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Detailed Solutions

ESE-2019 Mains Test Series

E & T Engineering Test No : 7

Section A : Signals and Systems + Electronic Measurements and Instrumentation

Q.1 (a) Solution:

Given that:

Applied voltage, $V = 150$ V ; Current, $I = 5$ A ; Power, $P = VI = 150 \times 5 = 750$ W

Now, $P = VI$

$$\frac{\partial P}{\partial V} = I = 5\text{ A} \; ; \quad \frac{\partial P}{\partial I} = V = 150\text{ V}$$

Uncertainties, $W_V = 0.1$ V and $W_I = 0.05$ A

$$\text{Uncertainty in power} = \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 W_V^2 + \left(\frac{\partial P}{\partial I}\right)^2 W_I^2}$$

$$= \sqrt{(5)^2\,(0.1)^2 + (150)^2\,(0.05)^2} = \pm 7.52\text{ W}$$

Power dissipated in resistance,

$$P = (750 \pm 7.52)\text{ W}$$

$$\text{Uncertainty in power (in \%)} = \pm\frac{7.52}{750}\times 100 \simeq 1\%$$

Q.1 (b) Solution

Given data:

Anode voltage, $E_a = 2000$ V

Length of deflecting plates, $l_d = 1.5\text{ cm} = 1.5 \times 10^{-2}$ m

Distance between deflecting plates, $d = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$

Distance between screen and the center of the deflecting plates,

$$L = 50\text{ cm} = 0.5\text{ m}$$

(i) Velocity of beam, $$v_{ox} = \sqrt{\frac{2eE_a}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2000}{9.1\times10^{-31}}} = 26.5 \times 10^6\text{ m/sec}$$

(ii) Deflection sensitivity, $$S = \frac{Ll_d}{2dE_a} = \frac{0.5\times1.5\times10^{-2}}{2\times5\times10^{-3}\times2000} = 0.375\text{ mm/V}$$

(iii) Deflection factor, $$G = \frac{1}{S} = \frac{1}{0.375} = 2.66\text{ V/mm}$$

Q.1 (c) Solution

$$V(z) = X(z) + \frac{1}{2}z^{-1}V(z)$$

$$V(z) = \frac{X(z)}{1-\frac{1}{2}z^{-1}}$$

$$Y(z) = 2[3X(z) + V(z)] + 2z^{-1}V(z) = 6X(z) + 2(1+z^{-1})V(z)$$

$$= \left(6 + \frac{2(1+z^{-1})}{1-0.5z^{-1}}\right)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{8-z^{-1}}{1-0.5z^{-1}}$$

By taking inverse z-transform, we get,

$$h(n) = 8(0.5)^n u(n) - (0.5)^{n-1}u(n-1)$$

Q.1 (d) Solution:

Given,

$$x(t) = \text{rect}\,[t - 1/2]$$

$$x(t) = \begin{cases} 1 & ;\ 0 \le t \le 1 \\ 0 & ;\ \text{otherwise} \end{cases}$$

$\because$ $$F[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$\therefore$ $$= \int_0^1 1\,e^{-j\omega t}\,dt = \frac{1}{j\omega}\left(1-e^{-j\omega}\right)$$

$$= \frac{2}{\omega}\left[\frac{e^{j\omega/2}-e^{-j\omega/2}}{2j}\right]\times e^{-j\omega/2}$$

$$F[x(t)] = \frac{\sin\omega/2}{\omega/2}\, e^{-j\omega/2}$$

and

$$x(-t) = \begin{cases} 1 & ;\ -1 \le t \le 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

$$F[x(-t)] = \int_{-\infty}^{\infty} x(-t)e^{-j\omega t}dt = \int_{-1}^{0} 1\cdot e^{-j\omega t}dt = \frac{1}{-j\omega}\left(e^{-j\omega t}\right)_{-1}^{0}$$

$$= \frac{1}{j\omega}(e^{j\omega} - 1) = \frac{1}{j\omega}\left[e^{j\omega/2} - e^{-j\omega/2}\right]e^{j\omega/2}$$

$$= \frac{2}{\omega}\left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right]e^{j\omega/2}$$

$$F[x(-t)] = \frac{\sin\omega/2}{\omega/2}\, e^{j\omega/2}$$

Therefore,

$$F[x(t) + x(-t)] = \frac{\sin\omega/2}{\omega/2}\,[e^{-j\omega/2} + e^{j\omega/2}]$$

$$= \frac{\sin\omega/2}{\omega/2}\, 2\cos\omega/2 = 2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$$

Q.1 (e) Solution:

$$x[n] = \{-2, 0, 1, \underset{\uparrow}{0}, 3, 0, 3, 2, 1, 0, -1\}$$

(i)

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

∴

$$X(0) = \sum_{n=-\infty}^{\infty} x[n] = 7$$

(ii) Using central ordinate theorem;

$$\int_{-\pi}^{\pi} X(\Omega)d\Omega = 2\pi x[0] = 0$$

(iii)

$$X(\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\pi} = \sum_{n=-\infty}^{\infty} x[n](-1)^n$$

$$= 2 - 1 - 3 - 3 + 2 - 1 + 1 = -3$$

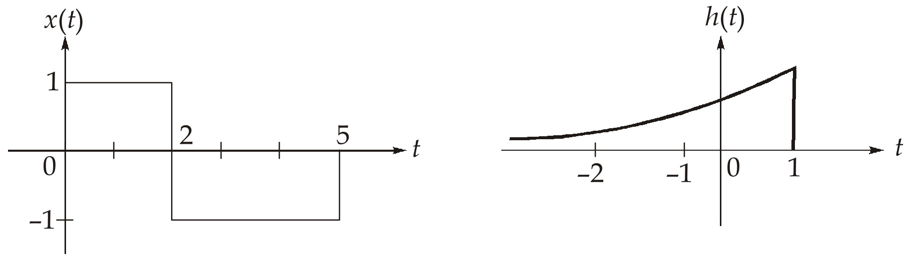
(iv) Using Parseval's theorem,

$$\int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= 2\pi[4 + 1 + 9 + 9 + 4 + 1 + 1] = 58\,\pi$$

Q.2 (a) Solution:

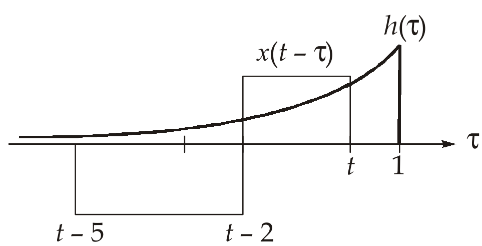
Given that, $h(t) = e^{2t}\,u(1-t)$ and $x(t) = u(t) - 2u(t-2) + u(t-5)$



$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{2\tau}\,u(1-\tau)[u(t-\tau)-2u(t-2-\tau)+u(t-5-\tau)]\,d\tau$$

- **For $t \le 1$,**



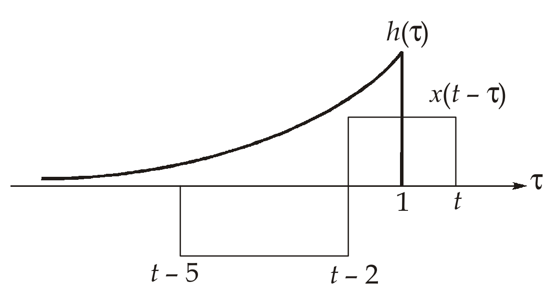
$$y(t) = \int_{t-5}^{t} e^{2\tau}[u(t-\tau)-2\,u(t-2-\tau)+u(t-5-\tau)]\,d\tau$$

$$= \int_{t-5}^{t-2} (-1)\,e^{2\tau}d\tau + \int_{t-2}^{t} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)} - e^{2(t-5)}\right] + \frac{1}{2}\left[e^{2t} - e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[1-2e^{-4}+e^{-10}\right]e^{2t}\ ;\ t<1$$

