DETAILED SOLUTIONS

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ESE 2023 : Prelims Exam
CLASSROOM TEST SERIES

GS & ENGINEERING APTITUDE

Test 1

Section A : Reasoning & Aptitude [All Topics]

Section B : Engineering Mathematics [All Topics]

1.	**(c)**	**11.**	**(b)**	**21.**	**(d)**	**31.**	**(c)**	**41.**	**(a)**
2.	**(b)**	**12.**	**(a)**	**22.**	**(d)**	**32.**	**(b)**	**42.**	**(a)**
3.	**(b)**	**13.**	**(a)**	**23.**	**(b)***	**33.**	**(a)**	**43.**	**(c)**
4.	**(a)**	**14.**	**(b)**	**24.**	**(a)**	**34.**	**(d)**	**44.**	**(d)**
5.	**(b)***	**15.**	**(c)**	**25.**	**(b)**	**35.**	**(d)**	**45.**	**(a)**
6.	**(c)**	**16.**	**(b)**	**26.**	**(b)**	**36.**	**(d)**	**46.**	**(d)**
7.	**(d)**	**17.**	**(a)**	**27.**	**(b)**	**37.**	**(b)**	**47.**	**(a)**
8.	**(b)**	**18.**	**(b)**	**28.**	**(a)**	**38.**	**(d)**	**48.**	**(b)**
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10.	**(b)**	**20.**	**(b)**	**30.**	**(a)**	**40.**	**(a)**	**50.**	**(a)**

*5 [*Answer key has been Updated*]

*23 [*Answer key has been Updated*]

DETAILED EXPLANATIONS

1. (c)

Let the distance of his home from the station is x km

According to question,

$$\frac{x}{20} - \frac{x}{25} = \frac{9}{60}$$

$$\frac{5x}{500} = \frac{9}{60}$$

$$x = \frac{900}{60} = 15\,\text{km}$$

2. (b)

If we multiply the given expression by 3, we get

$$\left(3x + \frac{1}{3x}\right) = 3$$

$$9x^2 + \frac{1}{9x^2} + 2 = 9$$

$$9x^2 + \frac{1}{9x^2} = 7$$

Required expression, $27x^3 + \dfrac{1}{27x^3} = (3x)^3 + \dfrac{1}{(3x)^3}$

$$= \left(3x + \frac{1}{3x}\right)\left((3x)^2 + \frac{1}{(3x)^2} - 1\right) = (3)(7-1)$$

$$= 3 \times 6$$

$$= 18$$

3. (b)

The number of vowels in the alphabet = 5

Number of consonants = 21

Number of non zero digits = 9

Chosen key can have as many options as the distinct values in the specific ring and the total number of possible combinations is the product of 3 numbers = 5 × 21 × 9 = 945

4. (a)

3	3	2	2	1	1
odd	even	odd	even	odd	even

First odd place can be filled with 3 vowels.

Second odd place can be filled with 2 vowels.

Third odd place can be filled with 1 vowels.

Similarly, even places will be filled by consonants.

$$\text{Number of ways} = 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 36 \text{ ways}$$

5. (b)

Let the original sum of money be ₹x after 6 years, this amount will become $2x$.

Please note that the money is doubling itself every 6 years. When we say 48 years from start, it means 48 – 6 = 42 years from the first maturity.

42 years is equivalent to $\frac{42}{6} = 7$ sets of 6 years each.

The amount on deposit is now ₹$2x$ and it will multiply itself 2 × 2 × 2 × 2 × 2 × 2 × 2 = 128 times of the amount put on deposit at the end of first 6 years. So option (b) is correct.

