34 Years
Previous Solved Papers

GATE 2021

Electronics Engineering

- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

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Publications

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GATE - 2021 : Electronics Engineering Topicwise Previous GATE Solved Papers (1987-2020)

1st Edition	:	2007
2nd Edition	:	2008
3rd Edition	:	2009
4th Edition	:	2010
5th Edition	:	2011
6th Edition	:	2012
7th Edition	:	2013
8th Edition	:	2014
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10th Edition	:	2016
11th Edition	:	2017
12th Edition	:	2018
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14th Edition	**:**	**2020**

Preface

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

B. Singh (Ex. IES)

The new edition of **GATE 2021 Solved Papers : Electronics Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

Contents

Unit .VIII

Signals and Systems

Contents

UNIT VIII Signals and Systems

Syllabus : Continuous-time signals: Fourier series and Fourier transform representations, sampling theorem and applications; Discrete-time signals: discrete-time Fourier transform (DTFT), DFT, FFT, Z-transform, interpolation of discrete-time signals; LTI systems: definition and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay, digital filter design techniques.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
1987	–	2	**4**
1988	–	6	**12**
1989	–	–	**–**
1990	–	4	**8**
1991	–	3	**6**
1992	–	3	**6**
1993	–	3	**6**
1994	4	–	**4**
1995	5	1	**7**
1996	2	2	**6**
1997	2	2	**6**
1998	12	–	**12**
1999	4	2	**8**
2000	3	3	**9**
2001	4	1	**6**
2002	4	2	**8**
2003	4	4	**12**
2004	3	6	**15**
2005	6	6	**18**
2006	3	3	**9**
2007	1	4	**9**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2008	2	8	**18**
2009	3	5	**13**
2010	3	2	**7**
2011	3	4	**11**
2012	2	3	**8**
2013	6	3	**12**
2014 Set-1	3	4	**11**
2014 Set-2	3	3	**9**
2014 Set-3	4	3	**10**
2014 Set-4	4	4	**12**
2015 Set-1	2	3	**8**
2015 Set-2	5	4	**13**
2015 Set-3	4	5	**14**
2016 Set-1	4	4	**12**
2016 Set-2	1	2	**5**
2016 Set-3	3	4	**11**
2017 Set-1	3	4	**11**
2017 Set-2	2	4	**10**
2018	3	3	**9**
2019	3	3	9
2020	2	4	10

1 Basics of Signals and Systems

1.1 An excitation is applied to a system at $t = T$ and its response is zero for $-\infty < t < T$. Such a system is a
(a) non-causal system (b) stable system
(c) causal system (d) unstable system
[1991 : 2 Marks]

1.2 Which of the following signals is/are periodic?
(a) $s(t) = \cos 2t + \cos 3t + \cos 5t$
(b) $s(t) = \exp(j8\pi t)$
(c) $s(t) = \exp(-7t)\sin 10\pi t$
(d) $s(t) = \cos 2t \cos 4t$
[1992 : 2 Marks]

1.3 A system with an input $x(t)$ and output $y(t)$ is described by the relation: $y(t) = t x(t)$. This system is
(a) linear and time-invariant
(b) linear and time varying
(c) non-linear & time-invariant
(d) non-linear and time-varying
[2000 : 1 Mark]

1.4 Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is
(a) 1 (b) –1
(c) 0 (d) $\frac{\pi}{2}$
[2001 : 1 Mark]

1.5 If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to
(a) E (b) $\frac{E}{2}$
(c) $2E$ (d) $4E$
[2001 : 1 Mark]

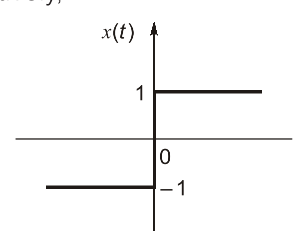
1.6 Let P be linearity, Q be time-invariance, R be causality and S be stability. A discrete-time system has the input-output relationship,

$$y(n) = \begin{cases} x(n) & n \ge 1 \\ 0, & n = 0 \\ x(n+1), & n \le -1 \end{cases}$$

where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties
(a) P, S but not Q, R
(b) P, Q, S but not R
(c) P, Q, R, S
(d) Q, R, S but not P **[2003 : 2 Marks]**

