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Detailed Solutions

ESE-2019
Mains Test Series

Electrical Engineering
Test No : 6

Section A : Power Electronics & Drives + Engineering Mathematics

Q.1 (a) Solution:

Number of series connected SCR,

$$\eta_s = \frac{V}{n_s V_1}$$

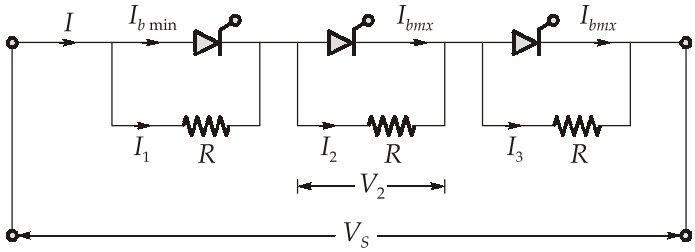
$$0.9 = \frac{11\times10^3}{n_s \times 1800}$$

$$n_s = 6.79 \approx 7$$

$$\eta = \frac{I}{n_p I_1}$$

$$0.9 = \frac{4\times10^3}{n_p \times 1000}$$

$$\Rightarrow \quad n_p = 4.44 \approx 5$$



Let there be n SCR. SCR_1 has minimum leakage current and the remaining SCR has

maximum leakage current I_{bmx}

Voltage across SCR_1 is,

$$V_{bm} = I_1R = (I - I_{b\,\min})R$$

voltage across each of the remaining SCRs

$$V_2 = I_2R$$

$$V_2 = (I - I_{bmx})R$$

$$V_S = V_{bm} + (n-1)V_2$$

$$V_S = V_{bm} + (n-1)\,(I - I_{bmx})R$$

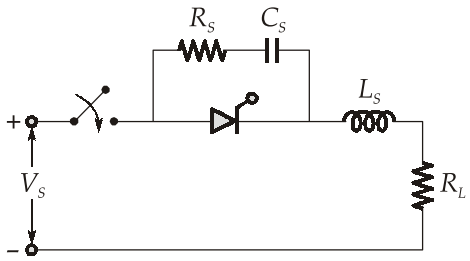
$$= V_{bm} + (n-1)\,(I_{bmin} + I_1)R - (n-1)I_{bmx}\,R$$

$$= V_{bm}\,(1 + n - 1) + (n-1)\,(I_{bmin} - I_{bmx})R$$

$$V_S = nV_{bm} - (n-1)\Delta I_bR$$

$$R = \frac{-V_S + nV_{bm}}{(n-1)\Delta I_b} = \frac{-11000 + 7 \times 1800}{(7-1) \times 12 \times 10^{-3}} = 22.22\ \text{k}\Omega$$

Q.1 (b) Solution:



$$V_S = L_S\left(\frac{di}{dt}\right)$$

$$500 = L_S\left(\frac{60}{2}\right)$$

$$L_S = \frac{500}{30} = 16.67\ \mu\text{H}$$

$$V_S = \frac{L_S}{R_S}\left(\frac{dV}{dt}\right)$$

$$R_S = \frac{500}{30 \times 500} \times \left(\frac{200}{2}\right) = \frac{100}{30} = 3.33\ \Omega$$

Before thyristor is turned on capacitor is charged to 500 V

$\therefore$

$$i_T = \frac{500}{3.33} + \frac{500}{20}$$

$$i_T = 150 + 25 = 175 \text{ A}$$

$\because$ $$\left(\frac{I_P}{2}\right) = \frac{250}{2} = 125 \text{ A}$$

$\because$ $$i_T > \left(\frac{I_P}{2}\right)$$

$\therefore$ $$\frac{500}{R_S} + 25 < 125$$

$$R_S \geq 5\ \Omega$$

Now, $$\xi = \frac{R_S}{2}\sqrt{\frac{C_S}{L_S}}$$

$$R_S^2 = \frac{28.17 \times 10^{-6}}{C_S}$$

$$R_S \geq 5$$

$\Rightarrow$ $$R_S^2 \geq 25$$

$$\frac{28.17 \times 10^{-6}}{C_S} \geq 25$$

$$C_S \leq 1.13 \times 10^{-6}$$

$$C_S \leq 1.13\ \mu\text{F}$$

Q.1 (c) Solution:

Average power loss during turn on

$$P = \frac{1}{T}\int_0^{t_{on}} v_0 i_a\, dt$$

$$= \frac{1}{T}\int_0^{t_{on}} \frac{-600}{t_{on}}(t - 5\times10^{-6}) \times \frac{100t}{t_{on}} dt$$

$$= \frac{6\times10^4}{T \cdot t_{on}^2}\int_0^{t_{on}} (5t\times10^{-6} - t^2)dt$$

$$= \frac{6\times10^4}{T \cdot t_{on}^2}\left[2.5t_{on}^2 \times 10^{-6} - \frac{t_{on}^3}{3}\right]$$

$$= \frac{6\times10^4}{T}\left[2.5\times10^{-6} - \frac{5\times10^{-6}}{3}\right]$$

$$= 0.05 \times 100$$

$$P = 5\ \text{W}$$

Q.1 (d) Solution:

The system of equations can be written in matrix form as

$$AX = B$$

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 4\\ 1 & 4 & 10\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ \eta\\ \eta^2\end{bmatrix}$$

Augmented matrix, $$[A:B] = \begin{bmatrix}1 & 1 & 1 & :1\\ 1 & 2 & 4 & :\eta\\ 1 & 4 & 10 & :\eta^2\end{bmatrix}$$

$$= (R_3 \to R_3 - R_2)\begin{bmatrix}1 & 1 & 1 & :1\\ 1 & 2 & 4 & :\eta\\ 0 & 2 & 6 & :\eta^2-\eta\end{bmatrix}$$

$$= (R_2 \to R_2 - R_1)\begin{bmatrix}1 & 1 & 1 & :1\\ 0 & 1 & 3 & :\eta-1\\ 0 & 2 & 6 & :\eta^2-\eta\end{bmatrix}$$

$$= (R_3 \to R_3 - 2R_2)\begin{bmatrix}1 & 1 & 1 & : & 1\\ 0 & 1 & 3 & : & \eta-1\\ 0 & 0 & 0 & : & \eta^2-\eta-2(\eta-1)\end{bmatrix}$$

For solution of the equation to exist

$$\eta^2 - \eta - 2\eta + 2 = 0$$

$$(\eta - 1)(\eta - 2) = 0$$

$$\eta = 1, 2$$

For $\eta = 1$, the augment matrix $[A:B]$ becomes

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 1$$
$$y + 3z = 0$$

Let, $z = K,$
$y = -3K,$
$x = 1 + 2K$

For $\eta = 1$, the solution is $[1 + 2K, -3K, K]$

For $\eta = 2$, the augment matrix, $[A : B]$ becomes

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + y + z = 1$$
$$y + 3z = 1$$

Let, $z = m,$
$y = 1 - 3m,$
$x = 2m$

For $\eta = 2$, the solution is $[2m, 1 - 3m, m]$

Q.1 (e) Solution:

Using trapezoidal rule,

$$\int_{-1}^{1} f(x)dx = \frac{h}{2}[f_1 + 2f_2 + f_3]$$

$\because$ $h = 1,$
$f_1 = 12,$
$f_2 = 7$
$f_3 = 6$

$\therefore$ $$\int_{-1}^{1} f(x)dx = \frac{1}{2}[12 + 2 \times 7 + 6] = \frac{1}{2} \times 32 = 16$$

Hence, approximate value of the function, using trapezoidal rule = 16

Since $f(x)$ is a second degree polynomial,

Let, $f(x) = ax^2 + bx + c$

$f(-1) = a(-1)^2 + b(-1) + c = 12$

$a - b + c = 12$ ...(i)

$f(0) = a(0) + b(0) + c = 7$

$c = 7$ ...(ii)

$f(1) = a(1)^2 + b(1) + c = 6$

$a + b + c = 6$ ...(iii)

Solving equation (i), (ii) and (iii),

$a = 2 \qquad b = -3$

and $c = 7$

$\therefore \quad f(x) = 2x^2 - 3x + 7$

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} (2x^2 - 3x + 7)dx$$

$$= \left[2\frac{x^3}{3} - \frac{3x^2}{2} + 7x\right]_{-1}^{1}$$

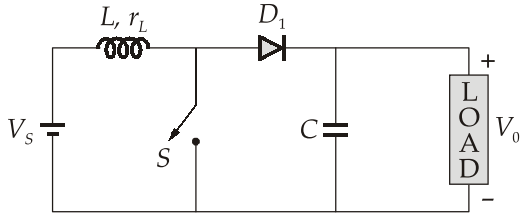
$$= \left(\frac{2}{3}(1)^3 - \frac{3}{2}(1)^2 + 7(1)\right) - \left(\frac{2}{3}(-1)^3 - \frac{3}{2}(-1)^2 + 7(-1)\right)$$

$$= \left(\frac{2}{3} - \frac{3}{2} + 7\right) - \left(-\frac{2}{3} - \frac{3}{2} - 7\right) = \frac{46}{3}$$

$\therefore \quad \text{Error} = \frac{46}{3} - 16 = -\frac{2}{3}$

Q.2 (a) Solution:

(i)



