MADE EASY

India's Best Institute for IES, GATE & PSUs

GATE 2020

Instrumentation Engineering

Questions and Solutions of afternoon session

Date of Exam : 1/2/2020

Scroll down to view

www.madeeasy.in

Corporate Office: 44-A/1, Kalu Sarai, New Delhi - 110016 | **Ph:** 011-45124612, 9958995830

Delhi | Hyderabad | Noida | Bhopal | Jaipur | Lucknow | Indore | Pune | Bhubaneswar | Kolkata | Patna

SECTION A : GENERAL APTITUDE

Q.1 P, Q, R and S are to be uniquely coded using α and β. If P is coded as $\alpha\alpha$ and Q as $\alpha\beta$, then R and S, respectively, can be coded as _____.

(a) $\beta\alpha$ and $\alpha\beta$
(b) $\alpha\beta$ and $\beta\beta$
(c) $\beta\alpha$ and $\beta\beta$
(d) $\beta\beta$ and $\alpha\alpha$

Ans. (c)

Given: Code of $P = \alpha\alpha$; Code of $Q = \alpha\beta$
Then, Code of $R = \beta\alpha$
and Code of $S = \beta\beta$

End of Solution

Q.2 I do not think you know the case well enough to opinions. Having said that, I agree with your other point. What does the phrase "having said that" means in the given text?

(a) contrary to what I have said
(b) in addition to what I have said
(c) as opposed to what I have said
(d) despite what I have said

Ans. (d)

End of Solution

Q.3 He is known for his unscrupulous ways. He always sheds ____ tears to deceive people.

(a) fox
(b) fox's
(c) crocodile's
(d) crocodile

Ans. (d)

End of Solution

Q.4 Select the word that fits the analogy:
Build : Building :: Grow : _____

(a) Grown
(b) Grew
(c) Growed
(d) Growth

Ans. (d)

End of Solution

Q.5 Crowd funding deals with mobilization of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.
Based on the above paragraph, which of the following is correct about crowd funding?

(a) Funds raised through unwilling contributions on web-based platforms.
(b) Funds raised through coerced contributions on web-based platforms.
(c) Funds raised through large contributions on web-based platforms.
(d) Funds raised through voluntary contributions on web-based platforms.

Ans. (d)

End of Solution

UPPSC

Combined State Engineering Services Exam, 2019

MADE EASY

India's Best Institute for IES, GATE & PSUs

Assistant Engineer | Total Posts : 692

Classroom Course

Streams: CE, ME, EE

Commencing from 10th Feb, 2020 | Classes at Delhi and Lucknow

- 650 Hrs of comprehensive course.
- General Studies and Hindi covered.
- Exclusive study materials will be provided as per requirement of UPPSC.

Admission open

Postal Course

Streams: CE, ME, EE

Enrollment open

- Technical theory books with practice questions.
- Previous years' solved papers.
- Practice books (MCQ) for technical subjects.
- General Studies theory book with practice questions.
- Hindi book with practice questions.

Admission open

Live/Online Classes

Streams: CE, ME, EE

Commencing from 10th Feb, 2020

- Useful for those candidates who are not able to join classroom programme.
- 650 Hrs of quality classes at your doorstep.
- Flexibility to learn at your own pace.
- Physical study materials will be dispatched at your address.

Admission open

Test Series (Online/Offline)

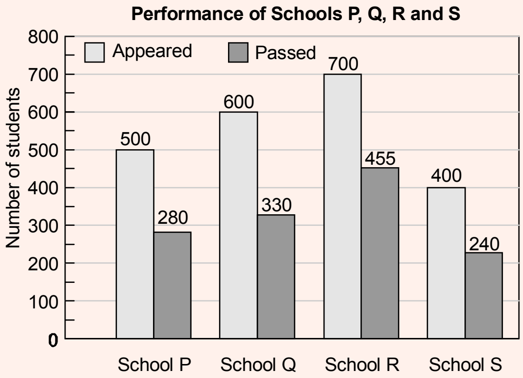
Streams: CE, ME, EE

20 Tests | Commencing from 23rd Feb, 2020

- Quality questions with detailed solutions.
- Comprehensive performance analysis.
- Tests on standard and pattern as per UPPSC examination.

Admission open

Q.6 The bar graph shows the data of students who appeared and passed in an examination for four schools P, Q, R and S. The average of success rates (in percentage) of these four schools is ____.



(a) 58.8% (b) 58.5%
(c) 59.0% (d) 59.3%

Ans. (c)

$$\text{Average percentage of passing students} = \frac{\dfrac{\text{Total no. of passed students}}{\text{Total no. of students}}}{4} \times 100$$

$$= \frac{\dfrac{280}{500}+\dfrac{330}{600}+\dfrac{455}{700}+\dfrac{240}{400}}{4} \times 100$$

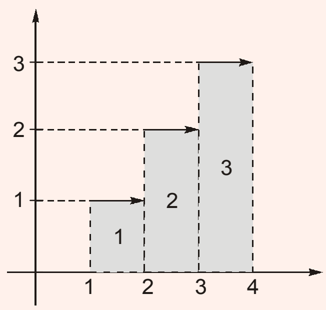
$$= \frac{0.56+0.55+0.65+0.60}{4} \times 100 = 59\%$$

End of Solution

Q.7 Define $[x]$ as the greatest integer less than or equal to x, for each $x \in (-\infty, \infty)$. If $y = [x]$ then area under y for $x \in [1, 4]$ is ____.

(a) 4 (b) 1
(c) 6 (d) 3

Ans. (c)



$$\text{Area} = 1 \times 1 + 1 \times 2 + 1 \times 3$$
$$= 1 + 2 + 3 = 6$$

End of Solution

Q.8 Jofra Archer, the England fast bowler, is _____ than accurate.

(a) more faster
(b) faster
(c) less fast
(d) more fast

Ans. (d)

End of Solution

Q.9 The sum of the first n terms in the sequence 8, 88, 888, 8888, ... is _____.

(a) $\frac{80}{81}(10^n - 1) - \frac{8}{9}n$
(b) $\frac{81}{80}(10^n - 1) + \frac{9}{8}n$
(c) $\frac{81}{80}(10^n - 1) - \frac{9}{8}n$
(d) $\frac{80}{81}(10^n - 1) + \frac{8}{9}n$

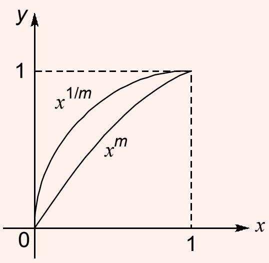
Ans. (a)

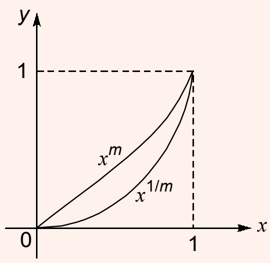
Put n = 1 in all options.

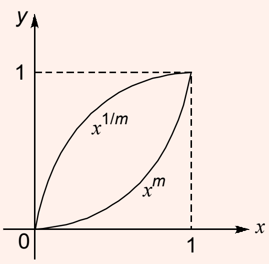
$$\frac{80}{81}\left[10^1 - 1\right] - \frac{8}{9}[1] = \frac{80}{9} - \frac{8}{9} = \frac{72}{9} = 8$$

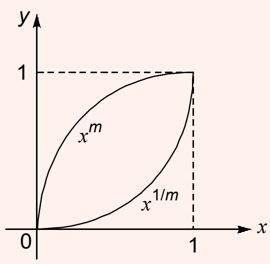
End of Solution

Q.10 Select the graph that schematically represents BOTH $y = x^m$ and $y = x^{1/m}$ properly in the interval $0 \le x \le 1$, for integer values of m, where $m > 1$.

(a)


(b)


(c)


(d)


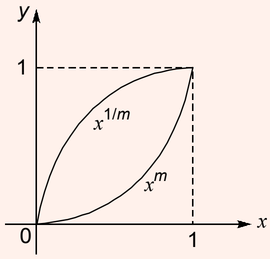
Ans. (c)

$$y = x^m \text{ and } y = x^{1/m}$$

for $m = 2$ in $0 \le x \le 1$

$$y = x^2 \quad \text{and} \quad y = \sqrt{x}$$

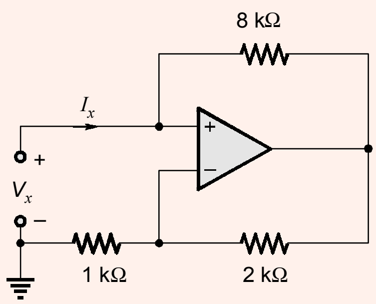
$\therefore$ graphs are shown as



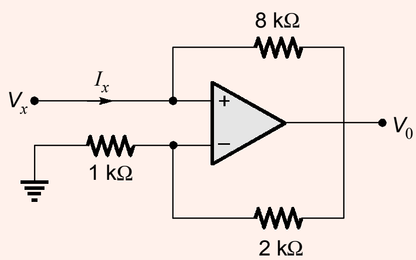
End of Solution

SECTION B : TECHNICAL

Q.1 Assume that the opamp in the circuit shown is ideal. The value of $\frac{V_x}{I_x}$ (in kΩ) is _____.



Ans. (–4)



$$V^+ = V^-$$

$$V_x = \frac{V_0 \times 1}{1+2}$$

$\Rightarrow$ $$V_0 = 3V_x \qquad ...(1)$$

$$I_x = \frac{V_x - V_0}{8} = \frac{V_x - 3V_x}{8} = \frac{-V_x}{4}$$

$\Rightarrow$ $$\frac{V_x}{I_x} = -4 \text{ k}\Omega$$

End of Solution

Q.2 The closed loop transfer function of a control system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$. For the input $r(t)$ = sin t, the steady state response $c(t)$ is _____.

