DETAILED SOLUTIONS

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ESE 2023 : Prelims Exam
CLASSROOM TEST SERIES

CIVIL ENGINEERING

Test 8

Section A : CPM PERT + Hydrology & Water Resource Engineering [All Topics]

Section B : Design of Steel Structure-I + Surveying and Geology-I [Part Syllabus]

Section C : Solid Mechanics-II [Part Syllabus]

1.	(b)	16.	(b)	31.	(d)	46.	(c)	61.	(b)
2.	(d)	17.	(*)	32.	(a)	47.	(c)	62.	(a)
3	(b)	18.	(d)	33.	(c)	48.	(b)	63.	(d)
4.	(a)	19.	(b)	34.	(b)	49.	(c)	64.	(b)
5.	(c)	20.	(d)	35.	(d)	50.	(*)	65.	(b)
6.	(d)	21.	(c)	36.	(a)	51.	(b)	66.	(b)
7.	(b)	22.	(c)	37.	(c)	52.	(b)	67.	(d)
8.	(c)	23.	(a)	38.	(a)	53.	(b)	68.	(b)
9.	(a)	24.	(d)	39.	(a)	54.	(d)	69.	(a)
10.	(d)	25.	(d)	40.	(c)	55.	(b)	70.	(c)
11.	(c)	26.	(c)	41.	(d)	56.	(d)	71.	(d)
12.	(c)	27.	(b)	42.	(d)	57.	(c)	72.	(a)
13.	(d)	28.	(c)	43.	(d)	58.	(d)	73.	(c)
14.	(a)	29.	(c)	44.	(d)	59.	(d)	74.	(d)
15.	(a)	30.	(c)	45.	(c)	60.	(b)	75.	(d)

- **Revised key/solution 30-11-2022 [Q.17, Q.50 marks to all]**

DETAILED EXPLANATIONS

1. (b)

According to central limit theorem, if there are various activities in a project network which follow β-distribution of frequency, then entire project is assumed to follow normal distribution of frequency.

So, the distribution curve for the time taken to complete each activity of a project resembles a β-distribution curve and distribution curve for the time taken to complete entire project in general resembles a normal distribution curve.

2. (d)

In ladder network, the duration will be the sum of duration of activity having maximum duration and $\frac{1}{n}$ of the sum of durations of remaining activities, where 'n' is the number of equal sub-parts.

$$\therefore \quad \text{Modified total duration} = 50 + \frac{1}{5}(10 + 5 + 35 + 10) = 62 \text{ days}$$

3 (b)

$$\text{Range of project duration} = (\text{Minimum time, maximum time})$$

$$\text{Minimum time} = T_E - 3\sigma = 25.6 \text{ days} \quad \text{...(i)}$$

$$\text{Maximum time} = T_E + 3\sigma = 60.4 \text{ days} \quad \text{...(ii)}$$

From (i) and (ii)

$$2T_E = 86$$

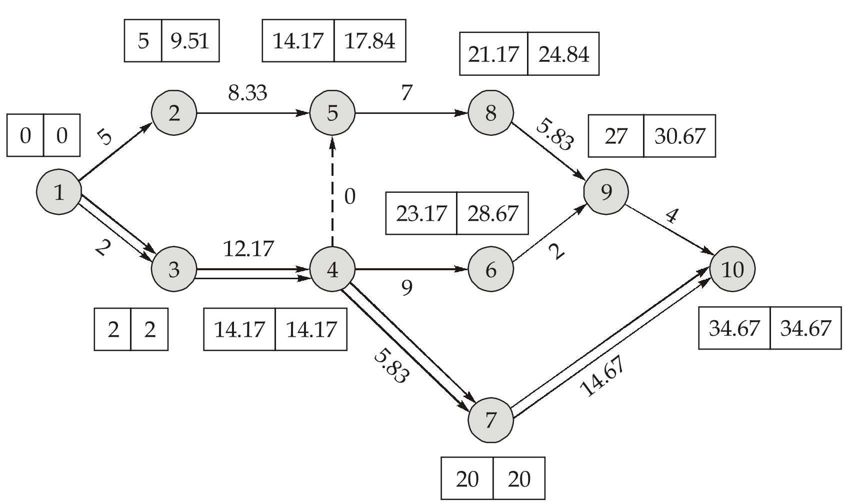
$$T_E = 43 \text{ days}$$

Note:

$$\text{Normal range of the project duration} = (T_E - \sigma) \text{ to } (T_E + \sigma)$$

$$\text{Maximum range of the project duration} = (T_E - 3\sigma) \text{ to } (T_E + 3\sigma)$$

4. (a)



∴ The expected duration of the project is 34.67 days.

5. (c)

Probability of completion of project on 20th day = (Probability of completion in 20 days) – (Probability of completion in 19 days)

Probability of completion in 20 days = 50%

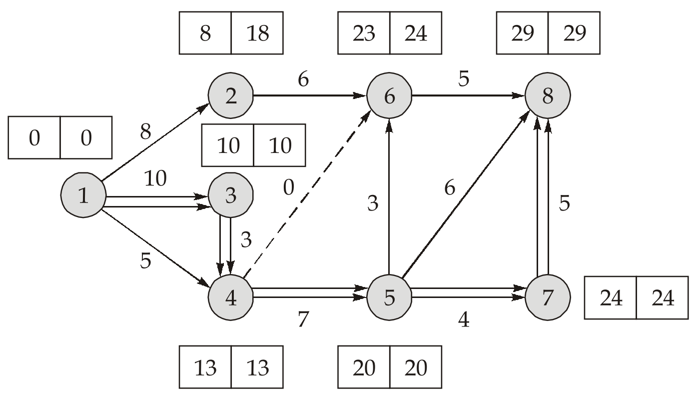
For probability of completion in 19 days,

$$Z = \frac{19-20}{3} = -\frac{1}{3} = -0.33$$

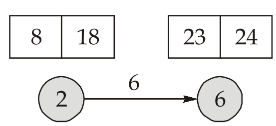
∴ Probability of completion in 19 days = 100 – 63.33 = 36.67%

