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UPPSC AE 2019
ASSISTANT ENGINEER

ELECTRICAL ENGINEERING

Test 2

Part Syllabus Test-2

Control Systems

ANSWER KEY

1.	(b)	**11.**	(c)	**21.**	(d)	**31.**	(c)	**41.**	(b)
2.	(b)	**12.**	(d)	**22.**	(d)	**32.**	(b)	**42.**	(c)
3.	(a)	**13.**	(d)	**23.**	(d)	**33.**	(c)	**43.**	(a)
4.	(a)	**14.**	(c)	**24.**	(a)	**34.**	(b)	**44.**	(b)
5.	(a)	**15.**	(c)	**25.**	(d)	**35.**	(c)	**45.**	(a)
6.	(d)	**16.**	(d)	**26.**	(a)	**36.**	(c)	**46.**	(c)
7.	(d)	**17.**	(d)	**27.**	(d)	**37.**	(c)	**47.**	(d)
8.	(b)	**18.**	(d)	**28.**	(c)	**38.**	(b)	**48.**	(c)
9.	(c)	**19.**	(b)	**29.**	(d)	**39.**	(c)	**49.**	(c)
10.	(d)	**20.**	(c)	**30.**	(c)	**40.**	(c)	**50.**	(c)

DETAILED EXPLANATIONS

1. (b)

The signal flow graph shown above has two forward paths and five loops.

Forward paths:

$$M_1 = G_1G_2G_3 \qquad \Delta_1 = 1$$
$$M_2 = G_3G_4 \qquad \Delta_2 = 1$$

Loops:

$$L_1 = -G_1H_1\,; \qquad L_2 = -G_2H_2$$
$$L_3 = G_4H_2H_1\,; \qquad L_4 = -G_4G_3H_3$$
$$L_5 = -G_1G_2G_3H_3$$

By applying Mason's gain formula,

$$\frac{x_6}{x_1} = \frac{G_1G_2G_3 + G_4G_3}{1 + G_1H_1 + G_2H_2 + G_4G_3H_3 + G_1G_2G_3H_3 - G_4H_2H_1}$$

2. (b)

The type of system is determined from the number of poles at origin for open loop transfer function.

3. (a)

$$\text{Transfer function} = \frac{K}{s^2 + 10s + K}$$
$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$
$$\omega_n = \sqrt{K}$$
$$2\,\xi\omega_n = 10$$
$$K = 100$$

The settling time for 2% criteria is,

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5 \times 10} = 0.8 \text{ sec}$$

4. (a)

Both phase margin and gain margin are positive for stable system.

5. (a)

The negative feedback improves stability and the sensitivity to the system is decreased. The sensitivity to the gain of the system reduces by a factor of $(1 + G(j\omega)\,H(j\omega))$.

6. (d)

First line having a slope of +20 dB/dec, therefore there is a term s in the numerator.

$$\text{T.F.} = \frac{Ks}{\frac{1}{10}(s+1)(s+10)} = \frac{K \cdot 10s}{(s+1)(s+10)}$$

To find gain K :

$$20 \log_{10}(1) + 20 \log_{10} K = 6$$
$$0 + 20 \log_{10} K = 6$$
$$K = 10^{0.3}$$

$$\therefore \quad \text{T.F.} = \frac{10^{1.3}(s)}{(s+1)(s+10)}$$

7. **(d)**

Phase difference between input and output = 30°

$$\omega = 2 \text{ rad/s}$$
$$\angle\text{T.F.} = 90° - \tan^{-1}\left(\frac{\omega}{P}\right) = 30°$$
$$\tan 60° = \frac{\omega}{P}$$
$$P = \frac{2}{\sqrt{3}}$$

8. **(b)**

The corresponding value of ω is found out from auxiliary equation in Routh array.

9. **(c)**

The introduction of time delay element decreases both phase margin and gain margin.

10. **(d)**

Intersection of asymptotes i.e.,

$$\text{Centroid} = \frac{\Sigma(\text{Real part of poles}) - \Sigma(\text{Real part of zeros})}{\text{No of poles} - \text{No. of zeros}}$$
$$= \frac{(-2-4-8)-(-5)}{4-1} = \frac{-14+5}{3} = \frac{-9}{3} = -3$$

