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ESE 2020 : Prelims Exam
CLASSROOM TEST SERIES

ELECTRICAL ENGINEERING

Test 6

Section A : Electrical Machines [All Topics]

Section B : Control Systems-1 + Engineering Mathematics-1 [Part Syllabus]

Section C : Electrical Circuits-2 + Digital Circuits-2 [Part Syllabus]

1.	(a)	**16.**	(b)	**31.**	(c)	**46.**	(d)	**61.**	(a)
2.	(b)	**17.**	(a)	**32.**	(c)	**47.**	(c)	**62.**	(d)
3.	(b)	**18.**	(b)	**33.**	(d)	**48.**	(d)	**63.**	(c)
4.	(b)	**19.**	(b)	**34.**	(c)	**49.**	(d)	**64.**	(d)
5.	(a)	**20.**	(b)	**35.**	(c)	**50.**	(c)	**65.**	(a)
6.	(b)	**21.**	(c)	**36.**	(c)	**51.**	(a)	**66.**	(c)
7.	(c)	**22.**	(c)	**37.**	(a)	**52.**	(b)	**67.**	(d)
8.	(d)	**23.**	(d)	**38.**	(a)	**53.**	(a)	**68.**	(b)
9.	(b)	**24.**	(c)	**39.**	(d)	**54.**	(a)	**69.**	(b)
10.	(c)	**25.**	(b)	**40.**	(c)	**55.**	(c)	**70.**	(a)
11.	(d)	**26.**	(b)	**41.**	(b)	**56.**	(b)	**71.**	(c)
12.	(d)	**27.**	(d)	**42.**	(a)	**57.**	(d)	**72.**	(d)
13.	(b)	**28.**	(b)	**43.**	(d)	**58.**	(c)	**73.**	(c)
14.	(a)	**29.**	(d)	**44.**	(c)	**59.**	(c)	**74.**	(c)
15.	(c)	**30.**	(c)	**45.**	(c)	**60.**	(c)	**75.**	(a)

DETAILED EXPLANATIONS
Section A : Electrical Machines

1. (a)

In a practical transformer the flux increases when leading load is applied and decreases when lagging load is applied.

2. (b)

We know, Power input, I^2R = 1200 W

$$R = \frac{1200}{(100)^2} = 0.12\ \Omega$$

The impedance, $$Z = \frac{V}{I} = \frac{60}{100} = 0.6\ \Omega$$

∴ The leakage reactance, $$X = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(0.6)^2 - (0.12)^2}$$

$$= 0.587 \approx 0.59\ \Omega$$

3. (b)

As rated output, hence torque is same in both cases. At 1000 rpm assuming back emf, E_{b2}

$$\frac{E_{b1}}{E_{b2}} = \frac{N_1}{N_2} = \frac{1500}{1000} = 1.5$$

$$E_{b2} = \frac{E_{b1}}{1.5} = \frac{240 - 0.4 \times 20}{1.5} = 154.66\ \text{V}$$

For current, I_a = 20 A

$$R_a = \frac{240 - 154.66}{20} = 4.26\ \Omega$$

Resistance to be added = 4.26 – 0.4

= 3.86 Ω

4. (b)

$$\text{Input power, } P = \frac{\text{Output power (in h.p.)} \times 746}{\eta}$$

$$= \frac{100 \times 746}{0.746} = 10^5\ \text{W}$$

$$\text{Input current} = \frac{\text{Power}}{\text{Volt}} = \frac{10^5}{500} = 200\ \text{A}$$

$$\text{The shunt field current} = \frac{500}{250} = 2\ \text{A}$$

The armature current = 200 – 2 = 198 A

The back emf $E_b = V - I_a R_a$

$= 500 - 198 \times 0.1$

$= 500 - 19.8 = 480.2$ V

$$E_b = \frac{P\phi N}{60}\frac{Z}{A} = \frac{4 \times 50 \times 10^{-3} \times N \times 540}{60 \times 2} = 480.2 \text{ V}$$

$N = 533.55$ rpm

5. (a)

An auto transformer has lower voltage regulation and lower per unit impedance which results in higher short circuit current.

$$\text{Copper saving} = \frac{1}{a_{\text{auto}}}$$

Also, $$\frac{\text{Conductive transfer}}{\text{Total transfer}} = \frac{1}{a_{\text{auto}}}$$

6. (b)

In case of power transformer, as load generally remains fixed hence voltage regulation is not as significant as compared to distribution transformer, where the load is varying between no load to full load. Similarly in case of all day efficiency also it is calculated for distribution transformer mostly.