∴ Probability of completion on 20th day = 50 – 36.67 = 13.33%

6. (d)



For activity 2 – 6,



EST = 8 EFT = 8 + 6 = 14

LST = 24 – 6 = 18 LFT = 24

Total float (F_T) = 18 – 8 = 24 – 14 = 10

Free float (F_F) = F_T – Slack at head event

= 10 – 1 = 9

Independent float (F_{ID}) = F_F – Slack at tail event

= 9 – (10) = –1

Interfering float (F_{IN}) = $F_T - F_F$ = 10 – 9 = 1

7. (b)

By sum of years digit method, depreciation at the end of m^{th} year

$$D_m = \left[\frac{n-m+1}{\frac{n(n+1)}{2}}\right](C_i - C_s)$$

$$= \left[\frac{5-m+1}{\frac{5(5+1)}{2}}\right](12000-2000) = \left(\frac{6-m}{15}\right)\times 10000$$

End of year (m)	Rate of depreciation	Depreciation per year (Rs.)	Total depreciation (Rs.)	Book value (Rs.)
0	–	–	–	12000
1	$\frac{5}{15}\times 10000$	3333	3333	8667
2	$\frac{4}{15}\times 10000$	2667	6000	6000

8. (c)

Effective drawbar pull = Available drawbar pull – Rolling resistance ± Grade resistance [upgrade (–); downgrade (+)]

$\Rightarrow$ 2000 = Available pull – (45 × 12) – $\left(\frac{2}{100}\times 12000\right)$

$\Rightarrow$ Available pull = 2780 kg

While operating on level surface with rolling resistance of 75 kg/tonne,

Effective drawbar pull = 2780 – (75 × 12) = 1880 kg

11. (c)

From water balance equation

$$\text{Inflow} - \text{Outflow} = \frac{d}{dt}(\text{Storage})$$

At maximum storage, $\frac{ds}{dt} = 0$

$\Rightarrow$ Inflow = Outflow

12. (c)

For a catchment, runoff coefficient is constant

$$C = \frac{\text{Runoff}}{\text{Precipitation}} = \frac{R}{P} = \text{Constant}$$

$$\therefore \quad \frac{R_1}{P_1} = \frac{R_2}{P_2}$$

$$\Rightarrow \quad \frac{12}{24} = \frac{R_2}{16}$$

$$\Rightarrow \quad R_2 = 8 \text{ mm}$$

13. (d)

Average method of rainfall measurement is useful when rain gauges are evenly distributed.

15. (a)

$$\because \quad \frac{1}{2} \times Q_P \times T_B \times 3600 = C_A \times 0.01$$

$$\Rightarrow \quad \frac{1}{2} \times 250 \times 36 \times 3600 = C_A \times 0.01$$

$$\Rightarrow \quad C_A = 1620 \text{ km}^2$$

16. (b)

$$Q_e = 2.778 \times \frac{A}{D}$$

$$= 2.778 \times \frac{300 \text{ km}^2}{4} = 208.35 \text{ m}^3/\text{s}$$

17. (*)

P(cm)	3	1
t(hr)	0 – 2	2 – 4
i(cm/hr)	1.5	0.5

For, $\phi \le i \Rightarrow$ Runoff = 0

$$P_e = 3 \text{ cm}, \quad t_e = 2 \text{ cm}$$

$$\phi = \frac{P_e - R}{t_e}$$

$$\Rightarrow \quad 1.5 = \frac{3 - R}{2}$$

$$\Rightarrow \quad R = 0 \text{ cm}$$

18. (d)

$$\text{Risk} = 1 - q^n$$

where, $q = 1 - p$

$$\Rightarrow \quad q = 1 - \frac{1}{T}$$

$$\therefore \quad \text{Risk} = 1 - \left(1 - \frac{1}{T}\right)^n$$

19. (b)

As per Sir Inglis formula

$$Q = \frac{124A}{\sqrt{A + 10.4}}$$

$$\Rightarrow \quad Q = \frac{124 \times 5.6}{\sqrt{5.6 + 10.4}} = \frac{124 \times 5.6}{4} = 173.6 \text{ m}^3/\text{s}$$

20. (d)

$$n = \text{Total number of available discharges}$$
$$m = \text{Order / rank of observation (discharges)}$$

1. California formula, $T_r = \dfrac{n}{m}$

2. Hazen's formula, $T_r = \dfrac{n}{m-0.5}$

3. Weibull's formula, $T_r = \dfrac{n+1}{m}$

4. Gumbel's formula, $T_r = \dfrac{n}{m+c-1}$

(c = Gumbel's correction)

21. (c)

The basic principles followed in slope-area method is

1. Area of flow is known.
2. Velocity of flow is evaluated using Manning's equation.
3. Water surface slope is observed.
4. Manning's n is known.

