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GATE 2022

CIVIL ENGINEERING

Exam held on
12/02/2022
(Afternoon Session)

Questions & Solutions

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SECTION - A GENERAL APTITUDE

Q.1 The movie was funny and I ________ .

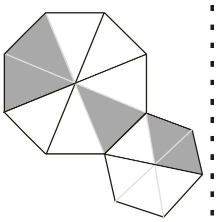
(a) could helped laughed
(b) couldn't help laughed
(c) couldn't help laughing
(d) could help laughing

Ans. (c)

The movies was funny and I couldn't help laughing.
i.e. movie was so funny that the person couldn't control his/her laugh.

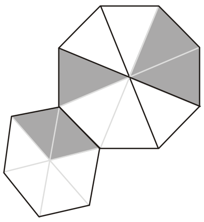
End of Solution

Q.2

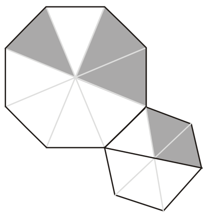


For the picture shown above, which one of the following is the correct picture representing reflection with respect to the mirror shown as the dotted line?

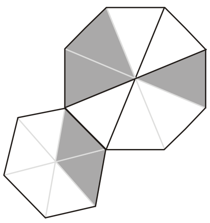
(a)



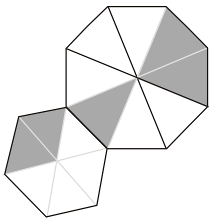
(b)



(c)



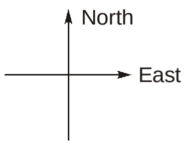
(d)



Ans. (d)

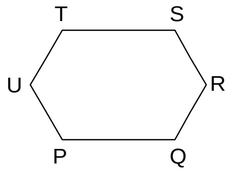
End of Solution

Q.3 An ant walks in a straight line on a plane leaving behind a trace of its movement. The initial position of the ant is at point P facing East.
The ant first turns 72° anticlockwise at P, and then does the following two steps in sequence exactly FIVE times before halting.

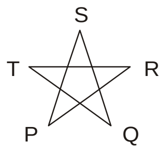


1. moves forward for 10 cm.
2. turns 144° clockwise

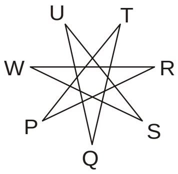
The pattern made by the trace left behind by the ant is

(a) 

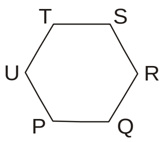
PQ = QR = RS = ST = TP = 10 cm

(b) 

SQ = QT = TR = RP = PS = 10 cm

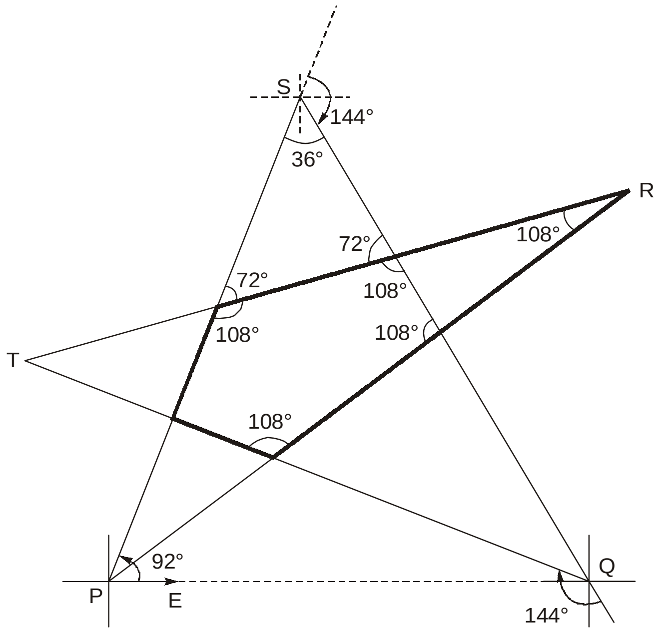
(c) 

SW = WR = RP = PT = TQ = QU = US = 10 cm

(d) 

PQ = QR = RS = ST = TU = UP = 10 cm

Ans. (b)



and PS = SQ = QT = TR = RP = 10 cm

End of Solution

Q.4 In a partnership business the monthly investment by three friends for the first six months is m the ratio 3 :4 : 5. After six months, they had to increase their monthly investments by 10%, 15% and 20%, respectively, of then initial monthly investment. The new investment ratio was kept constant for the next six months.

What is the ratio of their shares in the total profit (m the same order) at the end of the year such that the share is proportional to their individual total investment over the year?

(a) 63 : 86 : 110 (b) 33 : 46 : 60

(c) 22 : 23 : 24 (d) 22 : 33 : 50

Ans. (a)

$$\frac{(\text{Profit})_A}{(\text{Profit})_B} = \frac{(\text{Investment})_A}{(\text{Investment})_B} \times \frac{t_A}{t_B}$$

Given, ratio of initial investment of friends P, Q, R is 3 : 4 : 5 respectively.

Ratio, after increment in investment of P, Q, R.

P increase the investment by 5%.

Q increase the investment by 10%

R increase the investment by 20%

i.e. $3 + (3 \times 1.1) : 4 + (4 \times 1.5) : 5 + (5 \times 1.2)$

$= 6.3 : 8.6 : 11$

$= 63 : 86 : 110$

End of Solution

Q.5 A survey of 450 students about their subjects of interest resulted in the following outcome.

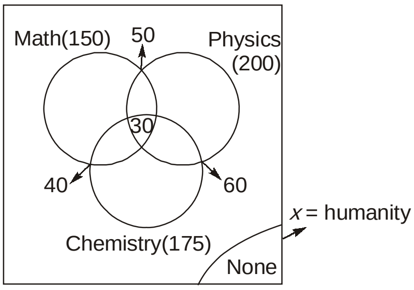
- 150 students are interested m Mathematics.
- 200 students are interested in Physics.
- 175 students are interested in Chemistry.
- 50 students are interested in Mathematics and Physics.
- 60 students are interested in Physics and Chemistry
- 40 students are interested in Mathematics and Chemistry.
- 30 students are interested in Mathematics, Physics and Chemistry.
- Remaining students are interested in Humanities.

Based on the above information, the number of students interested in Humanities is

(a) 45 (b) 10

(c) 40 (d) 30

Ans. (a)



$$T = n(M \cup P \cup C) + x$$

$$450 = [(150 + 200 + 175) - (50 + 60 + 40) + 30] + x$$

$$x = 45$$

End of Solution

Q.6 $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

What is the value of $\frac{x+z-y}{y}$ = ?

(a) 1.25 (b) 0.75
(c) 3.25 (d) 2.25

Ans. (a)

Given: $x : y : z = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$

As we need to find ratio of $(x + y + z)$ to y, so directly x, y, z values can be taken as

$$x = 6, \; y = 4, \; z = 3$$

$\therefore$ The value of $= \frac{x+z-y}{y} = \frac{6+3-4}{4} = \frac{5}{4} = 1.25$

End of Solution

Q.7 Both the numerator and the denominator of $\frac{3}{4}$ are increased by a positive $\frac{15}{17}$ integer, x, and those of — are decreased by the same integer. This operation results m the same value for both the fractions. What is the value of x?

(a) 3 (b) 4
(c) 1 (d) 2

Ans. (a)

Let's take the intger as x.

Now as per given statement,

$$\frac{3+x}{4+x} = \frac{15-x}{17-x}$$

By solving for $x = 3$

End of Solution

Q.8 Consider the following equations of straight lines:

Line L1 : $2x - 3y = 5$

Line L2 : $3x + 2y = 8$

Line L3 : $4x - 6y = 5$

Line L4 : $6x - 9y = 6$

Which one of the following is the correct statement?

(a) L1 is parallel to L2 and L1 is perpendicular to L3

(b) L2 is parallel to L4 and L2 is perpendicular to L1

(c) L4 is perpendicular to L2 and L4 is parallel to L3

(d) L3 is perpendicular to L4 and L3 is parallel to L2

Ans. (c)

Line L1 : $2x - 3y = 5$

$$y = \frac{2x-5}{3}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

Line L2 : $3x + 2y = 8$

$$y = \frac{8-3x}{2}$$

$$\frac{dy}{dx} = \frac{-3}{2}$$

Line L3 : $4x - 6y = 5$

$$y = \frac{4x-5}{6}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

Line L4 : $6x - 9y = 6$

$$y = \frac{6x-6}{9}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{L1} = \left(\frac{dy}{dx}\right)_{L3} = \left(\frac{dy}{dx}\right)_{L4} = \frac{2}{3}$$

and $$\left(\frac{dy}{dx}\right)_{L2} \times \left(\frac{dy}{dx}\right)_{L1} = \left(\frac{dy}{dx}\right)_{L2} \times \left(\frac{dy}{dx}\right)_{L3} = \left(\frac{dy}{dx}\right)_{L2} \times \left(\frac{dy}{dx}\right)_{L4} = -1$$

$\therefore$ L1, L3, L4 are parallel to each other and perpendicular to L2.

End of Solution

Q.9 In the last few years, several new shopping malls were opened in the city. The total number of visitors in the malls is impressive. However, the total revenue generated through sales in the shops in these malls is generally low.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

(a) More people are visiting the malls and spending more

(b) Fewer people are visiting the malls but spending more

(c) More people are visiting the malls but not spending enough

(d) Fewer people are visiting the malls and not spending enough

Ans. (c)

End of Solution

Q.10 Given below are two statements and four conclusions drawn based on the statements.

Statement 1: Some soaps are clean.

Statement 2: All clean objects are wet.

Conclusion I: Some clean objects are soaps.

Conclusion II: No clean object is a soap.

Conclusion III: Some wet objects are soaps.

Conclusion IV: All wet objects are soaps.

Which one of the following options can be logically inferred?

(a) Either conclusion I or conclusion II is correct

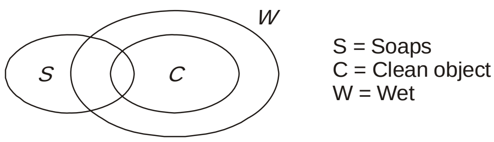
(b) Either conclusion III or conclusion IV is correct

(c) Only conclusion I and conclusion III are correct

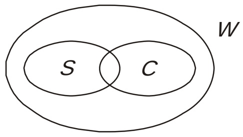
(d) Only conclusion I is correct

Ans. (c)

Statement 1: Vane diagram



Statement 2: Vane diagram



From the above vane diagram we can conculate that conclusion I and conclusion III are correct and conclusion II and IV are in correct.

