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ESE 2024 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 4

Section A : Control Systems [All Topics] + Engineering Mathematics [All Topics]

Section B : Electrical Circuits - 1 [Part Syllabus]

Section C : Digital Electronics - 1 [Part Syllabus] + Microprocessors - 1 [Part Syllabus]

ANSWER KEY

1.	(c)	**16.**	(a)	**31.**	(a)	**46.**	(d)	**61.**	(c)
2.	(d)	**17.**	(c)	**32.**	(b)	**47.**	(c)	**62.**	(b)
3.	(c)	**18.**	(b)	**33.**	(b)	**48.**	(b)	**63.**	(d)
4.	(c)	**19.**	(c)	**34.**	(b)	**49.**	(b)	**64.**	(d)
5.	(b)	**20.**	(a)	**35.**	(c)	**50.**	(b)	**65.**	(c)
6.	(d)	**21.**	(d)	**36.**	(d)	**51.**	(d)	**66.**	(b)
7.	(d)	**22.**	(d)	**37.**	(d)	**52.**	(b)	**67.**	(b)
8.	(d)	**23.**	(b)	**38.**	(c)	**53.**	(c)	**68.**	(d)
9.	(a)	**24.**	(a)	**39.**	(a)	**54.**	(a)	**69.**	(c)
10.	(a)	**25.**	(c)	**40.**	(a)	**55.**	(d)	**70.**	(b)
11.	(b)	**26.**	(b)	**41.**	(c)	**56.**	(c)	**71.**	(c)
12.	(c)	**27.**	(a)	**42.**	(d)	**57.**	(c)	**72.**	(c)
13.	(d)	**28.**	(c)	**43.**	(c)	**58.**	(d)	**73.**	(b)
14.	(a)	**29.**	(b)	**44.**	(c)	**59.**	(a)	**74.**	(c)
15.	(d)	**30.**	(b)	**45.**	(b)	**60.**	(d)	**75.**	(c)

DETAILED EXPLANATIONS
Section A : Control Systems + Engineering Mathematics

1. (c)

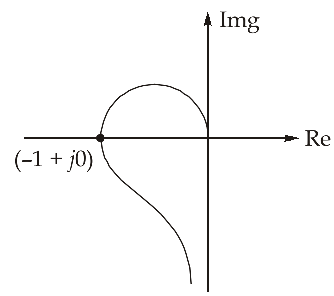
$$\text{Initial slope} = +40 \text{ dB/dec}$$

Hence two zeros at origin

At corner frequency 10 slope changing from +40 to –60 which indicates 5 poles at $s = -10$.

2. (d)

For a stable closed loop system, the gain at phase cross over frequency should be less than 1.



$$\text{Gain} < 20 \log 1 \text{ dB}$$

$\Rightarrow$ $$\text{Gain} < 0 \text{ dB}$$

3. (c)

$$G(s) = \frac{1}{(s+2)^2}$$

Closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{(s+2)^2+1} = \frac{1}{s^2+4s+5}$$

$\therefore$ $$\text{Closed loop poles} = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm j$$

4. (c)

Characteristic equation is

$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

The Routh array is

$$\begin{array}{c|cccc} s^6 & 1 & 8 & 20 & 16 \\ s^5 & 2 & 12 & 16 & \\ s^4 & 2 & 12 & 16 & \\ s^3 & 0 & 0 & 0 & \end{array}$$

Auxiliary polynomial

$$A(s) = s^4 + 6s^2 + 8$$

Solving for the roots of auxiliary polynomial

$$s^4 + 6s^2 + 8 = 0$$

$$(s^2 + 2)(s^2 + 4) = 0$$

$$s = \pm j\sqrt{2} \text{ and } \pm j2$$

These two pairs of roots one also the roots of the original characteristic equation. Thus, the characteristic equation has 4 roots on the imaginary axis of s-plane.

5. (b)

For a type-1 system, steady state error for

(i) step input = 0.

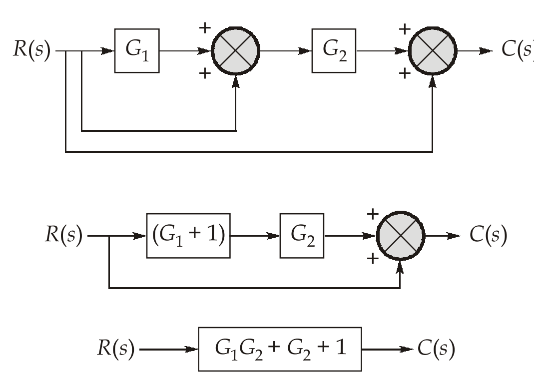
(ii) ramp input = $\dfrac{1}{K_v}$

$$K_v = \lim_{s\to 0} s\cdot G(s)H(s) = \lim_{s\to 0} s\cdot\frac{1}{s(0.5s+1)(0.2s+1)}$$

$$= \lim_{s\to 0}\frac{1}{(0.5s+1)(0.2s+1)} = 1$$

Hence, error, $e_{ss} = \dfrac{1}{K_v} = \dfrac{1}{1} = 1$

6. (d)



$$\frac{C(s)}{R(s)} = 1 + G_2 + G_1G_2$$

7. (d)

Output $Y(s)$ is given as, $Y(s) = \big[[R(s)-Y(s)]K + X(s)\big]\dfrac{1}{(s+1)}$

$$Y(s) + \frac{K}{(s+1)}Y(s) = \frac{KR(s)+X(s)}{(s+1)}$$

$$Y(s) = \frac{K[R(s)]}{(s+K+1)} + \frac{X(s)}{s+(K+1)}$$

Output due to disturbance, $Y_1(s)$

$$Y_1(s) = \frac{X(s)}{s+K+1}$$

Let $$X(s) = \frac{A}{s}$$

$$\lim_{t\to\infty} y_1(t) = \lim_{s\to 0} s\,Y_1(s) = \frac{A}{(K+1)}$$

As K increases, value of output due to disturbance decreases.