$P_S = P_0 + P_L$

$V_S I_L = V_0 I_D + I_L^2 r_L$

$V_S I_L = V_0 I_L (1 - D) + I_L^2 r_L$

$$V_S = V_0(1-D) + I_L r_L$$

$$I_D = I_L(1-D)$$

$$I_L = \frac{V_0}{R(1-D)}$$

$$V_S = \frac{V_0 r_L}{R(1-D)} + V_0(1-D)$$

$$V_0 = \frac{V_S}{(1-D)}\left[\frac{1}{1+\left[r_L / R(1-D)^2\right]}\right]$$

$$\eta = \frac{P_0}{P_0 + P_{\text{Loss}}} = \frac{\frac{V_0^2}{R}}{\frac{V_0^2}{R_L} + I_L^2 r_L}$$

$$\eta = \frac{\frac{V_0^2}{R}}{\frac{V_0^2}{R} + \left[\frac{V_0}{R(1-D)}\right]^2 r_L}$$

$$\eta = \frac{1}{\left[1 + \frac{r_L}{R(1-D)^2}\right]}$$

(ii)

$$D = 1 - \frac{V_S}{V_0} = 1 - \frac{5}{10} = 0.5$$

$$\eta = \frac{1}{1 + \frac{0.048}{10(1-0.5)^2}} \times 100 = 98.12\%$$

Q.2 (b) Solution:

(i) $\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{100\pi \times 0.1}{10}\right) = 72.34°$

$\because$ $\alpha(60°) < \tan^{-1}\left(\frac{\omega L}{R}\right)$

$\therefore$ load current is continuous.

(ii)

$$V_{0,\,avg} = \frac{2V_m}{\pi}\cos\alpha$$

$$= \frac{2\times 120\sqrt{2}\cos 60^\circ}{\pi} = 54\ \text{V}$$

$$I_{0\,avg} = \frac{V_0}{R} = \frac{54}{10} = 5.4\ \text{A}$$

(iii)

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\cos n\omega t\, d\omega t$$

$$a_n = \frac{V_m}{\pi}\int_{\alpha}^{\pi+\alpha} (\sin(1+n)\omega t + \sin(1-n)\omega t)\, d\,\omega t$$

$$= -\frac{V_m}{\pi}\left[\frac{\cos(1+n)\omega t}{(1+n)} + \frac{\cos(1-n)\omega t}{(1-n)}\right]_{\alpha}^{\pi+\alpha}$$

$$= -\frac{V_m}{\pi}\left[\begin{array}{l}\dfrac{1}{(1+n)}\left[\cos(1+n)(\pi+\alpha) - \cos(1+n)\alpha\right] \\ +\dfrac{1}{(1-n)}\left[\cos(1-n)(\pi+\alpha) - \cos(1-n)\alpha\right]\end{array}\right]$$

$$a_n = \frac{V_m}{\pi}\left[\frac{2\cos(1+n)\alpha}{(1+n)} + \frac{2\cos(1-n)\alpha}{(1-n)}\right];\ n = \text{even}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(1+n)\alpha}{(1+n)} + \frac{\cos(1-n)\alpha}{(1-n)}\right]$$

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\sin\omega t\, d\omega t$$

$$= \frac{V_m}{\pi}\int_{\alpha}^{\pi+\alpha} \left[\cos(1-n)\omega t - \cos(1+n)\omega t\right] d\omega t$$

$$b_n = \frac{V_m}{\pi}\left[\frac{\sin(1-n)\omega t}{(1-n)} - \frac{\sin(1+n)\omega t}{1+n}\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi}\left[\begin{array}{l}\left[\dfrac{\sin(\pi+\alpha)(1-n)-\sin(1-n)\alpha}{(1-n)}\right] \\ -\dfrac{1}{(1+n)}\left[\sin(1+n)(\pi+\alpha)-\sin(1+n)\alpha\right]\end{array}\right]$$

$$b_n = \frac{-V_m}{\pi}\left[\frac{2\sin(1-n)\alpha}{(1-n)}+\frac{2\sin(1+n)\alpha}{(n+1)}\right],\ n = \text{even}$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(1+n)\alpha}{(n+1)}+\frac{\sin(1-n)\alpha}{(1-n)}\right]$$

$$a_2 = \frac{2\times 120\sqrt{2}}{\pi}\left[\frac{\cos\frac{3\pi}{3}}{3}+\frac{\cos\frac{-\pi}{3}}{(-1)}\right] = \frac{2V_m}{\pi}\left(\frac{-5}{6}\right)$$

$$b_2 = \frac{2V_m}{\pi}\left[\frac{\sin\left(\frac{3\pi}{3}\right)}{3}+\frac{\sin\left(\frac{-\pi}{3}\right)}{-1}\right] = \frac{2V_m}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$$V_2 = \sqrt{a_2^2+b_2^2} = \frac{2V_m}{\pi}\sqrt{\frac{25}{36}+\frac{3}{4}} = 129.85\ \text{V}$$

$$Z_2 = 10 + j100\pi \times 0.1 \times 2 = 63.62\angle 81°\ \Omega$$

$$I_2 = \frac{129.85}{63.62} = 2.04\ \text{A}$$

$$I_{\text{rms}} = \sqrt{5.4^2+\left(\frac{2.04}{\sqrt{2}}\right)^2} = 5.58\ \text{A}$$

Power absorbed by the load,

$$P = I_{\text{rms}}^2 R = 5.58^2 \times 10 = 311.36\ \text{W}$$

Q.2 (c) Solution:

Here 2-sets are given with numbers n_1 and n_2 their mean is known to be m_1 and m_2 respectively. Their standard deviation is known to be s_1 and s_2 respectively.

According to given conditions,

$$m_1 = \frac{\Sigma x_1}{n_1} \Rightarrow \Sigma x_1 = m_1 n_1 \qquad \left[\text{Mean} = \frac{\text{Sum}}{\text{No. of item}}\right]$$

and $$m_2 = \frac{\Sigma x_2}{n_2} \Rightarrow \Sigma x_2 = m_2 n_2$$

$$\sigma_1^2 = \frac{\Sigma x_1^2}{n_1} - m_1^2 \Rightarrow \Sigma x_1^2 = \left(\sigma_1^2 + m_1^2\right) n_1$$

and $$\sigma_2^2 = \frac{\Sigma x_2^2}{n_2} - m_2^2 \Rightarrow \Sigma x_2^2 = \left(\sigma_2^2 + m_2^2\right) n_2$$

Now both the sets are grouped together to form a set of $(n_1 + n_2)$ number.

Let m_3 and σ_3 are mean and standard deviation for the grouped set.

$$m_3 = \frac{\Sigma x}{n_1 + n_2} = \frac{\Sigma x_1 + \Sigma x_2}{n_1 + n_2}$$

$$\Rightarrow \quad m_3 = \frac{m_1 n_1 + m_2 n_2}{n_1 + n_2}$$

Now, $$\sigma_3^2 = \frac{\Sigma x^2}{n_1 + n_2} - m_3^2 = \frac{\Sigma x_1^2 + \Sigma x_2^2}{n_1 + n_2} - m_3^2$$

$$= \frac{\left(\sigma_1^2 + m_1^2\right) n_1 + \left(\sigma_2^2 + m_2^2\right) n_2}{n_1 + n_2} - \frac{\left(m_1 n_1 + m_2 n_2\right)^2}{\left(n_1 + n_2\right)^2}$$

$$= \frac{\sigma_1^2 n_1 + \sigma_2^2 n_2}{n_1 + n_2} + \frac{m_1^2 n_1 + m_2^2 n_2}{n_1 + n_2} - \frac{m_1^2 n_1^2 + m_2^2 n_2^2 + 2 m_1 m_2 n_1 n_2}{\left(n_1 + n_2\right)^2}$$

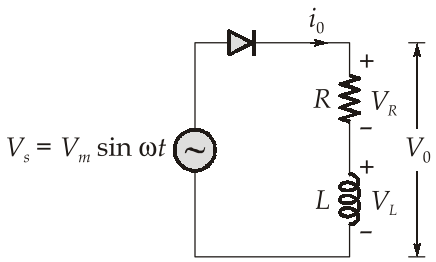
$$= \frac{\sigma_1^2 n_1 + \sigma_2^2 n_2}{n_1 + n_2} + \frac{\left(m_1^2 n_1 + m_2^2 n_2\right)\left(n_1 + n_2\right) - m_1^2 n_1^2 - m_2^2 n_2^2 - 2 m_1 m_2 n_1 n_2}{\left(n_1 + n_2\right)^2}$$

$$= \frac{\sigma_1^2 n_1 + \sigma_2^2 n_2}{n_1 + n_2} + \frac{m_1^2 n_1 n_2 + m_2^2 n_1 n_2 - 2 m_1 m_2 n_1 n_2}{\left(n_1 + n_2\right)^2}$$

$$= \frac{\sigma_1^2 n_1 + \sigma_2^2 n_2}{n_1 + n_2} + \frac{n_1 n_2 \left(m_1^2 + m_2^2 - 2 m_1 m_2\right)}{\left(n_1 + n_2\right)^2}$$

$$\sigma_3^2 = \frac{\sigma_1^2 n_1 + \sigma_2^2 n_2}{n_1 + n_2} + \frac{n_1 n_2}{\left(n_1 + n_2\right)^2} \left(m_1 - m_2\right)^2$$