End of Solution

■■■■

SECTION - B TECHNICAL

Q.11 A 100 mg of HNO_3 (strong acid) is added to water, bringing the final volume to 1.0 liter. Consider the atomic weights of H, N, and O, as 1 g/mol. 14 g mol. and 16 g/mol, respectively. The final pH of this water is (Ignore the dissociation of water.)

(a) 6.5
(b) 2.8
(c) 8.5
(d) 3.8

Ans. (b)

$$HNO_3 \rightarrow H^+ + NO_3^- \qquad \{\text{Molecular mass } HNO_3 = 63g\}$$

1 mole of HNO_3 produces 1 mole of H^+ means, 64g of HNO_3 produce 1 g of H^+

$\therefore$ 100 mg of HNO_3 will produce

$$\frac{1}{63} \times 100 = 1.587 \text{ mg of } H^+$$

$$= 1.587 \times 10^{-3} \text{ g of } H^+ \text{ in 1 lit}$$

$$pH = -\log_{10}[H^+]$$

$$= -\log_{10}[1.587 \times 10^{-3}]$$

$$= 2.8$$

End of Solution

Q.12 Match the following in Column X with Column Y:

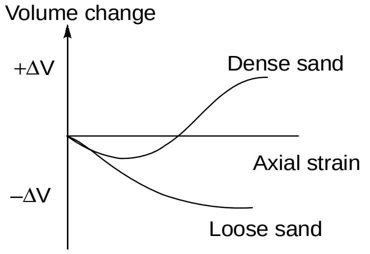
Column X	Column Y
P. In a triaxial compression test, with increase of axial strain in loose sand under drained shear condition. the volumetric strain.	I. decreases
Q. In a triaxial compression test, with increase of axial strain in loose sand under undrained shear condition, the excess pore water pressure.	II. increase
R. In a triaxial compression test, the pore pressure parameter "B" for a saturated soil.	III. remains same
S. For shallow strip footing in pure saturated clay, Terzaghi's bearing capacity factor (N_q) due to surcharge.	IV. is always 0.0
	V. is always 1.0
	VI. is always 0.5

Which one of the following combinations is correct?

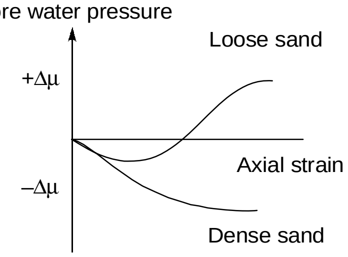
(a) (P)-(I), (Q)-(II), (R)-(V), (S)-(VI)
(b) (P)-(I), (Q)-(III), (R)-(VI), (S)-(IV)
(c) (P)-(I), (Q)-(I), (R)-(IV), (S)-(V)
(d) (P)-(I), (Q)-(II), (R)-(V), (S)-(V)

Ans. (d)

(P) Volume change



(Q) Pore water pressure



(R) Skempton parameter B value for saturated cohesive soil is 1.

(S) For clay, $N_q = 1$, $N_c = 5.7$ and $N_r = 0$ $(\because \phi = 0)$

End of Solution

Q.13 In a city, the chemical formula of biodegradable fraction of municipal solid waste (MSW) is $C_{100}H_{250}O_{80}N$. The waste has to be treated by forced-aeration composting process for which air requirement has to be estimated.

Assume oxygen in air (by weight) = 23% and density of air = 1.3 kg/m^3. Atomic mass: C = 12, H = 1, O = 16, N = 14.

C and H are oxidized completely whereas N is converted only into NH_3 during oxidation.

For oxidative degradation of 1 tonne of the waste, the required theoretical volume of air (in m^3/tonne) will be (round off the nearest integer)

(a) 1092
(b) 4749
(c) 1418
(d) 8025

Ans. (b)

$$\underset{2744g}{C_{100}H_{250}O_{80}N} + \underset{3896g}{121.75O_2} \rightarrow 100CO_2 + 123.5H_2O + NH_3$$

$\therefore$ if 1 tonne (or 1000 kg) of waste is present,

$$\text{oxygen required} = \left(\frac{3896}{2744}\right)\times 100\,\text{kg} = 1419.825\,\text{kg}$$

Now, 1.3 kg of air contains = 0.23 × 1.3 kg = 0.299 kg of oxygen

$$\therefore 1419.825 \text{ kg of oxygen will be contained by } = \left(\frac{1.3}{0.299}\right)\times 1419.825$$

$$= 6173.152 \text{ kg of air}$$

$$\text{Volume of air} = \frac{6173.152}{1.3} = 4748.57 \text{ m}^3$$

End of Solution

Q.14 Let y be a non-zero vector of size 2022 × 1. Which of the following statement(s) is/are TRUE?

(a) yy^T is symmetric matrix
(b) yy^T is invertible
(c) yy^T has a rank of 2022
(d) y^Ty is an eigen value of yy^T.

Ans. (a, d)

Let, $$y = \begin{bmatrix}1\\1\\1\end{bmatrix}_{3\times1}, \quad y^T = \begin{bmatrix}1 & 1 & 1\end{bmatrix}_{1\times3}$$

$$yy^T = \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}_{3\times3} \quad \text{and} \quad y^Ty = [3]_{1\times1}$$

Here, Rank of matrix $\rho(y) = \rho(y^T) = \rho(yy^T) = \rho(y^Ty) = 1$

Eigen value of $yy^T = 3, 0, 0$

Here, y^Ty is eigen value of yy^T i.e. 3.

- yy^T is symmetrical matrix.
- $\because$ Determinant of yy^T is zero.
  $\therefore$ A^{-1} will not exist.
- Rank of yy^T is 1.

End of Solution

Q.15 A pair of six-faced dice is rolled twice, The probability that the sum of the outcomes in each roll equals 4 in exactly two of the three attempts is ______ (round off to three decimal places).

Ans. (0.019)

$$A = \{(1, 3), (3, 1), (2, 2)\}$$

$$P(A) = \frac{3}{36} = \frac{1}{12}$$

$$P(\bar{A}) = \frac{11}{12}$$

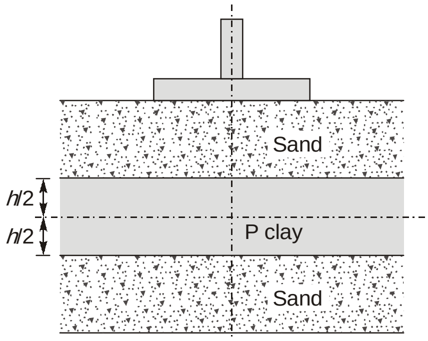
$$\text{Required probability} = P(AA\bar{A}) + P(A\bar{A}A) + P(\bar{A}AA)$$

$$= \frac{1}{12}\times\frac{1}{12}\times\frac{11}{12} + \frac{1}{12}\times\frac{11}{12}\times\frac{1}{12} + \frac{11}{12}\times\frac{1}{12}\times\frac{1}{12}$$

$$= 3\times\left(\frac{1}{12}\right)^2\times\frac{11}{12} = 0.019$$

End of Solution

Q.16 A saturated compressible clay layer of thickness *h* is sandwiched between two sand layers, as shown in the figure. Initially, the total vertical stress and pore water pressure at point P. which is located at the mid-depth of the clay layer were 150 kPa and 25 kPa. respectively. Construction of a building caused an additional total vertical stress of 100 kPa at P. When the vertical effective stress at P is 175 kPa. the percentage of consolidation in The clay layer at P is ________ . (in integer)



Ans. (50)

We know degree of consolidation,

$$\%U = \frac{u_1 - u}{u_i} \times 100$$

u_i = initial excess pore water pressure i.e. 100 kPa

Initial total stress, σ = 150 kPa

Initial effective stress, $\sigma_0' = \sigma - u = 150 - 25 = 125$ kPa

Total stress after construction of building $= 150 + 100 = 250$ kPa

When the vertical effective stress at P is 175 kPa, the change in effective stress is $175 - 125 = 50$ kPa, so during this, the change in pore water pressure is

$$\therefore \quad \Delta\sigma = \Delta\sigma' + \Delta u$$

$$(250 - 150) = (175 - 125) + \Delta u$$

$$\therefore \quad \Delta u = 50 \text{ kPa}$$

$$\therefore \quad \%u = \frac{100 - 50}{100} \times 100 = 50\%$$

End of Solution

Q.17 A single-lane highway has a traffic density of 40 vehicles km If the time-mean speed and space-mean speed are 40 kmph and 30 kmph, respectively, the average headway (in seconds) between the vehicles is

(a) 2.25
(b) 6.25×10^{-4}
(c) 3.00
(d) 8.33×10^{-4}

Ans. (c)

$q = kV$

q = Flow of traffic (veh/m)

V = Space mean speed of vehicle

k = Density of traffic (veh/km)

$\therefore \quad q = 40 \times 30 = 1200$ veh/hr

$\therefore \quad q = \dfrac{3600}{h_t}$

$\therefore \quad h_t = 3$ sec. (time headway)

End of Solution

Q.18 Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S, given by $1 \le x \le 3$. The first and second derivatives are $f'(x)$ and $f''(x)$.

Consider the following statements:

I. The given polynomial is zero at the boundary points $x = 1$ and $x = 3$.

II. There exists one local maxima of $f(x)$ within the domain S.

III. The second derivative $f''(x) > 0$ throughout the domains S.

IV. There exists one local minima $f(x)$ within the domain S.

(a) Only statements II and IV are correct.

(b) Only statements I and IV are correct.

(c) Only statements I. II and III are correct.

(d) Only statements I. II and IV are correct.

Ans. (d)

Given: $f(x) = x^3 - 6x^2 + 11x - 6$

$= (x-1)(x-2)(x-3)$ ... (i)

- At $x = 1$ and $x = 3$ $f(x) = 0$

$\therefore$ Statement 1 is correct

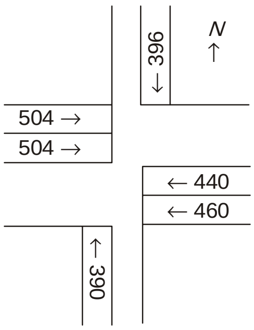
$$f'(x) = 3x^3 - 12x^2 + 11$$

$$f''(x) = 6x - 12 = 6(x-2)$$

- Now, $f''(x) < 0$, if $x < 2$. This is condition for maxima which is, within the domin S.
- Now, $f''(x) > 0$, if $x > 2$. This is condition for minima which is, within the domin S.
- But, $f''(x)$ is not greater than zero throughout the domains S.