6. (c)

Let profit be ₹x

Partner 1 who invested ₹3750 will get profit of

$$0.3x + (0.4x) \times \frac{3750}{6300} = 0.5381x$$

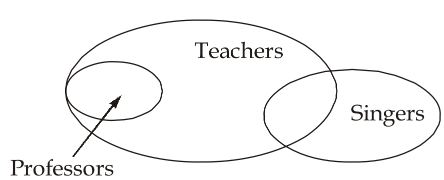
Partner 2 who invested ₹2550 will get profit of

$$0.3x + (0.4x) \times \frac{2550}{6300} = 0.4619x$$

Partner 1 gets ₹1280 more than partner 2

$$0.5381x = 0.4619x + 1280$$
$$0.0762x = 1280$$
$$x = 16797.9$$

7. (d)



From a look at the Venn diagram we get that it is not necessary that some or all professors are singers. Hence, none of the conclusions follow.

8. (b)

Let the number of students be x. Then,

Number of students of or above 8 years = (100 – 20)% of x
= 80% of x

$$\therefore \quad 80\% \text{ of } x = 48 + \frac{2}{3} \times 48 = 80$$

$$\frac{80}{100}x = 80$$
$$x = 100$$

9. (b)

Suppose C gets ₹x. Then, B gets ₹$(x + 8)$ and A gets ₹$(x + 15)$.

Then, $x + (x + 8) + (x + 15) = 53$

$$x = 10$$

$\therefore$ $A : B : C = (10 + 15) : (10 + 8) : 10$

$$= 25 : 18 : 10$$

10. (b)

Let 1 man's 1 day's work $= x$ and 1 boy's 1 day's work $= y$

Then, $2x + 3y = \frac{1}{10}$ ...(i)

$3x + 2y = \frac{1}{8}$ ...(ii)

Solving equation (i) and (ii),

$$x = \frac{7}{200} \text{ and } y = \frac{1}{100}$$

$\therefore$ (2 men + 1 boy)'s 1 day's work $= 2\times\frac{7}{200}+1\times\frac{1}{100}$

$$= \frac{16}{200} = \frac{2}{25}$$

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days

11. (b)

Let B be closed after x minutes. Then,

Port filled by (A + B) in x minutes + Part filled by A in $(18 - x)$ minutes = 1

$\therefore$ $x\left(\frac{1}{24}+\frac{1}{32}\right)+(18-x)\times\frac{1}{24} = 1$

$$\frac{7x}{96}+\frac{18-x}{24} = 1$$

$$\frac{7x+72-4x}{96} = 1$$

$$3x + 72 = 96$$
$$3x = 24$$
$$x = 8$$

Hence, B must be closed in 8 minutes.

12. (a)

$$\text{Let rate upstream} = x \text{ km/h}$$
$$\text{and rate downstream} = y \text{ km/hr}$$

Then $$\frac{40}{x} + \frac{55}{y} = 13 \quad \text{...(i)}$$

and $$\frac{30}{x} + \frac{44}{y} = 10 \quad \text{...(ii)}$$

Multiplying (ii) by 4 and (i) by 3 and subtracting, we get

$$\frac{11}{y} = 1$$
$$y = 11$$

Substituting $y = 11$ in equation (i), we get

$$x = 5$$

$$\text{Rate in still water} = \frac{1}{2}(11 + 5) = 8 \text{ kmph}$$

$$\text{Rate of current} = \frac{1}{2}(11 - 5) = 3 \text{ kmph}$$

13. (a)

A square and a rectangle with equal areas will satisfy the relation $P_1 < P_2$.

$$a^2 = lb$$
$$P_1 = 4a = 4\sqrt{lb}$$
$$P_2 = 2(l + b)$$
$$A.M. \geq G.M.$$

$$\frac{l+b}{2} \geq \sqrt{lb} \qquad \left(\frac{l+b}{2} = \sqrt{lb} \text{ only when } l = b\right)$$

$$\therefore \quad \frac{l+b}{2} > \sqrt{lb}$$

$$P_2 > P_1$$

14. (b)

$$\text{Volume of bowl} = \left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right) \text{cm}^3 = 486\pi \text{ cm}^3$$

$$\text{Volume of 1 bottle} = \left(\pi \times \frac{3}{2} \times \frac{3}{2} \times 4\right) \text{cm}^3 = 9\pi \text{ cm}^3$$