Q.3 (a) Solution:



$$V_m \sin \omega t = Ri(t) + \frac{Ldi(t)}{dt}$$

The solution of the above equation is,

$$i(t) = i_f(t) + i_n(t)$$

forced response, $$i_f(t) = \frac{V_m \sin \omega t}{Z}$$

$$i_f(t) = \frac{V_m \sin(\omega t - \theta)}{|Z|}$$

$$Z = R + j\omega L$$
$$= 100 + j2\pi \times 50 \times 0.1$$
$$= 104.82\angle 17.44° \ \Omega$$

$$i_f(t) = \frac{100}{104.82}\sin(\omega t - 17.44°)$$
$$= 0.954 \sin (\omega t - 17.44°)$$

Natural response,

$$Ri(t) + \frac{Ldi(t)}{dt} = 0$$

Solution of this equation,

$$i_n(t) = A e^{-t/\tau};$$

$$\tau = \frac{L}{R}$$

Final solution of current,

$$i(t) = \frac{V_m}{|Z|}\sin(\omega t - \theta) + Ae^{-Rt/L}$$

Using initial condition,

$$i(0) = \frac{V_m}{|Z|}\sin(0-\theta) + A = 0$$

$$A = \frac{V_m}{|Z|}\sin\theta = 0.954 \sin 17.44 = 0.286$$

At extinction angle, $\omega t = \beta$

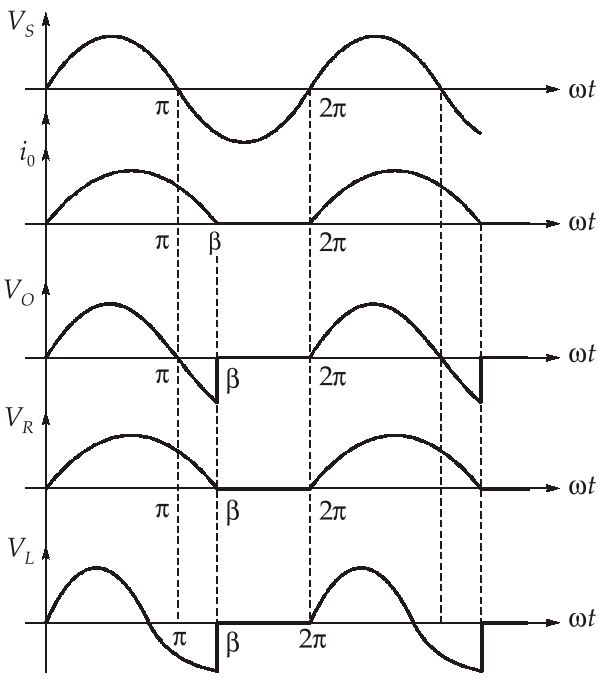
$$i(\beta) = \frac{V_m}{|Z|}\sin(\beta-\theta) + Ae^{-\frac{\beta}{\omega\tau}} = 0$$

$$0.954\sin(\beta - 17.44) + 0.954\sin 17.44 e^{-\beta/\omega\tau} = 0$$

$$\sin(\beta - 17.44) + 0.3e^{-3.183\,\beta} = 0$$

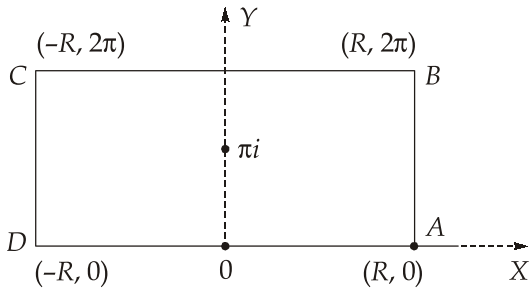
$$\beta = 197.44°$$

$$i(t) = \begin{cases} 0.954\sin(\omega t - 17.44°) + 0.286e^{-3.183\omega t} & 0 < \omega t < 197.44° \\ 0 & 197.44° < \omega t < 360° \end{cases}$$



Q.3 (b) Solution:

Consider $\int_c \frac{e^{\alpha z}}{e^z+1}dz = \int_c f(z)dz$ where C is a rectangle $ABCD$ with vertices at $(R, 0)$, $(R, 2\pi)$, $(-R, 2\pi)$ and $(-R, 0)$. [R being positive]



$f(z)$ has finite poles given by,

$$e^z = -1 = e^{(2n+1)\pi i}$$

or, $$z = (2n+1)\pi i ,$$

Where, $$n = 0, \pm1, \pm2, \pm3, ...$$

$\because$ the only pole inside the rectangle is $z = \pi i$,

$\therefore$ By residue theorem,

$$\int_c f(z)dz = 2\pi i \operatorname{Res} f(\pi i)$$

$$= 2\pi i \left[\frac{e^{\alpha z}}{\frac{d}{dz}(e^z+1)}\right]_{z=\pi i}$$

$$= \frac{2\pi i\, e^{\alpha\pi i}}{e^{\pi i}} = -2\pi i\, e^{\alpha\pi i} \qquad \left(\because e^{\pi i} = -1\right) \qquad ...(i)$$

Also, $$\int_c f(z)dz = \int_{AB} f(z)dz + \int_{BC} f(z)dz + \int_{CD} f(z)dz + \int_{DA} f(z)dz$$

$$= \int_0^{2\pi} f(R+iy)idy + \int_R^{-R} f(x+2\pi i)dx + \int_{2\pi}^0 f(-R+iy)idy + \int_{-R}^{R} f(x)dx$$

($\because z = R + iy$ along AB, $z = x + 2\pi i$ along BC, $z = -R + iy$ along CD and $z = x$ along DA)

$$\int_c f(z)dz = i\int_0^{2\pi} \frac{e^{\alpha(R+iy)}}{e^{R+iy}-1}dy - \int_{-R}^{R} \frac{e^{\alpha(x+2\pi i)}}{e^{x+2\pi i}+1}dx - i\int_0^{2\pi} \frac{e^{\alpha(-R+iy)}}{e^{-R+iy}+1}dy + \int_{-R}^{R} \frac{e^{\alpha x}}{e^x+1}dx$$

...(ii)

$\because$ for any two complex numbers z_1 and z_2,

If $\quad |z_1| \geq |z_2|$

$\Rightarrow \quad |z_1 + z_2| \geq |z_1| - |z_2|$

So that, $\quad \left|e^{R+iy} + 1\right| \geq e^R - 1$

Also, $\quad \left|e^{\alpha(R+iy)}\right| = e^{\alpha R}$

$\therefore \quad \left|\dfrac{e^{\alpha(R+iy)}}{e^{R+iy}+1}\right| \leq \dfrac{e^{\alpha R}}{e^R - 1}$ which $\to 0$ as $R \to \infty$ $\quad (\because \alpha > 1)$

and $\quad \left|\dfrac{e^{\alpha(-R+iy)}}{e^{-R+iy}+1}\right| \leq \dfrac{e^{-\alpha R}}{1 - e^{-R}}$ which also $\to 0$ as $R \to \infty$ $\quad (\because \alpha < 0)$

Hence as $R \to \infty$, first and third integrals in equation (ii) approach zero, we get

$$\int_c f(z)dz = -e^{2\alpha\pi i}\int_{-\infty}^{\infty}\frac{e^{\alpha x}}{e^x+1}dx + \int_{-\infty}^{\infty}\frac{e^{\alpha x}}{e^x+1}dx$$

$$= \left(1 - e^{2\alpha\pi i}\right)\int_{-\infty}^{\infty}\frac{e^{\alpha x}}{e^x+1}dx \qquad \ldots(iii)$$

$\therefore$ From (i) and (iii),

$$\left(1 - e^{2\alpha\pi i}\right)\int_{-\infty}^{\infty}\frac{e^{\alpha x}}{e^x+1}dx = -2\pi i e^{\alpha\pi i}$$

$\therefore$ Equating real parts,

$$\int_{-\infty}^{\infty}\frac{e^{\alpha x}}{e^x+1}dx = \frac{\pi}{\sin\alpha\pi}$$

Q.3 (c) Solution:

Let,

$$F(\alpha) = \int_0^{\alpha}\frac{\log(1+\alpha x)}{1+x^2}dx \qquad \ldots(i)$$

Using Leibnitz's rule,

$$F'(\alpha) = \int_0^{\alpha}\frac{\partial}{\partial\alpha}\left(\frac{\log(1+\alpha x)}{1+x^2}\right)dx + \frac{d(\alpha)}{d\alpha}\frac{\log(1+\alpha^2)}{1+\alpha^2} - 0$$

$$= \int_0^{\alpha} \frac{x}{(1+\alpha x)(1+x^2)}dx + \frac{\log(1+\alpha^2)}{1+\alpha^2} \quad \ldots(ii)$$

