GATE 2023

INSTRUMENTATION ENGINEERING

Questions & Solutions

Exam held on 11th Feb, 2023 Forenoon Session

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SECTION - A GENERAL APTITUDE

Q.1 The village was nestled in a green spot, _______ the ocean and the hills.

(a) through (b) in

(c) at (d) between

Ans. (d)

End of Solution

Q.2 Disagree : Protest : : Agree : _______.

(By word meaning)

(a) Refuse (b) Pretext

(c) Recommend (d) Refute

Ans. (c)

End of Solution

Q.3 A 'frabjous' number is defined as a 3 digit number with all digits odd, and no two adjacent digits being the same. For example, 137 is a frabjous number, while 133 is not. How many such frabjous numbers exist?

(a) 125 (b) 720

(c) 60 (d) 80

Ans. (d)

End of Solution

Q.4 Which one among the following statements must be TRUE about the mean and the median of the scores of all candidates appearing for GATE 2023?

(a) The median is at least as large as the mean.

(b) The mean is at least as large as the median.

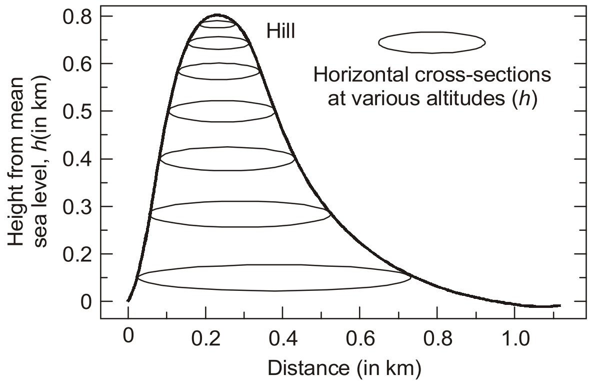
(c) At most half the candidates have a score that is larger than the median.

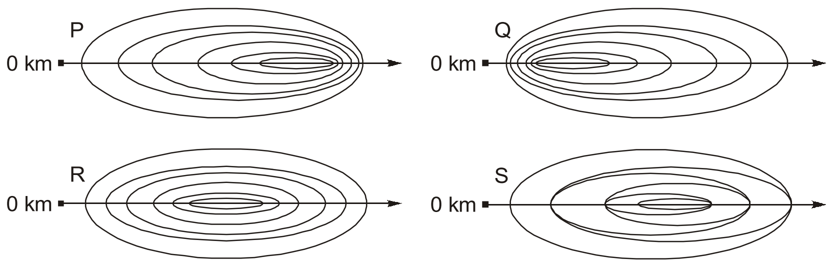
(d) At most half the candidates have a score that is larger than the mean.

Ans. (c)

End of Solution

Q.5 In the given diagram, ovals are marked at different heights (h) of a hill. Which one of the following options P, Q, R, and S depicts the top view of the hill?





(a) P
(b) Q
(c) R
(d) S

Ans. (b)

End of Solution

Q.6 Residency is a famous housing complex with many well-established individuals among its residents. A recent survey conducted among the residents of the complex revealed that all of those residents who are well established in their respective fields happen to be academicians. The survey also revealed that most of these academicians are authors of some best-selling books.

Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(a) Some residents of the complex who are well established in their fields are also authors of some best-selling books.
(b) All academicians residing in the complex are well established in their fields.
(c) Some authors of best-selling books are residents of the complex who are well established in their fields.
(d) Some academicians residing in the complex are well established in their fields.

Ans. (d)

End of Solution

Q.7 Ankita has to climb 5 stairs starting at the ground, while respecting the following rules:

1. At any stage, Ankita can move either one or two stairs up.
2. At any stage, Ankita cannot move to a lower step.

Let $F(N)$ denote the number of possible ways in which Ankita can reach the N^{th} stair For example, $F(1)=1$, $F(2)= 2$, $F(3) = 3$.

The value of $F(5)$ is _______.

(a) 8
(b) 7
(c) 6
(d) 5

Ans. (a)

End of Solution

Q.8 The information contained in DNA is used to synthesize proteins that are necessary for the functioning of life. DNA is composed of four nucleotides: Adenine (A), Thymine (T), Cytosine (C), and Guanine (G). The information contained in DNA can then be thought of as a sequence of these four nucleotides: A, T, C, and G. DNA has coding and non-coding regions. Coding regions—where the sequence of these nucleotides are read in groups of three to produce individual amino acids—constitute only about 2% of human DNA. For example, the triplet of nucleotides CCG codes for the amino acid glycine, while the triplet GGA codes for the amino acid proline. Multiple amino acids are then assembled to form a protein.

Based only on the information provided above, which of the following statements can be logically inferred with certainty?

(i) The majority of human DNA has no role in the synthesis of proteins.
(ii) The function of about 98% of human DNA is not understood.

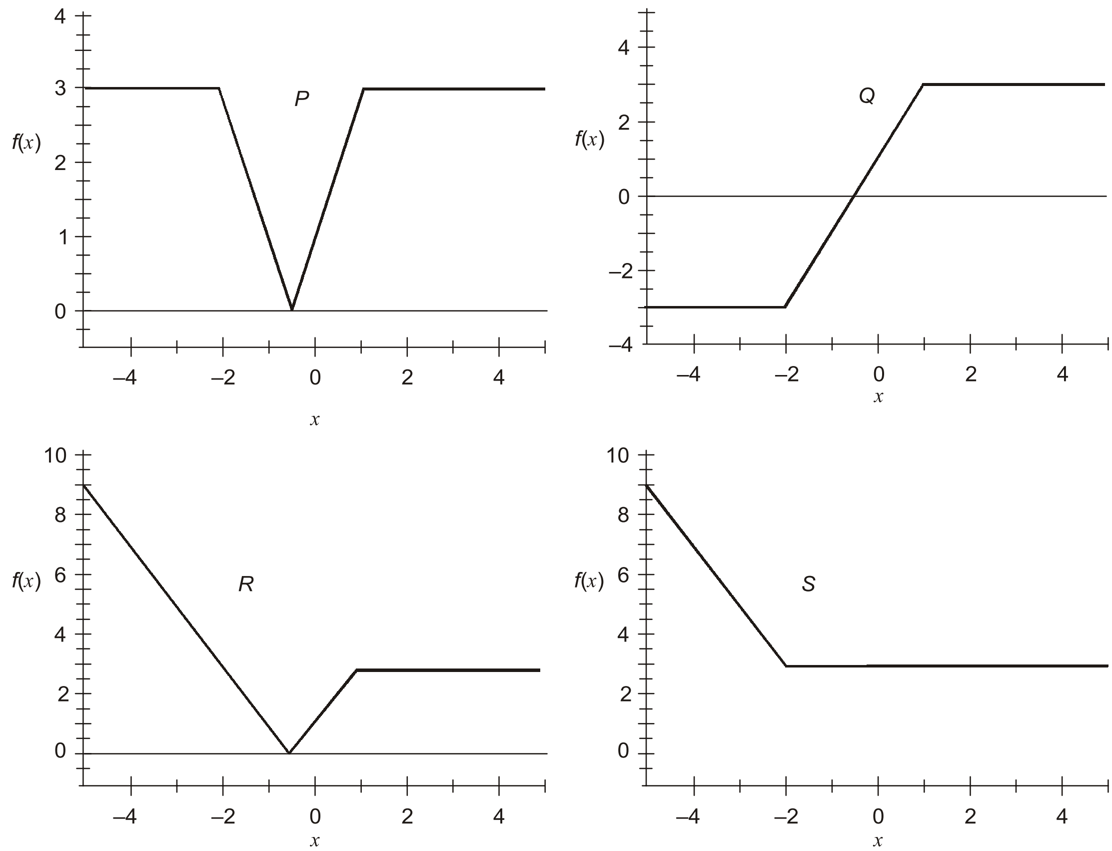
(a) only (i)
(b) only (ii)
(c) both (i) and (ii)
(d) neither (i) nor (ii)

Ans. (d)

End of Solution

Q.9 Which one of the given figures *P*, *Q*, *R* and *S* represents the graph of the following function?

$$f(x) = \left| |x + 2| - |x - 1| \right|$$

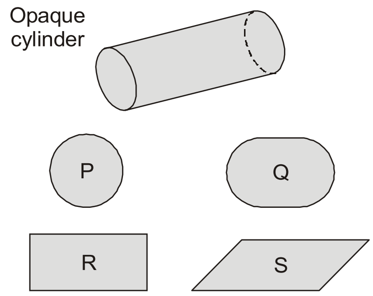


(a) P
(b) Q
(c) R
(d) S

Ans. (a)

End of Solution

Q.10 An opaque cylinder (shown below) is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The cylinder can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows *P*, *Q*, *R*, and *S* is NOT possible?



(a) P
(b) Q
(c) R
(d) S

Ans. (d)

End of Solution

■■■■

SECTION - B TECHNICAL

Q.11 Choose solution set S corresponding to the systems of two equations

$x - 2y + z = 0$

$x - z = 0$

Note: . R denotes the set of real numbers

(a) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

(b) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(c) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(d) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

Ans. **(a)**

Given : $x - 2y + z = 0$

$x - z = 0$

Solution is given by solving also equations.

	x		y		z	
–2		1		1		–2
0		–1		1		0

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{2} = K$$

$$\Rightarrow \quad x = K,\ y = K,\ z = K$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ K \in R$$

End of Solution

Q.12 Inductance of a coil is measured as 10 mH, using an LCR meter, when no other objects are present near the coil. The LCR meter uses a sinusoidal excitation at 10 kHz. If a pure copper sheet is brought near the coil, the same LCR meter will read ____________.

(a) less than 10 mH
(b) 10 mH
(c) more than 10 mH
(d) less than 10 mH initially and then stabilizes to more than 10 mH

Ans. (a)

For LCR meter,

$$\text{frequency, } f = \frac{1}{2\pi\sqrt{LC}}$$

$$L = \frac{1}{(2\pi)^2 f^2 C}$$

where,

$$C = \frac{\in A}{d} = \frac{\in_0 \in_r A}{d}$$

$$L \propto \frac{1}{\in_r}$$

by introducing copper sheet near the coil, inductor L reduces

$$\frac{L_1}{L_2} = \frac{\in_{r_1}}{\in_{r_2}}$$

$$L_2 = \frac{L_1}{\in_{r_2}} = \frac{10 \text{ mH}}{\in_{r_2}}$$

Since, $\in_{r_2} > 1$ for copper.

