DETAILED SOLUTIONS

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ESE 2022 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 4

Section A : Control Systems [All Topics] + Engineering Mathematics [All Topics]

Section B : Electrical Circuits - 1 [Part Syllabus]

Section C : Digital Electronics - 1 [Part Syllabus] + Microprocessors - 1 [Part Syllabus]

1.	(b)	**16.**	(d)	**31.**	(b)	**46.**	(c)	**61.**	(b)
2.	(c)	**17.**	(d)	**32.**	(b)	**47.**	(d)	**62.**	(d)
3.	(c)	**18.**	(c)	**33.**	(d)	**48.**	(b)	**63.**	(c)
4.	(b)	**19.**	(b)	**34.**	(c)	**49.**	(a)	**64.**	(c)
5.	(b)	**20.**	(d)	**35.**	(b)	**50.**	(b)	**65.**	(d)
6.	(d)	**21.**	(c)	**36.**	(d)	**51.**	(a)	**66.**	(b)
7.	(b)	**22.**	(b)	**37.**	(c)	**52.**	(a)	**67.**	(b)
8.	(a)	**23.**	(d)	**38.**	(c)	**53.**	(d)	**68.**	(b)
9.	(d)	**24.**	(a)	**39.**	(d)	**54.**	(d)	**69.**	(c)
10.	(b)	**25.**	(a)	**40.**	(b)	**55.**	(b)	**70.**	(c)
11.	(a)	**26.**	(a)	**41.**	(c)	**56.**	(d)	**71.**	(c)
12.	(a)	**27.**	(c)	**42.**	(a)	**57.**	(c)	**72.**	(d)
13.	(b)	**28.**	(a)	**43.**	(b)	**58.**	(a)	**73.**	(b)
14.	(b)	**29.**	(a)	**44.**	(a)	**59.**	(c)	**74.**	(c)
15.	(b)	**30.**	(c)	**45.**	(c)	**60.**	(a)	**75.**	(c)

DETAILED EXPLANATIONS
Section A : Control Systems

1. **(b)**

The accuracy of open loop system depends on the calibration of the input. Any departure from predetermined calibration effects the output.

2. **(c)**

Given, $r(t) = tu(t)$

or $R(s) = \frac{1}{s^2}$

$$E(s) = \frac{1}{s^2} \cdot \frac{s}{s^2 + 6s + 36}$$

By final value theorem, $e_{ss} = \lim_{s \to 0} sE(s)$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{s}{s^2 + 6s + 36}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + 6s + 36} = \frac{1}{36} \text{unit}$$

3. **(c)**

The poles are located at s = 0, –1, –4

The zero is located at s = –2

$$G(s) = \frac{K(s+2)}{s(s+1)(s+4)}$$

It is given at s = 1, the value of $G(s)$ is 3.6

$$G(1) = 3.6 = \frac{K(1+2)}{(1)(2)(5)} = \frac{K \times 3}{10}$$

$$K = \frac{3.6 \times 10}{3} = 12$$

$$G(s) = \frac{12(s+2)}{s(s+1)(s+4)}$$

4. **(b)**

Sensitivity of overall transfer function w.r.t. feedback path transfer function

$$S_H^M = -\frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{\frac{-25}{s(s+2)} \times 0.25}{1 + \frac{25 \times 0.25}{s(s+2)}}$$

$$= \frac{-6.25}{s^2 + 2s + 6.25}$$

5. **(b)**

Static error coefficients are associated with type of input applied to a closed loop control system.

6. **(d)**

The frequency at which Nyquist plot crosses the unit circle is called gain crossover frequency.

7. **(b)**

Closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad \text{Where } G(s) = \text{forward path transfer function}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{2}{s(s+3)}}{1+\frac{2}{s(s+3)}\cdot 1} = \frac{2}{s^2+3s+2} = \frac{2}{(s+2)(s+1)}$$

As the input is unit step,

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s}\cdot\frac{2}{(s+2)(s+1)}$$

By partial fraction,

$$\frac{2}{s(s+2)(s+1)} = \frac{K_1}{s}+\frac{K_2}{s+1}+\frac{K_3}{s+2}$$

$$K_1 = 1,\; K_2 = -2,\; K_3 = 1$$

$\because$ $$C(s) = \frac{1}{s}-\frac{2}{s+1}+\frac{1}{s+2}$$

By taking inverse Laplace transform

$$c(t) = 1 - 2e^{-t} + e^{-2t}$$

8. **(a)**

For given transfer function,

The characteristic equation:

$$1 + G(s)H(s) = s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4$$

Forming the Routh table for above characteristic equation

$$\begin{array}{c|ccc} s^5 & 1 & 3 & 2 \\ s^4 & -2 & -6 & -4 \\ s^3 & 0 & 0 & \end{array}$$

The row corresponding s^3 has all terms as zero.