Rewriting equation (ii) using partial fractions,

$$\int_0^{\alpha} \frac{x\,dx}{(1+\alpha x)(1+x^2)} = -\frac{\alpha}{1+\alpha^2}\int_0^{\alpha}\frac{dx}{1+\alpha x} + \frac{1}{2(1+\alpha^2)}\int_0^{\alpha}\frac{2x}{1+x^2}dx + \frac{\alpha}{1+\alpha^2}\int_0^{\alpha}\frac{dx}{1+x^2}$$

$$= -\frac{1}{1+\alpha^2}\left|\log(1+\alpha x)\right|_0^{\alpha} + \frac{1}{2(1+\alpha^2)}\left|\log(1+x^2)\right|_0^{\alpha} + \frac{\alpha}{1+\alpha^2}\left|\tan^{-1}x\right|_0^{\alpha}$$

$$= \frac{-\log(1+\alpha^2)}{1+\alpha^2} + \frac{\log(1+\alpha^2)}{2(1+\alpha^2)} + \frac{\alpha\tan^{-1}\alpha}{1+\alpha^2}$$

Substituting these values in equation (ii),

$$F'(\alpha) = \frac{\log(1+\alpha^2)}{2(1+\alpha^2)} + \frac{\alpha\tan^{-1}\alpha}{1+\alpha^2}$$

Now, integrating both sides w.r.t to α,

$$F(\alpha) = \frac{1}{2}\int \log(1+\alpha^2)\cdot\frac{1}{1+\alpha^2}d\alpha + \int\frac{\alpha\tan^{-1}\alpha}{1+\alpha^2}d\alpha$$

Integrating by parts,

$$F(\alpha) = \frac{1}{2}\left[\log(1+\alpha^2)\tan^{-1}\alpha - \int\frac{2\alpha}{1+\alpha^2}\tan^{-1}\alpha\,d\alpha\right] + \int\frac{\alpha\tan^{-1}\alpha}{1+\alpha^2}d\alpha + C$$

$$F(\alpha) = \frac{1}{2}\log(1+\alpha^2)\tan^{-1}\alpha + C$$

$\because$ $F(0) = 0$ (from equation (i))

$\therefore$ $C = 0$

$\Rightarrow$ $$F(\alpha) = \frac{1}{2}\log(1+\alpha^2)\tan^{-1}\alpha$$

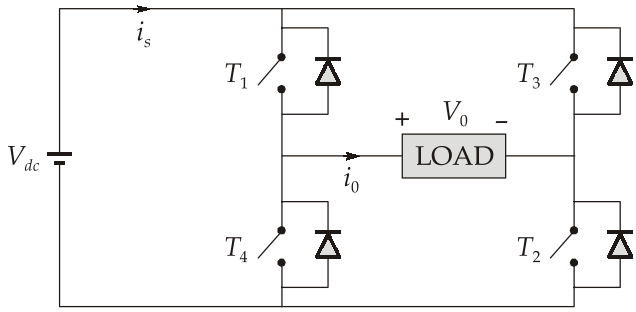
$$\int_0^{\alpha}\frac{\log(1+\alpha x)}{1+x^2}dx = \frac{1}{2}\log(1+\alpha^2)\tan^{-1}\alpha$$

Putting, $\alpha = 1$,

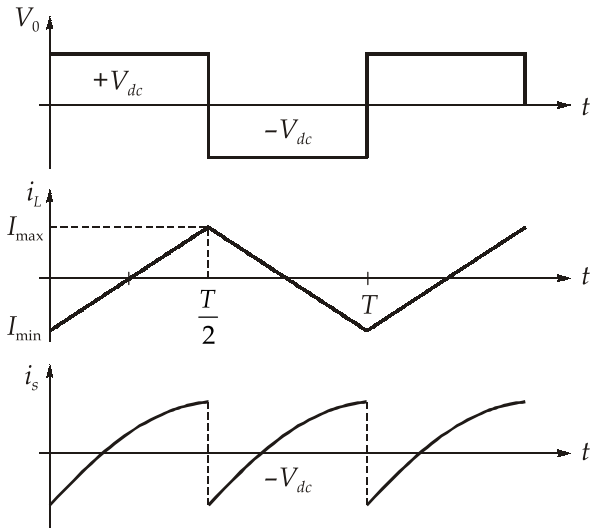
$$\int_0^1 \frac{\log(1+x)}{1+x^2}dx = \frac{1}{2}\log(1+1^2)\tan^{-1}1$$

$$= \frac{\pi}{8}\log_e 2 \qquad \text{Hence, proved}$$

Q.4 (a) Solution:



(i) When S_1 and S_2 are on voltage across the load is $+V_{dc}$ and current in the load rises.



$$0 \le t < \frac{T}{2}$$

$$i_L(t) = \frac{V_{dc}}{R} + Ae^{-t/\tau}$$

$$i_L(0) = I_{min}$$

$$I_{\min} = \frac{V_{dc}}{R} + Ae^{-0/\tau}$$

$$A = \left(I_{\min} - \frac{V_{dc}}{R}\right)$$

$$\frac{T}{2} < t < T$$

$$i_L(t) = \frac{-V_{dc}}{R} + Be^{-\frac{(t-T/2)}{\tau}}$$

$$i_L\left(\frac{T}{2}\right) = I_{\max} = \frac{-V_{dc}}{R} + B$$

$$B = \left(I_{\max} + \frac{V_{dc}}{R}\right)$$

$$i_L(t) = \begin{cases} \frac{V_{dc}}{R} + \left(I_{\min} - \frac{V_{dc}}{R}\right)e^{-t/\tau} & 0 < t < \frac{T}{2} \\ \frac{-V_{dc}}{R} + \left(I_{\max} + \frac{V_{dc}}{R}\right)e^{\frac{-(t-T/2)}{\tau}} & \frac{T}{2} < t < T \end{cases}$$

$$i_L\left(\frac{T}{2}\right) = I_{\max} = -I_{\min} = \frac{V_{dc}}{R} + \left(I_{\min} - \frac{V_{dc}}{R}\right)e^{-T/2\tau}$$

$$I_{\min}\left(1 + e^{-T/2\tau}\right) = \frac{-V_{dc}}{R}\left(1 - e^{-T/2\tau}\right)$$

$$I_{\max} = -I_{\min} = \frac{V_{dc}}{R}\frac{\left(1 - e^{-T/2\tau}\right)}{\left(1 + e^{-T/2\tau}\right)}$$

$$\frac{T}{2\tau} = \frac{1000 \times 10}{60 \times 2 \times 25 \times 1} = \left(\frac{10}{3}\right)$$

$$I_{\max} = 10\frac{\left(1 - e^{-10/3}\right)}{\left(1 + e^{-10/3}\right)} = 9.311 \text{ A}$$

$$i_L(t) = \begin{cases} 10 + (-9.31 - 10)e^{-400t} \\ -10 + (9.31 + 10)e^{-(t-T/2)400} \end{cases}$$

$$= \begin{cases} 10 - 19.31e^{-400t} \\ -10 + 19.31e^{-400(t-T/2)} \end{cases}$$

$$P_{dc} = V_{dc} I_{s\,avg} = I_{i\,rms}^2 R$$

$$I_{s\,avg} = \frac{1}{T/2} \int_0^{T/2} (10 - 19.31e^{-400t})\, dt$$

$$= 120 \int_0^{1/120} (10 - 19.31e^{-400t}) dt = 4.414 \text{ Amp}$$

$$I_{L\,rms} = \sqrt{120 \int_0^{1/120} (10 - 19.31e^{-400t})^2 dt}$$

$$= \sqrt{44.13} = 6.643 \text{ Amp}$$

$$P_{dc} = I_{L,\,rms}^2 R = 44.13 \times 10 = 441.3 \text{ W}$$

Q.4 (b) Solution:

(i) The voltage waveform is symmetrical about π as well as $\frac{\pi}{2}$.

As this voltage waveform has quarter-wave symmetry,

$$a_n = 0$$

$$b_n = \frac{4V_s}{\pi} \left[\int_0^{\alpha_1} \sin n\omega t\, d(\omega t) - \int_{\alpha_1}^{\alpha_2} \sin n\omega t\, d\omega t + \int_{\alpha_2}^{\pi/2} \sin n\,\omega t\, d(\omega t) \right]$$

$$b_n = \frac{4V_s}{\pi} \left[-\left(\frac{\cos n\omega t}{n} \right)_0^{\alpha_1} + \left(\frac{\cos n\,\omega t}{n} \right)_{\alpha_1}^{\alpha_2} - \left(\frac{\cos n\,\omega t}{n} \right)_{\alpha_2}^{\pi/2} \right]$$

$$= \frac{4V_s}{n\pi} \left[-\cos n\alpha_1 + 1 + \cos n\alpha_2 - \cos n\alpha_1 + \cos n\alpha_2 - \cos \frac{n\pi}{2} \right]$$

$$b_n = \frac{4V_s}{n\pi} \left[1 - 2\cos n\alpha_1 + 2\cos n\,\alpha_2 \right],\ n = \text{odd}$$

$$b_3 = b_5 = 0$$

$$b_7 = \frac{4\times V_S}{7\pi}\left[1-2\cos(7\times 23.62°)+2\cos(7\times 33.3°)\right] = 0.3154\ V_s$$

$$b_9 = \frac{4V_s}{9\pi}\left[1-2\cos(9\times 23.62°)+2\cos(9\times 33.3°)\right] = 0.5201\ V_s$$

$$b_{11} = \frac{4V_s}{11\pi}\left[1-2\cos(11\times 23.62°)+2\cos(11\times 33.3°)\right] = 0.3867\ V_s$$

(ii)

$$b_1 = \frac{4V_s}{\pi}\left[1-2\cos(23.62°)+2\cos(33.3°)\right] = 1.0684\ V_s$$

Amplitude of fundamental component of unmodulated output voltage wave is,

$$b_{1n} = \frac{4V_s}{\pi} = 1.2732\ V_s$$

$$\frac{b_1}{b_{1n}} = \frac{1.0684}{1.2732} = 0.8391$$

$$\%\ \text{deration} = (1-0.8391)\times 100 = 16.09\%$$

Main disadvantages:

- Amplitude of fundamental wave form is less than that of unmodulated waveform. Thus inverter is derated.
- No. of commutations per cycle is very large as compared to unmodulated waveform. So, it leads to more switching losses in the thyristors.