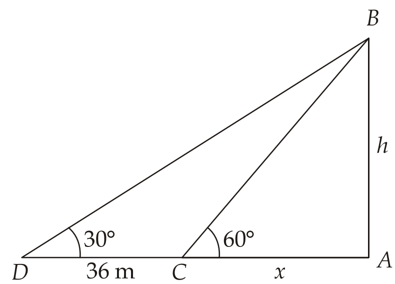
$$\text{Number of bottles} = \frac{486\pi}{9\pi} = 54$$

15. (c)

The hands of a clock coincide 11 times in every 12 hours (since between 11 and 1, they coincide only once i.e. at 12 O'clock).

∴ The hands coincide 22 times in a day.

16. (b)



Let AB be the tree and AC be the river. Let C and D be the two positions of the man.

Then, $\angle ACB = 60°, \angle ADB = 30°$ and $CD = 36$ m

Let $AB = h$ meters and $AC = x$ metres

Then $AD = (36 + x)$ metres

$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{h}{36+x} = \frac{1}{\sqrt{3}}$$

$$h = \frac{36+x}{\sqrt{3}} \quad \text{...(i)}$$

$$\frac{AB}{AC} = \tan 60° = \sqrt{3}$$

$$\frac{h}{x} = \sqrt{3}$$

$$h = \sqrt{3}x \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\frac{36+x}{\sqrt{3}} = \sqrt{3}x$$

$$x = 18 \text{ m}$$

So, the breadth of the river = 18 m

17. (a)

Let Age of A, B and C are A, B and C respectively.

$$(A + B) - (B + C) = 12$$

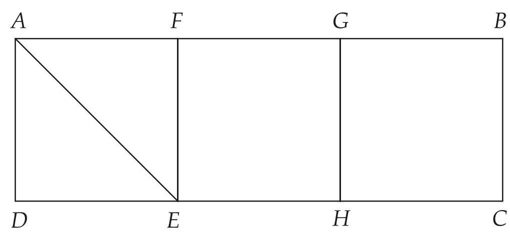
$$A - C = 12$$

18. (b)

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}+\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}} = \frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^a+x^b+x^c}+\frac{x^c}{x^c+x^b+x^a}$$

$$= 1$$

19. (b)



$$\begin{aligned}\text{Area of rectangle } ADEF &= 2 \times \text{Area of triangle } ADE \\ \text{Area of rectangle } ABCD &= 3 \times \text{Area of rectangle } ADEF \\ &= 6 \times \text{Area of triangle } ADE \\ 240 &= 6 \times \text{Area of triangle ADE} \\ \text{Area of triangle } ADE &= 40 \text{ cm}^2\end{aligned}$$

20. (b)

$$\text{Sushil} + \text{Mukesh} = 200 \quad \ldots(1)$$
$$\text{Asim} + \text{Rajesh} = \text{Sushil} \quad \ldots(2)$$
$$\text{Mukesh} = 4\text{ Rajesh} \quad \ldots(3)$$
$$\text{Rajesh} = \text{Asim} - 20$$
$$\Rightarrow \quad \text{Asim} = \text{Rajesh} + 20 \quad \ldots(4)$$

From equation (1) and (3),

$$\text{Sushil} + 4\text{ Rajesh} = 200 \quad \ldots(5)$$

From equation (2) and (4),

$$\text{Rajesh} + \text{Rajesh} + 20 = \text{Sushil}$$
$$\text{Sushil} = 2\text{ Rajesh} + 20 \quad \ldots(6)$$

From equation (5) and (6),

$$\begin{aligned}6\text{ Rajesh} + 20 &= 200 \\ 6\text{ Rajesh} &= 180 \\ \text{Rajesh} &= 30 \\ \text{Asim} &= 30 + 20 = 50\end{aligned}$$

21. (d)

Total income project in the year 2007 = 150

When the income in the year 2002 was 100.