- **For $1 < t \le 3$ or $(t-2) \le 1 < t$,**



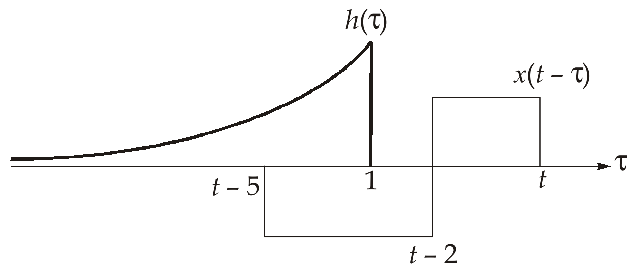
$$y(t) = \int_{t-5}^{t-2} -e^{2\tau}d\tau + \int_{t-2}^{1} e^{2\tau}d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-2)} - e^{2(t-5)}\right] + \frac{1}{2}\left[e^{2} - e^{2(t-2)}\right]$$

$$y(t) = \frac{1}{2}\left[-2e^{-4} + e^{-10}\right]e^{2t} + \frac{1}{2}e^{2} \; ; 1 < t < 3$$

- **For $3 < t \le 6$ or $(t - 5) \le 1 < (t - 2)$,**

$$y(t) = \int_{t-5}^{1} -e^{2\tau}d\tau = \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right] \; ; \; 3 < t < 6$$



- **For $t - 5 > 1$ or $t > 6$,**

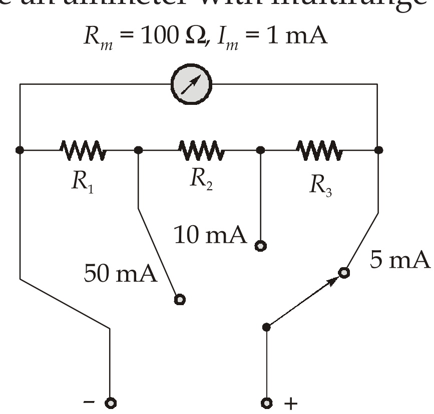
$$y(t) = 0 \; ; \; t > 6$$

The total solution is given by,

$$y(t) = \begin{cases} \frac{1}{2}\left[1 - 2e^{-4} + e^{-10}\right]e^{2t} & ; \quad t \le 1 \\ \frac{1}{2}\left[-2e^{-4} + e^{-10}\right]e^{2t} + \frac{1}{2}e^{2} & ; \quad 1 < t \le 3 \\ \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right] & ; \quad 3 < t \le 6 \\ 0 & ; \quad t > 6 \end{cases}$$

Q.2 (b) Solution:

The Aryton shunt to provide an ammeter with multirange can be given by,



- For 0 - 5 mA range,

$$I_m = 1 \text{ mA and } I_{sh} = 5 \text{ mA} - 1 \text{ mA} = 4 \text{ mA}$$
$$R_{sh} I_{sh} = I_m R_m$$
$$R_{sh} = R_1 + R_2 + R_3$$

So,
$$R_1 + R_2 + R_3 = \frac{100 \times 10^{-3}}{4 \times 10^{-3}} = 25\,\Omega \quad \text{...(i)}$$

- For 0 - 10 mA range,

$$I_m = 1 \text{ mA and } I_{sh} = 10 \text{ mA} - 1 \text{ mA} = 9 \text{ mA}$$
$$R_{sh} I_{sh} = I_m(R_m + R_3)$$
$$R_{sh} = R_1 + R_2$$

So,
$$R_1 + R_2 = \frac{(100\,\Omega + R_3)}{9} \quad \text{...(ii)}$$

From equations (i) and (ii), we get,

$$25\,\Omega - R_3 = \frac{100\,\Omega + R_3}{9}$$
$$10R_3 = 25 \times 9 - 100 = 125\,\Omega$$
$$R_3 = 12.5\,\Omega$$

So,
$$R_1 + R_2 = 25 - R_3 = 12.5\,\Omega \quad \text{...(iii)}$$

- For 0 - 50 mA range,

$$I_m = 1 \text{ mA and } I_{sh} = 50 \text{ mA} - 1 \text{ mA} = 49 \text{ mA}$$
$$I_{sh} R_{sh} = I_m(R_m + R_3 + R_2)$$
$$R_{sh} = R_1$$

So,
$$R_1 = \frac{100\,\Omega + 12.5\,\Omega + R_2}{49} = \frac{112.5\,\Omega + R_2}{49} \quad \text{...(iv)}$$

From equations (iii) and (iv), we get,

$$12.5\,\Omega - R_2 = \frac{112.5\,\Omega + R_2}{49}$$
$$50R_2 = 12.5\,\Omega \times 49 - 112.5\,\Omega = 500\,\Omega$$
$$R_2 = 10\,\Omega$$

From equation (iii), we get,

$$R_1 = 12.5\,\Omega - R_2 = 2.5\,\Omega$$

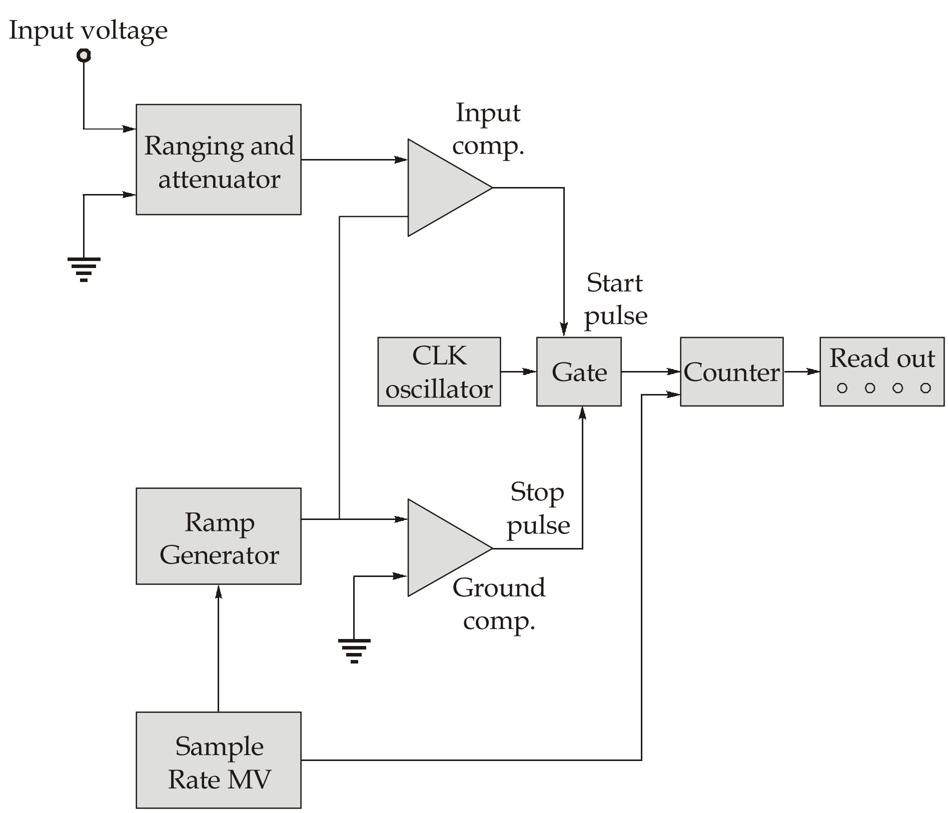
- So, the values of shunt resistors required for the desired Aryton shunt are, $R_1 = 2.5\,\Omega$, $R_2 = 10\,\Omega$ and $R_3 = 12.5\,\Omega$

Q.2 (c) Solution:

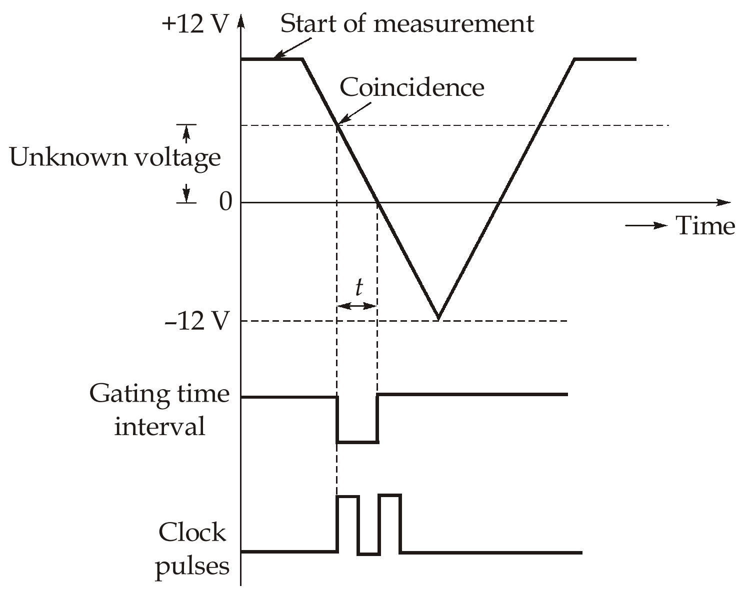
Ramp type DVM

The operating principle of a ramp type digital voltmeter is to measure, the time that a linear ramp voltage takes to change from level of input voltage to zero voltage (or vice-versa). This time intervals is measured with an electronic time interval counter and the count is displayed as a number of digits on electronic indicating tubes of the output readout of the voltmeter. The ramp voltage value is continuously compared with the voltage being measured (unknown voltage).

At the instant the value of ramp voltage is equal to that of unknown voltage a coincidence circuit, called an input comparator, generates a pulse which opens a gate. The ramp voltage continues to decrease till it reaches zero voltage. At this instant another comparator called ground comparator generates a pulse and close the gate. The time elapsed between opening and closing of the gate is 't' as indicate in figure. During this time interval pulses from a clock pulse generator pass through the gate and are counted and displayed.



Block diagram of a ramp DVM

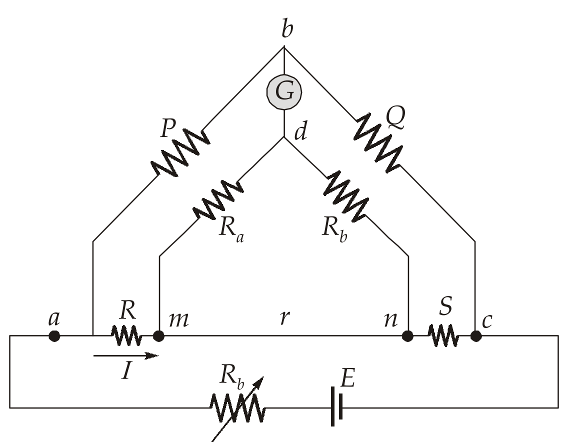


Timing diagram showing voltage to time conversion

Q.3 (a) Solution:

Kelvin's Double Bridge:

Resistances having a value under 1 Ω is known as low resistances. The resistance of leads and contacts, though small, are appreciable in comparison in the case of low resistances.



The Kelvin's Double bridge incorporates the idea of a second set of ratio arms – hence the name double bridge and the use of four terminal resistors for the low resistance arms. The first of ratio arms is P and Q. The second set of ratio arms R_a and R_b is used to connect the galvanometer to a point d at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistance r between the known resistance R, and the standard resistance S.

The ratio R_a/R_b is made equal to P/Q. Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E_{ab} is equal to the voltage drop E_{amd}.

Now, $$E_{ab} = \frac{P}{P+Q} \cdot E_{ac}$$

and $$E_{ac} = I\left[R + S + \frac{(R_a + R_b)r}{R_a + R_b + r}\right] \quad \text{... (i)}$$

and $$E_{amd} = I\left[R + \frac{R_a}{R_a + R_b}\left\{\frac{(R_a + R_b)r}{R_a + R_b + r}\right\}\right] = I\left[R + \frac{R_a r}{R_a + R_b + r}\right] \quad \text{... (ii)}$$

For zero galvanometer deflection,

or $$\frac{P}{P+Q} \cdot I\left[R + S + \frac{(R_a + R_b)r}{R_a + R_b + r}\right] = I\left[R + \frac{R_a r}{R_a + R_b + r}\right]$$

or $$R = \frac{P}{Q} \cdot S + \frac{R_b r}{R_a + R_b + r}\left[\frac{P}{Q} - \frac{R_a}{R_b}\right] \quad \text{... (iii)}$$

Now if $\frac{P}{Q} = \frac{R_a}{R_b}$ equation (iii) becomes

$$R = \frac{P}{Q} \cdot S$$

As the expression of R does not involve r hence the effect of r (contact resistance of leads) is eliminated.

From given question,

Here, $$\frac{R_4}{R_2} = \frac{R_b}{R_a}$$

Therefore, $$\frac{R_4}{R_2} = \frac{R_b}{R_a} = \frac{1}{1000}$$

since, $$R_1 = 0.5\, R_2$$

$$R_2 = \frac{5}{0.5} = 10\ \Omega$$

Therefore, $$\frac{R_4}{10} = \frac{1}{1000}$$

$$R_4 = 10 \times \frac{1}{1000} = 0.01\ \Omega$$

Q.3 (b) Solution:

(i) $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n)$

Taking the DTFT of the above equation, we get,

$$Y(e^{j\omega}) - \frac{3}{4}Y(e^{j\omega})e^{-j\omega} + \frac{1}{8}Y(e^{j\omega})e^{-j2\omega} = 2X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}\right) = 2X(e^{j\omega})$$

The frequency response of the system is,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}$$

$$= \frac{2}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Using the partial fraction expansion, we get,

$$H(e^{j\omega}) = \frac{4}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} - \frac{2}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

By taking the inverse DTFT of the above equation, the impulse response of the system can be given by,

$$h(n) = 4\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{4}\right)^n u(n)$$

(ii) The DTFT of the input $x(n) = \left(\frac{1}{4}\right)^n u(n)$ can be given by,

$$X(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

$$Y(e^{j\omega}) = H(e^{j\omega})\,X(e^{j\omega})$$

$$= \frac{4}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)} - \frac{2}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2}$$

Using the partial fraction expansion, we get,

$$Y(e^{j\omega}) = \frac{8}{\left(1-\frac{1}{2}e^{-j\omega}\right)} - \frac{4}{\left(1-\frac{1}{4}e^{-j\omega}\right)} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

Considering the following DTFT pairs,

$$a^n u(n) \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\omega}}$$

$$(n+1)\, a^n u(n+1) \xleftrightarrow{DTFT} \frac{1}{(1-ae^{-j\omega})^2}$$

So, by taking the inverse DTFT of $Y(e^{j\omega})$, the response of the system for the given input will be,

$$y(n) = 8\left(\frac{1}{2}\right)^n u(n) - 4\left(\frac{1}{4}\right)^n u(n) - 2(n+1)\left(\frac{1}{4}\right)^n u(n+1)$$

Q.3 (c) Solution:

Given that, $f_c = 1\text{ kHz and } f_s = 5\text{ kHz}$

So, $$\omega_c = 2\pi f_c T = \frac{2\pi f_c}{f_s} = \frac{2\pi \times 1000}{5000} = \frac{2\pi}{5}$$

Therefore, $$H_d(e^{j\omega}) = \begin{cases} 1\,; & |\omega| \le \frac{2\pi}{5} \\ 0\,; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-2\pi/5}^{2\pi/5} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-2\pi/5}^{2\pi/5} = \frac{\sin\left(\frac{2\pi}{5}n\right)}{\pi n}$$