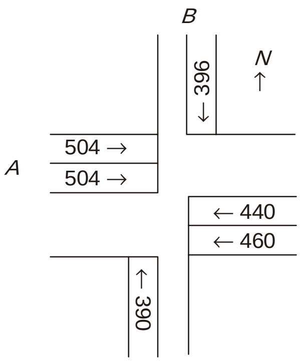
End of Solution

Q.19 The lane configuration with lane volumes in vehicles per hour of a four-arm signalized intersection is shown in the figure. There are only two phases: the first phase is for the East-West and the West-East through movements, and the second phase is for the North-South and the South-North through movements. There are no turning movements. Assume that the saturation flow is 1800 vehicles per hour per lane for each lane and the total lost time for the first and the second phases together is 9 seconds.



The optimum cycle length (in second), as per the Webster's method, is ______ . (round off to the nearest integer).

Ans. (37)



$$C_0 = \frac{1.5L+5}{1-Y}$$

Average flow over lane A and B.

$$q_A = \text{max.}(504,\ 460) = 504 \text{ veh.}$$

$$q_B = \text{max.}(360,\ 396) = 396 \text{ veh.}$$

Saturation flow over lane A and B.

Given, $S_A = S_B = 1800$ veh/hr

$$Y = y_A + y_B$$

$$= \frac{q_A}{S_A} + \frac{q_B}{S_B}$$

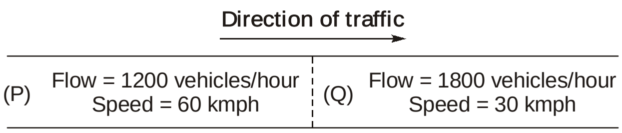
$$= \frac{504}{1800} + \frac{396}{1800} = \frac{1}{2}$$

Los time = 9s (given)

$\therefore$ Cycle length, $C_o = \dfrac{1.5(9)+5}{1-\dfrac{1}{2}} = 37s$

End of Solution

Q.20 Assuming that traffic on a highway obeys the Greenshields model, the speed of a shockwave between two traffic streams (P) and (Q) as shown in the schematic is ________ kmph. (in integer)



Ans. (60)

$$q_P = 1200 \text{ veh/hr}$$

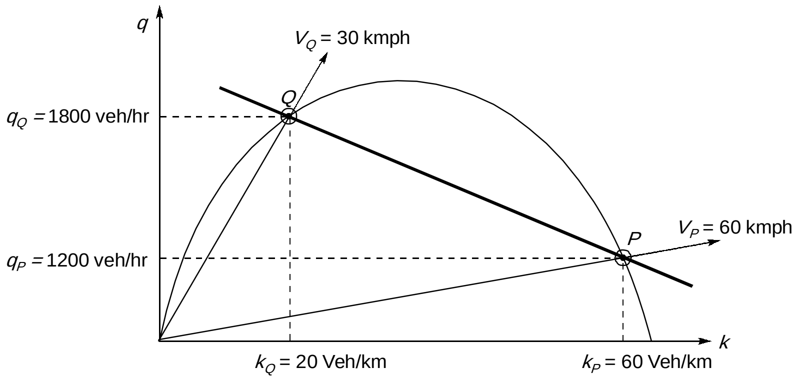
$$V_P = 60 \text{ kmph}$$

$\therefore$ $k_P = \dfrac{q}{v} = \dfrac{1200}{60} = 20$ veh/km

Similarly, $q_Q = 1800$ veh/hr

$$V_Q = 30 \text{ kmph}$$

$\therefore$ $k_Q = \dfrac{q}{v} = \dfrac{1800}{30} = 60$ veh/km



Speed of shock wave (*PQ*) is, slope of line *PQ*,

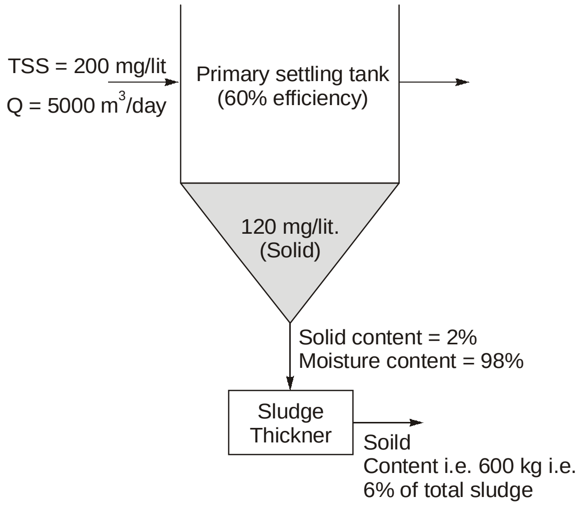
$$\text{Speed of shock wave} = \frac{q_Q - q_P}{k_P - k_Q}$$

$$= \frac{1800 - 1200}{60 - 20} = 15\,\text{kmph}$$

End of Solution

Q.21 A sewage treatment plant receives sewage at a flow rate of 5000 m^3/day. The total suspended solids (TSS) concentration in the sewage at the inlet of primary clarifier is 200 mg/L. After the primary treatment, the TSS concentration in sewage is reduced by 60 %. The sludge from the primary clarifier contains 2 % solids concentration. Subsequently, the sludge is subjected to gravity thickening process to achieve a solids concentration of 6%. Assume that the density of sludge, before and after thickening, is 1000 kg/m^3. The daily volume of the thickened sludge (in m^3 /day) will be__________. (round off to the nearest integer)

Ans. (10)



Solid content in thickener

$$\frac{5000 \times 10^3\, l/d \times 120\,\text{mg/lit}}{10^6} = 600\,\text{kg/day} \quad \text{i.e., 6\% of total mass of sludge}$$

$\therefore$ Total mass of sludge $= \dfrac{600 \times 100}{6} = 10000\,\text{kg/d}$

$\therefore$ Volume of sludge $= \dfrac{M_{sludge}}{\rho_{sludge}} = \dfrac{10000}{1000} = 10\ \text{m}^3/\text{day}$

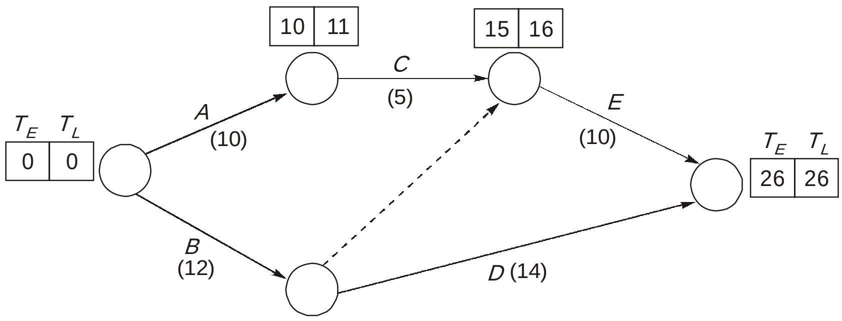
End of Solution

Q.22 The activities of a project are given in the following table along with their durations and dependency.

Activities	Duration (day)	Depends on
A	10	–
B	12	–
C	5	*A*
D	14	*B*
E	10	*B*, *C*

The total float of the activity E (in day) is ______ . (in integer)

Ans. (1)



$$F_T = T_L^j - T_E^i - t^{ij}$$
$$= 26 - 15 - 10$$
$$= 1$$

or

$$F_T = \text{LST} - \text{EST}$$
$$= 16 - 15 = 1$$
$$F_T = \text{LFT} - \text{EFT}$$
$$= 26 - 25 = 1$$

End of Solution

Q.23 If the magnetic bearing of the Sun at a place at noon is S 2° E. the magnetic declination (in degrees) at that place is

(a) 2° W
(b) 4° E
(c) 4° W
(d) 2° E

Ans. (d)

Magnetic bearing (MB) of Sun in WCB = 178°.

True bearing (TB) of Sun in WCB at noon must be 180°.

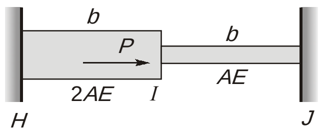
$$MB + \delta = TB$$

$$178° + \delta = 180°$$
$$\delta = 2°$$

$\because$ Declination (δ) is found to be positive so it is East declination of 2° i.e. 2°E.

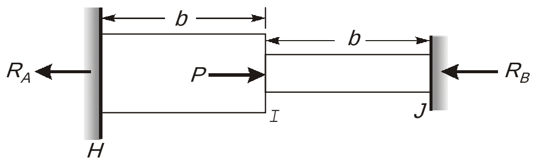
End of Solution

Q.24 Consider two linearly elastic rods HI and IJ. each of length *b*, as shown in the figure. The rods are co-linear, and confined between two fixed supports at if and J. Both the rods are initially stress free, The coefficient of linear thermal expansion is *a* for both the rods. The temperature of the rod IJ is raised by ΔT. whereas the temperature of rod HI remains unchanged. An external horizontal force P is now applied at node *I*. It is given that $a = 10^{-6}$ °C^{-1}, ΔT = 50° C, *b* = 2m, AE = 10^6N. The axial rigidities of the rods HI and U are 2 *AE* and *AE*, respectively.



To make the axial force in rod HI equal to zero, the value of the external force P (in *N*) is ________ . (rounded off to the nearest integer).

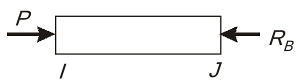
Ans. (50)



$$R_A + R_B = P$$

$\because$ There is no axial force in rod HI $\quad$ ($\therefore R_A = 0$)

Now check for rod *IJ*



$\therefore \quad R_B = P$

Now as rod IJ is fixed from both end, so net deflection due to increase in temperature will be zero.

$$b\alpha T + \left[-\frac{Pb}{AE}\right] = 0$$

$$b\alpha T = \frac{Pb}{AE}$$

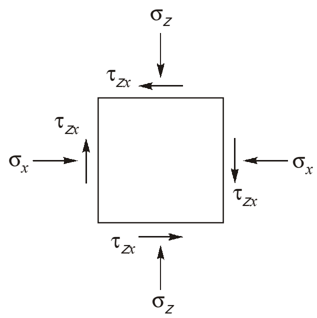
$$P = AE\alpha T$$

$\therefore \quad P = 10^6 \times 10^{-6} \times 50$

$$P = 50 \text{ N}$$

End of Solution

Q.25 Stresses acting on an infinitesimal soil element are shown in the figure (with $\sigma_z > \sigma_x$). The major and minor principal stresses are σ_1 and σ_3, respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?



