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GATE 2021

MECHANICAL ENGINEERING

Questions & Solutions

Exam held on **14/02/2021**
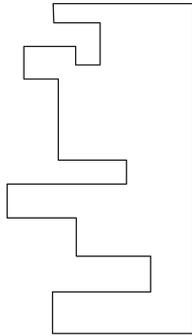**Forenoon Session**

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SECTION-A   GENERAL APTITUDE

Q.1   Consider the following sentences:
1.   After his surgery, Raja hardly could walk.
2.   After his surgery, Raja could barely walk.
3.   After his surgery, Raja barely could walk.
4.   After his surgery, Raja could hardly walk.
Which of the above sentences are grammatically CORRECT?
(a) 1 and 2   (b) 1 and 3
(c) 2 and 4   (d) 3 and 4

Ans.   **(c)**
Hardly/Scarcely/Barely have same sense that is negative and they are used after the verb (could barely and could hardly).

End of Solution

Q.2   Oxpeckers and rhinos manifest a symbiotic relationship in the wild. The oxpeckers warn the rhinos about approaching poachers, thus possibly saving the lives of the rhinos. Oxpeckers also feed on the parasitic ticks found on rhinos.
In the symbiotic relationship described above, the primary benefits for oxpeckers and rhinos respectively are,
(a) Oxpeckers get a food source, rhinos have no benefit.
(b) Oxpeckers save their habitat from poachers while the rhinos have no benefit.
(c) Oxpeckers get a food source, rhinos may be saved from the poachers.
(d) Oxpeckers save the lives of poachers, rhinos save their own lives.

Ans.   **(c)**
Option (a) and (b) are weekend by expression 'rhinos have no benefit'. Oxpeckers do not save life of poachers, so option (d) is incorrect.
Hence, option (c) is correct.

End of Solution

Q.3   Ms. X came out of a building through its front door to find her shadow due to the morning sun falling to her right side with the building to her back. From this, it can be inferred that building is facing _____.
(a) South   (b) West
(c) East   (d) North

Ans.   **(a)**
Morning sun is falling from east then the shadow will fall to the west. So west should be on the right side of Ms. X. So Ms. X came out towards south. Hence, her building is facing south.

End of Solution

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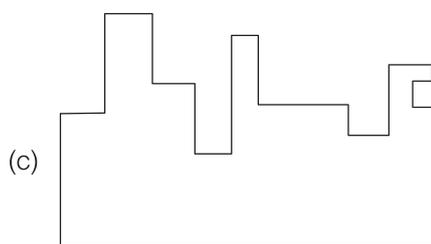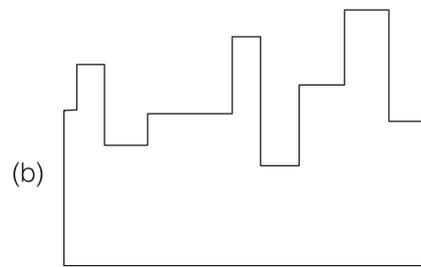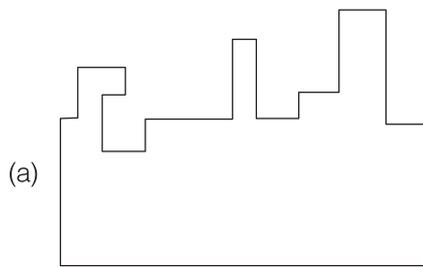*Forenoon Session*

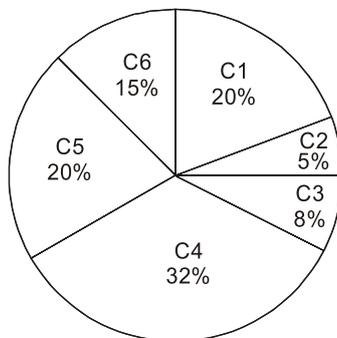India's Best Institute for IES, GATE & PSUs

Q.4

A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the above piece.

(a)

(b)

(c)

(d)

Ans. (d)

End of Solution

Q.5

C6 15%
C1 20%
C5 20%
C2 5%
C3 8%
C4 32%
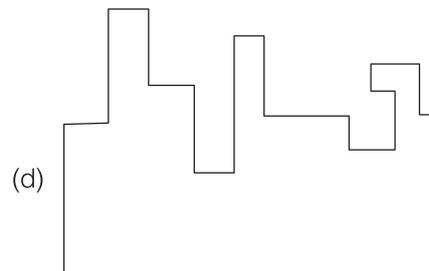
Company	Ratio
C1	3 : 2
C2	1 : 4
C3	5 : 3
C4	2 : 3
C5	9 : 1
C6	3 : 4

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The distribution of employees at the rank of executives, across different companies C1, C2, ..., C6 is presented in the chart given above. The ratio of executives with a management degree to thos without a management degree in each of these companies is provided in the table above. The total number of executives across all companies is 10000.

The total number of management degree holders among the executives in companies C2 and C5 together is _____.

(a) 225
(b) 1900
(c) 2500
(d) 600

Ans. **(b)**

Number of employee in C2 company = $\dfrac{5}{100} \times 10000 = 500$

Number of management degree holder employee in C2 = $\dfrac{1}{5} \times 500 = 100$

Number of employee in C5 company = $\dfrac{20}{100} \times 10000 = 2000$

Number of management degree holder employee in C5 = $\dfrac{9}{10} \times 2000 = 1800$

Total management degree holder employee = 100 + 1800 = 1900

End of Solution

Q.6 The number of hens, ducks and goats in farm P are 65, 91 and 169, respectively. The total number of hens, ducks and goats in a nearby farm Q is 416. The ratio of hens : ducks : goats in farm Q is 5 : 14 : 13. All the hens, ducks and goats are sent from farm Q to farm P.

The new ratio of hens : ducks : goats in farm P is _____.
(a) 5 : 7 : 13
(b) 21 : 10 : 26
(c) 5 : 14 : 13
(d) 10 : 21 : 26

Ans. **(d)**
In farm P,
Hens = 65, Ducks = 91, Goats = 169
In farm Q,
Hens : Ducks : Goats
5 : 14 : 13

$$\text{Hens} = \dfrac{5}{32} \times 416 = 65$$

$$\text{Ducks} = \dfrac{14}{32} \times 416 = 182$$

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$$\text{Goats} = \frac{13}{32} \times 416 = 169$$

∵ From farm *d*, hens, ducks and goats are sent to farm P,

∴ Total hens = 65 + 65 = 130

Total ducks = 91 + 182 = 273

Total goats = 169 + 169 = 338

New ratio = 130 : 273 : 338

= 10 : 21 : 26

End of Solution

Q.7 "The increased consumption of leafy vegetables in the recent months is a clear indication that the people in the state have begun to lead a healthy lifestyle".

Which of the following can be logically inferred from the information presented in the above statement?
(a) Consumption of leafy vegetables may not be the only indicator of healthy lifestyle.
(b) The people in the state have increased awareness of health hazards causing by consumption of junk foods.
(c) Leading a healthy lifestyle is related to a diet with leafy vegetables.
(d) The people in the state did not consume leafy vegetables earlier.

Ans. **(c)**

The last sentence of the passage is reflecting in option (c) only.

End of Solution

Q.8
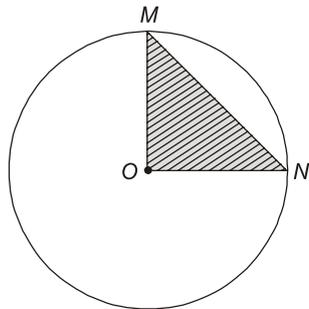
In the above figure, O is the center of the circle and, M and N lie on the circle.
The area of the right triangle MON is 50 cm^2.
What is the area of the circle in cm^2?
(a) 2 π (b) 100 π
(c) 50 π (d) 75 π

Ans. **(b)**

∵ Area of ∆MON = 50 cm^2

$$\frac{1}{2} \times r \times r = 50$$

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$$r^2 = 100$$
$$\text{Area of circle} = \pi r^2$$
$$= \pi \times 100$$
$$= 100\,\pi$$

<div style="text-align:right">End of Solution</div>

Q.9 Five persons P, Q, R, S and T are sitting in a row not necessarily in the same order. Q and R are separated by one person, and S should not be seated adjacent to Q. The number of distinct seating arrangements possible is

(a) 4   (b) 10

(c) 16   (d) 8

Ans. **(c)**

The possible seating arragements are

QPRST, QTRSP

QPRTS, QTRPS

PQTRS, TQPRS

SPQTR, STQPR

RPQTS, RTQPS

SRPQT, SRTQP

SPRTQ, STRPQ

PSRTQ, TSRPQ

Hence, total seating arragements are 16.

<div style="text-align:right">End of Solution</div>

Q.10 If $\begin{cases} "\oplus" \text{ means}"-", \\ "\otimes" \text{ means}"\div", \\ "\Delta" \text{ means}"+", \\ "\nabla" \text{ means}"\times", \end{cases}$

then, the value of expression $\Delta 2 \oplus 3\Delta\big((4 \otimes 2)\nabla 4\big) =$

Ans. **(7)**

$$\Delta 2 \oplus 3\Delta\big((4 \otimes 2)\nabla 4\big) \; = +2 - 3 + ((4 \div 2) \times 4)$$
$$= +2 - 3 + (2 \times 4)$$
$$= +2 - 3 + 8$$
$$= 7$$

<div style="text-align:right">End of Solution</div>

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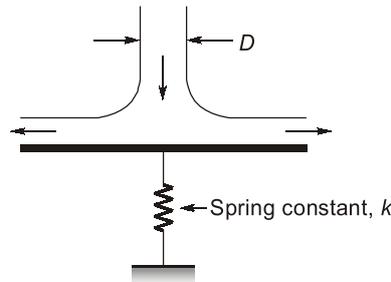Detailed Solutions
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SECTION-B   TECHNICAL

Q.1 A cylindrical jet of water (density = 1000 kg/m³) impinges at the center of a flat, circular plate and spreads radially outwards, as shown in the figure. The plate is resting on a linear spring with a spring constant k = 1 kN/m. The incoming jet diameter is D = 1 cm.

If the spring shows a steady deflection of 1 cm upon impingement of jet, then the velocity of the incoming jet is _____ m/s. [Round off to one decimal place]

Ans. 11.3 (11.2 to 11.4)
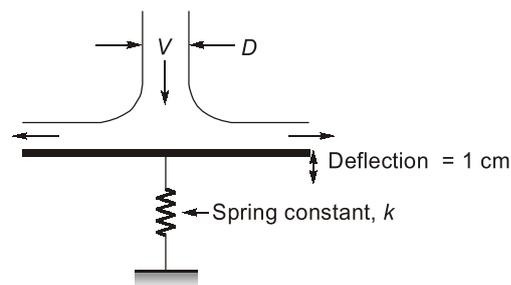
$$\delta = 1 \text{ cm}$$
$$D = 1 \text{ cm}$$
$$\rho = 1000 \text{ kg/m}^3$$
$$K = 1 \text{ kN-m}$$

Force due to jet = Spring force
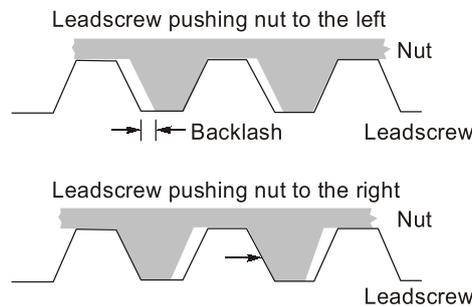$$\rho AV^2 = kx$$

$$10^3 \times \frac{\pi}{4} \times (0.01)^2 \times V^2 = 1 \times 10^3 \times 0.01$$

$$V = 11.28 \text{ m/s} \simeq 11.3 \text{ m/s}$$

End of Solution

Q.2 The XY table of a NC machine tool is to move from P(1, 1) to Q(51, 1); all coordinates are in mm. The pitch of the NC drive leadscrew is 1 mm. If the backlash between the leadscrew and the nut is 1.8°, then the total backlash of the table on moving from P to Q is _____ mm [Round off to two decimal places]

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Ans. 0.25 (0.20 to 0.30)

Leadscrew pushing nut to the left — Nut — Backlash — Leadscrew

Leadscrew pushing nut to the right — Nut — Leadscrew

$P(1, 1)$ to $P(51, 1)$ is 50 mm distance. As pitch of the leadscrew is 1 mm. It has to move 50 rotation and in each rotation, backlash is 1.8 degree. Therefore total backlash = 1.8 × 50 degree.