∴ Annual compound rate of growth is 8.5%

22. (d)

We will be required to get the value of n such that $\frac{n(n+1)}{2} = 100$

If $n = 13$

$$\frac{n(n+1)}{2} = 91$$

which means that 100th digit will be occupied by 14th set of digits or it will be equal to 4.

23. (b)

We are given that all boys play cricket and some play football. So those who play football must be playing cricket as well. Hence C1 follows. Some boys play cricket only. This is also true so C2 also follows.

Hence, both conclusion C1 and C2 follow.

24. (a)

Let numerator be x and denominator be y.

then,
$$x = y - 4$$
$$8(x - 2) = y + 1$$

The above equations can be written as

$$y - x = 4 \quad \text{...(i)}$$
$$8x - y = 17 \quad \text{...(ii)}$$
$$\Rightarrow \quad x = 3 \text{ and } y = 7$$

Hence, the required fraction $\frac{x}{y} = \frac{3}{7}$

25. (b)

Ratio of number of men, women and children = $\frac{18}{6} : \frac{10}{5} : \frac{12}{3}$ = $3x : 2x : 4x$

$$\therefore \quad (3x + 2x + 4x) = 18$$
$$x = 2$$

Therefore, number of women = 4

Share of all women = $\frac{10}{40} \times 4000$ = ₹1000

$\therefore$ Share of each women = $\frac{1000}{4}$ = ₹250

26. (b)

Method	Order
Bisection	1
Regula Falsi	1
Secant	1.62
Newton Raphson	2

27. (b)

Eigen values of $(A + 4I)$ are $\alpha + 4$, $\beta + 4$

Eigen values of $(A + 4I)^{-1}$ $\frac{1}{\alpha+4}$ and $\frac{1}{\beta+4}$

28. (a)

Given matrix A is $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$$\text{Determinant of } A,\ |A| = 1$$

$$\text{Adj } A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\therefore \quad \text{Inverse of matrix } A,\ A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

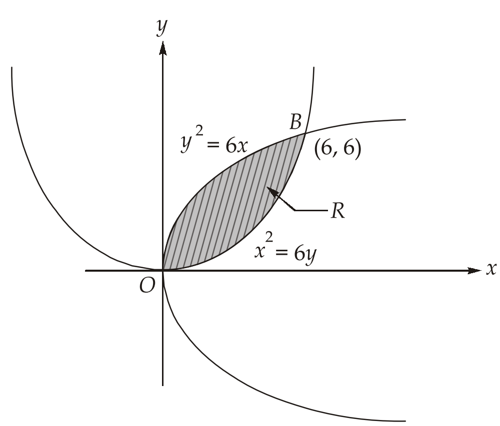
29. (b)

In Poisson distribution,

$$\text{Mean} = \text{Variance} = \mu$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \sqrt{\mu}$$

30. (a)



$$\text{Area of region } R = \int_0^6 \left(\sqrt{6x} - \frac{x^2}{6} \right) dx$$

$$= \left| 2\sqrt{6}\frac{x^{3/2}}{3} - \frac{x^3}{18} \right|_0^6 = \frac{2\sqrt{6}(6)^{3/2}}{3} - \frac{(6)^3}{18}$$

$$= 12 \text{ sq. units}$$

31. (c)

Both statements are correct.