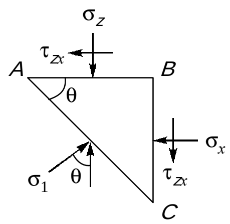
(a) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_3}\right)$

(b) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_3 - \sigma_x}\right)$

(c) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$

(d) $\tan^{-1}\left(\dfrac{\tau_{zx}}{\sigma_1 + \sigma_x}\right)$

Ans. **(c)**



$$\Sigma F_x = 0$$

$$\sigma_x(BC) - \tau_Z \times (AB) + \sigma_1 \sin\theta = 0$$

$$\sigma_x\left(\frac{AC\sin\theta}{\cos\theta}\right) + \tau_{zx}\left(\frac{AC\cos\alpha}{\cos\theta}\right) = \sigma_1\frac{\sin\theta AC}{\cos\theta}$$

$$\sigma_x \tan\theta + \tau_{zx} = \sigma_1 \tan\theta$$

$$\tan\theta\,(\sigma_1 - \sigma_2) = \tau_{zy}$$

$$\tan\theta = \left(\frac{\tau_{zx}}{\sigma_1 - \sigma_x}\right)$$

End of Solution

Q.26 Match the following attributes of a city with the appropriate scale of measurements.

Attribute	Scale of measurement
(P) Average temperature (℃) of a city	(I) Inverval
(Q) Name of city	(II) Oridnal
(R) Population density of a city	(III) Nominal
(S) Ranking of a city based on ease of business	(VI) Ratio

Which one of the following combinations is correct?

(a) (P)-(II), (Q)-(I), (R)-(IV), (S)-(III)
(b) (P)-(I), (Q)-(II), (R)-(III), (S)-(II)
(c) (P)-(I), (Q)-(III), (R)-(IV), (S)-(II)
(d) (P)-(II), (Q)-(III), (R)-(IV), (S)-(I)

Ans. (c)

- Nominal data are a type of categorical data; that is, they are used to represent named quality.
- Ordinal data are a type of categorical data. That is they are used to represent named qualities of things, so that they can be sorted in order in their rank.
- Interval data are a type of categorical data. That is, they represent measured quantities of things.
- Ratio data are a type of categorical data that is, they represent measured quantity of things.

End of Solution

Q.27 A concentrically loaded isolated square footing of size 2 m x 2 m carries a concentrated vertical load of 1000 kN. Considering Boussinesq's theory of stress distribution, the maximum depth (in m) of the pressure bulb corresponding to 10 % of the vertical load intensity will be ________. (round off to two decimal all places)

Ans. (2.18)

$$\sigma_z = 0.1\ Q \text{ (given)}$$

$$\because \quad \sigma_z = k_B \frac{Q}{Z^2}$$

$$\sigma_z = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \frac{Q}{Z^2}$$

$$0.1Q = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \times \frac{Q}{Z^2}$$

at $\frac{r}{z} = 0$

$$Z = 2.18 \text{ m}$$

End of Solution

Q.28 A flood control structure having an expected life of n years is designed by considering a flood of return period T years. When $T = n$, and $n \to \infty$, the structure's hydrologic risk of failure in percentage is _______ . (rounded off to one decimal place)

Ans. (63.2)

$$R = 1 - q^n$$

$$\Rightarrow \quad R = 1 - (1 - P)^n$$

$$R = 1-\left[1-\frac{1}{T}\right]^n$$

Given than $\quad T = n$ and $n \to \infty$

$$\Rightarrow \quad R = 1-\left[\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n\right]$$

$$\Rightarrow \quad R = 1-\left[\lim_{n\to\infty}\left\{1+\left(-\frac{1}{n}\right)\right\}^n\right] \qquad \text{...(i)}$$

We know that

$$\lim_{n\to\infty}(1+x)^{1/x} = e,$$

So let's take $\quad \dfrac{-1}{n} = y$

$$\therefore \quad n = \frac{-1}{y}$$

at $n = \infty$; $y = 0$

$$\text{Now } \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \lim_{y\to 0}(1+y)^{-1/y}$$

$$= \lim_{y\to 0}(1+y)^{\frac{1}{y}(-1)}$$

$$= \left[\lim_{y\to 0}(1+y)^{\frac{1}{y}}\right]^{-1}$$

$$= e^{-1}$$

Put in eq. (i)

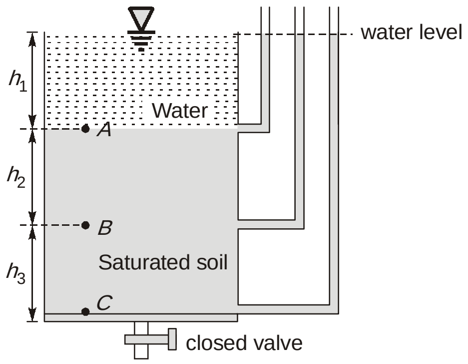
$$\therefore \quad R = 1 - e^{-1}$$

$$R = 1-\frac{1}{e}$$

$$R = 0.6321 \qquad \text{i.e. } 63.2\%$$

End of Solution

Q.29 A soil sample is underlying a water column of height h_1, as shown in the figure. The vertical effective stresses at point A, B and C are σ_A', s_B', and s_C' respectively. Let γ_{sat} and γ' be the saturated and submerged unit weights of the soil sample, respectively and γ_w be the unit weight of water. Which one of the following expressions correctly represents the sum $(\sigma_A' + \sigma_B' + \sigma_C')$?



(a) $(h_1 + h_2 + h_3)\gamma_{sat}$
(b) $(2h_2 + h_3)\gamma'$
(c) $(h_2 + h_3)(\gamma_{sat} - \gamma_w)$
(d) $(h_1 + h_2 + h_3)g'$

Ans. (b)

$$\sigma'_A = \gamma_w h_1 - \gamma_w h_1 = 0$$

$$\sigma'_B = \gamma_w h_1 + \gamma_{sat} h_2 - \gamma_w (h_1 + h_2) = \gamma' h_2$$

$$\sigma'_C = \gamma_w h_1 + \gamma_{sat}(h_2 + h_3) - \gamma_w (h_1 + h_2 + h_3) = \gamma'(h_2 + h_3)$$

$$\therefore \quad \sigma'_A + \sigma'_B + \sigma'_C = 0 + \gamma h_2 + \gamma'(h_2 + h_3)$$

$$= \gamma'[2h_2 + h_3]$$

End of Solution

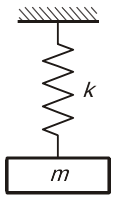
Q.30 To design an optimum municipal solid waste collection route, which of the following is/are NOT desired:

(a) Waste collection on congested roads, should not occur during rush hours in morning or evening,
(b) The last collection point on a route should be as close as possible to the waste disposal facility.
(c) Collection should occur in the uphill direction.
(d) Collection vehicle should not travel twice down the same street in a day.

Ans. (c, d)

End of Solution

Q.31 An undamped spring-mass system with mass m and spring stiffness k is shown in the figure. The natural frequency and natural period of this system are ω rad/s and T s. respectively. If the stiffness of the spring is doubled and the mass is halved, then the natural frequency and the natural period of the modified system, respectively, are



(a) ω rad/s and T s
(b) ω/2 rad/s and $2T$ s
(c) 4 ω rad/s and $T/4$ s
(d) 2 ω rad/s and $T/2$ s

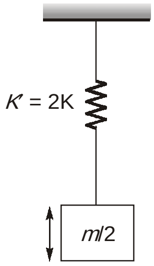
Ans. (d)

Natural frequency, $(\omega_n) = \sqrt{\frac{k}{m}}$ rad/s

Time period, $T_n = \frac{2\pi}{\omega_n}$

For new system $m' = \frac{m}{2}$

$k' = 2k$



∴ New natural frequency

$$\omega_n' = \sqrt{\frac{k'}{m'}} = \sqrt{\frac{2k}{m/2}} = \sqrt{\frac{4k}{m}}$$

∴ $\omega_n' = 2\sqrt{k/m}$ ∵ $\sqrt{k/m} = \omega_n$

∴ $\omega_n' = 2\omega_n$ ⇒ Frequency will be double.

Now, new time period

$$T_n' = \frac{2\pi}{\omega_n'} = \frac{2\pi}{2\omega_n}$$

∴ $\frac{2\pi}{w_n} = T_n$

∴ $T_n' = \frac{T_n}{2}$

Time period will become half.

End of Solution

Q.32 A sample of air analyzed at 25 °C and 1 atm pressure is reported to contain 0.04 ppm of SO_2. Atomic mass of S = 32. O = 16. The equivalent SCb concentration (in μg/m^3) will be ________. (round off to the nearest integer)

Ans. (105)

∴ $PV = nRT$

$$\frac{P_1V_1}{nRT_1} = \frac{P_2V_2}{nRT_2}$$

$\therefore$ Pressure is same in both case ($P_1 = P_2$)

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_1 = 273 \text{ K}$$

$$T_2 = 273 + 25 = 298 \text{ K}$$

$$\frac{V_1}{273} = \frac{V_2}{298}$$

$$T_2 = 273 + 25 = 298$$

$\therefore$ 1 mole of gas occupies 22.4 lit of volume at 0°C temperature and 1 atm pressure.

$$\frac{22.4}{237} = \frac{V_2}{298}$$

$$\therefore \quad V_2 = 24.45 \text{ lit} = 1 \text{ mole of gas at } 25°\text{C}$$

$$\therefore \quad 10^6 \text{ m}^3 \text{ of air} = 0.04 \text{ m}^3 \text{ of } SO_2$$

$$= 40 \text{ lit of } SO_2$$

$$\therefore \quad 10^6 \text{ m}^3 \text{ of air} = \frac{40}{24.45} = 1.636 \text{ mole of } SO_2$$

$$10^6 \text{ m}^3 \text{ of air} = 1.636 \times 64 \text{ g of } SO_2$$

$$1 \text{ m}^3 \text{ of air} = 104.7 \text{ }\mu\text{g of } SO_2 \simeq 105 \text{ }\mu\text{g of } SO_2$$

End of Solution

Q.33 Read the following statements:

(P) While designing a shallow footing in sandy soil, monsoon season is considered for critical design in terms of bearing capacity,

(Q) For slope stability of an earthen dam, sudden drawdown is never a critical condition.

(R) In a sandy sea beach, quicksand condition can arise only if the critical hydraulic gradient exceeds the existing hydraulic gradient.

(S) The active earth thrust on a rigid retaining wall supporting homogeneous cohesionless backfill will reduce with the lowering of water table in the backfill.

Which one of the following combinations is correct?