In one rotation i.e. 360° rotation - 1 mm movement take place

In 1.8 × 50° = $\dfrac{1.8 \times 50}{360}$ mm = 0.25 mm

End of Solution

Q.3 In a grinding operation of a metal, specific energy consumption is 15 J/mm^3. If a grinding wheel with a diameter of 200 mm is rotating at 3000 rpm to obtain a material removal rate of 6000 mm^3/min, then the tangential force on the wheel is _____ N. [Round off to two decimal places]

Ans. 47.746 (45 to 50)

$$E_{sp} = 15 \text{ J/mm}^3$$
$$\text{MRR} = 6000 \text{ mm}^3/\text{min}$$
$$N = 3000 \text{ rpm}$$
$$D = 200 \text{ mm}$$

$$E_{sp} = \frac{FcV}{LdV}$$

$$15 = \frac{F \times \dfrac{m}{\min}}{600 \dfrac{mm^3}{\min}}$$

$$V = \frac{\pi DN}{1m} = \frac{\pi(200)(3m)}{1000} = 600 \times 3.14$$

$$15 = \frac{F \times 600 \times 3\pi y}{6000}$$

$$F = \frac{150}{\pi} = 47.746 \text{ N} \simeq 47.75 \text{ N}$$

End of Solution

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Q.4 Consider a vector p in 2-dimensional space. Let its direction (counter-clockwise angle with the positive x-axis) be θ. Let p be an eigen vector of a 2×2 matrix A with corresponding eigen value λ, $\lambda > 0$. If we denote the magnitude of a vector v by $\|v\|$, identify the VALID statement regarding p', where $p' = Ap$.

(a) Direction of $\mathsf{p}' = \theta, \|\mathsf{p}'\| = \lambda \|\mathsf{p}\|$

(b) Direction of $\mathsf{p}' = \lambda\theta, \|\mathsf{p}'\| = \|\mathsf{p}\|$

(c) Direction of $\mathsf{p}' = \theta, \|\mathsf{p}'\| = \|\mathsf{p}\|/\lambda$

(d) Direction of $\mathsf{p}' = \lambda\theta, \|\mathsf{p}'\| = \lambda \|\mathsf{p}\|$

Ans. **(a)**

$\because$ A is a 2×2 matrix and P is the eigen vector of matrix A with corresponding eigen value λ.

Given: $P' = P$

$AP = \lambda P$

Hence, $P' = \lambda P$

$$\|P'\| = \|\lambda P\| = \lambda \|P\|$$
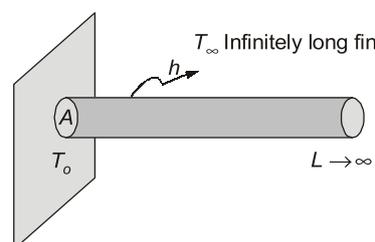
But direction of vector P' will be same as vector P.

End of Solution

Q.5 An infinitely long pin fin, attached to an isothermal hot surface, transfers heat at a steady rate of $\dot{Q}_1$ to the ambient air. If the thermal conductivity of the fin material is doubled, while keeping everything else constant, the rate of steady-rate heat transfer from the fin becomes $\dot{Q}_2$. The ratio $\dfrac{\dot{Q}_2}{\dot{Q}_1}$ is

(a) 2

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{\sqrt{2}}$

(d) $\sqrt{2}$

Ans. **(d)**

Fin problem:


$$q = \sqrt{hPkA}\,(T_o - T_\infty)\ \text{wait}$$

If k gets doubled q increases by $\sqrt{2}$ times.

End of Solution

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Q.6 An air-conditioning system provides a continuous flow of air to a room using an intake duct and an exit duct, as shown in the figure. To maintain the quality of the indoor air, the intake duct supplies a mixture of fresh air with a cold air stream. The two stream are mixed in an insulated mixing chamber located upstream of the intake duct. Cold air enters the mixing chamber at 5°C, 105 kPa with a volume flow rate of 1.25 m³/s during steady state operation. Fresh air enters the mixing chamber at 34°C and 105 kPa. The mass flow rate of the fresh air is 1.6 times of the cold air stream. Air leaves the room through the exit duct at 24°C.
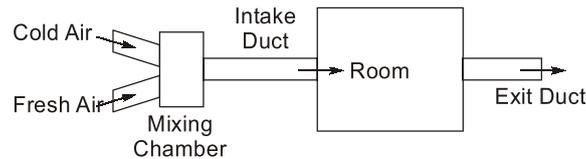
Assuming the air behaves as an ideal gas with c_p = 1.005 kJ/kgK and R = 0.287 kJ/kgK, the rate of heat gain by the air from the room is _____ kW. (Round off to two decimal places)

Ans. **4.96 (4.90 to 5.10)**

1 : Cold air

2 : Hot air

$$P_1 V_1 = \dot{m}_1 R T_1$$

$$105 \times 1.25 = \dot{m}_1 \times 0.287 \times 278$$

$$\dot{m}_1 = 1.645 \text{ kg/sec}$$

$$\dot{m}_2 = 1.6 \times 1.645 = 2.632 \text{ kg/sec}$$

$$\dot{m}_3 = 4.277 \text{ kg/sec}$$

After mixing : 0

$$\dot{m}_1 t_1 + \dot{m}_2 t_2 = \dot{m}_3 t_3$$

$$1.645 \times 5 + 2.632 \times 34 = 4.277 t_3$$

$$t_3 = 22.85 \text{ °C}$$

Heat gain : $= h_4 - h_3$

$$= \dot{m}_3 c_p (t_4 - t_3)$$

$$= 4.277 \times 1.005(24 - 22.85)$$

$$= 4.963 \text{ kW}$$

End of Solution

Q.7 An uninsulated cylindrical wire of radius 1.0 mm produces electric heating at the rate of 5.0 W/m. The temperature of the surface of the wire is 75°C when placed in air at 25°C. When the wire is coated with PVC of thickness 1.0 mm, the temperature of the surface of the wire reduces to 55°C. Assume that the heat generation rate from the wire and the convective heat transfer coefficient are same for both uninsulated wire and the coated wire. The thermal conductivity of PVC is _____ W/mK (Round off to two decimal places)

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Ans. 0.11 (0.10 to 0.12)
For uninsulated wire:

Given: $T_\infty = 25°C$, $R_1 = 1$ mm, $\dot{q}_{gen} = 5$ W/m, $T_{s_1} = 7.5°C$
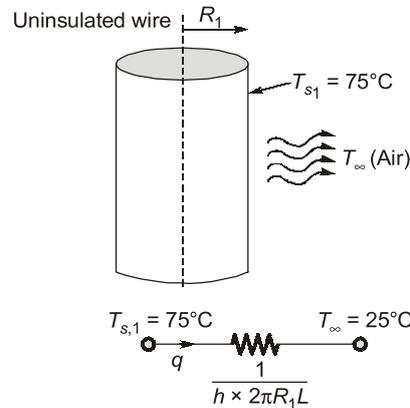
$$q = \dot{q}_{gen} \times L = \frac{T_{s1} - T_\infty}{\dfrac{1}{h \times 2\pi R_1 L}}$$

$$5 \times L = h \times (2\pi \times 0.001)L \times (75 - 25)$$
$$h = 15.91 \text{ W/m}^2\text{K}$$

For Insulated wire:
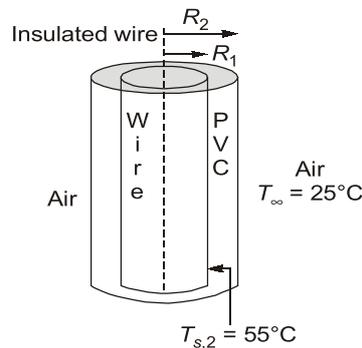
$R_2 = 2$ mm

Heat transfer rate after insulation kept on wire,

$$q = q_{\text{gen, wire}} \times L = 5 \times L$$
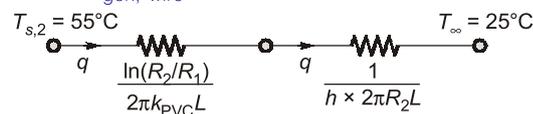
$$q = 5 \times L = \frac{55 - 25}{\dfrac{\ln(2/1)}{2\pi k_{PVC}L} + \dfrac{1}{15.91 \times 2 \times \pi \times 0.002 \times L}}$$

$$k_{PVC} = 0.1103 \text{ W/m-K}$$
$$= 0.11 \text{ W/m-K}$$

End of Solution

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Q.8 A right solid circular cone standing on its base on a horizontal surface is of height H and base radius R. The cone is made of a material with specific weight w and elastic modulus E. The vertical deflection at the mid-height of the cone due to self-weight is given by

(a) $\dfrac{wRH}{8E}$

(b) $\dfrac{wH^2}{8E}$

(c) $\dfrac{wRH}{6E}$

(d) $\dfrac{wH^2}{6E}$

Ans. **(b)**
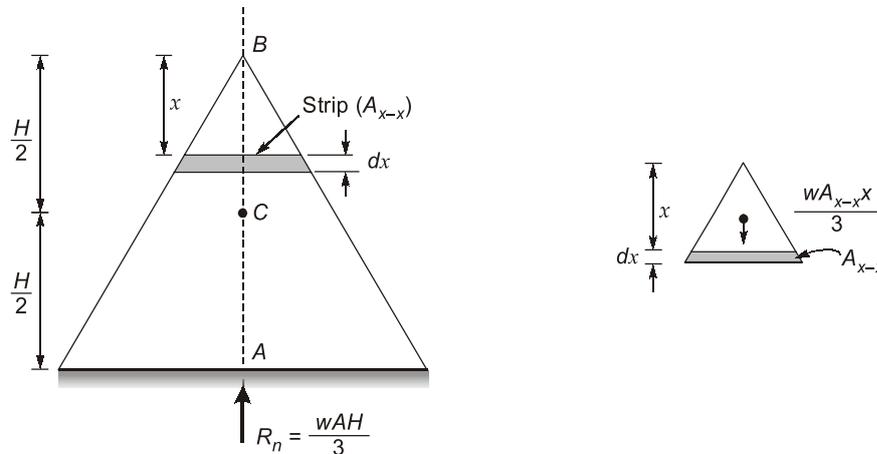
$$P_{\text{x-x}} = \frac{(-)wA_{x-x}(x)}{3}$$

Contraction of small strip $= (\delta l)_{\text{strip}}$

$$= \frac{P_{x-x}dx}{(AE)_{x-x}}$$

$$= \frac{w(A_{x-x})(x)dx}{3(A_{x-x})E} = \frac{wx}{3E}dx$$

Contraction of conical bar at mid height $\left(\text{i.e. } x = \dfrac{H}{2}\right)$

$$= \int_{H/2}^{H} (\delta l)_{\text{strip}}$$

$$= \frac{w}{3E} \int_{H/2}^{H} xdx$$

$$= \frac{w}{6E}\left[H^2 - \frac{H^2}{4}\right] = \frac{wH^2}{8E}$$

End of Solution

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Q.9 If $y(x)$ satisfies the differential equation

$$(\sin x)\frac{dy}{dx} + y\cos x = 1$$

subject to the condition $y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$, then $y\left(\dfrac{\pi}{6}\right)$ is