$\therefore$ We have to form auxiliary equation using row corresponding to s^4

$$P(s) = -2s^4 - 6s^2 - 4$$

So,
$$\frac{dP(s)}{ds} = -8s^3 - 12s \text{ or } -2s^3 - 3s$$

Now using the coefficient of $\frac{dP}{ds}$ corresponding to s^3 row

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 2 \\
s^4 & -2 & -6 & -4 \\
s^3 & -2 & -3 & \\
s^2 & -3 & -4 & \\
s^1 & \frac{9-8}{-3} = \frac{-1}{3} & 0 & \\
s^0 & -4 & &
\end{array}$$

$\therefore$ there is one sign change in s^4 row and hence one root lies on right half of s-plane.

Hence option (a) is correct.

9. (d)

$$\dot{\phi}(t) = A\,\phi(t) \text{ is the correct relation}$$

We know,
$$\phi(t) = e^{At}$$
$$\dot{\phi}(t) = Ae^{At}$$

So,
$$\dot{\phi}(t) = A\,\phi(t)$$

10. (b)

Given,
$$\frac{C(s)}{R(s)} = \frac{K(s+1)}{(s+1-j2)(s+1+j2)} = \frac{K(s+1)}{s^2+2s+5}$$

$$\frac{C(s)}{R(s)} = \frac{K(s+1)}{s^2+2s+5}$$

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s}\cdot\frac{K(s+1)}{s^2+2s+5}$$

$$C_{ss} = \lim_{s\to 0} sC(s) = \lim_{s\to 0} s \times \frac{1}{s}\cdot\frac{K(s+1)}{s^2+2s+5}$$

$$= \lim_{s\to 0}\frac{K(s+1)}{s^2+2s+5} = \frac{K}{5}$$

$$\therefore \qquad C_{ss} = 0.8$$

$$0.8 = \frac{K}{5}$$

or $$K = 4$$

$$\frac{C(s)}{R(s)} = \frac{4(s+1)}{s^2+2s+5}$$

11. (a)

Absolute and relative stability of only minimum phase system can be determined from the Bode plot.

12. (a)

Given characteristic equation,

$$s^3 + 4s^2 + (K+4)s + 5K = 0$$

$$s^3 + 4s^2 + Ks + 4s + 5K = 0$$

$$s^3 + 4s^2 + 4s + K(s+5) = 0$$

$$1 + \frac{K(s+5)}{s^3+4s^2+4s} = 0$$

$$1 + G(s)H(s) = 0$$

Now, comparing with standard characteristic equation

(where $G(s)H(s)$ = Open loop transfer function)

$$\text{O.L.T.F.} = \frac{K(s+5)}{s(s^2+4s+4)} = \frac{K(s+5)}{s(s+2)^2}$$

13. (b)

Given, natural frequency of oscillation, $\omega_n = 8$ rad/sec

Damping ratio, $\xi = 0.2$

Second order characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Resonant frequency, $$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$\omega_r = 8\sqrt{1-2\times(0.2)^2} = \frac{8\sqrt{23}}{5}$$

Resonant peak, $$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.2\sqrt{1-0.04}}$$

$$= \frac{1}{0.4\sqrt{0.96}} = \frac{25}{4\sqrt{6}}$$

14. (b)

We know, characteristic equation:

$$1 + G(s)H(s) = 0$$

$$1+\frac{K}{s(s+6)^2} = 0$$

Rearranging, $s(s+6)^2 + K = 0$

$$K = -s(s+6)^2$$
$$= -s(s^2 + 36 + 12s)$$
$$= -(s^3 + 12s^2 + 36s)$$

$$\Rightarrow \quad \frac{dK}{ds} = -(3s^2 + 24s + 36) = -3(s^2 + 8s + 12)$$
$$= -3(s+2)(s+6) = 0$$
$$s = -2,\ s = -6$$

$\therefore$One of the breakaway point will be $s = -2$

15. (b)

Given, $$A = \begin{bmatrix} 0 & 1 \\ -24 & -10 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 \\ 24 & s+10 \end{bmatrix}$$

$$|sI - A| = s(s+10) + 24 = 0$$

$$s^2 + 10s + 24 = 0$$

$$(s+4)(s+6) = 0$$

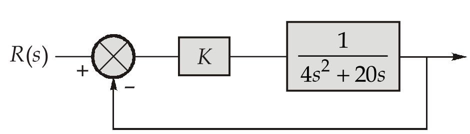
Eigen values are –4 and –6.