Q.4 (c) (i) Solution:

Since the particle is moving in x-y plane,

$$\Rightarrow \quad z = 0\ [\text{always}]$$

So,
$$\vec{F} = (2x-y)\hat{i}+(x+y)\hat{j}+(3x-2y)\hat{k}$$

$$\text{Work done} = \int \vec{F}\cdot d\vec{r}$$

$$d\vec{r} = dx\,\hat{i}+dy\,\hat{i}+0\,\hat{k}$$

$$\vec{F}\cdot d\vec{r} = (2x-y)\,dx+(x+y)\,dy$$

The equation of ellipse is given as

$$4x^2+9y^2 = 36$$

$$\frac{x^2}{(3)^2}+\frac{y^2}{(2)^2} = 1$$

Let,
$$x = 3\cos\theta$$

$$y = 2\sin\theta$$
$$dx = -3\sin\theta\, d\theta$$
$$dy = 2\cos\theta\, d\theta$$

θ varies from 0 to 2π

Substituting these values, we get

$$\int \vec{F}\cdot d\vec{r} = \int_0^{2\pi} (6\cos\theta - 2\sin\theta)(-3\sin\theta d\theta) + (3\cos\theta + 2\sin\theta)(2\cos\theta d\theta)$$

$$= \int_0^{2\pi} (-18\sin\theta\cos\theta + 6\sin^2\theta + 6\cos^2\theta + 4\sin\theta\cos\theta)\, d\theta$$

$$= \int_0^{2\pi} (-14\sin\theta\cos\theta + 6)\, d\theta = \int_0^{2\pi} (-7\sin 2\theta + 6)\, d\theta$$

$$= \left[7\frac{\cos 2\theta}{2} + 6\theta\right]_0^{2\pi} = 6 \times 2\pi = 12\pi$$

Work done = 12π J

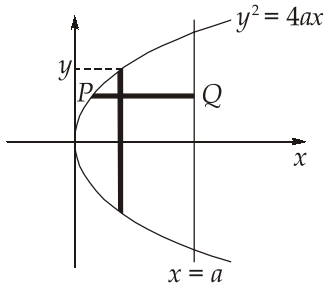
Q.4 (c) (ii) Solution:

The parabola is as shown in figure below.

Latus rectum is $x = a$

It cuts the parabola at, $y^2 = 4a \times a$

$$y = \pm 2a$$



Volume generated is given by $= \int \pi(PQ)^2\, dy = \int_{-2a}^{2a} \pi(a^2 - x^2)\, dy$

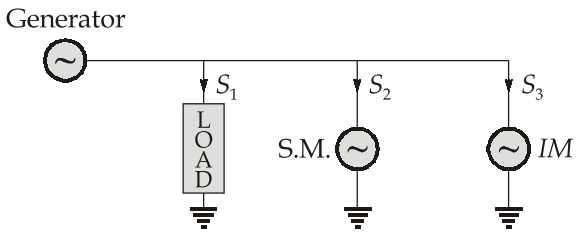
$$= 2\int_0^{2a} \pi(a^2 - x^2)\, dy = 2\pi\int_0^{2a}\left(a^2 - \frac{y^4}{16a^2}\right)dy$$

$$= 2\pi\left[a^2 y - \frac{y^5}{80a^2}\right]_0^{2a} = 2\pi\left(a^2 \times 2a - \frac{32a^5}{80a^2}\right)$$

$$= 2\pi\left(2a^3 - \frac{2a^3}{5}\right) = \frac{16\pi a^3}{5}$$

Section B : Basic Electronics Engg.-1 + Analog Electronics-1 + Electrical Materials-1 + Electrical Machines-2

Q.5 (a) Solution:



Lighting and heating load, $S_1 = P_1 + jQ_1 = (100 + j0)$

$= 100$ kVA

Induction motor load, $P_{IM} = 1000 \times 746 = 746$ kW

$$S_{IM} = \frac{746\angle\phi}{0.7} = 1065.71\angle\cos^{-1}(0.7)$$

$= 1065.71\angle 45.6°$ kVA

Input to induction motor, $S_3 = \frac{S_{IM}}{\eta} = \frac{1065.71\angle 45.6°}{0.85}$

$= 1253.78\angle 45.6°$ kVA

Synchronous motor load, $P_2 = \frac{300 \times 746}{0.92} = 243.26$ kW

$S = S_1 + S_2 + S_3$

$|S|\angle\cos^{-1} 0.95 = 100 + 243.26 + jQ_2 + 877.22 + j895.79$

$|S|\angle 18.2° = 1220.48 + j(Q_2 + 895.79)$

$|S| \times 0.95 = 1220.48$

$|S| = 1284.71$

$|S| \sin\phi = Q_2 + 895.79$

$1284.71 \sin(18.2) = Q_2 + 895.79$

$Q_2 = -494.52$ kVAR

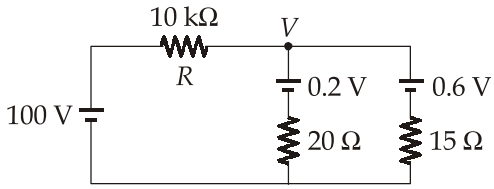
This much kVAR is delivered by synchronous motor

$$S_2 = 243.26 + j494.52$$
$$= 551.81\angle 63.81° \text{ kVA}$$

and
$$\text{power factor} = \cos\left(\tan^{-1}\left(\frac{Q}{P}\right)\right)$$
$$= \cos\left(\tan^{-1}\left(\frac{494.52}{243.26}\right)\right) = 0.442 \text{ leading}$$

Q.5 (b) Solution:

In the circuit if we assume that both the diodes are forward biased so we have circuit as,



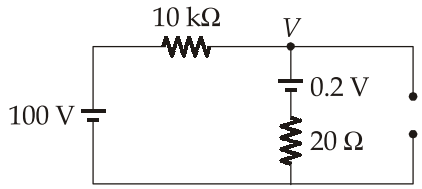
Applying nodal to the circuit,

$$\frac{V-100}{10k}+\frac{V-0.2}{20}+\frac{V-0.6}{15} = 0$$
$$V\left[\frac{1}{10k}+\frac{1}{20}+\frac{1}{15}\right]-\left[\frac{100}{10k}+\frac{0.2}{20}+\frac{0.6}{15}\right] = 0$$
$$V = 0.5138 \text{ V}$$

from the value of V it is clear that D_2 will remain OFF (reverse biased) so our assumption is wrong. Assume only D_1 is ON.

So, we have,

$$I_{D1} = \frac{100-0.2}{10k+20} = 9.96 \text{ mA}$$
$$V = 0.3992 \text{ V}$$



Means D_2 remains off so,

$$I_{D_2} = 0 \text{ Amp}$$

and
$$I_{D_1} = 9.96 \text{ mA}$$

Q.5 (c) Solution:

(i) In body centered cubic structure, the atoms touch each other along the diagonal of the cube as shown in figure. Therefore, the length of the diagonal of cube is $4r$.

$$AC = \sqrt{AB^2 + BC^2}$$

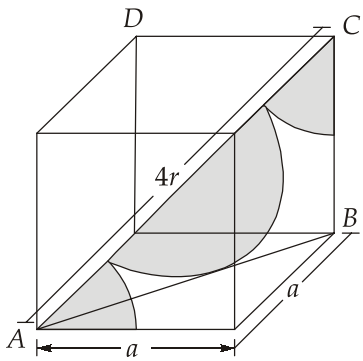
$$AB = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$BC = x \qquad \text{(Let } a = x\text{)}$$

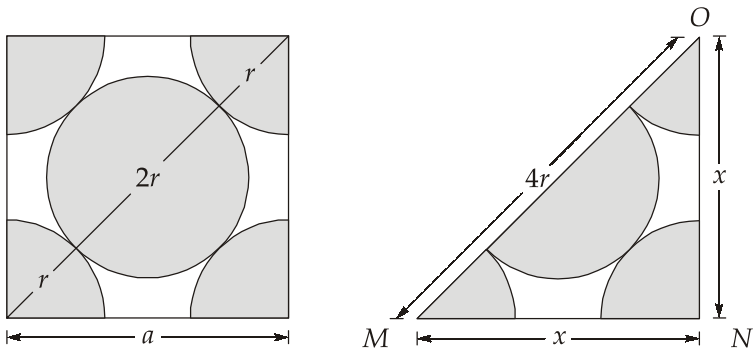
$$AC = 4r = \sqrt{(x\sqrt{2})^2 + x^2} = x\sqrt{3}$$

$$r = \frac{x\sqrt{3}}{4}$$

$$x = \frac{4r}{\sqrt{3}}$$



In face centered cubic structure, the atoms touch each other along the diagonal of any face of the cube as shown in figure. The length of the diagonal of the face = $4r$ as shown in figure below,



$$MO^2 = MN^2 + ON^2$$

$$(4r)^2 = x^2 + x^2 = 2x^2$$

or, $$x = \frac{4r}{\sqrt{2}}$$

or, $$r = \frac{x\sqrt{2}}{4}$$

or, $$r^2 = \frac{2x^2}{16} = \frac{x^2}{8}$$

$$r = \frac{x}{2\sqrt{2}}$$

So, $$x = 2\sqrt{2}r$$

(ii) Imperfections result due to deviation from an orderly periodic array. The study of defect is especially useful when studying the properties of materials which are structure sensitive.

