DETAILED SOLUTIONS

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ESE 2022 : Prelims Exam
CLASSROOM TEST SERIES

CIVIL ENGINEERING

Test 14

Section A : Flow of Fluids, Hydraulic Machines and Hydro Power
Section B : Design of Concrete and Masonry Structures - 1
Section C : Structural Analysis - 2

1.	(d)	16.	(a)	31.	(a)	46.	(a)	61.	(c)
2.	(c)	17.	(d)	32.	(b)	47.	(b)	62.	(a)
3.	(b)	18.	(a)	33.	(b)	48.	(b)	63.	(a)
4.	(a)	19.	(b)	34.	(a)	49.	(b)	64.	(b)
5.	(c)	20.	(d)	35.	(a)	50.	(b)	65.	(a)
6.	(d)	21.	(d)	36.	(c)	51.	(b)	66.	(d)
7.	(d)	22.	(c)	37.	(d)	52.	(b)	67.	(c)
8.	(c)	23.	(c)	38.	(d)	53.	(b)	68.	(b)
9.	(d)	24.	(d)	39.	(d)	54.	(c)	69.	(a)
10.	(b)	25.	(b)	40.	(c)	55.	(b)	70.	(a)
11.	(c)	26.	(c)	41.	(a)	56.	(a)	71.	(c)
12.	(b)	27.	(b)	42.	(d)	57.	(d)	72.	(c)
13.	(a)	28.	(d)	43.	(b)	58.	(d)	73.	(c)
14.	(d)	29.	(c)	44.	(b)	59.	(a)	74.	(d)
15.	(d)	30.	(a)	45.	(a)	60.	(c)	75.	(a)

DETAILED EXPLANATIONS

1. (d)

For a Newtonian fluid, $$\tau = \mu \frac{du}{dy}$$

Since the space between shaft and the sleeve is very small, i.e., the oil film is thin, it can be preasumed that

$$\frac{du}{dy} = \frac{u}{t}$$

Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

$\therefore$ $$\tau = \frac{F}{A} = \mu \frac{u}{t}$$

$\Rightarrow$ $$F = A\mu \frac{u}{t}$$

$\therefore$ $$F \propto u$$

$\therefore$ $$\frac{F_1}{u_1} = \frac{F_2}{u_2}$$

$\Rightarrow$ $$\frac{0.5 \text{ kN}}{60 \text{ cm/s}} = \frac{F_2}{300 \text{ cm/s}}$$

$\Rightarrow$ $$F_2 = 2.5 \text{ kN}$$

2. (c)

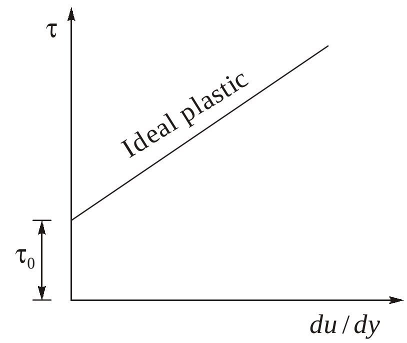
Gauge pressure inside the bubble,

$$P_i = 150 \text{ N/m}^2$$

Gauge presure outside the bubble,

$$P_o = \rho_0 gh$$
$$= 0.85 \times 10^3 \times 9.81 \times 0.0125$$
$$= 104.23 \text{ N/m}^2$$
$$\Delta P = P_i - P_o = 150 - 104.23 = 45.77 \text{ N/m}^2$$
$$\Delta P = \frac{4\sigma}{d}$$
$$\sigma = \frac{45.77 \times 1.5 \times 10^{-3}}{4} = 0.0172 \text{ N/m}$$

3. (b)



$$\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)$$

4. (a)

Gauge pressure, $P_g = \rho g h = \gamma h$

$$= 10 \times 10^3 \times 20$$

$$= 2 \times 10^5 \text{ N/m}^2 = 200 \text{ kPa}$$

Atmospheric pressure, $P_{atm} = 760$ mm of Hg

$$= (13.6 \times 10^3 \times 9.81) \times 0.76$$

$$= 101.4 \text{ kPa}$$

∴ Absolute pressure $= P_g + P_{at}$

$$= 200 + 101.4$$

$$= 301.4 \text{ kPa}$$

5. (c)

Centre of pressure, $h_{cp} = \bar{h} + \dfrac{I_{CG}\sin^2\theta}{A\bar{h}}$

For vertical plate, $h_{cp} = \bar{h} + \dfrac{I_{CG}}{A\bar{h}}$

$$I_{CG} = \frac{bh^3}{12} = \frac{(4\sqrt{2})\times(2\sqrt{2})^3}{12}\times 2 = \frac{64}{3} = 21.33 \text{ m}^4$$

$$\bar{h} = 8 + \frac{21.33}{(4\times 4)8} = 8 + 0.167 = 8.167 \text{ m}$$

∴ The centre of pressure is (8.167 – 8) = 0.167 m below the centroid of the plate

6. (d)

Distance between metacentre and centre of buoyancy

$$\text{BM} = \frac{I_{\min}}{V_{immersed}}$$

$$\Rightarrow \quad BG + MG = \frac{I_{min}}{V_{immersed}}$$

where, I_{min} = Moment of inertia of top view of floating body about longitudinal axis

$V_{immersed}$ = Volume of body immersed in liquid

7. (d)

- Stream line is an imaginary line drawn through the flow field in a manner such that the velocity vector of the fluid at each and every point on the streamline is tangent to the streamline at that instant and thats why there is no velocity component in the direction normal to the stream lines.
- A path line represents the trace or trajectory of a fluid particle over a period of time.
- A streak line is the instantaneous picture of the positions of all the fluid particles that have passed through a fixed point in the flow field.