The rectangular window, for the length of impulse response of 7, can be given by,

$$w_R(n) = \begin{cases} 1\,; & -3 \le n \le 3 \\ 0\,; & \text{otherwise} \end{cases}$$

The filter coefficients can be given by,

$$h(n) = h_d(n)\, w_R(n)$$

$$h(0) = h_d(0)w_R(0) = \lim_{n \to 0} \frac{\sin\frac{2\pi}{5}n}{\pi n} = \frac{2}{5} = 0.40$$

$$h(1) = h(-1) = \frac{\sin\frac{2\pi}{5}}{\pi} = 0.3027$$

$$h(2) = h(-2) = \frac{\sin\frac{4\pi}{5}}{2\pi} = 0.0935$$

$$h(3) = h(-3) = \frac{\sin\frac{6\pi}{5}}{3\pi} = -0.0624$$

$$h(n) = \{-0.0624, 0.0935, 0.3027, \underset{\uparrow}{0.40}, 0.3027, 0.0935, -0.0624\}$$

The transfer function of the filter is,

$$H(z) = -0.0624z^3 + 0.0935z^2 + 0.3027z + 0.40 + 0.3027z^{-1} + 0.0935z^{-2} - 0.0624z^{-3}$$

But this filter is non-causal, which is practically non-realizable. The transfer function of the practically realizable filter can be given by,

$$H'(z) = z^{-3}H(z)$$

$$= -0.0624 + 0.0935z^{-1} + 0.3027z^{-2} + 0.40z^{-3} + 0.3027z^{-4} + 0.0935z^{-5} - 0.0624z^{-6}$$

Q.4 (a) Solution:

Given, impulse response, $h[n] = \begin{cases} a^n; & n \geq 0 \\ 0; & n < 0 \end{cases}$

Also we can write,

$$h[n] = a^n u[n]$$

$$x[n] = \begin{cases} 1; & 0 \leq n \leq N-1 \\ 0; & \text{otherwise} \end{cases}$$

Also we can write,

$$x[n] = u[n] - u[n-N]$$

by taking z-transform,

$$H(z) = \frac{1}{1-az^{-1}}; \quad |z| > |a|$$

$$X(z) = \frac{1-z^{-N}}{1-z^{-1}}; \qquad \text{All } z \text{ (by shifting property)}$$

Therefore, $$Y(z) = X(z)H(z)$$

$$= \frac{1}{1-az^{-1}} \cdot \frac{1-z^{-N}}{1-z^{-1}} = \frac{1-z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

$$= \frac{1}{(1-z^{-1})(1-az^{-1})} - \frac{z^{-N}}{(1-z^{-1})(1-az^{-1})}$$

The ROC is $|z| > |a|$. Consider

$$P(z) = \frac{1}{(1-z^{-1})(1-az^{-1})} = \frac{A}{1-z^{-1}} + \frac{B}{1-az^{-1}}$$

For, $$A = \left.\frac{1}{1-az^{-1}}\right|_{z^{-1}=1} = \frac{1}{1-a}$$

$$B = \left.\frac{1}{1-z^{-1}}\right|_{z^{-1}=\frac{1}{a}} = \frac{1}{1-a^{-1}}$$

$\therefore$ $$P(z) = \frac{\frac{1}{1-a}}{1-z^{-1}} + \frac{\frac{1}{1-a^{-1}}}{1-az^{-1}}$$

Therefore, $$p(n) = \frac{1}{1-a}u[n] + \frac{1}{1-a^{-1}}a^n u[n]$$

Now, note that,

$$Y(z) = P(z)\left[1-z^{-N}\right]$$

by taking inverse z-transform

$$y[n] = p(n) - P[n-N]$$

$$y[n] = \frac{1}{1-a}\{u[n]-u[n-N]\} + \frac{1}{1-a^{-1}}\left\{a^n u[n] - a^{n-N}u[n-N]\right\}$$

This may write as,

$$y[n] = \begin{cases} 0 & ; \quad n < 0 \\ \frac{(a^n - a^{-1})}{(1-a^{-1})} & ; \quad 0 \le n \le N-1 \\ \frac{a^n(1-a^{-N})}{(1-a^{-1})} & ; \quad n > N-1 \end{cases}$$

Q.4 (b) Solution:

(i)

Given: Currents, $I_{FSD} = 50$; $I_{MFSD} = 5$; Resistance, $R = 0.09\ \Omega$; Inductance, $L = 90\ \mu H$

Multiplying power of shunt, $m = \dfrac{I_{FSD}}{I_{MFSD}} = \dfrac{50}{5} = 10$

In order that the meter may read correctly at all frequencies, the time constants of meter and shunt circuits should be equal. Under this condition multiplying power,

$$m = 1 + \frac{R}{R_{sh}}$$

$\therefore$ Shunt resistance,
$$R_{sh} = \frac{R}{m-1} = \frac{0.09}{10-1} = 0.01\ \Omega$$

Also,
$$\frac{L}{R} = \frac{L_{sh}}{R_{sh}}$$

$\therefore$ Inductance of shunt,

$$L_{sh} = \frac{L}{R} R_{sh} = \frac{90}{0.09} \times 0.01 = 10\ \mu H$$

With DC, the current through the meter for a total current of 50 A is,

$$I_{MFSD} = \frac{R_{sh}}{R + R_{sh}} \times I_{FSD} = \frac{0.01}{0.09 + 0.01} \times 50 = 5\ A$$

With 50 Hz, the current through the meter for a total current of 50 A is,

$$I_{MFSD} = \frac{R_{sh}}{\sqrt{(R + R_{sh})^2 + \omega^2 L^2}} \times I$$

$$= \frac{0.01}{\sqrt{(0.09 + 0.01)^2 + (2\pi \times 50 \times 90 \times 10^{-6})^2}} \times 50 = 4.81\ A$$

Since the meter reading is proportional to the current,

$$\text{Error} = \frac{4.81 - 5}{5} \times 100 = -3.8\%$$

or meter reads 3.8% low.

(ii)

Voltage across instrument for full scale deflection = 100 mV.

Current in instrument for full scale deflection,

$$I = \frac{V}{R} = \frac{100 \times 10^{-3}}{20} = 5 \times 10^{-3}\ A$$

Deflecting torque,
$$T_d = NBldI = 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3}$$
$$T_d = 375 \times 10^{-6} \times B\ \ \Omega\text{-m}$$

∴ Controlling torque for a deflection, $\theta = 120°$

$$T_c = K\theta = 0.375 \times 10^{-6} \times 120 = 45 \times 10^{-6} \text{ N-m}$$

At steady state position, $T_d = T_c$

or, $375 \times 10^{-6} \times B = 45 \times 10^{-6}$

∴ Flux density in the air gap,

$$B = \frac{45\times10^{-6}}{375\times10^{-6}} = 0.12 \text{ Wb/m}^2$$

Q.4 (c) Solution:

Given, Permittivity $\in_o = 8.85 \times 10^{-12}$ F/m ; Plates area $A = 500$ mm^2 = 500×10^{-6} m^2

Separation $d = 0.2$ mm = 0.2×10^{-3} m.

Initial capacitance, $$C = \frac{\in_o A}{d} = \frac{8.85\times10^{-12}\times500\times10^{-6}}{0.2\times10^{-3}} \text{ F}$$

$$C = 22.125 \text{ pF}$$

(i) Change in displacement,

$$\Delta d = 0.2 - 0.18 = 0.02 \text{ mm}$$

Capacitance after application of displacement,

$$C + \Delta C = \frac{8.85\times10^{-12}\times500\times10^{-6}}{0.18\times10^{-3}} \text{ F} = 24.583 \text{ pF}$$

Change in capacitance, $\Delta C = 24.583 - 22.125 = 2.458$ pF

$$\text{Ratio} = \frac{\frac{\Delta C}{C}}{\frac{\Delta d}{d}} = \frac{\frac{(2.458)}{(22.125)}}{\frac{(0.02)}{(0.2)}} = 1.111$$

(ii) Initially the displacement between the plates is 0.2 mm. Since the thickness of mica is 0.01 mm, the length of air gap between the plates = 0.2 – 0.01 = 0.19 mm.