This effects the properties like fracture strength plasticity, electrical resistivity and thermal conduction.

Point Defects:

A point defect is a very localized defect in the regularity of a lattice and usually does not spread over more than one or two lattice spacing. The structural point defect can be classified into three types, i.e., vacancies, interstitial, and impurities.

- Vacancies defect: In vacancies defect, there may be missing atom in a crystal. A vacancy may result because of imperfect packing during original crystallization or may arise from thermal vibrations of atoms at elevated temperature.
- Interstitial defect: In interstitial defect, an atom occupies a definite position in the lattice such that it is normally not occupied in a perfect crystal. The position occupied by a normal atom or a foreign atom lies between the atoms of the ideal crystal.
- Impurity defect: In impurity defect, foreign atoms are introduced into a crystal lattice either as an interstitial atom or as a substitution atom. Therefore defects may be classified as interstitial impurity defect and substitution impurity defect.

Line Defects:

Line defects are basically one-dimensional defects or dislocations. Due to dislocations, lattice distortions are centered around a line. Dislocations are produced due to solidification of the crystalline solids by vacancy condensation or by atomic mismatch in the solid solutions. Dislocations are mainly of two types i.e., the edge dislocation and the screw dislocation.

- Edge dislocation: In edge dislocation an extra half plane of atoms is inserted in a crystal either above or below the slip plane. Edge dislocation may be either positive or negative.
- Screw dislocation: The screw dislocation can be created in a perfect crystal by applying upward or downward shear stresses to the regions of perfect crystal, which have been separated by a cutting plane. Screw dislocation may be either positive or negative.

Q.5 (d) Solution:

We know that, $$V_D = \eta V_T \ln\left|\frac{I}{I_0}\right|$$

for $I = 10$ mA, V_D is 0.7 V then,

$$0.7 = \eta V_T \ln\left|\frac{10\,\text{m}}{I_0}\right| \quad \text{...(i)}$$

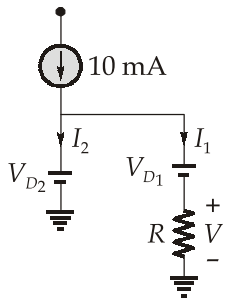
for $I = 100$ mA, V_D is 0.8 V then,

$$0.8 = \eta V_T \ln\left|\frac{100\,\text{m}}{I_0}\right| \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii), we get,

$$0.1 = \eta V_T \ln|10|$$

$$\eta V_T = 43.43\ \text{mV} \quad \text{...(iii)}$$



$$V_{D2} - V_{D1} - V = 0$$

$$V = V_{D2} - V_{D1}$$

$$80\ \text{mV} = \eta V_T \ln\left|\frac{I_2}{I_0} \times \frac{I_0}{I_1}\right|$$

$$\ln\left|\frac{I_2}{I_1}\right| = \frac{80}{\eta V_T}$$

$$I_2 = 6.3I_1 \quad \text{...(iv)}$$

and $$I_1 + I_2 = 10\ \text{mA} \quad \text{...(v)}$$

So from equation (iv) and (v),

$$I_1 = 1.369\ \text{mA}$$

$$I_1 \times R = V$$

$$1.369\ \text{m} \times R = 80\ \text{m}$$

The value of resistance, $R = 58.4\ \Omega$

Q.5 (e) Solution:

The different performance criteria which are considered for selecting liquids dielectric are: stability of liquids, oxidation, electrical stresses and thermal stresses.

The insulating liquids are organic liquids which are used wholly in electrical context as coolants and dielectrics and are liquids whose presence either along or in conjugation with solid insulating and constructional materials, permit the design of electrical transmission and distribution equipment at minimum cost with an acceptable performance and life.

An ideal insulating liquid must have following properties:

- high electric strength, impulse strength and volume resistivity
- low dielectric dissipation factor
- high or low dielectric constant (depending on application)
- high specific heat and thermal conductivity
- excellent chemical stability and gas absorbing properties
- low viscosity, low density, low volatility, low solvent power and high flash point
- good arc quenching properties
- nonflammable and nontoxic.

Q.6 (a) Solution:

(i)

$$R_1 = (r_1 + r_2 + ... r_n + r_a)$$

$$R_1 = \frac{V_t}{I_{\max}}$$

$$I_{\min} = \left(\frac{V_t - E_1}{R_1}\right)$$

Now, r_1 is cut out, then, $R_2 = (r_2 + r_3 + \ldots r_n + r_a)$

$$= \left(\frac{V_t - E_1}{I_{max}}\right)$$

$$\frac{R_2}{R_1} = \frac{I_{min}}{I_{max}}$$

Similarly, $$\frac{R_3}{R_2} = \frac{I_{min}}{I_{max}}$$

$\Rightarrow$ $$\frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \ldots = \frac{r_a}{R_n} = \frac{I_{min}}{I_{max}}$$

$$\left(\frac{r_a}{R_1}\right) = \left(\frac{I_{min}}{I_{max}}\right)^n$$

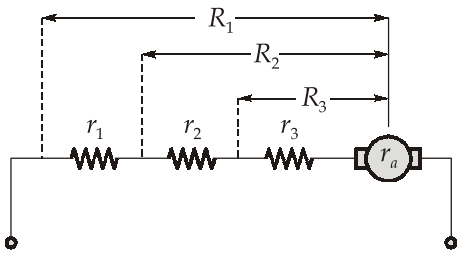
$$n = \frac{\log\left(\frac{r_a}{R_1}\right)}{\log\left(\frac{I_{min}}{I_{max}}\right)}$$

$$R_1 = \frac{250}{350 \times 2} = 0.357$$

$$n = \frac{\log\left(\frac{0.05}{0.357}\right)}{\log\left(\frac{1}{2}\right)} = 2.84$$

$\therefore$ no. of steps to be cut = 3

(ii)



$R_1 = 0.357\ \Omega$

$E_1 = V_t - I_{min} R_1$

$= 250 - 350 \times 0.357 = 125.05$ V

$$R_2 = r_2 + r_3 + \ldots + r_a = \frac{250 - 125.05}{700} = 0.1785\ \Omega$$

$$r_1 = R_1 - R_2 = 0.357 - 0.1785 = 0.1785\ \Omega$$

$$E_2 = 250 - 350 \times 0.1785 = 187.5\ \text{V}$$

$$R_3 = r_3 + r_a = \frac{250 - 187.5}{700} = 0.0893\ \Omega$$

$$r_2 = R_2 - R_3 = 0.1785 - 0.0893 = 0.0892$$

$$E_3 = 250 - 0.0893 \times 350 = 218.75\ \text{V}$$

$$r_3 = R_3 - r_a$$

$$= 0.0893 - 0.05 = 0.0393\ \Omega$$

Now,
$$I_A = \frac{V_t - E_3}{r_a} = \frac{250 - 218.75}{0.05} = 625\ \text{A}$$

$r_1 = 0.1785\ \Omega$,

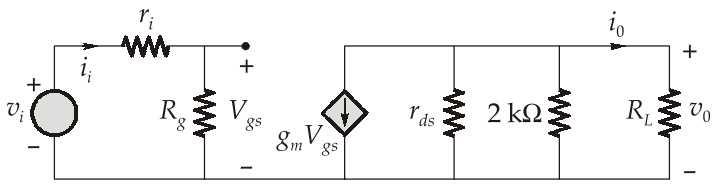
$r_2 = 0.0892\ \Omega$

and $r_3 = 0.0393\ \Omega$

are cut out when E reaches 125.05 V, 187.5 V and 218.75 V respectively.

Q.6 (b) Solution:

AC equivalent circuit is drawn in figure below,



In the above circuit,
$$R_g = \frac{800k \times 200k}{800k + 200k} = 160\ \text{k}\Omega$$

$$Z_{\text{in}} = \frac{v_i}{i_i} = r_i + R_g$$

$$Z_{\text{in}} = 165\ \text{k}\Omega$$

From above circuit,

$$v_0 = -g_m V_{gs}\left(r_{ds} \parallel 2k \parallel 2k\right)$$

$$V_{gs} = \frac{v_i \times R_g}{R_g + r_i} = \frac{v_i \times 160k}{160k + 5k} = \frac{v_i \times 160}{165}$$

So we have,

$$v_0 = \frac{-v_i \times 160}{165} \times g_m \times (r_{ds} \parallel 2k \parallel 2k)$$

$$\frac{v_0}{v_i} = \frac{-160}{165} \times 2 \times 10^{-3} \times (30k \parallel 2k \parallel 2k)$$

$$A_V = \frac{v_0}{v_i} = -1.8768$$

$$i_0 = \frac{v_0}{R_L}$$

and

$$i_1 = \frac{v_i}{R_g + r_i}$$

$$A_I = \frac{i_0}{i_1} = \frac{v_0 / R_L}{v_i / (R_g + r_i)}$$

$$A_I = A_V \times \frac{R_g + r_i}{R_L}$$

$$A_I = -1.8768 \times \frac{160k + 5k}{2k}$$

$$A_I = -1.8768 \times \frac{165}{2}$$

$$A_I = -154.836$$

Q.6 (c) Solution

"Elemental solid dielectrics" are the materials consisting of single type of atoms (e.g. Diamond, Sulphur, Germanium etc.) These materials contain neither ions nor permanent dipoles and thus it exhibits only electronic polarization.