(a) $\frac{1}{\sqrt{2}}\cos t$

(b) $\frac{1}{\sqrt{2}}\sin\left(t+\frac{\pi}{4}\right)$

(c) 1

(d) $\frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$

Ans. (d)

$$\text{Transfer function} = \frac{1}{s+1} = G(s)$$

$$G(j\omega) = \frac{1}{j\omega+1} = \frac{1}{\sqrt{\omega^2+1}}\angle -\tan^{-1}\omega$$

$$\text{Input} = \underset{A}{1}\sin(\underset{\omega}{(1)}t + \underset{\phi_1}{0°})$$

$$\text{Output} = B\sin(\omega t + \phi_2)$$

where $B = A|G(j\omega)| = 1\times\frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

and $\phi_2 = \phi_1 + \angle G(j\omega) = -\tan^{-1} 1 = -45°$

$\therefore$ Steady state o/p $= \frac{1}{\sqrt{2}}\sin\left(t-\frac{\pi}{4}\right)$

End of Solution

Q.3 Consider the signal $x(t) = e^{-|t|}$. Let $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ be the Fourier transform of $x(t)$. The value of $X(j0)$ is _____.

Ans. (2)

$$x(t) = e^{-|t|} \Longleftrightarrow X(j\omega) = \frac{2}{1+\omega^2}$$

By using area under frequency domain property,

$$\text{Area of } x(t) = X(j0)$$

$\Rightarrow$ $X(j0)$ = Area of $x(t)$

$$= \int_{-\infty}^{\infty} x(t)dt$$

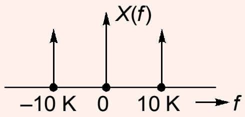
$$= \int_{-\infty}^{0} e^t dt + \int_{0}^{\infty} e^{-t}dt = 1 + 1 = 2$$

End of Solution

Q.4 A sinusoid of 10 kHz is sampled at 15 k samples/s. The resulting signal is passed through an ideal low pass filter (LPF) with cut-off frequency of 25 kHz. The maximum frequency component at the output of the LPF (in kHz) is _____.

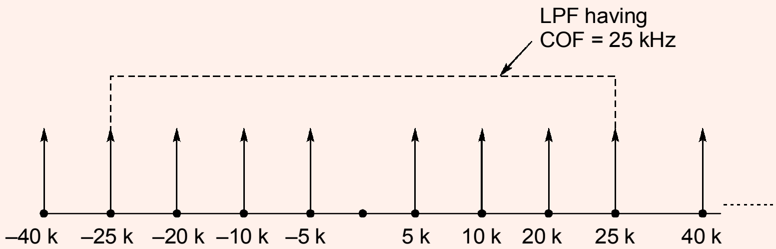
Ans. (25)

Given signal is sinusoidal signal having frequency ⇒ 10 kHz



Given, $f_s = 15$ kHz

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$



After passing through Ideal LPF having Cut-off frequency = 25 km
frequencies present at filter output are → 5 k, 10 k, 20 k, 25 k, ...
Max frequency present at filter output → 25 kHz

Method : 2

Given, $f_m = 10$ kHz

After sampling, frequencies present will be → $\pm f_m \pm nf_s$: n = 0, 1, 2, 3, 4, ...

i.e. ± 10 k, ± 10 k ± 15 k, ± 10 k ± 30 k, ...

After passing through ideal LPF having Cut-off frequency = 25 kHz, heighest free component present at filter output = 25 kHz.

End of Solution

Q.5 A phase lead network has the transfer function $G(s) = \frac{1+0.2s}{1+0.05s}$. The angular frequency at which the maximum phase shift for the network occurs is _____.

(a) 20 rad/s
(b) 200 rad/s
(c) 100 rad/s
(d) 10 rad/s

Ans. (d)

$$G(s) = \frac{1+0.2s}{1+0.05s}$$

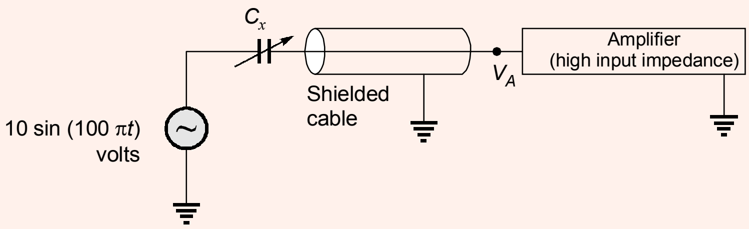
$$Z = 5$$

$$P = 20$$

$$\omega_n = \sqrt{Z.P} = 10 \text{ rad/s}$$

End of Solution

Q.6 The capacitance C_x of a capacitive type sensor is (1000 x) pF, where x is the input to the sensor. As shown in the figure, the sensor is excited by a violate 10 sin (100 πt) V. The other terminal of the sensor is tied to the input of a high input impedance amplifier through a shielded cable, with shield connected to ground. The cable capacitance is 100 pF. The peak of the voltage V_A at the input of the amplifier when x = 0.1 (in volts) is _____.



Ans. **(5)**

The capacitance of the capacitive sensor

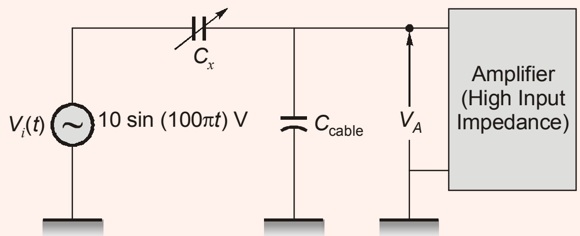
$$C_x = (1000x)\ \text{Pf}$$

x value is = 0.1

$$C_x = (1000 \times 0.1)\ \text{Pf} = 100\ \text{Pf}$$

Cable capacitance = 100 Pf

Electrical equivalent of given measurement system.



As the amplifier has high input impedance there is no loading effect.

So, Peak value of $V_A = V_i \times \dfrac{\dfrac{1}{j\omega C_{\text{cable}}}}{\dfrac{1}{j\omega C_{\text{cable}}} + \dfrac{1}{j\omega C_x}}$

$$= 10 \times \frac{C_x}{C_x + C_{\text{cable}}} = 10 \times \frac{100 \times 10^{-12}}{(100+100) \times 10^{-12}} = 5\ \text{V}$$

End of Solution

Q.7 Given $f(A, B, C, D) = \Sigma m(0, 1, 2, 6, 8, 9, 10, 11) + \Sigma d(3, 7, 14, 15)$ is a Boolean function, where m represents min-term and d represents don't-cares. The minimal sum of products expression for f is _____.

(a) $f = \overline{B} + C$

(b) $f = A\overline{B} + CB$

(c) $f = \overline{D} + A$

(d) $f = \overline{A}B + \overline{C}D$

Ans. (a)

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 6, 8, 9, 10, 11) + \Sigma d(3, 7, 14, 15)$$

AB \ CD	00	01	11	10
00	1	1	x	1
01			x	1
11			x	x
10	1	1	1	1

$C + \overline{B}$

$$f = \overline{B} + C$$

End of Solution

Q.8 A set of linear equations is given in the form $Ax = b$, where A is a 2 × 4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a unique solution by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose and inverting the matrix A^TA? Answer is _____.

(a) Yes, can obtain a unique solution provided the matrix A is will conditioned.

(b) Yes, it is always possible to get a unique solution for any 2 × 4 matrix A.

(c) Yes, can obtain a unique solution provided the matrix A^TA is well conditional.

(d) No, it is not possible to get a unique solution for any 2 × 4 matrix A.

Ans. (b)

Matrix A has rank 2

Matrix A^T has ranks 2

$\Rightarrow AA^T$ has rank 2

$$\therefore \quad \left|AA^T{}_{4\times4}\right| = 0$$

Hence system cannot have unique solution.

End of Solution

Q.9 Let $f(x) = \dfrac{1}{z+a}, a > 0$. The value of the integral $\oint f(x)dx$ over a circle C with centre $(-a, 0)$ and radius $R > 0$ evaluated in the anti-clockwise direction is _____.