$\Rightarrow \quad L_2 < 10$ mH

End of Solution

Q.13 Which of the following flow meters offers the lowest resistance to the flow?

(a) Turbine flow meter
(b) Orifice flow meter*
(c) Venturi meter
(d) Electromagnetic flow meter

Ans. (d)

Electromagnetic flow meter offers lowest resistance to the flow.

End of Solution

Q.14 Pair the quantities (p) to (s) with the measuring devices (i) to (iv).

(i) Linear Variable Differential Transformer (LVDT)	(p) Torque
(ii) Thermistor	(q) Pressure
(iii) Strain gauge	(r) Linear position
(iv) Diaphragm	(s) Temperature

(a) (i) - (r), (ii) - (s), (iii) - (q), (iv) - (p)
(b) (i) - (p), (ii) - (s), (iii) - (r), (iv) - (q)
(c) (i) - (r), (ii) - (s), (iii) - (p), (iv) - (q)
(d) (i) - (q), (ii) - (s), (iii) - (p), (iv) - (r)

Ans. (c)

End of Solution

Q.15 Capacitance '*C*' of a parallel plate structure is calculated as 20 pF using $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ where ε_0 is the permittivity of free space, ε_r is the relative permittivity of the dielectric, *A* is the overlapping area of the electrodes and *d* is the distance between them. The value of *C* is then measured using an LCR meter. If the meter is assumed to be ideal and it introduces no error due to cable capacitance, which one of the following readings is likely to be correct?

(a) 20.5 pF
(b) 20 pF
(c) 19.5 pF
(d) 10 pF

Ans. (a)

End of Solution

Q.16 The table shows the present state $Q(t)$, next state $Q(t+1)$, and the control input in a flip-flop. Identify the flip-flop.

$Q(t)$	$Q(t+1)$	Input
0	0	0
0	1	1
1	0	1
1	1	0

(a) T flip-flop
(b) D flip-flop
(c) SR flip-flop
(d) JK flip-flop

Ans. **(a)**

When, Input = 0

$Q(t+1) = Q(t)$

When, Input = 1

$Q(t+1) = \bar{Q}(t)$

Here, the output toggle for input = 1

So, it is a T-flip-flop.

End of Solution

Q.17 Match the Exclusive-OR (XOR) operations (i) to (iv) with the results (p) to (s), where X is a Boolean input.

(i) $X \oplus X$	**(p)** 1
(ii) $X \oplus \bar{X}$	**(q)** 0
(iii) $X \oplus 0$	**(r)** $\bar{X}$
(iv) $X \oplus 1$	**(s)** X

(a) (i) - (q), (ii) - (r), (iii) - (s), (iv) - (p)
(b) (i) - (q), (ii) - (r), (iii) - (p), (iv) - (s)
(c) (i) - (p), (ii) - (s), (iii) - (q), (iv) - (r)
(d) (i) - (q), (ii) - (p), (iii) - (s), (iv) - (r)

Ans. **(d)**

Truth table for EXOR gate

Inputs A	Inputs B	Output Y	
0	0	0	→ Same inputs
1	1	1	← Different inputs
1	0	1	← Different inputs
1	1	0	→ Same inputs

Inputs A	Inputs B	Output Y
0	0 →	0
0	1 →	1
1	0	1
1	1	0

If one input of EXOR gate is 0, then the output is same as second input.

Inputs A	Inputs B	Output Y
0	0	0
0	1	1
1	0 →	1
1	1 →	0

If one output of EXOR gate is 1, then the output is the complement of second input.

(1) → (Q)
(2) → (P)
(3) → (S)
(4) → (R)

End of Solution

Q.18 A light emitting diode (LED) emits light when it is _____ biased. A photodiode provides maximum sensitivity to light when it is _____ biased.

(a) forward, forward
(b) forward, reverse
(c) reverse, reverse
(d) reverse, forward

Ans. (b)

LED emits light when it is **forward** biased.

Photodiode provides maximum sensitivity to light when it is operated in **reverse** bias.

End of Solution

Q.19 $F(z) = \frac{1}{1-z}$, when expanded as a power series around $z = 2$, would result in $F(z) = \sum_{k=0}^{\infty} a_k (z-2)^k$ with the region of convergence (ROC) $|z-2| < 1$. The coefficients a_k, ≥ 0, are given by the expression ________ .

(a) $(-1)^k$
(b) $(-1)^{k+1}$
(c) $\left(\frac{1}{2}\right)^k$
(d) $\left(\frac{-1}{2}\right)^{k+1}$

Ans. (b)

Power series expansion for

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots\ldots \quad \text{when } |x| < 1$$

Now, given that,

$$F(z) = \frac{1}{1-z} = -\left[\frac{1}{1+(z-2)}\right]$$

$$= -[1 - (z-2) + (z-2)^2 - (z-2)^3 + \ldots\ldots], \quad \text{when } |z-2| < 1$$

$$= -\sum_{K=0}^{\infty} (-1)^K (z-2)^K, \quad |z-2| < 1$$

$$= \sum_{K=0}^{\infty} (-1)^{K+1} (z-2)^K, \quad |z-2| < 1$$

$$= \sum_{K=0}^{\infty} a_K (z-2)^K, \quad |z-2| < 1$$

$\therefore \quad a_K = (-1)^{K+1}$

End of Solution

Q.20 The solution $x(t)$, $t \geq 0$, to the differential equation $\ddot{x} = -k\dot{x}, k > 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ is

(a) $x(t) = 2e^{-kt} + 2kt - 1$
(b) $x(t) = 2e^{-kt} + 1$
(c) $x(t) = 1$
(d) $x(t) = 2e^{-kt} - kt - 1$

Ans. (c)

$$\text{D.E}: \frac{d^2x}{dt^2} + k\frac{dx}{dt} = 0, \; k > 0$$

$(D_2 + KD)\,x = 0$ is a homogeneous linear differential equation

Solution is $x = x_{CF}$

given by $f(m) = m^2 + km = 0$

$\Rightarrow \quad m = 0, -k$

$\therefore \quad X = X_{CF} = C_1e^{0t} + C_2e^{-kt}$

Put $t = 0$, $x = 1$

We get $C_1 + C_2 = 1$

$$\frac{dx}{dt} = -kC_2e^{-kt}$$

Put $t = 0$, $\frac{dx}{dt} = 0$

We get $C_2 = 0 \;\Rightarrow\; C_1 = 1$

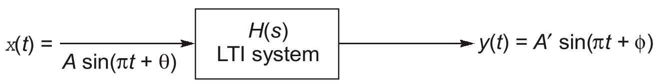
End of Solution

Q.21 A system has the transfer-function $\frac{Y(s)}{X(s)} = \frac{s-\pi}{s+\pi}$. Let $u(t)$ be the unit-step function. The input $x(t)$ that results in a steady-state output $y(t) = \sin\pi t$ is ______ .

(a) $x(t) = \sin(\pi t)u(t)$
(b) $x(t) = \sin\left(\pi t + \frac{\pi}{2}\right)u(t)$
(c) $x(t) = \sin\left(\pi t - \frac{\pi}{2}\right)u(t)$
(d) $x(t) = \cos\left(\pi t + \frac{\pi}{2}\right)u(t)$

Ans. (c)

We know,



where, $A' = A \cdot |H(\omega)|_{\omega=\omega_o}$

$$\phi = \theta + \angle H(\omega)|_{\omega=\omega_o}$$

We have, $\omega_o = \pi$, $A' = 1$, $\phi = 0$

Now, $$|H(\omega)|_{\omega=\pi} = \frac{\sqrt{\omega^2+\pi^2}}{\sqrt{\omega^2+\pi^2}} = 1$$

$$\angle H(\omega)|_{\omega=\pi} = 180° - \tan^{-1}\left(\frac{\omega}{\pi}\right) - \tan^{-1}\left(\frac{\omega}{\pi}\right)$$

$$= 180° - 45° - 45°$$

$$= 90°$$

$\therefore$ $$A = 1$$

$$\theta = -90°$$

Now, $$x(t) = \sin(\pi t - 90°) = \sin\left(\pi t - \frac{\pi}{2}\right)$$

End of Solution

Q.22 Choose the fastest logic family among the following:

(a) Transistor-Transistor Logic
(b) Emitter-Coupled Logic
(c) CMOS Logic
(d) Resistor-Transistor Logic

Ans. (b)

End of Solution

Q.23 What is $\lim_{x\to\infty} f(x)$, where $f(x) = x\sin\frac{1}{x}$?

(a) 0
(b) 1
(c) ∞
(d) Limit does not exist

Ans. (a)

$$\lim_{x\to 0} x\sin\frac{1}{x} = \lim_{x\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{y\to\infty}\frac{\sin y}{y} = \frac{\text{finite}}{\infty} = 0$$

End of Solution

Q.24 The number of zeros of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right-half plane is ______.

Ans. **(2)**

$s^3 + 2s^2 + 5s + 80$

Forming RH array

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 2 & 80 \\ s^1 & \frac{10-80}{2} = -35 & \\ s^0 & 80 & \end{array}$$

$\therefore$ There are two sign change.

Hence, 2 open loop zeros in RHS.

End of Solution

Q.25 The number of times the Nyquist plot of $G(s)H(s) = \dfrac{1}{2}\dfrac{(s-1)(s-2)}{(s+1)(s+2)}$ encircles the origin is _________ .

Ans. **(2)**

$$G(s)H(s) = \frac{(s-1)(s-2)}{2(s+1)(s+2)}$$

$\therefore$ Number of poles which are in RHS, $P = 0$

Number of zeros which are in RHS, $Z = 2$

(Here it is asking for number of encirclement around the origin).

$\therefore$ Encirclement, $N = P - Z$

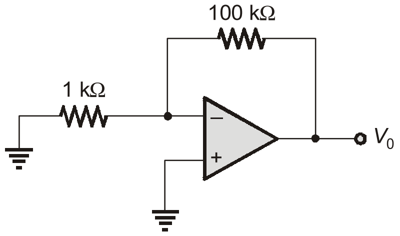
$= 0 - 2$

$N = -2$

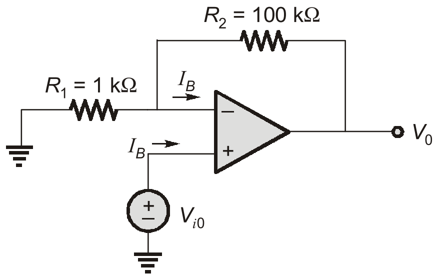
$\therefore$ 2 clockwise encirclement.