(a) (P)-False, (Q)-True, (R)-False, (S)-False
(b) (P)-False, (Q)-True, (R)-True, (S)-True
(c) (P)-True, (Q)-False, (R)-False, (S)-False
(d) (P)-True, (Q)-False, (R)-True, (S)-True

Ans. (c)

- Stability of slope will defiantly affected by sudden draw down condition even, it is one of the reason of failure of slope.
- Active thrust decrease when water table is lowered.

For eg:. When WT is at G/L then active pressure at base is $k_a\gamma' H + \gamma_w H$ and when there is no WT than active thrust at base is $k_a\gamma H$.

- Quick sand condition will not occur if $i_c > i$.

End of Solution

Q.34 Read the following statements relating to flexure of reinforced concrete beams:

I. In over-reinforced sections, the failure strain in concrete reaches earlier than the yield strain in steel.

II. In under-reinforced sections, steel reaches yielding at a load lower than the load at which the concrete reaches failure strata.

III. Over-reinforced beams are recommended in practice as compared to the under-reinforced beams.

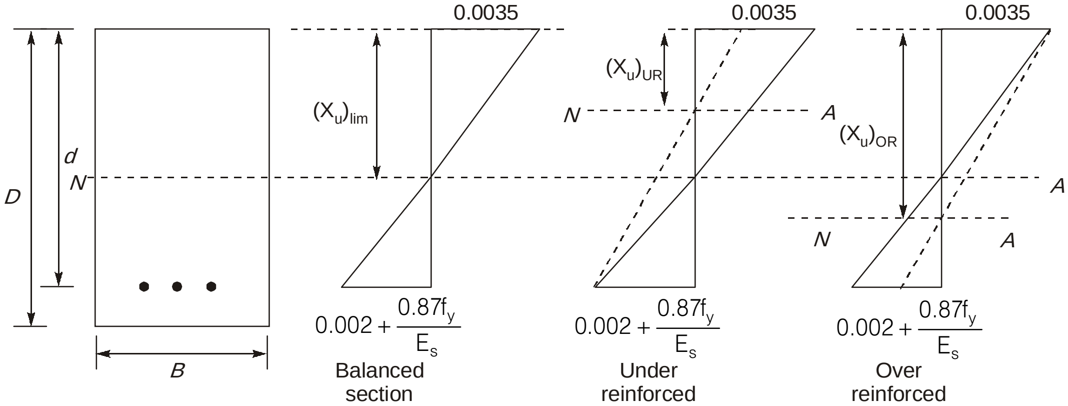
IV. In balanced sections, the concrete reaches failure strain earlier than the yield strain in tensile steel.

Each of the above statements is either True or False.

Which one of the following combinations is correct?

(a) (I)-True, (II)-True, (III)-False, (IV)-False
(b) (I)-False, (II)-True, (III)-True, (IV)-False
(c) (I)-False, (II)-False, (III)-True, (IV)-False
(d) (I)-True, (II)-True, (III)-False, (IV)-True

Ans. (a)



End of Solution

Q.35 The components of pure shear strain in a sheared material are given in the matrix form:

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Here, Trace (ε) = 0. Given, P = Trace (ε^8) and Q = Trace (ε^{11}).

The numerical value of ($P + Q$) is _______ . (in integer)

Ans. (32)

$$\varepsilon = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{(Given)}$$

Eigen value of ε

$$\lambda_1 + \lambda_2 = 0$$
$$\lambda_1 \lambda_2 = -2$$

$\therefore$ $$\lambda_1 = \sqrt{2} \quad \lambda_2 = -\sqrt{2}$$

$\therefore$ $$\text{Eigen value of } \varepsilon^8 = \left(\sqrt{2}\right)^8 \text{ and } \left(-\sqrt{2}\right)^8 = 16 \text{ and } 16$$

$$\varepsilon^{11} = \left(\sqrt{2}\right)^{11} \text{ and } \left(-\sqrt{2}\right)^{11}$$
$$= 32\sqrt{2} \text{ and } -32\sqrt{2}$$

$$\text{Trace of } \varepsilon^8 = 16 + 16 = 32 \text{ i.e. } P$$

$$\text{Trace of } \varepsilon^{11} = 32\sqrt{2} - 32\sqrt{2} = 0 \text{ i.e. } Q$$

$\therefore$ $$P + Q = 32$$

End of Solution

Q.36 A process equipment emits, 5 kg/h of volatile organic compounds (VOCs). If a hood placed over the process equipment captures 95% of the VOCs, then the fugitive emission in kg/h is

(a) 0.25
(b) 4.75
(c) 2.50
(d) 0.48

Ans. (a)

If hood captures 95% of VOCs then the fugitive emission in kg/h is

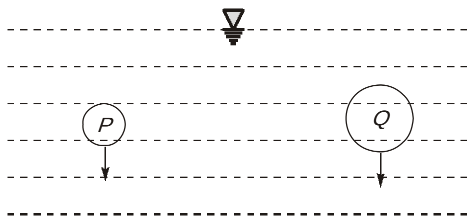
$$5(1 - 0.95) = 0.25$$

End of Solution

Q.37 Two discrete spherical particles (P and Q) of equal mass density are independently released in water. Particle P and particle Q have diameters of 0.5 mm and 1.0 mm. respectively. Assume Stoke's law is valid.

The drag force on particle Q will be ________ times the drag force on particle P. (round off to the nearest integer).

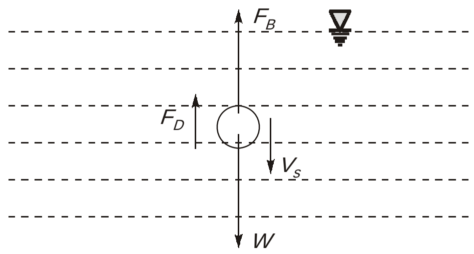
Ans. (8)



ρ = density of water
ρ_b = density of particle
Diameter of $P = D_1 = 0.5$ mm
Diameter of $Q = D_2 = 1$ mm

$$\therefore \quad \frac{F_{D2}}{F_{D1}} = \frac{\text{Drag force for particle } Q}{\text{Drag force for particle } P} = ?$$

Let's take basic case of Stoke's law i.e. a particle of diameter d moving, downward in fluid.



F_B = Buoyancy force
F_D = Drag force
V_s = Setting velocity
W = weigth of particle

$$W_g = F_B + F_D$$
$$mg - F_B = 3\pi\mu D V_s$$
$$(\rho_b - \rho)V_{\text{sphere}} = 3\pi\mu D V_s$$

$\because$ Volume of sphere, $= \frac{4}{3}\pi r^3$

$$\therefore \quad V_s = \frac{(\rho_b - \rho)D^2 g}{18\mu}$$
$$V_s = \frac{(\rho_b - \rho)g}{18\mu}D^2$$

Now,
$$\frac{F_{D_2}}{F_{D_1}} = \frac{3\pi\mu D_2 V_{s_2}}{3\pi\mu D_1 V_{s_1}}$$
$$\frac{F_{D_2}}{F_{D_1}} = \frac{D_2}{D_1} \times \frac{D_2^2}{D_1^2}$$
$$\frac{F_{D_2}}{F_{D_1}} = \left(\frac{D_2}{D_1}\right)^3 = \left(\frac{1}{0.5}\right)^3$$
$$\therefore \quad \frac{F_{D_2}}{F_{D_1}} = \frac{\text{Drag force for particle } Q}{\text{Drag force for particle } P} = 8$$

End of Solution

Q.38 The function $f(x, y)$ satisfies the Laplace equation

$$\Delta^2 f(x, y) = 0$$

on a circular domain of radius $r = 1$ with its center at point P with coordinates $x = 0$, $y = 0$. The value of this function on the circular boundary of this domain is equal to 3. The numerical value of f/(0, 0) is:

(a) 1
(b) 0
(c) 2
(d) 3

Ans. **(d)**

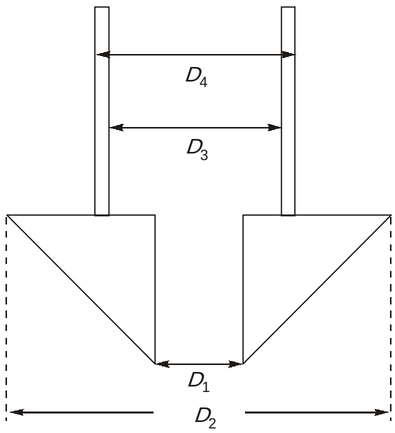
According to given condition given function $f(x, y)$ is nothing but constant function i.e. $f(x, y) = 3$ because this is the only function whose value is 3 at any point on the boundary of unit circle and it is also satisfying Laplace equation, so

$$f(0, 0) = 3$$

End of Solution

Q.39 The inside diameter of a sampler tube is 50 mm, The inside diameter of the cutting edge is kept such that the inside Clearance Ratio (ICR) is 1.0 % to minimize the friction on the sample as the sampler tube enters into the soil. The inside diameter (in mm) of the cutting edge is ________ . (round off to two decimal places)

Ans. **(49.50)**



$$C_i = \frac{D_3 - D_1}{D_1} \times 100$$

$$1 = \frac{50 - D_1}{D_1} \times 100$$

$$D_1 = 49.50 \text{ mm}$$

End of Solution

Q.40 In a solid waste handling facility, the moisture contents (MC) of food waste, paper waste, and glass waste were found to be MCf. MCp, and MCg, respectively. Similarly, the energy contents (EC) of plastic waste, food waste, and glass waste were found to be ECpp. ECf, and ECg. respectively. Which of the following statements) is/are correct?

(a) MCf < MCp < MCg
(b) ECpp < ECf < ECg
(c) ECpp > ECf > ECg
(d) MCf > MCp > MCg

Ans. (c, d)

End of Solution

Q.41 P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(a) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.
(b) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$.
(c) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$.
(d) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$.

