of air}$$

$$\text{Net power output of plant} = \dot{m}_a \times 0.36932 \times (W_T)$$

$$= 25 \times 0.36932 \times 380.25 = 3510.85 \text{ kW}$$

$$\text{(iv) Thermal efficiency} = \frac{W_{\text{net}}}{Q_{\text{supplied}}} = \frac{0.36932 \times 380.25}{0.01642 \times 42000}$$

$$= 0.20363 \text{ or } 20.636\%$$

Result:

(i) Gas required per kg of air compressed in compressor drive turbine to supply power to compressor = 0.6471 kg

(ii) Mass of the fuel burned per kg of air = 0.01642 kg

(iii) Net power output of plant = 3510.85 kW

(iv) Thermal efficiency of plant = 20.363%

Q.7 (b) Solution:

For 100 kg of coal, let 'a' kilomoles of oxygen are supplied.



By carbon balance:

$$b + d = \frac{88}{12}$$

$$b + d = 7.333 \quad \dots(1)$$

By hydrogen balance, $g = 2.2$

$$\text{By oxygen balance, } a = b + \frac{d}{2} + e + \frac{g}{2} = b + \frac{d}{2} + e + \frac{2.2}{2}$$

$$a = b + \frac{d}{2} + e + 1.1 \quad \dots(2)$$

By nitrogen balance, $f = 3.76a$... (3)

From dry flue gas analysis:

$$\%CO_2 = \frac{b}{(b+d+e+f)} \times 100$$

$$\frac{13.2}{100} = \frac{b}{b+d+e+f} \quad \dots(4)$$

$$\%O_2 = \frac{e}{b+d+e+f} \times 100$$

$$\frac{3.2}{100} = \frac{e}{b+d+e+f} \quad \dots(5)$$

Dividing equation (4) by equation (5),

$$\frac{13.2}{3.2} = \frac{b}{e}$$

$$b = (4.125)e \quad \dots(6)$$

From equation (1), (2) and (6),

$$a = 7.333 - d + 0.5d + \left(\frac{7.333 - d}{4.125} \right) + 1.1$$

$$a = (10.2107 - 0.7424d) \quad \dots(7)$$

$$f = 3.76a$$

$$f = 38.3922 - 2.7914d$$

From equation (1), (6) and (7),

Now,
$$b + d + e + f = 7.333 + \left(\frac{7.333 - d}{4.125} \right) + 38.3922 - 2.7914d$$

$$b + d + e + f = 47.5029 - 3.0338d \quad \dots(8)$$

Now, from equation (4),

$$\frac{13.2}{100} = \frac{b}{47.5029 - 3.0338d}$$

$$0.132[47.5029 - 3.0338d] = 7.333 - d$$

$$(0.59954)d = 1.06262$$

$$d = 1.7724$$

from equation (1), $b = 7.333 - 1.7724 = 5.5606$

from equation (7), $a = 10.2107 - 0.7424 \times 1.7724$

$$a = 8.8949$$

from equation (3), $f = 33.4448$

from equation (6), $e = \frac{b}{4.125} = \frac{5.5606}{4.125} = 1.348$

Now, $a = 8.8949$, $b = 5.5606$, $d = 1.7724$, $e = 1.348$, $f = 33.4448$

Mass of air supplied for 100 kg coal = $\frac{8.8949 \times 32}{0.23} = 1237.55$ kg

Air contains 23% oxygen by weight

or

Mass of air supplied for 100 kg coal = $(8.8949 \times 32 + 28 \times 33.4448) = 1221.10$ kg

Hence, mass of air supplied for 1 kg coal = 12.211 to 12.375 kg

Total moles of dry flue gases = $b + d + e + f$

$$= 5.5606 + 1.7724 + 1.348 + 33.4448 = 42.1258$$

$$\% \text{CO}_2 \text{ by volume} = \frac{b \times 100}{b + d + e + f} = \frac{5.5606 \times 100}{42.1258} = 13.2\%$$

$$\% \text{CO by volume} = \frac{d \times 100}{b + d + e + f} = \frac{1.7724 \times 100}{42.1258} = 4.2074\%$$

$$\% \text{O}_2 \text{ by volume} = \frac{e \times 100}{b + d + e + f} = \frac{1.348 \times 100}{42.1258} = 3.20\%$$

$$\% \text{N}_2 \text{ by volume} = \frac{F \times 100}{42.1258} = \frac{33.4448 \times 100}{42.1258} = 79.3926\%$$