16. (d)

For the given block diagram,

Transfer function of inner feedback loop,

$$\frac{G(s)}{1+G(s)H(s)} = \frac{\dfrac{1}{s(1+4s)}}{1+\dfrac{19s}{s(1+4s)}} = \frac{1}{4s^2+s+19s}$$



$$\text{Transfer function, } T(s) = \frac{\dfrac{K}{4s^2+20s}}{1+\dfrac{K}{4s^2+20s}} = \frac{K}{4s^2+20s+K}$$

$$= \frac{\frac{K}{4}}{s^2 + 5s + \frac{K}{4}}$$

Comparing with standard second order form, $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

So, $2\xi\omega_n = 5$

for $\xi = 0.5$, $2 \times 0.5\ \omega_n = 5$

$\Rightarrow$ $\omega_n = 5$

$\therefore$ $\frac{K}{4} = \omega_n^2 = 25$

$K = 100$

17. **(d)**

The transfer function from steady state model can be written as

$$T(s) = C[sI - A]^{-1} B$$

$$[sI - A]^{-1} = \frac{\text{adj}[sI - A]}{|sI - A|}$$

$$[sI - A] = \begin{bmatrix} s+5 & -1 \\ 0 & s+4 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+5)(s+4)}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ s+5 \end{bmatrix}$$

$$T(s) = \frac{1 + 4(s+5)}{(s+4)(s+5)} = \frac{4s+21}{(s+4)(s+5)}$$

If two systems are connected in parallel

$$T'(s) = T(s) + T(s) = \frac{2(4s+21)}{(s+4)(s+5)}$$

18. **(c)**

Checking for controllability,

$$Q_C = [B : AB]$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } AB = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 4 \\ 1 & 1 \end{bmatrix}$$

So, $$|Q_C| = 0 - 4 = -4 \neq 0$$

$\therefore$ system is always controllable

Checking for observability

$$Q_0 = \left[C^T \ : \ A^T C^T\right]$$

$$C^T = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

and $$A^T C^T = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ 4a \end{bmatrix}$$

$\Rightarrow$ $$Q_0 = \begin{bmatrix} a & a \\ 0 & 4a \end{bmatrix}$$

$\Rightarrow$ $$|Q_0| = 4a^2$$

System is observable for non-zero values of a.

19. (b)

By introducing lag compensator in system, bandwidth is reduced.

20. (d)

Routh array can be written as

$$\begin{array}{c|ccc} s^4 & 1 & a & 12 \\ s^3 & 4 & 16 & \\ s^2 & a-4 & 12 & \\ s^1 & & & \\ s^0 & & & \end{array}$$

$\therefore$ Auxiliary equation :

$$(a - 4)s^2 + 12 = 0$$

Putting $s = j\omega$, $$(a - 4) = \frac{12}{\omega^2} = \frac{12}{4}$$

$$a - 4 = 3$$

$$a = 7$$

21. (c)

Given open loop transfer function,

$$G(s) = \frac{1}{s(2s+1)(s+1)} \quad , \quad H(s) = 1$$

So, $$G(s)H(s) = \frac{1}{s(2s+1)(s+1)}$$

By putting, $$s = j\omega$$

$$G(j\omega)\ H(j\omega) = \frac{1}{(j\omega)(2j\omega+1)(j\omega+1)}$$

The intersection with real axis occurs in negative half of the $j\omega$ plane when Im$\{G(j\omega)H(j\omega)\}$ = 0

$$\therefore \text{Im}\left\{\frac{(+1-2j\omega)(+1-j\omega)(-j\omega)}{\omega^2(4\omega^2+1)(1+\omega^2)}\right\} = 0$$

$$\text{Im}\{(\omega - 2\omega^3)j - (\omega^2 + 2\omega^2)\} = 0$$

So, $$\omega(1 - 2\omega^2) = 0$$

$$\omega = 0, \text{ and } \omega = \frac{1}{\sqrt{2}}$$

$$\therefore \text{ Value } G(j\omega)H(j\omega)\Big|_{\omega=\frac{1}{\sqrt{2}}} = \frac{1}{\left(j\frac{1}{\sqrt{2}}\right)\left(2j\frac{1}{\sqrt{2}}+1\right)\left(j\frac{1}{\sqrt{2}}+1\right)}$$

$$= \frac{1}{\frac{j^2}{2}-1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$$

22. (b)

Given, $$G(j\omega) = \frac{1}{j\omega(j\omega T+1)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{\omega^2T^2+1}}$$

$$\angle G(j\omega) = -90° - \tan^{-1}\left(\frac{\omega T}{1}\right)$$

At $\omega = 0$, $$|G(j\omega)| = \infty\ ;$$

$$\angle G(j\omega) = -90°$$

At $\omega = \infty$, $$|G(j\omega)| = 0$$

$$\angle G(j\infty) = -180°$$

23. (d)

For phase crossover, ω_{pc}

$$\angle G(j\omega_p)H(j\omega_p) = -180°$$

$$-90° - \tan^{-1} 2\omega_p - \tan^{-1} \omega_p = -180°$$

$$\Rightarrow \quad \tan^{-1} 2\,\omega_p + \tan^{-1} \omega_p = 90°$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\omega_p + \omega_p}{1-(2\omega_p)(\omega_p)}\right) = 90°$$