End of Solution

Q.26 The opamp in the circuit shown is ideal, except that it has an input bias current of 1 nA and an input offset voltage of 10 μV. The resulting worst-case output voltage will be ± _________ μV (rounded off to the nearest integer).



Ans. (1110)



KCL at inverting terminal:

$$\frac{0-V_{i0}}{R_1}+\frac{V_0-V_{i0}}{R_2} = I_B$$

$$\frac{V_0}{R_2} = I_B+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_{i0}$$

$$V_0 = I_BR_2+\left(1+\frac{R_2}{R_1}\right)V_{i0}$$

$$V_0 = 10^{-9}\times 100\times 10^3+\left(1+\frac{100}{1}\right)\times 10\times 10^{-6}$$

$$V_0 = 1110\ \mu\text{V}$$

End of Solution

Q.27 The force per unit length between two infinitely long parallel conductors, with a gap of 2 cm between them is 10 μN/m. When the gap is doubled, the force per unit length will be ________ μN/m (rounded off to one decimal place).

Ans. (5)

We know that,

Force is inversely proportional to 'd'

$$F \propto \frac{1}{d}$$

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}$$

$$\frac{10}{F_2} = \frac{4\text{ cm}}{2\text{ cm}}$$

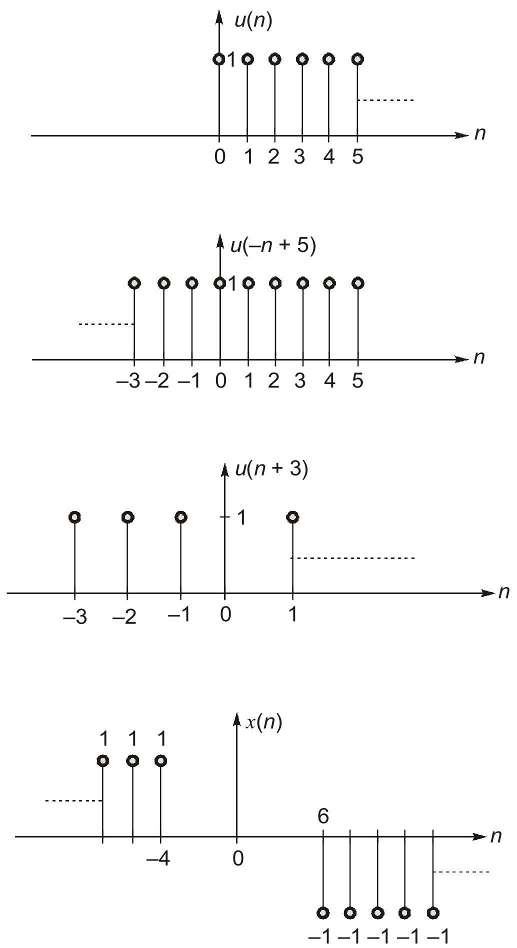
$$F_2 = \frac{10}{2} = 5\ \mu\text{N/m}$$

End of Solution

Q.28 Consider the discrete-time signal $x[n] = u[-n+5] - u[n+3]$, where $u[n] = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$.

The smallest n for which $x[n] = 0$ is ________ .

Ans. (–3)



Smallest 'n' for which $x[n] = 0$

$\therefore \quad n = -3$

End of Solution

Q.29 Let $y(t) = x(4t)$, where $x(t)$ is a contnuous-time periodic signal with fundamental period of 100 s. The fundamental period of $y(t)$ is _______ s (rounded off to the nearest integer).

Ans. (25)

Let $x(t) = \sin(\omega_o t)$

We have, $T = 100$ sec

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{100} \text{ rad/sec}$$

Now, $y(t) = x(4t) = \sin(4\omega_o t)$

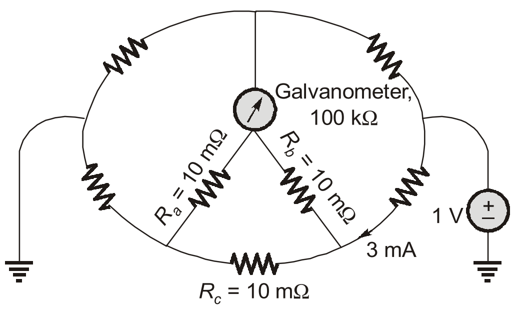
$$\therefore \quad \omega' = 4\omega_o = 4\times\frac{2\pi}{100}$$

$$\frac{2\pi}{T'} = 4\times\frac{2\pi}{100}$$

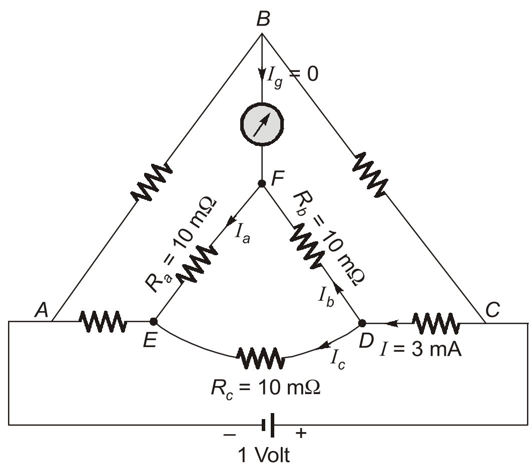
$$T' = 25 \text{ sec}$$

End of Solution

Q.30 When the bridge given below is balanced, the current through the resistor R_a is ________ mA (rounded off to two decimal places).



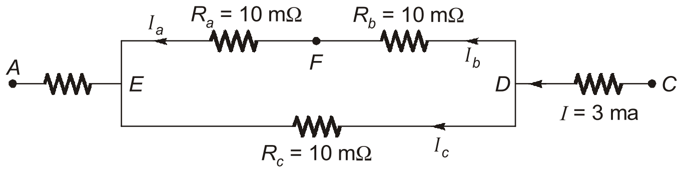
Ans. (1)



This is a Kelvin double bridge.

Under balanced condition $\Rightarrow$ $I_g = 0$

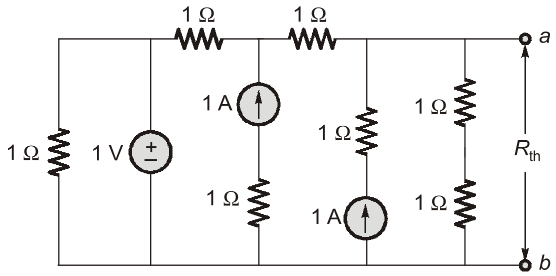
We can redraw the bridge ⇒



$$I_a = I_b = 3\text{ mA} \times \frac{10}{10+10+10} = 1\text{mA}$$

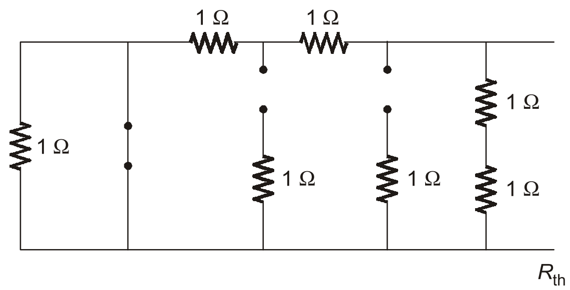
End of Solution

Q.31 In the circuit given, the Thevenin equivalent resistance R_{th} across the terminals 'a' and 'b' is ___________Ω (rounded off to one decimal place).



Ans. (1)

For Thevenin resistance, we need to deactivate all the independent source.



$$R_{th} = (1+1)\,||\,(1+1) = 2\,||\,2$$

$$= \frac{2\times 2}{2+2} = \frac{4}{4} = 1\,\Omega$$

$$R_{th} = 1\ \Omega$$

End of Solution

Q.32 X is a discrete random variable which takes values 0, 1 and 2. The probabilities are $P(X = 0) = 0.25$ and $P(X = 1) = 0.5$. With $E[\ .\ ]$ denoting the expectation operator, the value of $E[X] - [X^2]$ is _______ (rounded off to one decimal place).

Ans. **(–0.5)**

x is discrete R.V

W.K.T: $\sum_{i=1}^{n} P(x = x_i) = 1$

$\Rightarrow \quad 0.25 + 0.5 + k = 1$

$\Rightarrow \quad k = 1 - 0.75$

$k = 0.25$

$\therefore \quad E(x) = \sum_i x_i P_i$

$= 0 \times 0.25 + 1 \times 0.5 + 2 \times 0.25$

$0.5 + 0.5$

$E(x) = 1$

$E(x^2) = \sum_i x_i^2 P_i$

$= 0^2 \times 0.25 + 1^2 \times 0.5 + 2^2 \times 0.25$

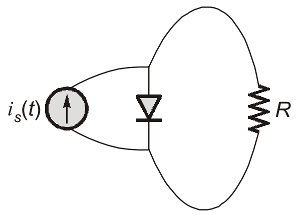
$= 0 + 0.5 + 1$

$E(x^2) = 1.5$

$\therefore \quad E(x) - E(x^2) = 1 - 1.5 = -0.5$

End of Solution

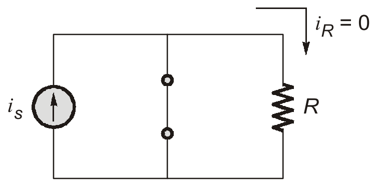
Q.33 The diode in the circuit is ideal. The current source $i_s(t) = \pi \sin(3000\pi\ t)$ mA. The magnitude of the average current flowing through the resistor R is ______mA (rounded off to two decimal places).



Ans. **(1)**

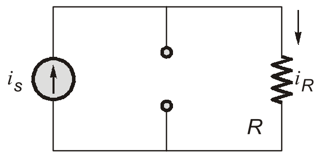
If i_s is positive : Diode is ON.

$i_R = 0$



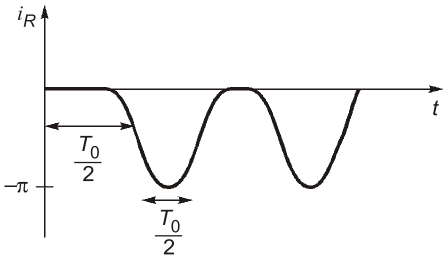
If i_s is negative : Diode is OFF.