8. (c)

$$u = 10\,xy$$
$$v = 5x^2$$
$$w = t^2x + z$$

Acceleration in y-direction:

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\Rightarrow \quad a_y = 0 + (10xy)(10x) + 5x^2(0) + (t^2x + z)(0)$$

$$\Rightarrow \quad a_y = 100x^2y$$

At point (1, 2, 3) and t = 1 unit,

$$a_y = 100 \times 1^2 \times 2 = 200 \text{ units}$$

9. (d)

$$\phi = 2xy$$

$$\therefore \quad u = -\frac{\partial \phi}{\partial x} = -2y$$

$$v = -\frac{\partial \phi}{\partial y} = -2x$$

Streamline function, $u = -\dfrac{\partial \psi}{\partial y}$

$$\Rightarrow \quad -2y = -\frac{\partial \psi}{\partial y}$$

$$\Rightarrow \quad \int 2y\,\partial y = \int \partial \psi$$

$$\Rightarrow \quad \psi = y^2 + f(x)$$

$$\therefore \quad \frac{\partial \psi}{\partial x} = f'(x) = v$$

$$\Rightarrow \qquad -2x = f'(x)$$

Integrating, both sides

$$-x^2 = f(x) + C'$$

$$\therefore \qquad f(x) = -x^2 + C$$

where $C = -C'$ = Another constant

Now, stream function,

$$\psi = y^2 - x^2 + C$$

At (1, 3), $\psi_1 = (9 - 1) = 8$ units

At (3, 3), $\psi_2 = (3^2 - 3^2) = 0$ units

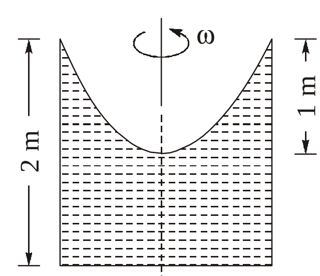
Now, discharge/flow per unit width

$$= |\psi_2 - \psi_1|$$
$$= 8 - 0 = 8 \text{ units}$$

11. (c)

A steady irrotational flow of an incompressible fluid is called potential flow.

13. (a)



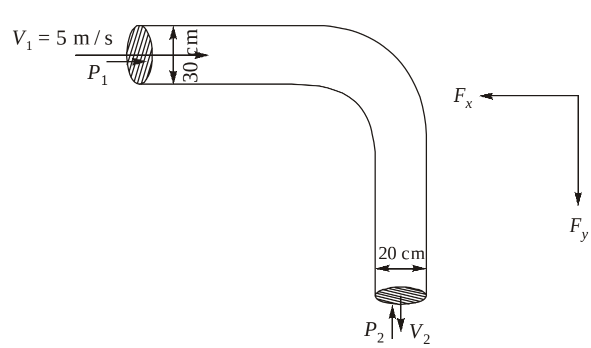
Original volume, $V_1 = \pi r^2 h = \pi r^2 \times 2$

Volume of water spilled out, $V_2 = \frac{1}{2}\pi r^2 h'$

$$= \frac{1}{2} \times \pi r^2 \times 1$$

Ratio, $$\frac{V_2}{V_1} = \frac{\frac{\pi r^2}{2}}{2\pi r^2} = \frac{1}{4}$$

15. (d)



$$\text{Net force in } x\text{-direction} = \text{Change in momentum in } x\text{-direction}$$

$\Rightarrow$ $$P_1A_1 - F_x = \rho Q(0 - V_1)$$

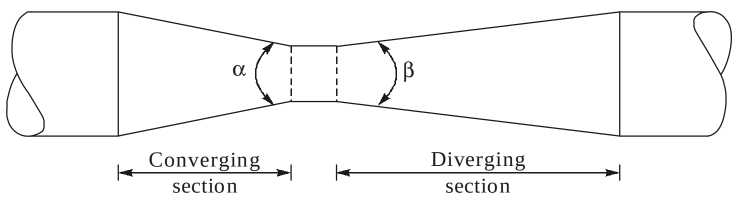
$\Rightarrow$ $$\left(80\times10^3\right)\times\frac{\pi}{4}(0.3)^2 - F_x = -\rho A_1V_1^2$$

$\Rightarrow$ $$\left(80\times10^3\right)\times\frac{\pi}{4}\times(0.3)^2 - F_x = -10^3\times\frac{\pi}{4}\times(0.3)^2\times5^2$$

$\Rightarrow$ $$F_x = \frac{\pi}{4}\times(0.3)^2\times10^3\times(80+25)\text{ N}$$

$\Rightarrow$ $$F_x = 7.42\text{ kN}$$

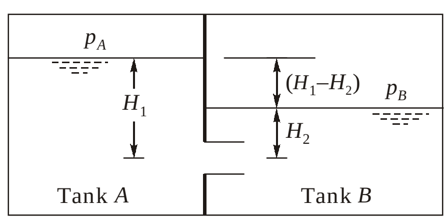
16. (a)



$$\alpha \simeq 19° \text{ to } 23°$$
$$\beta \simeq 5° \text{ to } 7°$$

18. (a)

Velocity of efflux is equal to the velocity of free fall from the surface of reservoir. This is known as Torricelli's theorem.



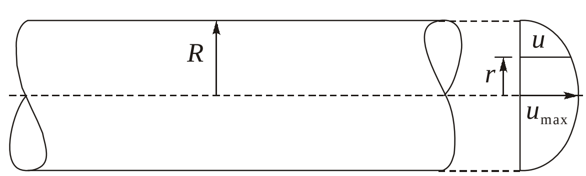
$$Q = C_d b\sqrt{2gh}$$

where, $$h = H_1 - H_2$$

$\therefore$ $$Q \propto \sqrt{H_1 - H_2}$$

19. (b)

For a laminar flow through a circular pipe.



$$u = U_{max}\left(1-\frac{r^2}{R^2}\right)$$

where, $U_{\max} = \frac{1}{4\mu}\left(-\frac{dP}{dx}\right)R^2$

$= \frac{1}{4\times 0.001}\times\{-(-10)\}\times(0.05)^2 = 6.25 \text{ m/s}$

$\therefore$ $u = 6.25\left\{1-\left(\frac{0.2}{5}\right)^2\right\} = 6.24 \text{ m/s}$

20. (d)

- If metacentric height is positive, i.e., when M is above G, then it represents the condition of stable equilibrium of floating bodies.
- If however, the metacentre M lies below G (in between B and G), the metacentric height is negative signifying unstable equilibrium.