(a) 0

(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{6}$

(d) $\dfrac{\pi}{2}$

Ans. **(b)**

$$\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$$

I.F. $= e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

$\Rightarrow \qquad y(\sin x) = \int \operatorname{cosec} x \sin x\, dx + c$

$\Rightarrow \qquad y\sin x = x + c$

$\Rightarrow \qquad \dfrac{\pi}{2}\sin\dfrac{\pi}{2} = \dfrac{\pi}{2} + c$

$\Rightarrow \qquad \dfrac{\pi}{2} = \dfrac{\pi}{2} + c \qquad \Rightarrow c = 0$

$\Rightarrow \qquad y\sin x = x$

$\Rightarrow \qquad y\sin\dfrac{\pi}{6} = \dfrac{\pi}{6}$

$\Rightarrow \qquad y\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$

$\Rightarrow \qquad y = \dfrac{\pi}{3}$

End of Solution

Q.10 A pressure measurement device fitted on the surface of a submarine, located at a depth H below the surface of an ocean, reads an absolute pressure of 4.2 MPa. The density of sea water is 1050 kg/m³, the atmospheric pressure is 101 kPa, and the acceleration due to gravity is 9.8 m/s². The depth H is _____ m. [Round off to the nearest integer]

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Ans. 398 (397 to 399)

$$P_A = 4.2 \text{ MPa} \qquad \text{[Absolute pressure]}$$
$$P_{atm} = 101 \text{ kPa}$$
$$\rho = 1050 \text{ kg/m}^3$$
$$g = 9.8 \text{ m/s}^2$$
$$P_A = P_{atm} + \rho g H$$
$$4.2 \times 10^6 = (101 \times 10^3) + [1050 \times 9.81 \times H]$$
$$H = 397.94 \text{ or } 398 \text{ m}$$

End of Solution

Q.11 A set of jobs A, B, C, D, E, F, G, H arrive at time $t = 0$ for processing on turning and grinding machines. Each jobs needs to be processed in sequence - first on the turning machine and second on the grinding machine, and the grinding must occur immediately after turning. The processing times of the jobs are given below

Job	A	B	C	D	E	F	G	H
Turning (minutes)	2	4	8	9	7	6	5	10
Grinding (minutes)	6	1	3	7	9	5	2	4

If the makespan is to be minimized, then the optimal sequence in which these jobs must be processed on the turning and grinding machines is
(a) G-E-D-F-H-C-A-B
(b) A-D-E-F-H-C-G-B
(c) A-E-D-F-H-C-G-B
(d) B-G-C-H-F-D-E-A

Ans. **(c)**
Sequencing,

A	②	6
B	4	①
C	8	③
D	9	⑦
E	⑦	9
F	6	⑤
G	5	②
H	10	④

A E D F H C G B

End of Solution

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Q.12 The loading the unloading response of a metal is shown in the figure. The elastic and plastic strains corresponding to 200 MPa stress, respectively are

(a) 0.01 and 0.01   (b) 0.02 and 0.02

(c) 0.01 and 0.02   (d) 0.02 and 0.01

Ans. **(d)**

Elastic strain : Which can be recovered = 0.03 − 0.01 = 0.02

Plastic strain : Permanent strain = 0.01

End of Solution

Q.13 A single jet Pelton wheel operates at 300 rpm. The mean diameter of the wheel is 2 m. Operating head and dimensions of jet are such that water comes out of the jet with a velocity of 40 m/s and flow rate of 5 m³/s. The jet is deflected by the bucket at an angle of 165°. Neglecting all losses, the power developed by the Pelton wheel is _____ MW. [Round off to two decimal places]

Ans. **2.65 (2.50 to 2.80)**

$$P = ? \qquad\qquad P = \dot{m}\left[V_{w_1} - V_{w_2}\right]u$$

$N = 300$ rpm

$D = 2$ m

$V_1 = 40$ m/s

$Q = 5$ m³/s

$\beta = 180 - 165 = 15°$

$$u = \frac{\pi DN}{60} = \frac{\pi \times 2 \times 300}{60}$$

$u_1 = u_2 = u = 31.42$ m/s

$V_{r1} = v_1 - u$

$V_{r1} = 40 - 31.42$

$V_{r1} = 8.58$

$V_{r1} = V_{r2} = 8.58$ m/s [neglecting blade friction]

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$$V_{r2}\cos\beta_2 = u_2 - V_{w2}$$
$$P = \rho Q[V_{w1} - V_{w2}]$$
$$= 2.65$$
$$P = 2.65 \text{ MW}$$

End of Solution

Q.14 A cantilever beam of length, L, and flexural ridigity, EI, is subjected to an end moment, M, as shown in the figure. The deflection of the beam at $x = \dfrac{L}{2}$ is

(a) $\dfrac{ML^2}{16EI}$

(b) $\dfrac{ML^2}{4EI}$

(c) $\dfrac{ML^2}{2EI}$

(d) $\dfrac{ML^2}{8EI}$

Ans. **(d)**

$$Y_C - Y_A = \left(\dfrac{A\overline{X}}{EI}\right)_{AC}$$

$$Y_C - 0 = \dfrac{1}{EI}\left[\dfrac{-ML}{2} \times \dfrac{L}{4}\right]$$

$$Y_C = \dfrac{ML^2}{8EI} \text{ (downward)}$$

End of Solution

Q.15 An orthogonal cutting operation is performed using a single point cutting tool with a rake angle of 12° on a lathe. During turning, the cutting force and the friction force are 1000 N and 600 N, respectively. If the chip thickness and the uncut thickness during turning are 1.5 mm and 0.75 mm, respectively, then the shear force is _____ N. [Round off to two decimal places]

Ans. **685.77 (625 to 750)**
$F_c = 1000$ N, $F = 600$ N, $t = 0.75$ mm, $t_c = 1.5$ mm, $\alpha = 12°$

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This is orthogonal turning operation

$$r = \frac{t}{t_c} = \frac{0.75}{1.5} = 0.5$$

$$\tan \phi = \frac{0.5\cos 12°}{1 - 0.5\sin 12°}$$

$\Rightarrow \qquad \phi = 28.63°$

Method - 1 : Using force relations

$$F = F_c \sin\alpha + F_t \cos\alpha$$

$$600 = 1000 \sin 12° + F_t \cos 12°$$

or $\qquad F_t = 400.85$ N

$\therefore \qquad F_s = F_c \cos\phi - F_t \sin\phi$

$\qquad\qquad = 1000 \cos 28.63° - 400.85 \sin 28.63°$

$\qquad\qquad = 685.66$ N

Method - 2 : Using Merchant circle

$$F_c = R\cos(\beta - \alpha)$$

$$F = R \sin \beta$$

or $\qquad \dfrac{F}{F_c} = \dfrac{R\sin\beta}{R\cos(\beta - \alpha)}$
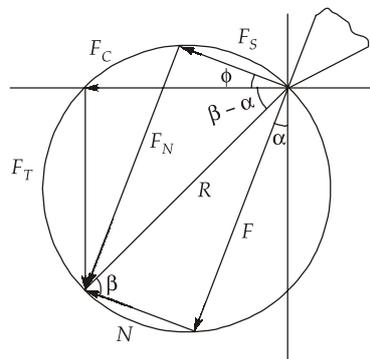
$$\frac{600}{1000} = \frac{\sin\beta}{\cos(\beta - 12)}$$

$0.6 \cos\beta \cos 12° + 0.6 \sin\beta \sin 12° = \sin\beta$

$\qquad 0.586888 \cos\beta = (1 - 0.6 \sin 12°) \sin\beta = 0.875253 \sin\beta$

$$\tan\beta = \frac{0.586888}{0.875253}$$

$\Rightarrow \qquad \beta = 33.84°$

$\therefore \qquad F = R \sin \beta$

$\qquad 600 = R \sin 33.84°$

$\Rightarrow \qquad R = 1077.44$ N

$\therefore \qquad F_s = R \cos(\phi + \beta - \alpha)$

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$$= 1077.44 \cos (28.63° + 33.84° - 12°)$$
$$= 685.77 \text{ N}$$

End of Solution

Q.16 In a machining operation, if a cutting tool traces the workpiece such that the directrix is perpendicular to the plane of the generatrix as shown in figure, the surface generated is _____
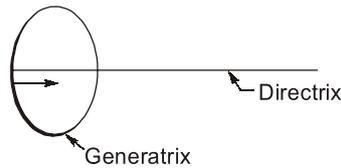
(a) spherical   (b) cylindrical
(c) surface of revolution   (d) plane

Ans. **(b)**
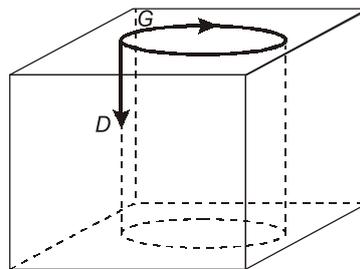
End of Solution

Q.17 Customers arrives at a shop according to the Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is
(a) 0.50   (b) 0.61
(c) 0.86   (d) 0.39

Ans. **(b)**

(i) Probability of zero in 3 min ($\lambda T = 0.5$)

$$P_1 \rightarrow P(0, 3) = \frac{(\exp)^{-0.5} \cdot (0.5)^0}{0!} = 0.606$$

(ii) Probability of 1 customer in 6 min ($\lambda T = 1$)

$$P_3 \rightarrow P(1, 6) = \frac{(\exp)^{-1} \cdot (1)^1}{1!} = e^{-1} = 0.3678$$

$$P_1 \times P_2 = P_3$$
$$0.606 \times P_2 = 0.3678$$
$$P_2 = 0.6069 \simeq 0.61$$

End of Solution

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Q.18 Consider the surface roughness profile as shown in the figure
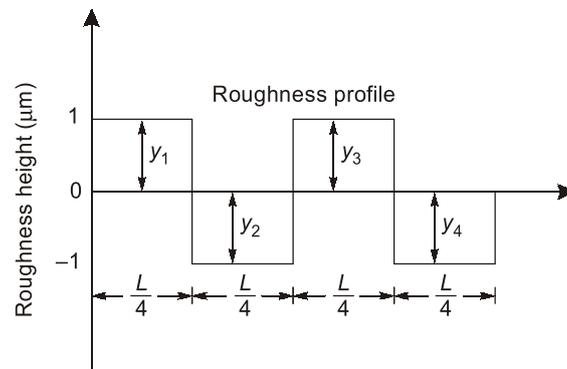
The center line average roughness (R_a in μm) of the measured length (L) is

(a) 1
(b) 4
(c) 2
(d) 0

Ans. **(a)**
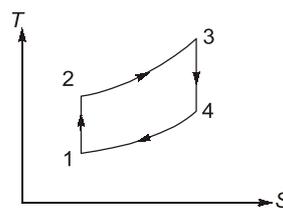
$$R_G = \frac{\sum_{i=1}^{n} y}{n} = \frac{4}{4} = 1$$

End of Solution

Q.19 In which of the following pairs of cycle, both cycles have at least one isothermal process?

