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ESE 2023 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 6

Section A : Electrical Machines [All Topics]

Section B : Control Systems-1 + Engineering Mathematics-1 [Part Syllabus]

Section C : Electrical Circuits-2 + Digital Circuits-2 [Part Syllabus]

ANSWER KEY

1.	(d)	**16.**	(d)	**31.**	(b)	**46.**	(d)	**61.**	(b)
2.	(b)	**17.**	(c)	**32.**	(a)	**47.**	(d)	**62.**	(a)
3.	(a)	**18.**	(a)	**33.**	(b)	**48.**	(c)	**63.**	(d)
4.	(c)	**19.**	(a)	**34.**	(d)	**49.**	(d)	**64.**	(a)
5.	(d)	**20.**	(c)	**35.**	(a)	**50.**	(d)	**65.**	(a)
6.	(b)	**21.**	(a)	**36.**	(c)	**51.**	(b)	**66.**	(d)
7.	(a)	**22.**	(b)	**37.**	(a)	**52.**	(c)	**67.**	(a)
8.	(d)	**23.**	(d)	**38.**	(d)	**53.**	(b)	**68.**	(a)
9.	(d)	**24.**	(a)	**39.**	(b)	**54.**	(a)	**69.**	(a)
10.	(c)	**25.**	(c)	**40.**	(c)	**55.**	(a)	**70.**	(b)
11.	(b)	**26.**	(b)	**41.**	(c)	**56.**	(a)	**71.**	(a)
12.	(d)	**27.**	(d)	**42.**	(d)	**57.**	(b)	**72.**	(d)
13.	(c)	**28.**	(d)	**43.**	(a)	**58.**	(b)	**73.**	(c)
14.	(a)	**29.**	(a)	**44.**	(c)	**59.**	(b)	**74.**	(c)
15.	(b)	**30.**	(c)	**45.**	(b)	**60.**	(a)	**75.**	(b)

Note: Answer key has been updated of Q.59.

DETAILED EXPLANATIONS
Section A : Electrical Machines

1. (d)

$$H = \frac{NI}{l} = \frac{200 \times 5}{20 \times 10^{-2}} = 5000 \text{ AT/m}$$

and
$$B = \mu_0 \mu_r H = \frac{\phi}{A}$$

$$\mu_r = \frac{0.8 \times 10^{-3} \times \pi}{10 \times 10^{-4} \times 4\pi \times 10^{-7} \times 5000} = 400$$

2. (b)

$$\frac{(V_\Delta)_{ph}}{(V_Y)_{ph}} = 5 \qquad \{\text{For } \Delta\text{-connection, } V_{L\text{-}L} = (V_\Delta)_{ph} = 100\sqrt{3} \text{ V}\}$$

$$\Rightarrow \quad \frac{100\sqrt{3}}{(V_Y)_{ph}} = 5$$

$$\Rightarrow \quad (V_Y)_{ph} = 20\sqrt{3} \text{ V}$$

$$(V_Y)_{line} = (20\sqrt{3}) \cdot \sqrt{3} = 20 \times 3 = 60 \text{ V}$$

3. (a)

We know,

$$\frac{T_{st}}{T_{max}} = \frac{2s_m}{s_m^2 + 1}$$

$$0.5 = \frac{2s_m}{s_m^2 + 1}$$

$$\Rightarrow \quad s_m^2 - 4s_m + 1 = 0$$

$$s_m = \frac{4 \pm \sqrt{16 - 4 \times 1}}{2}$$

As $0 < s_m < 1$ we take (+) sign only

$$s_m = \frac{4 - \sqrt{12}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

$$= 2 - 1.732 = 0.268$$

4. (c)

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Slip at maximum torque, } (s_m) = \frac{1500-1200}{1500} = 0.2$$

$$s_m = \frac{R_2'}{X_2'}$$

$$\Rightarrow \quad 0.2 = \frac{0.04}{X_2'}$$

$$\Rightarrow \quad X_2' = 0.2\ \Omega$$

To obtain maximum toque at starting,

Let, rotor resistance $= R_2' + R_{ext}$

At starting, $s = 1$

$$s_{mT} = \frac{R_2' + R_{ext}}{X_2'}$$

$$1 = \frac{R_2' + R_{ext}}{0.2}$$

$$\Rightarrow \quad R_{ext} + R_2' = 0.2\ \Omega$$

$$R_{ext} = 0.2 - 0.04 = 0.16\ \Omega$$

5. (d)

$$(Z_b)_\Delta = \frac{3V_s^2}{\text{kVA}} = \frac{3\times(200)^2}{20\times1000}\Omega = 6\ \Omega$$

$$\text{ohmic impedance} = (0.05 + j0.08) \times 6$$

$$= (0.3 + j0.48)\Omega$$

6. (b)