$i_R = i_s = \pi \sin 300\pi t$ mA



Average current through the resistor R,

$$i_R = \frac{\text{Peak value}}{\pi} = -\frac{\pi}{\pi} = -1\text{mA}$$

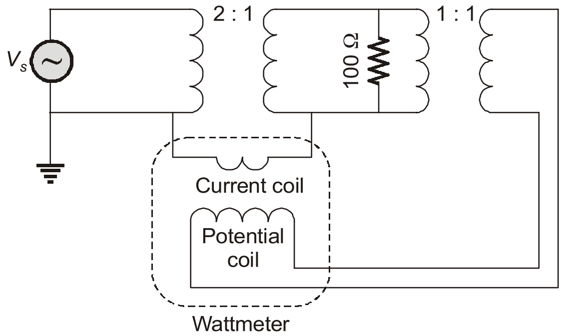


Magnitude of average current through the resistor R,

$$i_R = 1 \text{ mA}$$

End of Solution

Q.34 The full-scale range of the wattmeter shown in the circuit is 100 W. The turns ratio of the individual transformers are indicated in the figure. The RMS value of the ac source voltage V_s is 200 V. The wattmeter reading will be ________W (rounded off to the nearest integer).



Ans. (0)

Since transformer is act as isolation device, there is no current flowing through secondary winding of 1 : 1 transformer hence the current coil draws no current. Hence, the wattmeter reads 0 Watt reading.

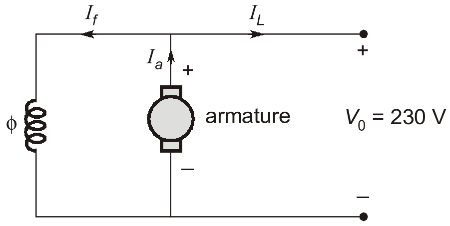
End of Solution

Q.35 The no-load steady-state output voltage of a DC shunt generator is 200 V when it is driven in the clockwise direction at its rated speed. If the same machine is driven at the rated speed but in the opposite direction, the steady-state output voltage will be ________V (rounded off to the nearest integer).

Ans. (0)

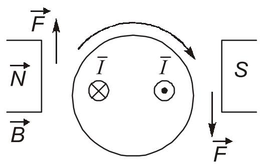
As due to change in direction polarity change, which cancel out residual field.

Alternate:



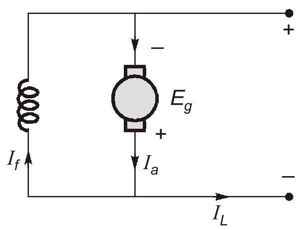
Using Fleming right hand rule.

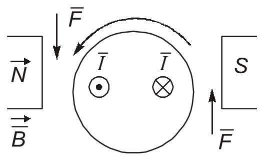
(a) Initially in clockwise direction



Now its characteristic curve B and H

(b) Now its direction is anticlockwise achieve this firstly its speed must be reduced to 0 but at 0 speed, the ϕ_f reaches residual flux ϕ_{res}.





Now I_f direction reversed so slowly the residual flux of ϕ_f becomes '0'.

$$E_g \propto \phi \cdot N$$

$$E_g = 0 \text{ (where } \phi = 0)$$

Steady-state output voltage will be 0 V.

End of Solution

Q.36 The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\,\delta(t - 4)$, where $\delta(t)$ is the continuous-time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi}{4}t\right)$, the output is ________.

(a) $0.5\cos\left(\frac{7\pi}{4}t\right)$

(b) $1.5\cos\left(\frac{7\pi}{4}t\right)$

(c) $0.5\sin\left(\frac{7\pi}{4}t\right)$

(d) $1.5\sin\left(\frac{7\pi}{4}t\right)$

Ans. (a)

We have; $h(t) = \delta(t) + 0.5\delta(t-4)$

Taking FT;

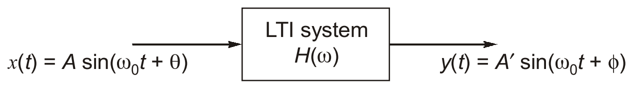
$$H(\omega) = 1 + 0.5e^{-j4\omega}$$

at $\omega = \frac{7\pi}{4}$

$$H(\omega) = 1 + 0.5e^{-j4\left(\frac{7\pi}{4}\right)}$$
$$= 1 + 0.5e^{-j7\pi}$$
$$= 0.5$$

$\therefore$ $$|H(\omega)|_{\omega=\frac{7\pi}{4}} = 0.5$$

$$\angle H(\omega)|_{\omega=\frac{7\pi}{4}} = 0$$



$$A' = A_0 |H(\omega)|_{\omega=\omega_0}$$

$$\phi = \theta + \angle H(\omega)|_{\omega=\omega_0}$$

$\therefore$ $$y(t) = 0.5\cos\left(\frac{7\pi}{4}t\right)$$

End of Solution

Q.37 The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is ______, where $u(t)$ denotes the continuous-time unit step signal.

(a) $\frac{e^{-5s}}{s+3}$, Real$\{s\} > -3$

(b) $\frac{e^{-5(s-3)}}{s-3}$, Real$\{s\} > -3$

(c) $\frac{e^{-5(s+3)}}{s+3}$, Real$\{s\} > -3$

(d) $\frac{e^{-5(s-3)}}{s+3}$, Real$\{s\} > -3$

Ans. (c)

We have, $x(t) = e^{-3t}u(t-5)$

We know, $u(t) \rightarrow \frac{1}{s}$

$$u(t-5) \rightarrow \frac{1}{s}\cdot e^{-5s}$$

$$e^{-3t}u(t-5) \rightarrow \frac{1}{(s+3)}\cdot e^{-5(s+3)}$$

$\therefore$ $$X(s) = \frac{e^{-5(s+3)}}{s+3}; \quad \text{Re}(s) > -3$$

End of Solution

Q.38 In a p-i-n photodiode, a pulse of light containing 8×10^{12} incident photons at wavelength $\lambda_0 = 1.55\ \mu m$ gives rise to an average 4×10^{12} electrons collected at the terminals of the device. The quantum efficiency of the photodiode at this wavelength is ______%.

(a) 50 (b) 54.2
(c) 62.5 (d) 80

Ans. (a)

Incident photons $= 8 \times 10^{12}$

Electron collected/generated $= 4 \times 10^{12}$

$$\text{Quantum efficiency of photodiode} = \frac{\text{Number of electron generated}}{\text{Number of photon incident}}$$

$$= \frac{4 \times 10^{12}}{8 \times 10^{12}} = \frac{4}{8} = \frac{1}{2} = 0.5$$

$= 50\%$

End of Solution

Q.39 Let $f(z) = j\dfrac{1-z}{1+z}$, where z denotes a complex number and j denotes $\sqrt{-1}$. The inverse function $f^{-1}(z)$ maps the real axis to the________.

(a) unit circle with centre at the origin (b) unit circle with centre not at the origin
(c) imaginary axis (d) real axis

Ans. (a)

Given : $$w = u + iv = \frac{i(1-z)}{(1+z)}$$

$$= \frac{i - ix - i^2 y}{1 + x + iy} = \frac{i(1-x) + y}{(1+x) + iy} \times \frac{(1+x) - iy}{(1+x) - iy}$$

$$u + iv = \frac{y(1-x) + y(1+x)}{(1+x)^2 + y^2} + i\frac{(1-x^2) - y^2}{(1+x)^2 + y^2}$$

To find image of real axis is w-plane.

We take $V = 0$

$$\Rightarrow \quad \frac{(1-x^2) - y^2}{(1+x)^2 + y^2} = 0$$

$$\Rightarrow \quad 1 - x^2 - y^2 = 0$$

$$\Rightarrow \quad x^2 + y^2 = 1$$

$\therefore$ Under the mapping of $f^{-1}(z)$ real axis in w-plane maps to unit circle in z-plane.

End of Solution

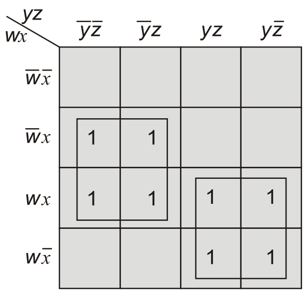
Q.40 The simplified form of the Boolean function $F(W, X, Y, Z) = \Sigma(4, 5, 10, 11, 12, 13, 14, 15)$ with the minimum number of terms and smallest number of literals in each term is________.

(a) $WX + \overline{W}X\overline{Y} + W\overline{X}Y$
(b) $WX + WY + X\overline{Y}$
(c) $X\overline{Y} + WY$
(d) $\overline{X}Y + \overline{W}\overline{Y}$

Ans. **(c)**

Given: $F(w, x, y, z) = \Sigma m(4, 5, 10, 11, 12, 13, 14, 15)$

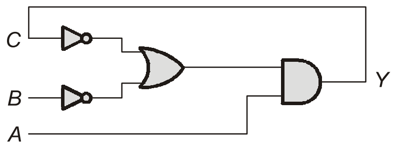
K-Map:



$F(w, x, y, z) = x\overline{y} + wy$

End of Solution

Q.41 For the given digital circuit, $A = B = 1$. Assume that AND, OR, and NOT gates have propagation delays of 10 ns, 10 ns, and 5 ns respectively. All lines have zero propagation delay. Given that $C = 1$ when the circuit is turned on, the frequency of steady-state oscillation of the output Y is ______.

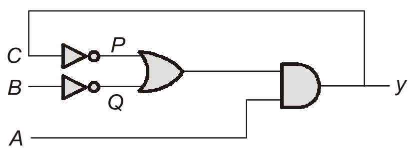


(a) 20 MHz
(b) 15 MHz
(c) 40 MHz
(d) 50 MHz

Ans. **(a)**

Given $A = B = 1$

$C = 1$ (Initially)



From circuit $P = \overline{C} = \overline{y}$

$Q = \overline{B}$

Next state of the output

$y^{+} = A \cdot (\overline{B} + \overline{C})$

$A = 1$ and $B = 1$

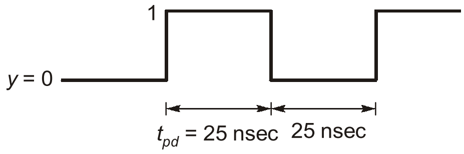
$$y^+ = 1\cdot(0+\bar{C}) = 0+\bar{y} = \bar{y}$$

$$y^+ = \bar{y}$$

Overall propagation delay of circuit,

$$t_{Pd} = 5 + 10 + 10 = 25 \text{ nsec}$$

So, after every 25 nsec the output will toggle.