1.7 Consider the sequence
$x[n] = \left[-4 - j5,\ \underset{\uparrow}{1 + j2},\ 4\right]$
The conjugate antisymmetric part of the sequence is
(a) $[-4 - j2.5,\ j2,\ 4 - j2.5]$
(b) $[-j2.5,\ 1,\ j2.5]$
(c) $[-j5,\ \ j2,\ \ 0]$
(d) $[-4,\ \ 1,\ \ 4]$ **[2004 : 2 Marks]**

1.8 The function $x(t)$ is shown in the figure. Even and odd parts of a unit-step function $u(t)$ are respectively,



(a) $\frac{1}{2}, \frac{1}{2}x(t)$ (b) $-\frac{1}{2}, \frac{1}{2}x(t)$
(c) $\frac{1}{2}, -\frac{1}{2}x(t)$ (d) $-\frac{1}{2}, -\frac{1}{2}x(t)$
[2005 : 1 Mark]

1.9 The power in the signal
$s(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$ is
(a) 40 (b) 41
(c) 42 (d) 82 **[2005 : 1 Mark]**

1.10 The Dirac delta function $\delta(t)$ is defined as
(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$
(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

[2006 : 1 Mark]

1.11 A system with input $x[n]$ and output $y[n]$ is given as $y[n] = \left(\sin\frac{5}{6}\pi n\right)x(n)$.

The system is

(a) linear, stable and invertible
(b) non-linear, stable and non-invertible
(c) linear, stable and non-invertible
(d) linear, unstable and invertible

[2006 : 2 Marks]

1.12 A Hilbert transformer is a

(a) non-linear system
(b) non-causal system
(c) time-varying system
(d) low-pass system [2007 : 2 Marks]

1.13 The input and output of a continuous time system are respectively denoted by $x(t)$ and $y(t)$. Which of the following descriptions corresponds to a casual system?

(a) $y(t) = x(t-2) + x(t+4)$
(b) $y(t) = (t-4)\,x(t+1)$
(c) $y(t) = (t+4)\,x(t-1)$
(d) $y(t) = (t+5)\,x(t+5)$ [2008 : 1 Mark]

1.14 Let $x(t)$ be the input and $y(t)$ be the output of a continuous time system. Match the system properties P_1, P_2 and P_3 with system relations R_1, R_2, R_3, R_4.

Properties

P_1: Linear but NOT time-invariant
P_2: Time-invariant but NOT linear
P_3: Linear and time-invariant

Relations

R_1: $y(t) = t^2 x(t)$
R_2: $y(t) = t|x(t)|$
R_3: $y(t) = |x(t)|$
R_4: $y(t) = x(t-5)$

(a) $(P_1, R_1), (P_2, R_3), (P_3, R_4)$
(b) $(P_1, R_2), (P_2, R_3), (P_3, R_4)$
(c) $(P_1, R_3), (P_2, R_1), (P_3, R_2)$
(d) $(P_1, R_1), (P_2, R_2), (P_3, R_3)$ [2008 : 2 Marks]

1.15 The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$. The system is

(a) time-invariant and stable
(b) stable and not time-invariant
(c) time-invariant and not stable
(d) not time-invariant and not stable

[2012 : 2 Marks]

1.16 For a periodic signal
$v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$,
the fundamental frequency in rad/s is

(a) 100 (b) 300
(c) 500 (d) 1500

[2013 : 1 Mark]

1.17 The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is

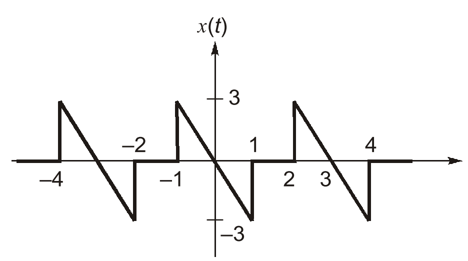
(a) 0 (b) 1
(c) 2 (d) 3

[2013 : 2 Marks]

1.18 A discrete-time signal $x[n] = \sin(\pi^2 n)$, n being an integer, is

(a) periodic with period π
(b) periodic with period π^2
(c) periodic with period $\pi/2$
(d) not periodic [2014 : 1 Mark, Set-1]

1.19 The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is ________.

[2015 : 1 Mark, Set-1]

1.20 Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_1[n] = \alpha\, 0.5^n\, u[n]$, where α is a positive real number and $u[n]$ is the unit step sequence. Assume

$$x_2[n] = \begin{cases} \sqrt{1.5} & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the value of α is _____.