(a) Brayton cycle and Rankine cycle
(b) Bell-Coleman cycle and Vapour compression refrigeration cycle
(c) Carnot cycle and Stirling cycle
(d) Diesel cycle and Otto cycle

Ans. **(c)**
1. Brayton

1 to 2 :   Isentropic
2 to 3 :   Isobaric
3 to 4 :   Isentropic
4 to 1 :   Isobaric

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2. VCRS
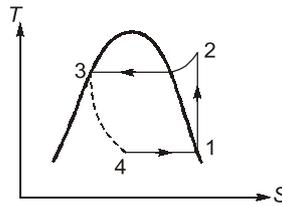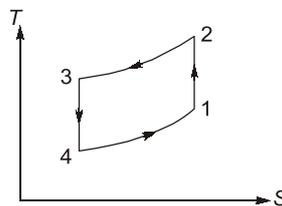

1 to 2 : Isentropic compression
2 to 3 : constant pressure ($P = C$)
3 to 4 : Isenthalpic expansion ($h_3 = h_4$)
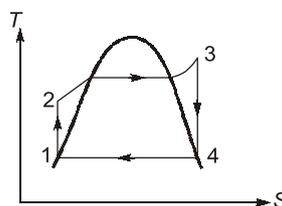4 to 1 : Constant pressure ($P = C$)

3. Carnot
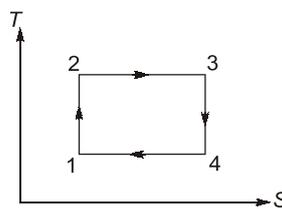
1 to 2 : Isentropic compression
2 to 3 : Isothermal heat addition
3 to 4 : Isentropic expansion
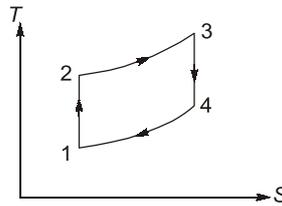4 to 1 : Isothermal heat rejection

4. Bell Coleman

1 to 2 : Isentropic compression
2 to 3 : Constant pressure
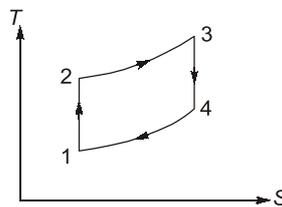3 to 4 : Isentropic expansion
4 to 1 : Constant pressure

5. Rankine

1 to 2 : Isentropic compression
2 to 3 : Constant pressure
3 to 4 : Isentropic expansion
4 to 1 : Constant pressure

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6. **Diesel**

1 to 2 : Isentropic compression
2 to 3 : Constant pressure $(P = C)$
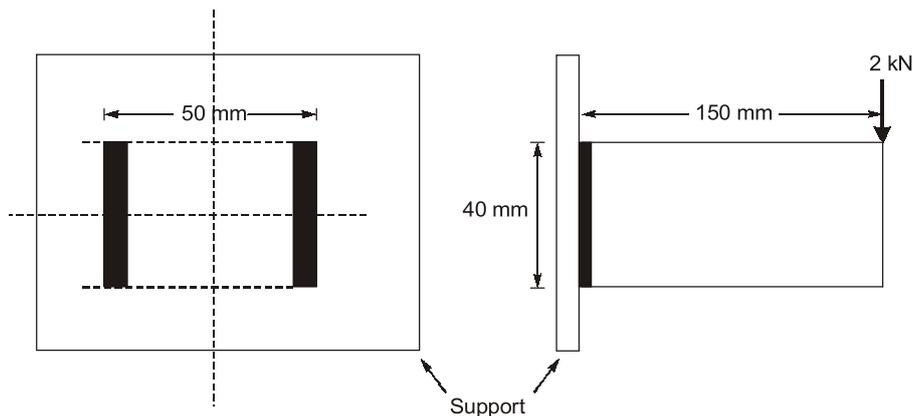3 to 4 : Isentropic expansion
4 to 1 : Constant volume

7. **Otto**

1 to 2 : Isentropic compression
2 to 3 : Constant pressure $(V = C)$
3 to 4 : Isentropic expansion
4 to 1 : Constant volume $(V = C)$

End of Solution

Q.20 A cantilever beam of rectangular cross-section is welded to a support by means of two fillet welds as shown in figure. A vertical load of 2 kN acts at free end of the beam.

Considering that the allowable shear stress in weld is 60 N/mm², the minimum size (leg) of the weld required is _____ mm [Round off to one decimal place]

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Ans. 6.65 (6.4 to 6.9)

$$\tau_{max} = \frac{2 \times 10^3}{0.707t(40) \times 2} = \frac{35.36}{t} \text{ MPa}$$

$$\sigma_{max} = \frac{M_{max}}{I_{NA}} \cdot \tau_{max} = \frac{2000 \times 150 \times 20}{\dfrac{0.707t(40)^3 \times 2}{12}}$$

$$\sigma_{max} = \frac{795.615}{t} \text{ MPa}$$

MSST, $\sqrt{\sigma_{max}^2 + 4\tau^2} \leq 2\left(\dfrac{S_{ys}}{N}\right)$

$$\sqrt{\left(\frac{795.615}{t}\right)^2 + 4\left(\frac{35.36}{t}\right)^2} \leq 2 \times 60$$

$$\frac{798.752}{t} \leq 2(60)$$

$$t = 6.65 \text{ mm}$$

End of Solution

Q.21 Consider a steam power plant operating on an ideal reheat Rankine cycle. The work input to the pump is 20 kJ/kg. The work output from the high pressure turbine is 750 kJ/kg. The work output from the low pressure turbine is 1500 kJ/kg. The thermal efficiency of the cycle is 50%. The enthalpy of saturated liquid and saturated vapour at condenser pressure are 200 kJ/kg and 2600 kJ/kg, respectively. The quality of steam at the exit of the low pressure turbine is _____ % [Round off to the nearest integer]

Ans. 93 (92 to 96)
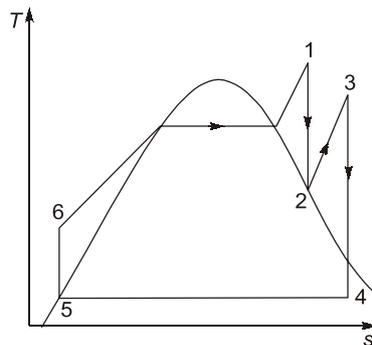
$$h_f = 200 \text{ kJ/kg}$$
$$h_g = 2600 \text{ kJ/kg}$$
$$w_p = 20 \text{ kJ/kg} = h_6 - h_5$$
$$h_1 - h_2 = 750 \text{ kJ/kg}$$
$$h_3 - h_4 = 1500 \text{ kJ/kg}$$

$$\eta = 0.5 = \frac{W_{NET}}{Q_s} = \frac{W_T - W_P}{Q_s}$$

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$$0.5 = \frac{750 + 1500 - 20}{Q_s}$$

$$Q_s = 4460 \ kJ/kg$$

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$$\Rightarrow \qquad \frac{Q_R}{Q_S} = 0.5$$

$$Q_R = 2230 \ kJ/kg$$

$$Q_R = h_4 - h_5$$

$$2230 = h_4 - 200$$

$$\Rightarrow \qquad h_4 = 2430 \ kJ/kg$$

$$h_4 = h_f + x(h_g - h_f)$$

$$2430 = 200 + x(2600 - 200)$$

$$x = 0.9291$$

$$x = 93\%$$

End of Solution

Q.22 The value of $\lim\limits_{x \to 0}\left(\dfrac{1 - \cos x}{x^2}\right)$ is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$

(c) 1   (d) $\dfrac{1}{4}$

Ans. **(a)**

$$\lim_{x \to 0}\left(\frac{1 - \cos x}{x^2}\right) = ? \qquad \left(\frac{0}{0} \ form\right)$$

$$\text{Applying L-H rule} = \lim_{x \to 0}\frac{\sin x}{2x}\left(\frac{0}{0}\right) = \lim_{x \to 0}\frac{\cos x}{2} = \frac{1}{2}$$
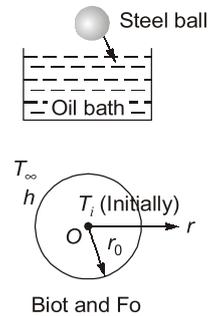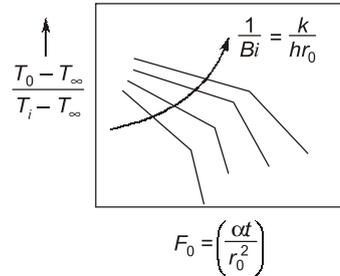
End of Solution

Q.23 A hot steel spherical ball is suddenly dipped into a low temperature oil bath.

Which of the following dimensionless parameters are required to determine instantaneous center temperature of the ball using a Heisler chart?
(a) Nusselt number and Grashoff number
(b) Reynolds number and Prandtl number
(c) Biot number and Fourier number
(d) Biot number and Froude number

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Ans. **(c)**
Unsteady state

$$\frac{T_0 - T_\infty}{T_i - T_\infty}$$ $$\frac{1}{Bi} = \frac{k}{hr_0}$$

Steel ball
Oil bath

T_∞
h
T_i (Initially)
O
r_0
r

$$F_0 = \left(\frac{\alpha t}{r_0^2}\right)$$

Biot and Fo

Q.24 In modern CNC machine tools, the backlash has been eliminated by
(a) slider crank mechanism
(b) rack and pinion
(c) ratchet and pinion
(d) preloaded ballscrews

Ans. **(d)**

Q.25 A solid sphere of radius 10 mm is placed at the centroid of a hollow cubical enclosure of side length 30 mm. The outer surface of the sphere is denoted by 1 and the inner surface of the cube is denoted by 2. The view factor F_{22} for radiation heat transfer is _____ [round off to two decimal places]

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Ans. 0.77 (0.76 to 0.78)
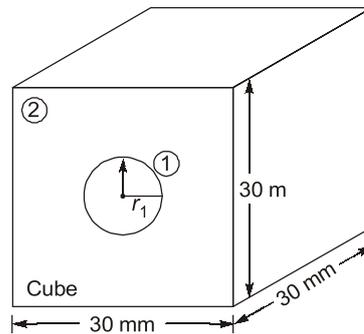
$$r_1 = 10 \text{ mm}$$
$$A_1 = 4\pi r_1^2$$
$$A_2 = 6 \times (30^2)$$
$$F_{12} = 1$$

$$A_1 F_{12} = A_2 F_{21} \quad \Rightarrow \quad F_{21} = \frac{A_1}{A_2} = \frac{4\pi \times (10)^2}{6 \times (30)^2} = 0.2327$$

$$F_{22} = 1 - F_{21} = 0.7672 \simeq 0.77$$

End of Solution

Q.26 Consider a single machine workstation to which jobs arrive according to a Poisson distribution with a mean arrival rate of 12 jobs/hour. The process time of the workstation is exponentially distributed with a mean of 4 minutes. The expected number of jobs at the workstation at any given point of time is _____ [round off to the nearest integer]

Ans. 4 (4 to 4)

$$\lambda = 12 \text{ per hour}, \quad \frac{1}{\mu} = 4 \text{ min/Jobs}, \quad \mu = 15 \text{ Jobs/hr}$$

$$\rho = \frac{12}{15} = \frac{4}{5}$$

$$L_s = \frac{\rho}{1-\rho} = \frac{\frac{4}{5}}{1-\frac{4}{5}} = \frac{\frac{4}{5}}{\frac{1}{5}}$$

$$\frac{4}{5} \times \frac{5}{1} = 4 \text{ Jobs}$$

End of Solution

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Q.27 A prismatic bar PQRST is subjected to axial loads as shown in the figure. The segments having maximum and minimum axial stresses, respectively, are
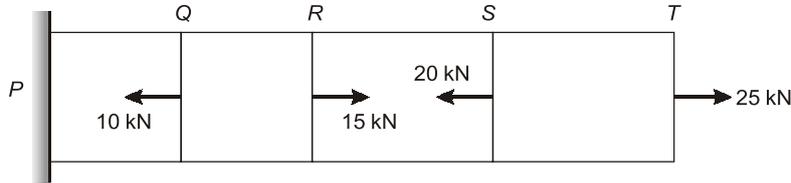
(a) ST and PQ
(b) QR and PQ
(c) ST and RS
(d) QR and RS

Ans. **(c)**
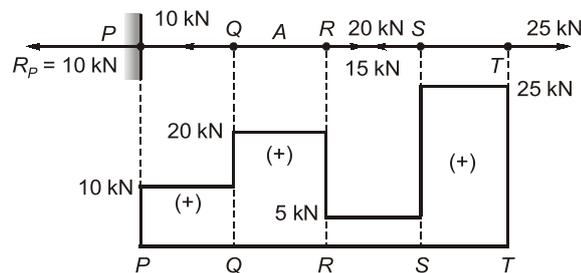
Axial Loading Diagram

$$P_{max} = P_{ST} = 25 \text{ kN}$$
$$P_{min} = P_{RS} = 5 \text{ kN}$$

Hence, maximum and minimum axial stresses are in ST and RS portions because of prismatic bar. Hence the answer is (c).