Initial capacitance of transducer,

$$C = \frac{\in_o A}{\frac{d_1}{\in_1} + \frac{d_2}{\in_2}} = \frac{8.85\times10^{-12}\times500\times10^{-6}}{\left(\frac{0.19}{1}+\frac{0.01}{8}\right)\times10^{-3}} = 23.137 \text{ pF}$$

When a displacement of 0.02 mm is applied the length of air gap is reduced to (0.19 – 0.02) = 0.17 mm.

∴ Capacitance with displacement applied,

$$= \frac{8.85\times10^{-12}\times500\times10^{-6}}{\left(\frac{0.17}{1}+\frac{0.01}{8}\right)\times10^{-3}} = 25.839 \text{ pF}$$

Change in capacitance, $\Delta C = 25.839 - 23.137 = 2.702$ pF

$$\text{Ratio} = \frac{\frac{\Delta C}{C}}{\frac{\Delta d}{d}} = \frac{\left(\frac{2.702}{23.137}\right)}{\left(\frac{0.02}{0.2}\right)} = 1.168$$

Section B : Electromagnetics-1 + Basic Electrical Engineering-1 + Analog Circuits-2 + Materials Science-2

Q.5 (a) Solution:

We have to verify,

$$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} \quad \text{...(i)}$$

taking L.H.S

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & z^2 & yz \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial x}z^2 - \frac{\partial}{\partial y}xz\right)\hat{a}_z + \left(\frac{\partial}{\partial y}(yz) - \frac{\partial}{\partial z}(z^2)\right)\hat{a}_x - \left(\frac{\partial}{\partial x}yz - \frac{\partial}{\partial z}xz\right)\hat{a}_y$$

$$= (z-2z)\hat{a}_x + x\hat{a}_y = -z\hat{a}_x + x\hat{a}_y \quad \text{...(ii)}$$

$$\therefore \quad \nabla\times(\nabla\times\vec{A}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z & x & 0 \end{vmatrix}$$

$$= 0\cdot\hat{a}_x - (+1)\hat{a}_y + \hat{a}_z = -\hat{a}_y + \hat{a}_z \quad \text{...(iii)}$$

now considering R.H.S

$$\nabla\cdot\vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$$

$$\nabla\cdot\vec{A} = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(z^2) + \frac{\partial}{\partial z}yz = z + y$$

$$\nabla(\nabla\cdot\vec{A}) = \frac{\partial}{\partial x}A\,\hat{a}_x + \frac{\partial}{\partial y}A\,\hat{a}_y + \frac{\partial}{\partial z}A\,\hat{a}_z$$

$$\therefore \quad = \frac{\partial}{\partial x}(z+y)\,\hat{a}_x + \frac{\partial}{\partial y}(z+y)\,\hat{a}_y + \frac{\partial}{\partial z}(z+y)\,\hat{a}_z$$

$$= \hat{a}_y + \hat{a}_z \quad \text{...(iv)}$$

$$\nabla^2 \vec{A} = \nabla^2 A_x \hat{a}_x + \nabla^2 A_y \hat{a}_y + \nabla^2 A_z \hat{a}_z$$

and

$$\nabla^2 \vec{A} = \left[\frac{\partial^2}{\partial x^2}(xz) + \frac{\partial^2}{\partial y^2}(xz) + \frac{\partial^2}{\partial z^2}(xz)\right]\hat{a}_x + \left[\frac{\partial^2}{\partial x^2}(z)^2 + \frac{\partial^2}{\partial y^2}(z)^2 + \frac{\partial^2}{\partial z^2}(z)^2\right]\hat{a}_y + \left[\frac{\partial^2}{\partial x^2}(yz) + \frac{\partial^2}{\partial y^2}(yz) + \frac{\partial^2}{\partial z^2}(yz)\right]\hat{a}_z$$

$$\nabla^2 \vec{A} = 2\hat{a}_y \quad \text{...(v)}$$

From equation (iv) and (v), we get,

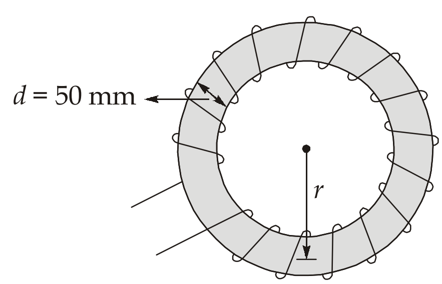
$$\nabla \cdot (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \hat{a}_y + \hat{a}_z - 2\hat{a}_y = \hat{a}_z - \hat{a}_y \quad \text{...(vi)}$$

∴ From equation (iii) and (vi),

$$\text{RHS} = \text{LHS} = \hat{a}_z - \hat{a}_y$$

Hence Proved.

Q.5 (b) Solution:



$$\text{Length} = 2\pi r = 2\pi \times 150 \times 10^{-3} \text{ m} = 0.942 \text{ m}$$

$$\text{Area of cross-section} = \pi r^2$$

$$= \pi \times (25 \times 10^{-3})^2 = 1.963 \times 10^{-3} \text{ m}^2$$

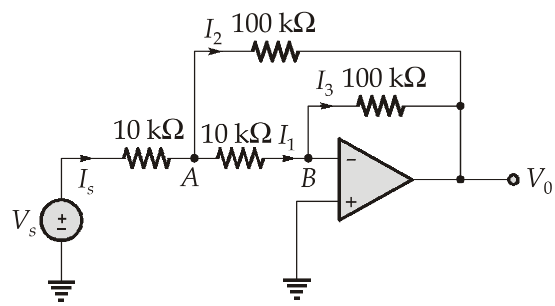
Inductance,

$$L = \frac{N^2}{\text{Reluctance}} = \frac{N^2}{l/\mu_o \mu_r A}$$

$$L = \frac{(500)^2}{\left[\dfrac{0.942}{4\pi \times 10^{-7} \times 1.963 \times 10^{-3}}\right]} \quad [\because \mu_r = 1 \text{ for air}]$$

$$L = 6.54 \times 10^{-4} \text{ H}$$

Q.5 (c) Solution:



Now, point B is at virtually ground potential and input current of op-amp is zero as $R_{in} = \infty$.

$$\frac{V_A}{10\times10^3} = \frac{-V_0}{100\times10^3}$$

$$V_A = \frac{-V_0}{10}$$

Now, applying KCL at node A,

$$I_s = I_1 + I_2$$

$$\frac{V_s - V_A}{10\times10^3} = \frac{V_A - V_B}{10\times10^3} + \frac{V_A - V_0}{100\times10^3}$$

Now, $V_B = 0$ and $V_A = \dfrac{-V_0}{10}$.

$$\frac{V_s + \left(\frac{V_0}{10}\right)}{10\times10^3} = -\frac{V_0}{10^5} - \frac{V_0}{10^6} - \frac{V_0}{10^5}$$

$$V_s = -0.1V_0 - 0.01V_0 - 0.1V_0 - 0.1V_0 = -0.31V_0$$

$$\therefore \quad \frac{V_0}{V_s} = \frac{-1}{0.31} \approx -3.226$$

Q.5 (d) Solution

The state of material at which resistivity reduces to zero is called superconductivity. The temperature at which there is transition from normal state to superconducting state is called transition temperature. Above critical temperature (T_c) the material is in the familiar normal state but below (T_c) it enters an entirely different superconducting state, resistance of these materials in the superconducting state is at least 10^{16} times smaller than their room temperature values.