[2015 : 2 Marks, Set-3]

1.21 A continuous time function $x(t)$ is periodic with period T. The function is sampled uniformly with a sampling period T_s. In which one of the following cases is the sampled signal periodic?

(a) $T = \sqrt{2}T_s$ (b) $T = 1.2T_s$
(c) Always (d) Never

[2016 : 1 Mark, Set-1]

1.22 Consider a single input single output discrete-time system with $x[n]$ as input and $y[n]$ as output, where the two are related as

$$y[n] = \begin{cases} n|x[n]|, & \text{for } 0 \le n \le 10 \\ x[n] - x[n-1], & \text{otherwise.} \end{cases}$$

Which one of the following statements is true about the system?

(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal nor stable

[2017 : 1 Mark, Set-1]

1.23 The input $x(t)$ and the output $y(t)$ of a continuous-time system are related as

$$y(t) = \int_{t-T}^{t} x(u)\,du$$

The system is

(a) linear and time-variant
(b) linear and time-invariant
(c) non-linear and time-variant
(d) non-linear and time-invariant

[2017 : 1 Mark, Set-2]

1.24 Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

(a) $\dfrac{d^3y}{dt^3} + a_1\dfrac{d^2y}{dt^2} + a_2\dfrac{dy}{dt} + a_3y$
$= b_3u + b_2\dfrac{du}{dt} + b_1\dfrac{d^2u}{dt^2}$
(with initial rest conditions)

(b) $y(t) = \int_0^t e^{\alpha(t-\tau)}\beta u(\tau)\,d\tau$

(c) $y = au + b, b \neq 0$

(d) $y = au$

[2018 : 1 Mark]

1.25 Consider the signal

$$f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right),$$

where t is in seconds. Its fundamental time period, in seconds, is _______.

[2019 : 1 Mark]

■■■

Answers Basics of Signals and Systems

1.1	(c)	1.2	(a, b, d)	1.3	(b)	1.4	(a)	1.5	(b)	1.6	(a)	1.7	(a)
1.8	(a)	1.9	(a)	1.10	(d)	1.11	(c)	1.12	(b)	1.13	(c)	1.14	(a)
1.15	(d)	1.16	(a)	1.17	(b)	1.18	(d)	1.19	(2)	1.20	(1.5)	1.21	(b)
1.22	(a)	1.23	(b)	1.24	(c)	1.25	(12)						

Explanations Basics of Signals and Systems

1.1 (c)

For the given system if the response is zero prior to the application of the excitation. Then such a system is called causal system.

1.2 (a, b, d)

(a) $s(t)$ is periodic as the ratio of any two frequencies $= \frac{p}{q}$ is rational

where p and q are integers.

(b) $s(t)$ is periodic with $\omega = 8\pi$

(d) $2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$$s(t) = \frac{1}{2}[\cos 2t + \cos 6t]$$

So, $s(t)$ is periodic with fundamental frequency 2 rad/sec.

1.3 (b)

$$ay_1(t) = atx_1(t)$$
$$ay_2(t) = atx_2(t)$$
$$a[y_1(t) + y_2(t)] = a[tx_1(t) + tx_2(t)]$$

∴ System is linear.

$$y(t - t_0) = (t - t_0)\,x(t - t_0)$$

Delay is introduced. So, time varying.