End of Solution

Q.28 Superheated steam at 1500 kPa, has a specific volume of 2.75 m³/kmol and compressibility factor (Z) of 0.95. The temperature of steam is _____ °C [Round off to the nearest integer]

(a) 249
(b) 471
(c) 522
(d) 198

Ans. **(a)**

Given: $P = 1500$ kPa, $V = 2.75$ m3/k-mol, $Z = 0.95$

$$P\bar{V} = Z \times n\bar{R}T$$

$$P\frac{\bar{V}}{n} = Z\bar{R}T$$

$$P\bar{V} = Z\bar{R}T$$

1500 KPa × 2.75 m³/K-mol = 0.95 × 8.314 kJ/K-molK × T

$$T = 522.26 \text{ K}$$
$$T = 522.26 - 273 = 249.26°C$$

End of Solution

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Q.29 A machine part in the form of cantilever beam is subjected to fluctuating load as shown in the figure. The load varies from 800 N to 1600 N. The modified endurance, yield and ultimate strengths of the material are 200 MPa, 500 MPa and 600 MPa, respectively

800 N to 1600 N   Beam cross-section

100 mm   20 mm   12 mm
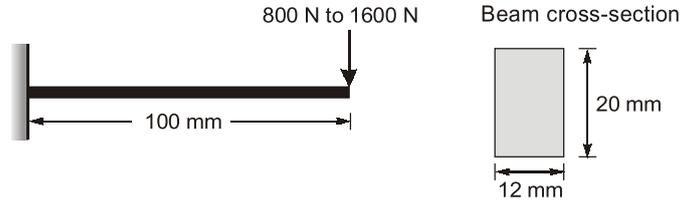
The factor of safety of the beam using modified Goodman criterion is _____ [round off to one decimal place]

Ans. **2 (1.9 to 2.1)**

1600 N to 800 N

100 mm

20 mm y 12 mm

A B

$\sigma_e = 200$ MPa
$S_{yt} = 600$ MPa
$S_{yt} = 500$ MPa
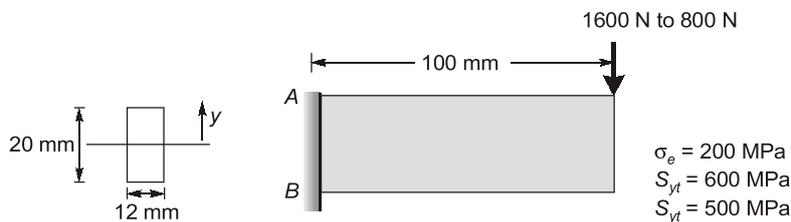
A : Critical Point

$$\sigma_{max,A} = \sigma_{b,max} \text{ at } A \text{ due to } 1600 \text{ N} = \frac{6\,M}{bd^2} = \frac{6 \times 1600 \times 200}{12 \times (2\sigma)^2}$$

$$\sigma_{max} = 200 \text{ MPa}$$

$$\sigma_{min,A} = \sigma_{b,max} \text{ at } A \text{ due to } 800 \text{ N}$$

$$= \frac{6 \times 800 \times 200}{12 \times (20)^2} = 100 \text{ MPa}$$

$$\text{Modified Goodman} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{\sigma_e} \le \frac{1}{N}$$

$$\sigma_m = \left| \frac{\sigma_{max} + \sigma_{min}}{2} \right| = 150 \text{ MPa}$$

$$\sigma_a = \left| \frac{\sigma_{max} - \sigma_{min}}{2} \right| = 50 \text{ MPa}$$

$$\frac{150}{600} + \frac{50}{200} \le \frac{1}{N}$$

$$N \le 2$$

$$N \approx 2$$

Langer, $$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \le \frac{1}{N}$$

$$\frac{150}{500} + \frac{50}{500} \le \frac{1}{N} \qquad (N \le 2.5)$$

$$N = 2.5$$
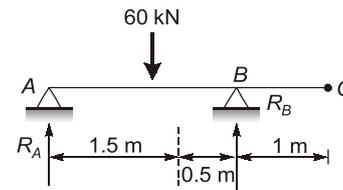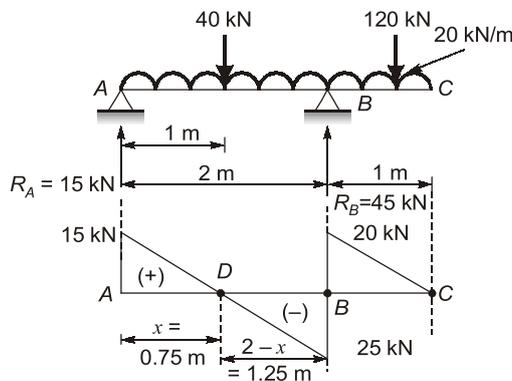
Modified Goodman = Safe result of [Goodman or Langer]

End of Solution

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Q.30 An overhanging beam PQR is subjected to uniformly distributed load 20 kN/m as shown in the figure.

Beam cross-section

20 kN/m

z --- P ------------------ Q -------- R --- z

2000 mm   1000 mm

100 mm

24 mm
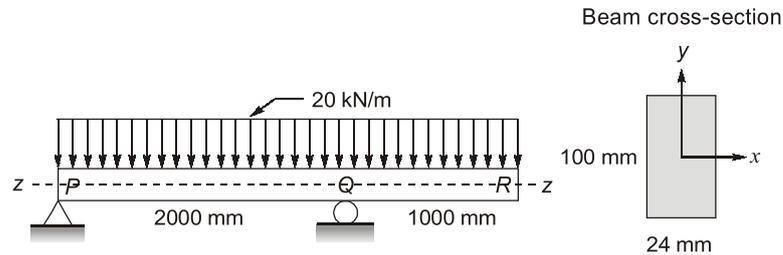
The maximum bending stress developed in the beam is _____ MPa [Round off to one decimal place]

Ans. 250 (249 to 251)

40 kN   120 kN 20 kN/m

A B C

1 m

R_A = 15 kN   2 m   1 m

R_B=45 kN

15 kN   20 kN

(+) D

A (−) B C

x = 0.75 m $2 - x$ = 1.25 m   25 kN

60 kN

A B C

R_A 1.5 m R_B

0.5 m   1 m

$$M_D - M_A = \frac{1}{2} \times 15 \times 0.75$$

$$M_D = 5.625 \text{ kN-m(s)}$$

$$\Sigma M_A = 60 \times 1.5 - R_B \times 2 = 0$$

$$R_B = 45 \text{ kN}(\uparrow); \quad R_A = 15 \text{ kN}(\uparrow)$$

Location of D:

$$\frac{15}{x} = \frac{25}{2 - x}$$

$$6 - 3x = 5x$$

$$= \frac{3}{4}\text{m}$$

$$M_C - M_B = \frac{1}{2} \times 20 \times 1$$

$$M_B = -10 \text{ kN-m}$$

$$M_B = 10 \text{ kN-m } (+1)$$

$$\text{Max. B.M.} = \text{Larger of } (M_B \text{ and } M_D)$$

$$M_B = 10 \text{ kN-m } (+1)$$

$$(\sigma_b)_{max} = \frac{M_{max}}{Z_{N.A}} = \frac{6 \times 10 \times 10^6}{24 \times 100^2} = 250 \text{ MPa}$$

End of Solution

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Q.31 Shear stress distribution on the cross-section of the coil wire in a helical compression spring is shown in the figure. This shear stress distribution represents
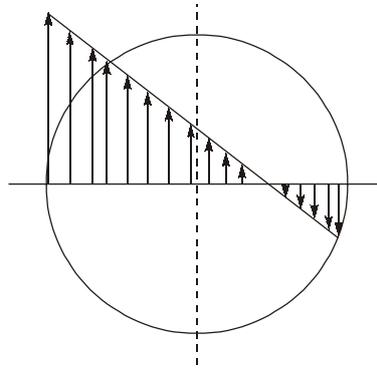
(a) Combined direct shear and torsional shear stress in the coil wire cross-section.
(b) Combined direct shear and torsional shear stress along with the effect of stress concentration at inside edge of the coil wire cross-section.
(c) Torsional shear stress in the coil wire cross-section.
(d) Direct shear stress in the coil wire cross-section.

Ans. **(a)**

$$\tau_d = \frac{W}{A} = \frac{4W}{\pi d^2}$$
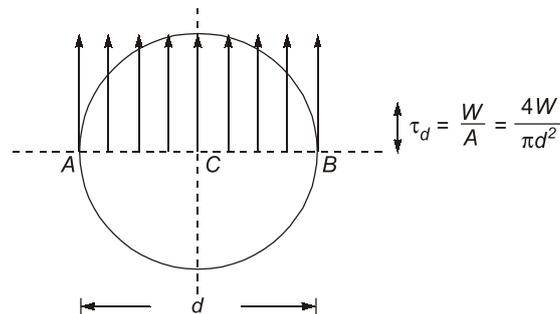
Fig. : Direct shear stress distribution across the cross-section of wire

$$(\tau_L)_{max} = \frac{WD/2}{\pi d^3/16} = \frac{8WD}{\pi d^3}$$
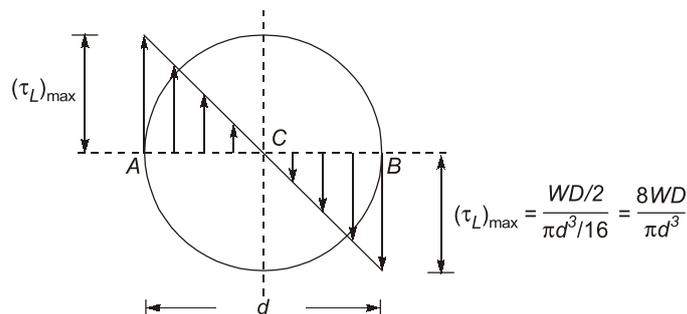
Fig. : Torsional shear stress distribution across the cross-section of wire

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$(\tau_R)_A = (\tau_t)_{Max} + \tau_d$

$(\tau_R)_B = (\tau_t)_{Max} - \tau_d$

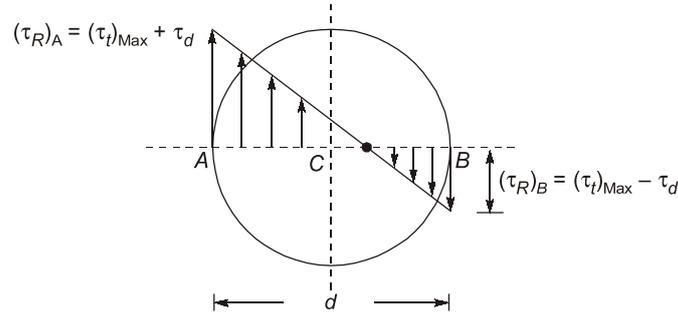**Fig. :** Resultant shear stress variation across the cross-section of wire (without considering stress concentration effect)

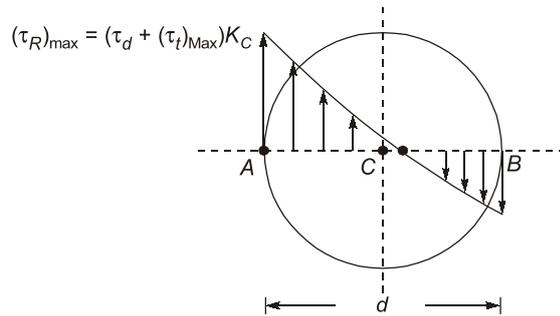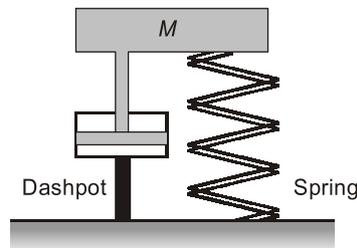$(\tau_R)_{max} = (\tau_d + (\tau_t)_{Max})K_C$

Fig. : Resultant shear stress variation considering stress concentration effect

End of Solution

Q.32 Consider a single degree of freedom system comprising a mass M, supported on a spring and a dashpot as shown in the figure.