(a) 0

(b) $2\pi i$

(c) $-2\pi i$

(d) $4\pi i$

Ans. (b)

Given $\quad f(z) = \dfrac{1}{z+a}, a > 0$ and singularity of $f(z)$ is $z = -a$

$$\oint_c f(z)dz = \oint_c \frac{1}{z+a}dz, \text{ where 'c' : } |z + a| = R$$

singularity ($z = -a$) lies inside 'c'

$\therefore$ By Cauchy integral formula

$$\oint_c \frac{1}{z-(-a)} dz = 2\pi i(1) = 2\pi i$$

End of Solution

Q.10 Consider the Signal $x[n] = \sin(2\pi n)\ u[n]$, where $u[n] = \begin{cases} 1 & n = 0,1,2,3,..... \\ 0 & \text{otherwise} \end{cases}$

The period of this signal $x[n]$ is

(a) 3 (b) 4

(c) 2 (d) 1

Ans. **(d)**

$$x(n) = \sin\ 2\pi n.u(n)$$

For $n = 0$, $x(0) = 0$

$n = 1$, $x(1) = \sin 2\pi = 0$

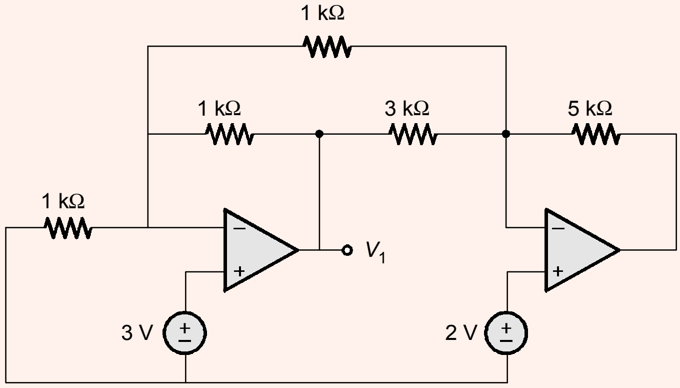
$n = 2$, $x(2) = \sin 4\pi = 0$

Thus, $x(n) = \{....\ 0,\ 0,\ \underset{\uparrow}{0},\ 0,\ 0,\ 0,\ .....\}$

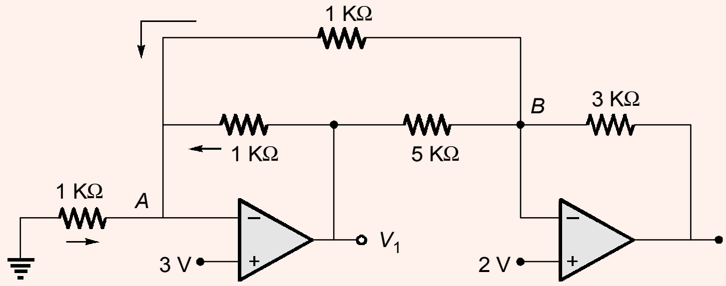
Hence, Time-period is 1.

End of Solution

Q.11 Assuming ideal opamps, the output voltage at V_1 in the figure shown (in volts) is _____.



Ans. **(7)**



$V_A = 3$ V; $V_B = 2$ V

KCL at Node A:

$$\frac{0-3}{1} + \frac{V_1 - 3}{1} + \frac{2-3}{1} = 0$$

$$\Rightarrow \quad V_1 = 7 \text{ V}$$

End of Solution

Q.12 Consider the recursive equation $X_{n+1} = X_n - h(F(X_n) - X_n)$, with initial condition $X_0 = 1$ and h > 0 being a very small valued scalar. This recursion numerically solves the ordinary differential equation _____.

(a) $\dot{X} = -F(X),\ X(0) = 1$
(b) $\dot{X} = F(X),\ X(0) = 1$
(c) $\dot{X} = F(X) + X,\ X(0) = 1$
(d) $\dot{X} = -F(X) + X,\ X(0) = 1$

Ans. (d)

Given recursive equation $x_{n+1} = x_n - h[F(x_n) - x_n]$

can be written at $x_{n+1} = x_n + h[x_n - F(x_n)]$

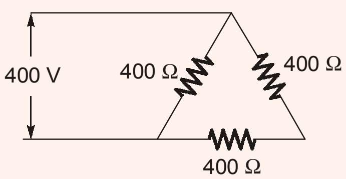
Clearly it is Euler forward numerical solution for the differential equation.

$$\frac{dx}{dt} = x - F(x)$$

End of Solution

Q.13 Three 400 Ω resistors are connected in delta and powered by a 400 V (rms), 50 Hz, balanced, symmetrical *R-Y-B* sequence, three-phase three-wire mains. The rms value of the line current (in amperes, rounded off to one decimal place) is _____.

Ans. (1.7)

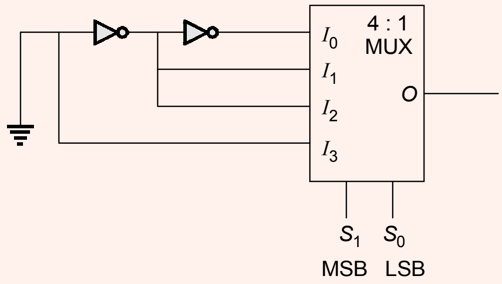


$$I_{Ph} = \frac{V_{Ph}}{R_{Ph}} = \frac{400}{400} = 1\text{A}$$

$$I_{Line} = \sqrt{3}\, I_{Ph} = \sqrt{3} \times 1 = 1.732 \text{ A}$$

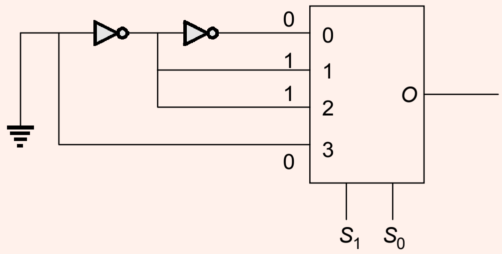
End of Solution

Q.14 The Boolean operation performed by the following circuit at the output O is _____.



(a) $O = S_0 \bullet \bar{S}_1$
(b) $O = S_1 \bullet \bar{S}_0$
(c) $O = S_1 + S_0$
(d) $O = S_1 \oplus S_0$

Ans. (d)



$$O = \bar{S}_1\bar{S}_0 . 0 + \bar{S}_1 S_0 . 1 + S_1\bar{S}_0 . 1 + S_1 S_0 . 0$$
$$= \bar{S}_1 S_0 + S_1 \bar{S}_0 = S_1 \oplus S_0$$

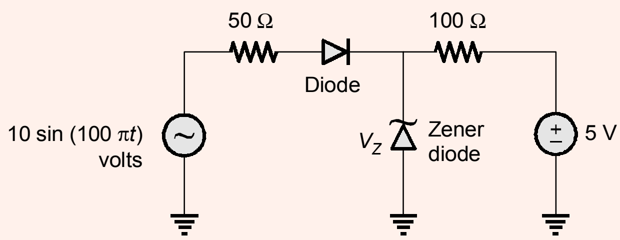
End of Solution

Q.15 A Q meter is best suited for the measurement of the _____.
(a) Quality factor of a capacitance.
(b) Distributed capacitance of a coil.
(c) Turns-ratio of a transformer
(d) Quality factor of piezoelectric sensor.

Ans. (b)

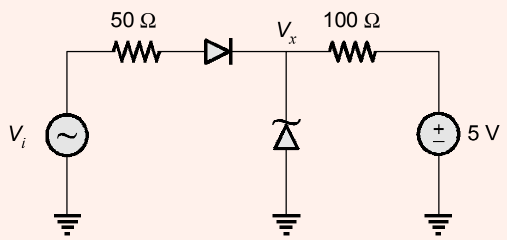
End of Solution

Q.16 If the diodes in the circuit shown are ideal and the breakdown voltage V_z of the Zener diode is 5 V, the power dissipated in the 100 Ω resistor (in watts) is _____.



(a) 225/100
(b) 1
(c) 0
(d) 25/100

Ans. (c)

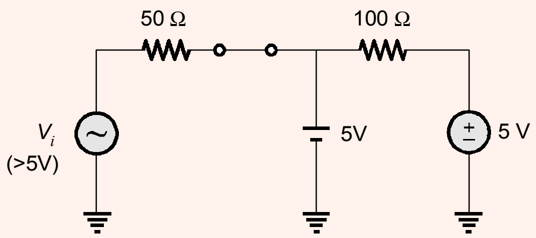


Let both diodes be OFF ⇒ V_x = 5 V

If V_i < 5 V then both diodes are OFF

∴ current through 100 Ω remains zero.

If V_i > 5 V then normal diode becomes ON and zener diode undergoes breakdown.



Now also current through 100 Ω is zero. Hence power dissipation in 100 Ω resistor is zero.

End of Solution

Q.17 If I is the current flowing through a Hall effect sensor and B is the magnetic flux density perpendicular to the direction of the current (in the plane of the Hall effect sensor), the Hall voltage generated is _____.

(a) Directly proportional to both I and B

(b) Inversely proportional to I and directly proportional to B

(c) Directly proportional to I and inversely proportional to B

(d) Inversely proportional to both I and B

Ans. (a)

The Hall voltage developed across the Hall sensor is given by

$$V_H = K_H \frac{IB}{t}$$

So, $$V_H \propto I; \quad V_H \propto B; \quad V_H \propto \frac{1}{t}$$

End of Solution

Q.18 A 200 mV full-scale dual-slope analog to digital converter (DS-ADC) has a reference voltage of 100 mV. The first integration time is set as 100 ms. The DS-ADC is operated in the continuous conversion mode. The conversion time of the DS-ADC for an input voltage of 123.4 mV (in ms, rounded off to one decimal place) is _____.

Ans. **(223.4)**

Dual Slope ADC

Reference voltage, $V_r = 100$ mV

For $V_A = 123.4$ mV

$$\text{Deintegration time} = \frac{V_A}{V_r} \times \text{ integration time} = \frac{123.4}{100} \times 100 \text{ ms}$$

So, Total conversion time = 100 ms + 123.4 ms = 223.4 ms

End of Solution

Q.19 A player throws a ball at a basket kept at distance. The probability that the ball falls into the basket in a single attempt is 0.1. The player attempts to throw the ball twice. Considering each attempt to be independent, the probability that this player puts the ball into the basket only in the second attempt (rounded of to two decimal places) is _____.

Ans. **(0.09)**

Given, the probability that the ball faces into the basket in a single attept = 0.1

i.e., probability that the base does not fall into basket = 1 – 0.1 = 0.9

Given each attempt is independent.

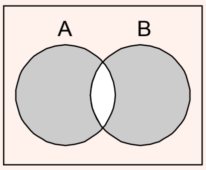
∴ Probability that the ball falls into basket only in second attempt

= P(does not fall) × P(falls)

= 0.9 × 0.1 = 0.09

End of Solution

Q.20 The Boolean expression for the shaded regions as shown in the figure is _____.