Mass of water vapour formed per kg coal = $2.2 \times \left(\frac{18}{100} \right) = 0.396$ kg water vapour

Q.7 (c) Solution:

By definition,

$$R = \frac{(h_1 - h_2) + (h_3 - h_5) + (h_6 - h_8) + (h_9 - h_{11}) + (h_{12} - h_{14})}{h_1 - h_{13}} \quad \dots(1)$$

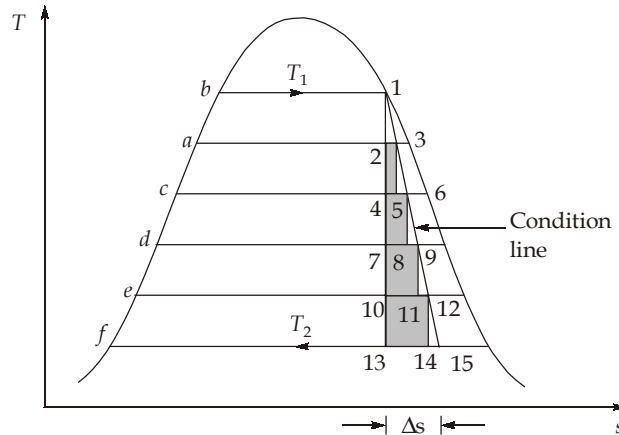
Now,

$$h_1 - h_2 = \text{area } 12 ab$$

$$h_2 - h_4 = \text{area } 24 ca$$

$$h_4 - h_7 = \text{area } 47 dc$$

and so on.



$$R = \frac{(h_1 - h_{13}) + \text{hatched area}}{h_1 - h_{13}}$$

$$R - 1 = \frac{\text{hatched area}}{h_1 - h_{13}}$$

For a large number of stages, hatched area = $\frac{1}{2}(T_1 - T_2) \Delta s$

$$\therefore R - 1 = \frac{1}{2}(T_1 - T_2) \frac{\Delta s}{h_1 - h_{13}}, \text{ where } \Delta s = s_{15} - s_{13} \quad \dots(2)$$

Again,

$$h_1 - h_3 = \eta_s(h_1 - h_2)$$

$$\begin{aligned} h_3 - h_2 &= (h_1 - h_2) - (h_1 - h_3) = (h_1 - h_2) - \eta_s(h_1 - h_2) \\ &= (1 - \eta_s)(h_1 - h_2) \end{aligned}$$

$$\begin{aligned} h_6 - h_4 &= (h_6 - h_5) + (h_5 - h_4) = (h_6 - h_5) + (h_3 - h_2) - \text{area } 2345 \\ &= (1 - \eta_s)(h_3 - h_5) + (1 - \eta_s)(h_1 - h_2) - \text{area } 2345 \\ &= (1 - \eta_s)(h_1 - h_2 + h_3 - h_5) - \text{area } 2345 \end{aligned}$$

$$\begin{aligned} h_9 - h_7 &= (h_9 - h_8) + (h_6 - h_4) - \text{area } 4687 \\ &= (1 - \eta_s)(h_6 - h_8) + (1 - \eta_s)(h_1 - h_2 + h_3 - h_5) - \text{area } 2345 - \text{area } 4687 \\ &= (1 - \eta_s)(h_1 - h_2 + h_3 - h_5 + h_6 - h_6 - h_8) - \text{area } 2345 - \text{area } 4687 \end{aligned}$$

Similarly,

$$h_{15} - h_{13} = (1 - \eta_s)(h_1 - h_2 + h_3 - h_5 + h_6 - h_8 + h_9 - h_{11} + h_{12} - h_{14})$$

- area 2345 - area 4687 - area (7 - 9 - 11 - 10) - area(10 - 12 - 14 - 13)

from eq. (1),

$$h_{15} - h_{13} = (1 - \eta_s)(h_1 - h_{13})R - \Sigma \text{ area}[2345 + 4687 + (7 - 9 - 1 - 10)] + (10 - 12 - 14 - 13)$$