1.4 (a)

$$\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right) dt = f(0) = \cos\left(\frac{3 \times 0}{2}\right) = \cos 0 = 1$$

1.5 (b)

$$E = \int_{-\infty}^{\infty} f(t)^2\, dt$$

$$E' = \int_{-\infty}^{\infty} f(2t)^2\, dt$$

$$= \int_{-\infty}^{\infty} f(p)^2 \frac{dp}{2} \left(2t = p;\ dt = \frac{dp}{2}\right)$$

$$E' = \frac{E}{2}$$

1.6 (a)

$$y(n - n_0) = x(n - n_0 + 1) \text{(time varying)}$$
$$y(n) = x(n + 1) \text{ (depends on future)}$$

i.e. $y(1) = x(2)$ (non causal)

For bounded input, system has bounded output. So it is stable.

$$y(n) = x(n),\ n \geq 1$$
$$= 0,\ n = 0$$
$$= x(n + 1),\ n \leq -1$$

So, system is linear.

1.7 (a)

$$x(n) = [-4 - j5,\ \underset{\uparrow}{1 + 2j},\ 4]$$

$$x^*(-n) = [4,\ \underset{\uparrow}{1 - 2j},\ -4 + j5]$$

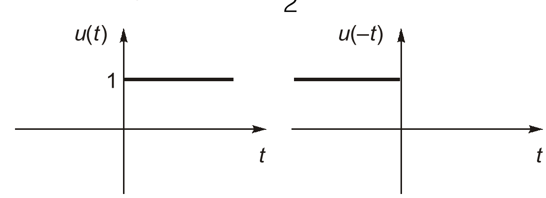
$$x_{CAS}(n) = \frac{x(n) - x^*(-n)}{2}$$

$$= [-4 - 2.5j,\ \underset{\uparrow}{2j},\ 4 - 2.5j]$$

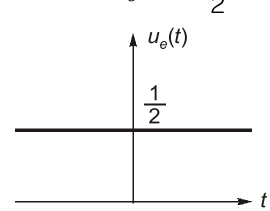
1.8 (a)

$$\text{Even part} = \frac{u(t) + u(-t)}{2}$$

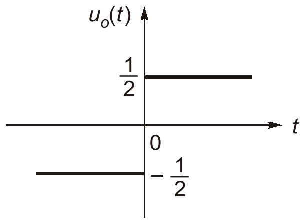
$$\text{Odd part} = \frac{u(t) - u(-t)}{2}$$



$$u_e(t) = \frac{1}{2}$$



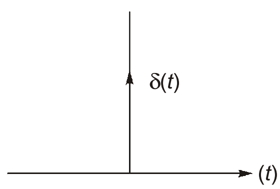
$$u_o(t) = \frac{x(t)}{2}$$



1.9 (a)

$$s(t) = 8\cos\left(\frac{\pi}{2} - 20\pi t\right) + 4\sin 15\pi t$$
$$= 8\sin 20\pi t + 4\sin 15\pi t$$
$$P = \frac{8^2}{2} + \frac{4^2}{2} = 32 + 8 = 40$$

1.10 (d)



1.11 (c)

$$y(n) = \left(\sin\frac{5}{6}\pi n\right)x(n)$$

Let $\quad x(n) = \delta(n)$

$\therefore \quad y(n) = \sin 0 = 0$ (bounded)

BIBO stable

1.12 (b)

The Hilbert transformer is characterised by the impulse response

$$h(t) = \frac{1}{\pi t} \quad t \in (-\infty, \infty)$$

$h(t) \neq 0$ for $t < 0$

Thus, the Hilbert transformer is a non-causal system.

1.13 (c)

A system is casual if the output at any time depends only on values of the input at the present time and in the past.

1.15 (d)

$$y = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$
$$y(t - t_0) = \int_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)\,d\tau$$

$y'(t)$ for input $x(t - t_0)$ is

$$y'(t) = \int_{-\infty}^{t} x(\tau - t_0)\cos 3\tau\,d\tau$$
$$y'(t) = \int_{-\infty}^{(t-t_0)} x(\tau)\cos 3(\tau + t_0)\,d\tau$$

$y'(t) \neq y(t - t_0)$ so system is not time invariant for input $x(\tau) = \cos(3\tau)$ (bounded input)

$$y(t) = \int_{-\infty}^{t} \cos^2(3\tau)\,d\tau \to \infty \text{ as } t \to \infty$$

So, for bounded i/p, o/p is not bounded therefore system is not stable.