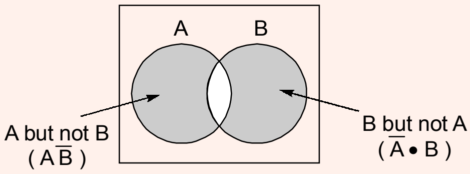
(a) $(A+\bar{B})\cdot(\bar{A}+B)$

(b) $(\bar{A}+B)\cdot(\bar{A}+B)$

(c) $(\bar{A}+\bar{B})\cdot(A+\bar{B})$

(d) $(A+B)\cdot(\bar{A}+\bar{B})$

Ans. (d)



$$f = A\overline{B} + A\overline{B}$$

$$f = (\overline{A}+\overline{B}).(A+B)$$

End of Solution

Q.21 The unit vectors along the mutually perpendicular x, y and z axes are $\hat{i}, \hat{j}$ and $\hat{k}$ respectively. Consider the plane z = 0 and two vector $\vec{a}$ and $\vec{b}$ on that plane such that $\vec{a} \neq \alpha\vec{b}$ for any scalar α. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is _____.

(a) $\hat{i} - \hat{j}$
(b) $-\hat{j}$
(c) $\hat{k}$
(d) $\hat{i}$

Ans. (c)

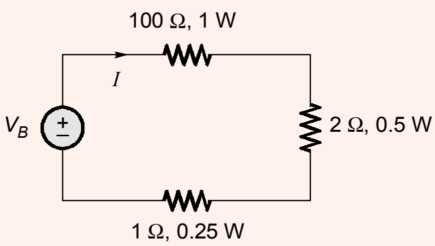
Given z = 0 and $\overline{a} \neq \alpha\overline{b}$

So, it is clear that both $\overline{a}, \overline{b}$ are ine xy-plane.

$\therefore$ The vector perpendicular to both $\overline{a}, \overline{b}$ will be in the direction of z-axis.

End of Solution

Q.22 In the circuit shown below, the safe maximum value for the current I is _____.



(a) 0.5 A
(b) 0.1 A
(c) 1.0 A
(d) 0.05 A

Ans. (b)

Maximum current for 100 Ω, 1 W; $I = \sqrt{\frac{P}{R}} = 0.1$ A

Maximum current for 2 Ω, 0.5 W; $I = \sqrt{\frac{0.5}{2}} = 0.5$ A

Maximum current for 1 Ω, 0.25 W; $I = \sqrt{0.25} = 0.5$ A

The safe maximum value of current I which can flow through series resistances is 0.1 A.

End of Solution

Q.23 A differentiator has a transfer function whose
(a) magnitude decreases linearly with frequency
(b) magnitude remains constant
(c) phase increase linearly with frequency
(d) magnitude increases linearly with frequency

Ans. (d)

$$y(t) = \frac{dx(t)}{dt}$$

Taking Fourier Transform

$$Y(\omega) = j\omega X(\omega)$$

$$\Rightarrow \quad H(\omega) = \frac{Y(\omega)}{X(\omega)} = j\omega$$

$$\Rightarrow \quad |H(\omega)| = \omega$$

$$\angle H(\omega) = 90^\circ$$

Magnitude increases linaerly and phase is constant.

End of Solution

Q.24 Two 100 Ω resistors having tolerance 3% and 4% are connected in series. The effective tolerance of the series combination (in %, rounded off to one decimal place) is _____.

Ans. (3.5)

Given,

$$R_1 = 100\ \Omega \pm 3\%$$

$$R_2 = 100\ \Omega \pm 4\%$$

$$R = R_1 + R_2 = 100 + 100 = 200\ \Omega$$

Here, $\frac{\delta R_1}{R_1} \times 100 = 3$

$$\delta R_1 = \frac{100 \times 3}{100} = 3\ \Omega$$

$$\frac{\delta R_2}{R_2}\times 100 = 4$$

$$\delta R_2 = \frac{100\times 4}{100} = 4\ \Omega$$

We know, $\delta R = \delta R_1 + \delta R_2 = 3 + 4 = 7\ \Omega$

$\therefore$ $\% \in_r \text{ in } R = \frac{\delta R}{R}\times 100 = \frac{7}{200}\times 100 = 3.5\%$

End of Solution

Q.25 A second order system has closed loop poles located as $s = -3 \pm j4$. The time t at which the maximum value of the step response occurs (in seconds, rounded off to two decimal places) is _____.

Ans. (0.78)

Given, Poles $= -3 \pm j4 = \xi\omega_n \pm \omega_n\sqrt{1-\xi^2}$

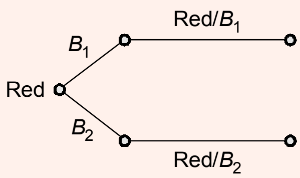
So, $\omega_n\sqrt{1-\xi^2} = 4$

and since $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.78$

End of Solution

Q.26 Consider two identical bags B_1 and B_2 each containing 10 balls of identical shapes and sizes. Bag B_1 contains 7 Red and 3 Green balls, while bag B_2 contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B_1 (rounded off to one decimal place) is _____.

Ans. (0.7)



$$P(B_1) = \frac{1}{2}$$

$$P(B_2) = \frac{1}{2}$$

$$P\left(\frac{\text{Red}}{B_1}\right) = \frac{7}{10}$$

$$P\left(\frac{\text{Red}}{B_2}\right) = \frac{3}{10}$$

By Baye's Theorem

$$P\left(\frac{B_1}{\text{Red}}\right) = \frac{P(B_1 \cap \text{Red})}{P(\text{Red})}$$

$$= \frac{P(B_1).P\left(\frac{\text{Red}}{B_1}\right)}{P(B_1).P\left(\frac{\text{Red}}{B_1}\right) + P(B_2)\,P\left(\frac{\text{Red}}{B_1}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{7}{10}}{\left(\frac{1}{2}\times\frac{7}{10}\right)+\left(\frac{1}{2}\times\frac{3}{10}\right)}$$

$$= \frac{\frac{7}{20}}{\frac{1}{2}} = \frac{7}{10} = 0.7$$

End of Solution

Q.27 Consider the following state variable equations :

$$\dot{x}_2(t) = x_2(t)$$

$$\dot{x}_2(t) = -6x_1(t) - 5x_2(t)$$

The initial conditions are $x_1(0) = 0$ and $x_2(0) = 1$. At $t = 1$ second, the value of $x_2(1)$ (rounded off to two decimal places) is _____.

Ans. (–0.12)

$$\dot{x}_1(t) = x_2(t)$$

$$Sx_1(s) - x_1(0)_0 = x_2(s)$$

$$\therefore \quad x_1(s) = \frac{x_2(s)}{s}$$

$$\dot{x}_2(t) = -6x_1(t) - 5x_2(t)$$

$$Sx_2(s) - x_2(0) = -6x_1(s) - 5x_2(s)$$

$$Sx_2(s) - 1 = -\frac{6x_2(s)}{s} - 5x_2(s)$$

$$x_2(s)\left(s + \frac{6}{s} + 5\right) = 1$$

$$x_2(s) = \frac{s}{s^2+5s+6}$$

$$x_2(s) = \frac{-2}{s+2}+\frac{3}{s+3}$$

$$x_2(t) = -2e^{-2t} + 3e^{-3t}$$

$$\therefore \quad x_2(1) = -2e^{-2} + 3e^{-3}$$

$$= -0.12$$

End of Solution

Q.28 A metallic strain gauge of resistance R_x with a gauge factor of 2 is bonded to a structure made of a metal with modulus of elasticity of 200 GN/m^2. The value of R_x is 1 kΩ when no stress is applied. R_x is a part of a quarter bridge with three identical fixed resistors of 1 kΩ each. The bridge is excited from a DC voltage of 4 V. The structure is subjected to a stress of 100 MN/m^2. Magnitude of the output of the bridge (in mV, rounded off to two decimal places) is _____.

Ans. (1)

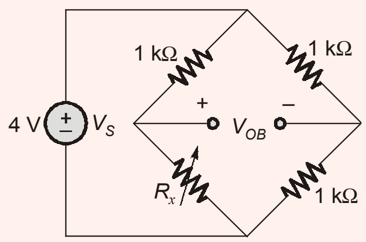
Given, The initial resistance of strain gauge (R_x) is

$$R_x = 1 \text{ k}\Omega$$

Youngs Modulus of Elasticity (E) = 200×10^9 N/m^2

Applied stress = 100×10^6 N/m^2

Quarter bridge configuration



Output voltage of quarter bridge for small strain

$$V_{OB} = \frac{V_S}{4}\times\frac{\Delta R_x}{R_x}$$

$$\Delta R_x = R_x G_f \in$$

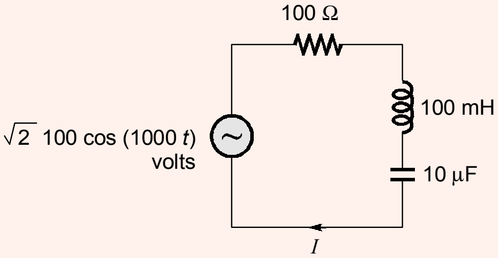
(R_x = No load resistance; G_f = Gauge factor; $\in$ = Applied strain)

$$V_{OB} = \frac{V_S}{4}\times G_f \in \; = \; \frac{4}{4}\times 2\times\left[\frac{\text{Stress}}{\text{Youngs Modulus}}\right]$$

$$= 2\times\frac{100\times10^6}{200\times10^9} = 1\times10^{-3} \text{ V} \; = \; 1 \text{ mV}$$

End of Solution

Q.29 The rms value of the phasor current I in the circuit shown (in amperes) is _____.