7. (c)

Given,

Iron loss of transformer, $P_i = 900$ W

Copper loss of transformer, $P_{cu} = 1600$ W

Let the maximum efficiency occurs at x times of full load, then

$$x = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{900}{1600}} = \frac{3}{4} = 0.75$$

For maximum efficiency output power at unity power factor,

$P_{\text{out}} = 0.75 \times 40 \times 1$

$= 30$ kW

Total losses, $P_{\text{loss}} = P_i + x^2 P_{\text{cu}}$

$$= 900 + \left(\frac{3}{4}\right)^2 1600$$

$$= 900 + \frac{9}{16} \times 1600 = 1800 \text{ W or } 1.8 \text{ kW}$$

So, maximum efficiency, $$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} = \frac{30}{30 + 1.8} = \frac{30}{31.8}$$

$= 0.9433$ or 94.33%

8. (d)

Considering the second winding,

The resistance when referred to primary side

$$Z_2' = (Z_2)\left(\frac{N_1}{N_2}\right)^2 = 20\left(\frac{5}{1}\right)^2 = 20 \times 25 = 500\ \Omega$$

Similarly for third winding,

The resistance when referred to primary winding

$$Z_3' = (Z_3)\left(\frac{N_1}{N_3}\right)^2 = 80\left(\frac{5}{4}\right)^2$$

$$= \frac{80 \times 25}{16} = 125\ \Omega$$

Total resistance on primary side,

$$Z_1 = Z_2' \parallel Z_3'$$

$$= \frac{500 \times 125}{625} = 100\ \Omega$$

$$\text{Current drawn from supply} = \frac{V}{Z} = \frac{20}{100} = 0.2 \text{ A}$$

$$\therefore \text{Power drawn by supply, } P_t = V \times I \cos\phi$$

$$= 20 \times 0.2 \times 1 = 4 \text{ W}$$

9. (b)

Phase voltage on primary side (Y) of transformer,

$$V_{PY} = \frac{11000}{\sqrt{3}} \text{V}$$

On secondary side (Δ), $V_{P\Delta} = V_{L\Delta}$

$$= \frac{11000}{\sqrt{3} \times 10} = \frac{1100}{\sqrt{3}} \text{V}$$

Phase current on secondary side,

$$I_{P\Delta} = 11 \times 10 = 110 \text{ A}$$

Line current on secondary side,

$$I_{L\Delta} = 110\sqrt{3} \text{ A}$$

$$\therefore \quad \text{Output kVA} = \sqrt{3} \times \frac{1100}{\sqrt{3}} \times 110 \times \sqrt{3} = 121\sqrt{3} \text{ kVA}$$

10. (c)

Load share of transformer,

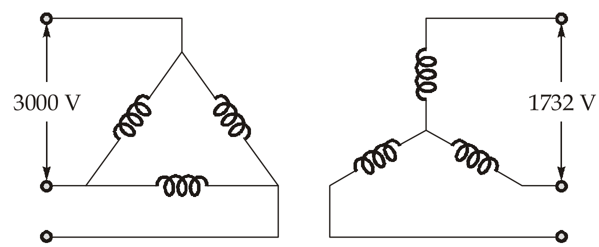
$$T_1 = \frac{(0.6 + j2.4) \times 400\angle -\cos^{-1} 0.8}{(1 + j4) + (0.6 + j2.4)}$$

$$= \frac{(0.6 + j2.4) \times 400\angle -36.89^\circ}{1.6 + j6.4}$$

$$= \frac{0.6(1 + j4) \times 400\angle -36.89^\circ}{1.6(1 + j4)} = 150\angle{-36.89^\circ} \text{ kVA}$$

11. (d)

Given 3-ϕ, transformer can be represented as



Primary side resistance, R_1 = 0.3 Ω/phase

Secondary side resistance, R_2 = 0.02 Ω/phase

Primary line current, $I_{l1} = \frac{1800 \times 1000}{3000\sqrt{3}} = \frac{600}{\sqrt{3}}\text{A}$