$\therefore$ K should be very high value.

8. (d)

$$T = \frac{C(s)}{R(s)} = \frac{\frac{5K}{s(s+1)}}{1+\frac{5K}{s(s+1)}} = \frac{5K}{s^2+s+5K}$$

Sensitivity with respect to K.

$$S_K^T = \frac{\partial T}{\partial K} \times \frac{K}{T}$$

$$= \frac{(s^2+s+5K)5 - 5K\times 5}{(s^2+s+5K)^2} \times \frac{K}{5K}(s^2+s+5K)$$

$$= \frac{s^2+s+5K-5K}{(s^2+s+5K)}$$

$$S_K^T = \frac{s^2+s}{s^2+s+5K}$$

given $s = j2$, $K = 1$

$$\left|S_K^T\right| = \left|\frac{-4+2j}{-4+2j+5}\right| = \left|\frac{-4+2j}{2j+1}\right|$$

$$\left|S_K^T\right| = 2$$

9. (a)

$$\begin{array}{l|lll}
s^5 & 2 & 4 & 2 \\
s^4 & 1 & 2 & 1 \\
s^3 & 4 & 4 & \\
s^2 & 1 & 1 & \\
s^1 & 2 & 0 & \\
s^0 & 1 & 0 &
\end{array}$$

$A(s) = s^4 + 2s^2 + 1$

$\frac{dA}{ds} = 4s^3 + 4s$

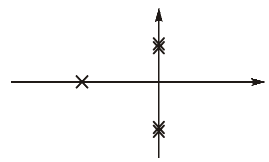
$A(s) = s^2 + 1$

$\frac{dA}{ds} = 2s$

There is no sign change. Therefore, no root lie in right hand side of s-plane.

But there are two rows which are zero completely. Hence, multiple roots are symmetrically located w.r.t origin.

As s^3 row is zero completely, hence 4 roots are symmetrically located.



Therefore, exactly one root lie in the left half of s-plane.

$\therefore$ Ans. is (a)

10. (a)

$$\frac{C(s)}{R(s)} = 1+\frac{8}{s(s+2)}\left[\frac{G_c(s)}{1+G_c(s)}\right]$$

$$= 1+\frac{8\times 2(s+4)}{s(s+2)[1+2(s+4)]}$$

For DC gain remove the poles and zeros at origin and put $s = 0$.

$$K = \left.\frac{s(s+2)[1+2(s+4)]+16(s+4)}{(s+2)[1+2(s+4)]}\right|_{s=0}$$

$$K = \frac{0+16(0+4)}{2(1+8)} = \frac{16\times 4}{2\times 9}$$

$$K = \frac{32}{9} = 3.55$$

$$K = 3.55$$

11. (b)

Using R-H criterion for characteristic equation:

$$s^2(s+2)(s+5)+K(2s+1) = 0$$

$$(s^3+2s^2)(s+5)+K(2s+1) = 0$$

$$s^4+5s^3+2s^3+10s^2+2sK+K = 0$$

$$s^4+7s^3+10s^2+2sK+K = 0$$

s^4	1	10	K
s^3	7	$2K$	0
s^2	$\frac{70-2K}{7}$	K	
s^1	$\frac{\left(\frac{70-2K}{7}\right)2K-7K}{\left(\frac{70-2K}{7}\right)}$		
s^0	K		

For stable system, $K > 0$,

$$\frac{70-2K}{7} > 0$$

$$K < 35$$

and $$\frac{\left(\frac{70-2K}{7}\right)2K - 7K}{\left(\frac{70-2K}{7}\right)} > 0$$

$$(70 - 2K)2K - 49K > 0$$

$$K < \frac{91}{4}$$

$$K < 22.75$$

∴ Range of K, $0 < K < 22.75$

12. (c)

The transfer function of second order system has the form

$$T(s) = \frac{K}{(s+1)(s+4)}$$

Put, $s = j\omega$ $$T(j\omega) = \frac{K}{(j\omega+1)(j\omega+4)}$$

$$\text{Phase } \phi = \angle T(j\omega) = -90° \text{ given}$$

∴ $$-90° = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

$$90° = \tan^{-1}\left[\frac{\omega + \frac{\omega}{4}}{1 - \frac{\omega^2}{4}}\right]$$

$$1 - \frac{\omega^2}{4} = 0$$

$$\omega^2 = 4$$

$$\omega = 2 \text{ rad/sec}$$

13. (d)