22. (c)

$$\because \quad C_0 + C_1 + C_2 = 1$$
$$\Rightarrow \quad C_0 = 1 - C_1 - C_2$$
$$= 1 - 0.45 - 0.7 = -0.15$$

23. (a)

$$\text{Sodium absorption ratio (SAR)} = \frac{Na^+}{\sqrt{\dfrac{Ca^{++} + Mg^{++}}{2}}} = \frac{33}{\sqrt{\dfrac{4.4+3.6}{2}}} = 16.5$$

For SAR between : 10 to 18,
Water is classified as medium sodium water [S2]
For electrical conductivity between 750 to 2250 micro-mhos/cm,
Water is classified as high salinity water [C3]

24. (d)

Given, Field capacity = 23%

$$G_s = 1.63$$
$$d = 0.6 \text{ m}, \quad Q = 80 \text{ m}^3/\text{hour}$$
$$\because \quad d_w = \frac{\gamma_d}{\gamma_w} \times d \times [FC - M]$$

Here, $M = 0.3 \times \text{F.C.}$

$= 0.3 \times 0.23 = 0.069$

$\therefore \quad d_w = G_s \cdot d \cdot [\text{FC} - M]$

$= 1.63 \times 0.6 \times [0.23 - 0.069]$

$= 0.157 \text{ m} = 157 \text{ mm}$

$$\text{Time required } (t) = \frac{\text{Area} \times d_w}{\text{Discharge}} = \frac{35 \times 265 \times 0.157}{80} = 18.2 \text{ hours}$$

25. (d)

- A ridge canal is aligned along a watershed and it can command area on both banks.
- A canal aligned nearly parallel to the contours of the area is called a contour canal. The contour canal can irrigate only on one side.

26. (c)

$$\text{Leaching requirement } [L_R] = \frac{E_{ci}}{E_{cd}} = \frac{E_{ci}}{2E_{ce}}$$

$$= \frac{1.2}{2 \times 12} \times 100 = 5\%$$

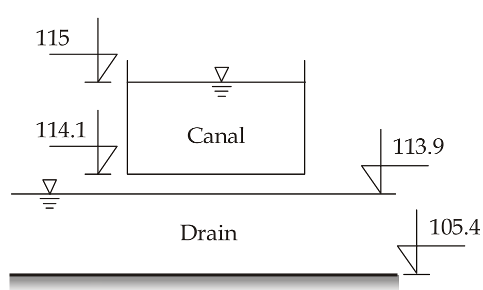
Also, $$L_R\,(\%) = \frac{D_{d \times 100}}{D_i} = \frac{D_i - D_c}{D_i} \times 100$$

$$\Rightarrow \quad 5 = \frac{D_i - 75}{D_i} \times 100$$

$$\Rightarrow \quad 5D_i = 100D_i - 7500$$

$$\Rightarrow \quad D_i = 78.947 \text{ mm} \simeq 79 \text{ mm}$$

28. (c)



Bed level of canal = 114.1 m

Fully supply level of canal = 114.1 + 0.9 = 115 m

Bed level of drain = 105.4 m

High flood level of drainage = 105.4 + 8.5 = 113.9 m

As bed level of canal is at higher level than the high flood level of drainage, so aqueduct is the structure to be adopted.

29. (c)

$$\text{Length of creep } (L_c) = 2 \times 8 + 3 + 6 + 10 \times 2$$
$$= 16 + 9 + 20 = 45 \text{ m}$$
$$\text{Head causing flow, } H = 4.5 \text{ m}$$
$$\text{Hydraulic gradient, 'i'} = \frac{H}{L_c} = \frac{4.5}{45} = 1 \text{ in } 10$$

30. (c)

Resisting shear against movement of particle (τ_c), is given as

$$\tau_c = 0.056\, \gamma_w \cdot d\, (S_s - 1)$$
$$= 0.056 \times 9.81 \times 0.005\, [2.7 - 1]$$
$$= 4.6 \times 10^{-3} \text{ kN/m}^2$$

31. (d)

$$\text{Duty } [D] = 8.64 \times \frac{B}{\Delta}$$

Here, B = Transplantation period = 15 days

Δ = Depth of irrigation water actually applied in the field = 60 – 10 = 50 cm

D = Duty on field in ha/cumec

$$\therefore \quad D = \frac{8.64 \times 15}{0.5} = 259.2 \text{ ha/cumec}$$

Now, losses in canal are 10% which means 1 cumec of water discharged at the head of water course will be reduced to 0.9 cumec at the head of the field and hence will irrigate $259.2 \times 0.9 = 233.28$ he/cumec.

Hence the duty of water at the head of distributary is 233.28 he/cumec

32. (a)

For no tension to be developed: $$B = \frac{H}{\sqrt{S_c - C}} = \frac{200 - 100}{\sqrt{2.4 - 0.4}} = 70.7 \text{ m}$$

For dam to be safe in sliding: $$B = \frac{H}{\mu[S_c - C]}$$
$$= \frac{200 - 100}{0.7[2.4 - 0.4]} = 71.43 \text{ m} \simeq 71.4 \text{ m}$$

The value of B chosen should be greater of the above two values i.e.

$$\therefore \quad B = 71.4 \text{ m}$$

33. (c)

Channel is flowing through a material which can be scoured as easily as it can be deposited. Such soil is known as incoherent alluvium.

34. (b)

As per Lacey's theory, $V = \sqrt{\frac{2}{5}R \cdot f}$

$\therefore$ $f = \frac{V^2 \times 5}{2 \times R} = \frac{(1.2)^2 \times 5}{2 \times 3} = 1.2$

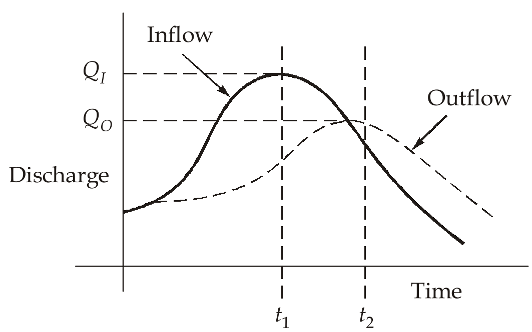
35. (d)

Khanna's module outlet is rigid module outlet.