$$VI_0 \cos\phi_0 = 600$$

$$1000 \times 1 \times \cos\phi_0 = 600$$

$$\cos\phi_0 = 0.6$$

$$\sin\phi_0 = 0.8$$

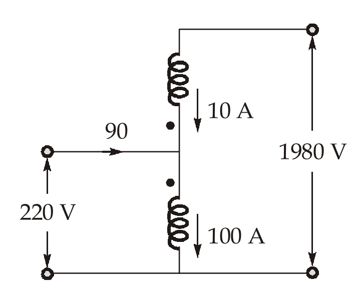
$$\text{Magnetizing current, } I_m = I_0 \sin\phi_0$$

$$= 1 \times 0.8 = 0.8\text{ A}$$

7. (a)

$$\text{Primary current, } I_P = \frac{22000}{220} = 100\text{ A}$$

$$\text{Secondary current, } I_S = \frac{22000}{2200} = 10\text{ A}$$



When connected as 220/1980 V auto transformer,

kVA rating of the transformer = 1980 × 10 = 19.8 kVA

8. (d)

Given, $V_1 = 250$ V,

$V_2 = 500$ V

No-load current oh hv side = $5 \times \dfrac{250}{500} = 2.5$ A

At no load, the pf remains same on both sides.

9. (d)

$$X_d = \frac{(V_{\max})_{ph}}{I_{\min}} = \frac{120}{\sqrt{3} \times 8} = 8.66\,\Omega$$

$$X_q = \frac{(V_{\min})_{ph}}{I_{\max}} = \frac{90}{\sqrt{3} \times 15} = 3.46\,\Omega$$

10. (c)

$$m = \frac{\text{slot}}{\text{poles} \times \text{phase}} = \frac{180}{4 \times 3} = 15$$

$$\beta = \frac{180}{\text{slots/pole}} = \frac{180}{180/4} = 4°$$

So, the distribution factor,

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m \sin\left(\frac{\beta}{2}\right)} = \frac{\sin\left(\frac{15 \times 4°}{2}\right)}{15 \sin\left(\frac{4°}{2}\right)} = \frac{\sin 30°}{15 \sin 2°} = \frac{1}{30 \sin 2°}$$

11. (b)

$P_i = 200$ W

$P_{cu} = 800$ W

Load at which maximum efficiency occur

$$x = \sqrt{\frac{200}{800}} = \frac{1}{2}$$

p.f. = 0.6

$$\eta_{\max} = \frac{5000 \times \frac{1}{2} \times 0.6}{5000 \times \frac{1}{2} \times 0.6 + 2 \times 200}$$

$$= \frac{1500}{1500 + 400} = \frac{15}{19} = 78.94\%$$

12. (d)

At half load 'cu' loss = 200 W

$$\left(\frac{1}{2}\right)^2 I^2R = 200 \text{ W}$$

$$(I^2R)_{\text{at full-load}} = 800 \text{ W}$$

$$R_{pu} = \frac{800}{500\times 1000} = 0.0016 \text{ p.u.}$$

$$X_{pu} = 0.025 \text{ p.u.}$$

$$\text{V.R.} = 0.0016 \times 0.6 + 0.025 \times 0.8$$

$$= 0.02096 = 2.09\%$$

13. (c)

We know that eddy current loss (P_e)

$$P_e \propto f^2B_m^2 \propto (fB_m)^2 \propto V^2 \qquad \left(\because V = \sqrt{2}\,\pi f\,(B_m A)\right) \text{ i.e. } V \propto B_m f$$

$$P_{e1} \text{ at } 400 \text{ V} = 500 \text{ W}$$

$$P_{e2} \text{ at } 200 \text{ V} = ?$$

$$\frac{500}{P_{e2}} = \frac{(400)^2}{(200)^2}$$

$$P_{e2} = 125 \text{ W}$$

Hence, P_{e2} at 200 V is 125 W.

14. (a)

- Power transformer is used for the transmission purpose at heavy load, so its efficiency is greater than distribution transformer.
- Iron weight/Cu weight is less in power transformer.
- Power transformer generally operated at full load, hence it is designed such that copper losses are minimal. However, a distribution transformer operated at loads less than full load for most of time. Hence it is designed such the core losses are minimal.

15. (b)

Here, $N_r = 970$ rpm, N_s must be 1000 rpm

$$s = \frac{N_s - N_r}{N_s} = \frac{1000-970}{1000} = 0.03$$

$$\text{Input power} = 40 \text{ kW}$$

$$\text{air-gap power, } P_g = 40 - 2 = 38 \text{ kW}$$

$$\text{mechanical power, } P_m = P_g(1-s) = 38\,(1-0.03) = 38 \times 0.97 = 36.86 \text{ kW}$$

$$\text{shaft power} = 36.86 - 0.5 = 36.36$$

$$\text{Efficiency} = \frac{\text{Shaft power}}{\text{Input power}} = \frac{36.36}{40} = 90.9\%$$

16. (d)

- Size of single-phase is larger because of high magnetizing current.
- Copper losses are high due to single winding carrying all the current while in poly-phase windings share the current.
- Poor power factor is due to high magnetizing current in single phase.