32. (b)

$$\vec{F} = \sin y\hat{i} + x(1+\cos y)\hat{j}$$

$$\int_c \vec{F}\cdot\vec{dr} = \int_c \left[\sin y\hat{i} + x(1+\cos y)\hat{j}\right]\left(\hat{i}dx + \hat{j}dy\right)$$

$$= \int_c \sin y\,dx + x(1+\cos y)\,dy$$

On applying Green's theorem, we have

$$\oint_c (\phi\,dx + \psi dy) = \iint_s \left(\frac{\partial\psi}{\partial x} - \frac{\partial\phi}{\partial y}\right)dx\,dy$$

$$= \iint_s \left[(1+\cos y) - \cos y\right]dx\,dy$$

where s is the circular plane surface of radius a

$$= \iint_s dx\,dy = \text{Area of circle} = \pi a^2$$

33. (a)

Cauchy-Riemann's equation in polar form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$$\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$$

34. (d)

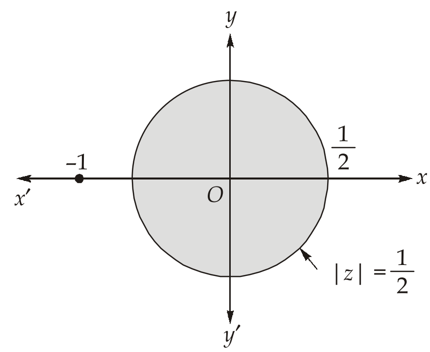
Pole of the integrand is given by

$$z + 1 = 0$$

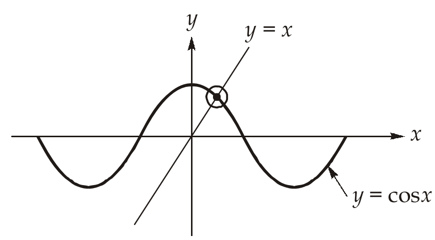
$$z = -1$$

The given circle $|z| = \frac{1}{2}$ with centre at $z = 0$ and radius $\frac{1}{2}$ does not enclose any singularity of the given function. Thus, by Cauchy's theorem,

$$\int_c \frac{3z^2 + 7z + 1}{z+1}dz = 0$$



35. (d)



There is only one point of intersection, so there is only one solution.

36. (d)

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + \left(\frac{d^3y}{dx^3}\right)^{3/2} = 0$$

$$1 + \left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}-\frac{2}{3}} = 0$$

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{5}{6}} = -1$$

Raising power 6 on both sides,

$$\left(\frac{d^3y}{dx^3}\right)^{5} = 1$$

Thus, degree of differential equation is 5.

37. (b)

By Simpson's $\frac{1}{3}$ rule,

$$I = \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + ... + y_{n-2}) + 4(y_1 + y_3 + ... + y_{n-1})\right]$$

$$= \frac{0.2}{3}\left[(1.3863 + 1.6487) + 2(1.4816 + 1.5686) + 4(1.4351 + 1.5261 + 1.6094)\right]$$

$$= 1.8278472$$

38. **(d)**

All statements are correct.

39. **(d)**

All statements are correct.

40. **(a)**

If $r = r' < n$; the equations are consistent and there are infinite number of solutions.

If $r = r' = n$; the equations are consistent and there is a unique solution.

41. **(a)**

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= \left(10xy + 2.5z^2\right)\hat{i} + (5x^2 - 10yz)\hat{j} + (-5y^2 + 5zx)\hat{k}$$

$$= 12.5\hat{i} - 5\hat{j} \text{ at } P(1, 1, 1)$$

Also direction of the given line is

$$\hat{A} = \frac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$$

Hence, the required directional derivative = $\nabla\phi\,\hat{A}$

$$= \left(12.5\hat{i} - 5\hat{j}\right)\cdot\frac{\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{3}$$

$$= \frac{(25 + 10)}{3} = \frac{35}{3} = 11\frac{2}{3}$$

42. **(a)**

We can also write

$$\left(ye^x dx - e^x dy\right) + 2xy^2 dx = 0$$

Multiplying throughout by $\frac{1}{y^2}$, follows

$$\frac{ye^x dx - e^x dy}{y^2} + 2x\,dx = 0$$

$$d\left(\frac{e^x}{y}\right) + 2xdx = 0$$

Integrating, we get

$$\frac{e^x}{y} + x^2 = c \text{ which is required solution}$$

43. (c)