For a large number of stages the summation of the areas becomes the triangle

1 - 15 - 13 - 1.

$$h_{15} - h_{13} = (1 - \eta_s)(h_1 - h_{13})R - \frac{1}{2}(T_1 - T_2)\Delta s$$

Again,

$$h_{15} - h_3 = T_2\Delta s$$

$$T_2\Delta s = (1 - \eta_s)(h_1 - h_{13})R - \frac{1}{2}(T_1 - T_2)\Delta s$$

$$\Delta s \left(T_2 + \frac{T_1}{2} - \frac{T_2}{2} \right) = (1 - \eta_s)(h_1 - h_{13})R$$

\therefore

$$\frac{\Delta s}{h_1 - h_{13}} = \frac{(1 - \eta_s)R}{(T_1 + T_2)/2}$$

Substituting eq. (2),

$$R - 1 = \frac{1}{2}(T_1 - T_2) \frac{(1 - \eta_s)R}{(T_1 + T_2)/2}$$

On rearranging,

$$R = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)} \text{ Proved.}$$

If $\eta_s = 1$, the reheat factor becomes unity.

The actual reheat factor will be less than this ideal value because in enthalpy drop due to reheat will be less than the triangle 1 - 15 - 13 - 1 by the unhatched portions.

Q.8 (a) Solution:

For gas turbine,

$$P_1 = 1 \text{ bar}, T_1 = 10^\circ\text{C} = 283 \text{ K}$$

$$P_2 = 5 \text{ bar}, (\eta_s)_c = 80\%$$

Now,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma}$$

$$T_2 = 283 \times (5)^{(1.4-1)/1.4}$$

$$T_2 = 448.22 \text{ K}$$

$$\text{Isentropic efficiency, } \eta_i = \frac{T_2 - T_1}{T_2' - T_1}$$

$$(T_2' - T_1) = \frac{(448.22 - 283)}{0.8} = 206.525 \text{ K}$$

$$T_2' = 283 + 206.525 = 489.525 \text{ K}$$

Given; Power consumed by compressor = Power produced by high pressure turbine

$$\dot{m}(c_p)_a (T_2' - T_1) = \dot{m}(c_p)_g (T_3 - T_4')$$

$$\left(\frac{\gamma R}{\gamma - 1} \right)_c (489.525 - 283) = \left(\frac{\gamma R}{\gamma - 1} \right)_T (1200 - T_4')$$

$$\left(\frac{1.4}{1.4 - 1} \right) (489.525 - 283) = \frac{1.333}{1.333 - 1} (1200 - T_4')$$

$$\{R_c = R_T\}$$

$$180.574 = 1200 - T_4'$$

$$T_4' = 1019.426 \text{ K}$$

$$(\eta_i)_T = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$T_3 - T_4 = \frac{1200 - 1019.426}{0.85}$$

$$T_4 = 1200 - \left(\frac{1200 - 1019.426}{0.85} \right)$$

$$T_4 = 987.56 \text{ K}$$

Now pressure ratio in high pressure turbine:

$$\frac{T_3}{T_4} = \left(\frac{5}{P_4} \right)^{(\gamma-1)/\gamma}$$

$$\frac{1200}{987.56} = \left(\frac{5}{P_4} \right)^{(1.333-1)/1.333}$$

$$P_4 = \frac{5}{2.18135} = 2.292 \text{ bar}$$

Now, in low pressure turbine,

$$P_4 = 2.292 \text{ bar}, T_4' = 1019.426 \text{ K}$$

$$P_5 = 1 \text{ bar}$$

$$\frac{T_4'}{T_5} = \left(\frac{P_4}{P_5} \right)^{\gamma-1/\gamma}$$

$$\frac{1019.426}{T_5} = (2.292)^{1.333-1/1.333}$$

$$T_5 = 828.647 \text{ K}$$

$$\text{Isentropic efficiency, } \eta_i = \frac{T_4' - T_5'}{T_4 - T_5}$$

$$(1019.426 - 828.647) \times 0.85 = 1019.426 - T_5'$$

$$T_5' = 857.264 \text{ K}$$

Let mass flow rate of air is \dot{m} kg/s.