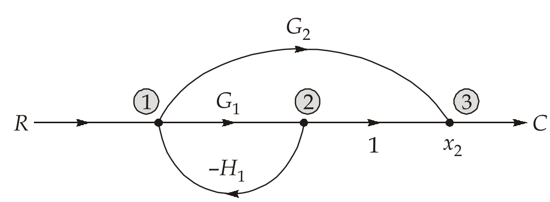
$$\Rightarrow \quad \frac{3\omega_p}{1-2\omega_p^2} = \tan 90° = \infty$$

$$\Rightarrow \quad 1 - 2\,\omega_p^{\,2} = 0$$

$$\Rightarrow \quad \omega_p = \frac{1}{\sqrt{2}}$$

24. (a)

Drawing signal flow graph



Forward paths : $P_1 = G_1$;

$P_2 = G_2$

Loop : $L_1 = -G_1H_1$

Loop touches both forward paths so $\Delta_1 = \Delta_2 = 1$

and $\Delta = 1 - L_1 = 1 + G_1H_1$

So, $$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1H_1}$$

25. (a)

2-digit positive integers are 10, 11, 12 ... 99. Thus there are 90 such numbers. Since out of these, 30 numbers are multiple of 3, therefore the probability that a randomly chosen positive 2-digit integer is a multiple of 3, is $\frac{30}{90}$.

26. (a)

Directional derivative = $\nabla\phi$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2 - y^2 + 2z^2)$$

$$= 2x\hat{i} - 2y\hat{j} + 4z\hat{k}$$

Directional derivative at the point $P(1, 2, 3)$

$$= 2\hat{i} - 4\hat{j} + 12\hat{k}$$

Now, $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = 4\hat{i} - 2\hat{j} + \hat{k}$

Directional derivative along $PQ = \nabla\phi \cdot (\widehat{PQ})$

$$= (2\hat{i} - 4\hat{j} + 12\hat{k}) \cdot \frac{(4\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{16+4+1}}$$

$$= \frac{8+8+12}{\sqrt{21}} = \frac{28}{\sqrt{21}}$$

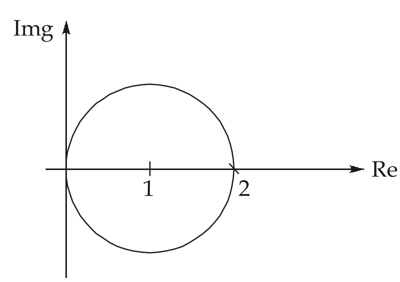
27. (c)

Poles of the integrand are given by putting the denominator equal to zero

$$z^2 - 1 = 0$$

$\Rightarrow$ $$z^2 = 1$$

$\Rightarrow$ $$z = \pm 1$$



The circle with centre $z = 1$ and radius encloses a simple pole at $z = 1$

Using Cauchy Integral formula

$$\int_C \frac{3z^2 + z}{z^2 - 1} dz = 2\pi i \left[\frac{3z^2 + z}{z+1}\right]_{z=1}$$

$$= 2\pi i\left(\frac{3+1}{1+1}\right) = 4\pi i$$

28. (a)

Given, $x^2 - N = 0$

Taking, $f(x) = x^2 - N$

We have, $f'(x) = 2x$

Then Newton-Raphson's formula gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - N}{2x_n} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

29. (a)

Condition for orthogonality of matrix

$$QQ^T = Q^TQ = I$$

$$Q^T = Q^{-1}$$

30. (c)

Given surface is $\phi = 2xz^2 - 3xy - 4x - 7$ and $P = (1, -1, 2)$

The normal vector of $\phi(x, y, z)$ at point P is given by $(\nabla\phi)_p$.

Now, $\nabla\phi = (2z^2 - 3y - 4)\hat{i} + (-3x)\hat{j} + 4xz\hat{k}$

$\therefore$ $(\nabla\phi)_p = 7\hat{i} - 3\hat{j} + 8\hat{k}$

31. (b)

$$(D^2 + 6D + 9)y = 5e^{3x}$$

Auxiliary equation is $m^2 + 6m + 9 = 0$

$$(m + 3)^2 = 0$$

$$m = -3, -3$$

$$\text{C.F.} = (C_1 + C_2x)e^{-3x}$$

$$\text{P.I.} = \frac{1}{D^2 + 6D + 9} \times (5e^{3x})$$

$$= 5 \cdot \frac{e^{3x}}{(3)^2 + 6 \times 3 + 9} = \frac{5e^{3x}}{36}$$

The complete solution is

$$y = (C_1 + C_2x)e^{-3x} + \frac{5e^{3x}}{36}$$

32. (b)

Given matrix, $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

Applying $R_3 \to R_3 + R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = Number of non zero rows in echalon form = 2

33. (d)

$$\nabla \cdot \vec{F} = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}\right]$$

$$= -2 + 2x - 2x + 2 = 0$$

Thus, $\vec{F}$ is solenoidal

$$\text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (y^2 - z^2 + 3yz - 2x) & (3xz + 2xy) & (3xy - 2xz + 2z) \end{vmatrix}$$

$$= (3x - 3x)\hat{i} - (-2z + 3y - 3y + 2z)\hat{j} + (3z + 2y - 2y - 3z)\hat{k}$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

Thus $\vec{F}$ is irrotational.