$$L^{-1}\left(\frac{1}{s^2+4s+5}\right) = L^{-1}\left(\frac{1}{(s+2)^2+1}\right) = e^{-2t}\sin t$$

$$L^{-1}\left\{\frac{d}{ds}\left(\frac{1}{s^2+4s+5}\right)\right\} = (-1)^1 t\cdot e^{-2t}\sin t$$

$$L^{-1}\left\{\frac{-(2s+4)}{(s^2+4s+5)^2}\right\} = -t\cdot e^{-2t}\sin t$$

$$L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\} = \frac{1}{2}te^{-2t}\sin t$$

44. (d)

The probability that A can solve the problem is $\frac{1}{2}\cdot$ Similarly the probabilities that B and C cannot solve the problem are $1-\frac{1}{3}$ and $1-\frac{1}{4}$.

∴ The probability that A, B and C cannot solve the problem is $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{4}$$

45. (a)

$$Z_1 = 7+(7\sqrt{3})i$$

$$Z_2 = \frac{4}{\sqrt{3}}+4i$$

$$\text{Argument of } (Z_1) = \theta_1 = \tan^{-1}\left(\frac{7\sqrt{3}}{7}\right)$$

$$\theta_1 = 60°$$

$$\text{Argument of } (Z_2) = \theta_2 = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$$

$$\theta_2 = 60°$$

$$\text{Argument of } \frac{Z_1}{Z_2} = \text{Argument of } (Z_1) - \text{Argument of } (Z_2)$$

$$\theta_3 = 60° - 60° = 0$$

46. (d)

$$\text{The poles of } f(z) = \frac{\sin z}{\cos z} \text{ are given by } \cos z = 0$$

i.e. $$z = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

of these poles, $z = \frac{\pi}{2}$, and $\frac{-\pi}{2}$ only are within the circle $|z| = 4$

$$\therefore \quad \text{Res}\, f\left(\frac{\pi}{2}\right) = \underset{z\to\frac{\pi}{2}}{lt}\left(z-\frac{\pi}{2}\right)f(z) = \underset{z\to\frac{\pi}{2}}{lt}\frac{\left(z-\frac{\pi}{2}\right)\sin z}{\cos z}$$

$$= \underset{z\to\frac{\pi}{2}}{lt}\frac{\left(z-\frac{\pi}{2}\right)\cos z+\sin z}{-\sin z} = -1$$

Similarly, $\text{Res}\, f\left(-\frac{\pi}{2}\right) = -1$

Hence, by residue theorem,

$$\oint_c f(z)\,dz = 2\pi i\left\{\text{Res}\, f\left(\frac{\pi}{2}\right)+\text{Res}\, f\left(-\frac{\pi}{2}\right)\right\}$$

$$= 2\pi i\,(-1-1) = -4\pi i$$

47. (a)

If x is a random variable, then

$$\sum_{i=0}^{6} P(x_i) = 1$$

$$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$

$$k = \frac{1}{49}$$

48. (b)

$$f(x) = \frac{x^3}{3} - x$$

We will find the first and second derivative.

$f'(x) = \frac{3x^2}{3} - 1 = x^2 - 1$ and $f''(x) = 2x$ to determine maximum value of x,

Putting $f'(x) = x^2 - 1 = 0$ gives $x = 1$ or -1

For $x = -1$ only, $f''(x) < 0$ which means maximum value of the function exists for $x = -1$.

49. (b)

Given that the partial differential equation is parabolic

$\therefore \quad B^2 - 4AC = 0$ Here, $A = 2$, $C = 2$

$$\therefore \quad B^2 - 4 \times 2 \times 2 = 0$$

$$B^2 - 16 = 0$$

$$B^2 = 16$$

50. (a)

For solenoidal vector field,

$$\nabla \cdot \vec{F} = 0$$

$$\frac{\partial\left(y^2 - z^2 + 3yz - 2x\right)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

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