11. **(c)**

The number of root-locus segments ending at infinity are equal to n-m, where

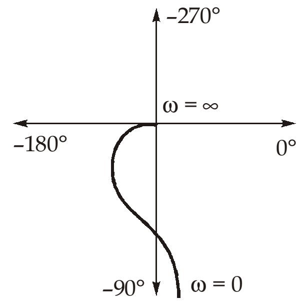
n = number of open-loop poles

and m = number of open-loop zeros

12. **(d)**

$$G(j\omega) = \frac{1+j4\omega}{j\omega(1+j\omega)(1+2j\omega)}$$
$$|G(j\omega)| = \frac{\sqrt{1+16\omega^2}}{\omega\cdot\sqrt{1+\omega^2}.\sqrt{1+4\omega^2}}$$
$$\angle G(j\omega) = \tan^{-1}(4\omega) - 90° - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

	$\omega = 0$	$\omega = \infty$
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	$-90°$	$-180°$



14. (c)

Method I: (Analytical method)

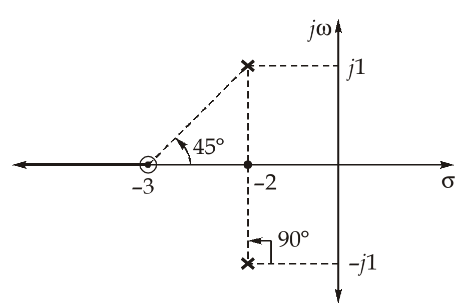
$$\arg\left|G(s)H(s)\right|_{s=-2+j1} = \arg\left|\frac{K(s+3)}{(s+2+j1)}\right|_{s=-2+j1}$$

$$= \arg\left|\frac{K(1+j1)}{2j}\right| = \frac{\angle 45°}{\angle 90°} = \angle -45°$$

$$\theta_d = 180° + \arg\left|G(s)H(s)\right| \text{ for poles}$$

$$\theta_d = 180° - 45° = 135°$$

Method II: (Graphical method)



$$\theta_d = 180° - (90° - 45°)$$
$$= 135°$$

15. (c)

The total number of root locus branches which tends to infinity is $P - Z$.

16. (d)

Of the four choices, the first two are ruled out because they possess pole/zero in the right half of plane.

For $\dfrac{e^{-3s}}{s}$,

$$\Rightarrow \qquad \frac{e^{-3j\omega}}{j\omega} = -3\omega - \frac{\pi}{2}$$

For minimum phase,

$$\angle F(j\omega)\big|_{\omega=\infty} = -(P-Z)\frac{\pi}{2}$$

The above function is not minimum phase function because

at $\omega = \infty$, $\angle F(j\omega) = -3(\infty) - \frac{\pi}{2} = -\infty$ radian $\neq -(P-Z)\frac{\pi}{2}$

For $\dfrac{s}{(s+1)(s+2)}$,

$$\Rightarrow \quad \frac{j\omega}{(j\omega+1)(j\omega+2)}$$

$$\angle F(j\omega) = 90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$\angle F(j\omega)\big|_{\omega=\infty} = 90° - 90° - 90° = -90°$$

$$= -(P-Z)\frac{\pi}{2}$$

Hence option (d) is correct.

17. (d)

(i) When transfer function has at least one pole or zero in the RHS of s-plane, it is called **non-minimum phase transfer function.**

(ii) When transfer function has no pole or zero in the RHS of s-plane. It is called **minimum phase transfer function.**

18. (d)

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$|Q_C| = \left|\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\right| = 0$$

Hence system is not controllable

Observability:

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$CA = \begin{bmatrix} -1 & -1 \end{bmatrix}$$

$$|Q_0| = \left|\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\right| = 0$$

Hence system is not observable.

19. (b)

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 \\ 0 & s+3 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\text{Adj}(sI - A)}{|sI - A|} = \frac{\begin{bmatrix} s+3 & 1 \\ 0 & s \end{bmatrix}}{s(s+3)}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

Taking inverse Laplace transform

$$\phi(t) = \begin{bmatrix} 1 & \frac{1}{3} - \frac{1}{3}e^{-3t} \\ 0 & e^{-3t} \end{bmatrix}$$

20. (c)

Maximum phase shift occurs at,

$$\omega_{max} = \sqrt{\omega_1 \times \omega_2}$$

$$= \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec}$$

21. (d)

For the given signal flow graph

Forward paths:

$$P_1 = G_1G_2G_3$$

gains $\quad P_2 = G_3G_4$

Number of possible forward paths: 2

Individual loop gain:

$$L_1 = H_1H_2G_3G_4$$

$$L_2 = -G_1H_2H_3$$

$$L_3 = G_1G_2G_3H_1H_2$$

Number of individual loops for given signal flow graph is equal to 3.