36. (a)

- While preparing network diagram, we concentrate mainly on technological constraints (like one activity cannot start until the other is over etc.), and assume that the resources are unlimited.
- In most of the real-life projects we have resource constraints.
- A number of methods have been developed to solve this problem. These methods can broadly be classified under two categories: (i) Resource leveling, (ii) Resource smoothing

37. (c)



From graph,

$t_1 < t_2$

$\Rightarrow$ $Q_I > Q_O$

38. (a)

Rainfall records of float type and weighing bucket type gauges give plot of mass curve of rainfall. Slope of mass curve gives rainfall intensity at different intervals.

39. (a)

Exit gradient as per Khosla's theory,

$$G_E = \frac{H}{d} \cdot \frac{1}{\pi\sqrt{\lambda}}$$

{where H = head causing flow, d = length of downstream pile}

when, $d = 0$

G_E = Infinite

Thus a downstream pile is a must for preventing piping failure.

40. (c)

Silt excluder is a preventive measure while silt extractor is a curative measure.

41. (d)

Refer IS 800 : 2007 Clause 10.3.6

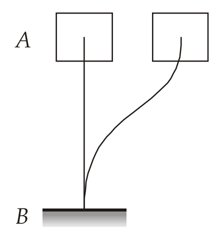
42. (d)

Refer IS 800 : 2007 Clause 10.5.7.3

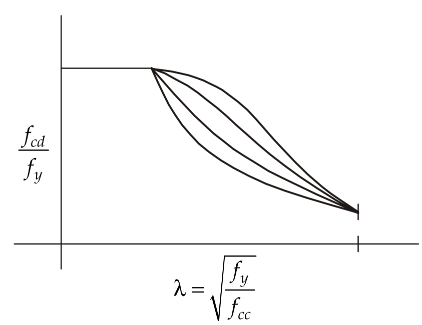
43. (d)

Column *AB*:

End	Translation	Rotation
A	Free	Restrained
B	Restrained	Restrained



44. (d)



f_{cd} = Design compressive strength

f_y = Yield stress

λ = Non-dimension effective slenderness ratio

f_{cc} = Euler's buckling stress

45. (c)

Effective slenderness ratio of the laced column is increased by 5 percent.

$$\left(\frac{kL}{r_0}\right)_{\text{laced column}} = 1.05 \times \left(\frac{kL}{r_0}\right)_{\text{column}}$$

47. (c)

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\times 1.25}[2\times A_{nb}]$$

$$= \frac{400}{\sqrt{3}\times 1.25}\left[2\times 0.78\times\frac{\pi}{4}\times 16^2\right]\text{N} = 57.95 \text{ kN} \simeq 58 \text{ kN}$$

48. (b)

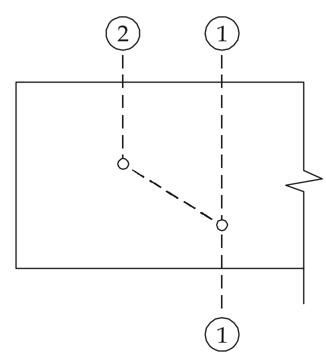
Angle between fusion faces	*k*
60° – 90°	0.7
91° – 100°	0.65
101° – 106°	0.6
107° – 113°	0.55

49. (c)

Refer IS 800 : 2007, Table 3

50. (*)

Net section rupture, $$T_{dn} = 0.9A_n \times \frac{f_u}{\gamma_{m_1}}$$



1 – 1 : $$A_n = (100 + 75 + 50 - 20) \times 10 = 2050 \text{ mm}^2$$

2 – 1 : $$A_n = \left(100+75+50-20\times 2+\frac{50^2}{4\times 75}\right)\times 10 = 1933.33 \text{ mm}^2$$

$$A_{min} = 1933.33 \text{ mm}^2$$

$$T_{dn} = \frac{0.9\times 1933.33\times 410}{1.25}\text{N} = 570.719 \text{ kN} \simeq 570.72 \text{ kN}$$

51. (b)

Number of observations = 5

$$\text{Most probable value, } '\bar{x}' = \frac{48°26'10'' + 48°26'12'' + 48°26'14'' + 48°26'16'' + 48°26'18''}{5}$$

$$= 48°\ 26'\ 14''$$

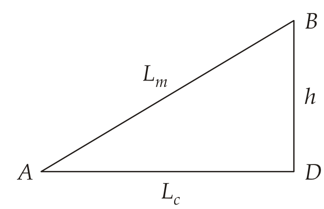
$$\text{Standard deviation } '\sigma' = \sqrt{\frac{(-4'')^2 + (-2'')^2 + (0)^2 + (2'')^2 + (4'')^2}{5-1}} = \sqrt{\frac{40}{4}} = \sqrt{10}$$

$$\text{Standard error of mean} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2} = 1.41$$

52. (b)

Let the correct length be 'L_c'

In ΔABD,



$$L_c = \sqrt{L_m^2 - h^2}$$

$$\text{Correction} = \text{True value} - \text{Measured value}$$

$$= L_c - L_m$$

$$= \sqrt{L_m^2 - h^2} - L_m$$

53. (b)

Reference line on map marked as 5 cm now measuring 4.95 cm indicates that map has shrunk. Also the chain used was 20 cm too short.