Hence statement 1, 2 and 3 are correct.

17. (c)

$$V_{OC} = 2000 \text{ V}$$
$$I_{SC} = 250 \text{ A}$$

Synchronous impedance, $Z_S = \dfrac{\text{Open circuit voltage}}{\text{Short circuit current}} = \dfrac{2000}{250} = 8\,\Omega$

Internal voltage drop $= 200 \times 8 = 1600$ V

18. (a)

If field excitation is such that $E_f \cos\phi > V$ the motor is said to be over-excited and it draws leading current.

19. (a)

Load saturation curve is drawn between terminal voltage versus field current.

20. (c)

Given, $R_W = 0.6\ \Omega$

We know,

Current rating ∝ No. of parallel path

$$\text{Voltage rating} \propto \frac{1}{\text{No. of parallel path}}$$

As, $(\text{Power})_{\text{lap winding}} = (\text{Power})_{\text{wave-winding}}$

$$I_L^2 R_L = I_W^2 R_W$$

$$\frac{R_L}{R_W} = \frac{I_W^2}{I_L^2} = \frac{A_W^2}{A_L^2}$$

For lap winding : $A_L = P = 4$

For wave winding : $A_W = 2$

$$\frac{R_L}{0.6} = \frac{2^2}{4^2}$$

$$R_L = \frac{1}{4} \times 0.6 = 0.15\ \Omega$$

21. (a)

$$\text{Power} = E_a I_a = K\,\phi\,\omega_m\,I_a$$

For series motor ($\phi \propto I_a$)

$$P = K'\omega_m I_a^2 = \text{const.}$$

$$\frac{\omega_{m1}}{\omega_{m2}} = \frac{I_{a2}^2}{I_{a1}^2} = \frac{1}{0.111} = 9$$

$\Rightarrow$ $$I_{a2} = 3I_{a1}$$

$$\phi_2 = 3\phi_1$$

We know,

$$\text{back emf, } E_b = K\,\phi\,\omega_m$$

$$\frac{E_{b1}}{E_{b2}} = \frac{\phi_1\omega_{m1}}{\phi_2\omega_{m2}} = \frac{\phi_1 \times 1}{3\phi_1 \times 0.111} = 3$$

$\Rightarrow$ $$E_{b2} = 0.33\,E_{b1}$$

22. (b)

Here, $P = 4$, $I_L = 145$ A, $I_f = 5$ A

$$I_a = I_L + I_f = 145 + 5 = 150 \text{ A}$$

$$\phi_{\text{elec}} = \frac{P}{2}\cdot\theta_{\text{mech}} = \frac{4}{2}\times 5^\circ = 10^\circ \text{ electrical}$$

Cross-magnetizing ATs/pole

$$= \frac{180 - 2\theta_{\text{elec.}}}{180}\times\frac{I_a}{A}\frac{Z}{2P}$$

$$= \frac{180 - 2\times 10}{180}\times\frac{150}{4}\times\frac{480}{2\times 4} = 2000 \text{ ATs/pole}$$

23. (d)

Peripheral velocity of commutator,

$$V_p = \frac{\pi DN}{60}\text{m/s} = \pi\cdot 30\cdot\frac{1000}{60}\text{cm/s} = 500\pi \text{ cm/s}$$

$$\text{Time of commutation} = \frac{\text{Brush width}}{V_p} = \frac{3.14}{500\pi}\text{sec} = 2 \text{ msec}$$

24. (a)

$$I_f = \frac{400}{200} = 2 \text{ A},$$

$$I_L = 42 \text{ A},$$

$$r_a = 0.1\ \Omega$$

$$I_a = I_L - I_f = 42 - 2 = 40 \text{ A}$$

For shunt motor,

$$T \propto I_a \propto N^2 \quad \text{(given)}$$

$$\frac{I_{a1}}{I_{a2}} = \frac{N_1^2}{N_2^2}$$

$$\frac{40}{I_{a2}} = \left(\frac{1000}{600}\right)^2$$

$$I_{a2} = (0.6)^2 \times 40 = 14.4 \text{ A}$$

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} = \frac{V - I_{a1} r_a}{V - I_{a2}(r_a + r_{\text{ext}})}$$

$$\frac{400 - 40 \times 0.1}{400 - 14.4 \times (0.1 + r_{\text{ext}})} = \frac{1000}{600}$$

$$396 \times 0.6 = 400 - 14.4(0.1 + r_{\text{ext}})$$

$$0.1 + r_{\text{ext}} = \frac{400 - 237.6}{14.4} = 11.27\ \Omega$$

$$r_{\text{ext}} = 11.17\ \Omega$$

25. (c)

In closed slot,

1. Net air gap is low, so reluctance is less.
   So, magnetizing current is less.
   Hence power factor improved.
2. Reluctance is less so leakage reactance is more.
3. As leakage flux is more than useful flux hence starting torque will be less.