$$T(s) = C[(sI - A]^{-1}B$$

$$sI - A = \begin{bmatrix} s+3 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$\left(\text{Where } A = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)}\begin{bmatrix}(s+2) & 0\\ 0 & (s+3)\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{(s+3)} & 0\\ 0 & \frac{1}{(s+2)}\end{bmatrix}$$

$$[sI - A]^{-1}B = \begin{bmatrix}\frac{1}{(s+3)} & 0\\ 0 & \frac{1}{(s+2)}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{(s+3)}\\ \frac{1}{(s+2)}\end{bmatrix}$$

$$T(s) = C[(sI - A]^{-1}B = [1 \quad 0]\begin{bmatrix}\frac{1}{(s+3)}\\ \frac{1}{(s+2)}\end{bmatrix}$$

$$T(s) = \frac{1}{(s+3)}$$

14. (a)

$$[Q_c] = [B \quad AB]$$

Here, $A = \begin{bmatrix}0 & 1\\ 2 & -3\end{bmatrix}, B = \begin{bmatrix}0\\1\end{bmatrix}, C = [1 \quad 1]$

$$AB = \begin{bmatrix}1\\3\end{bmatrix}$$

$$[Q_c] = \begin{bmatrix}0 & 1\\ 1 & 3\end{bmatrix}$$

$$|Q_c| \neq 0$$

$\therefore$ System is controllable

$$[Q_0] = [C \quad CA]^T$$

$$[CA] = [1 \quad 1]\begin{bmatrix}0 & 1\\ 2 & -3\end{bmatrix} = [2 \quad -2]$$

$$[Q_0] = \begin{bmatrix}1 & 1\\ 2 & -2\end{bmatrix}$$

$$|Q_0| = -2 - 2 = -4 \neq 0$$

$\therefore$ System is observable.

15. (d)

All statements are correct.

16. (a)

$$N = P - Z_+$$

$$N = 0 \rightarrow \text{No encirclement of } 1 + j0$$

Also, $P = 0$

Therefore, $Z_+ = 0$

17. (c)

Here in the given problem, $\omega_{pc} = 4$ rad/sec, also $\omega_{gc} = 4$ rad/sec, because polar plot cuts the unity circle at $\omega_{pc} = \omega_{gc} = 4$ rad/sec, so given system is just stable or marginally stable.

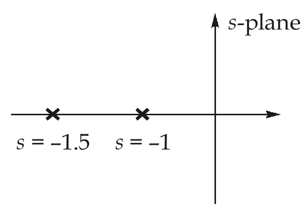
Also, G.M. = 0 dB

P.M. = 0°

18. (b)

It is a lag compensator, which reduces the bandwidth.

Pole zero plot



19. (c)

Settling, $t_s = \dfrac{4}{\xi\omega_n} = 7$

$$\xi\omega_n = \frac{4}{7}$$

Peak time, $t_p = \dfrac{\pi}{\omega_d}$

$$\omega_d = \frac{\pi}{t_p} = \frac{\pi}{2}$$

∴ System poles are

$$s = -\xi\omega_n \pm j\omega_d$$

$$s = -0.571 \pm j1.571$$

20. (a)

Given system is controllable canonical form

$$\frac{C(s)}{R(s)} = \frac{b(C_1 s + C_0)}{s^2 + a_1 s + a_0}$$

$$A = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}; \qquad B = \begin{bmatrix} 0 \\ b \end{bmatrix}; C = \begin{bmatrix} C_0 & C_1 \end{bmatrix}$$

$$\frac{C(s)}{R(s)} = \frac{1(s+2)}{s^2+4s+9}$$

$$A = \begin{bmatrix} 0 & 1 \\ -9 & -4 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

Alternate Solution:

$$\frac{C(s)}{R(s)} = C[sI - A]^{-1} B$$

Let, $$C = \begin{bmatrix} a & b \end{bmatrix}$$

Then, $$\frac{s+2}{s^2+4s+9} = \begin{bmatrix} a & b \end{bmatrix} \cdot \frac{1}{s^2+4s+9} \begin{bmatrix} s+4 & 1 \\ 9 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{s+2}{s^2+4s+9} = \frac{1}{s^2+4s+9} \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$\frac{s+2}{s^2+4s+9} = \frac{a+bs}{s^2+4s+9}$$

On comparing,

$$a = 2, b = 1$$

Therefore, $$C = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

21. (d)

$$\int_0^a \frac{x^7 dx}{\sqrt{a^2 - x^2}} dx \qquad \text{Put, } x = a \sin\theta$$

$$dx = a \cos\theta d\theta$$

when, $$x = 0, \quad \theta = 0$$

when, $$x = a, \quad \theta = \frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}} \frac{a^7 \sin^7\theta}{a\cos\theta} \cdot a\cos\theta \cdot d\theta = a^7 \int_0^{\frac{\pi}{2}} \sin^7\theta \, d\theta$$

By using gamma function

$$a^7 \frac{6 \cdot 4 \cdot 2}{7 \cdot 5 \; 3 \cdot 1} = \frac{16}{35} a^7$$

22. (d)

Given,

$$r \sin\theta d\theta + (r^3 - 2r^2 \cos\theta + \cos\theta)dr = 0$$

Let, $$M = r\sin\theta \text{ and } N = r^3 - 2r^2\cos\theta + \cos\theta$$

$$\frac{\partial M}{\partial r} = \sin\theta$$

$$\frac{\partial N}{\partial \theta} = 2r^2 \sin\theta - \sin\theta$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial \theta} - \frac{\partial M}{\partial r}\right) = 2\left(r - \frac{1}{r}\right)$$