Scale (s) : 10 m to 1 cm

$$\text{Measured area} = 94.03 \text{ cm}^2$$

$$\text{Considering shrinkage of map, area} = 94.03 \times \left(\frac{5}{4.95}\right)^2$$

$$\text{Considering error in chain length, true area on map} = 94.03 \times \left(\frac{5}{4.95}\right)^2 \times \left(\frac{19.8}{20}\right)^2 = 94.03 \text{ cm}^2$$

$$\text{True area on ground} = \text{True area on map} \times S^2$$

$$= 94.03 \times 10 \times 10 = 9403 \text{ m}^2$$

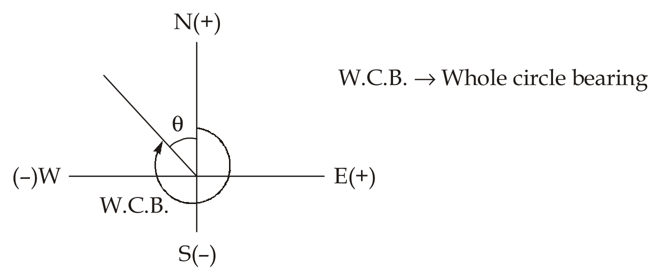
54. **(d)**

Let length of line be 'l' m

Latitude, $l \cos\theta = 57.8$ m

Departure, $l \sin\theta = -100$ m

As latitude is positive and departure is negative, line lies in N-W quadrant.



Dividing, we get

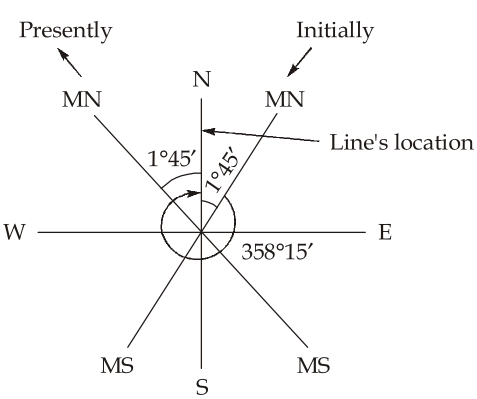
$$\tan\theta = \frac{100}{57.8} = 1.73$$

(Dont consider (–) sign as we have already located the position of line)

$$\theta = 60°$$

$$\text{Whole circle bearing} = 360° - 60° = 300°$$

55. **(b)**



$$\text{True bearing of line} = 1°45' + 358°15' = 360°$$

Now the magnetic meridian has moved 3°30′ towards west,

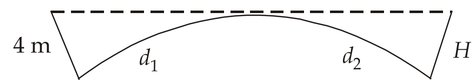
So, inclination of magnetic meridian = $3°30' - 1°45'$ W = $1°45'$ W

So, Current magnetic bearing = $1°45'$

56. (d)

$$\text{Distance, } d = 3.855\sqrt{h} \text{ km } (h \text{ i sin metres})$$

Consider height of light house as H.



$$d_1 + d_2 = 23.13 \text{ km (given)}$$

Also, $d_1 = 3.855\sqrt{4} = 7.71$ m and $d_2 = 3.855\sqrt{H}$

Substituting values

$$3.855\sqrt{H} = 23.13 - 7.71$$

$$\Rightarrow \quad \sqrt{H} = \frac{15.42}{3.855}$$

$$\Rightarrow \quad H = 16 \text{ m}$$

57. (c)

Main scale is divided into degrees and each degree into 6 parts.

$$\therefore \quad m = \frac{60'}{6} = 10' \text{ (value of 1 main scale division)}$$

Least count, L.C. = 20″

Let, $y = n$ so $x = n - 1$

(n – 1 divisions of main scale coincide with n divisions of vernier scale)

$$\text{L.C.} = \frac{m}{n}$$

$$\Rightarrow \quad 20'' = \frac{10'}{n}$$

$$\Rightarrow \quad n = \frac{10' \times 60}{20''} = 30''$$

So, $x = n - 1 = 29$ and $y = 30$

∴ 29 divisions of main scale coincide with 30 divisions of vernier scale.

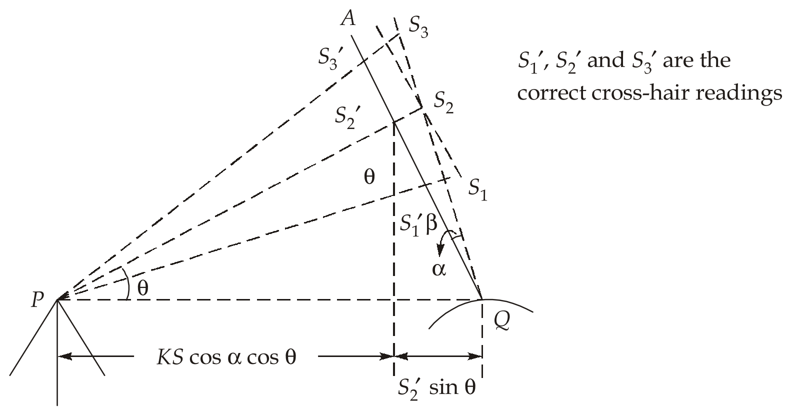
58. (d)

Let 'S' be the staff intercept in deviated position and S_1, S_2, and S_3 be the cross-hair reading on the staff as it is considered initially that staff was normal so horizontal distance 'D' measured is given by

$$D = KS\cos\theta + S_2\sin\theta$$

But actually the staff was deviated by an angle 'α'

Correct horizontal distance 'D_c' is computed as below:



$$\text{Correct staff intercept} = AB \approx S\cos\alpha$$

and $\quad S_2' = S_2\cos\alpha$

So, $\quad D_c = KS\cos\alpha\cos\theta + S_2\cos\alpha\sin\theta$

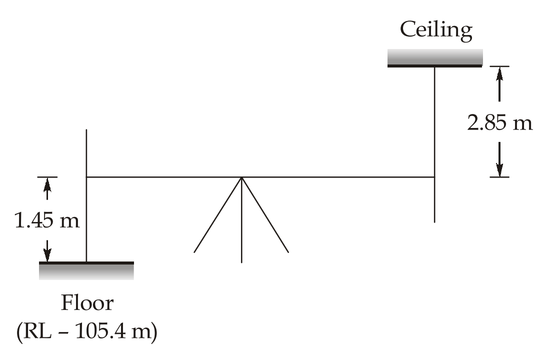
$$= D\cos\alpha$$

$$\text{Error} = \text{Measured value} - \text{True value}$$

$$= D - D\cos\alpha$$

$$= D(1-\cos\alpha)$$

59. (d)



$$\text{R.L of ceiling} = \text{R.L of floor} + 1.45 + 2.85$$

$$= 105.4 + 1.45 + 2.85 = 109.7\text{ m}$$

60. (b)