Hence statements 1, 2 and 4 are correct.

26. (b)

The voltage regulation calculated using the emf method will always be higher than the actual voltage regulation of an alternator i.e. why it is known as the pessimistic method.

27. (d)

3-ϕ alternator, $N_s = 720$ rpm, $P = 10$

$$N_s = \frac{120f}{P} = 720$$

$$f = \frac{720 \times 10}{120} = 60 \text{ Hz}$$

For induction motor :

$$P = 4, \qquad s = 0.05$$

$$N_r = N_s(1 - s)$$

$$= \frac{120 \times 60}{4}(1 - 0.05) = 1710 \text{ rpm}$$

28. (d)

$$P_{out} = 180 \text{ kW}, \qquad X_S = 10\ \Omega$$

$$V_{L\text{-}L} = 1200\sqrt{3}\text{ V}, \qquad V_{ph} = 1200 \text{ V}$$

$$I_a = \frac{180 \times 10^3}{\sqrt{3} \times 1200\sqrt{3} \times 1} = 50 \text{ A}$$

$$\vec{E}_f = \vec{V}_{ph} + jX_s\vec{I}$$

$$= \frac{1200\sqrt{3}}{\sqrt{3}} + j \times 10 \times 50\angle 0°$$

$$= 1200 + j500 = 1300\angle 22.61° \text{ V}$$

$$\text{Maximum power output} = \frac{3E_f V_{ph}}{X_s} = \frac{3 \times 1300 \times 1200}{10} = 468 \text{ kW}$$

29. (a)

For constant V/f :

Starting torque :

$$T_{st} \propto \frac{V^2}{f^3} \propto \left(\frac{V}{f}\right)^2 \frac{1}{f}$$

Maximum torque :

$$T_{max} \propto \frac{V^2}{f^2} \propto \left(\frac{V}{f}\right)^2$$

As frequency increases, starting torque decreases and maximum torque remain constant.

30. (c)

$$r_2 = 0.5\ \Omega,$$

$$X_2 = 5\ \Omega$$

$$s_m = \frac{r_2}{X_2} = \frac{0.5}{5} = 0.1$$

$$\frac{T_{st}}{T_m} = \frac{2s_m}{s_m^2 + 1} = \frac{2}{\frac{s_m}{1} + \frac{1}{s_m}} = \frac{2}{\frac{0.1}{1} + \frac{1}{0.1}} = \frac{2}{10.1}$$

$$T_m = 2\ T_{fL} \text{ (given)}$$

$$T_{st} = \frac{2}{10.1}(2T_{fL})$$

with star-delta starter, starting torque is one-third of that obtained by direct on line starter

$$T_{st} = \frac{1}{3} \times \frac{2}{10.1} \times 2T_{fL}$$

$$\frac{T_{st}}{T_{fL}} = \frac{4}{30.3} = 0.132$$

31. (b)

$$VI\cos\phi = 0.577$$
$$1 \times I \times 1 = 0.577$$
$$I = 0.577 \text{ p.u.}$$
$$E_f = 1 + j0.577$$
$$\delta_1 = \tan^{-1}\left(\frac{0.577}{1}\right) = 30°$$

As excitation is constant, so E_f is also constant with increases in mechanical power input. New power angle will be

$$\frac{P_1}{P_2} = \frac{\sin\delta_1}{\sin\delta_2}$$
$$\frac{0.577}{\sqrt{2/3}} = \frac{\sin 30°}{\sin\delta_2}$$
$$\delta_2 = 45°$$
$$\sin\delta_2 = \frac{1}{2}\times\sqrt{\frac{2}{3}}\times\sqrt{3} = \frac{1}{\sqrt{2}}$$
$$\delta_2 = 45°$$

32. (a)

$$\text{Efficiency} = 80\%$$
$$\text{Power output at full load} = 28.8 \text{ kW}$$
$$\text{Input power} = \frac{28.8}{0.8} = 36 \text{ kW}$$
$$\text{Input power} = \sqrt{3}V_{LL}I_{LL}\cos\phi = 36 \text{ kW}$$
$$\cos\phi = \frac{36\times10^3}{\sqrt{3}\times400\times50\sqrt{3}} = 0.6$$

33. (b)

$$\text{Load} = 600 \text{ kW at } 0.8 \text{ pf lagging}$$
$$\text{Load KVA} = \frac{600}{0.8} = 750 \text{ KVA}$$
$$\text{Load KVAR} = 750 \times \sin(\cos^{-1} 0.8)$$
$$= 750 \times 0.6 = 450 \text{ kVAR}$$

Hence reactive power supplied by the motor is 450 kVAR to operate alternator at unity power factor.