If the amplitude of the free vibration response reduces from 8 mm to 1.5 mm in 3 cycles, the damping ratio of the system is _____ [Round off to three decimal places]

Ans.   **0.088 (0.085 to 0.090)**

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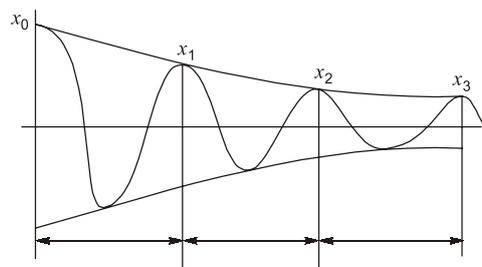
$$\frac{x_0}{x_3} = \frac{8}{1.5} = \frac{80}{15}$$

$$\frac{x_0}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} = \frac{80}{15}$$

$$e^\delta \cdot e^\delta \cdot e^\delta = \frac{80}{15}$$

$$e^{3\delta} = \ln\left(\frac{80}{15}\right) = 1.67397$$

$$\delta = 0.55799$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.55799$$

$$\xi = 0.088$$

<div align="right">**End of Solution**</div>

Q.33 Activities A, B, C and D form the critical path for a project with a PERT network. The means and variances of the activity duration for each activity are given below. All activity durations follow the Gaussian (normal) distribution, and are independent of each other.

Activity	A	B	C	D
Mean (days)	6	11	8	15
Variance (days^2)	4	9	4	9

The probability that the project will be completed within 40 days is _____. [Round off to two decimal places]
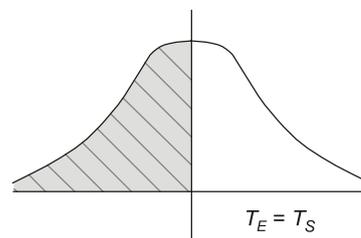
[Note : Probability is a number between 0 and 1]

Ans. **0.50 (0.5 to 0.5)**

PERT-CPM

$$T_S = 40 \text{ days}, \quad T_E = 6 + 11 + 8 + 15$$
$$T_E = 40 \text{ days},$$

$$2 = \frac{T_S - T_E}{\sigma} = 0 \quad \rightarrow 50\%$$

$$T_E = T_S$$

Probability of completing project in expected time is always 50%.

<div align="right">**End of Solution**</div>

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Q.34 The correct sequence of machining operations to be performed to finish a large diameter through hole is

(a) boring, drilling, reaming
(b) boring, reaming, drilling

(c) drilling, reaming, boring
(d) drilling, boring, reaming

Ans. **(d)**

Q.35 Consider a binomial random variable X. If $X_1, X_2, ... X_n$ are independent and identically distributed samples from the distribution of X with sum $Y = \sum_{i=1}^{n} X_i$, then the distribution of Y as $n \to \infty$ can be approximated as

(a) Exponential
(b) Bernoulli

(c) Binomial
(d) Normal

Ans. **(d)**

Q.36 The resistance spot welding of two 1.55 mm thick metal sheet is performed using welding current of 10000 A for 0.25s. The contact resistance at the interface of the metal sheets is 0.0001 Ω. The volume of weld nugget formed after welding is 70 mm^3. Considering the heat required to melt unit volume of metal is 12 J/mm^3, the thermal efficiency of the welding process is _____ %. [Round off to one decimal place]

Ans. **33.6 (30 to 36)**

$$\text{Thickness} = 1.55 \text{ mm}$$
$$I = 10000 \text{ A}, \ t = 0.25 \text{ s}$$
$$R = 0.0001 \ \Omega, \ V_n = 70 \text{ mm}^3$$
$$H_m = 12 \text{ J/mm}^3$$
$$\eta_m = ?$$
$$\eta_m = \frac{H_m}{H_s}$$
$$\text{Melting efficiency} = \text{Thermal efficiency}$$
$$\eta_m = \frac{H_m}{I^2 Rt} = \frac{12 \times 70}{(10000)^2 \times 0.0001 \times 0.25}$$
$$= 0.336$$
$$\eta_m = 33.6\%$$

Q.37 A true centrifugal casting operation needs to be performed horizontally to make copper tube sections with outer diameter of 250 mm and inner diameter of 230 mm. The value of acceleration due to gravity, g = 10 m/s^2. If a G-factor (ratio of centrifugal force to weight) of 60 is used for casting the tube, the rotational speed required is _____ rpm. [Round off to the nearest integer]

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Ans. **662 (660 to 664)**

Given: $D_0 = 250$ mm; $D_i = 230$ mm; $g = 10$ m/s^2; G-factor $= 60$

$$F_c = mr\omega^2$$
$$F_g = mg$$

$$\text{G-factor} = \frac{F_c}{F_g} = 60$$

$$\text{G-factor} = \frac{F_c}{F_g} = \frac{mr\omega^2}{mg} = \frac{D_0}{2g}\left(\frac{2\pi N}{60}\right)^2$$

$$N = \sqrt{\frac{2 \times \text{G-Factor} \times g}{D_0}} \times \left(\frac{60}{2\pi}\right)$$

$$N = \sqrt{\frac{2 \times 60 \times 10}{0.25}} \times \left(\frac{60}{2\pi}\right)$$

$$N = 661.93$$
$$N = 662 \text{ rpm}$$

End of Solution

Q.38 A tappet valve mechanism in an IC engine comprises a rocker arm ABC that is hinged at B as shown in the figure. The rocker is assumed rigid and it oscillates about the hinge B. The mass moment of inertia of the rocker about B is 10^{-4} kgm^2. The rocker arm dimensions are $a = 3.5$ cm and $b = 2.5$ cm. A pushrod pushes the rocker at location A, when moved vertically by a cam that rotates at N rpm. The pushrod pushes the rocker at location A, when moved vertically by a cam that rotates at N rpm. The pushrod is assumed massless and has a stiffness of 15 N/mm. At the other end C, the rocker pushes a valve against a spring of stiffness 10 N/mm. The valve is assumed massless and rigid.
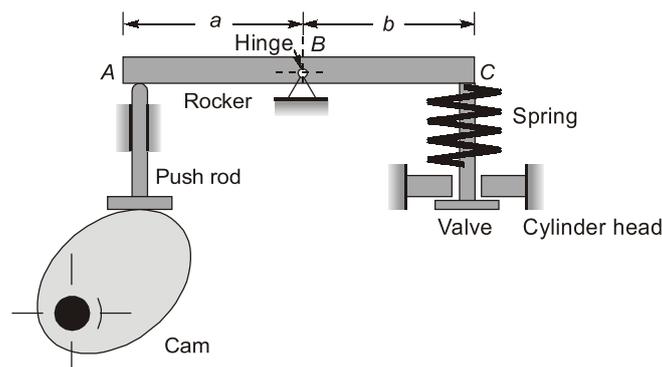
Resonance in the rocker system occurs when the cam shaft runs at a speed of _____ rpm. [Round off to the nearest integer]
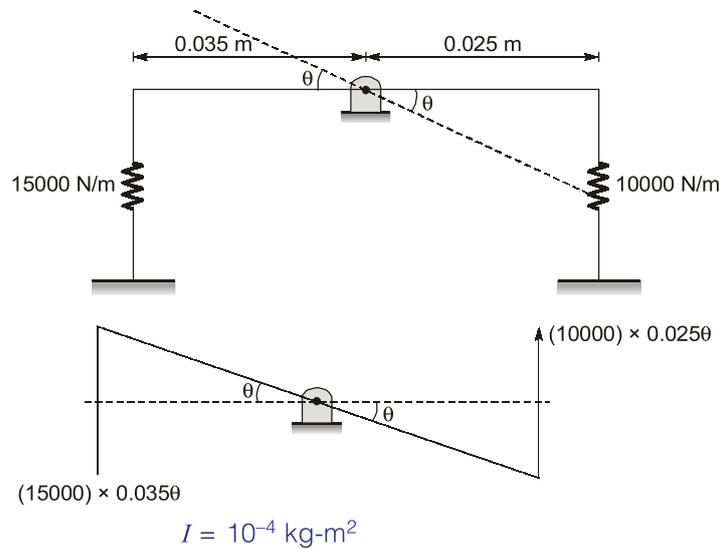
(a) 2369   (b) 790
(c) 496   (d) 4739

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Ans. (d)

$$I = 10^{-4} \text{ kg-m}^2$$

By D'Alembert Principle

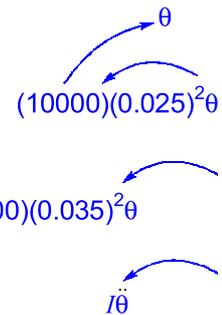$$I\ddot{\theta} + [10000 \times (0.025)^2 + 15000 \times (0.035)^2]\theta = 0$$

$$(10^{-4})\ddot{\theta} + (24.625)\theta = 0$$

$$\ddot{\theta} + \left(\frac{24.625}{10^{-4}}\right)\theta = 0$$
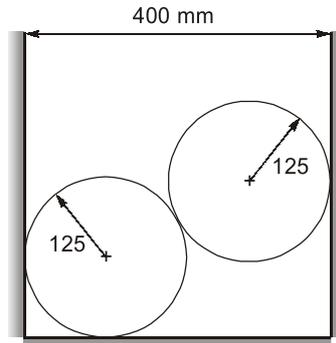
$\Rightarrow$ $\omega_n^2 = (246250)$

$\omega_n = 496.2358 \text{ rad/s}$

$\Rightarrow$ $N_C = \dfrac{496.2358 \times 60}{2\pi} = 4738.7031 \text{ r.p.m.}$

$(10000)(0.025)^2\theta$

$(15000)(0.035)^2\theta$

$I\ddot{\theta}$

End of Solution

Q.39 Two smooth identical spheres each of radius 125 mm and weight 100 N rest in a horizontal channel having vertical walls. The distance between vertical walls of the channel is 400 mm.