Frequency of steady state oscillations,

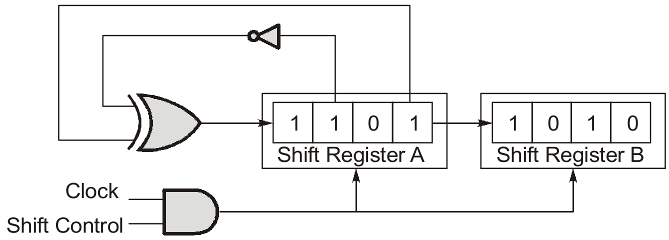
$$f = \frac{1}{50\times10^{-9}} = 20\times10^6 \text{ Hz}$$

$$f = 20 \text{ MHz}$$

End of Solution

Q.42 In the circuit shown, the initial binary content of shift register A is 1101 and that of shift register B is 1010. The shift registers are positive-edge triggered, and the gates have no delay.

When the shift control is high, what will be the binary content of the shift registers A and B after four clock pulses?



(a) A = 1101, B = 1101
(b) A = 1110, B = 1001
(c) A = 0101, B = 1101
(d) A = 1010, B = 1111

Ans. (c)

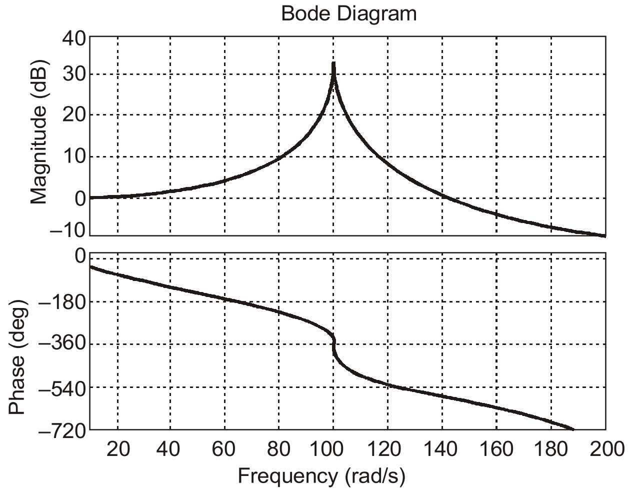
	Register A	Register B
	1 1 0 1	1 0 1 0
Clock 1:	1 1 1 0	1 1 0 1
Clock 2:	0 1 1 1	0 1 1 0
Clock 3:	1 0 1 1	1 0 1 1
Clock 4:	0 1 0 1	1 1 0 1

After 4th clock pulse,

$$A = 0\ 1\ 0\ 1$$
$$B = 1\ 1\ 0\ 1$$

End of Solution

Q.43 The magnitude and phase plots shown in the figure match with the transfer-function _______.



(a) $\dfrac{10000}{s^2 + 2s + 10000}$

(b) $\dfrac{10000}{s^2 + 2s + 10000}e^{-0.05s}$

(c) $\dfrac{10000}{s^2 + 2s + 10000}e^{-0.5\times10^{-12}s}$

(d) $\dfrac{100}{s^2 + 2s + 100}$

Ans. (b)

From the figure it is clear that there should be phase lag.

Hence, option (a) and (c) could be answer.

From the figure, at ω = 100 rad/sec, Phase is –360°.

Now considering option (c) only delay element

$$e^{-0.05s} = e^{-0.05\times100j} = e^{-j5}$$

Hence, phase is = –5 radians = –286.5°

and from the rest of transfer function $\dfrac{1000}{s^2 + 2s + 10000}$

at $\omega = \omega_n$ = 100 rad/sec phase is –90°.

= –286.5 – 90°

= –376.5°

Hence, option (c) satisfy the graph.

End of Solution

Q.44 A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0,\ \forall\ t < 0$. From the list given below, selec ALL the signals whose continous-time Fourier transform is purely imaginary.

(a) $x(t) + x(-t)$

(b) $x(t) - x(-t)$

(c) $j(x(t) + x(-t)$

(d) $j(x(t) - x(-t)$

Ans. (b, c)

If $x(t)$ is real, then its Fourier transform will be complex conjugate.

i.e., $x(t) \longrightarrow X(\omega)$

$x(-t) \longrightarrow X(-\omega) = X^*(\omega)$

Option (a) → $x(t) + x(-t) \xleftrightarrow{FT} X(\omega) + X(-\omega) = X(\omega) + X^*(\omega) \Rightarrow$ Real

Option (b) → $j[(x(t) + x(-t)]$

$= j[X(\omega) + X(-\omega)]$ ... Taking Fourier Transform

$= j(X(\omega) + X^*(\omega))$

= Purely Imaginary

Option (c) → $x(t) - x(-t) \xleftrightarrow{F.T} X(\omega) - X(-\omega) = X(\omega) - X^*(\omega) \Rightarrow$ Purely imaginary

Option (d) → $j[x(t) - x(-t)] \xleftrightarrow{F.T} j[X(\omega) - X^*(\omega)] \Rightarrow$ Purely Real

End of Solution

Q.45 A silica-glass fiber has a core refractive index of 1.47 and a cladding refractive index of 1.44. If the cladding is completely stripped out and the core is dipped in water having a refractive index of 1.33, the numerical aperture of the modified fiber is _____ (rounded off to three decimal places)

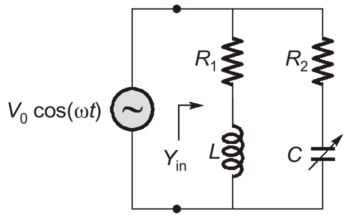
Ans. (0.626)

Numerical Aperture, N.A = $\sqrt{n_{core}^2 - n_{water}^2} = \sqrt{1.47^2 - 1.33^2}$

N.A = 0.626

End of Solution

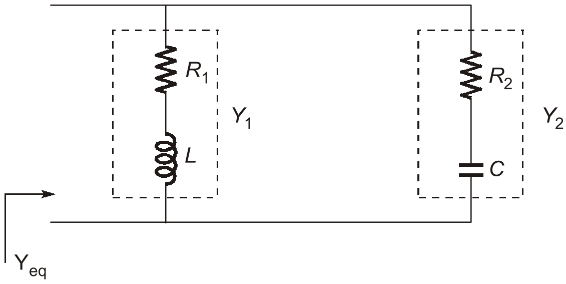
Q.46 In the circuit shown, $\omega = 100\,\pi$ rad/s, $R_1 = R_2 = 2.2\ \Omega$ and $L = 7$ mH. The capacitance C for which Y_{in} is purely real is _______ mF. (rounded off to two decimal places)



Ans. (1.45)

Given : $R_1 = R_2 = 2.2\ \Omega$

$L = 7$ mH, $\omega = 100\pi$ rad/sec



$$Y_{eq} = Y_1 + Y_2$$

$$= \frac{1}{R_1 + j\omega L} + \frac{1}{R_2 - \frac{j}{\omega C}}$$

$$= \frac{(R_1 - j\omega L)}{\sqrt{R_1^2 + (\omega L)^2}} + \frac{R_2 + \frac{j}{\omega C}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2}}$$

For resonant frequency,

$$I_m\{Y_{eq}\} = 0$$

$$\frac{-j\omega L}{\sqrt{R_1^2 + (\omega L)^2}} + \frac{\frac{j}{\omega C}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2}} = 0$$

$$\omega L\sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{1}{\omega C}\sqrt{R_1^2 + (\omega L)^2}$$

$$\omega^2 L^2\left(R_2^2 + \frac{1}{\omega^2 C^2}\right) = \frac{1}{\omega^2 C^2}(R_1^2 + \omega^2 L^2)$$

$$\omega^2 L^2 R_2^2 + \frac{L^2}{C^2} = \frac{R_1^2}{\omega^2 C^2} + \frac{L^2}{C^2}$$

$$\omega^2 L^2 R_2^2 = \frac{R_1^2}{\omega^2 C^2}$$

$$C^2 = \frac{R_1^2}{R_2^2} \cdot \frac{1}{\omega^4 L^2}$$

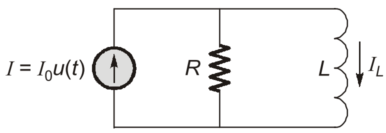
$$C = \frac{R_1}{R_2} \cdot \frac{1}{\omega^2 L}$$

$$= \frac{2.2}{2.2} \times \frac{1}{(100\pi)^2 \times 7 \times 10^{-3}}$$

$$C = 1.45 \times 10^{-3} \text{ F}$$

End of Solution

Q.47 The R-L circuit with R = 10 kΩ and L = 1 mH is excited by a step current $I_0 u(t)$. At $t = 0^-$, there is a current $I_L = I_0/5$ flowing through the inductor. The minimum time taken for the current through the inductor to reach 99% of its final value is _____ μs (rounded off to two decimal places).