Two independent conditions of superconductivity are
1. Zero resistivity.
2. Perfect diamagnetism.

Properties

(i) At room temperature, the resistivity 'ρ' of superconducting materials are greater than other elements.
(ii) All thermo electric effects disappear in superconducting state.
(iii) When a sufficient strong magnetic field is applied to superconductor below critical temperature T_c, its superconducting property is destroyed.
(iv) When current is passed through the superconducting material the heat loss (I^2R) is zero.
(v) The magnetic flux density in superconductor is zero.

Q.5 (e) Solution

The magnetization for a paramagnetic spin system is given by,

$$M = \frac{Np_B^2\mu_0 H}{kT}$$

Magnetization per spin in Bohr magnetons becomes

$$M' = \frac{M}{Np_B} = \frac{p_B \mu_0 H}{kT}$$

Where, p_B = Bohr magnetron

$= 9.27 \times 10^{-24}$ Am^2

∵ $\mu_0 = 4\pi \times 10^{-7}$ H/m

$H = 10^6$ A/m

k – Boltzmann's constant = 1.38×10^{-23} J/K

T – Temperature = 300 K (given)

So,
$$M' = \frac{9.27\times10^{-24}\times4\pi\times10^{-7}\times10^{6}}{1.38\times10^{-23}\times300}$$

$= 2.81 \times 10^{-3}$ Bohr magneton

Q.6 (a) Solution

In the free space, the velocity of the wave is 3×10^8 m/s.

So,
$$\beta = \frac{\omega}{3\times10^8} = \frac{2\pi\times400\times10^6}{3\times10^8} = \frac{8\pi}{3} \text{ rad/sec}$$

The direction cosines of the wave normal are,

$$\cos\phi_x = \cos 60° = \frac{1}{2}, \quad \cos\phi_y = \cos 45° = \frac{1}{\sqrt{2}}$$ and $\cos\phi_z$ is unknown

For a wave normal, $\cos^2\phi_x + \cos^2\phi_y + \cos^2\phi_z = 1.$

So, $$\cos^2\phi_z = 1-\frac{1}{4}-\frac{1}{2}=\frac{1}{4} \Rightarrow \cos\phi_z = \frac{1}{2} \qquad \because \phi_z < 90°$$

The unit vector along the direction of the wave normal is,

$$\hat{n} = \frac{1}{2}\hat{x}+\frac{1}{\sqrt{2}}\hat{y}+\frac{1}{2}\hat{z}$$

The electric field in the free space can be given by,

$$\vec{E} = \vec{E}_0\, e^{-j\beta\hat{n}\cdot\vec{r}} = \vec{E}_0\, e^{-j\frac{8\pi}{3}\left(\frac{x}{2}+\frac{y}{\sqrt{2}}+\frac{z}{2}\right)}$$

Here, $\vec{E}_0$ is a constant vector perpendicular to the wave normal.

$$\vec{E}_0 = E_{0x}\hat{x}+E_{0y}\hat{y}+E_{0z}\hat{z}$$

Since, the wave is linearly polarized E_{0x}, E_{0y} and E_{0z} should be in phase. Therefore, without loosing generality, let us assume them to be real.

Since, for uniform plane wave $\vec{E}_0$ is perpendicular to $\hat{n}$, we have

$$\hat{n}\cdot\vec{E}_0 = \frac{E_{0x}}{2}+\frac{E_{0y}}{\sqrt{2}}+\frac{E_{0z}}{2}=0 \qquad \text{...(i)}$$

It is given that, $\left|\vec{E}_0\right| = 10\text{ V/m}$ and $E_{0x} = 2E_{0y}$

So, $$E_{0x} = 2E_{0y} \qquad \text{...(ii)}$$

$$E_{0x}^2+E_{0y}^2+E_{0z}^2 = 100 \qquad \text{...(iii)}$$

By solving the equations (i), (ii) and (iii), we get,

$$E_{0x} = \pm 4.9\text{ V/m},\ E_{0y} = \pm 2.45 \text{ and } E_{0z} = \mp 8.37\text{ V/m}$$

So, two different solutions exist for $\vec{E}$.

$$\vec{E} = (4.9\hat{x}+2.45\hat{y}-8.37\hat{z})\, e^{-j\frac{8\pi}{3}\left(\frac{x}{2}+\frac{y}{\sqrt{2}}+\frac{z}{2}\right)}\ \text{V/m}$$

or $$\vec{E} = -(4.9\hat{x}+2.45\hat{y}-8.37\hat{z})\, e^{-j\frac{8\pi}{3}\left(\frac{x}{2}+\frac{y}{\sqrt{2}}+\frac{z}{2}\right)}\ \text{V/m}$$

Q.6 (b) Solution:

Given: Iron losses = 200 W ; Secondary current = I_2 ; $V_2 I_2 \cos\phi_2 = 20 \times 10^3$ W

$$I_2 = \frac{20\times10^3}{V_2} \qquad [\because \text{ Assuming pure resistive as given in question}]$$

$$I_2 = \frac{20\times10^3}{220} = 90.91 \text{ A}$$

Equivalent resistance referred to secondary,

$$R_{eq} = 2.1\times\left(\frac{N_2}{N_1}\right)^2 + 0.026 = 0.0514\ \Omega$$

$$\text{Total copper loss} = I_2^2\, R_{eq} = (90.91)^2 \times 0.0514 = 424.8 \text{ W}$$

(i)

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$$

$$\text{Efficiency at full load and 0.5 p.f. lagging} = \frac{20\times10^3\times0.5}{(20\times10^3\times0.5)+200+424.8}$$

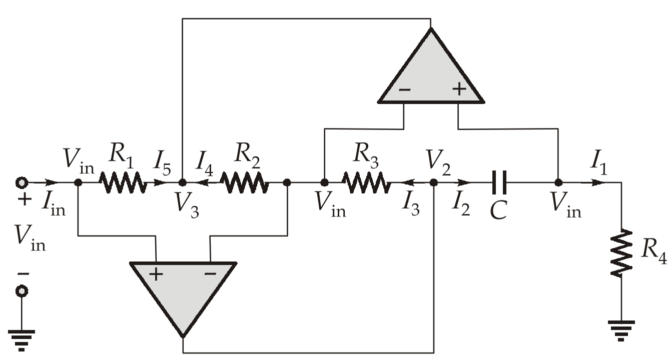
$$\eta = 94.12\%$$

(ii) Efficiency at half load and 0.8 p.f. leading

$$\eta = \frac{\frac{1}{2}\times20\times10^3\times0.8}{\left(\frac{1}{2}\times20\times10^3\times0.8\right)+200+424.8(0.5)^2} = 96.31\%$$

Q.6 (c) Solution

Let us calculate the equivalent impedance of the op-amp circuit in parallel to C_{in}, as follows:



Now, $$I_1 = \frac{V_{in}}{R_4}$$

Thus, $$I_2 = I_1 = \frac{V_{in}}{R_4}$$

$\Rightarrow$ $$V_2 = V_{in} + \frac{V_{in}}{sCR_4}$$

$\Rightarrow$ $$I_3 = \frac{V_{in}}{sR_4R_3C}$$

Now, $$I_3 = I_4 = \frac{V_{in}}{sCR_4R_3}$$

$$V_3 = V_{in} - \frac{V_{in}R_2}{sCR_4R_3}$$

Thus, $$I_5 = \frac{V_{in}R_2}{sCR_4R_3R_1}$$

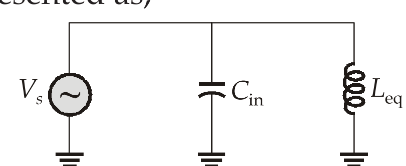
$\Rightarrow$ $$I_5 = I_{in}$$

$$I_{in} = \frac{V_{in}R_2}{sCR_4R_3R_1}$$

$\Rightarrow$ $$Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = s\left[\frac{CR_4R_3R_1}{R_2}\right] = sL_{eq}$$

where, $$L_{eq} = \left[\frac{CR_4R_3R_1}{R_2}\right] = \frac{100\times10^{-9}\times(10^4)^3}{10^4} = 10\text{ H}$$