1.16 (a)

$\omega_1 = 100$

$\omega_2 = 300$

$\omega_3 = 500$

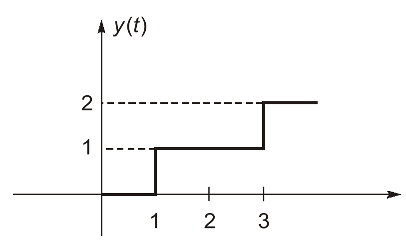
H.C.F. of (ω_1, ω_2 and ω_3) = H.C.F. (100, 300, 500)

$\omega = 100$ rad/sec.

1.17 (b)

Step response = Integration of impulse response.

$$\int_{-\infty}^{t} \delta(t-1)\,dt = u(t-1)$$
$$\int_{-\infty}^{t} \delta(t-3)\,dt = u(t-3)$$



at $\quad t = 2$

$y(t) = 1$

1.18 (d)

$x[n] = \sin(\pi^2 n)$

$\omega_o = \pi^2$

$$\therefore \quad N = \frac{2\pi}{\omega_o} \cdot m$$

where m is the smallest integer that converts $\frac{2\pi}{\omega_o}$ into a integer value.

$$\therefore \quad N = \frac{2\pi}{\pi^2} \cdot m = \frac{2}{\pi} \cdot m$$

So, there exists no such integer value of m which could make the N integer, so the system is not periodic.

1.19 (2)

$$x(t) = -3t, \quad -1 < t < 1$$

$$x\left(\frac{t-1}{2}\right) = -\frac{3}{2}(t-1) \qquad -1 < t < 3$$

and $T = 6$

$$\text{Average power} = \frac{1}{6}\int_{-1}^{3}\left(-\frac{3}{2}(t-1)\right)^2 dt = 2$$

1.20 (1.5)

$$\text{Energy of } x_1[n] = \sum_{n=-\infty}^{\infty} |x_1[n]|^2$$

$$= \sum_{n=0}^{\infty} \alpha^2\left(\frac{1}{4}\right)^n = \alpha^2 \cdot \frac{1}{1-\frac{1}{4}} = \alpha^2 \cdot \frac{4}{3}$$

Energy of $x_2[n] = 1.5 + 1.5 = 3$

$$\Rightarrow \quad \alpha^2 \frac{4}{3} = 3$$

$$\Rightarrow \quad \alpha^2 = \frac{9}{4}$$

$$\Rightarrow \quad \alpha = 1.5$$

1.21 (b)

A signal is said to be periodic if $\frac{T}{T_s}$ is a rational number.

Here, $T = 1.2\, T_s$

$\Rightarrow \quad \frac{T}{T_s} = \frac{6}{5}$ Which is a rational number

1.22 (a)

Since present output does not depend upon future values of input, the system is causal and also every bounded input produces bounded output, so it is stable.

1.23 (b)

Given that, $y(t) = \int_{t-T}^{t} x(u)\,du$

- Since the given system satisfies both homogeneity and additivity properties, the system is linear.
- Check for time invariance:

$$y(t - t_o) = \int_{t-t_o-T}^{t-t_o} x(u)\,du$$

- When the applied input is $x(t - t_o)$,

$$y_1(t) =$$

$$\int_{t-T}^{t} x(u-t_o)\,du = \int_{t-t_o-T}^{t-t_o} x(\tau)\,d\tau$$

$$= y(t - t_o)$$

$\Rightarrow$ System is time invariant

1.24 (c)

$y = au + b$, $b \neq 0$ is a non-linear system.