Let us consider 'α_e' is the polarizability per atom in such materials then,

$$\text{Polarization } (\vec{P}) = N\alpha_e \vec{E}_i \quad \text{...(i)}$$

where, N = Number of atoms/m^3

and E_i = The internal field seen by the atom

we have also,

$$\vec{E}_i = \vec{E} + \frac{\gamma}{\varepsilon_0}\vec{P} \quad \text{...(ii)}$$

where, γ = proportionality constant which is referred to as the internal field constant.

E = Applied field.

Here, we assume for macroscopic relation,

Then,

$$\text{Polarization } (\vec{P}) = \varepsilon_0\chi_e\vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E} \quad \text{...(iii)}$$

From equation (i) and (ii) we get,

$$(\vec{P}) = N\alpha_e\left[\vec{E} + \frac{\gamma}{\varepsilon_0}\vec{P}\right]$$

Now, using equation (iii),

$$\text{or} \quad \varepsilon_0\chi_e\vec{E} = N\alpha_e\left[\vec{E} + \frac{\gamma}{\varepsilon_0}\left(\varepsilon_0\chi_e\vec{E}\right)\right]$$

$$\text{or} \quad \varepsilon_0\chi_e = N\alpha_e\left[1 + \gamma\chi_e\right]$$

$$\text{or} \quad \chi_e = \frac{N\alpha_e}{\varepsilon_0}\cdot\left[1 + \gamma\chi_e\right]$$

Since, susceptibility $(\chi_e) = \varepsilon_r - 1$ so,

$$\text{or} \quad \chi_e = \frac{N\alpha_e}{\varepsilon_0}\cdot\gamma\chi_e + \frac{N\alpha_e}{\varepsilon_0}$$

$$\text{or} \quad (\varepsilon_r - 1)\left[1 - \frac{\gamma N\alpha_e}{\varepsilon_0}\right] = \frac{N\alpha_e}{\varepsilon_0}$$

$$\text{or} \quad \varepsilon_r = \frac{(N\alpha_e/\varepsilon_0)}{\left(1 - \frac{\gamma N\alpha_e}{\varepsilon_0}\right)} + 1$$

$$\text{or} \quad \varepsilon_r = \frac{1 + \left(\frac{N\alpha_e}{\varepsilon_0}\right) - \gamma\left(\frac{N\alpha_e}{\varepsilon_0}\right)}{1 - \gamma\left(\frac{N\alpha_e}{\varepsilon_0}\right)}$$

$$\Rightarrow \quad \varepsilon_r = \frac{1 + (1-\gamma)\left(\frac{N\alpha_e}{\varepsilon_0}\right)}{1 - \gamma\left(\frac{N\alpha_e}{\varepsilon_0}\right)} \quad \text{...(iv)}$$

If the internal field $\vec{E}_i$ is taken as the Lorentz field then for a cubical lattice structure, $\gamma = \frac{1}{3}$ then equation (iv) may reduce to

$$\varepsilon_r = \frac{1+\frac{2}{3}\left(\frac{N\alpha_e}{\varepsilon_0}\right)}{1-\frac{1}{3}\left(\frac{N\alpha_e}{\varepsilon_0}\right)} \quad \ldots(v)$$

Assuming, $\frac{N\alpha_e}{\varepsilon_0} = x$ then equation (v) may be,

$$\varepsilon_r = \frac{1+\frac{2}{3}x}{1-\frac{1}{3}x} \quad \ldots(vi)$$

After solving equation (vi) we get,

$$x = \frac{3(\varepsilon_r - 1)}{(\varepsilon_r + 2)}$$

The equation (vi) may be written as,

$$\varepsilon_r = \frac{1+\frac{2}{3}\left\{\frac{3(\varepsilon_r - 1)}{(\varepsilon_r + 2)}\right\}}{1-\frac{1}{3}\left\{\frac{3(\varepsilon_r - 1)}{(\varepsilon_r + 2)}\right\}}$$

$\because$ $$x = \frac{N\alpha_e}{\varepsilon_0} = \frac{3(\varepsilon_r - 1)}{(\varepsilon_r + 2)}$$

Final equation is,

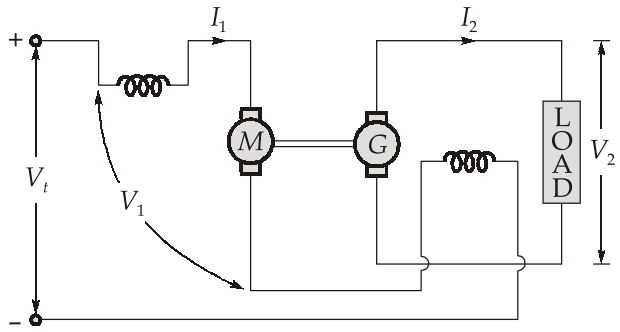
$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \quad \ldots(vii)$$

This equation (vii) is well known **"CLAUSIUS-MOSOTTI RELATION".**

This Clausius-Mossotti relation contains the following assumptions:

- The polarizability of molecules is isotropic.
- The arrangement of molecules is isotropic.
- Polarization of the molecules is by elastic displacement only.
- Absence of short range (non-dipolar) interactions.

Q.7 (a) Solution:



Power input to whole set $= V_t I_1$

$= (400 + 25 + 25) \times 50$

$= 450 \times 50 = 22.5$ kW

Total losses in the whole set, $W = V_t I_1 - V_2 I_2$

$= 22.5 - 350 \times 40 \times 10^{-3}$

$W = 8.5$ kW

No load rotational losses $= W$ – ohmic losses

$W_0 = 8500 - 50^2 \times 0.5 - 25 \times 50 - 25 \times 50 - 40^2 \times 0.5$

$W_0 = 3950$ W

$$\text{Motor efficiency, } \eta_m = 1 - \frac{\frac{W_0}{2} + I_1^2 r_{am} + V_f I_L}{V_1 I_1}$$

$$\eta_m = \left[1 - \frac{\left(\frac{3950}{2} + 50^2 \times 0.5 + 25 \times 50\right)}{425 \times 50}\right] \times 100 = 78.94\%$$

Generator efficiency, $P_{\text{losses}} = 40^2 \times 0.5 + 25 \times 50 + 3950 \times 0.5$

$= 4025$ W

$P_{\text{in}} = 350 \times 40 + 4025$

$= 18025$ W

$$\eta_G = \left(1 - \frac{4025}{18025}\right) \times 100 = 77.67\%$$

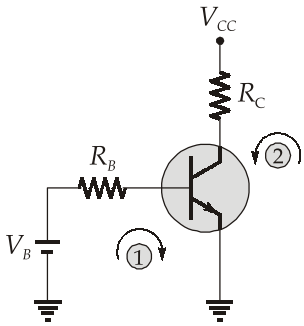
Q.7 (b) Solution:

On performing DC analysis of circuit,

$$V_B = \frac{V_{CC} \times R_2}{R_1 + R_2} = \frac{12 \times 6.3}{93.7 + 6.3}$$

$$V_B = 0.756 \text{ V}$$

$$R_B = \frac{R_1 \times R_2}{R_1 + R_2} = 5.9 \text{ k}\Omega$$



Applying KVL in loop-1, we get

$$V_B - I_B R_B - 0.7 = 0$$

$$I_B = 9.49 \, \mu\text{A}$$

$$I_{CQ} = \beta \times I_B = 0.949 \text{ mA} \approx 0.95 \text{ mA}$$

Applying KVL in loop-2,

$$V_{CC} - I_{CQ} R_C - V_{CEQ} = 0$$

$$V_{CEQ} = 6.31 \text{ V}$$

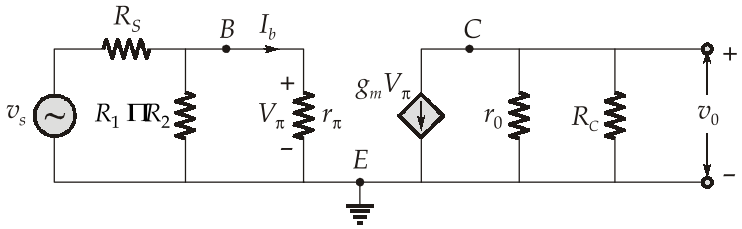
For AC analysis of the circuit,

$$r_\pi = \frac{V_T \times \beta}{I_{CQ}} = \frac{26 \times 100}{0.95} \approx 2.74 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.95}{26} = 36.5 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{0.95} = 105 \text{ k}\Omega$$