Ans. **(0.438)**

The current equation for inductor

$$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$i_L(0^+) = \frac{I_o}{5}$$

$$i_L(\infty) = I_o$$

$$i_L(t) = I_o + \left[\frac{I_o}{5} - I_o\right]e^{-t/\tau} = I_o - \frac{4I_o}{5}e^{-t/\tau}$$

The inductor current reaches to 99% of its final value.

i.e. $$i_L(t) = 0.99\, I_o$$

$$0.99\, I_o = I_o\left[1 - \frac{4}{5}e^{-t/\tau}\right]$$

$$\frac{4}{5}e^{-t/\tau} = 0.01$$

$$t = -\tau \ln\left(\frac{0.05}{4}\right) = 4.382\tau$$

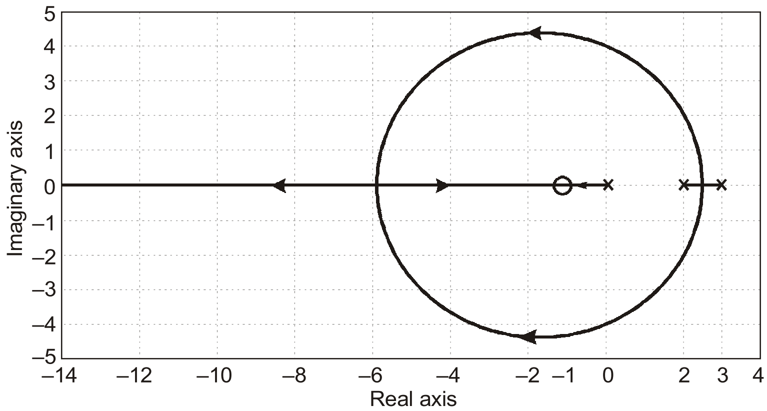
$$\tau = \frac{L}{R} = \frac{1\times 10^{-3}}{10\times 10^{3}} = 10^{-7} \text{ sec}$$

$$t = 4.382 \times 10^{-7} \text{ sec}$$

$$t = 0.4382\ \mu\text{s}$$

End of Solution

Q.48 Consider a standard negative feedback configuration with $G(s) = \frac{1}{(s-2)(s-3)}$ and the controller $C(s) = K_P + \frac{K_1}{s} + K_D s$. The root-locus of $G(s)C(s)$ is presented in the figure below. The gain $C(j\omega)$ = 2 at ω =1 rad/s. The value of K_D is _______ (rounded off to one decimal place).



Ans. (1)

$$G(s) = \frac{1}{(s-2)(s-3)}$$

$$C(s) = K_P + \frac{K_i}{s} + K_D S$$

$$G(s)C(s) = \frac{sK_P + K_i + K_D s^2}{s(s-2)(s-3)}$$

Two zeros are at (–1)

$$(s + 1)^2 = 0$$

$$s^2 + 2s + 1 = 0 \quad \text{...(1)}$$

Compared with

$$K_D s^2 + K_P s + K_i = 0 \quad \text{...(2)}$$

$$K_D = 1,\ K_P = 2 \text{ and } K_i = 1$$

$$K_D = 1$$

as per given $C(s) = K_P + \frac{K_i}{s} + K_D \cdot S$

$$C(j\omega) = 2 + \frac{1}{j\omega} + j\omega$$

$$C(j1) = 2 - j + j = 2$$

End of Solution

Q.49 How many five-digit numbers can be formed using the integers 3, 4, 5 and 6 with exactly one digit appearing twice?

Ans. (240)

We have 5 digits out of which 2 are identical.

$$\text{Number of ways} = \frac{5!}{2!} = \frac{120}{2} = 60$$

Out of four digit exactly one digit appearing twice.

$$\text{Number of ways} = {}^4C_1 = 4$$

$$\text{Total 5-digit number} = 4 \times 60 = 240$$

End of Solution

Q.50 The phase margin of the transfer function $G(s) = \dfrac{2(1-s)}{(1+s)^2}$ is __________degrees. (rounded off to the nearest integer).

Ans. (0)

$$G(s) = \frac{2(1-s)}{(1+s)^2}$$

Phase margin is calculated at gain crossover frequency ω_{gc}.
ω_{gc} is the frequency at which the magnitude of $G(j\omega)H(j\omega)$ is 1.

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = \frac{2\sqrt{1+\omega_{gc}^2}}{\left(1+\omega_{gc}^2\right)} = \frac{2}{\sqrt{1+\omega_{gc}^2}} = 1$$

$$2 = \sqrt{1+\omega_{gc}^2}$$

$$4 = 1+\omega_{gc}^2$$

$$\omega_{gc} = \sqrt{3} \text{ rad/sec}$$

Now

$$\phi \text{ at } \omega_{gc} = \tan^{-1}[-\omega_{gc}] - 2\tan^{-1}[\omega_{gc}]$$

$$= -3\tan^{-1}[\omega_{gc}]$$

$$= -3\tan^{-1}\left(\sqrt{3}\right)$$

$$= -180°$$

$\therefore$

$$\text{Phase margin} = 180 + \phi \text{ at } \omega_{gc}$$

$$= 180° - 180° = 0°$$

End of Solution

Q.51 A wire-wound 'resistive potentiometer type' angle sensor with 72 turns is used in an application. The first turn of the potentiometer is connected to ground while its last turn is connected to 3.6 V. The width of the wiper covers two turns ensuring make-before-break. The output (wiper) voltage when the wiper is on top of both the turns 35 and 36 is _______V (rounded off to two decimal places).

Ans. (1.78)

End of Solution

Q.52 The two secondaries of a linear variable differential transformer (LVDT) showed a magnitude of 2 V (RMS) for zero displacement position of the core. It is noted that the phase of one of the secondaries has a deviation of one degree from the expected phase. Other than this deviation, the LVDT is ideal.
If the differential output sensitivity of the LVDT is 1 mV (RMS)/1 μm, the output for zero displacement is __________ μm (rounded off to one decimal place).

Ans. (34.9)

For an LVDT,

$$\text{Output voltage} = \overline{E_{S1}} - \overline{E_{s2}}$$

Given there is deviation of 1° and rms value is 2 V.

Voltage at zero displacement

$$V_0 = 2\angle 0° - 2\angle 1°$$
$$V_0 = 2 - 2\cos 1 - j2\sin 1$$
$$V_0 = 3.04609 \times 10^{-4} - j[349.04812 \times 10^{-4}]$$
$$V_0 = 3.04609 \times 10^{-4} - j[349.04812 \times 10^{-4}]$$
$$V_0 = 349.061411 \times 10^{-4} \angle -89.5$$
$$|V_0| = 3.49061411 \times 10^{-2}$$

So given sensitivity is 1 mV/1 μm

$$\text{Output voltage} = |V_0| \times S$$

$$\text{displacement} = \frac{3.49061411 \times 10^{-2}}{1 \times 10^{-3} / 10^{-6}\text{m}}$$

$$\text{displacement}\,(\mu\text{m}) = 3.49061411 \times 10^{1}\ \mu\text{m}$$
$$= 34.906\ \mu\text{m}$$

End of Solution

Q.53 Five measurements are made using a weighing machine, and the readings are 80 kg, 79 kg, 81 kg, 79 kg and 81 kg. The sample standard deviation of the measurement is ____________ kg (rounded off to two decimal places).

Ans. (1)

Given,

X(kg)	80	79	81	79	81
$\bar{X}$	80	80	80	80	80

where,

$$\bar{X} = \frac{\text{Sum}}{N};\ N = 5$$

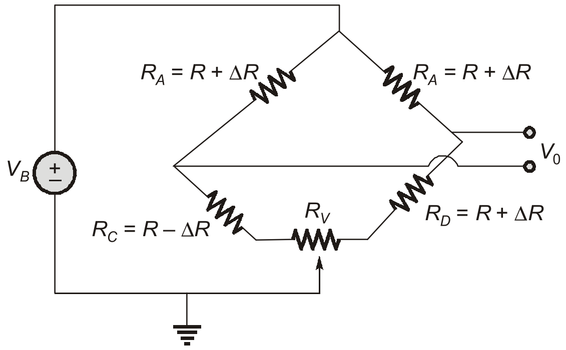
$$= \frac{80 + 79 + 81 + 79 + 81}{5} = \frac{400}{5} = 80$$

$$\text{S.D} = \sqrt{\frac{\sum_{i=1}^{5}(X - \bar{X})^2}{N - 1}} = \sqrt{\frac{(1)^2 + (1)^2 + (1)^2 + (1)^2}{4}}$$

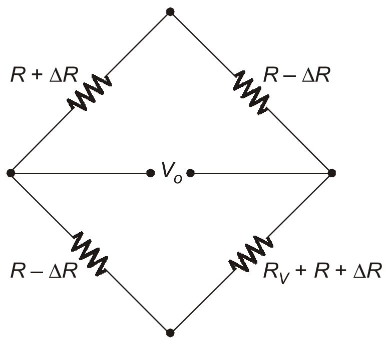
$$\text{S.D} = 1\ \text{kg}$$

End of Solution

Q.54 Four strain gauges R_A, R_B, R_C and R_D, each with nominal resistance R, are connected in a bridge configuration. When a force is applied, R_A and R_D increase by ΔR and R_B and R_C decrease by ΔR as shown. A potentiometer with total resistance R_v is connected as shown. If $R = 100\ \Omega$, and $\Delta R = 1\ \Omega$, the minimum value of resistance R_v required to balance the bridge is ______ Ω (rounded off to two decimal places).



Ans. (–3.96)



Given that $R = 100\ \Omega$, $\Delta R = 1\ \Omega$

$$V_o = E\left[\frac{R+\Delta R}{R+\Delta R+R-\Delta R} - \frac{R-\Delta R}{R-\Delta R+(R_V+R+\Delta R)}\right]$$

$$V_o = E\left[\frac{R+\Delta R}{2R} - \frac{R-\Delta R}{2R+R_V}\right]$$

For a balanced bridge $\Rightarrow V_b = 0$

$$0 = E\left[\frac{R+\Delta R}{2R} - \frac{R-\Delta R}{2R+R_V}\right]$$

$$\frac{R+\Delta R}{2R} = \frac{R-\Delta R}{2R+R_V} \Rightarrow 2R + R_V = 2R\left[\frac{R-\Delta R}{R+\Delta R}\right]$$

$$R_V = 2R\left[\frac{R-\Delta R}{R+\Delta R} - 1\right] = 2R\left[\frac{R-\Delta R-R-\Delta R}{R+\Delta R}\right]$$

$$= 2R\left[-\frac{2\Delta R}{R+\Delta R}\right] = \frac{-4R\Delta R}{R\left[1+\frac{\Delta R}{R}\right]} = -\frac{4\Delta R}{1+\frac{\Delta R}{R}} \qquad \left(\frac{\Delta R}{R}\text{ neglected}\right)$$

$$= -4\Delta R \Rightarrow \text{minimum value}$$

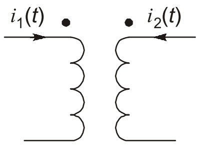
$$R_V = \frac{-4\times 1}{1+\frac{1}{100}} = \frac{-4}{1.01} = -3.96\ \Omega$$

End of Solution

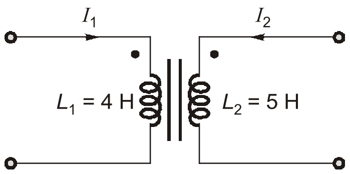
Q.55 A sinusoidal current of $i_1(t)$ = 1 sin(200πt) mA is flowing through a 4 H inductor which is mutually coupled to another 5 H inductor carrying

$$i_2(t) = 2\sin(200\pi t)\text{ mA}$$

as shown in the figure. The coupling coefficient between the inductors is 0.6. The peak energy stored in the circuit is ________ μJ (rounded off to two decimal places).