1.25 (12)

$$f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$$

$$\omega_1 = \pi$$

$$\omega_2 = \frac{2\pi}{3}$$

$$\omega_3 = \frac{\pi}{2}$$

$$\omega_0 = \text{GCD}\left(\pi, \frac{2\pi}{3}, \frac{\pi}{2}\right) = \frac{\pi}{6}$$

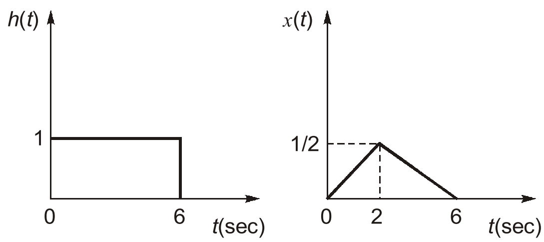
Fundamental period,

$$N = \frac{2\pi}{\omega_o} = \frac{2\pi}{(\pi/6)} = 12$$

■■■

2 LTI Systems Continuous and Discrete (Time Domain)

2.1 The impulse response and the excitation function of a linear time invariant causal system are shown in figure (a) and (b) respectively. The output of the system at $t = 2$ sec. is equal to



(a) 0 (b) 1/2
(c) 3/2 (d) 1 **[1990 : 2 Marks]**

2.2 Let $h(t)$ be the impulse response of a linear time invariant system. Then the response of the system for any input $u(t)$ is

(a) $\int_0^t h(\tau)u(t-\tau)d\tau$

(b) $\frac{d}{dt}\int_0^t h(\tau)u(t-\tau)d\tau$

(c) $\int_0^t \left[\int_0^t h(\tau)u(t-\tau)d\tau\right]dt$

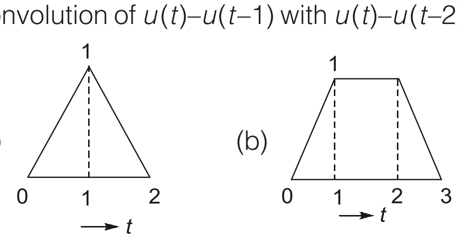
(d) $\int_0^t h^2(\tau)u(t-\tau)d\tau$ **[1995 : 1 Mark]**

2.3 The unit impulse response of a linear time invariant system is the unit step function $u(t)$. For $t > 0$, the response of the system to an excitation $e^{-at}\,u(t)$, $a > 0$ will be

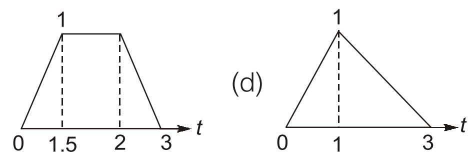
(a) $a\,e^{-at}$ (b) $(1/a)\,(1 - e^{-at})$
(c) $a(1 - e^{-at})$ (d) $1 - e^{-at}$
[1998 : 1 Mark]

2.4 Let $u(t)$ be the step function. Which of the waveforms in the figure corresponds to the convolution of $u(t)-u(t-1)$ with $u(t)-u(t-2)$?

(a) (b)



(c) (d)



[2000 : 2 Marks]

2.5 The impulse response functions of four linear systems S_1, S_2, S_3, S_4 are given respectively by

$h_1(t) = 1$ $h_2(t) = u(t)$

$h_3(t) = \dfrac{u(t)}{t+1}$ $h_4(t) = e^{-3t}u(t)$

Where $u(t)$ is the unit step function. Which of these systems is time invariant, causal, and stable?

(a) S_1 (b) S_2
(c) S_3 (d) S_4
[2001 : 2 Marks]

2.6 Convolution of $x(t+5)$ with impulse function $\delta(t-7)$ is equal to

(a) $x(t-12)$ (b) $x(t+12)$
(c) $x(t-2)$ (d) $x(t+2)$
[2002 : 1 Mark]

2.7 The impulse response $h[n]$ of a linear time-invariant system is given by

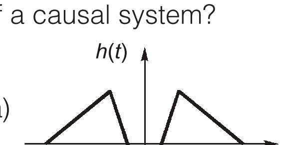
$h[n] = u[n+3] + u[n-2] - 2u[n-7]$

where $u[n]$ is the unit step sequence. The above system is

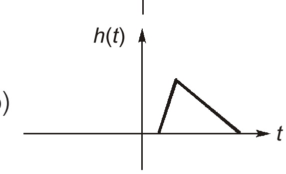
(a) stable but not causal
(b) stable and causal
(c) causal but unstable
(d) unstable and not causal
[2004 : 1 Mark]