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400 mm

125

125

All dimensions are in mm

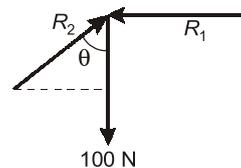The reaction at the point of contact between two spheres is _____ N. [Round off to end one decimal place]

Ans.   **125 (124 to 126)**

400 mm

B   125

θ

A   θ

125   150   C
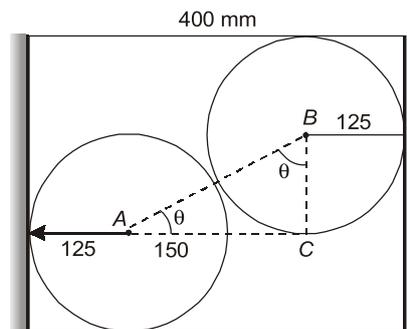
$$BC = 250^2 - 150^2$$

$$\cos\theta = \frac{200}{250}$$

$$\theta = 36.869°$$
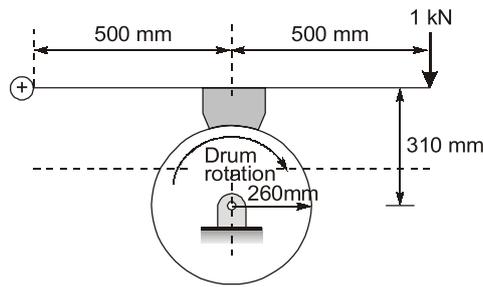

R_2 R_1

θ

100 N

$$R_2 \cos\theta = 100$$

$$R_2 = \frac{100 \times 250}{200} = 125 \text{ N}$$

End of Solution

Q.40   A short shoe drum (radius 260 mm) brake is shown in the figure. A force of 1 kN is applied to the lever. The coefficient of friction is 0.4.

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The magnitude of the torque applied by the brake is _____ N.m. [Round off to one decimal place]

Ans.   **200 (199 to 201)**
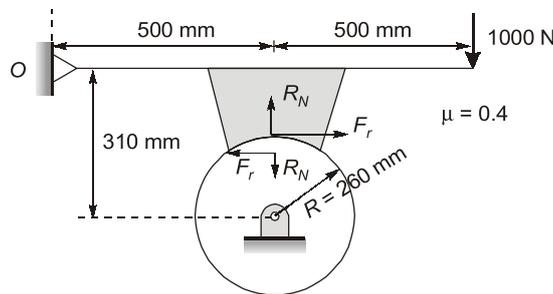
Taking moment about 'O'

$R_N(500) + F_Y[310 - 260] - 1000 \times 1000 = 0$

$R_N(500) + 0.4(R_N)(50) - 1000 \times 1000 = 0$

$R_N = 1923.076 \text{ N}$

$F_r = \mu R_N = 769.23 \text{ N}$

$T_f = F_r \times R = 200 \text{ N-m}$

End of Solution

Q.41 Consider a reciprocating engine with crank radius R and connecting rod of length L. The secondary unbalance force for this case is equivalent to primary unbalance force due to a virtual crank of _____.

(a) radius $\dfrac{R^2}{4L}$ rotating at twice the engine speed

(b) radius $\dfrac{L}{2}$ rotating at twice the engine speed

(c) radius $\dfrac{R}{4}$ rotating at half the engine speed

(d) radius $\dfrac{L^2}{4R}$ rotating at half the engine speed

Ans.   **(a)**

End of Solution

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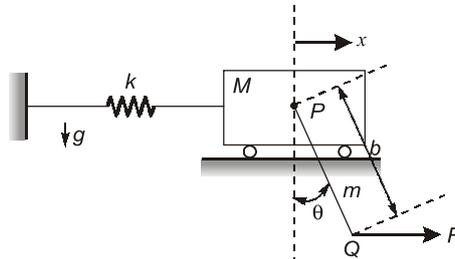14-02-2021
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Q.42 Consider a two degree of freedom system as shown in the figure, where PQ is a rigid uniform rod of length, b and mass, m.
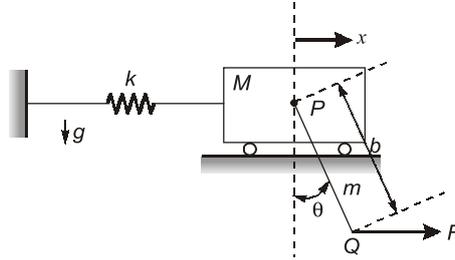
Assume that the spring deflects only horizontally and force F is applied horizontally at Q. For this system, the Lagrangian, *L* is

(a) $\frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$

(b) $\frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$

(c) $\frac{1}{2}M\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta + Fb\sin\theta$

(d) $\frac{1}{2}M\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2$
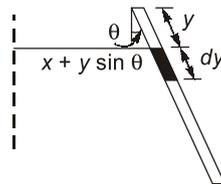
Ans. **(a)**

For mass, *M* $T = \frac{1}{2}M\dot{x}^2$

$V = \frac{1}{2}Kx^2$

For mass, *m*

$dm = \frac{m}{b}dy$

Displacement = $x + y\sin\theta$

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Velocity $= \dot{x} + y\cos\theta\dot{\theta}$

$$dT = \frac{1}{2}dMVel^2 = \frac{1}{2}dm\left[\dot{x}^2 + y^2\dot{\theta}^2\cos^2\theta + 2\dot{x}\dot{\theta}y\cos\theta\right]$$

$$T = dT = \frac{1}{2}\frac{m}{b}\int(\dot{x}^2 + y^2\dot{\theta}^2 \times 1 + 2\dot{x}\dot{\theta}y\cos\theta)\,dy$$

$$T = \frac{1}{2}\frac{m}{b}\left(\dot{x}^2 b + \frac{b^3}{3}\dot{\theta}^2 + 2\dot{x}\dot{\theta}\cos\theta\frac{b^2}{2}\right)$$

$$T = \frac{1}{2}\frac{m}{b}\left(\dot{x}^2 b + \frac{b^3}{3}\dot{\theta}^2 + \dot{x}\dot{\theta}b^2\cos\theta\right)$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{mb^2}{6}\dot{\theta}^2 + \frac{m}{2}\dot{x}\dot{\theta}b\cos\theta$$

$$V = -mg\frac{b}{2}\cos\theta$$

For both masses, M and m

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{mb^2}{6}\dot{\theta}^2 + \frac{m}{2}b\dot{\theta}\dot{x}^2\cos\theta$$

$$V = \frac{1}{2}kx^2 - mg\frac{b}{2}\cos\theta$$
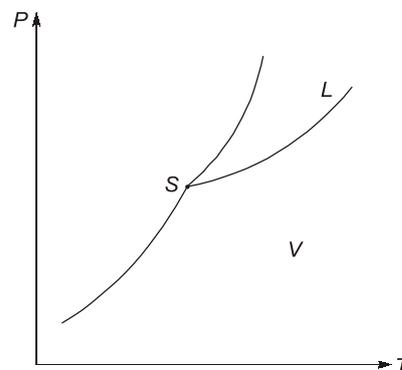
$$L = T - V$$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{mb^2\dot{\theta}^2}{6} - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$$

End of Solution

Q.43 In the vicinity of the triple point, the equation of liquid-vapour boundary in the P-T phase diagram for ammonia is ln P = 24.38 – 3063/T, where P is pressure (in Pa) and T is temperature (in K). Similarly, the solid-vapour boundary is given by ln P = 27.92 – 3754/T. The temperature at the triple point is _____ K. [Round off to one decimal place]

Ans. 195.2 (195.1 to 195.3)

Liquid vapour, $\quad \ln P = 24.38 - \dfrac{3063}{T}$

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Solid vapour, $\ln P = 27.92 - \dfrac{3754}{T}$

At triple point temperature of solid, liquid and vapour is same.

∴ equating

$$24.38 - \dfrac{3063}{T} = 27.92 - \dfrac{3754}{T}$$

Multiplying by T on both sides

$$24.38T - 3063 = 27.92T - 3754$$
$$3.54T = 691$$
$$T = 195.197 \text{ K}$$

<div style="text-align:right">**End of Solution**</div>

Q.44 Let $f(x) = x^2 - 2x + 2$ be a continuous defined on $x \in [1, 3]$. The point x at which the tangent of $f(x)$ becomes parallel to the straight line joining f(1) and f(3) is

(a) 1   (b) 2

(c) 0   (d) 3

Ans. **(b)**

By Lagrangian mean value theorem,

$$\dfrac{f(3) - f(1)}{3 - 1} = f'(c)$$

$\Rightarrow$ $\dfrac{5 - 1}{3 - 1} = 2x - 2$

$\Rightarrow$ $2x - 2 = 4$

$\Rightarrow$ $x = 2 \in (1, 3)$

<div style="text-align:right">**End of Solution**</div>

Q.45 The fundamental thermodynamic relation for a rubber band is given by dU = TdS + τdL, where T is the absolute temperature, S is the entropy, τ is the tension in the rubber band, and L is the length of the rubber band. Which one of the following relations is CORRECT?

(a) $T = \left(\dfrac{\partial U}{\partial S}\right)_\tau$ (b) $T = \left(\dfrac{\partial U}{\partial S}\right)_L$

(c) $\left(\dfrac{\partial T}{\partial S}\right)_L = \left(\dfrac{\partial \tau}{\partial L}\right)_S$ (d) $\left(\dfrac{\partial T}{\partial L}\right)_S = \left(\dfrac{\partial \tau}{\partial S}\right)_L$

Ans. **(d)**

$$dU = Tds + \tau dL$$

T = Absolute temperature, S = Entropy, τ = Tension, L = Length

Comparing with $dz = Mdx + Ndy$

$$M = \left(\dfrac{\partial z}{\partial x}\right)_y$$

$$N = \left(\dfrac{\partial z}{\partial y}\right)_x$$

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Since z is exact differential

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Since U is exact differential

$$T = \left(\frac{\partial U}{\partial S}\right)_L \qquad\qquad \text{Since } M = \left(\frac{\partial z}{\partial x}\right)_y$$

$$T = \left(\frac{\partial U}{\partial L}\right)_S \qquad\qquad \text{Since } N = \left(\frac{\partial z}{\partial y}\right)_x$$

$$\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$$

End of Solution

Q.46 Robot Ltd. wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time period is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units.

Using $\phi^{-1}(0.95) = 1.64$, where ϕ represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfilment probability for the lead time period is _____ units. [Round off to two decimal places]

Ans. **36.67 (36.5 to 36.8)**

$$\text{Safety stock} = Z \times \sigma$$

$$\sigma = \sqrt{10^2 + 10^2 + 10^2 + 10^2 + 10^2} = 10\sqrt{5}$$

$$= 22.3606$$

$$\text{Safety stock} = 1.64 \times 22.3606$$

$$= 36.67$$

End of Solution

Q.47 The relative humidity of ambient air at 300 K is 50% with a partial pressure of water vapour equal to p_v. The saturation pressure of water at 300 K is p_{sat}. The correct relation for the air-water mixture is

(a) $p_v = 0.622\ p_{sat}$ (b) $p_v = 2\ p_{sat}$

(c) $p_v = 0.5\ p_{sat}$ (d) $p_v = p_{sat}$

Ans. **(c)**

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{sat}}$$

$$0.5 = \frac{p_v}{p_{sat}}$$

$$p_v = 0.5\ p_{sat}$$

End of Solution

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Q.48 The figure shows an arrangement of a heavy propeller shaft in a ship. The combined polar mass moment of inertia of the propeller and the shaft is 100 kg.m². The propeller rotates at ω = 12 rad/s. The waves acting on the ship hull induces a rolling motion as shown in the figure with an angular velocity of 5 rad/s. The gyroscopic moment generated on the shaft due to the motion described is _____ N.m. [Round off to the nearest integer]
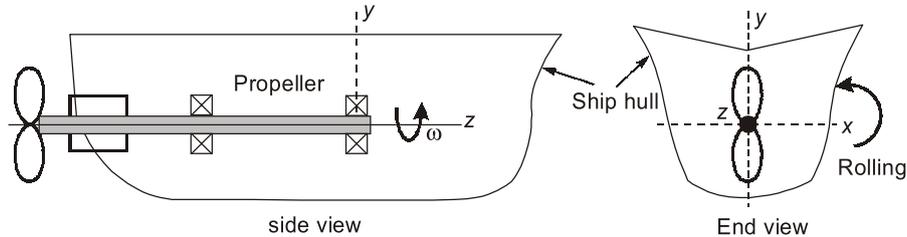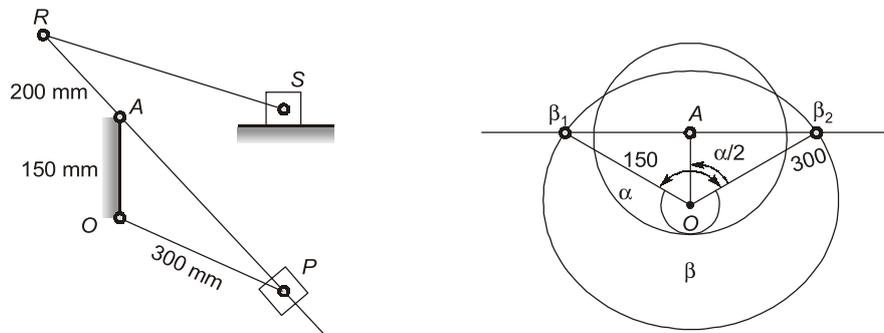