Power developed by low pressure turbine is the net power output of plant.

$$\text{Power developed} = \dot{m} c_p (T_4' - T_5')$$

$$80 = \dot{m} \times \left(\frac{\gamma R}{\gamma - 1} \right) (1019.426 - 857.264)$$

$$\dot{m} = \frac{(1.333 - 1) \times 80}{1.333 \times 0.287 \times (1019.426 - 857.264)}$$

$$\dot{m} = 0.4294 \text{ kg/s}$$

$$\text{Thermal efficiency of the cycle, } \eta = \frac{W_{\text{net}}}{\text{Heat input}}$$

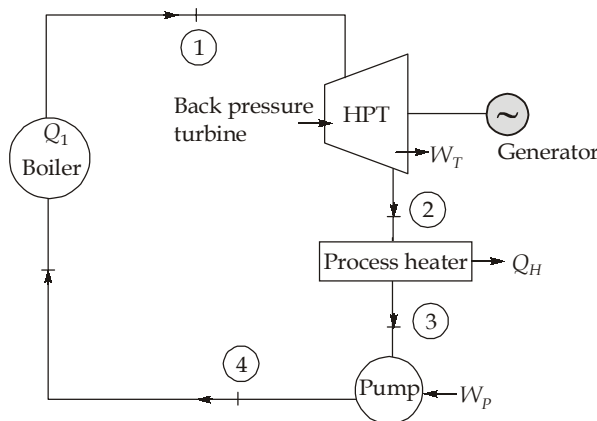
$$\eta = \frac{80}{\dot{m} \left[(c_p)_g T_3 - (c_p)_a T_2 \right]} = \frac{80}{0.4294 \left[\left(\frac{\gamma R}{\gamma - 1} \right)_g \times 1200 - \left(\frac{\gamma R}{\gamma - 1} \right)_a \times 489.525 \right]}$$

$$= \frac{80}{0.4294 \left[\left(\frac{1.333 \times 0.287}{1.333 - 1} \right) \times 1200 - \left(\frac{1.4 \times 0.287}{1.4 - 1} \right) \times 489.525 \right]}$$

Thermal efficiency of cycle, $\eta = 0.2100 = 21.00\%$

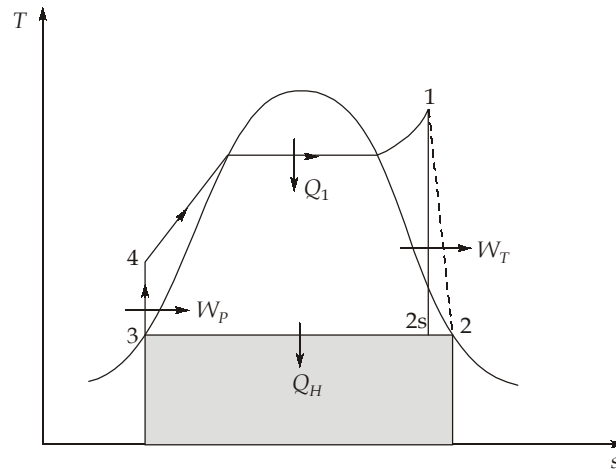
Q.8 (b) Solution:

Back pressure turbine: By modifying the initial steam pressure and exhaust pressure, it is possible to generate the required power and make available the required quantity of exhaust steam at the desired temperature for process work. In the figure, the exhaust steam from the turbine is utilized for process heating, the process heater is used by replacing the condenser of the ordinary Rankine cycle. The pressure at exhaust from the turbine is the saturation pressure corresponding to the temperature desired in the process heater. Such a turbine is called a back pressure turbine. A plant producing both electrical power and process heat simultaneously is called a cogeneration plant when the process steam is the basic need, and the power is produced incidently as a by-product, the cycle is often called a by-product power cycle.



Cogeneration plant with a back pressure turbine

T-s diagram of a by product power cycle with a back pressure turbine is shown in figure. If W_T is the turbine output, Q_H is the process heat required and mass flow rate of steam is \dot{m}_s .



We know that, turbine work output, $W_T = \dot{m}_s \times (h_1 - h_2)$

$$\text{Process heat, } Q_H = \dot{m}_s \times (h_2 - h_3)$$

Now,

$$Q_H = \frac{W_T}{(h_1 - h_2)} \times (h_2 - h_3)$$

If the total heat input is Q_1 to the cogeneration plant, W_T part of it is converted into shaft work, or electricity. The remaining energy ($Q_1 - W_T$), which would otherwise have been a waste, as in the Rankine cycle, is utilized as process heat.