22. (c)

Convective acceleration, $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$

$= (2x^2+y^2)\frac{\partial}{\partial x}(2x^2+y^2) + (-4xy)\frac{\partial}{\partial y}(2x^2+y^2)$

$= (2x^2+y^2)\times 4x + (-4xy)\times 2y$

$= 8x^3 + 4xy^2 - 8xy^2$

$= 8x^3 - 4xy^2$

At (1, 2) $a_x = 8 \times 1^3 - 4 \times 1 \times (2)^2$

$= 8 - 16 = -8$ units

23. (c)

Displacement thickness, $\delta^* = \int_0^\delta \left(1-\frac{u}{U_\infty}\right)\times dy$

$= \int_0^\delta \left(1-\frac{y}{\delta}\right)\times dy$

$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \delta - \frac{\delta}{2} = \frac{\delta}{2}$

$\therefore$ $\frac{\delta^*}{\delta} = \frac{1}{2} = 0.5$

24. (d)

For a RC at critical depth of flow.

Velocity, $V_c = \sqrt{gy_c}$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left\{\frac{(4/2.5)^2}{10}\right\}^{1/3} = \left(\frac{32}{125}\right)^{1/3}$$

$$= \frac{2}{5} \times 4^{1/3} = \frac{2}{5} \times 1.6 = 0.64 \text{ m}$$

$$V_c = \sqrt{10 \times 0.64} = \sqrt{6.4}$$

$$= \frac{8}{\sqrt{10}} = \frac{8}{3.162} = 2.53 \text{ m/s}$$

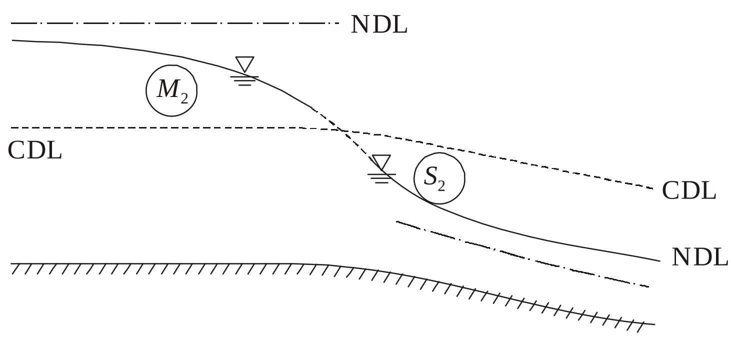
25. (b)

Ratio of sequent depths,

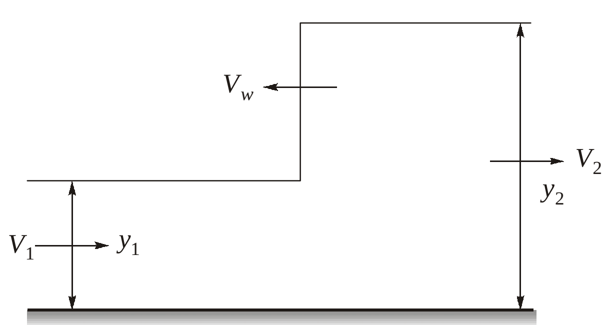
$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 \times F_{r1}^2}\right]$$

$$= \frac{1}{2}\left[-1 + \sqrt{1 + 8 \times (10)^2}\right] = 13.65$$

26. (c)



27. (b)



Celerity,

$$C = V_w + V_1$$

$$\Rightarrow \quad \sqrt{\frac{g}{2} \times \frac{y_2}{y_1} \times (y_1 + y_2)} = V_w + V_1$$

$$\Rightarrow \quad \sqrt{\frac{9.81}{2} \times \frac{0.6}{0.3}(0.6+0.3)} = 3 + V_1$$

$$\Rightarrow \quad V_1 \simeq 0 \text{ m/s}$$

From continuity equation,

$$\Rightarrow \quad By_2(V_2 + V_w) = By_1(V_1 + V_w)$$

$$\Rightarrow \quad 0.6(V_2 + 3) = 0.3\,(0 + 3)$$

$$\Rightarrow \quad V_2 = \frac{0.3 \times 3}{0.6} - 3 = -1.5 \text{ m/s}$$

Magnitude of velocity at upstream and downstream 0 and 1.5 m/s.

28. (d)

Average boundary shear stress,

$$\tau_w = \gamma_w RS$$

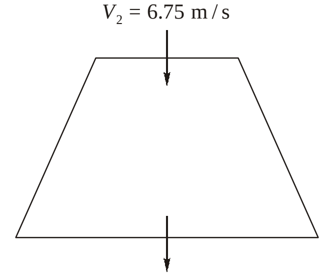
$$R = \frac{A}{P} = \frac{3 \times 2.5}{3 + 2 \times 2.5} = \frac{7.5}{8} = 0.94 \text{ m}$$

$$\therefore \quad \tau_w = 9.81 \times 0.94 \times 0.0002 \times 10^3 = 1.84 \text{ N/m}^2$$

29. (c)

Any GVF profile will have atleast one control section.

30. (a)



Efficiency of draft tube:

$$\eta_d = \frac{\dfrac{V_2^2}{2g} - \dfrac{V_3^2}{2g} - \dfrac{0.2V_3^2}{2g}}{\dfrac{V_2^2}{2g}} = \frac{\dfrac{V_2^2}{2g} - \dfrac{1.2V_3^2}{2g}}{\dfrac{V_2^2}{2g}}$$

$$\Rightarrow \quad \eta_d = 1 - 1.2\left(\frac{V_3}{V_2}\right)^2$$

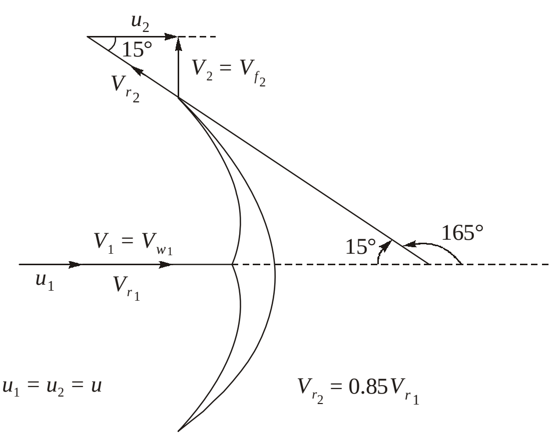
$$= 1 - 1.2\left(\frac{3}{6.75}\right)^2$$

$$= 0.763 \simeq 76.3\%$$

31. (a)