The adjustments in transit theodolite are of two types:
1. Permanent adjustment
2. Temporary adjustment

The permanent adjustments are made to establish the fixed relationships between the fundamental line of the instrument and once made, they last for a long time. These include
1. Adjustment of the horizontal plate levels.
2. Collimination adjustment.
3. Horizontal axis adjustment.
4. Adjustment of the telescope level
5. Vertical circle index adjustment

The temporary adjustments are made at each setup of the instrument before taking readings. These includes
1. Setting up of theodolite over the station
2. Levelling up the theodolite
3. Elimination of parallax
4. Focusing

61. (b)

Least count of Surveyor's compass is 15 minutes and needle used is of edge bar type.
For prismatic compass, needle used is of broad type and its least count is 30 minutes.

62. (a)

$$f_{cc} = \text{Elastic critical stress}$$

For low slenderness ratio, f_{cc} will be very high

$$f_{cc} = \frac{\pi^2 E}{\lambda^2}$$

Now,

$$\sigma_{ac} = \frac{0.6 f_{cc} f_y}{\left((f_{cc})^n + (f_y)^n\right)^{1/n}}$$

$$(f_{cc})^n >> (f_y)^n$$

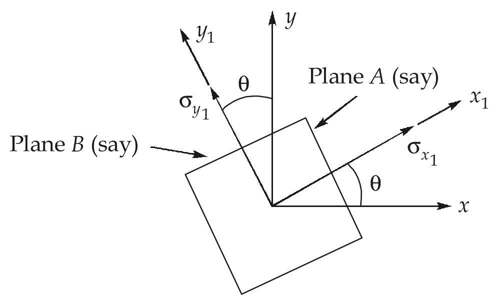
$$\sigma_{ac} \simeq \frac{0.6 f_{cc} f_y}{\left((f_{cc})^n\right)^{1/n}} = 0.6 f_y$$

∴ Failure stress depends on yield stress only.

63. (d)

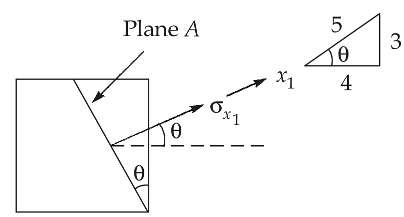
Anallactic lens is a convex lens, provided between the eye piece and the object glass at a fixed distance from the object glass. It is provided to make the additive constant equal to zero.

64. (b)



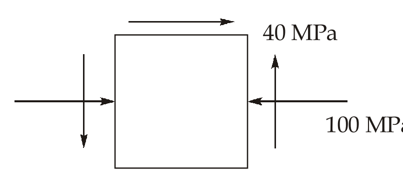
- The new axes (x_1, y_1) are normal to the planes in which the stresses are to be determined.
- σ_{x_1} acts on plane A.
- Angle between plane A and the vertical will be same as the angle between their normals.

$$\sigma_{x_1} = \tau_{xy} \sin 2\theta$$
$$= \tau_{xy} (2 \sin\theta \cos\theta)$$
$$= 26 \times 2 \times \frac{3}{5} \times \frac{4}{5}$$
$$\sigma_{x_1} = 24.96 \text{ MPa}$$



65. (b)

$$\tau_{max} = \sqrt{\left(\frac{100}{2}\right)^2 + 40^2}$$
$$= \pm\sqrt{50^2 + 40^2}$$
$$= \pm\sqrt{4100}$$
$$= \pm 10\sqrt{41} \text{ MPa}$$



66. (b)

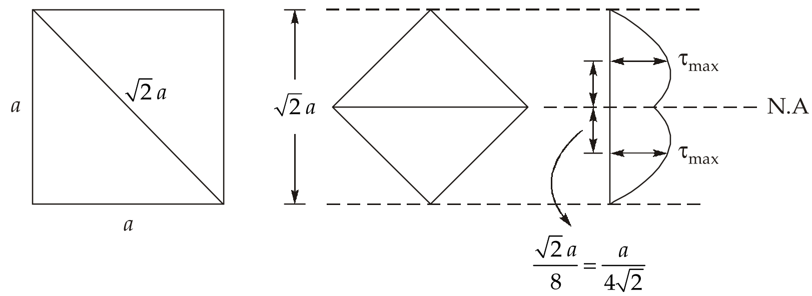
Solid circular shaft,

$$\text{Torsional strain energy per unit volume} = \frac{\tau_{max}^2}{4G}$$

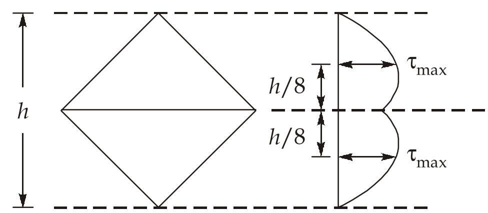
Hollow circular shaft,

$$\text{Torsional strain energy per unit volume} = \frac{\tau_{max}^2}{4G} \times \frac{R_o^2 + R_i^2}{R_o^2}$$

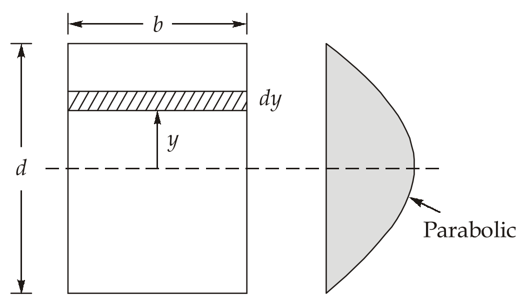
67. (d)