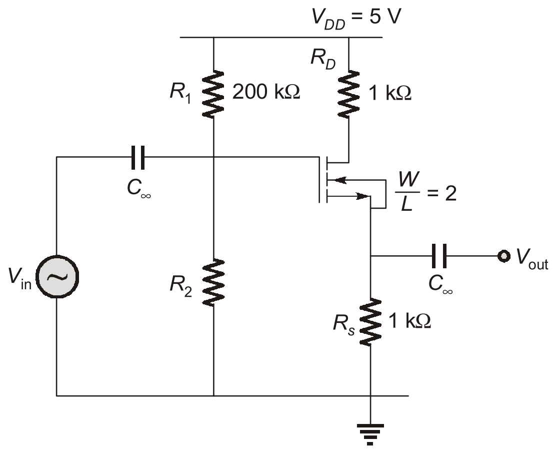
Ans. (17.37)



$$i_1(t) = 1\sin(200\ \pi t)\text{ mA}$$
$$i_2(t) = 2\sin(200\ \pi t)\text{ mA}$$
$$M = K_1\sqrt{L_1L_2}$$
$$M = 0.6\sqrt{4\times 5} = 2.683\text{ H}$$
$$W_{max} = \frac{1}{2}L_1 I_{1max}^2 + \frac{1}{2}L_2 I_{2max}^2 + MI_{1max}I_{2max}$$
$$= \frac{1}{4}(4)1^2\times 10^{-6} + \frac{1}{2}(5)(2)^2\times 10^{-6} + 2.683(1)(2)10^{-6}$$
$$W_{max} = 17.366\ \mu\text{J}$$

End of Solution

Q.56 The figure below shows a feedback amplifier constructed using an nMOS transistor. Assume that $\mu_n C_{ox}$ = 1 mA/V^2, threshold voltage V_T = 1 V and W/L = 2. The bias voltage at the drain terminal is 4 V. The capacitors C_∞ offer zero impedance at the signal frequency. The ratio V_{out}/V_{in} is ___________(rounded off to two decimal places).



Ans. **(0.67)**

Given : $V_D = 4$ V, $\frac{W}{L} = 2$, $V_T = 1$ V

$\mu_n C_{ox} = 1$ mA/V^2

Given amplifier is common drain amplifier.

Now, $$g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)$$

$$V_{GS} = V_G - V_S$$

$$V_{GS} = \frac{5 \times 300}{500} - I_D R_S$$

$$V_{GS} = 3 - I_D R_S$$

Also, $$I_D = \frac{5 - V_D}{R_D} = \frac{5-4}{1} = 1 \text{ mA}$$

∴ $$V_{GS} = 3 - 1$$

$$V_{GS} = 2 \text{ Volt}$$

Hence, $$g_m = 1 \times 10^{-3} \times 2[2 - 1]$$

$$g_m = 2 \text{ mA/V}$$

∴ For common drain amplifier,

$$\frac{V_{out}}{V_{in}} = \frac{g_m R_s}{1 + g_m R_s} \quad \text{(Taking } r_{ds} = \infty)$$

$$= \frac{2 \times 1}{1 + 2 \times 1}$$

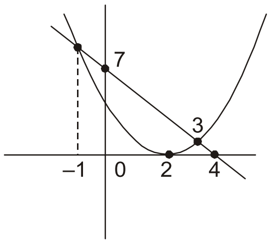
$$\frac{V_{out}}{V_{in}} = \frac{2}{3} \text{ V/V}$$

End of Solution

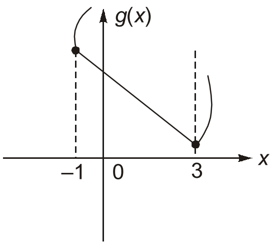
Q.57 Consider the real-valued function $g(x) = \max\{(x-2)^2, -2x+7\}$, where $x \in (-\infty, \infty)$. The minimum value attained by $g(x)$ is _______ (rounded off to one decimal place).

Ans. (1)

$$g(x) = \max\{(x-2)^2, -2x+7\}$$



Then $g(x)$



From the figure:

Minimum value of $g(x)$ is at

$$x = 3$$

$$\therefore \quad g_{min}(x) = g(3)$$

$$= (3-2)^2 \text{ or } (-2x+7)_{x=3}$$

$$= 1$$

End of Solution

Q.58 A short-circuit test is conducted on a single-phase transformer by shorting its secondary. The frequency of input voltage is 1 kHz. The corresponding wattmeter reading, primary current and primary voltage are 8 W, 2 A and 6 V respectively. Assume that the no-load losses and the no-load currents are negligible, and the core has linear magnetic characteristics. Keeping the secondary shorted, the primary is connected to a 2 V (RMS), 1 kHz sinusoidal source in series with a $\frac{1}{2\pi\sqrt{5}}$ mF capacitor. The primary current (RMS) will be _____ A (rounded off to two decimal places).

Ans. (1)

$$P_m = I^2 R_{eq}$$

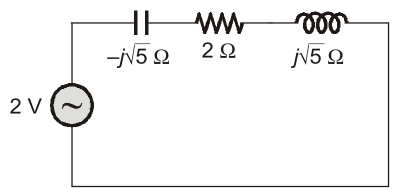
$$8 = 2^2 \times R_{eq}$$

$$R_{eq} = 2\ \Omega$$

$$Z_{eq} = \frac{6}{2} = 3\ \Omega$$

$$X_{eq} = \sqrt{3^2 - 2^2}$$

$$= \sqrt{5}\ \Omega$$



$$C = \frac{1}{2\pi\sqrt{5}}\text{mF},$$

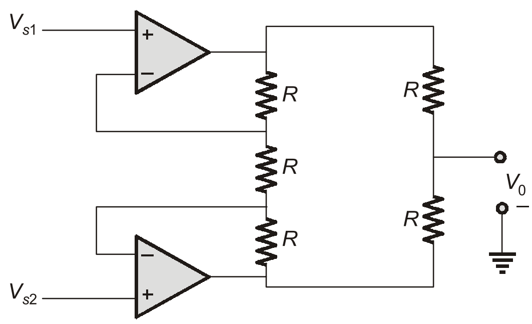
$$X_C = -j\frac{1}{2\pi \times 1 \times 10^3 \times \frac{1}{2\pi\sqrt{5}} \times 10^{-3}}$$

$$= -j(\sqrt{5})$$

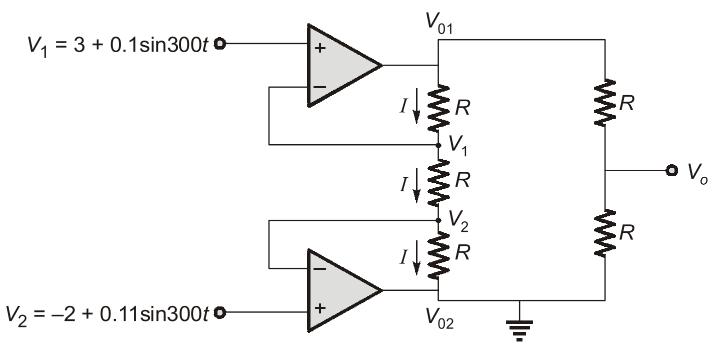
$$I_{\text{primary}} = \frac{2}{2} = 1\text{ A}$$

End of Solution

Q.59 The opamps in the circuit are ideal. The input signals are V_{S1} = 3 + 0.10sin(300t) V and V_{S2} = –2 + 0.11 sin(300t) V.
The average value of the voltage V_0 is ______V (rounded off to two decimal places).



Ans. (0.5)



$$V_{0_1} = IR + V_1$$

$$= \frac{(V_1 - V_2)}{R}R + V_1 = 2V_1 - V_2 \qquad \text{...(i)}$$

$$V_{o_2} = V_2 - IR = V_2 - \frac{(V_1 - V_2)}{R} \times R$$

$$= 2V_2 - V_1 \quad \text{...(ii)}$$

$$V_o = \frac{V_{o_1} \times R}{R+R} + \frac{V_{o_2} \times R}{R+R} = \frac{V_{o_1} + V_{o_2}}{2}$$

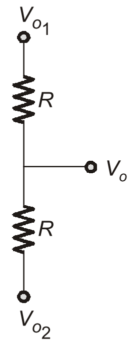
$$= \frac{2V_1 - V_2 + 2V_2 - V_1}{2} = \frac{V_1 + V_2}{2}$$

$$V_o = \frac{V_1 + V_2}{2} = \frac{3 + 0.10\sin 300t - 2 + 0.11\sin 300t}{2}$$

$$= \frac{1 + 0.21\sin 300t}{2} = \frac{1}{2} + \frac{0.21}{2}\sin 300t$$

$$= 0.5 + 0.105\sin 300t$$

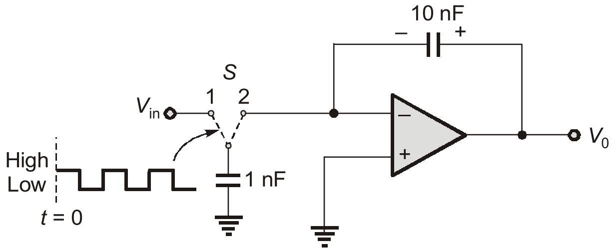
$$V_{o(avg)} = 0.5 \text{ V}$$



End of Solution

Q.60 In the circuit shown, the input voltage V_{in} = 100 mV. The switch and the opamp are ideal. At time t = 0, the initial charge stored in the 10 nF capacitor is 1 nC, with the polarity as indicated in the figure. The switch S is controlled using a 1 kHz square-wave voltage signal V_s as shown. Whenever V_s is 'High', S is in position '1' and when V_s is 'Low', S is in position '2'.