Therefore, I.F. if

$$\text{I.F.} = e^{\int 2\left(r-\frac{1}{r}\right)\cdot dr} = \frac{e^{r^2}}{r^2}$$

23. (b)

$$\int_{-\infty}^{\infty} e^{-ax^2}\cdot dx = \int_{-\infty}^{\infty} e^{-(\sqrt{a}\,x)^2}$$

Let, $\sqrt{a}x = t$

$$dx = \frac{dt}{\sqrt{a}}$$

$$\int_{-\infty}^{\infty} e^{-(\sqrt{a}\,x)^2} dx = \int_{-\infty}^{\infty} e^{-t^2}\frac{dt}{\sqrt{a}} = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} e^{-t^2} dt$$

$$\frac{\sqrt{\pi}}{\sqrt{a}} = \sqrt{\frac{\pi}{a}}$$

24. (a)

Here A is 3 × 3 matrix.

So, $$\text{Rank } A = \text{Rank }(\alpha V_1 V_1^T + \beta V_1 V_2^T)$$
$$\leq \text{Rank }(\alpha V_1 V_1^T) + \text{Rank }(\beta V_2 V_2^T)$$
$$\leq 1 + 1 \text{ (because min rank is 1)}$$
$$\leq 2$$

and matrix is of 3 × 3.

So, one of eigen value must be 0.

25. (c)

$$f(x) = 8\ln x - x^2 + 3$$

$$f'(x) = \frac{8}{x} - 2x = \frac{8-2x^2}{x}$$

$$8 - 2x^2 = 0 \quad \text{so,} \quad x = \pm 2$$

$$f(1) = 8\ln 1 - 1 + 3 = 2$$

$$f(2) = 8\ln 2 - 4 + 3 = 4.54$$

$$f(e) = 8 \ln e - e^2 + 3 = 3.61$$

So, maximum value of $f(x)$ at $x = 2$

26. (b)

Biggest possible circle that can be inscribed in the given square would be touching all the four sides of square.

$$\text{Required probability} = \frac{\text{Area of circle}}{\text{Area of square}}$$

$$= \frac{\pi(5\times 5)}{(10\times 10)} = \frac{\pi}{4} = 0.785$$

27. (a)

$$y^2 + z^2 = 4x, \quad x = 5$$

$$V = \int_0^5 dx \int_{-2\sqrt{x}}^{2\sqrt{x}} dy \int_{-\sqrt{4x-y^2}}^{\sqrt{4x-y^2}} dz \Rightarrow 4\int_0^5 dx \int_0^{2\sqrt{x}} dy \int_0^{\sqrt{4x-y^2}} dz$$

$$\Rightarrow \quad 4\int_0^5 dx \int_0^{2\sqrt{x}} dy [z]_0^{\sqrt{4x-y^2}} \Rightarrow 4\int_0^5 dx \int_0^{2\sqrt{x}} dy \sqrt{4x-y^2}$$

$$\Rightarrow \quad 4\int_0^5 dx \left[\frac{y}{2}\sqrt{4x-y^2} + \frac{4x}{2}\sin^{-1}\frac{y}{2\sqrt{x}}\right]_0^{2\sqrt{x}}$$

$$\Rightarrow \quad 4\int_0^5 \left[0 + 2x\left(\frac{\pi}{2}\right)\right] dx \Rightarrow 4\pi\int_0^5 x\,dx \Rightarrow 4\pi\left[\frac{x^2}{2}\right]_0^5 = 50\pi$$

28. (c)

For poisson random variable

$$P(X = x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}$$

$$P(X = 0) = P(X = 2)$$

$$e^{-\lambda} = \frac{e^{-\lambda}\cdot\lambda^x}{2!}$$

$$\Rightarrow \quad \lambda = \sqrt{2}$$

$$P(X \le 1.3) = P(X = 0) + P(X = 1)$$

$$= e^{-\sqrt{2}} + e^{-\sqrt{2}}(\sqrt{2})^1$$

$$= e^{-\sqrt{2}}\left(\sqrt{2}+1\right)$$

29. (b)

$$\vec{T} = \vec{r} \times \vec{F}$$

$$\vec{T} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 6 \end{vmatrix} = (18-1)\hat{i} - (12+1)\hat{j} + (-2-3)\hat{k}$$

$$= 17\hat{i} + 13\hat{j} - 5\hat{k}$$

$$|\vec{T}| = \sqrt{17^2 + 13^2 + 5^2} = 21.97 \text{ N-m}$$

30. (b)

Applying L Hospital rule:

$$\lim_{x\to 0} \frac{ae^{ax} + ae^{-ax}}{\frac{b}{1+bx}}, \text{ By putting } x = 0$$

$$\frac{ae^0 + ae^0}{b} = \frac{a+a}{b} = \frac{2a}{b}$$

31. (a)

$$\text{Rank} \leq \min(A, B)$$

$$\min(2, 3) = \text{Highest possible rank} = 2$$

If rank of A = 2, it will consistent.

But in order to inconsistent, maximum rank of A is '1'.