Given: Dia. of Pelton wheel = 1.1 m

H = 900 m



From inlet velocity triangle:

$$V_{r_1} = V_1 - u$$

Also, $$V_{r_2} \cos 15° = u$$

$\Rightarrow$ $$0.85\,(V_1 - u)\cos 15° = u$$

$\Rightarrow$ $$0.85\left(C_v\sqrt{2gH} - u\right)\cos 15° = u$$

$\Rightarrow$ $$0.85\left(0.97\sqrt{2\times 9.81\times 900} - u\right)0.97 = u$$

$\Rightarrow$ $$0.85\,(0.97 \times 4.43 \times 30 - u)\,0.97 = u$$

$\Rightarrow$ $$0.825\,(0.97 \times 4.43 \times 30 - u) = u$$

$\Rightarrow$ $$u = \frac{0.825\times 0.97\times 4.43\times 30}{1.825} = 58.27 \text{ m/s}$$

But $$u = \frac{\pi DN}{60} = 58.27$$

$\Rightarrow$ $$N = \frac{58.27\times 60}{\pi\times 1.1} = 1011.7 \simeq 1012 \text{ rpm}$$

32. (b)

Given: Total head, H = 90 m

Design speed, N = 1000 rpm

Specific speed, N_s = 30

Discharge through each pump, $Q = 0.25 \text{ m}^3/\text{s}$

$$N_s = \frac{N\sqrt{Q}}{(H)^{3/4}}$$

$$\Rightarrow \quad 30 = \frac{1000 \times \sqrt{0.25}}{(H_m)^{3/4}}$$

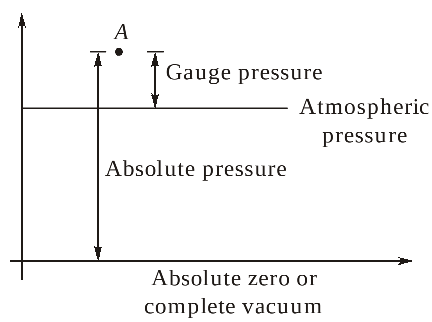
$$\Rightarrow \quad H_m = \left(\frac{1000 \times \sqrt{0.25}}{30}\right)^{4/3} = 42.57 \text{ m} \simeq 42.6 \text{ m}$$

$$\therefore \quad \text{Number of pump stages} = \frac{90}{42.6} = 2.11 \simeq 3$$

33. (b)

Volute casing: In this type of casing, the area of flow gradually increases from the impeller outlet to the delivery pipe so as to reduce the velocity of flow. Thus, the increase in pressure occurs in volute casing.

34. (a)



$$\therefore \quad P_{abs} = P_{atm} + P_g$$

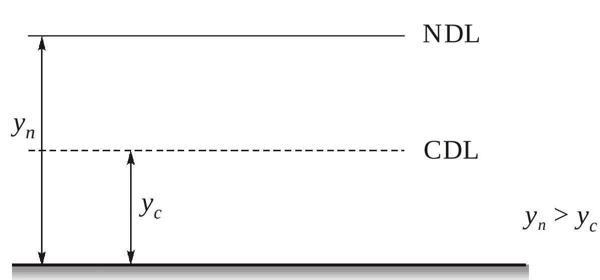
36. (c)

In Kaplan turbine frictional losses are low because of small number of blades used.

37. (d)

The term alternate depths is used in the open channel flow to denote the depths having the same specific energy for a given discharge.

38. (d)



Mild slope

Subcritical flow occurs at normal depth in a mild sloped channel.

Since $\quad y_n > y_c$

39. (d)

For continuous beam, if, width of support > $\frac{L_{clear}}{12}$,

then, effective length of intermediate span is equal to clear length.

So, $$450 \text{ mm} > \left(\frac{4800}{12} = 400 \text{ mm}\right)$$

$\therefore$ $$L_{eff} = L_{clear} = 4.8 \text{ m}$$

40. (c)

Limit state design is based on the probabilistic concept. Thus the actual load and actual stresses may differ from the calculated load and calculated stresses. However the partial safety factor and load factor provide adequate safety to the structure against collapse.

41. (a)

- Failure of over-reinforced section occurs due to primary compression failure.
- Under-reinforced section fails due to secondary compression.

42. (d)

The minimum reinforcement in edge strip, parallel to that edge shall be 0.15% for mild steel and 0.12% for HYSD bars, of the cross-section area of concrete.

43. (b)

According to Clause 29.1 of IS 456: 2000, beam shall be deemed to be a deep beam when the ratio of effective span to overall depth i.e. l/D is less than,

1. 2 for simply supported beam.
2. 2.5 for continuous beam.

44. (b)

Positive moment tension reinforcement:

Atleast $\frac{1}{3}$rd of the tensile reinforcement in simply supported member and $\frac{1}{4}$th of tensile reinforcement in continuous member shall extend along the same face into the support to a length of $\frac{L_d}{3}$.

45. (a)

$$b_f = \frac{L_o}{\frac{L_o}{b}+4} + b_w = \frac{5000}{\frac{5000}{1000}+4} + 300 = 855.56 \text{ mm}$$

46. (a)

τ_c is increased by a factor of δ, where

$$\delta = \left(1 + \frac{3P_u}{f_{ck}BD}\right) \le 1.5$$

So, $$\delta = 1+\frac{3\times800\times10^3\times1.5}{25\times500\times600} = 1.48 < 1.5$$

$\therefore$ $$\delta = 1.48$$

48. (b)

Any section of the wall will be subjected to tension and bending both.