AC equivalent hybrid π circuit,



$$v_0 = -g_m V_\pi \times (r_0 \,||\, R_c)$$

$$V_\pi = V_s\left[\frac{R_1 \| R_2 \| r_\pi}{R_s + R_1 \| R_2 \| r_\pi}\right]$$

$$V_0 = -g_m\left[\frac{R_1 \| R_2 \| r_\pi}{R_s + R_1 \| R_2 \| r_\pi}\right] \times (r_0 \| R_c)V_s$$

$$A_V = \frac{V_0}{V_s}$$

$$= -36.5 \times 10^{-3} \times \left[\frac{93.7k \| 6.3k \| 2.74k}{0.5k + 93.7k \| 6.3k \| 2.74k}\right] \times (105k \| 6k)$$

$$A_V = -163.48$$

$$R_i = (R_1 \| R_2 \| r_\pi) = (93.7k \| 6.3k \| 2.74k) = 1.87\ \text{k}\Omega$$

$$R_0 = r_0 \| R_c = (105k \,||\, 6k)$$

$$R_0 = 5.68\ \text{k}\Omega$$

Q.7 (c) Solution:

(i) Rated :

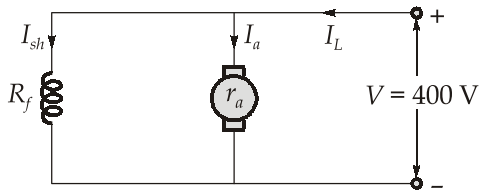
$V = 400$ V,

$I_L = 30$ A

$\omega = 100$ rad/s

$r_a = 1\ \Omega$,

$R_f = 200\ \Omega$



$$I_a = I_L - I_{sh}$$

$$= 30 - \frac{400}{200} = 28\ \text{A}$$

$$E = V - I_a r_a = 400 - 28 \times 1 = 372\ \text{V}$$

During plugging,

$$I_{ap} = 1.5 \times I_{a\ \text{rated}}$$

$$= 1.5 \times 28 = 42\ \text{A}$$

$$I_{ap} = \frac{E+V}{R+r_a} = 42$$

$$R + 1 = \frac{372+400}{42} = 18.38$$

$$R = 17.38\ \Omega$$

Now, $E = K\phi\omega$

$$K\phi = \frac{372}{100} = 3.72\ \text{V/rad-s}$$

(ii) At the instant of plugging,

$$I_a = 42\ \text{A}$$

$$T = K\phi I_a$$

$$= 42 \times 3.72 = 156.24\ \text{Nm}$$

(iii) When the motor is approaching zero speed,

$$E_b = 0$$

$$I_a = \frac{V}{R+r_a} = \frac{400}{17.38+1} = 21.76\ \text{A}$$

$$T = K\phi I_a = 3.72 \times 21.76$$

$$= 80.95\ \text{Nm}$$

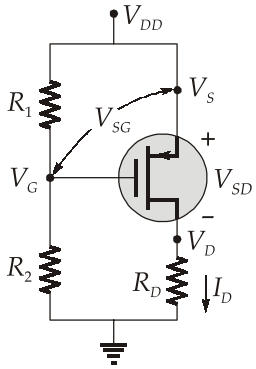
Q.8 (a) Solution:

From the given circuit,

$$V_G = \frac{V_{DD} \times R_2}{R_1 + R_2} = \frac{5 \times 50k}{50k + 50k}$$

$$V_G = 2.5\ \text{V}$$

$$V_{SG} = V_S - V_G = 5 - 2.5 = 2.5\ \text{V}$$



Assume that MOSFET is in saturation region then,

$$I_D = K_P(V_{SG} + V_{TP})^2$$
$$= 0.2\,(2.5 - 0.8)^2$$
$$I_D = 0.578\ \text{mA}$$

and $$V_{SD} = V_{DD} - I_D R_D = 5 - 0.578 \times 7.5$$
$$V_{SD} = 0.665\ \text{V}$$

for MOSFET to be in saturation,

$$V_{SD} \geq V_{SG} + V_{TP}$$
$$V_{SG} + V_{TP} = 2.5 - 0.8 = 1.7\ \text{V}$$

It can be inferred that our assumption is wrong, MOSFET is not in saturation,

Now for non-saturation region,

$$I_D = K_P[2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2]$$

Where, $$V_{SD} = V_{DD} - I_D R_D$$
$$I_D = K_P[2(V_{SG} + V_{TP})\,(V_{DD} - I_D R_D) - (V_{DD} - I_D R_D)^2]$$
$$I_D = 0.2[2(2.5 - 0.8)\,(5 - I_D \times 7.5k) - (5 - I_D \times 7.5k)^2]$$
$$I_D = 0.515 \times 10^{-3}\ \text{Amp}$$
$$V_{SD} = 5 - 0.515 \times 7.5$$
$$= 1.14\ \text{mV}$$

Q.8 (b) Solution:

Given, $$X_d = 0.9\ \text{p.u.}$$
$$X_q = 0.65\ \text{p.u.}$$

$$T = \frac{P}{\omega_{sm}} = \frac{P}{1} = P$$

$$T = \frac{EV_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$

$$= \frac{1\times 1}{0.9}\sin\delta + \frac{1^2}{2}\left(\frac{1}{0.65} - \frac{1}{0.9}\right)\sin 2\delta$$

$$T = 1.11\sin\delta + 0.214\sin 2\delta$$

$$\frac{dT}{d\delta} = 1.11\cos\delta + 0.428\cos 2\delta = 0$$

$$2.6\cos\delta + 2\cos^2\delta - 1 = 0$$

$$\cos\delta = 0.31, -1.61$$

$$\delta = \cos^{-1}(0.31) = 71.91°$$

$$V_t \sin\delta = I_q X_q$$

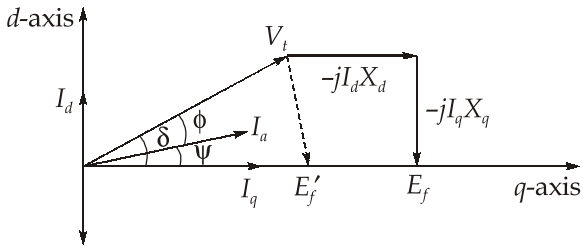
$$I_q = \frac{1\times\sin 71.91}{0.65} = 1.462 \text{ p.u.}$$

$$V_t \cos\delta = E_f - I_d X_d$$

$$I_d = \frac{1-\cos 71.91}{0.9} = 0.766 \text{ p.u.}$$

$$I_a = \sqrt{I_d^2 + I_q^2} = 1.651 \text{ p.u.}$$

$$\psi = \tan^{-1}\left(\frac{I_d}{I_q}\right) = \tan^{-1}\left(\frac{0.766}{1.462}\right) = 27.65°$$



$$\text{Power factor} = \cos\phi$$

$$\text{p.f.} = \cos(\delta - \psi)$$

$$= \cos(71.91 - 27.65)$$

$$= 0.716 \text{ lagging}$$

Q.8 (c) Solution:

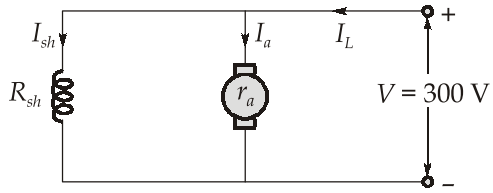
$$\eta = \left(1 - \frac{P_{\text{losses}}}{P_{\text{in}}}\right)\times 100$$

$$0.9 = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}}$$

$$\frac{P_{\text{loss}}}{P_{\text{out}}} + 1 = \frac{1}{0.9}$$

$$P_{\text{loss}} = \frac{1}{9}\times 15\times 0.85\times 10^3 = 1416.67 \text{ W}$$

$$I_{sh} = \frac{V}{R_{\text{sh}}} = \frac{300}{150} = 2 \text{ A}$$



At maximum efficiency,

$$\text{Constant losses} = \text{Variable losses}$$

$$= \frac{1416.67}{2} = 708.33 \text{ W}$$

$$P_{\text{input}} \text{ at max } \eta = 0.85 \times 15 \times 10^3 + 1416.67$$

$$= 14166.67 \text{ W}$$

$$\text{Line current } I_{L1} = \frac{14166.67}{300} = 47.22 \text{ A}$$

$$I_{a1} = 47.22 - 2$$

$$= 45.22 \text{ A}$$

$$I_{a1}^2 r_a = 708.33$$

$$r_a = 0.346\ \Omega$$

$$\text{When line current, } I_{L2} = 72 \text{ A}$$

$$I_{a2} = 72 - 2 = 70 \text{ A}$$

$$P_{\text{losses}} = 708.34 + 70^2 \times 0.346 = 2403.74 \text{ W}$$

$$\eta = \left(1 - \frac{2403.74}{300 \times 72}\right) \times 100 = 88.87\%$$

$$E_1 = 300 - 45.22 \times 0.346 = 284.35 \text{ V}$$

$$E_2 = 300 - 70 \times 0.346 = 275.78 \text{ V}$$

$$\frac{E_1}{E_2} = \frac{N_1 \phi_1}{N_2 \phi_2}$$

$$N_2 = \frac{800 \times 275.78}{284.35} = 775.9 \text{ rpm}$$

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