32. (b)

A is skew symmetric,

Then, $A = -A^T$

So, $(A \cdot A)^T = A^T \cdot A^T = (-A) \cdot (-A) = A \cdot A$

$A \cdot A$ is symmetric matrix.

33. (b)

Since, P is very small, so will make the use of Poission's distribution

$$\lambda = nP = \frac{1}{100} \times 100 = 1$$

$$P(X \geq 2) = 1 - \{P(X=0) + P(X=1)\}$$

$$P(X = 0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\lambda}$$

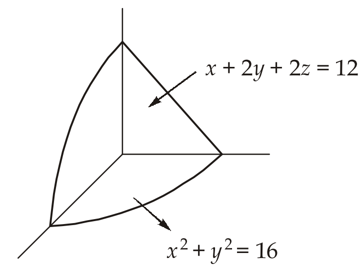
$$P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-\lambda} \cdot \lambda$$

$$P(X \geq 2) = 1 - e^{-1} - e^{-1} = \frac{e-2}{e}$$

34. (b)

$$x + 2y + 2z = 12$$

Rewriting equation of plane



$$\Rightarrow \quad z = \frac{12 - x - 2y}{2}$$

We have, $$z_x = \frac{-1}{2}, \quad z_y = -1$$

$$\text{Surface area} = \iint_R \sqrt{1 + z_x^2 + z_y^2}\, dx\, dy$$

$$= \iint \sqrt{1 + 1 + \frac{1}{4}}\, dx\, dy$$

$$\frac{3}{2}\iint dx\, dy \Rightarrow \text{in Polar coordinates} = \frac{3}{2}\int_0^{\frac{\pi}{2}}\int_0^4 r\, dr\, d\theta$$

$$= \frac{3}{2}\int_0^{\frac{\pi}{2}}\left[\frac{r^2}{4}\right]_0^4 d\theta \Rightarrow \frac{3}{2}\int_0^{\frac{\pi}{2}} 2 \times 4\, d\theta$$

$$\Rightarrow \quad 2 \times \frac{3}{2} \times \frac{4\pi}{2} = 6\pi$$

35. (c)

$$E(X) = nP$$

$$n = 100$$

$$P = P(3) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$E(X) = \frac{100}{3} = 33.33$$

36. (d)

$$I = \int_{-2}^{2} \left|1 - x^4\right| dx$$

The given function is an even function i.e.

$$f(x) = f(-x)$$

$$I = 2\int_0^2 \left|1 - x^4\right| \cdot dx$$

$$= 2\left\{\int_0^1 (1 - x^4)\, dx + \int_1^2 (x^4 - 1)\, dx\right.$$

$$= 2\left\{\left[x-\frac{x^5}{5}\right]_0^1+\left[\frac{x^5}{5}-x\right]_1^2\right\}=2\left\{1-\frac{1}{5}+\frac{32}{5}-2-\frac{1}{5}+1\right\}$$

$$I = 12$$

37. (d)

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$

$$|A| \cdot (A^{-1}) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}$$

$$= \frac{2}{-6}, \frac{2}{3}$$

$$= -0.33,\ 0.67$$

38. (c)

Given equation is,

$$\frac{d^2y}{dx^2}+p\frac{dy}{dx}+qy = 0$$

$$(D^2 + pD + q)y = 0$$

$$\therefore \quad D^2 + pD + q = 0$$

Its solution is $y = c_1e^{-x} + c_1e^{-3x}$

So the roots of $D^2 + pD + q = 0$ are $\alpha = -1$ and $\beta = -3$

sum of roots $= -p = -1 - 3$

$$p = 4$$

Product of roots $= q = (-1)(-3) = 3$

$$(D^2 + 4D + 4)y = 0$$

$$(D + 2)^2 = 0$$

$$D = -2, -2$$

$$y = (c_1x + c_2)e^{-2x}$$

$$y = c_1x\, e^{-2x} + c_2e^{-2x}$$

39. (a)

$$(D^2 + 2D + 1)y = 0$$

$$\text{C.F.} = (c_1 + c_2x)e^{-x}$$

$$y = c_1e^{-x} + c_2xe^{-x}$$

$$y(0) = 1,$$

$$1 = c_1$$

$$y' = -c_1e^{-x} + c_2(e^{-x} - x\,e^{-x})$$

$$y'(0) = -1, -1 = -c_1 + c_2$$
$$c_1 = 1 \text{ and } c_2 = 0$$
$$\therefore \quad y = e^{-x}$$
$$\text{At } x = 2, \quad y = e^{-2}$$

40. (a)

Its solution is of the type $u = f(x)$, i.e. dependent variable is u.

Hence equation is linear and non-homogeneous.