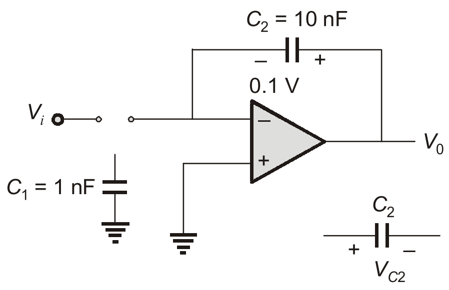
At t = 20 ms, the magnitude of the voltage V_0 will be _______ mV (rounded off to the nearest integer).



Ans. (100)

For $t \le 0$

Charge stored in C_2 is 1 nC

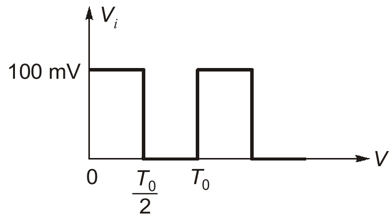


$$V_{C2} = \frac{Q}{C_2} = \frac{1\text{nC}}{10\text{ nF}} = 0.1\text{ V}$$

Considering given polarity,

$$V_{C2} = -0.1\text{ V}$$

$$T_0 = \frac{1}{f_0} = 1\text{msec}$$



For $0 < t < 0.5$ msec

C_1 is connected to V_i

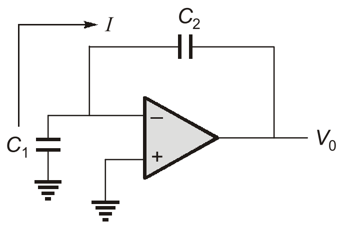
C_1 charges upto 100 mV

For 0.5 msec $< t < 1$ msec

C_1 is connected to inverting terminal.

Consider V^- at virtual ground.

C_1 should instantaneously discharge to 0.



$$\Delta V_{C1} = 100\text{ mV} - 0 = 100\text{ mV}$$

C_1 discharges through C_2

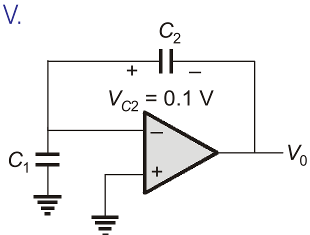
Charge lost by C_1 = Charge supplied to C_2

$$C_1 \times \Delta V_{C1} = C_2 \times \Delta V_{C2}$$

$$\Delta V_{C2} = \frac{C_1}{C_2} \times \Delta V_{C1} = \frac{1\text{nF}}{10\ nF} \times 100\text{ mV} = 0.01\text{ V}$$

In one cycle of input, voltage across C_2 changes by 0.01 V.

At t = 20 msec : Input has 20 cycles



$\Rightarrow$ Change in $V_{C2} = 20 \times 0.01 = 0.2$ V

$$V_{C2}|_{t\,=\,20\text{ ms}} = V_{C2}|_{t\,=\,0} + \Delta V_{C2}$$

$$= -0.1 + 0.2$$

$$= 0.1\text{ V}$$

$$V^- - V_0 = 0.1\text{ V}$$

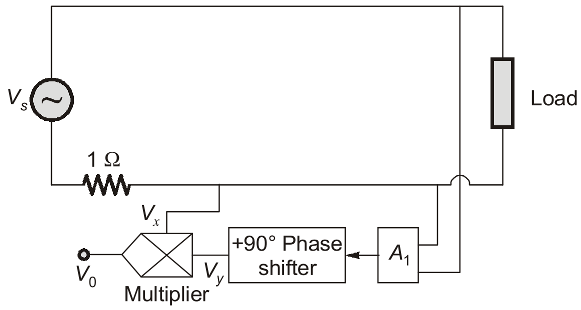
$$0 - V_0 = 0.1\text{ V}$$

$\Rightarrow$ $V_0 = -0.1$ V

$$|V_0| = 0.1\text{ V}$$

End of Solution

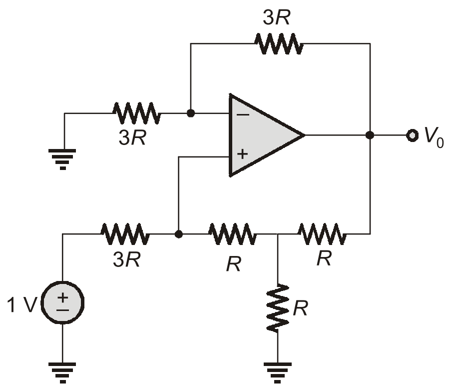
Q.61 In the diagram shown, the frequency of the sinusoidal source voltage V_S is 50 Hz. The load voltage is 230 V (RMS), and the load impedance is $\frac{230}{\sqrt{2}} + j\frac{230}{\sqrt{2}}\Omega$. The value of attenuator $A_1 = \frac{1}{50\sqrt{2}}$. The multiplier output voltage $V_0 = \frac{V_x V_y}{1\text{ V}}$, where V_x and V_y are the inputs. The magnitude of the average value of the multiplier output V_0 is ___________ V (rounded off to one decimal place).



Ans. (2.4)

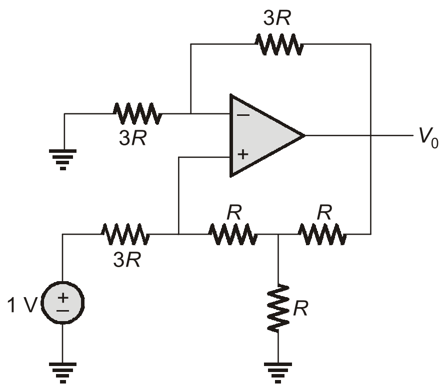
End of Solution

Q.62 In the circuit shown, assuming an ideal opamp, the value of the output voltage V_0 = ____ V (rounded off to one decimal place).



Ans. (2)

Assuming op-amp is not saturated.



$$\frac{V_A - 1}{4R} + \frac{V_A}{R} + \frac{V_A - V_o}{R} = 0$$

$$\frac{V_A - 1}{4} + V_A + V_A - V_o = 0$$

$$9V_A - 1 - 4V_o = 0$$

$$V_A = \frac{1}{9} + \frac{4}{9}V_o$$

∴ Current flowing through $3R\ \Omega$.

$$i = \frac{1 - V_A}{4R} = \frac{1}{4R}\left[1 - \left(\frac{1}{9} + \frac{4V_o}{9}\right)\right]$$

∴ Voltage at non-inverting terminal

$$V_+ = 1 - \frac{1}{4R} \times 3R\left[1 - \left(\frac{1}{9} + \frac{4V_o}{9}\right)\right]$$

$$V_+ = 1 - \frac{3}{4}\left[\frac{8}{9} - \frac{4V_o}{9}\right]$$

$$V_+ = 1 - \frac{2}{3} + \frac{V_o}{3} = \frac{1}{3} + \frac{V_o}{3}$$

Using virtual short concept

$$V_+ = V_- = \frac{1}{3} + \frac{V_o}{3}$$

Applying KCL at inverting node

$$\frac{0 - V_-}{3R} = \frac{V_- - V_o}{3R}$$

$$-V_- = V_- - V_o$$

$$V_o = 2V_-$$

$$V_o = 2\left[\frac{1}{3} + \frac{V_o}{3}\right]$$

$$3V_o = 2 + 2V_o$$

$$V_o = 2 \text{ Volt}$$

End of Solution

Q.63 The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is _______ (rounded off to the nearest integer).

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

Ans. (–8)

Given : $\rho(A_{3\times2}) = 1$

So, $C_2 = KC_1$

$\Rightarrow$ $4 = K(1)$

$\Rightarrow$ $K = 4$

Also, $\alpha = -3K$ and $6 = K\beta$

$\therefore$ $$\frac{\alpha}{\beta} = -\frac{1}{2}K^2 = -\frac{1}{2}\times 4^2 = -8$$

End of Solution

Q.64 A 1.999 V True RMS 3-1/2 digit multimeter has an accuracy of ±0.1% of reading ±2 digits. It is used to measure 100 A (RMS) current flowing through a line using a 100:5 ratio, Class-1 current transformer with a burden of 0.1 Ω ±0.5 %.
The worst-case absolute error in the multimeter output is ________ V (rounded off to three decimal places).

Ans. (0.011)

Given that $3\frac{1}{2}$ DMM, $V_{FSD} = 1.999$ Volts

$1 = 100$ Amps, Burden $\Rightarrow Z_e = 0.1\ \Omega \pm 0.5\%$

Output voltage $= I * Z_e = 100$ Amp $\times$ 0.1 Ω, Reading = 10 Volt

Error = ±0.1% reading + 2 digits

$$= \pm\left[\frac{0.1}{100}\times 10 \text{ volt} + 2 \text{ digits}\right]$$

$= \pm[0.01 + 2 \text{ digits}]$

$$1 \text{ digit} = \frac{V_{FSD}}{2000} = \frac{1.999}{2000} = 0.0009995$$

2 digits = 2 × 0.0009995 = 0.00199

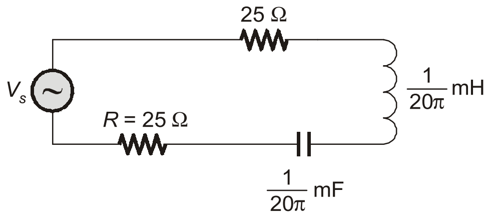
Error = ±[0.01 + 0.00199] = ±0.0119

Output voltage = 10 Volt ± 0.0119

Absolute Error = ±0.011

End of Solution

Q.65 The voltage source $V_s = 10\sqrt{2}\sin(20000\pi t)$ V has an internal resistance of 50 Ω. The RMS value of the current through R is_______mA (rounded off to one decimal place).



Ans. (100)

Given, $V_s = 10\sqrt{2}\sin(20000\pi t)$ V

$$\text{Current, } I = \frac{V_s}{50 + 25 + 25 + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{V_s}{100 + j20000\pi \times \frac{10^{-3}}{20\pi} + \frac{1}{j20000\pi \times \frac{10^{-3}}{20\pi}}}$$

$$= \frac{V_s}{100 + j1 - j1}$$

$$= \frac{V_s}{100}$$

$$I = \frac{10\sqrt{2}}{100}\sin(20000\pi t)$$

$$\therefore \quad I_{RMS} = \frac{\frac{10\sqrt{2}}{100}}{\sqrt{2}} = 0.1 = 100 \text{ mA}$$

End of Solution

*This option did not appear in GATE 2023 Examination. It appeared as "Turbine flow meter", that is, "Turbine flow meter" option was repeated.

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