The cogeneration plant efficiency is given by,

$$\eta_{co} = \frac{W_T + Q_H}{Q_1}$$

If η_e is electric plant efficiency.

η_h is process heat generator efficiency.

For separate generation of electricity and steam, the heat added per unit total energy output is,

$$\frac{e}{\eta_e} + \frac{1-e}{\eta_h}$$

where,

e = electricity fraction of total energy output

$$= \frac{W_T}{W_T + Q_H}$$

The combined efficiency η_c for separate generation is given by,

$$\eta_c = \frac{1}{\frac{e}{\eta_e} + \frac{1-e}{\eta_h}}$$

Cogeneration is beneficial if the efficiency of the cogeneration plant is greater than that of separate generation.

Back pressure turbines are quite small with respect to their power output because they have no great volume of exhaust to cope with, the density being high. Hence, cheap in terms of cost per MW compared to condensing sets of the same power.

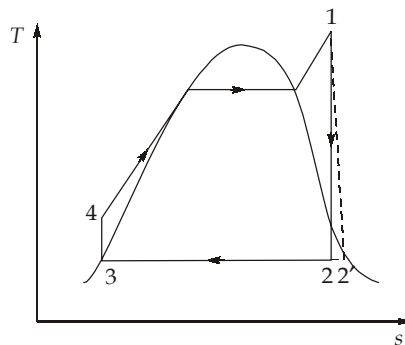
Uses: Besides their use in process industries and photochemical installations, back pressure turbines are used for desalination of sea water, and also for driving compressors and feed pumps.

Q.8 (c) Solution:

Given, $h_1 = 3038 \text{ kJ/kg}$, $s_1 = 6.912 \text{ kJ/kg-K}$

Now, condenser pressure = 76 - 72.5

$$= 3.5 \text{ cm of Hg} = \frac{3.5}{76} \times 1.013 = 0.04665 \text{ bar}$$



Now, We know that,

$$S_1 = S_2$$

$$6.912 = 0.4644 + x(8.413 - 0.4644)$$

$$\frac{6.912 - 0.4644}{8.413 - 0.4644} = x$$

$$x = 0.8111$$

Now,

$$h_2 = (h_f) + xh_{fg}$$

(At 0.04665 bar pressure)

$$= 133 + 0.8111 \times 2426$$

$$h_2 = 2100.73 \text{ kJ/kg}$$

$$\text{Heat supplied} = h_1 - h_4 = h_1 - h_3 \quad (\text{Pump work is neglected})$$

$$= 3038 - 133 = 2905 \text{ kJ/kg}$$

$$\text{Theoretical work done by turbine} = 3038 - 2100.73 = 937.27 \text{ kJ/kg}$$

$$\text{Actual steam consumption} = \frac{50000}{2 \times 3600} = 6.944 \text{ kg/s}$$

$$\text{Efficiency of turbine} = \frac{(\Delta h)_{\text{coupling}}}{(\Delta h)_{\text{isent}}}$$

$$(\eta)_{\text{turbine}} = \frac{4500}{6.944 \times 937.27} = 0.6914 = 69.14\%$$

We know that,

$$\dot{m}_f (\text{kg/kWh}) \times \text{LCV} (\text{kg/kg}) \times \eta_b = \dot{m}_{sa} (\text{kg/kWh}) \times \text{heat supplied (kJ/kg) of steam}$$

$$\dot{m}_f \times 29000 \times 0.92 = \frac{6.944 \times 1.05 \times 3600 \times 2905}{4500}$$

$$\dot{m}_f = \frac{6.944 \times 1.05 \times 3600 \times 2905}{4500 \times 0.92 \times 29000}$$

$$\dot{m}_f = 0.635 \text{ kg/kWh}$$

$$\text{Fuel cost} = \text{Rs.} \frac{0.635 \times 3500}{1000}$$

$$\text{Fuel cost} = \text{Rs.} 2.222/\text{kWh}$$

In actual steam power plant mass flow rate of steam will be 5% more than the mass flow rate required during testing.