41. (c)

$$x = e^{-x}$$
$$f(x) = x - e^{-x} = 0$$
$$f'(x) = 1 + e^{-x}$$
$$x_{K+1} = x_K - \frac{f(x_K)}{f'(x_K)} = x_K - \frac{x_K - e^{-x_K}}{1+e^{-x_K}}$$
$$= \frac{e^{-x_K} + x_K - x_K + x_K e^{-x_K}}{1+e^{-x_K}}$$
$$= (1+x_K)\frac{e^{-x_K}}{1+e^{-x_K}}$$

42. (d)

$$z^2 + 4 = 0$$
$$\Rightarrow \quad z^2 = -4$$
$$\Rightarrow \quad z = \pm 2i$$

43. (c)

$$\frac{\sin z}{z^8} = \frac{z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \frac{z^9}{9!} + \ldots}{z^8}$$
$$= \frac{1}{z^7} - \frac{1}{3!z^5} + \frac{1}{5!z^3} - \frac{1}{7!z} + \frac{1}{9!} \ldots$$
$$\operatorname*{Res}_{z\to 0}\frac{\sin z}{z^8} = \text{Coefficient of } \frac{1}{z} = \frac{-1}{7!}$$

44. (c)

The number of sign changes of the elements of the first column of Routh array represents the number of roots of characteristic equation lying to the right side of $s = 0$ axis.

45. (b)

Transfer function having zero in the right hand side of s-plane are called non-minimum phase transfer function. Statement-II is also true but not the correct explanation of statement-I.

Section B : Electrical Circuits-1

46. (d)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{28\times 28}{4\times 20}$$

$$= \frac{7\times 14}{10} = 9.80 \text{ W}$$

47. (c)

Slope of the characteristic is negative between $-\infty$ to 0.

So characteristic is of active element.

48. (b)

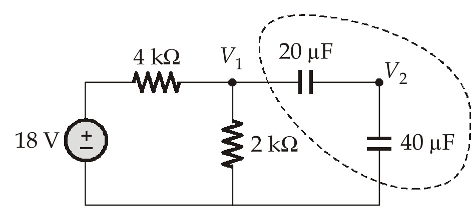
Voltage across the capacitor,

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} I\,dt$$

Hence on integrating square wave, we will get a triangular wave.

49. (b)

At steady state both capaciters will act as open circuit



By applying voltage divider rule

The voltage, $V_1 = 18\times\frac{2}{4+2} = 6 \text{ V}$

Therefore, $V_2 = \frac{20}{20+40}\times 6 = \frac{1}{3}\times 6 = 2\text{ V}$

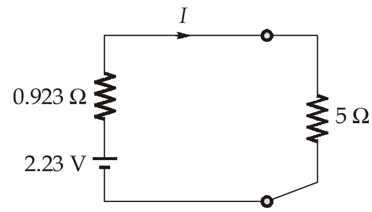
50. (b)

Using Millman's theorem, the equivalent voltage is given by

$$V_{eq} = \frac{V_1G_1 + V_2G_2 - V_3G_3}{G_1+G_2+G_3} = \frac{3\times\frac{1}{2}+8\times\frac{1}{3}-7\times\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 2.23 \text{ V}$$

$$R_{eq} = \frac{1}{G} = \frac{1}{G_1+G_2+G_3} = \frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 0.923\ \Omega$$

The equivalent circuit is shown in figure



$$I = \frac{V_{eq}}{R_{eq}+R_L} = \frac{2.23}{0.923+5} = 0.3765 \text{ A}$$

51. (d)

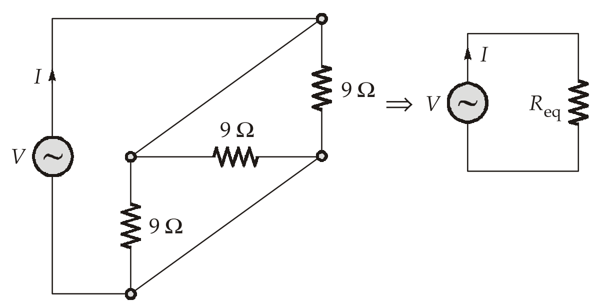
We have, $V = 6\cos 0.5 \times 10^6 t$ and $\omega = 0.5 \times 10^6$ rad/s

$L = 1$ mH $\Rightarrow X_L = \omega L = 0.5 \times 10^6 \times 10^{-3} = 0.5 \times 10^3\ \Omega$

$C = 4$ nF $\Rightarrow X_C = \dfrac{1}{\omega C} = \dfrac{1}{0.5\times10^6\times4\times10^{-9}} = 0.5\times10^3\ \Omega$

$\therefore \quad X_C = X_L$

Equivalent circuit will be,



$$R_{eq} = 9\,\Omega \| 9\,\Omega \| 9\,\Omega = 3\,\Omega$$

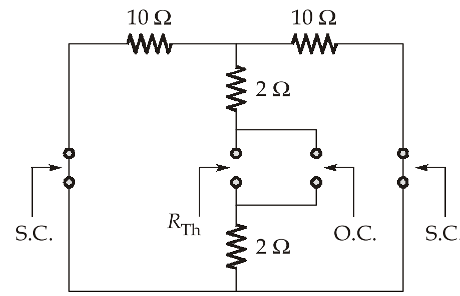
$$\therefore \quad I = \frac{V}{R_{eq}} = \frac{6\cos 0.5\times10^6 t}{3} = 2\cos 0.5\times10^6 t \text{ A}$$

52. (b)

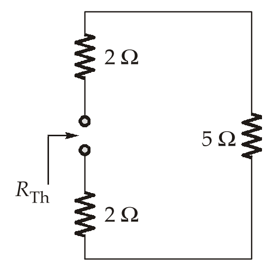
Calculation of Thevenin's Resistance:

- Replace independent voltage source with short circuit.
- Replace independent current source with open circuit.
- Remove load resistor across which Thevenin's resistance is to be calculated.