34. (d)

$$f = 50 \text{ Hz},$$
$$P = 6$$
$$N_S = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$
$$s_f = \frac{1000 - 960}{1000} = 0.04$$
$$s_b = 2 - s_f = 2 - 0.04 = 1.96$$

Rotor resistance in the backward branch

$$= \frac{R_2}{2s_b} = \frac{15.68}{2 \times 1.96} = 4\Omega$$

35. (a)

Stepping frequency or pulse rate

= Pulse per second (PPS)

If α is step angle,

$$\text{Motor speed, } n = \frac{\alpha f}{360} \text{rps}$$

Given, $N = 750 \text{ rpm} = \frac{750}{60} \text{rps}$

$$f = 250 \text{ step/sec}$$

So, $\frac{750}{60} = \frac{\alpha \times 250}{360}$

$$\alpha = 18°$$

36. (c)

A large power transformer in loaded condition carry large amount of current and voltages. So its very difficult to arrange such high rating voltmeter, ammeter and wattmeter. Therefore, the testing under full load current is not possible.

38. (d)

The flux $\phi \propto \frac{E}{Nf}$

For constant flux, E, N and f should be constant.

40. (c)

By over-excitation, the synchronous machine would behave as capacitor.

Section B : Control Systems-1 + Engineering Mathematics-1

41. (c)

- Node represents a system variable which is equal to the sum of all incoming signals at the node. Outgoing signals do not affect the value of the node variable.
- Input node is a node with only outgoing branches and this condition is always met.
- Output node is a node with only incoming branches. However, this condition is not always met. An additional branch with unit gain may be introduced in order to meet the specified conditions.

42. (d)

A node variable represents the signal equal to the sum of all incoming signals at the node

$$\therefore \quad x_4 = G_2x_2 + G_3x_3$$
$$x_7 = (G_6 + G_7)x_5 + G_8x_6$$
$$x_6 = G_3x_4$$
$$x_5 = G_4x_3$$

44. (c)

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$= \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{s(s+60)(s+10)}$$

$$= \frac{1}{s}\cdot\frac{600}{(s^2+70s+600)} = R(s)\cdot\frac{600}{s^2+70s+600}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore \quad \omega_n = \sqrt{600} = 24.49 \text{ rad/sec}$$

$$\xi = \frac{70}{2\omega_n} = \frac{70}{2\times 24.49} = 1.43$$

45. (b)

$$G(s) = \frac{K}{s(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+2s+K}$$

$$2\xi\,\omega_n = 2$$

$$\omega_n = \frac{1}{\xi} = \frac{1}{0.4} = 2.5$$

$$K = 2.5^2 = 6.25$$

$$e_{ss} = \frac{1}{1+K_p} + \frac{4}{K_v}$$

$$K_p = \lim_{s \to 0} G(s) = \infty$$

$$K_v = \lim_{s \to 0} sG(s) = \frac{6.25}{2}$$

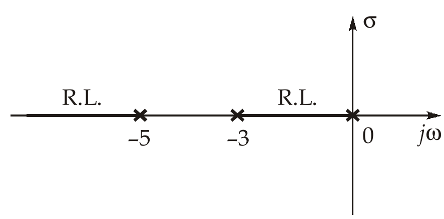
$$e_{ss} = \frac{1}{1+\infty} + \frac{4 \times 2}{6.25} = 1.28$$

46. (d)

- Type number is calculated by open loop transfer function but order can be specified for any transfer function.
- The type number is given by the number of poles of open loop transfer function lying at origin of s-plane but the order is given by number of poles of transfer function.

47. (d)

The segments of the real axis between 0 and –3, –5 and –∞, lie on the root locus.



48. (c)

$$G(s) = \frac{K}{s^2 + 10s + 100}$$

Two poles at $s = -5 \pm j8.66$

No. of asymptotes $= P - Z = 2 - 0 = 2$

$$\theta_1 = \frac{180}{2-0} = 90°$$

$$\theta_2 = \frac{3 \times 180}{2-0} = 270°$$

$$\text{Centroid} = \frac{-5-5-0}{2-0} = -5$$

Angle of departure = 180° – 90° = 90°

All of the these conditions are matched with the option (c).

49. (d)

Let us consider,

$$s + 1 = z$$

$$\Rightarrow \quad s = z - 1$$

Put value of s in the characteristic equation,

$$(z-1)^3 + 10(z-1)^2 + 20(z-1) + 30 = 0$$

$$z^3 - 3z^2 + 3z - 1 + 10z^2 - 20z + 10 + 20z - 20 + 30 = 0$$

$$z^3 + 7z^2 + 3z + 19 = 0$$

z^3	1	3
z^2	7	19
z^1	$\frac{7\times3-19}{7} > 0$	0
z^0	19	

∴ system is stable, since there is no sign change in the first column.