Note:



68. (b)



Shear force carried by the element $= \tau \cdot b\,dy$

But
$$\tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

Shear force carried by upper $\frac{1}{4}$th portion $= \int_{d/4}^{d/2} \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right) b\,dy$

$$= \frac{6V}{d^3}\left(\frac{d^2}{4}y - \frac{y^3}{3}\right)_{d/4}^{d/2}$$

$$= \frac{6V}{d^3}\left(\frac{d^3}{8} - \frac{d^3}{24} - \frac{d^3}{16} + \frac{d^3}{192}\right)$$

$$= \frac{6V}{d^3}\left(\frac{(24 - 8 - 12 + 1)d^3}{192}\right) = \frac{5V}{32}$$

Note: Shear-stress distribution in rectangular section

$$\tau = \frac{VA\bar{y}}{Ib}$$

$$A = b \times \left(\frac{d}{2} - y\right)$$

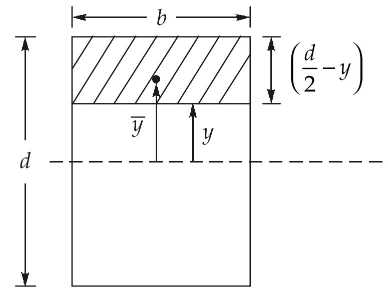
$$\bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2}$$

$$\Rightarrow \quad \bar{y} = \frac{d}{4} + \frac{y}{2} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$I = \frac{bd^3}{12}$$

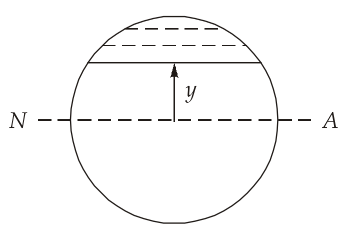
$$\tau = \frac{V \times b\left(\frac{d}{2} - y\right)\frac{1}{2}\left(\frac{d}{2} + y\right)}{\frac{bd^3}{12} \times b}$$

$$\Rightarrow \quad \tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$



Similarly shear-stress distribution in circular cross-section

$$\tau = \frac{4V}{3\pi R^4}(R^2 - y^2)$$



where R is radius of the circular cross-section.

69. (a)

Maximum principal stress theory,

$$\sigma_1 \leq \left(\frac{f_y}{\text{FOS}}\right)$$

Let the area of the bolt be 'x' mm^2

$$\text{Normal stress on the bolt} = \frac{8000}{x} \text{ N/mm}^2$$

$$\text{Shear stress on the bolt} = \frac{3000}{x} \text{ N/mm}^2$$

Maximum principal stress,
$$\sigma_1 = \frac{4000}{x} + \sqrt{\left(\frac{4000}{x}\right)^2 + \left(\frac{3000}{x}\right)^2}$$

$$\Rightarrow \quad \sigma_1 = \frac{4000}{x} + \frac{5000}{x}$$

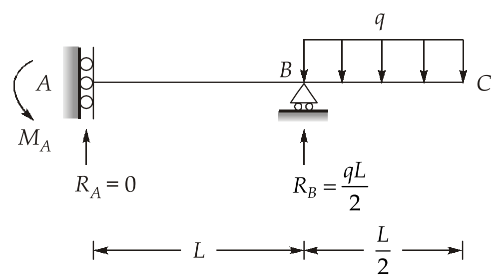
$$\Rightarrow \quad \frac{9000}{x} = \frac{270}{3}$$

$$\Rightarrow \quad x = 100 \text{ mm}^2$$

$$\therefore \quad \frac{\pi}{4}d^2 = 100$$

$$\Rightarrow \quad d = \frac{20}{\sqrt{\pi}}$$

70. (c)



$$\Sigma F_y = 0$$

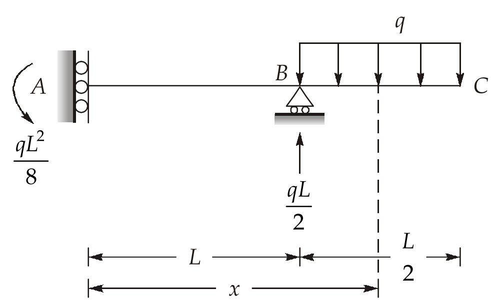
$$\Rightarrow \quad R_B - \frac{qL}{2} = 0$$

$$\Rightarrow \quad R_B = \frac{qL}{2}$$

By moment equilibrium

$$-M_A - \frac{qL}{2} \times L + \frac{qL}{2}\left(\frac{5L}{4}\right) = 0$$

$$\Rightarrow \quad M_A = -\frac{qL^2}{2} + \frac{5qL^2}{8} = \frac{qL^2}{8}$$



$$EI\,\frac{d^2y}{dx^2} = -\frac{qL^2}{8}x^0 + \frac{qL}{2}(x-L) - \frac{q(x-L)^2}{2}$$

$$EI\,\frac{dy}{dx} = -\frac{qL^2}{8}x + \frac{qL}{4}(x-L)^2 - \frac{q(x-L)^3}{6} + C_1$$

At, $x = 0$ Slope, $\left(\frac{dy}{dx}\right) = 0, \Rightarrow C_1 = 0$

$$\therefore \quad EI\,\frac{dy}{dx} = -\frac{qL^2}{8}x + \frac{qL}{4}(x-L)^2 - \frac{q(x-L)^3}{6}$$