Ans. (a, c)

$\because$ P and Q are not invertible, so

$$[PQ]^{-1} \neq Q^{-1}\ P^{-1}$$

If P and Q are invertible then,

$$[QP]^{-1} = P^{-1}\ Q^{-1}$$

If P and Q are invertible, then

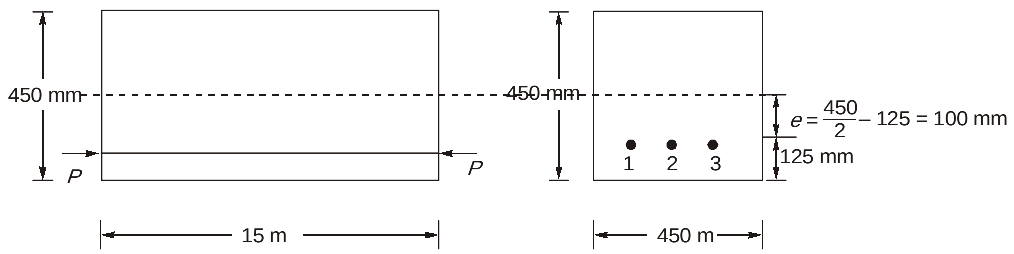
$$[PQ]^{-1} = Q^{-1}\ P^{-1}$$

End of Solution

Q.42 A post-tensioned concrete member of span 15 m and cross-section of 450 mm × 450 mm is prestressed with three steel tendons, each of cross-sectional area 200 mm^2. The tendons are tensioned one after another to a stress of 1500 MPa. All the tendons are straight and located at 125 mm from the bottom of the member. Assume the prestress to be the same in all tendons and the modular ratio to be 6. The average loss of prestress, due to elastic deformation of concrete, considering all three tendons is

(a) 28.32 MPa
(b) 7.08 MPa
(c) 14.16 MPa
(d) 42.48 MPa

Ans. (c)



Given:

Area of steel of each bar = 200 mm^2

Initial prestress = 1500 N/mm^2

∴P-force in each tendon, $P_u = p_o A_s$

$$P = 1500 \times \frac{200}{1000} = 300 \text{ kN in each wire}$$

As per given condition,

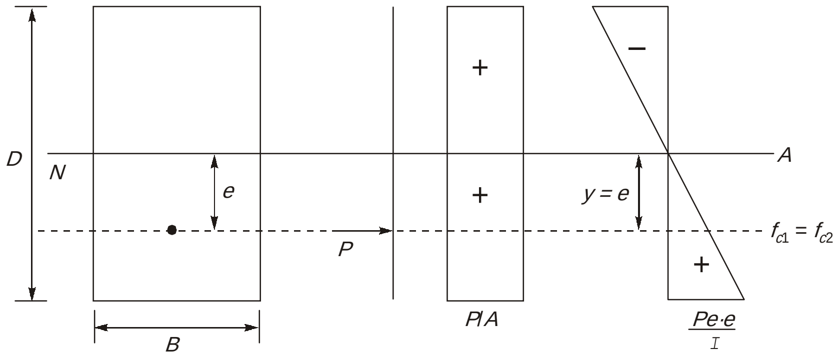
Loss of stress in 3rd wire = 0

Loss in wire-2, is due to P_3

Loss in wire-1, is due to P_2 and P_3 both

(Neglecting effect of self weight, because self weight will result in gain of stress, not loss)

Now, Loss of stress due to elastic shortening in wire-2,



$$f_{c1} = f_{c2} = \frac{P_3}{A} + \frac{P_3 e.e}{I} = \frac{P_3}{A} + \frac{P_3 e^2}{I}$$

$$f_{c1} = f_{c2} = \frac{300 \times 1000}{450 \times 450} + \frac{300 \times 1000 \times 100^2}{\left(450 \times \frac{450^3}{12}\right)}$$

$$f_{c1} = f_{c2} = 2.36 \text{ N/mm}^2$$

Loss of stress in wire-2 = $m(fc)_{avg.}$ = 6 × 2.36 = 14.16 N/mm^2

Loss of stress due to elastic shortening in wire-1, due to $P_2 + P_3 = 2P$ = 600 kN

$$f_{c1} = f_{c2} = \frac{P_2 + P_3}{A} + \frac{P_2 + P_3}{I}(e^2)$$

$$= \frac{2P}{A} + \frac{2P}{I}(e^2) = 2\left(\frac{P}{A} + \frac{P}{I}e^2\right)$$

= 2 × (f_{c1} for wire no. 2)

= 2 × 2.36 = 4.72 N/mm^2

Loss in stress in wire-1 = $m(fc)_{avg.}$ = 6 × 4.72 = 28.32 N/mm^2

$$\text{Average loss of stress in all three wires} = \frac{0 + 14.16 + 28.32}{3} = 14.16 \text{ N/mm}^2$$

End of Solution

Q.43 Match all the possible combinations between Column *X* (Cement compounds) and Column *Y* (Cement properties):

Column X	Column Y
(i) C_3S	(P) Early age strength
(ii) C_2S	(Q) Later age strength
(iii) C_3A	(R) Flash setting
	(S) Highest heat of hydration
	(T) Lowest heat of hydration

Which one of the following combinations is correct?

(a) (i) - (P), (ii) - (Q) and (T), (iii) - (R) and (S)

(b) (i) - (Q) and (T), (ii) - (P) and (S), (iii) - (R)

(c) (i) - (P), (ii) - (Q) and (R), (iii) - (T)

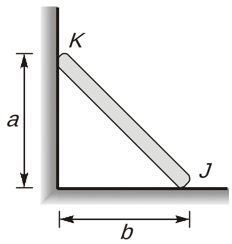
(d) (i) - (T), (ii) - (S), (iii) - (P) and (Q)

Ans. (a)

- C_3S is responsible for early age strength i.e. within 7 days.
- C_2S is responsible for progressive strength and releases low heat of hydration i.e. 260 J/g.
- C_3A is responsible for flash setting within 24 hours and also releases maximum heat of hydration i.e. 865 J/g.

End of Solution

Q.44 A uniform rod *KJ* of weight *w* shown in the figure rests against a frictionless vertical wall at the point K and a rough horizontal surface at point J, It is given that $w = 10$ kN, $a = 4$ in and $b = 3$ m.

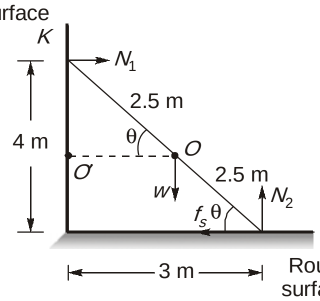


The minimum coefficient of static friction that is required at the point J to hold the rod in equilibrium is____________. (round off to three decimal places)

Ans. (0.375)

Free body diagram:

Given: $w = 10$ kN



$$KJ = \sqrt{4^2 + 3^2} = 5\text{m}$$

$\therefore$ $KO = OJ = 2.5 \text{ m}, \; WO' = 2.5\cos\theta$

For equilibrium,

$$\Sigma \vec{F}_{ext} = 0$$

$\Rightarrow$ $\Sigma \vec{F}_x = 0$ i.e. $N_1 = F_s$

$\Rightarrow$ $\Sigma \vec{F}_y = 0$ i.e. $N_2 = w$

For equilibrium,

$$(\Sigma \vec{\tau}_{ext})_{\text{about point } K} = 0$$

$$w \times 2.5\cos\theta + F_s(4) = N_2 \times 3$$

$\because$ $\cos\theta = \dfrac{3}{5}$ and $N_2 = w = 10 \text{ kN}$

$\therefore$ $$10\left(2.5 \times \frac{3}{5}\right) + 4F_s = 3(10)$$

$\therefore$ $$F_s = \frac{15}{4}\text{kN}$$

We know that

$$F_s \le (F_s)_{max}$$
$$F_s \le \mu_s N_2$$
$$\frac{15}{4} \le \mu_s \times w$$
$$\mu_s \ge \frac{15}{4 \times 10}$$
$$\mu \ge 0.375$$

$\therefore$ $(\mu)_{min} = 0.375$

End of Solution

Q.45 A group of total 16 piles are arranged in a square grid format, The center-to-center spacing (s) between adjacent piles is 3 m. The diameter (d) and length of embedment of each pile are 1 m and 20 m, respectively, The design capacity of each pile is 1000 kN in the vertical downward direction. The pile group efficiency η_g is given by

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$$

where m and n are number of rows and columns in the plan grid of pile arrangement,

and $\theta = \tan^{-1}\left(\dfrac{d}{s}\right)$.

The design value of the pile group capacity (in kN) in the vertical downward direction is _______ . (round off to the nearest integer)

Ans. (11085)

$$\eta_g = 1 - \frac{\theta}{90}\left\{\frac{m(n-1)+n(m-1)}{mn}\right\}$$

$$\theta = \tan^{-1}\left(\frac{d}{s}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$$

$$\eta_g = \frac{Q_{ug}}{nQ_{up}}$$

$$\therefore \quad Q_{ug} = \left[1 - \frac{18.43}{90}\left\{\frac{4(4-1)+4(4-1)}{4\times 4}\right\}\right](16\times 1000) = 11085\,\text{kN}$$

End of Solution

Q.46 A hydraulic jump takes place in a 6 in wide rectangular channel at a point where the upstream depth is 0.5 m (just before the jump). If the discharge in the channel is 30 m^3/s and the energy loss in the jump is 1.6 m. then the Froude number computed at the end of the jump is ________. (round off to two decimal places)

(Consider the acceleration due to gravity as 10 m/s^2.)

Ans. (*)

Method I:

Given,
Q = 30 m^3/s (discharge in channel)
B = 6 m (width of channel)
E_L = 1.6 m (Energy loss)
y_1 = 0.5 m (depth before jump)
F_2 = ? (Provide number after jump)

$$V_1 = \frac{Q}{By_1} = \frac{30}{6\times 0.5} = 10\,\text{m/s}$$

$$F_{r1} = \frac{V_1}{\sqrt{gy_1}} = \frac{10}{\sqrt{10\times 0.5}} = 4.47$$

We know

$$\frac{y_2}{y_1} = \frac{-1+\sqrt{1+8F_{r1}^2}}{2}$$

$$y_2 = 2.92\text{ m}$$

$$F_{r2}^2 = \frac{Q^2T}{gA_2^3} = \frac{30^2\times 6}{10\times(6\times 2.95)^3} = 0.100$$

F_{r2} = 0.32 [This should be the answer]

But Method II :

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(2.922 - 0.5)^3}{4 \times 2.922 \times 0.5}$$

$E_L = 2.43$ m [This is not equal to energy loss given]

So, now as per given energy loss

$$E_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$1.6 = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$1.6 \times 4 \times 0.5 \times y_2 = (y_2 - 0.5)^3$$

$$y_2 = 2.5 \text{ m}, -0.0527 \text{ m}, -0.947 \text{ m}$$

$$\therefore \quad y_2 = 2.5 \text{ m}$$

$$F_{r_2} = \sqrt{\frac{Q^2T}{gA_2^3}} = \sqrt{\frac{30^2 \times 6}{10 \times (6 \times 2.5)^3}} = 0.4$$

Note : We are getting two different values of Froude number from given data.