Hence all roots are more negative than $s = -1$.

50. (d)

All statements are correct.

51. (b)

Given, matrix $A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ ...(i)

$$\bar{A} = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix}$$

$$A^\theta = (\bar{A})' = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix} \quad \text{...(ii)}$$

On adding (i) and (ii), we get

$$A + A^\theta = \begin{bmatrix} 2 & 2-2i & 4-6i \\ 2+2i & 4 & 2i \\ 4+6i & -2i & 14 \end{bmatrix}$$

Substracting (ii) from (i), we get

$$A - A^{\theta} = \begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

$$\text{Hermitian matrix} = \frac{1}{2}(A + A^{\theta})$$

$$= \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix}$$

$$\text{Skew-Hermitian matrix} = \frac{1}{2}(A - A^{\theta})$$

$$= \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} + \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

52. (c)

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = -1 + 2 = 1$$

$$|A^{509} - 5A^{508}| = |A^{508}(A - 5I)|$$

$$= |A^{508}||A - 5I|$$

$$= |A|^{508} \cdot |A - 5I|$$

$$= |A - 5I|$$

$$= \begin{vmatrix} 1-5 & -2 \\ 1 & -1-5 \end{vmatrix} = \begin{vmatrix} -4 & -2 \\ 1 & -6 \end{vmatrix} = 24 + 2 = 26$$

53. (b)

Solving the equation of the given curve,

$$y = \frac{3x^2}{4}$$

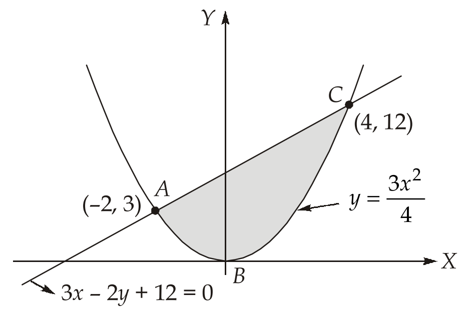
$$3x - 2y + 12 = 0$$

$$3x - 2 \times \frac{(3x^2)}{4} + 12 = 0$$

$$3x^2 - 6x - 24 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$



$x = 4$, $x = -2$ which gives $y = 12$ and $y = 3$

The required area of ABC

$$= \int_{-2}^{4} \frac{12 + 3x}{2} dx - \int_{-2}^{4} \frac{3x^2}{4} dx$$

$$= \left(6x + \frac{3x^2}{4}\right)_{-2}^{4} - \frac{3x^3}{12}\Big|_{-2}^{4} = 27 \text{ square unit}$$

54. (a)

Let, $f_1 = x^2 + y^2 + z^2 - 9 = 0$

and $f_2 = x^2 + y^2 - z - 3 = 0$

Then $N_1 = \nabla f_1$ at (2, –1, 2)

$$= \left(2x\,\hat{a}_x + 2y\,\hat{a}_y + 2z\,\hat{a}_z\right) \text{ at } (2, -1, 2)$$

$$= 4\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z \quad \text{...(i)}$$

$N_2 = \nabla f_2$ at (2, –1, 2)

$$= (2x\,\hat{a}_x + 2y\,\hat{a}_y - \hat{a}_z) \text{ at } (2, -1, 2)$$

$$= 4\hat{a}_x - 2\hat{a}_y - \hat{a}_z \quad \text{...(ii)}$$

The angle θ between the two surfaces at a point is angle between their normal at that point and

$$\cos\theta = \frac{N_1 \cdot N_2}{n_1 n_2} = \frac{(4\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z)\cdot(4\hat{a}_x - 2\hat{a}_y - \hat{a}_z)}{\sqrt{(16+4+16)}\sqrt{(16+4+1)}}$$

$$= \frac{4(4) + (-2)(-2) + 4(-1)}{6\sqrt{12}} = \frac{16}{6\sqrt{21}}$$

Hence, the required angle,

$$\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$

55. (a)

We have, $$\frac{dx}{dy} + \frac{x}{y\log y} = \frac{1}{y} \qquad ...(i)$$

$$\text{Integrating factor} = e^{\int \frac{1}{y\log y}dy} = e^{\log(\log y)} = \log y$$

Thus the solution of equation (i) is,

$$x(\text{I.F.}) = \int \frac{1}{y}(I.F.)dy + C$$

$$x \log y = \int \frac{1}{y}\log y\, dy + C$$

$$= \frac{1}{2}(\log y)^2 + C$$

$$x = \frac{1}{2}\log y + C(\log y)^{-1}$$

56. (a)