49. (b)

Type of staircase	Width of staircase
Service stair for maintenance, catwalks and two floor residential buildings.	1 - 1.2 m
Staircase for offices, apartment, etc.	1.2 - 1.6 m
Staircases for public houses, etc.	1.2 - 2m

50. (b)

Interaction equation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

Exponential power, α_n depends on the ratio $\frac{P_u}{P_{uz}}$:

$\frac{P_u}{P_{uz}}$	α_n
< 0.2	1
0.2 to 0.8	1 to 2
> 0.8	2

So, $$\frac{P_u}{P_{uz}} = \frac{4000}{5500} = 0.727$$

$$\alpha_{0.727} = 1+\frac{1}{0.6}[0.727-0.2]$$

$$= 1.878 \simeq 1.88$$

51. (b)

The effect of minimum eccentricity is deemed to be incorporated.

52. (b)

Average shear stress for shorter span is given by,

$$V_u = \frac{w_u \times L_x}{3} = \frac{15\times3}{3} = 15\,\text{kN}$$

53. (b)

Since, $BM_u < BM_{u,\ lim}$ section is under reinforced

$$M_{u,\ lim} = 0.133 f_{ck} bd^2$$
$$= 0.133 \times 20 \times 200 \times (450)^2 \times 10^{-6}$$
$$= 107.73 \text{ kNm}$$

$$A_{st\ req.} = \frac{0.5 \times f_{ck} \times B \times d}{f_y}\left(1-\sqrt{1-\frac{4.6 \times M_u}{f_{ck}Bd^2}}\right)$$

$$A_{st\ req.} = \frac{0.5 \times 20 \times 200 \times 450}{500}\left(1-\sqrt{1-\frac{4.6 \times 25 \times 10^6}{20 \times 200 \times 450^2}}\right)$$

$$= 132.66 \text{ mm}^2$$

$$\text{min., } A_{st} = \frac{0.85Bd}{f_y} = \frac{0.85 \times 200 \times 450}{500} = 153 \text{ mm}^2$$

Here, $A_{st,\ min.} > A_{st\ req.}$

So, $A_{st,\ provided} = A_{st\ min.} = 153 \text{ mm}^2$

54. (c)

Minimum shear reinforcement in the forms of stirrups is provided to improve the dowel action of longitudinal bars.

55. (b)

$$(x_u)_{lim} = 0.48 \times d = 0.48 \times 500 = 240 \text{ mm}$$

Assuming $(x_u)_{act} < 160 \text{ mm } (D_f)$

$\therefore$ $C = T$

$\Rightarrow$ $0.36 f_{ck} B_{eff}\, x_u = 0.87 f_y A_{st}$

$\Rightarrow$ $0.36 \times 20 \times 1200 \times x_u = 0.87 \times 415 \times 1964$

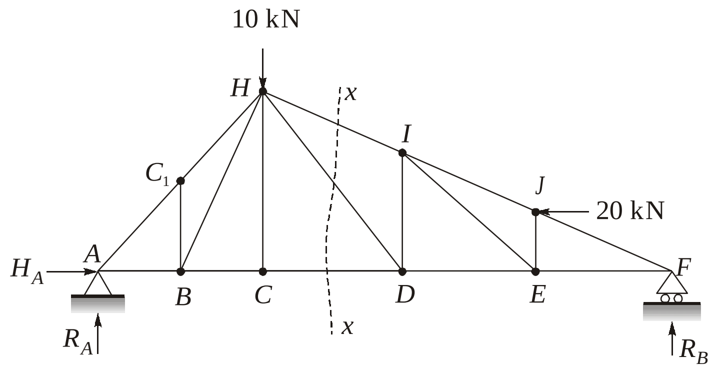
$\Rightarrow$ $x_u = 82.07 \text{ mm} < D_f (= 160 \text{ mm})$ (O.K)

$\therefore$ NA lies in flange.

58. (d)

$$D_s = m + r_e - 2j$$
$$= 19 + 3 - 2 \times 11$$
$$= 22 - 22 = 0$$
$$D_k = 2j - r_e$$
$$= 2 \times 11 - 3 = 19$$

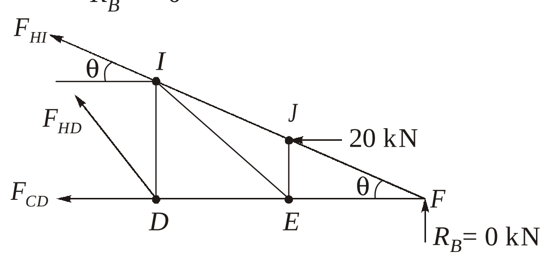
59. (a)



$$\Sigma M_A = 0$$

$$\Rightarrow \quad -R_B \times 13 - 20 \times 2 + 10 \times 4 = 0$$

$$\Rightarrow \quad R_B = 0$$



$$\Sigma M_D = 0$$

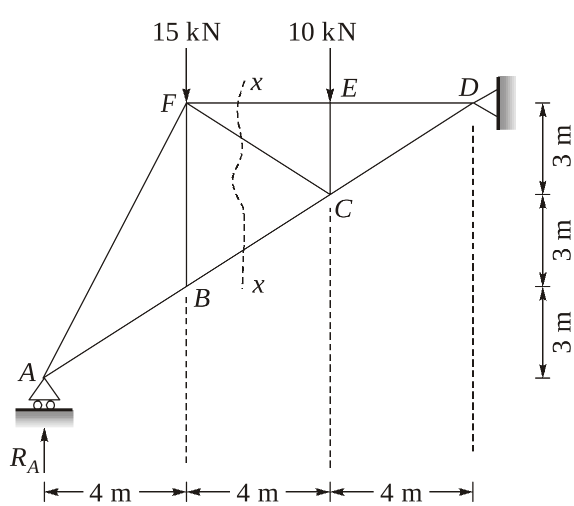
$$\Rightarrow \quad F_{HI} \times \cos\theta \times 3 + 20 \times 2 = 0$$

$$\cos\theta = \frac{3}{\sqrt{9+4}} = \frac{3}{\sqrt{13}} = 0.832$$

$$\therefore \quad F_{HI} \times 0.832 \times 3 + 40 = 0$$

$$\Rightarrow \quad F_{HI} = -16.025 \text{ kN} \simeq 16 \text{ kN (Compressive)}$$

60. (c)

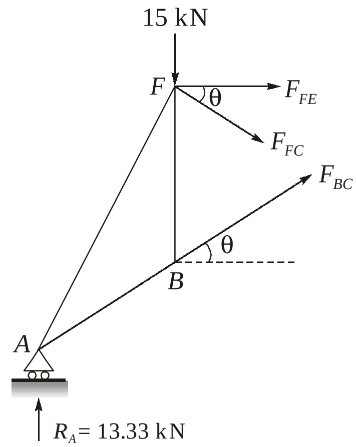


$$\Sigma M_D = 0$$

$$R_A \times 12 - 15 \times 8 - 10 \times 4 = 0$$

$$\Rightarrow \quad R_A = 13.33 \text{ kN}$$

Cutting a vertical section (x-x) as shown in the figure.