Thus an equivalent model of the given circuit can be represented as,



$\therefore$ $$f = \frac{1}{2\pi\sqrt{L_{eq}C_{in}}}$$

$$\frac{100\times10^3}{\pi} = \frac{1}{2\pi\sqrt{10\times C_{in}}}$$

$$200\times10^3 = \frac{1}{\sqrt{10\times C_{in}}}$$

$$10\times C_{in} = \left(\frac{10^{-3}}{200}\right)^2 = (5\times10^{-6})^2$$

$$C_{in} = \frac{25\times10^{-12}}{10} = 2.5\times10^{-12} = 2.5\text{ pF}$$

Q.7 (a) Solution:

We have, $V = 100e^{-50x}\sin 50y$ V

(i) As we know,

$$\vec{E} = -\nabla V$$

$$= -\frac{\partial}{\partial x}\left[100e^{-50x}\sin 50y\right]\hat{a}_x - \frac{\partial}{\partial y}\left[100e^{-50x}\sin 50y\right]\hat{a}_y - \frac{\partial}{\partial z}\left[100e^{-50x}\sin 50y\right]\hat{a}_z$$

$$= 5000(e^{-50x}\sin 50y\,\hat{a}_x - e^{-50x}\cos 50y\,\hat{a}_y) \quad ...(i)$$

Also, $\vec{D} = \varepsilon_0\vec{E}$

$\therefore$ $\nabla\cdot\vec{D} = \varepsilon_0\cdot\nabla\cdot\vec{E}$ ...(ii)

$$= \varepsilon_0\frac{\partial}{\partial x}(5000\,e^{-50x}\sin 50y) + \frac{\partial}{\partial y}(-5000\,e^{-50x}\cos 50y)$$

$$= 5000\,\varepsilon_0\left[-50\,e^{-50x}\sin 50y + 50e^{-50x}\sin 50y\right]$$

$$= 0$$

(ii) $V = 100e^{-50x}\sin 50y$ V

For $y = 0$, $V = 0$

Since, V is constant and independent of x and z.

$\therefore$ $y = 0$, is an equipotential surface.

(iii) $\vec{E}$ (at $y = 0$), from equation (i), we have,

$$\vec{E} = 5000\,e^{-50x}\left[\sin 50y\,\hat{a}_x - \cos 50y\,\hat{a}_y\right]$$

For $y = 0$, $\vec{E} = 5000\,e^{-50x}\left[\sin 0\,\hat{a}_x - \cos 0\,\hat{a}_y\right] = -5000\,e^{-50x}\,\hat{a}_y$

$\therefore$ $\vec{E}$ at $y = 0$ is perpendicular to the plane $y = 0$.

(iv) $\vec{D} = \varepsilon_0\vec{E} = -5000\,\varepsilon_0\,e^{-50x}\,\hat{a}_y$ C/m^2

and $\rho_s = \left|\vec{D}\right| = 5000\,\varepsilon_0\,e^{-50x}$ C/m^2

$$\because \qquad Q = \int \rho_s\, ds = \int_0^{\infty}\int_0^{1} \rho_s\, dx\, dz$$

$$\therefore \qquad = \int_0^{\infty}\int_0^{1} \varepsilon_0\, 5000\, e^{-50x}\, dx\, dz$$

$$= 5000\,\varepsilon_0 \left(\frac{e^{-50x}}{-50}\right)_0^{\infty} \cdot (z)_0^1$$

$$= 5000\,\varepsilon_0 \left(\frac{1}{50}\right) \times 1 = 100\,\varepsilon_0$$

or $\qquad Q = 0.8854$ nC

Since, $y < 0$ is the conductor interior, this charge should be negative.

(v)

$$W_E = \frac{1}{2}\int \varepsilon_0 |E|^2\, dV$$

$$= \frac{\varepsilon_0}{2}\int_0^1\int_0^1\int_0^1 \left|\vec{E}\right|^2 dx\, dy\, dz$$

$$= \frac{\varepsilon_0}{2}\int_0^1\int_0^1\int_0^1 (5000 e^{-50x})^2 (\sin^2 50y + \cos^2 50y)\, dx dy dz \text{ J}$$

$$= \frac{\varepsilon_0}{2} \times 25 \times 10^6 \int_0^1\int_0^1\int_0^1 e^{-100x}\, dx\, dy\, dz \text{ J}$$

$$= 12.5 \times 10^6\, \varepsilon_0 \left(\frac{e^{-100x}}{-100}\right)_0^1 (y)_0^1\, (z)_0^1 \text{ J}$$

$$= 12.5 \times 10^6 \times 8.854 \times 10^{-12} \times \left(\frac{1 - e^{-100}}{100}\right) \text{J} = 1.10675\ \mu\text{J}$$

Q.7 (b) Solution:

(i)

$$E_2 = \frac{50}{\sqrt{3}} = 28.867 \text{ V}$$

At standstill with slip-rings short circuited,

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{28.867}{\sqrt{(0.4)^2 + (2)^2}} = 14.15 \text{ A}$$

(ii) The total resistance in the rotor circuit is 5.4 Ω per phase.

$$\therefore \qquad I_2 = \frac{E_2}{\sqrt{(R_2+r)^2+X_2^2}} = \frac{28.867}{\sqrt{(5.4)^2+4}} = 5.01 \text{ A}$$

(iii) Full load slip $= s = \dfrac{N_s - N_r}{N_s}$

where, $$N_s = \frac{120\times f}{P} = \frac{120\times 50}{4} = 1500 \text{ rpm}$$

$$s = \frac{1500-1440}{1500} = 0.04$$

$\therefore$ Rotor Emf = 28.867 × 0.04 = 1.155 V/ph

(iv) $$I_2 = \frac{sE_2}{\sqrt{R^2+(sX_2)^2}} = \frac{1.155}{\sqrt{(0.4)^2+(0.04\times 2)^2}}$$

$$I_2 = 2.83 \text{ A}$$

(v) Rotor power factor at full load $= \dfrac{R_2}{\sqrt{R_2^2+(sX_2)^2}} = \dfrac{0.4}{\sqrt{(0.4)^2+(0.04\times 2)^2}}$

$$= 0.98 \text{ (lagging)}$$

Q.7 (c) Solution:

(i) The equation for i_y current can be given as

$i_y = I_y + B_0 + B_1\cos(\omega t) + B_2(\cos(2\omega t) + B_3\cos(3\omega t) + \ldots\ldots\ldots\ldots$

Thus, the given current equation for i_y with that, we obtain

$$I_y + B_0 = 35 \text{ mA}$$

$$B_1 = 17 \text{ mA}$$

$$B_2 = 10 \text{ mA}$$

and $$B_3 = 0.3 \text{ mA}$$

Now, the total distortion or distortion factor 'D' is computed.

$$D = \sqrt{D_2^2 + D_3^2}$$

$$= \sqrt{\left(\frac{B_2}{B_1}\right)^2 + \left(\frac{B_3}{B_1}\right)^2} = \sqrt{\left(\frac{10}{17}\right)^2 + \left(\frac{0.3}{17}\right)^2} = 0.5885$$

Thus, the total distortion is 58.85% of the fundamental component.