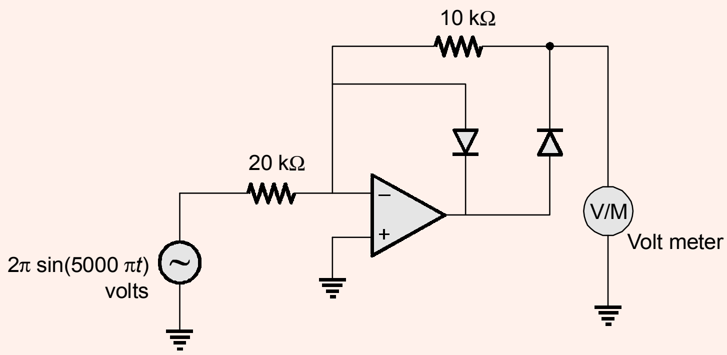


Ans. (1)

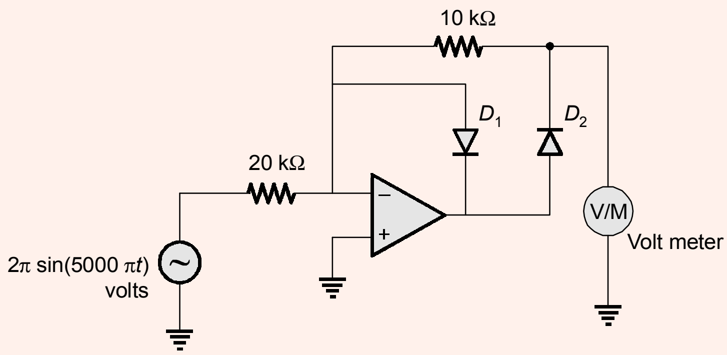
$$R = 100\ \Omega$$
$$X_L = \omega L = 100\ \Omega$$
$$X_C = \frac{1}{\omega C} = 100\ \Omega$$
$$Z = 100 + j(100 - 100)$$
$$|Z| = 100\ \Omega$$
$$I = \frac{V_{rms}}{Z} = \frac{\left(\sqrt{2} \times 100\right)/\sqrt{2}}{100} \Rightarrow 1\text{ A}$$

End of Solution

Q.30 Assuming that the opamp used in the circuit shown is ideal, the reading of the 1 Hz bandwidth, permanent magnet moving coil (PMMC) type voltmeter (in volts) is _____.



Ans. (1)



Case-1: For +ve half cycle

$$V_0 = -\text{ve as opamp is inverting}$$

D_1 is FB and D_2 is RB.

From the concept of virtual ground

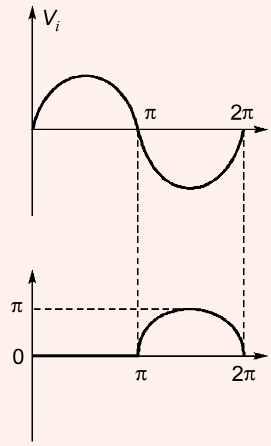
$$V_0 = 0$$

Case-2 : For –ve half cycle

$$V_0 = +\text{ve}$$

D_1 is RB and D_2 is FB.

$$\therefore \quad V_0 = -(-2\pi)\times\frac{10}{20} = +\pi$$



Here V_0 is the peak voltage

From the waveform

As a PMMC reads the average value

$$\therefore \quad V_{avg} = \frac{V_P}{\pi} = \frac{\pi}{\pi} = 1\text{ V}$$

End of Solution

Q.31 A 1000/1 A, 5 VA, UPF bar-primary measuring current transformer has 1000 secondary turns. The current transformer exhibits a ratio error of – 0.1% and a phase error of 3.438 minutes when the primary current is 1000 A. At this operating condition, the rms value of the magnetization current of the current transformer (in amperes, rounded off to two decimal places) is _____.

Ans. (1.00)

In a current transformer we know that the phase angle error

$$\beta = \frac{180}{\pi}\frac{I_m}{nI_s}$$

Here $\beta = 3.438\text{ mins} = \frac{3.438}{60}\text{deg}$

$$n = 1000$$

$$I_s = 1$$

$$\therefore \quad \frac{3.438}{60} = \frac{180}{\pi}\times\frac{I_m}{1000\times 1}$$

$$I_m = \frac{3.438\times 1000\times\pi}{60\times 180}$$

$$= \frac{108.0079}{108} = 1.00\text{ A}$$

End of Solution

Q.32 Consider the function $f(x, y) = x^2 + y^2$. The minimum value of the function attains on the line $x + y = 1$ (rounded off to two decimal places) is _____.

Ans. (0.5)

$$f(x, y) = x^2 + y^2 \text{ with respect to line } x + y = 1$$

from constraint $x + y = 1 \Rightarrow y = 1 - x$

Then
$$f(x, y) = x^2 + (1 - x)^2$$
$$= x^2 + 1 + x^2 - 2x$$
$$f(x, y) = \phi(x) = 2x^2 - 2x + 1 \text{ to be minimum}$$

find stationary point

$$\phi'(x) = 4x - 2 = 0$$
$$\Rightarrow \quad x = 1/2$$

Then,
$$y = 1 - x = 1 - \frac{1}{2} = \frac{1}{2}$$

$\therefore$ stationary point is $\left(\frac{1}{2}, \frac{1}{2}\right)$

and
$$\phi''(x) = 4 > 0$$

$\Rightarrow$ $f(x, y)$ is minimum at point $\left(\frac{1}{2}, \frac{1}{2}\right)$

Hence, minimum value of $f(x, y) = x^2 + y^2$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

End of Solution

Q.33 Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. One of the eigen vectors of M is

(a) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Ans. (b)

$\because$ $C_1 + C_2 + C_3 = 0 \Rightarrow \lambda = 0$

We know that, $MX = \lambda X$

$\Rightarrow$ $MX = 0$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consider two equations

$$x - y - 0 \cdot z = 0$$
$$x - 2y + z = 0$$

Solving $$\frac{x}{-1} = \frac{y}{-1} = \frac{z}{-1} = K$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

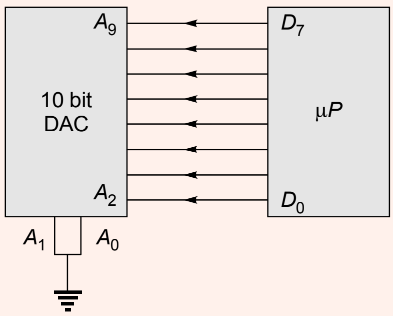
Hence one of the eigen vector is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

End of Solution

Q.34 The address lines A_9 .... A_2 of a 10 bit, 1.023 V full-scale digital to analog converter (DAC) is connected to the data lines D_7 to D_0 of an 8-bit microprocessor, with A_1 and A_0 of the DAC grounded. Now, D_7 .... D_0 is changed from 1010 1010 to 1010 1011. The corresponding change in the output of the DAC (in mV, rounded off to one decimal places) is _____.

Ans. **(4)**

V_{FS} = 1.023 10 bit DAC



1010101000 = initial input of DAC
1010101100 = changed input of DAC
0000000100 = change in input

Output voltage of DAC,

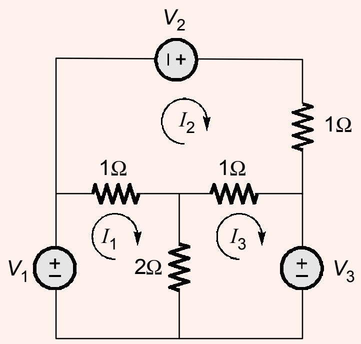
$$V_{DAC} = \frac{V_{FS}}{2^{10} - 1} \times \text{(Decimal equivalent of binary input)}$$

Change in output of DAC for corresponding input change in

$$\Delta V_{DAC} = \frac{1.023}{1023} \times 4 = 4 \text{ mV}$$

End of Solution

Q.35 I_1, I_2 and I_3 in the figure below are mesh currents. The correct set of mesh equations for these currents, in matrix form, is _____.



(a) $\begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

Ans. (c)

From inspection

$$3I_1 - I_2 - 2I_3 = V_1 \quad ...(1)$$
$$-I_1 + 3I_2 - I_3 = V_2 \quad ...(2)$$
$$-2I_1 - I_2 + 3I_3 = -V_3 \quad ...(3)$$

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

End of Solution

Q.36 Consider the finite sequence $X = (1, 1, 1)$. The Inverse Discrete Fourier Transform (IDFT) of X is given as $(x(0), x(1), x(2))$. The value of $x(2)$ is _____.

Ans. (0)

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)e^{j\frac{2\pi}{N}Kn}$$

$$= \frac{1}{3}\sum_{K=0}^{2} X(K)e^{j\frac{2\pi}{3}Kn} \quad [\because X(K) = \{X(0), X(1), X(2)\} = \{1,1,1\}]$$

$$= \frac{1}{3}\sum_{K=0}^{2} e^{j\frac{2\pi}{3}Kn} = \frac{1}{3}\left[1 + e^{j\frac{4\pi}{3}} + e^{j\frac{8\pi}{3}}\right]$$

$$= \frac{e^{j\frac{4\pi}{3}}}{3}\left[e^{j\frac{4\pi}{3}} + 1 + e^{-j\frac{4\pi}{3}}\right] = \frac{e^{j\frac{4\pi}{3}}}{3}\left[1 + 2\cos\frac{4\pi}{3}\right]$$

$$= \frac{e^{j\frac{4\pi}{3}}}{3}\left[1 + 2\left(-\frac{1}{2}\right)\right] = 0$$

End of Solution

Q.37 The loop transfer function of negative feedback system is $G(s)H(s) = \frac{2(s+1)}{s^2}$. The phase margin of the system (in degrees, rounded off to one decimal place) is _____.