22. (d)

The transfer function of the PI controller is

$$G_c(s) = K_p + \frac{K_I}{s}$$

23. (d)

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{8}{s^2 + 3s + 8}$$

$$\therefore \quad \omega_n^2 = 8$$

$$\omega_n = \sqrt{8} = 2.82$$

$$2\xi\omega_n = 3$$

$$\xi = \frac{3}{2\omega_n} = \frac{3}{2 \times 2.82} = 0.53$$

Since $\xi < 1$, it is an underdamped system.

24. (a)

The characteristics equation is

$$1 + G(s) = 0$$

$$\therefore \quad 1 + \frac{K}{s(s+6)^2} = 0$$

$$s(s + 6)^2 + K = 0$$

$$K = -s(s + 6)^2 = -s(s^2 + 12s + 36) = -(s^3 + 12s^2 + 36s)$$

$$\frac{dK}{ds} = -(3s^2 + 24s + 36)$$

$$\frac{dK}{ds} = -3(s + 2)\ (s + 6)$$

Put, $$\frac{dK}{ds} = 0$$

$$s_1 = -2,\ s_2 = -6$$

$s = -2$ is the breakaway point.

25. (d)

Slope changes from +20 dB/decade to –60 dB/decade hence number of poles are 4.

26. (a)

For maximum peak overshoot $M_p \propto \frac{1}{\xi}$

$\xi = 0.50$ for option (a) which is least among all options. Therefore correct option is (a).

27. (d)

Let us calculate the response as follows:

$$\frac{Y(s)}{R(s)} = \frac{K}{s\tau + K + 1}$$

$$Y(s) = \frac{1}{s} \cdot \frac{K}{s\tau + K + 1} \qquad \left[\because R(s) = \frac{1}{s}\right]$$

Therefore, $$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot\frac{K}{s\tau+K+1} = \frac{K}{K+1}$$

$$\frac{K}{K+1} = 0.8$$

$$1+\frac{1}{K} = \frac{1}{0.8}$$

$$\frac{1}{K} = 0.25$$

$$K = 4$$

28. (c)

The PD control improves the transient part and the PI control improves the steady-state part. A combination of PI and PD control improves the overall response of the system.

29. (d)

In the pole zero form,

$$G(s)\,H(s) = \frac{K(s+z_1)(s+z_2)\ldots}{s^n(s+p_1)(s+p_2)\ldots}$$

the type of the system is 'n' and order of the system is the highest power of s in the denominator.

30. (c)

Settling time at 2% of tolerance band of the system,

$$t_s = \frac{4}{\xi\omega_n}$$

Settling time at 5% of tolerance band of the system,

$$t_s = \frac{3}{\xi\omega_n}$$

31. (c)

$$\text{Gain margin} = \frac{1}{\text{Gain}}$$

32. (b)

$$c(t) = t^2 e^{-t}$$

$$C(s) = \frac{2}{(s+1)^3}$$

$$R(s) = \frac{1}{s}$$

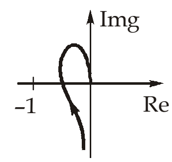
Transfer function, $$G(s) = \frac{C(s)}{R(s)} = \frac{2/(s+1)^3}{1/s}$$

$$\Rightarrow \quad G(s) = \frac{2s}{(s+1)^3}$$

33. (c)

$$G(s)H(s) = \frac{5}{s(1+0.1s)(1+0.01s)}$$

Nyquist diagram is



34. (b)

When a pole is added at negative real axis, the tail of the plot remains at same position whereas the head of plot shifts by 90° in clockwise direction.

35. (c)

For a minimum phase system to be stable, both phase margin and gain margin should be positive.

36. (c)

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times 0.421\sqrt{1-(0.421)^2}}$$

$$M_r = 1.30$$

37. (c)

Routh array is

s^4	1	3	K
s^3	2	2	
s^2	2	K	
s^1	$2-K$	0	
s^0	K		

For oscillations, $2 - K = 0$

$\Rightarrow$ $K = 2$

For oscillations,

$$2s^2 + K = 0$$

Putting $s = j\omega$ and $K = 2$,

$$-2\omega^2 + 2 = 0$$

$\Rightarrow$ $\omega^2 = 1$

$\Rightarrow$ $\omega = 1$ rad/sec

38. (b)

Signal flow graph is mainly used for finding transfer function with the help of mason gain formula.