Let, $$x = \sqrt[3]{N}$$

$$x^3 = N$$

$$x^3 - N = 0$$

Let, $$f(x) = x^3 - N = 0$$

$$f'(x) = 3x^2$$

By Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2}$$

57. (b)

Given differential equation,

$$\frac{dy}{dx} = f(x, y) \qquad ...(i)$$

$y = \phi(x)$ be the solution of (i)

$$y = f(x) \qquad ...(ii)$$

$(x_0, y_0), (x_1, y_1)$ .... (x_n, y_n), (x_{n+1}, y_{n+1}) be the points on the curve of (ii),

x_0, x_1 ..... x_n, x_{n+1} are equi spaced at equal interval h

$$y_{n+1} = \phi(x_{n+1})$$

$$= \phi(x_n + h)$$

$$= \phi(x_n) + h\phi'(x_n) + \frac{1}{2}h^2\phi''(x_n) + ...$$

$$= \phi(x_n) + h\phi'(x_n)$$

$$= \phi(x_n) + h\, f(x_n, y_n)$$

$$y_{n+1} = y_n + h\, f(x_n, y_n)$$

58. (b)

Put, $\sin^{-1} x = \theta$

or $x = \sin\theta$

So that, $dx = \cos\theta\, d\theta$

Also when, $x = 0$, $\theta = 0$;

when $x = 1$, $\theta = \frac{\pi}{2}$;

$$\therefore \int_0^1 \frac{\sin^{-1} x}{x} dx = \int_0^{\pi/2} \theta \cdot \frac{\cos\theta}{\sin\theta} d\theta$$

$$= \left[\theta \cdot \log\sin\theta\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \log\sin\theta\, d\theta$$

$$= -\int_0^{\pi/2} \log\sin\theta\, d\theta = -\left(\frac{-\pi}{2}\log 2\right) = \frac{\pi}{2}\log 2$$

59. (b)

Let, $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$

$$f'(x) = 12x^3 - 6x^2 - 12x + 6$$

$$= 6(x^2 - 1)(2x - 1)$$

$$f'(x) = 0 \text{ when } x = \pm 1, \frac{1}{2}$$

So in the interval (0, 2) $f(x)$ can have maximum or minimum at $x = \frac{1}{2}$ or 1

$$f''(x) = 36x^2 - 12 - 12 = 12(3x^2 - x - 1)$$

So that, $f''\left(\frac{1}{2}\right) = -9$ and $f''(1) = 12$

$$f(1) = 3(1)^4 - 2(1)^3 - 6(1)^2 + 6(1) + 1$$

$$= 2$$

and also $f(0^+) = 1^+$

So minimum value of $f(x)$ is at $x = 0^+$

Section C : Electrical Circuits-2 + Digital Circuits-2

61. (b)

The energy delivered to the inductor

$$W = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 - MI_1I_2$$

$$= 0.5 \times 4 \times 81 + 0.5 \times 9I_2^2 - 3 \times 9 \times I_2$$

$$= 162 + 4.5I_2^2 - 27I_2$$

For finding the minimum energy, we get $\frac{dW}{dI_2}$

$$\frac{dW}{dI_2} = 9I_2 - 27 = 0$$

$$I_2 = 3 \text{ A}$$

The corresponding minimum stored energy

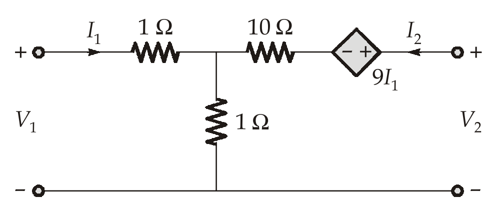
$$W = 4.5 \times (3)^2 - 27 \times 3 + 162$$

$$= 121.5 \text{ J}$$

62. (a)

By source transformation techniques

Applying KVL to Mesh-1



$$V_1 = 2I_1 + I_2 \quad \text{...(i)}$$

Applying KVL to mesh-2,

$$V_2 = 9I_1 + 10I_2 + (I_1 + I_2)$$

$$= 10I_1 + 11I_2 \quad \text{...(ii)}$$

Comparing equation (i) and (ii) with z-parameter equation, we get

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 10 & 11 \end{bmatrix} \Omega$$

63. (d)

All statements are correct.