Ans. (65.5)

$$G(s)H(s) = \frac{2(s+1)}{s^2}$$

$$\omega_{gc} = 2.2$$

$$PM = 180° + [-180° + \tan^{-1}(\omega_{gc})] = 65.5°$$

End of Solution

Q.38 The real power drawn by a balanced load connected to a 400 V, 50 Hz, balanced, symmetrical 3-phase, 3-wire, RYB sequence main is measured using the two-wattmeter method. Wattmeter W_1 is connected in the R line and wattmeter W_2 is connected in the B line. The line current is measured as $\frac{1}{\sqrt{3}}$ A. If the wattmeter W_1 reads zero, the reading on W_1 (in watts) is _____.

Ans. (200)

In a 2 wattmeter method is one of the wattmeter reads '0' then ϕ = 60° and the second wattmeter indicates the total power consumed by the load
and

$$\therefore \quad P = P_1 + P_2 = \frac{3}{2} V_{PH} I_{PH}$$

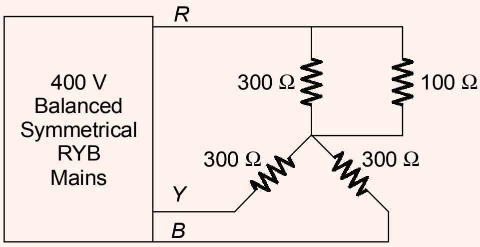
As $$I_{PH} = I_L = \frac{1}{\sqrt{3}}$$

$$V_{PH} = \frac{400}{\sqrt{3}}$$

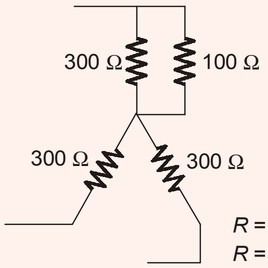
$$\therefore \quad P = \frac{3}{2} \times \frac{400}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{400}{2} = 200 \text{ watts}$$

End of Solution

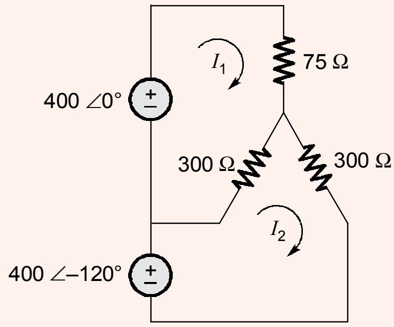
Q.39 In the circuit shown, the rms value of the voltage across the 100 Ω, resistor (in volts) _____.



Ans. (115.47)



$R = 300 \,||\, 100$
$R = 75\ \Omega$



Apply KVL in each loop

$$375\, I_1 - 300\, I_2 = 400\angle 0^\circ \quad \ldots(1)$$
$$-300\, I_1 + 600\, I_2 = 400\angle -120^\circ \quad \ldots(2)$$

Solving (1) and (2), $I_1 = 1.5396\angle -30^\circ$

$$V = 75 \times I_1 \Rightarrow 115.47\angle -30^\circ$$

The voltage across 100 Ω resistor is 115.47 V.

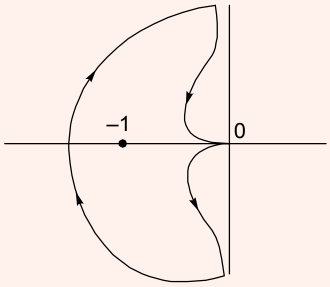
End of Solution

Q.40 The loop transfer function of a negative feedback system is $G(s)H(s) = \dfrac{1}{s(s-2)}$. The Nyquist plot for the above system _____.

(a) does not encircle $(-1 + j0)$ point
(b) encircles $(-1 + j0)$ point once in the clockwise direction
(c) encircles $(-1 + j0)$ point twice in the counterclockwise direction
(d) encircles $(-1 + j0)$ point once in the counterclockwise direction

Ans. (b)

$$G(s)H(s) = \frac{1}{s(s-2)}$$



From Nyquist plot

$$N = -1$$
$$P = 1$$

$\therefore$ Nyquist plot encircles (–1, 0) once in clockwise direction.

End of Solution

Q.41 Let $g[n] = \begin{cases} 1 & n = 0 \\ 0 & n = \pm 1, \pm 2, \pm 3, \ldots.. \end{cases}$ and $h[n] = \begin{cases} 1 & n = 0, 3, 6, 9, \ldots.. \\ 0 & \text{otherwise} \end{cases}$. Consider

$y[n] = h[n] \otimes g[n]$, where $\otimes$ denotes the convolution operator. The value of $y[2]$ is ______.

Ans. (0)

$$g(n) = \{\ldots., 0, 0, 1, 0, 0, \ldots.\} = \delta(n)$$
$$h(n) = \{1, 0, 0, 1, 0, 0, 1, 0, 0, \ldots..\}$$

Now, $y(n) = g(n) * h(n) = \delta(n) * h(n) = h(n)$

Therefore, $y(2) = h(2) = 0$

End of Solution

Q.42 Consider the differential equation $\frac{dx}{dt}$ = sin (x), with the initial condition $x(0) = 0$. The solution to this ordinary differential equation is ______.

(a) $x(t) = 0$
(b) $x(t) = \sin(t)$
(c) $x(t) = \sin(t) - \cos(t)$
(d) $x(t) = \cos(t)$

Ans. (a)

$$\frac{dx}{dt} = \sin(x)$$

Using Variable Seperable Method:

$$\int \frac{dx}{\sin x} = \int dt$$

$$\log[\text{cosec } x - \cot x] = t + c$$

$$\text{cosec } x - \cot x = e^{t+c}$$

$$\frac{1-\cos x}{\sin x} = e^{t+c}$$

$$\tan\frac{x}{2} = e^{t+c}$$

$$\frac{x}{2} = \tan^{-1}[e^{t+c}]$$

Put $t = 0$, $x = 0$

$$0 = \tan^{-1}[e^{0+c}]$$

$\Rightarrow$ $c = -\infty$

$\therefore$ solution is $\frac{x}{2} = \tan^{-1}[e^{t-\infty}]$

but as $t \geq 0$ and finite

We get, $\frac{x}{2} = \tan^{-1}[e^{-\infty}]$

$$\frac{x}{2} = 0 \Rightarrow x(t) = 0$$

End of Solution

Q.43 The loop transfer function of a negative feedback system is given by

$G(s)H(s) = \frac{K}{s(s+2)(s+6)}$, where K > 0. The value of K at the breakaway point of the root locus for the above system (rounded off to one decimal place) is _____.

Ans. (5)

$$G(s)H(s) = \frac{K}{s(s+2)(s+6)}$$

Breakaway point = –0.9

$$\therefore \quad \left|\frac{K}{s(s+2)(s+6)}\right| = 1$$

$$S = -0.9$$

$$\therefore \quad K = 5$$

End of Solution

Q.44 The system shown in figure (a) has a time response $y(t)$ to an input $r(t) = 10\ u(t)$ as shown in figure (b), $u(t)$ being the unit step input. Both K, τ are positive. The gain K of the system is _____.

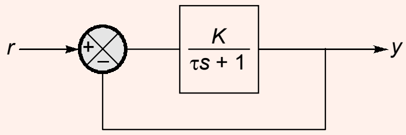


Fig. (a)

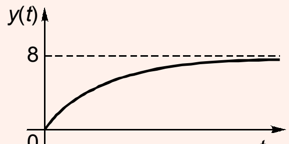


Fig. (b)

Ans. (4)

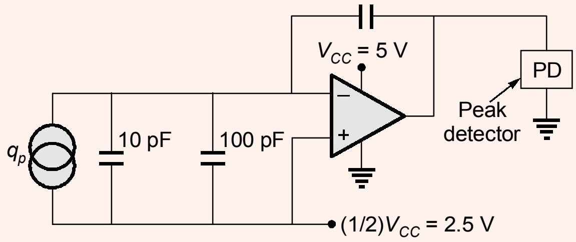
$$\text{CLTF} = \frac{K}{\tau s + 1 + K}$$

$$\text{DC gain} = \frac{K}{1+K} = \frac{8}{10}$$

$\therefore$ $K = 4$

End of Solution

Q.45 The circuit shown uses ideal opamp powered from a supply $V_{CC} = 5$ V. If the charge q_p generated by the piezoelectric sensor is of the form $q_p = 0.1 \sin(10000\ \pi t)\ \mu$C, the peak detector output after 10 cycles of q_p (in volts, rounded off to one decimal place) is _____.



Ans. (3.5)

The potential at $V_+ = V_- = 2.5$ V

because of virtual ground current drawn by opamp is 0 from q_p

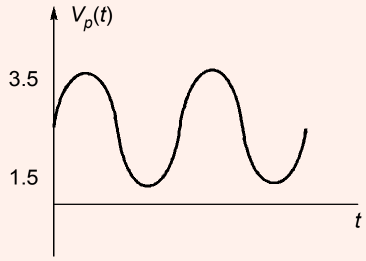
So, peak dectector output $V_p(t) = 2.5 - V_c(t)$

$$2.5 - V_p(t) = \frac{1}{C} q_p(t)$$

$$2.5 - V_p(t) = \frac{1}{100 \times 10^{-9}} \times 0.1 \sin(10^4 \pi t) \times 10^{-6}$$

$$2.5 - V_p(t) = \sin(10^4 \pi t)$$

$$\therefore \quad V_p(t) = 2.5 - \sin(10^4 \pi t)$$

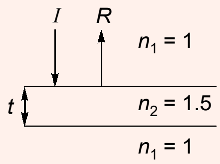


$$V_p(t) = 3.5 \text{ V}$$

After 10 cycles, peak value detected by peak detector should be 3.5 Volts.