Ans. 0 (0 to 0)

End of Solution

Q.49 The Whitworth quick return mechanism is shown in the figure with link lengths as follows: OP = 300 mm, OA = 150 mm, AR = 160 mm, RS = 450 mm.
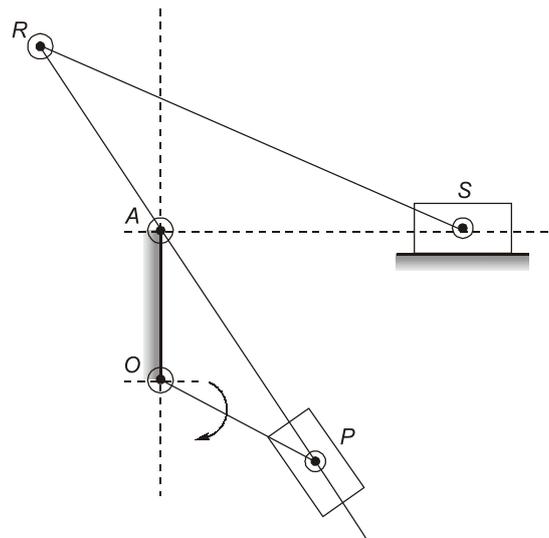
The quick return ratio for the mechanism is _____. [Round off to one decimal place]

Ans. 2 (1.9 to 2.1)

$$\cos\frac{\alpha}{2} = \frac{150}{300} = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{\alpha}{2} = 60°$$

$$\alpha = 120°$$

$$\beta = (360° - 120°) = 240°$$

$$QRR = \left(\frac{\beta}{\alpha}\right) = \frac{240}{120} = 2$$

End of Solution

Q.50 A rigid insulated tank is initially evacuated. It is connected through a valve to a supply line that carries air at a constant pressure and temperature of 250 kPa and 400 K respectively. Now the valve is opened and air is allowed to flow into the tank until the pressure inside the tank reaches to 250 kPa at which point the valve is closed. Assume that the air behaves as a perfect gas with constant properties (c_p = 1.005 kJ/kgK, c_v = 0.718 kJ/kgK, R = 0.287 kJ/kgK). Final temperature of the air inside the tank is _____ K. [Round off to one decimal place]

Ans. 559.9 (555 to 565)

P_i = 1500 kPa T_i = 400 K
Control Volume
P_2 = 1500 kPa
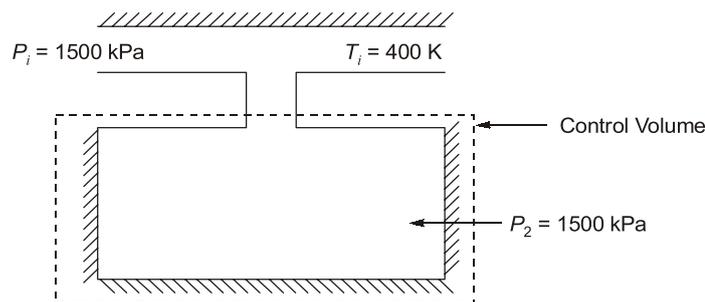
Let, 1 = Initial condition of tank

2 = Final condition of tank

Applying unsteady state energy equation

$$U_2 - U_1 = m_i h_i + Q - m_e h_e - W_{CV}$$

$$U_1 = 0, \ Q = 0, \ m_e h_e = 0, \ W_{CV} = 0$$

$$U_2 = m_i h_i$$

$$m_2 u_2 = (m_2 - m_1) \times h_i$$

$$u_2 = h_i \qquad\qquad (\because m_1 = 0)$$

$$C_V T_2 = C_P T_i$$

$$T_2 = \frac{C_P}{C_V} T_i$$

$$T_2 = \left(\frac{1.005}{0.718}\right) \times 400$$

$$T_2 = 559.9 \ K$$

End of Solution

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Q.51 Let C represent the unit circle centered at origin in the complex plane, and complex variables, $z = x + iy$. The valve of the contour integral $\oint_C \dfrac{\cosh 3z}{2z} dz$ (where integration is taken counter clockwise) is

(a) πi (b) $2\pi i$

(c) 0   (d) 2

Ans. **(a)**

Pole of $f(c)$ is $z = 0$ simple pole.

Reduce at $z = 0$,

$$\mathop{R_b}_{\tau=0} f(z) = \lim_{z \to 0} (z - 0)f(z) = \lim_{z \to 0} \frac{\cosh(3z)}{2} = \lim_{z \to 0} \frac{e^{3z} + e^{-3z}}{2 \times 2} = \frac{1}{2}$$

By Cauchy Riemann theorem,

$$I = 2\pi \left(\frac{1}{2} \right) = \pi i$$

End of Solution

Q.52 A 200 mm wide plate having a thickness of 20 mm is fed through a rolling mill with two rolls. The radius of each roll is 300 mm. The plate thickness is to be reduced to 18 mm in one pass using a roll speed of 50 rpm. The strength coefficient (K) of the work material flow curve is 300 MPa and the strain hardening exponent, n is 0.2. The coefficient of friction between the rolls and the plate is 0.1. If the friction is sufficient to permit the rolling operation then the roll force will be _____ kN. [Round off to the nearest integer]

Ans. **780.309 (775 to 784)**

$$b = 200 \text{ mm}, \; h_I = 20 \text{ mm}, \; h_F = 18 \text{ mm}, \; r = 300 \text{ mm}$$

$$\therefore \qquad \sigma_{mf} = \frac{k \varepsilon^n}{n+1}$$

$$\because \qquad \varepsilon = \ln\frac{h_F}{h_I} = \ln\left(\frac{18}{20}\right) = -0.105 = 0.105$$

$$\because \qquad \sigma_{mf} = \frac{300(0.105)^{0.2}}{0.2+1} = 159.28 \text{ MPa}$$

$$\because \qquad L_P = \sqrt{R\Delta h} = \sqrt{300 \times 2} = 24.49 \text{ mm}$$

$$A_P = b \times L_P = 200 \times 24.49$$
$$= 4898.97 \text{ mm}^2$$
$$F = \sigma_{mf} \times A_P$$
$$= 159.28 \times 4898.97$$
$$= 780309.45 \text{ N}$$
$$= 780.309 \text{ kN}$$

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Method II:

Ans. 956 (950 to 960)

$$h_{\text{avg}} = \frac{h_0 + h_f}{2} = \frac{20 + 18}{2} = 19 \text{ mm}$$

$$L = \sqrt{R\Delta h} = \sqrt{300(20 - 18)} = 24.49 \text{ mm}$$

$$\therefore \qquad \Delta = \frac{h_{avg}}{L} = \frac{19}{24.49} = 0.77 < 1$$

Therefore friction is important

Now, $$\varepsilon_f = \ln\left(\frac{h_f}{h_0}\right) = \ln\left(\frac{18}{20}\right) = -0.1054$$
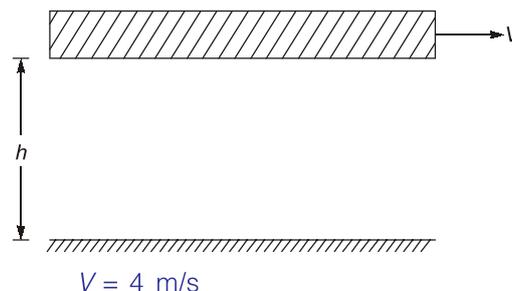
$$= 0.01054 \text{ (compression)}$$

$$\overline{\sigma}_0 = \frac{K\varepsilon_f^n}{1+n} = \frac{300 \times (0.1054)^{0.1054}}{1 + 0.2}$$

$$= 159.4 \text{ MPa (compression)}$$

$$F = 1.15\sqrt{R\Delta h} \times b \times \overline{\sigma}_0 \times \left(1 + \frac{\mu\sqrt{R\Delta h}}{h_0 + h_f}\right)$$

$$= 1.15\sqrt{300 \times 2} \times 200 \times 159.4 \times \left(1 + \frac{0.1 \times \sqrt{300 \times 2}}{20 + 18}\right)$$

$$= 955.9 \text{ kN}$$

End of Solution

Q.53 Consider fully developed, steady state incompressible laminar flow of a viscous fluid between two large parallel horizontal plates. The bottom plate is fixed and the top plate moves with a constant velocity of U = 4 m/s. Separation between the plates is 5 mm. There is no pressure gradient in the direction of flow. The density of fluid is 800 kg/m³, and the kinematic viscosity is 1.25 × 10⁻⁴ m²/s. The average shear stress in the fluid is _____ Pa. [Round off to the nearest integer]

Ans. 80 (79 to 81)

V = 4 m/s

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$$\rho = 800 \text{ kg/m}^3$$
$$\nu = 1.25 \times 10^{-4} \text{ m}^2/\text{s}$$
$$h = 5 \text{ mm}$$
$$\tau = \mu \cdot \frac{du}{dy}$$
$$\tau = [800 \times 1.25 \times 10^{-4}] \times \frac{4}{5 \times 10^{-3}}$$
$$\tau = 80 \text{ N/m}^2$$
$$= 80 \text{ Pa}$$

End of Solution

Q.54 The Dirac-delta function $\left(\delta\left(t - t_0\right)\right)$ for t, $t_0 \in \mathbb{R}$, has the following property

$$\int_a^b \phi(t)\delta\left(t - t_0\right)dt = \begin{cases} \phi(t_0) & a < t_0 < b \\ 0 & \text{otherwise} \end{cases}$$

The Laplace transform of the Dirac-delta function $\delta(t - a)$ for $a > 0$; $\mathcal{L}\left(\delta(t - a)\right) = F(s)$ is

(a) e^{sa} (b) ∞

(c) e^{-sa} (d) 0

Ans. **(c)**

$$\because \quad \int_0^\infty f(t)\delta(t - a)dt = f(a)$$

$$\therefore \quad L\left\{\delta(t - a)\right\} = \int_0^\infty e^{-st}\delta(t - a)dt = e^{-as}$$

End of Solution

Q.55 The ordinary differential equation $\frac{dy}{dt} = -\pi y$ subject to an initial condition $y(0) = 1$ is

solved numerically using the following scheme:

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

where h is the time step, $t_n = $ nh, and $n = 0, 1, 2, \ldots\ldots$ This numerical scheme is stable for all values of h in the interval _____.

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(a) $0 < h < \dfrac{2}{\pi}$ (b) for all $h > 0$

(c) $0 < h < 1$ (d) $0 < h < \dfrac{\pi}{2}$

Ans.   **(a)**

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

$$y_{n+1} = -\pi h y_n + y_n = \left(-\pi h + 1\right) y_n$$

It is recursion relation between y_{n+1} and y_n
so solution will be stable if

` $\left| -\pi h + 1 \right| < 1$

$$-1 < -\pi h + 1 < 1$$
$$-2 < -\pi h < 0$$
$$0 < \pi h < 2$$

$$0 < h < \frac{2}{\pi}$$

Therefore option (a) is correct.

End of Solution