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$$\Sigma M_F = 0$$

$$\Rightarrow \quad R_A \times 4 - (F_{BC} \cos\theta) \times 6 = 0$$

$$\Rightarrow \quad F_{BC} \times \frac{4}{5} \times 6 = 13.33 \times 4$$

$$\Rightarrow \quad F_{BC} = 11.11 \text{ kN}$$

$$\Sigma F_y = 0$$

$$13.33 - 15 + F_{BC} \sin\theta - F_{FC} \sin\theta = 0$$

$$-1.67 + 11.11 \times \frac{3}{5} - F_{FC} \times \frac{3}{5} = 0$$

$$\Rightarrow \quad F_{FC} = 8.33 \text{ kN}$$

$$\Sigma F_x = 0$$

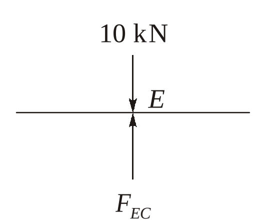
$$\Rightarrow \quad F_{BC} \cos\theta + F_{FC} \cos\theta + F_{FE} = 0$$

$$\Rightarrow \quad 11.11 \times \frac{4}{5} + 833 \times \frac{4}{5} + F_{FE} = 0$$

$$\Rightarrow \quad 8.89 + 6.66 + F_{FE} = 0$$

$$F_{FE} = -15.55 \text{ kN}$$

Considering joint E

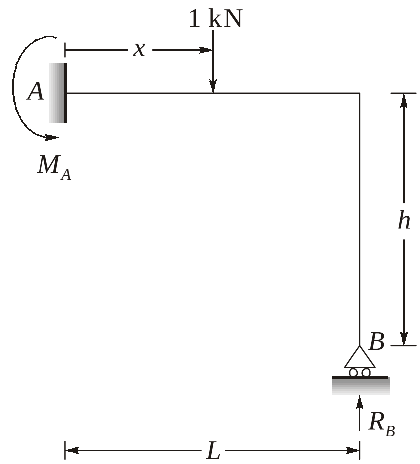


$$\Sigma F_y = 0$$

$$\Rightarrow \quad 10 - F_{EC} = 0$$

$$\Rightarrow \quad F_{EC} = 10 \text{ kN}$$

61. (c)



$$M_A = y_2 f(x)$$

$$\Sigma M_A = 0$$

$$\Rightarrow \quad -M_A - R_B \times L + 1 \times x = 0$$

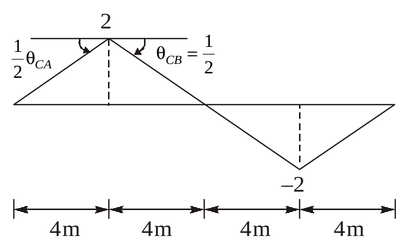
$$\Rightarrow \quad R_B \times L = x - M_A$$

$$= x - f(x)$$

$$\therefore \quad R_B = y_2 = \frac{x}{L} - \frac{f(x)}{L}$$

62. (a)

Applying Muller Braslau principle, apply unit rotation at point C,

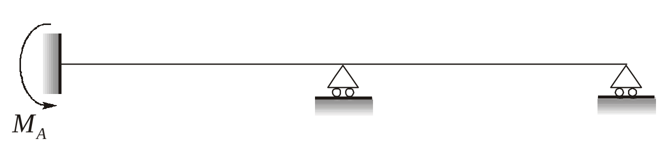


$$\theta_{CA} + \theta_{CB} = 1$$

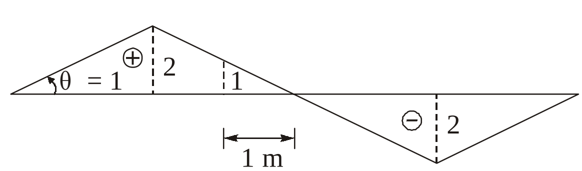
$$\Rightarrow \quad 2\theta_{CA} = 1$$

$$\Rightarrow \quad \theta_{CA} = \frac{1}{2}$$

63. (a)



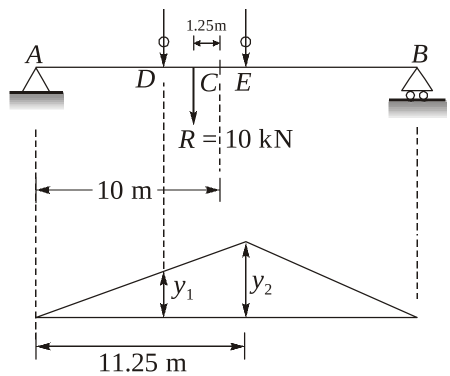
ILD for M_A



$$M_A = 2\times\left(\frac{1}{2}\times2\times2\right)+10\times1-3\times\left(\frac{1}{2}\times2\times2\right)$$

$$= 4 + 10 - 6 = 8 \text{ kNm}$$

64. (b)