34. (c)

$$\lim_{n\to\infty} \frac{1 + 2 + 3 + ... + n}{n^2} = \lim_{n\to\infty} \frac{n(n+1)}{2n^2}$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(1 + \frac{1}{n}\right) = \frac{1}{2}$$

35. (b)

Given, $f(x) = x(x - 1)(x - 2)$

$$f(a) = 0$$

$$f(b) = \frac{1}{2}\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right) = \frac{3}{8}$$

$$f'(x) = 3x^2 - 6x + 2$$

$\Rightarrow$ $f'(c) = 3c^2 - 6c + 2$

Given, $f(b) - f(a) = (b - a)\, f'(c)$

$$\frac{3}{8} - 0 = \left(\frac{1}{2} - 0\right)(3c^2 - 6c + 2)$$

$$\frac{3}{8} = \frac{1}{2}(3c^2 - 6c + 2)$$

$$3 = 12c^2 - 24c + 8$$

$$12c^2 - 24c + 5 = 0$$

Hence, $$c = \frac{24 \pm \sqrt{(24)^2 - 12 \times 5 \times 4}}{24} = 1 \pm 0.764$$

$$= 1.764 \text{ and } 0.236$$

Hence, $c = 0.236$, since it only lies between 0 and $\frac{1}{2}$.

36. (d)

All of the given statements are correct.

37. (c)

The probability that A can solve the problem is $\frac{1}{2}$.

The probability that A can not solve the problem is $1 - \frac{1}{2}$.

Similarly the probabilities that B and C cannot solve the problem are $1 - \frac{1}{3}$ and $1 - \frac{1}{4}$.

$\therefore$ The probability that A, B and C cannot solve the problem (i.e. problem cannot be solved) is

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Hence the probability that the problem will be solved, i.e. at least one student will solve it

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{3}{4}$$

38. (c)

Given, $$X(s) = \left[\frac{5s + 3}{s^2 + 8s + 20}\right]$$

Using initial value theorem,

$$x(0^+) = \lim_{s \to \infty} sX(s) = \lim_{s \to \infty}\left[\frac{s(5s + 3)}{s^2 + 8s + 20}\right] = \lim_{s \to \infty}\left[\frac{5 + \frac{3}{s}}{1 + \frac{8}{s} + \frac{20}{s^2}}\right] = \frac{5}{1} = 5$$

39. **(d)**

- If $b^2 - 4ac > 0$ then the equation is called hyperbolic.
- If $b^2 - 4ac = 0$, then the equation is called parabolic.
- If $b^2 - 4ac < 0$, then the equation is called elliptic.

40. **(b)**

If X is a random variable then

$$\sum_{i=0}^{6} p(x_i) = 1$$

$K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$$K = \frac{1}{49}$$

$$\therefore \quad P(X < 4) = K + 3K + 5K + 7K = 16K$$

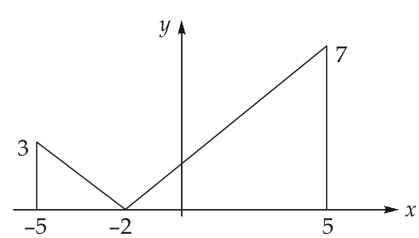
$$\because \quad K = \frac{1}{49}$$

$$\therefore \quad P(X < 4) = \frac{16}{49}$$

41. **(c)**

In case of Simpson's $\frac{1}{3}$ rule the interval is split up into n sub-intervals, with n an even number.

42. **(a)**



$$\text{Area} = \frac{1}{2}\times 3\times 3+\frac{1}{2}\times 7\times 7 = 29$$

Alternate Solution:

Since for $-5 \le x \le -2$, $x + 2 \le 0$

$$\therefore \quad |x+2| = -(x+2)$$

and for $-2 \le x \le 5$, $x + 2 \ge 0$

$$\therefore \quad |x+2| = x+2$$

$$\therefore \quad \int_{-5}^{5} |x+2|dx = \int_{-5}^{-2} |x+2|dx + \int_{-2}^{5} |x+2|dx = \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)dx$$

$$= \left[\frac{-x^2}{2} - 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= (-2+4) - \left(\frac{-25}{2} + 10\right) + \left(\frac{25}{2} + 10\right) - (2-4)$$

$$= 2 + \frac{5}{2} + \frac{25}{2} + 10 + 2 = 29$$

43. (b)