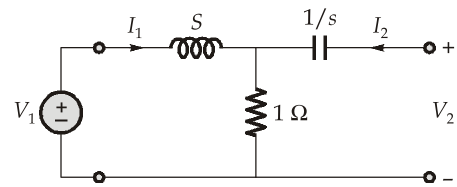
64. (a)

In the s-domain,

$$1\text{ H} \Rightarrow \quad sL = s,$$

$$1\text{ F} \Rightarrow \quad \frac{1}{sC} = \frac{1}{s}$$

To get g_{11} and g_{21}, we open-circuit the output port and connect voltage source V_1 to the input port,



$$I_1 = \frac{V_1}{s+1}$$

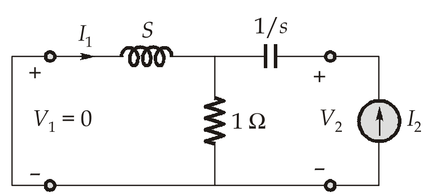
$$g_{11} = \frac{I_1}{V_1} = \frac{1}{s+1}$$

By voltage division,

$$V_2 = \frac{1}{s+1}V_1$$

$$g_{21} = \frac{V_2}{V_1} = \frac{1}{s+1}$$

Obtain g_{12} and g_{22}, we short-circuit the input port and connect a current source I_2 to the output port



By current division,

$$I_1 = -\frac{1}{s+1}I_2$$

or

$$g_{12} = \frac{I_1}{I_2} = \frac{-1}{s+1}$$

$$V_2 = I_2\left(\frac{1}{s} + s \parallel 1\right)$$

$$g_{22} = \frac{V_2}{I_2} = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

Thus, $$[g] = \begin{bmatrix} \frac{1}{s+1} & \frac{-1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$$

65. (a)

Using nodal analysis in conjuction with the ideal transformer equations. First at port-1,

$$I_1 = V_1 + (V_1 - V)$$
$$= 2V_1 - V$$
$$= 2V_1 - \frac{1}{a}V_2$$

Now considering that $aI_2 = -I$, a node equation at the primary of the transformer yields

$$I_2 = -\frac{1}{a}I = \frac{-1}{a}[-V + (V_1 - V)] = -\frac{1}{a}V_1 + \frac{2}{a^2}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-1}{a} \\ \frac{-1}{a} & \frac{2}{a^2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

66. (d)

All statements are correct.

67. (a)

The total number of possible trees,

$$T = \left| A \cdot A^T \right|$$

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

$$[A^T] = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$[A \cdot A^T] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$[A \cdot A^T] = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\left| A \cdot A^T \right| = 3(6 - 1) + 1(-3 - 1) - 1\,(1 + 2) = 15 - 4 - 3$$
$$= 8$$

68. (a)

An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self inductance.

69. (a)

The transfer function is,

$$H(s) = \frac{V_0}{V_i} = \frac{R \parallel \frac{1}{sC}}{sL + R \parallel \frac{1}{sC}}$$

$$R \parallel \frac{1}{sC} = \frac{\frac{R}{sC}}{R + \frac{1}{sC}} = \frac{R}{1 + sRC}$$

$$H(s) = \frac{R}{s^2 RLC + sL + R}$$

Put $s = j\omega$

$$H(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

Since $H(0) = 1$ and $H(\infty) = 0$, we conclude that the circuit is lowpass filter.

70. (b)

$$\text{Resolution} = \frac{V_{FS}}{2^n - 1}$$

$$\frac{0.098}{100} \times V_{FS} = \frac{V_{FS}}{2^n - 1}$$

$$9.8 \times 10^{-4} = \frac{1}{2^n - 1}$$

$$2^n - 1 = \frac{1}{9.8 \times 10^{-4}} \approx 1020$$

$$2^n \approx 1020$$

$$n \approx 10 \text{ bit}$$

71. (a)

$$\text{Step size} = \left(\frac{R_F}{8R}\right)V_{\text{in}}$$

$$0.1\text{ V} = \left(\frac{1\text{K}}{8R}\right)\times 1$$

$$8R = \left(\frac{1\text{K}}{8R}\right)\times 1$$

$$R = \frac{10}{8} = 1.25\text{ k}\Omega$$

72. (d)

$$\text{Overall MOD} = 3 \times 5 \times 10 \times 15 = 2250$$

73. (c)

The clear signal is $\overline{Q_C \cdot Q_D}$

Here counter is cleared after 0011

So, number of states counted are 0 to 3.

MOD of counter is 4.

74. (c)

The counter given is the asynchronous counter in which, the clock is applied to first flip flop only and the remaining flip flop take the clock from the previous flip flops outputs.

We know that first flip flop divides the frequency by 2. Same applies for next flip also.

The output of Q_2 is taken at 3rd flip-flop. So division of frequency will be $\frac{f_{\text{clk}}}{8}$.

$\therefore$ Frequency of the waveform of $Q_3 = \frac{10}{8} = 1.25$ MHz.

75. (b)

From circuit,

$$D_1 = Q_0$$

and $$D_0 = \overline{Q_1 \oplus Q_0}$$

Present State		Next State	
Q_1	Q_0	$Q_1^+ = D_1$	$Q_0^+ = D_0$
0	0	0	1
0	1	1	0
1	0	0	0

❍❍❍❍