2.8 Which of the following can be impulse response of a causal system?

(a)



(b)



$$Y(s) = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]$$

Taking inverse Laplace transform

$$y(t) = \frac{1}{a}\left[1 - e^{-at}\right]$$

2.4 (b)

$$u(t) - u(t-1) = \frac{1}{s}\left[1 - e^{-s}\right]$$

$$u(t) - u(t-2) = \frac{1}{s}\left[1 - e^{-2s}\right]$$

Convolution in time domain = Multiplication in s-domain

$$\Rightarrow \quad \frac{1}{s^2}\left[1 - e^{-s}\right]\left[1 - e^{-2s}\right]$$

$$= \frac{1}{s^2}\left[1 - e^{-2s} - e^{-s} + e^{-3s}\right]$$

$= tu(t) - (t-1)\,u(t-1) - (t-2)\,u(t-2) + (t-3)\,u(t-3)$

$= r(t) - r(t-1) - r(t-2) + r(t-3)$

2.5 (d)

$h_1(t) \neq 0 \quad$ for $t < 0$

Therefore s_1 is non causal

$h_2(t) = u(t) = 0$ for $t < 0$

$$\int_{-\infty}^{\infty} h_2(t)\,dt = \int_{-\infty}^{\infty} u(t)\,dt = \int_{0}^{\infty} dt = \infty$$

Therefore s_2 is unstable and causal

$$h_3(t) = \frac{u(t)}{t+1}$$

at $\quad t = -1$

$h_3(t) = \infty$, $h_3(t) = 0$ for $t < 0$

Therefore s_3 is unstable and causal

$h_4(t) = e^{-3t}\,u(t)$

s_4 is time invariant, causal and stable.

2.6 (c)

$$x(t+5) * \delta(t-7) = x(t+5-7)$$
$$= x(t-2)$$

2.7 (a)

$$\sum_{k=-\infty}^{\infty} h(k) = \sum_{k=-3}^{\infty} u(k+3) + \sum_{k=2}^{\infty} u(k-2)$$
$$-2\sum_{k=7}^{\infty} u(k-7)$$
$$= \sum_{k=-3}^{6} 1 + \sum_{k=2}^{6} 1$$

$= 10 + 5 = 15 < \infty$

For bounded input, bounded output. So system is stable.

Response depends on future value of input signal i.e. $u(n+3)$. So, system is not causal.

2.8 (b)

For a causal system

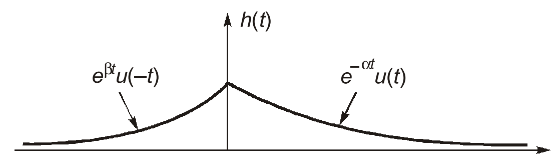
$h(t) = 0$ for $t < 0$

2.9 (d)

$$h(t) = e^{\alpha t}u(t) + e^{\beta t}u(-t)$$

For the system to be stable, $\int_{-\infty}^{\infty} h(t)dt < \infty$

For the above condition, $h(t)$ should be as shown below.



Therefore $\alpha < 0$ & $\beta > 0$.

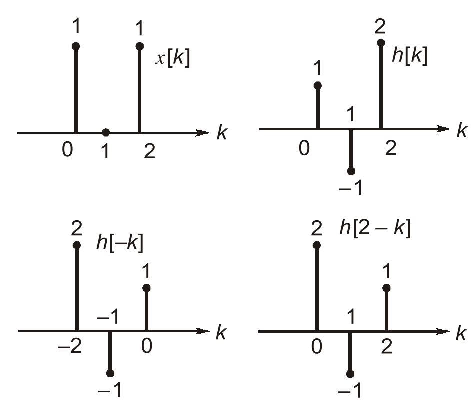
2.10 (d)

$L_1 = 3$, $L_2 = 3$

$\Rightarrow$ Number of samples in output $= L_1 + L_2 - 1 = 5$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]\,h[2-k]$$



$\Rightarrow \quad y(2) = 3$

2.11 (b)

$h(n) = 2^n\,u(n-2)$

For causal system $h(n) = 0$ for $n < 0$.

Hence given system is causal.