ILD for BM at *E*

$$\frac{y_2}{11.25} = \frac{y_1}{(11.25-5)}$$

$$\Rightarrow \quad y_1 = \frac{4.92}{11.25}\times6.25 = 2.73$$

Absolute maximum bending moment

$$= 5 \times 2.73 + 5 \times 4.92$$

$$= 38.25 \text{ kNm}$$

65. (a)

$$k_s = \frac{1}{\frac{1}{k_1}+\frac{1}{k_2}}+k_3+k_4+k_5$$

$$= \frac{1}{\left(\frac{1}{10}+\frac{1}{15}\right)}+18+18+18$$

$$= \frac{1}{\frac{(3+2)}{30}}+18\times3$$

$$= 6 + 54 = 60 \text{ N/cm}$$

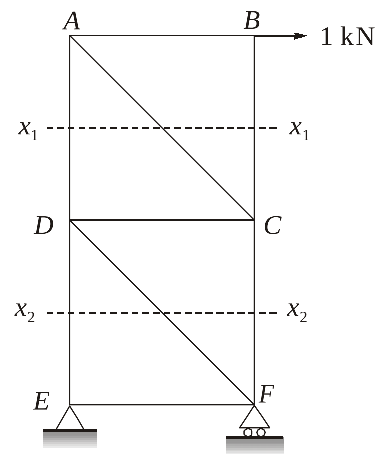
$$\omega = \sqrt{\frac{k_s}{W}}$$

$$g = 981 \text{ cm/sec}^2$$

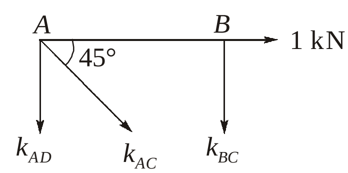
$$\therefore \qquad \omega = \sqrt{\frac{60 \times 981}{20}} = 54.25 \text{ radian/sec}$$

$$\therefore \qquad f = \frac{\omega}{2\pi} = \frac{54.25}{2 \times \pi} = 8.63 \text{ cycles per second}$$

66. (d)



Pass a section x_1-x_1 as shown in figure.

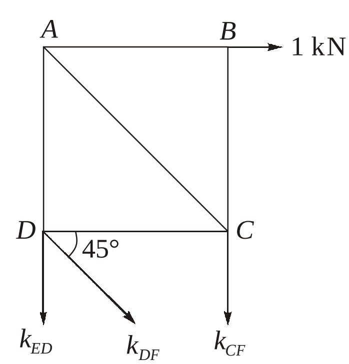


$$\Sigma F_x = 0$$

$$\Rightarrow \qquad k_{AC} \times \cos 45° + 1 = 0$$

$$\Rightarrow \qquad k_{AC} = -\frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = -\sqrt{2} \text{ kN}$$

Pass a section x_1-x_2 as shown in the figure



$$\Sigma F_x = 0$$

$$\Rightarrow \quad k_{DF} \cos 45° + 1 = 0$$

$$\Rightarrow \quad k_{DF} = -\frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = -\sqrt{2} \text{ kN}$$

$$\delta_{BH} = \sum_{i=1}^{n} k_i \left(\frac{P_i L_i}{A_i E_i} + L_i \alpha_i \Delta T_i + \lambda_i\right)$$

$$\Rightarrow \quad \delta_{BH} = \sum_{i=1}^{n} k_i \left(L_i \alpha_i \Delta T_i\right)$$

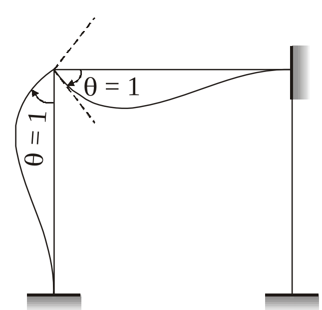
$$= k_{AC}\left(L\alpha\Delta T_{AC}\right) + k_{DF}\left(L\alpha\Delta T_{DF}\right)$$

$$= \left(-\sqrt{2}\right)\left(2\sqrt{2} \times \frac{1}{150000} \times 20\right) + \left(-\sqrt{2}\right)\left(2\sqrt{2} \times \frac{1}{150000} \times 30\right)$$

$$= (-0.533 - 0.8) \text{ mm}$$

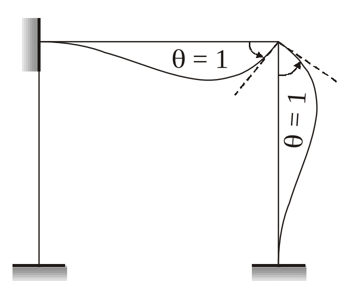
$$= -1.33 \text{ mm}$$

67. (c)



$$k_{11} = \frac{4(EI)}{5} + \frac{4(2EI)}{6} = \left(\frac{4}{5} + \frac{4}{3}\right)EI = \frac{32}{15}EI$$

$$k_{21} = \frac{2(2EI)}{6} = \frac{2}{3}EI$$



$$k_{12} = \frac{2 \times 2EI}{6} = \frac{2}{3}EI$$

$$k_{22} = \frac{32}{15}EI$$

$$\therefore \quad \text{Stiffness matrix} = EI\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = EI\begin{bmatrix} \frac{32}{15} & \frac{2}{3} \\ \frac{2}{3} & \frac{32}{15} \end{bmatrix}$$

68. (b)

$$\text{Flexibility matrix } (A) = \frac{L^3}{4EI}\begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix}$$

$$\text{Let} \quad (P) = \begin{bmatrix} 6 & 5 \\ 5 & 8 \end{bmatrix}$$

$$\therefore \quad \text{adj(P)} = \text{Transpose of co-factor of [P]}$$

$$= \begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix}$$

$$\text{Determinant of matrix (P)} = |P| = 6 \times 8 - 5 \times 5 = 23$$

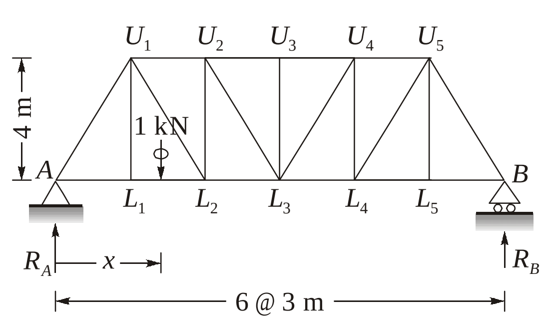
$$\therefore \quad \text{Stiffnes matrix} = \text{Inverse of flexibility matrix (A)}$$

$$= \frac{4EI}{L^3}[P]^{-1}$$

$$= \frac{4EI}{L^3}\frac{\text{adj}(P)}{|P|}$$

$$= \frac{4EI}{L^3}\cdot\frac{1}{23}\begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix} = \frac{4EI}{23L^3}\begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix}$$