Now, equivalent circuit



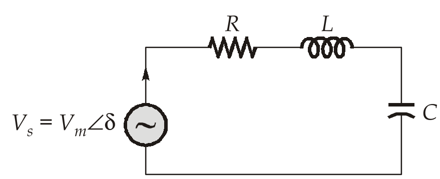
- 10 Ω is in parallel with 10 Ω.



$\Rightarrow$ $$R_{Th} = 5 + 2 + 2 = 9\ \Omega$$

53. (c)

In series RLC circuit,



At resonance condition, $I_m = \dfrac{V_m}{R}$

At lower cut-off frequency,

$$I = \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{V_m}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} = \frac{V_m}{\sqrt{2}R}$$

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

At lower cut off frequency

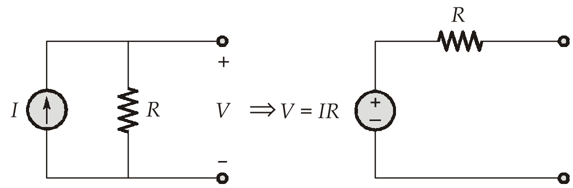
$$\omega L - \frac{1}{\omega C} = -R$$

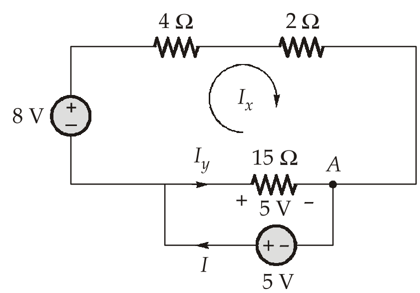
Power factor angle, $\phi = \tan^{-1}\left(\dfrac{\omega L - \dfrac{1}{\omega C}}{R}\right) = \tan^{-1}(-1) = -45°$

Power factor = $\cos\phi = \cos(-45°) = 0.707$ leading

54. (a)

From the source transformation theorem,





$$I_x = \frac{8+5}{6} = \frac{13}{6}\text{ A}$$

$$I_y = \frac{5}{15} = \frac{1}{3}\text{ A}$$

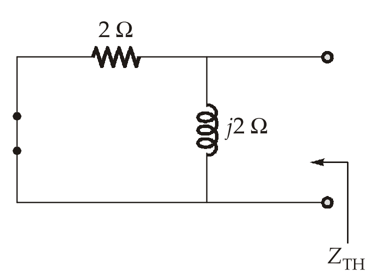
KCL at node A: $I_x + I_y = I$

$$\frac{13}{6} + \frac{1}{3} = I$$

$$I = \frac{13+2}{6} = \frac{15}{6} = \frac{5}{2} = 2.5\text{ Amp}$$

55. (d)

Here, R_L is a resistive load so, for maximum power transfer to R_L.



$$R_L = |Z_{TH}|$$

$$Z_{TH} = 2 \,||\, (j2)$$

$$= \frac{2 \times j2}{2 + j2} = \frac{j2}{1+j}$$

$$= \frac{j2(1-j)}{(1)^2 - (j)^2} = 1 + j\,\Omega$$

$$Z_{TH} = \sqrt{2}\angle 45°\,\Omega$$

$$R_L = |Z_{TH}| = \sqrt{2}\,\Omega$$

56. **(c)**

- Reciprocity theorem is based on the symmetry of impedance or admittance matrix.
- Superposition theorem is also applicable for the circuit having initial conditions.

57. **(c)**

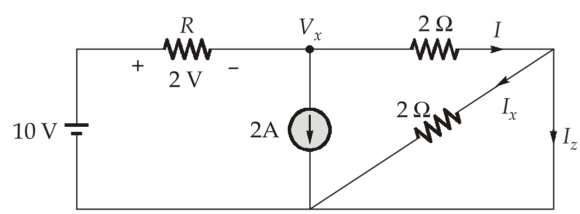
For maximum power transfer to the load, the source resistance must be minimum i.e. zero

$$R = 0\ \Omega$$

58. **(d)**

With D.C. source, inductor behaves as short circuit and capacitor behaves as open circuit at steady state.

∴ The simplified circuit can be

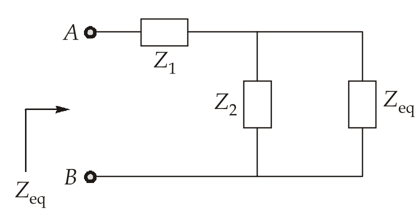


Using KVL, $V_x = 8$ V

$$I = \frac{V_x}{2} = 4 \text{ Amp}$$

$$I = I_x + I_z = I_z \qquad ...[\because I_x = 0]$$

59. **(a)**



$$Z_1 = j16\ \Omega - j7\ \Omega = j9\ \Omega$$

$$Z_2 = j6 - j2 = j4\ \Omega$$

$$Z_{eq} = j9 + \frac{j4Z_{eq}}{j4 + Z_{eq}}$$

Only option (a) satisfied this condition

$$Z_{eq} = j12\ \Omega$$

60. **(d)**

Reciprocity theorem is applicable to a network:

1. Containing R, L and C elements.
2. Should be initially relaxed system.
3. With only independent source present.

Section C : Digital Electronics - 1 + Microprocessors - 1

61. **(c)**

From the given figure, we can conclude that

$$Y_0 = X_1 \odot X_0$$
$$Y_1 = X_1 \odot X_1 = 1 \text{ always}$$
$$Y_2 = X_2 \odot X_1$$

Now for given input,

$$X_2 = 0,\ X_1 = 0,$$
$$X_0 = 1$$
$$Y_0 = 0,\ Y_1 = 1$$
$$Y_2 = 1$$

So, $(Y_2Y_1Y_0) = (110)_2$

62. **(b)**

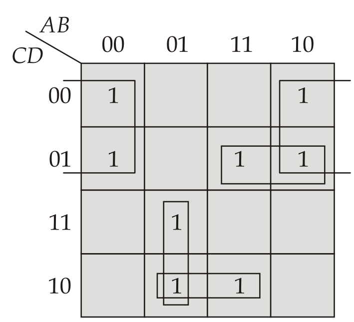
The MOD of the counter is 2^8. So, after 256 pulse the cycle repeats or after 1024 clock pulses the counter's count is 0.

The state of the counter at 1028 is 1032 – 1024 = 8

$\therefore$ The state is $(00001000)_2 = (8)_{10}$

63. **(d)**

From given expression, *K*-map is



Hence, no. of essential prime implicant are $\bar{A}BC, BC\bar{D}, A\bar{C}D$.

64. **(d)**

Base-6 number system

The number present are

$$0 \rightarrow 0\,0\,0$$
$$1 \rightarrow 0\,0\,1$$
$$2 \rightarrow 0\,1\,0$$
$$3 \rightarrow 1\,0\,1$$
$$4 \rightarrow 1\,1\,0$$
$$5 \rightarrow 1\,1\,1$$

As the code is given to be self complementory, so code of 0 and 5 are complementory of each other,

Therefore, code of 4 = 1 1 0

65. (c)

From the MUX,

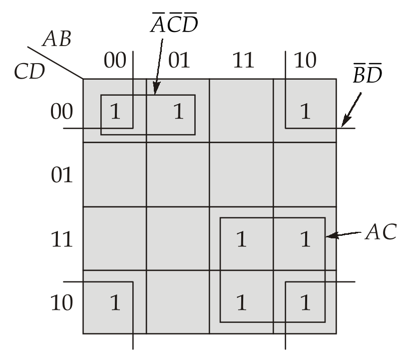
$$Y = I_0\bar{S}_1\bar{S}_0 + I_1\bar{S}_1S_0 + I_2S_1\bar{S}_0 + I_3S_1S_0$$

$$= A\bar{B} + \bar{B}\cdot B$$

$$Y = A\bar{B}$$

66. (b)

The *K*-map can be grouped as



$$F = \bar{A}\bar{C}\bar{D} + \bar{B}\bar{D} + AC$$

67. (b)

∵ We know that,

$$X \odot X = 1$$

$$X \odot \bar{X} = 0$$

The output of first X-NOR is

$$0 \odot X = \bar{X}$$

Now, the output of second XNOR gate would be

$$\bar{X} \odot X = 0$$

For 10 such X-NOR gates in cascade, the final output,

$$Y = 0$$

68. (d)

RAM is volatile memory i.e. data are lost when power goes off.

69. (c)

TRAP → is a interrupt

HLDA → is in response to HOLD signal sent from Mp to DMA controller for direct memory access.

RESET $\rightarrow$ To initialize microprocessor from (0000)H

ALE $\rightarrow$ address latch enable, to demultiplex address bus and data bus.

70. **(b)**

END is an assembler directive/pseudo instruction.

71. **(c)**

I/O and power failure are related to external interrupts like DMA and TRAP.

72. **(c)**

MVI A, 52 H	||	$A \leftarrow 52$ H
MOV B, A	||	$B \leftarrow 52$ A
STC	||	Set the carry ; CY = 1
CMC	||	Complement the carry i.e. CY = 0
RAR	||	Rotate accumulator right through carry

CY = 0

$A \rightarrow$ $\boxed{0\,1\,0\,1\ \ 0\,0\,1\,0}$

$A \rightarrow$ $\boxed{0\,0\,1\,0\ \ 1\,0\,0\,1}$ and CY = 0

XRA B || Perform EX-OR operation with B and store in A

$$\begin{array}{r l} A \rightarrow & 0010\ 1001 \\ B \rightarrow & 0101\ 0010 \\ \hline & 0111\ 1011 \\ \hline \end{array}$$

$A \rightarrow$ 7B H

73. **(b)**

SIM can mask RST 5.5, RST 6.5 and RST 7.5 and also can set SOD pin (serial output data line).

74. **(c)**

$$F = A \oplus B \oplus C = \sum m(1, 2, 4, 7)$$

To determine the inputs a table is constructed as shown below:

	I_0 $\bar{A}\bar{B}$	I_1 $\bar{A}B$	I_2 $A\bar{B}$	I_3 AB
$\bar{C}$	0	2	4	6
C	1	3	5	7
	C	$\bar{C}$	$\bar{C}$	C

Hence, $I_0 = I_3 = C$

$I_2 = I_1 = \bar{C}$

75. (c)

By using the combination of NAND and NOR gates, we can realize any Boolean function but not with the minimum number of logic gates. Hence. Statement(II) is incorrect.

❍❍❍❍