End of Solution

Q.47 Water is flowing in a horizontal, frictionless, rectangular channel. A smooth hump is built on the channel floor at a section and its height is gradually increased to reach choked condition in the channel. The depth of water at this section is y_2 and that at its upstream section is y_1. The correct statement(s) for the choked and unchoked conditions in the channel is/are

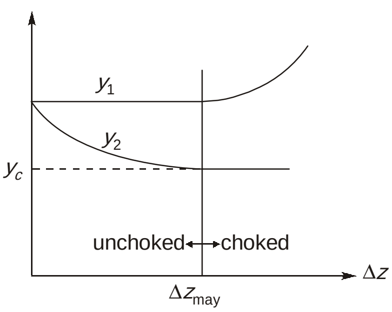
(a) In choked condition, y_2 is equal to the critical depth if the flow is supercritical or sub critical.

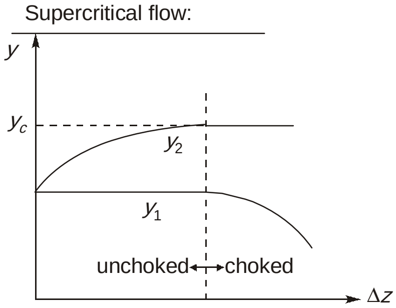
(b) In choked condition, decreases if the flow is supercritical and increases if the flow is subcritical.

(c) In unchoked condition, y_1 remains unaffected when the flow is supercritical or subcritical.

(d) In chocked conditions, y_1 increases if the flow is supercritical and decreases if the flow is subcritical.

Ans. (a, b, c)





End of Solution

Q.48 The error in measuring the radius of a 5 cm circular rod was 0.2%, If the cross-sectional area of the rod was calculated using this measurement, then the resulting absolute percentage error in the computed area is _______. (round off to two decimal places)

Ans. (0.40)

$$A = \pi r^2$$

For percentage error,

$$\partial A = 2\pi r \partial r$$

$$\frac{\partial A}{A} = \frac{2\pi r \partial r}{A}$$

$$\frac{\partial A}{A} = \frac{2\pi r \partial r}{\pi r^2}$$

$$\frac{\partial A}{A} = 2\left[\frac{\partial r}{r}\right] \qquad \left\{\frac{\partial r}{r} = 0.2\% \text{ given}\right\}$$

$$\therefore \quad \frac{\partial A}{A} = 2 \times 0.2 = 0.40\%$$

End of Solution

Q.49 It is given that an aggregate mix has 260 grams of coarse aggregates and 240 grams of fine aggregates. The specific gravities of the coarse and fine aggregates are 2.6 and 2.4. respectively. The bulk specific gravity of the mix is 2.3. The percentage air voids in the mix is _________. (round off to the nearest integer)

Ans. (8)

$$G_t = \frac{\text{Total weight}}{V_{CA} + V_{FA}}$$

$$= \frac{260+240}{\frac{260}{2.6}+\frac{240}{2.4}} = 2.5$$

$$V_{av} = \frac{G_t - G_m}{G_t} \times 100 \qquad \{G_m = 2.3\}$$

$$= \frac{2.5-2.3}{2.5} \times 100 = 8\%$$

End of Solution

Q.50 For a traffic stream, V is the space mean speed, k is the density, q is the flow. V_f is the free flow speed, and k_f is the jam density. Assume that the speed decreases linearly with density,

Which of the following relation(s) is/are correct?

(a) $q = k_j k - \left(\frac{k_j}{V_f}\right) k^2$

(b) $q = k_j V - \left(\frac{k_j}{V_f}\right) V^2$

(c) $q = V_f V - \left(\frac{V_f}{k_j}\right) V^2$

(d) $q = V_f k - \left(\frac{V_f}{k_j}\right) k^2$

Ans. (b, d)

We know as per greenshield traffic model

$$V = V_f\left[1-\frac{k}{k_j}\right] = V_f - \left(\frac{V_f}{k_j}\right)k$$

Where, V_f = Free and low speed

k_j = jam density

k = density

V = speed of vehicle

q = flow of traffic

We know that $q = Vk$

$$q = \left[V_f - \left(\frac{V_f}{k_j}\right)k\right]k$$

$$q = V_f k - \frac{V_f}{k_j}k^2 \quad \text{...(i)}$$

Now, we know that $q = Vk$

$$\therefore \quad k = \frac{q}{V}$$

Put in equation (i)

$$q = V_f\frac{q}{V} - \frac{V_f}{k_j}\left(\frac{q^2}{V^2}\right)$$

$$1 = \frac{V_f}{V} - \frac{V_f}{V^2}\frac{q}{k_j}$$

$$\frac{V_f q}{V^2 k_j} = \frac{V_f}{V} - 1$$

$$q = \frac{V^2 k_j}{V_f}\left[\frac{V_f}{V} - 1\right]$$

$$\therefore \quad q = k_j V - \left(\frac{k_j}{V_f}\right)V^2$$

End of Solution

Q.51 The dimension of dynamic viscosity is:

(a) $M\,L^{-2}\,T^{-2}$

(b) $M\,L^{0}\,T^{-1}$

(c) $M\,L^{-1}\,T^{-2}$

(d) $M\,L^{-1}\,T^{-1}$

Ans. (d)

$$\text{Unit of dynamic viscosity} = \frac{kg}{m.s} \text{ or } \frac{Ns}{m^2}$$

$$= ML^{-1}T^{-1}$$

End of Solution

Q.52 The base length of the runway at The mean sea level (MSL) is 1500 m. If the runway is located at an altitude of 300 m above the MSL, the actual length (in m) of the runway To be provided is ________. (round off to the nearest integer)

Ans. (1605)

Basic runway length = 1500 m (at MSL)

Elevation of airport = 300 m above MSL

We know as per elevation correction the length of should be increase by 7% per 300 m elevation above MSL.

$$\text{Correction due to elevation} = \frac{0.07}{300} \times 300 \times 1500 = 105 \text{ m}$$

$\therefore$ Corrected length = 1605 m

End of Solution

Q.53 For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (v), and shear modulus (G) is

(a) $G = \dfrac{E}{2(1+v)}$

(b) $G = \dfrac{E}{(1+2v)}$

(c) $E = \dfrac{G}{2(1+v)}$

(d) $E = \dfrac{G}{(1+2v)}$

Ans. (a)

$E = 2G(1 + \mu)$

G = Shear modulus

μ = Poission's ratio

E = Young's modulus

End of Solution

Q.54 A pump with an efficiency of 80% is used to draw groundwater from a well for irrigating a flat field of area 108 hectares. The base period and delta for paddy crop on this field are 120 days and 144 cm, respectively. Water application efficiency in the field is 80%. The lowest level of water in the well is 10 m below the ground. The minimum required horse power (h.p.) of the pump is ________. (round off to two decimal places)

(Consider 1 h.p. = 746 W; unit weight of water = 9810 N/m^3)

Ans. (30.82)

$$\frac{A}{Q} = 8.64\frac{B}{\Delta}$$

Q = discharge required by crop

A = 108 hac

Δ = 1.44 m

B = 120 days

$$\therefore \quad Q = \frac{A\Delta}{8.64B} = \frac{108 \times 1.44}{8.64 \times 120} = 0.15 \text{ m}^3/\text{s}$$

$$\text{Water required to be pumped} = \frac{0.15}{0.8} = 0.1875\,\text{m}^3/\text{s}$$

$$\text{Power required by pump} = \frac{\gamma QH}{0.8}$$

$$= \frac{9810\,\text{N/m}^3 \times 0.1875\,\text{m}^3/\text{s} \times 10}{0.8 \times 746} = 30.82\ \text{h.p.}$$

End of Solution

Q.55 Match **Column X** with **Column Y**:

Column X	Column Y
(P) Horton eqution	(I) Design of alluvial channel
(Q) Penman method	(II) Maximum flood dischage
(R) Chezy's formula	(III) Evapotranspiration
(S) Lacey's theory	(IV) Infiltration
(T) Dicken's formula	(V) Flow velocity

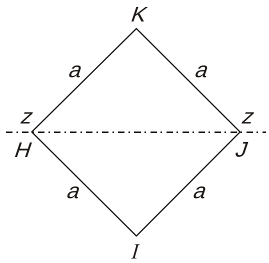
Which one of the following combinations is correct?

(a) (P)-(IV), (Q)-(III), (R)-(II), (S)-(I), (T)-(V)
(b) (P)-(III), (Q)-(IV), (R)-(I), (S)-(V), (T)-(II)
(c) (P)-(IV), (Q)-(III), (R)-(V), (S)-(I), (T)-(II)
(d) (P)-(III), (Q)-(IV), (R)-(V), (S)-(I), (T)-(II)

Ans. (c)

End of Solution

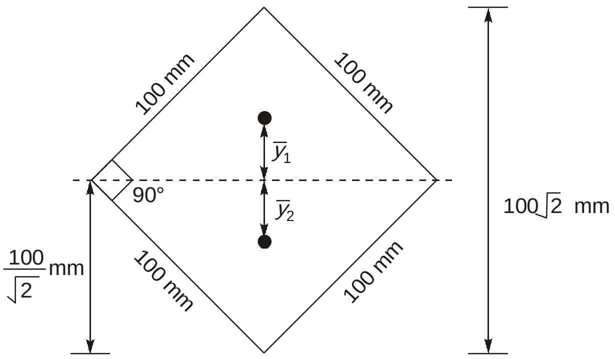
Q.56 For the square steel beam cross-section shown hi the figure, the shape factor about Z - Z axis is S and the plastic moment capacity is M_p. Consider yield stress f_y = 250 MPa and a = 100 mm.



The values of S and M_P (rounded off to one decimal place) are

(a) S = 2.0, M_P = 100.2 kN-m
(b) S = 1.5, M_P = 58.9 kN-m
(c) S = 2.0, M_P = 58.9 kN-m
(d) S = 1.5, M_P = 100.2 kN-m

Ans. (c)



$$M_P = f_y Z_P$$

$\therefore$ $$Z_P = \frac{F}{2}(\bar{y}_1 + \bar{y}_2)$$

$$Z_P = \frac{100 \times 100}{2}\left[\frac{100}{3\sqrt{2}} + \frac{100}{3\sqrt{2}}\right]$$

$$Z_P = 235.702 \times 10^3 \text{ mm}^3$$

$\therefore$ $$M_p = F_y Z_P$$
$$= 250 \times 235.702 \times 10^3 \times 10^{-6}$$
$$M_P = 58.925 \text{ kN-m}$$

Shape factor, $$SF = \frac{M_p}{M_y}$$

$\because$ $$M_y = f_y Z_e$$

$$= 250 \times \frac{a^3}{6\sqrt{2}} = 250 \times \frac{(100)^3}{6\sqrt{2}} = 29.46 \text{ kNm}$$

$\therefore$ $$SF = \frac{58.925}{29.46} = 2$$

End of Solution

Q.57 Which of the following statement(s) is/are correct?