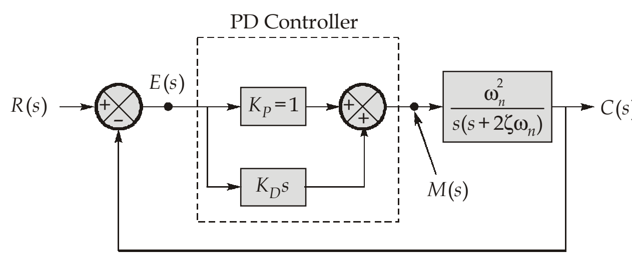
39. (c)

Maximum phase shift, $\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right)$

$$\phi_m = \sin^{-1}\left(\frac{2/3}{4/3}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

40. (c)

PD Controller:



$$\frac{M(s)}{E(s)} = K_P + K_D s$$

$$\frac{C(s)}{R(s)} = \frac{(K_P + K_D s)\omega_n^2}{s^2 + (2\xi\omega_n + K_D\omega_n^2)s + K_P\omega_n^2}$$

Characteristic equation is

$$s^2 + (2\xi\omega_n + K_D\omega_n^2)s + \omega_n^2 = 0$$

$\because$ $K_P = 1$

Comparing with $s^2 + 2\xi'\omega_n's + \omega_n'^2 = 0$

$$\omega_n' = \omega_n$$

$$\xi' = \xi + \frac{K_D\omega_n}{2}$$

Thus ω_n remains fixed but ξ increases.

41. (b)

(i) Integral controller improves the steady state response.
(ii) Derivative controller improves the transient response.

42. (c)

Time for peak overshoots are

$$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}} \qquad n = 1, 3, 5, \ldots$$

For first peak overshoot, $n = 1$

$$t_{p1} = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}}$$

For second peak overshoot, $n = 3$

$$t_{p2} = \frac{3\pi}{\omega_n\sqrt{1-\xi^2}}$$

43. (a)

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+3)(s+2)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \dfrac{1}{s+3} & \dfrac{1}{(s+3)(s+2)} \\ 0 & \dfrac{1}{(s+2)} \end{bmatrix}$$

The steady state value is

$$= \lim_{s \to 0} s \cdot [sI - A]^{-1} x(0)$$

$$= \lim_{s \to 0} s \cdot \begin{bmatrix} \dfrac{1}{s+3} & \dfrac{1}{(s+2)(s+3)} \\ 0 & \dfrac{1}{(s+2)} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$= \lim_{s \to 0} s \cdot \begin{bmatrix} \dfrac{10}{s+3} - \dfrac{10}{(s+2)(s+3)} \\ -\dfrac{10}{s+2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Hence, $A = 0$; $B = 0$

44. (b)

$$A = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Roots can be determined from the characteristic equation.

i.e. $\quad |sI - A| = 0$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} = 0$$

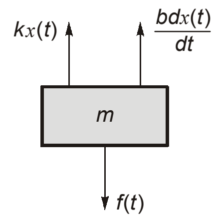
$$\begin{vmatrix} s & 1 \\ -1 & s+2 \end{vmatrix} = 0 \Rightarrow s^2 + 2s + 1 = 0$$

$$(s+1)^2 = 0$$

Thus it can be determined that the system is critically damped.

45. (a)

Free body diagram of mass m is shown below:



At balance,

$$f(t) - \frac{b\,dx(t)}{dt} - kx(t) = \frac{md^2\,x(t)}{dt^2}$$

or $$\frac{m\,d^2x(t)}{dt^2} + \frac{bdx(t)}{dt} + kx(t) = f(t)$$

46. (c)

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 3 & A \\ s^1 & \frac{12-A}{3} & 0 \\ s^0 & A & \end{array}$$

$$12 - A > 0$$
$$A < 12 \text{ and } A > 0$$
$$0 < A < 12$$

47. (d)

From circuit,

$$E_0(s) = \frac{sL}{R + sL} \cdot E_i(s)$$

$$\frac{E_0(s)}{E_i(s)} = \left(\frac{s}{s + R/L}\right)$$

48. (c)

The characteristic equation is

$$1 + \left(\frac{s-5}{s+4}\right) \cdot K = 0$$

$$(s + 4) + K(s - 5) = 0$$

$$s(1 + K) + (4 - 5\,K) = 0$$

$$\begin{array}{c|c} S^1 & 1+K \\ S^0 & 4-5K \end{array}$$

$$4 - 5\,K \geq 0$$
$$5\,K \leq 4$$

$\Rightarrow$ $$K \leq \frac{4}{5}$$

49. (c)

Resonant frequency;

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

For $\omega_r = 0$;

$$\sqrt{1-2\xi^2} = 0$$

$$\xi = 0.707$$

50. (c)

$$C(s) = R(s)\ .\ H(s)$$

$$1 = \frac{1}{s}\times H(s)$$

$$H(s) = s$$

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