Here, $M = 1 + 2xy\cos x^2 - 2xy$

and $N = \sin x^2 - x^2$

$$\frac{\partial M}{\partial y} = 2x\cos x^2 - 2x$$

and $$\frac{\partial N}{\partial x} = 2x\cos x^2 - 2x$$

the equation is exact and its solution is

$$\int_{(y\text{ constant})} M dx + \int (\text{terms } N \text{ not containing } x) = C$$

$$\Rightarrow \int_{(y\text{ constant})} (1 + 2xy\cos x^2 - 2xy)dx = C$$

$$x + y\sin x^2 - yx^2 = C$$

44. (a)

- The eigen values of a Hermitian matrix are real.
- The eigen values of a Skew-Hermitian matrix are purely imaginary or zero.

Suppose A is Hermitian matrix, then

$$(\overline{iA})^T = (\bar{i}\,\bar{A})^T = (-i\,\bar{A})^T = -i\bar{A}^T$$

Since transposition does not effect i,

Thus $$(\overline{iA})^T = -i\bar{A}^T = -i\,\mathrm{A}$$

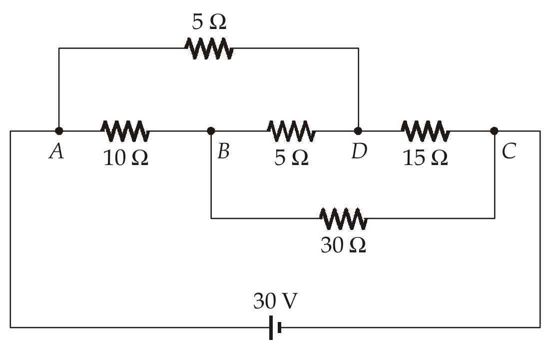
45. (c)

The number of sign changes of first column = +ve real parts or roots on right s-plane.

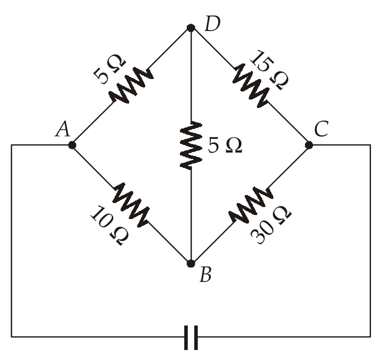
Section B : Electrical Circuit - 1

46. (c)

The given circuit can be rearranged as follows:



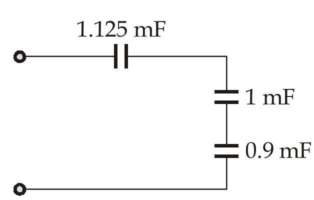
On rearranging again,



The above circuit is balanced Wheatstone's bridge. Hence current I through 5 Ω resistor i.e. between B and D is 0.

47. (d)

The circuit can be redrawn as,

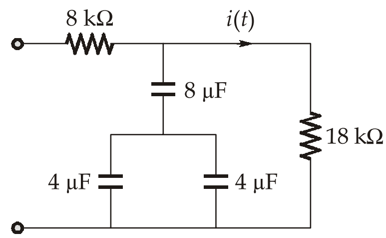


Combine series capacitance,

$$C_{eq} = \frac{1}{\frac{1}{1.125}+\frac{1}{1}+\frac{1}{0.9}} = 0.33 \text{ mF}$$

48. (b)

For time constant calculation, the circuit can be redrawn as



Where, $R_{eq} = 18\ \text{k}\Omega$

and $C_{eq} = 4\ \mu\text{F}$

$\therefore$ Time constant, $\tau = R_{eq}C_{eq} = 72$ msec

49. (a)

Applying KVL in the left most loop

$$50 - 300I_1 - 500 \times 0.6I_1 = 0$$
$$50 - 600I_1 = 0$$
$$I_1 = \frac{50}{600} = \frac{1}{12}\text{A}$$

Current though 500 Ω resistance

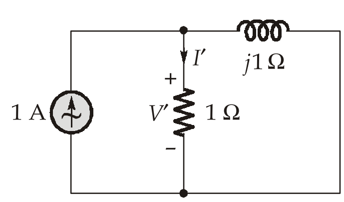
$$= 0.6 \times \frac{1}{12} = \frac{6}{10} \times \frac{1}{12} = \frac{1}{20}\text{A}$$

Power absorbed by 500 Ω resistance

$$= 500 \times \frac{1}{20} \times \frac{1}{20} = 1.25\ \text{W}$$

50. (b)

When the 1 A current source is acting alone:



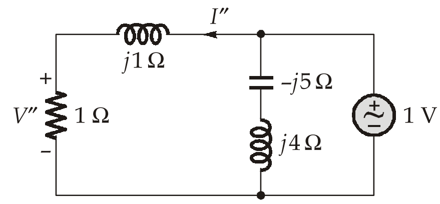
Applying current division rule,

The current through 1 Ω resistor is

$$I' = 1 \times \frac{j1}{1+j1} = \frac{j1}{1+j1}$$

The voltage across the resistor, $R = 1\ \Omega$ is $V' = \dfrac{j}{1+j}$

When the 1 V voltage source is acting alone



The current through the resistor,

$$I'' = \frac{1}{1+j}$$

The voltage across the resistor,

$$R = 1\ \Omega$$

$$V'' = I'' \times 1 = \frac{1}{1+j}$$

So by the superpostion theorem, total voltage across the resistor when both the sources are acting simultaneously is,

$$V = (V' + V'') = \frac{j}{1+j} + \frac{1}{1+j} = 1\,\text{V}$$

51. (a)

By KVL for the right hand side mesh,

$$V_{OC} = V_x = (-40I_0) \times 50$$

$$= -2000I_0 \qquad \text{...(i)}$$

From the left hand side loop,

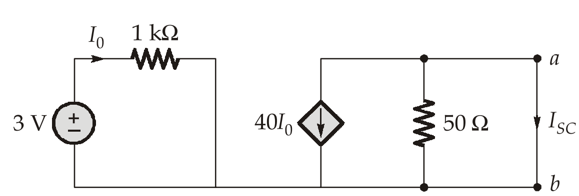
$$I_0 = \frac{3-2V_x}{1000} = \frac{3-2V_{OC}}{1000} \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$V_{OC} = -2000\left(\frac{3-2V_{OC}}{1000}\right) = -6 + 4V_{OC}$$

$$V_{OC} = 2\ \text{V}$$

To determine the short circuit current, we short-circuit the terminals a and b.



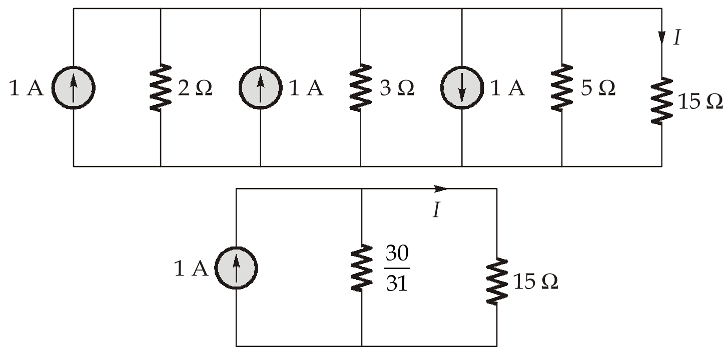
$$I_{SC} = -40I_0 = -40 \times \frac{3}{1000} = -0.12\ \text{A}$$

∴ Thevenin's resistance,

$$R_{\text{Th}} = \frac{V_{OC}}{I_{SC}} = \frac{2}{0.12} = -16.67\ \Omega$$

52. (a)

Applying source transformation,



Applying current division rule,

$$I = \frac{1\times\frac{30}{31}}{15+\frac{30}{31}} = \frac{\frac{30}{31}}{\frac{495}{31}} = 0.0606 \text{ A}$$

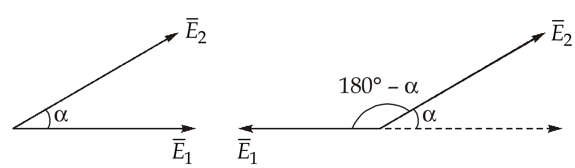
53. (d)

Tellegen's theorem is applicable for any lumped network having elements which are linear or nonlinear, active or passive time varying or time invariant.

54. (d)

All of the given statements are correct.

55. (b)



When two sources are connected in series,

$$\sqrt{E_1^2+E_2^2+2E_1E_2\cos\alpha} = 200$$

$$\sqrt{E^2+E^2+2E^2\cos\alpha} = 200$$

$$2E^2 + 2E^2\cos\alpha = 40000 \quad \text{...(i)}$$

When one source is reversed,

$$\sqrt{E_1^2+E_2^2-2E_1E_2\cos\alpha} = 15$$

$$2E^2 - 2E^2\cos\alpha = 225 \quad \text{...(ii)}$$

Adding equation (i) and (ii),

$$4E^2 = 40225$$

$$E^2 = 10056.25$$

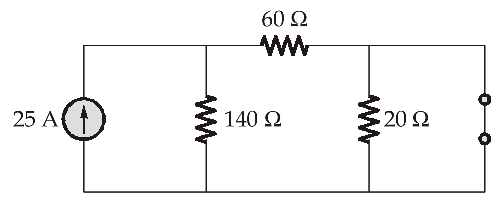
$$E = 100.28 \text{ V}$$

56. **(d)**

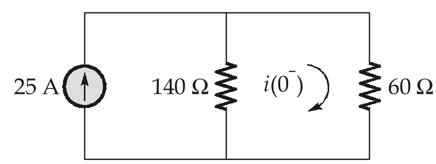
All of the statements are correct.