End of Solution

Q.46 As shown in figure, a slab of finite thickness t with refractive index $n_2 = 1.5$, has air ($n_1 = 1$) above and below it. Light of free space wavelength 600 mm is incident normally from air as shown. For a destructive interference to be observed at R, the minimum value of thickness of the slab t (in nm) is _____.



Ans. (300)

Given; Wavelength (λ) = 600 nm;

Refractive index of air (N_1) = 1

Refractive Index of slab (N_2) = 1.5

Thickness of slab = t mm

Path difference of light ray in given setup is

$$PD = (N_2 - N_1) \times t$$

for the minimum thickness to be found, central destructive fringe should be observed.

So, $$PD = (2n + 1)l/2$$

$$2 \times (N_2 - N_1) \times t = l/2 \quad (\because n = 0)$$

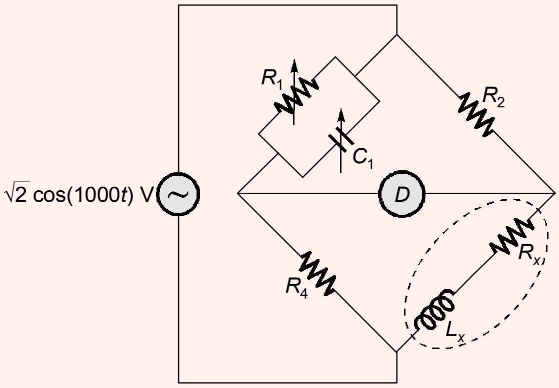
$$2 \times (1.5 - 1) \times t = l/2$$

$$t = l/2$$

$$t = 600/2 = 300 \text{ nm}$$

End of Solution

Q.47 In the Maxwell-Wien bridge shown, the detector D reads zero when C_1 = 100 nF and R_1 = 100 kΩ. The Q factor of the coil is _____.



Ans. (10)

Given,
$$C_1 = 100 \text{ nF}$$
$$R_1 = 100 \text{ k}\Omega$$
$$\omega = 1000$$

In the above circuit @ balance

$$Q_n = \omega C_1 R_1$$
$$= 1000 \times 100 \times 10^{-9} \times 100 \times 10^{3}$$
$$= 10$$

End of Solution

Q.48 A $6\frac{1}{2}$ digit timer-counter is set in the 'time period' mode of operation and the range is set as '*ns*'. For an input signal, the timer-counter displays 1000000. With the same input signal, the timer-counter is changed to 'frequency' mode of operation and the range is set as 'Hz'. The display will show the number _____.

Ans. (1000)

$6\frac{1}{2}$ digital voltmeter time period

1	0	0	0	0	0	0
10^6	10^5	10^4	10^3	10^2	10^1	10^0

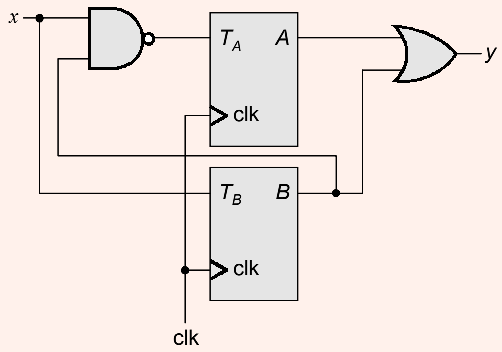
$$10^6\text{ns} = 10^{-3} \text{ sec}$$

Frequency $\dfrac{1}{10^{-3}}$ Hz $= 1000$ Hz

1 0 0 0

End of Solution

Q.49 Two T-flip flops are interconnected as shown in the figure. The present state of the flip flops are : $A = 1$, $B = 1$. The input x is given as 1, 0, 1 in the next three clock cycles. The decimal equivalent of $(ABy)_2$ with A being the MSB and y being the LSB, after the 3rd clock cycle is _____.



Ans. (7)

$T_A = \overline{X \cdot B}$ $\qquad$ $A^+ = \overline{X \cdot B} \oplus A$

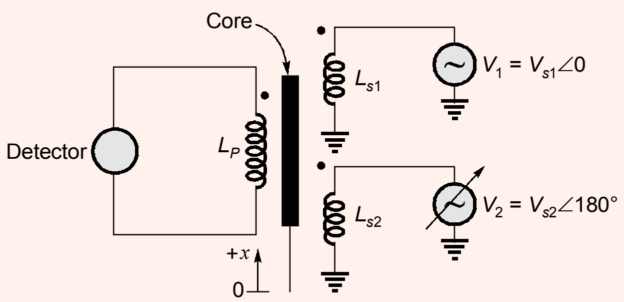
$T_B = X$ $\qquad$ $B^+ = X \oplus B$ $\qquad$ $Y = A + B$

X	A	B	A^+	B^+	y
1	1	1	1	0	1
0	1	0	0	0	0
1	0	0	1	1	1

So $\qquad (ABy)_2 = (111)_2 = 7$

End of Solution

Q.50 The mutual inductances between the primary coil and the secondary coils of a linear variable differential transformer (LVDT) shown in the figure are M_1 and M_2. Assume that the self-inductances L_{S1} and L_{S2} remain constant and are independent of x. When $x = 0$, $M_1 = M_2 = M_0$. When x is in the range ± 10 mm, M_1 and M_2 change linearly with x. At $x = +10$ mm or –10 mm, the change in the magnitude of M_1 and M_2 is 0.25 M_0. For a particular displacement $x = D$, the voltage across the detector becomes zero when $|V_2| = 1.25\,|V_1|$. The value of D (in mm, rounded off to one decimal place) is _____.



Ans. (10)

Given that,

Self-inductances L_{s1} and L_{s2} remain constant and are independent of x.

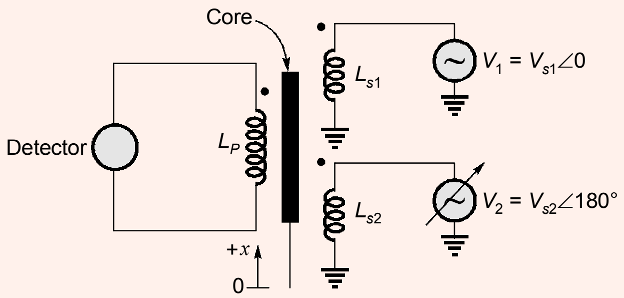
At $x = 0$, $M_1 = M_2 = M_0$

M_1 and M_2 change linearly with x. This means

$$M_1 = M_0 + k_1 x$$
$$M_2 = M_0 + k_2 x$$

at $x = +\ 10$ mm

$$M_1 = M_0 + k_1 \times 10 \times 10^{-3}$$
$$M_2 = M_0 + k_2 \times 10 \times 10^{-3}$$
$$M_1 - M_2 = (k_1 - k_2) \times 10 \times 10^{-3} = 0.25\ M_0$$
$$(k_1 - k_2) = 25\ M_0 \quad ...(i)$$



Voltage across the detector (V_D) = Volteage develpped across the primary coil (I_P)

$$V_D = V_{LP}$$

Voltage develped across the primarycoil depends on the primary current and current in the secondary coils.

So, $$V_D = V_{LP} = L_P \frac{di_p}{dt} + M_1 \frac{di_{s_1}}{dt} - M_2 \frac{di_{s_2}}{dt} \quad ...(ii)$$

($\because$ i_{s1} and i_{s2} are out of phase)

Accroding to the given information detector output voltage (V_D) = 0 at $x = D$ and there can't be any primary current (i_p)

So, equation (ii) can be written as

$$M_1 \frac{di_{s_1}}{dt} - M_2 \frac{di_{s_2}}{dt} = 0 \quad ...(iii)$$

From the diagram, voltages across secodnary coils can be written as

$$V_1 = L_{s_1} \frac{di_{s_1}}{dt}$$
$$V_2 = L_{s_2} \frac{di_{s_2}}{dt}$$

as L_{s_1}, L_{s_2} are not changing and initial voltages across both should be zero.

So, $$L_{s_1} = L_{s_2} = L_s$$

$$V_1 = L_s \frac{di_{s_1}}{dt} \quad \text{and} \quad V_2 = L_s \frac{di_{s_2}}{dt} \quad \text{... (iv)}$$

Consider equation (iii) xL_s

Then, $M_1 L_s \frac{di_{s_1}}{dt} - M_2 L_s \frac{di_{s_2}}{dt} = 0$

$$M_1 V_1 - M_2 V_2 = 0 \quad \text{...(v)}$$

at $x = D$

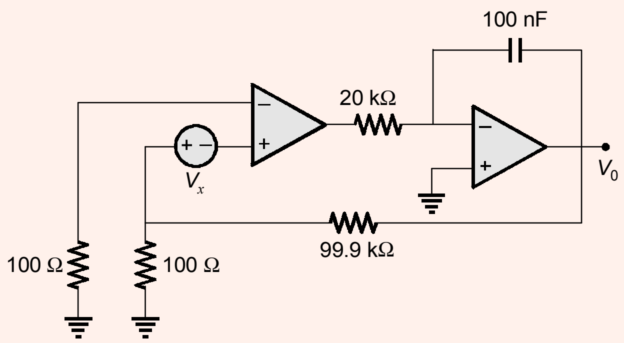
$$M_1 = M_0 + k_1 D$$
$$M_2 = M_0 + k_2 D$$
$$V_2 = 1.25\ V_1$$

From equation (v)

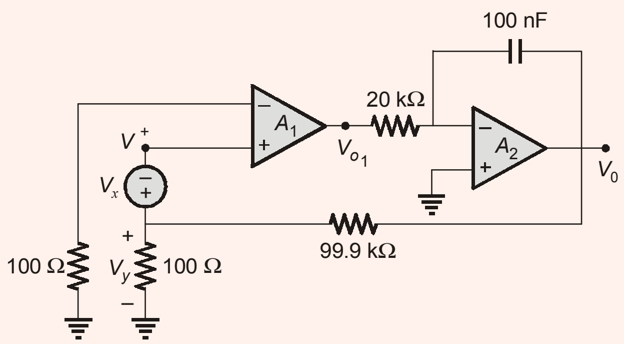
$$(M_0 + k_1 D)V_1 - (M_0 + k_2 D) \times 1.25 V_1 = 0$$
$$(k_1 - k_2)D = 0.25\ M_0$$
$$D = \frac{0.25M_0}{k_1 - k_2} = \frac{0.25M_0}{25M_0} = 10 \times 10^{-3} \text{ (From eq.(i))}$$
$$D = 10 \text{ mm}$$

End of Solution

Q.51 If the opamps in the circuit shown are ideal and V_x = 0.5 mV, the steady state value of V_0 (in volts, rounded off to two decimal places) is _____.