At, $x = \frac{3L}{2}$, $\frac{dy}{dx} = \theta_C$

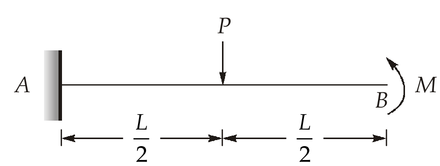
$$\therefore \quad EI\theta_C = -\frac{qL^2}{8}\left(\frac{3L}{2}\right)+\frac{qL}{4}\left(\frac{L}{2}\right)^2-\frac{q}{6}\left(\frac{L}{2}\right)^3$$

$$\Rightarrow \quad EI\theta_C = -\frac{3qL^3}{16}+\frac{qL^3}{16}-\frac{qL^3}{48}$$

$$\Rightarrow \quad EI\theta_C = \frac{(-9+3-1)qL^3}{48} = -\frac{7qL^3}{48}$$

$$\therefore \quad \theta_C = \frac{7qL^3}{48EI}$$

71. (d)



$$\delta_B = 0$$

$$\Rightarrow \quad \frac{P\left(\frac{L}{2}\right)^2}{6EI}\left(3L-\frac{L}{2}\right) = \frac{ML^2}{2EI}$$

$$\Rightarrow \quad \frac{5PL^3}{48EI} = \frac{ML^2}{2EI}$$

$$\Rightarrow \quad M = \frac{5PL}{24}$$

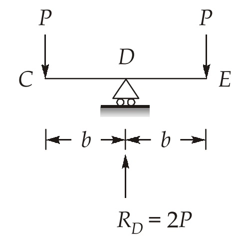
Slope, $$\theta_B = \frac{-P\left(\frac{L}{2}\right)^2}{2EI}+\frac{ML}{EI}$$

$$\Rightarrow \quad \theta_B = -\frac{PL^2}{8EI}+\frac{5PL^2}{24EI}$$

$$\Rightarrow \quad \theta_B = \frac{PL^2}{12EI}$$

72. (a)

Free body diagram of CDE:

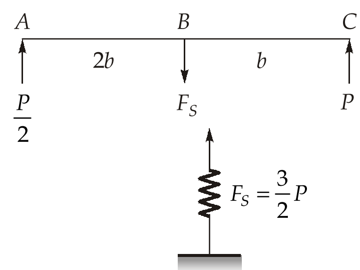


$$\Sigma M_C = 0$$

$$\Rightarrow \quad P \times 2b = R_D \times b$$

$$\Rightarrow \quad R_D = 2P$$

Free body diagram of *ABC*:



$$\Sigma M_A = 0$$

$$\Rightarrow \quad P \times 3b = F_S \times 2b$$

$$\Rightarrow \quad F_S = \frac{3}{2}P$$

$$\text{Total strain energy } (U) = U_{AB} + U_{BC} + U_{CD} + U_{DE} + U_{\text{spring}}$$

$$U_{BC} = U_{CD} = U_{DE} = \int_0^b \frac{(Px)^2\,dx}{2EI}$$

$$= \frac{P^2}{2EI}\left(\frac{x^3}{3}\right)_0^b = \frac{P^2 b^3}{6EI}$$

$$U_{AB} = \int_0^{2b} \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI} = \frac{P^2}{8EI}\left(\frac{x^3}{3}\right)_0^{2b} = \frac{P^2 b^3}{3EI}$$

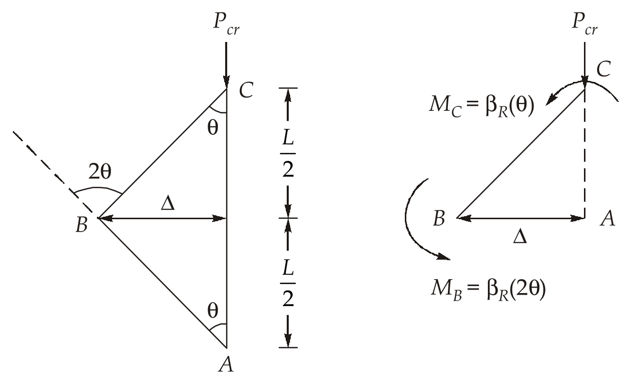
$$U_{\text{spring}} = \frac{1}{2} \times F_S \times \Delta_{\text{spring}} = \frac{1}{2} \times F_S \times \frac{F_S}{k} = \frac{1}{2} \times \left(\frac{3P}{2}\right)^2 \times \frac{b^3}{EI} = \frac{9P^2 b^3}{8EI}$$

$$\therefore \quad \text{Total strain energy } (U) = 3\left(\frac{P^2 b^3}{6EI}\right) + \frac{P^2 b^3}{3EI} + \frac{9P^2 b^3}{8EI} = \frac{47P^2 b^3}{24EI}$$

$$\delta_E = \frac{\partial U}{\partial P} = \frac{47 \times 2P \times b^3}{24EI} = \frac{47Pb^3}{12EI}$$

$$\therefore \quad \delta_E = \frac{47Pb^3}{12EI}$$

73. (c)



$$P_{cr} \times \Delta = \beta_R(\theta) + \beta_R(2\theta)$$

$$\Rightarrow \quad P_{cr} \times \frac{L}{2} \times \theta = \beta_R(3\theta)$$

$$\Rightarrow \quad P_{cr} = 6\left(\frac{\beta_R}{L}\right)$$

74. (d)

Since the transformation equations are derived solely from equilibrium of an element, they are applicable to stresses in any kind of material, whether linear or non-linear, elastic or inelastic.

75. (d)

In a pressurized cylinder

$$\text{Circumferential stress } \sigma_1 = \frac{PD}{2t}$$

$$\text{Longitudinal stress } \sigma_2 = \frac{PD}{4t}$$

$$\sigma_1 = 2\sigma_2$$

∴ A longitudinal welded seam must be twice as strong as a circumferential seam.

❍❍❍❍