57. **(c)**

At $t = 0^-$, the inductor acts as a short circuit,



Simplifying the network,



Applying current division,

$$i(0^-) = 25 \times \frac{140}{140+60} = 25 \times \frac{140}{200} = 17.5 \text{ A}$$

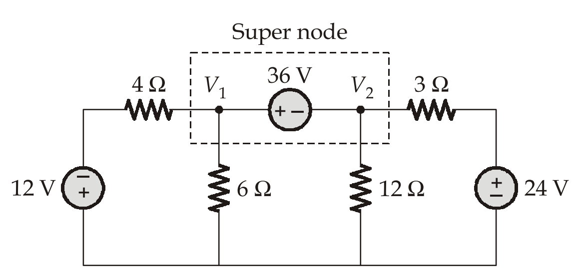
Since current through the inductor cannot change instantaneously,

$\therefore \quad i(0^+) = 17.5$ A

58. **(a)**

- Voltage division allows us to calculate what fraction of the total voltage across a series string of resistors is dropped across any one resistor.
- Current division allows us to calculate what fraction of the total current into a parallel string of resistors flows through any one of the resistors.

59. **(c)**



Applying KCL at supernode

$$\frac{V_1+12}{4} + \frac{V_1}{6} + \frac{V_2}{12} + \frac{V_2-24}{3} = 0$$

$$\frac{3V_1+36+2V_1+V_2+4V_2-96}{12} = 0$$

$$5V_1 + 5V_2 = 60$$

$$V_1 + V_2 = 12 \quad \text{...(i)}$$

also $$V_1 - V_2 = 36 \quad \text{...(ii)}$$

On solving equation (i) and (ii),

We get, $$V_1 = 24 \text{ V}$$

60. (a)

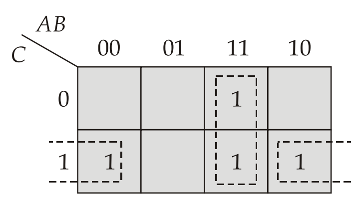
There is no current through a capacitor if the voltage across it is not changing with time. A capacitor is therefore an open circuit to dc.

Section C : Digital Electronics-1 + Microprocessors-1

61. (b)

Given, $Y(A, B, C) = AB + C\overline{B} + AC$

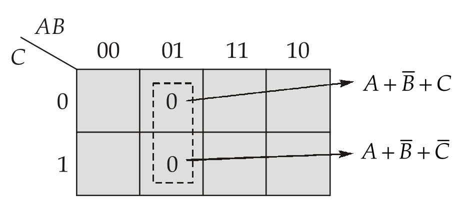
By drawing K-map for given expression



$$Y(A, B, C) = C\overline{B} + AB$$

62. (d)

By K-map



Simplified expression is $A+\overline{B}$

63. (c)

The carry output of full adder = $AB + AC_{in} + BC_{in}$

64. (c)

Given, $Y(A, B, C, D) = \Sigma m(1, 3, 7, 11, 15) + d(0, 2, 5)$

Here Σm represents SOP values and d represents don't care values

$\therefore$ Drawing K-map accordingly as below,

CD \ AB	00	01	11	10
00	×			
01	1	×		
11	1	1	1	1
10	×			

Simplified expression, $Y = CD + \bar{A}\bar{B}$

65. (d)

All given statements are advantages of multiplexer.

66. (b)

- The BCD adder output is in BCD form.
- BCD addition can be performed using binary addition only.

67. (b)

- The content of program counter is the address of the next instruction to be executed.
- The instruction register (IR) of 8085A stores opcode of instruction which is being executed.

68. (b)

$\overline{\text{MEMW}} \rightarrow$ Writing into memory
$\overline{\text{MEMR}} \rightarrow$ Reading from memory
$\overline{\text{IOR}} \rightarrow$ input port read
$\overline{\text{IOW}} \rightarrow$ output port write

69. (c)

Total number of T-states needed for execution of instruction STA 2065 is 13

Opcode fetch $\rightarrow$ 4T states and for subsequent operation $\rightarrow$ 9T states

70. (c)

MVI B, F2H is a two byte instruction where the first byte specifies the operation code and the second byte specifies the operand.

71. (c)

In tri state devices when enable pin is high (or activated state), the circuit functions as an ordinary digital circuit. When enable line is low, the device stays in the high impedance state.

72. (d)

All given statements are correct regarding RST instruction.

73. (b)

- It manages eight interrupts.
- Initialization control word is used to initialize 8259A, whereas operational control words are issued to control the operation of 8259A.

74. (c)

- Demultiplexer can be used as decoder by interchanging select lines with input.
- Demultiplexer is implemented using combination of AND gate and NOT gate.

75. (c)

The assembler can reserve memory locations for data or result.

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