(ii) Power delivered at fundamental frequency ω_0 is obtained from

$$P_1 = \frac{B_1^2 R_L}{2} = \frac{(17\times10^{-3})^2\times2.2\times10^3}{2}$$
$$= 0.3179 \text{ W} = 317.9 \text{ mW}$$

(iii) The total power delivered to the load is approximately equal to

$$P = (1 + D^2)P_1 = 1 + (0.5885)^2 \times 317.9 \text{ mW}$$
$$\approx 428 \text{ mW}$$

Q.8 (a) Solution:

Given, $$\vec{E}_1' = 120e^{-j4\pi z}\,\hat{a}_x \text{ V/m}$$

and $$\vec{E}_1'' = 30\angle 50^\circ\, e^{j4\pi z}\,\hat{a}_x \text{ V/m}$$

(i) From the given expression, we get,

Phase constant, $\beta = 4\pi$

where, $$\beta = \omega\sqrt{\mu\varepsilon}$$

For free space,

$$\sqrt{\mu\varepsilon} = \sqrt{\mu_0\varepsilon_0} = \sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi}\times10^{-9}} = 3.33\times10^{-9}$$

or $$\omega = \frac{\beta}{\sqrt{\mu_0\varepsilon_0}} = \frac{4\pi}{\sqrt{\mu_0\varepsilon_0}} = 3.769\times10^9 \text{ rad/s}$$

and $$f = \frac{\omega}{2\pi} = 599.85 \text{ MHz}$$

(ii) The reflection coefficient is,

$$\Gamma = \frac{|E_1''|}{|E_1'|} = \frac{30\angle 50^\circ}{120} = 0.25\angle 50^\circ$$

But reflection coefficient is,

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

where, η_1 = intrinsic impedance of free space

$\eta_1 = 377\ \Omega$

$$\therefore \qquad \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.25\angle 50°$$

$$\frac{\left(\frac{\eta_2}{\eta_1} - 1\right)}{\left(\frac{\eta_2}{\eta_1} + 1\right)} = 0.25\angle 50°$$

or $$\frac{\eta_2}{\eta_1} = 1.367\angle 22.2°$$

or $$\eta_2 = 515\angle 22.2°\ \Omega$$

or $$\eta_2 = 477 + j194.7\ \Omega$$

Hence, the intrinsic impedance in the region $z > 0$, which would cause the given reflected wave is,

$$\eta_2 = 477 + j194.7\ \Omega$$

(iii) The total field in region 1, is addition of main and reflected fields.

The total field in region 1,

$$\therefore \qquad \vec{E}_1 = E_1' + E_1''$$

$$= 120e^{-j4\pi z} + 30\angle 50°\ e^{j4\pi z}$$

$$= 120e^{-j4\pi z}\left(1 + 0.25e^{j8\pi z} \times e^{j50°}\right)$$

At maximum value,

$$8\pi z + 50° = 0°, \pm 360°, \pm 720°, ...$$

$$1440° z + 50° = 0°, \pm 360°, \pm 720°, ...$$

$\therefore$ We have, $$z = -3.47 \text{ cm} \quad \text{and} \quad -28.5 \text{ cm}$$

Q.8 (b) Solution:

(i)

The magnetic field produced by the solenoid,

$$H = \frac{NI}{l} = \frac{1000 \times 2.5}{0.25} = 10000 \text{ A/m}$$

The increase in magnetic induction when placed in oxygen

$$= 1.04 \times 10^{-8} \text{ Wb/m}^2 = \mu_0 M$$

This increase is due to magnetization (M)

$$M = \frac{1.04\times10^{-8}}{4\pi\times10^{-7}} = 8.276\times10^{-3}\text{ A/m}$$

Magnetic susceptibility, $$\chi_m = \frac{M}{H} = \frac{8.276\times10^{-3}}{10^4} = 8.276\times10^{-7}$$

(ii)

A nanoparticle of Si can be made by laser evaporation of a Si substrate in the region of a helium gas pulse. The beam of neutral clusters is photolyzed by a UV laser producing ionized clusters whose mass to charge ratio is then measured in a mass spectrometer. The most striking property of nanoparticles made of semiconducting elements is the pronounced changes in their optical properties compared to those of bulk material.

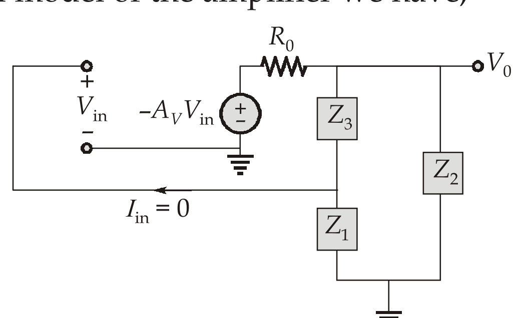
In a bulk semiconductor a bound electron-hole pair, called an exciton, can be produced by a photon having an energy greater than that of the bandgap of material. The photon excites an electron from the filled band to the unfilled band above. The separation between the hole and the electron is many lattice parameters.

The existence of the exciton has a strong influence on the electronic properties of the semiconductor and its optical absorption. The exciton can be modeled as a hydrogen-like atom and has energy levels with relative spacings analogous to the energy levels of the hydrogen atom but lower actual energies. Light induced transitions between these hydrogen like energy levels produce a series of optical absorptions.

When the nanoparticle becomes smaller than or comparable to the radius of the orbit of the electron-hole pair, there are two situations, called weak-confinement and the strong-confinement regimes. In the weak regime the particle radius is larger than the radius of the electron-hole pair, but the range of motion of the exciton is limited, which causes the blue shift of the absorption spectrum. When the radius of the particle is smaller than the orbit radius of the electron-hole pair, the motion of the electron and the hole become independent and the excitation does not exist. The hole and electron have their own set of energy levels. Here there is also a blue shift, and the emergence of a new set of absorption lines.

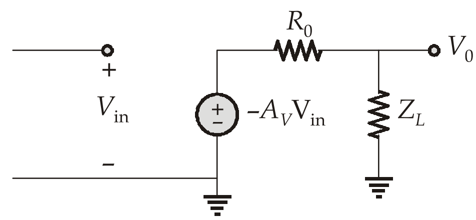
Q.8 (c) Solution:

Drawing the small signal model of the amplifier we have,



$\because \qquad I_{in} = 0;$

The above circuit can be reduced as

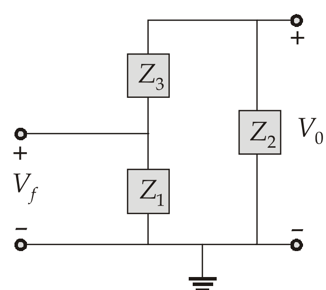


Thus, the overall gain of the amplifier,

$$A = \frac{V_0}{V_{in}} = \frac{-A_V Z_L}{Z_L + R_0}$$

where, $$Z_L = \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_2 + Z_3)}$$

For the feedback circuit,



The feedback gain, $$\beta = \frac{V_f}{V_0} = \frac{Z_1}{Z_1 + Z_3}$$

$\because$ The phase shift of the feedback circuit is negative.

$\therefore$ $$A\beta = \frac{-A_V Z_1 Z_L}{(R_0 + Z_L)(Z_1 + Z_3)} = \frac{-A_V Z_1 \left[\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right]}{\left[R_0 + \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}\right](Z_1+Z_3)}$$

$$= \frac{-A_V Z_1 Z_2}{R_0(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)}$$

Now, $Z_1 = jX_1$, $Z_2 = jX_2$ and $Z_3 = jX_3$

$\Rightarrow$ $$A\beta = \frac{A_V(X_1 X_2)}{jR_0(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

To produce sustained oscillations the phase shift of the loop gain $A\beta$ should be 0°.

Thus, $$R_0(X_1 + X_2 + X_3) = 0$$

$$\Rightarrow \quad X_1 + X_2 + X_3 = 0$$

$$(X_1 + X_3) = -X_2$$

$$\therefore \quad A\beta = \frac{-A_V X_1}{(X_1 + X_3)}$$

$$\Rightarrow \quad A\beta = \frac{A_V X_1}{X_2}$$

Hence X_1 and X_2 should be of the same type of reactance.

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