Ans. (0.50)



Given: $V_x = 0.5$ mV

For op-amp A_1: $V^- = 0$

$$V^+ = -V_x + V_y$$

$$V^+ = -0.5\times10^{-3} + \frac{V_o\times0.1}{0.1+99.1} = (-0.5 + V_o) \times 10^{-3}\ \text{V}$$

Let $V_o = 0$ at $t = 0 \Rightarrow V^+ = -0.5$ mV

$\Rightarrow$ Output of A_1 should be $-V_{sat}$

Op-amp A_2 is integrator

$$V_o = -\frac{1}{RC}\int_t V_{o_1}\,dt = \frac{-1}{20\times10^3\times10^{-7}}\int_t -V_{sat}\,dt$$

$$V_o = 500 \times V_{sat} \times t$$

If $t\uparrow$ then $V_o\uparrow \Rightarrow V_y\uparrow \Rightarrow V^+\downarrow$

If V_y becomes +0.5 mV, then

$$V_y = \frac{V_o}{1000}$$

$\Rightarrow$ $V_o = 1000\ V_y = 0.5$ V

If V_y becomes slightly greater than 0.5 mV then V^+ becomes slightly positive. Hence V_{o_1} becomes $+V_{sat}$.

$\Rightarrow$ $V_o = -500 \times V_{sat} \times t$

i.e. V_o should becomes a negative ramp.

As $V_o\downarrow \Rightarrow V_y\downarrow \Rightarrow V^+\downarrow$

If V_y again becomes 0.5 mV or slightly less then V^+ becomes negative.

$\Rightarrow$ V_{o_1} becomes $-V_{sat}$ and V_o comes back to 0.5 V.

Thus output V_o is forced to remain constant at 0.5 V.

End of Solution

Q.52 A straight line drawn on an x-y plane intercepts the x-axis at –0.5 and the y-axis at 1. The equation that describes this line is _____.

(a) $y = x - 0.5$

(b) $y = 2x + 1$

(c) $y = 0.5x - 1$

(d) $y = -0.5x + 1$

Ans. (b)

Given line x-intercept is $-\frac{1}{2}$ and y-intercept is 1

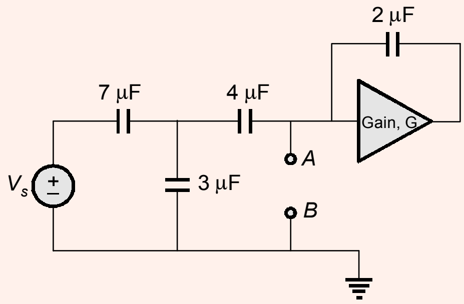
Then the line equation is $\frac{x}{-\frac{1}{2}} + \frac{y}{1} = 1$

$\Rightarrow$ $-2x + y = 1$

$\Rightarrow$ $y = 2x + 1$

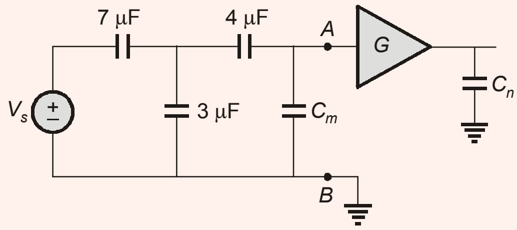
End of Solution

Q.53 A circuit consisting of capacitors, DC voltage source and an amplifier having a voltage gain $G = -5$ is shown in the figure. The effective capacitance across the nodes A and B (in μF, rounded off to one decimal place) is _____.



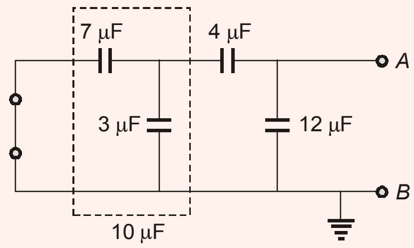
Ans. **(14.86)**

Apply Miller's theorem to $C = 2\ \mu F$



$$C_m = C(1 + A_V) = 2 \times (1 + 5) = 12\ \mu F$$

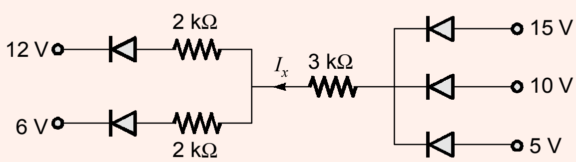
Disable V_S



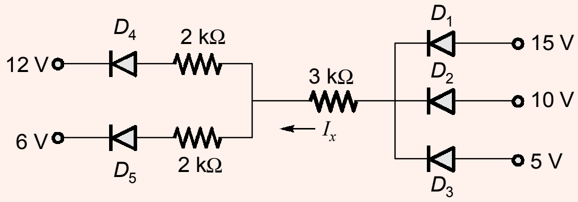
$$C_{AB} = \frac{10 \times 4}{10 + 4} + 12 = 14.86\ \mu F$$

End of Solution

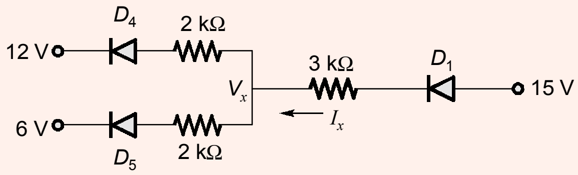
Q.54 Assume the diodes in the circuit shown are ideal. The current I_x flowing through the 3 kΩ resistor (in mA, rounded off to one decimal place) is _____.



Ans. (1.8)



D_1, D_2, D_3 are in common cathode connection. Only D_1 can conduct D_2 and D_3 remain OFF. Let D_4, D_5 be ON.



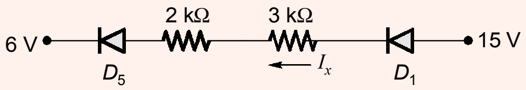
KCL at V_x Node:

$$I_x = I_4 + I_5$$

$$\frac{15 - V_x}{3} = \frac{V_x - 12}{2} + \frac{V_x - 6}{2} = \frac{2V_x - 18}{2}$$

$$\frac{15 - V_x}{3} = V_x - 9 \quad \Rightarrow \quad V_x = 10.5 \text{ V}$$

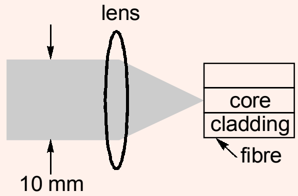
$\because \quad V_x < 12 \text{ V} \quad \Rightarrow \quad D_4$ must be OFF



$$I_x = \frac{15 - 6}{2 + 3} = 1.8 \text{ mA}$$

End of Solution

Q.55 A laser beam of 10 mm beam diameter is focused onto an optical fibre using a thin biconvex lens as shown in the figure. The refractive index of the lens is 1.5. The refractive indices of the core and cladding of the fibre are 1.55 and 1.54 respectively. The minimum value of the focal length of the lens to attain the maximum coupling to the fibre (in mm, rounded off to one decimal place) is _____.



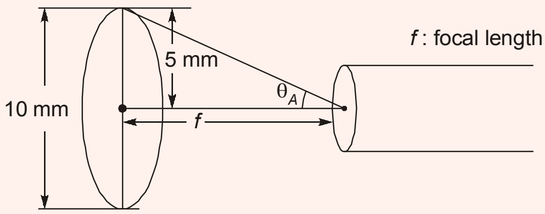
Ans. (28)

Given, the diameter of Laser beam = 10 mm

Refractive index of lens = 1.5

Refractive index of core (n_1) = 1.55

Refractive index of cladding (n_2) = 1.54



$$\text{Acceptance angle } (\theta_a) = \sin^{-1}\left[\frac{NA}{N_m}\right]$$

(NA → Numerical aperture; N_m → Surrounding medium (air))

$$\theta_A = \sin^{-1}\left(\frac{NA}{n_0}\right)$$

$$\theta_A = \sin^{-1}\left[\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right] = \sin^{-1}\left[\sqrt{\frac{(1.55)^2 - (1.54)^2}{1}}\right]$$

$$\theta_A = \sin^{-1}(0.1757839) = 10.124°$$

from the figure, $\tan\theta_A = \dfrac{5\text{ mm}}{f}$

$$f = \frac{5\text{ mm}}{\tan\theta_A} = \frac{5\text{ mm}}{\tan(10.124°)} = 28\text{ mm}$$

End of Solution

■■■■