69. (a)



$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_A + R_B = 1$$

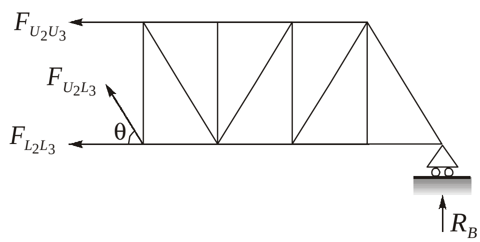
$$\Sigma M_B = 0$$

$$\Rightarrow \quad R_A \times 18 - 1(18 - x) = 0$$

$$\Rightarrow \quad R_A = \left(\frac{18-x}{18}\right)$$

$$\therefore \quad R_B = 1-\frac{18-x}{18} = \frac{x}{18}$$

When the load is on the left side of U_2L_3 $(0 \le x \le 6)$



$$\Sigma F_y = 0$$

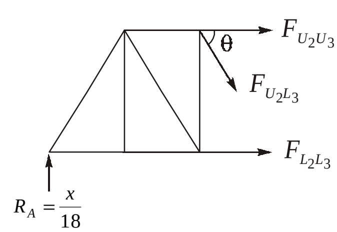
$$\Rightarrow \quad F_{U_2L_3}\sin\theta + R_B = 0$$

$$\Rightarrow \quad F_{U_2L_3} = -\frac{R_B}{\sin\theta} = -\frac{x}{18\sin\theta} = \frac{-x\times 5}{18\times 4} = -\frac{5x}{18\times 4}$$

At $x = 6$ m

$$F_{U_2L_3} = -\frac{5\times 6}{18\times 4} = -\frac{5}{12} = -0.42 \text{ kN}$$

When the load is in the right side of U_3L_3 $(9 \le x \le 18)$



$$\Sigma F_y = 0 \qquad \left(\sin\theta = \frac{4}{5}\right)$$

$$\Rightarrow \quad F_{U_2L_3}\sin\theta - \frac{x}{18} = 0$$

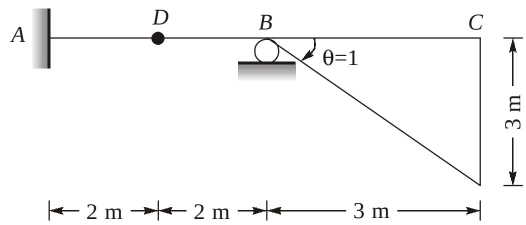
$$\Rightarrow \quad F_{U_2L_3} = \frac{x\times 5}{18\times 4}$$

At $x = 9$ m,

$$F_{U_2L_3} = \frac{9}{18}\times\frac{5}{4} = \frac{5}{8} = +0.625 \text{ kN}$$

70. (a)

Using Muller Braslaeu principle



ILD for bending moment at support *B*.

72. (c)

The deflection at the centre is given by

$$\delta_C = \frac{5}{384}\frac{wL^4}{EI}$$

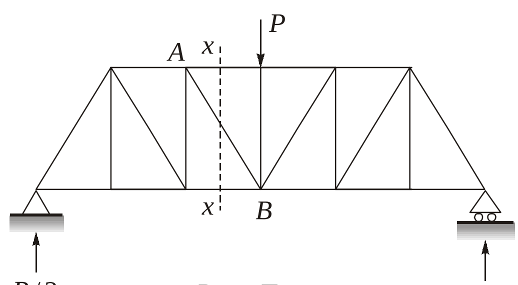
Stiffness,
$$k = \frac{384EI}{5L^3}$$

$$w = \sqrt{\frac{kg}{wL}} = \sqrt{\frac{384EI}{5L^3}\frac{g}{wL}}$$

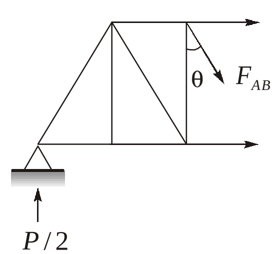
$$= \sqrt{\frac{384EIg}{5wL^4}}$$

∴ Natural frequency,
$$f = \frac{w}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{384EIg}{5wL^4}}$$

73. (c)



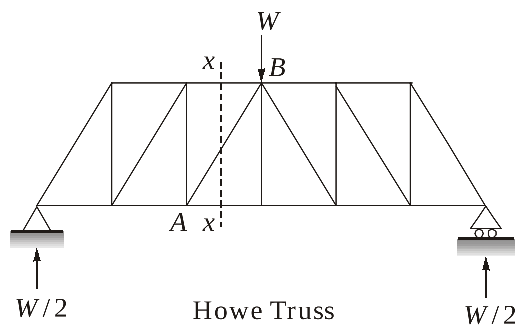
Pratt Truss



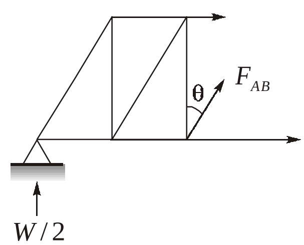
$$\Sigma F_y = 0$$

$$\Rightarrow \quad \frac{P}{2} - F_{AB}\cos\theta = 0$$

$$\Rightarrow \quad F_{AB} = +\frac{P}{2}\sec\theta \text{(Tensile)}$$



Howe Truss



$$\Sigma F_y = 0$$

$$\Rightarrow \quad \frac{W}{2} + F_{AB}\cos\theta = 0$$

$$\Rightarrow \quad F_{AB} = -\frac{W}{2\cos\theta}$$

$$= \frac{W}{2\cos\theta}\text{(Compressive)}$$

As in Howe truss, the diagonal member carries compressive force, so it may undergo buckling. Therefore, Pratt truss is better than Howe truss.

74. (d)

Stiffness matrix method is not a compatibility method.

■■■■