(a) If a structure is acted upon by two force system P_a and P_b, in equilibrium separately, the external virtual work done by a system forces P_b during the deformations caused by another system of forces P_a is equal to the external virtual work done by the P_a system during the deformation caused by the P_b system.

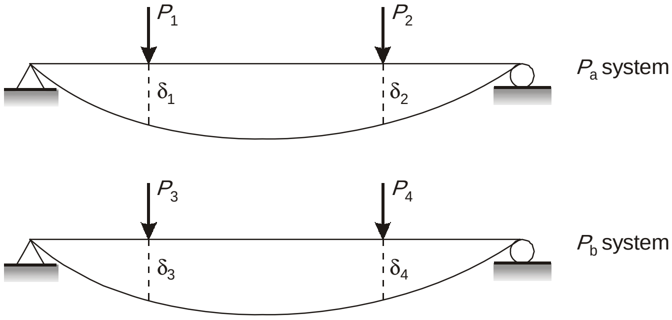
(b) If a linearly elastic structure is subjected to a set of loads, the partial derivative of the total strain energy with respect to the load at any point is equal to the deflection at that point.

(c) The shear force in a conjugate beam loaded by the M/EI diagram of the real beam is equal to the corresponding deflection of the real beam.

(d) If a linearly elastic structure is subjected to a set of loads the partial derivative of the total strain energy with respect to the deflection at any point is equal to the load applied at that point,

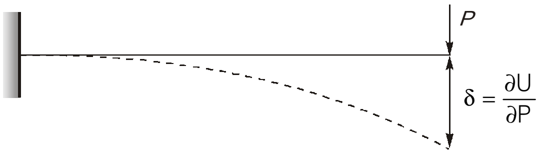
Ans. **(a, b, d)**

(a) As per Bett's theorem



$$P_1\delta_3 + P_2\delta_4 = P_3\delta_1 + P_4\delta_2$$

(b) As per Castigliano Theorem-II



U = total strain energy

P = Load applied

(c) Not correct.

The shear force in conjugate beam loaded by M/EI diagram of real beam equal to corresponding slope of real beam.

(d) Correct.

$$\frac{\partial U}{\partial \delta} = P$$

As per Castigliano Theorem-I.

End of Solution

Q.58 $\int\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\right)dx$ is equal to

(a) $-\frac{1}{1-x^2}$ + Constant

(b) $-\frac{1}{1-x}$ + Constant

(c) $\frac{1}{1+x^2}$ + Constant

(d) $\frac{1}{1+x}$ + Constant

Ans. **(*)**

End of Solution

Q.59 A parabolic vertical crest curve connects two road segments with grades +1.0% and –2.0%, If a 200 m stopping sight distance is needed for a driver at a height of 1.2 m to avoid an obstacle of height 0.15 m, then the minimum curve length should be _____m, (round off to the nearest integer)

Ans. (270.90)

Assume, $L_s > SSD$

$$L_s = \frac{NS^2}{2[\sqrt{H}+\sqrt{h}]^2}$$

$$= \frac{[0.01-(-0.02)]\times 200^2}{2[\sqrt{1.2}+\sqrt{0.15}]^2}$$

$= 272.90$ m $>$ SSD (∴ Assumption was correct)

End of Solution

Q.60 In a certain month, the reference crop evapotranspiration at a location is 6 mm day, If the crop coefficient and soil coefficient are 1.2 and 0.8, respectively, the actual evapotranspiration in mm/day is

(a) 8.00
(b) 7.20
(c) 6.80
(d) 5.76

Ans. (d)

Given : $Et_0 = 6$ mm/day, $K_c = 1.2$, $K_s = 0.8$

We know that based on Basal's equation of reference evapotranspiration.

$$Et_c = K_c \cdot K_s\ Et_0$$

$$\Rightarrow \quad Et_c = 1.2 \times 0.8 \times 6$$

$= 5.76$ mm/day

End of Solution

Q.61 At a municipal waste handling facility, 30 metric ton mixture of food waste, yard waste, and paper waste was available. The moisture content of this mixture was found to be 10%. The ideal moisture content for composting this mixture is 50%, The amount of water to be added to this mixture to bring its moisture content to the ideal condition is _______ metric ton, (in integer)

Ans. (24)

Total municipal waste = 30 MT

∴ Moisture content (given) = 10% i.e. 3 MT

∴ Solid mass = 30 – 0.1(30) = 27 MT

Ideally moisture content of mix = 50% (given)

∴ Total mass of waste ideally must be = (T) = 27 + 0.5 (T)

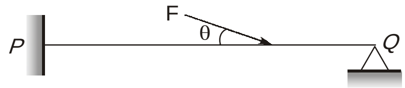
$\therefore$ Total mass = 54 MT (in which 27 MT is water)
= 54 MT

$\therefore$ Amount of water added = 27 − 3 = 24 MT

($\therefore$ 3 MT water is already added)

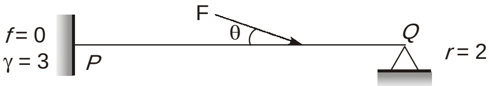
End of Solution

Q.62 Consider a beam *PQ* fixed at *P*, hinged at *Q*, and subjected to a load *F* as shown in figure (not drawn to scale). The static and kinematic degrees of indeterminacy, respectively, are



(a) 2 and 0
(b) 2 and 2
(c) 1 and 2
(d) 2 and 1

Ans. **(d)**



Static indeterminacy, SI = $r - 3$
= (3 + 2) − 3 = 2

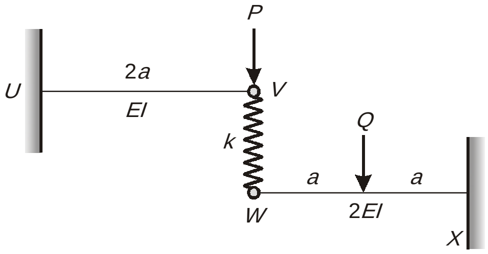
Kinematic indeterminacy = 0 + 1 = 1

{at A degree of freedom is zero}

{at B, degree of freedom is 1}

End of Solution

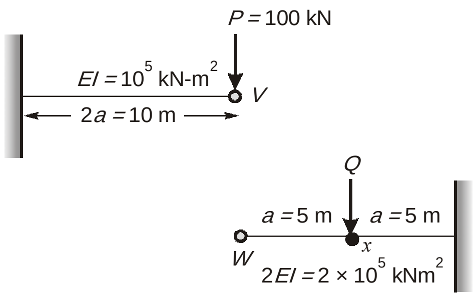
Q.63 The linearly elastic planar structure shown in the figure is acted upon by two vertical concentrated forces. The horizontal beams *UV* and *WX* are connected with the help of the vertical linear spring with spring constant k = 20 kN/m. The fixed supports are provided at U and X. It is given that flexural rigidity $EI = 10^5$ kN-m^2, P = 100 kN. and a = 5 m. Force Q is applied at the center of beam *WX* such that the force in the spring *VW* becomes zero.



The magnitude of force Q (in kN) is _______ . (round off to the nearest integer)

Ans. **(640)**

Given that there is no force in spring, so deformation in spring will be zero.
From here it can be concluded that distance travelled by V and W are equal.



$$\delta_V = \delta_W$$

$$\delta_V = \delta_x + Q_x L_{wx}$$

$$\frac{P(2a)^3}{3EI} = \frac{Qa^3}{3(2EI)} + \frac{Qa^2}{2(2EI)}(a)$$

$$\frac{100(10)^3}{3(10^5)} = \frac{Q(5)^3}{3(2\times10^5)} + \frac{Q(5)^2(5)}{2(2\times10^5)}$$

$\therefore$ $Q = 640$ kN

End of Solution

Q.64 In a triaxial unconsolidated undrained (UU) test on a saturated clay sample, the cell pressure was 100 kPa, If the deviatoric stress at failure was 150 kPa, then the undrained shear strength of the soil is__________kPa. (in integer)

Ans. (75)

UU test for clay $(\phi = 0)$

$$\sigma_d = 150 \text{ kPa}$$

$$\sigma_3 = 100 \text{ kPa}$$

$$\sigma_1 = \sigma_3 + \sigma_d = 250 \text{ kPa}$$

$\therefore$

$$\sigma_1 = \sigma_3 \tan^2\left(45+\frac{\phi}{2}\right) + 2C\tan\left(45+\frac{\phi}{2}\right)$$

$$250 = 100 + 2C$$

$$C = 75 \text{ kPa}$$

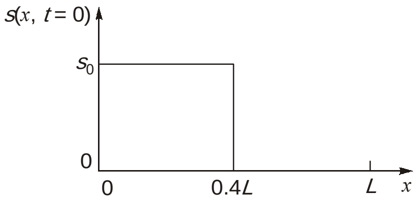
Shear strength, $\tau = C + \bar{\sigma}_n \tan\phi = 75 \text{ kPa}$

End of Solution

Q.65 The concentration $s(x, t)$ of pollutants, in a one-dimensional reservoir at position x and time t satisfies the diffusion equation

$$\frac{\partial s(x,t)}{\partial t} = D\frac{\partial^2 s(x,t)}{\partial x^2}$$

on the domain $0 \le x \le L$, where D is the diffusion coefficient of the pollutants. The initial condition $s(x, 0)$ is defined by the step-function shown in the figure.



The boundary conditions of the problem are given by $\dfrac{\partial s(x,t)}{\partial x} = 0$ at the boundary points $x = 0$ and $x = L$ at all times. Consider $D = 0.1\ m^2/s$, $s_0 = 5$ μmol/m and $L = 10$ m. The steady-state concentration $\bar{s}\left(\dfrac{L}{2}\right) = s\left(\dfrac{L}{2}, \infty\right)$ at the center $x = \dfrac{L}{2}$ of the reservoir (in μmol/m) is _______ . (in integer)

Ans. (2)

From figure $s(x, t)$

at $x = 0 \Rightarrow s(0, t) = s_0 = 5$

at $x = L \Rightarrow s(L, t) = 0$

The steady state solution of diffusion equation is given by

$$s(x) = \left(\frac{s(L) - s(0)}{L}\right)x + s(0)$$

$$s\left(\frac{L}{2}\right) = \left(\frac{0-5}{L}\right)\left(\frac{L}{2}\right) + 5$$

$$= \frac{-5}{2} + 5 = 2.5$$

End of Solution

■■■■