Primary phase current, $I_{\text{ph1}} = \frac{600}{\sqrt{3} \times \sqrt{3}} = 200\text{A}$

Secondary line current, $I_{l2} = \frac{1800 \times 1000}{\sqrt{3} \times 1732} = 600\text{A}$

Secondary phase current, I_{ph2} = 600 A

copper loss on primary side $= 3I_1^2R_1$

$= 3 \times (200)^2 \times 0.3$

$= 36000$ W

Copper loss on secondary side $= 3 \times I_2^2R_2 = 3 \times (600)^2 \times 0.02$

$= 21600$ W

$$\frac{\text{Ratio of primary side copper loss}}{\text{Ratio of secondary side copper loss}} = \frac{36000}{21600} = 1.67$$

12. (d)

All are correct.

13. (b)

The machine whose set point is increased will assume more load.

14. (a)

Stiffness is improved by decreasing the synchronous reactance.

15. (c)

An overexcited synchronous motor delivers reactive power and operates at leading power factor.

16. (b)

Inverted V curves are obtained by plotting power factor vs field current.

17. (a)

ASA method gives accurate result for both cylindrical and salient pole generator.

18. (b)

$$X_d = \frac{\text{Maximum voltage per phase}}{\text{Minimum current per phase}} = \frac{\frac{2820}{\sqrt{3}}}{275} = 5.92\ \Omega$$

19. (b)

$$\text{Power input to motor} = \frac{\text{Motor output}}{\eta} = \frac{50}{0.92} = 54.35\text{ kW}$$

$$\text{Armature current, } I = \frac{54.35 \times 1000}{\sqrt{3} \times 400 \times 0.8} = 98.055\text{ A} \approx 98\text{ A}$$

20. (b)

The generator described above is Y connected, so the direct current in the resistance test flows through two windings.

Hence,
$$2R_A = \frac{V_{DC}}{I_{DC}}$$

$$R_A = \frac{V_{DC}}{2I_{DC}} = \frac{10\text{V}}{(2)(25)} = 0.2\ \Omega$$

21. (c)

$$P = V\,I\cos\phi$$

$$I\cos\phi = \text{constant} \qquad \text{(for same voltage and load)}$$

$$\Rightarrow \quad I_1\cos\phi_1 = I_2\cos\phi_2$$

$$200 \times 1 = I_2 \times 0.5$$

$$I_2 = 400\text{ A}$$

22. (c)

Both given statements are correct.

23. (d)

$$\text{Area of conductor, } A = 2\pi rl = 2\pi \times 1 \times 0.6$$

$$\phi = B.A = B(1.2\,\pi) = 1.2\pi B$$

Induced emf,
$$e = \frac{\phi}{T} = \frac{\phi N}{60} = \frac{1.2\pi(B) \times 800}{60} = 3600\,\pi$$

Where N is speed in rpm,

$$B = \frac{3600\pi \times 60}{1.2 \times \pi \times 800} = 225\text{ Wb/m}^2$$

24. (c)

Required flux density in the core,

$$B = \frac{\phi}{A} = \frac{0.012}{120 \times 10^{-4}} = 1 \text{ T}$$

Magnetic flux intensity, $H = 144$ A-T/m

The magnetomotive force,

$$\mathcal{F} = Ni = Hl_c$$

$$= (144)\ (0.60) = 86.4 \text{ A-T}$$

The required current, $i = \frac{\mathcal{F}}{N} = \frac{86.4}{200} = 0.432$ A

25. (b)

Voltage drop in armature is

$$I_{a1}R_a = 50 \times 0.5 = 25 \text{ V}$$

The emf generated is

$$E_1 = 250 + 25 = 275 \text{ V}$$

Immediately after the change of flux, the speed remains same,

Thus, $$\frac{E_2}{E_1} = \frac{\phi_2}{\phi_1}$$

$\Rightarrow$ $$E_2 = \frac{0.029}{0.03} \times 275 = 265.8 \text{ V}$$

Armature current immediately after the change of flux is

$$I_a = \frac{265.8 - 250}{0.5} = \frac{15.8}{0.5} = 31.6 \text{ A}$$

26. (b)

- The effect of armature reaction in case of dc motor is that it shifts MNA in the direction which is against the rotation of motor from GNA.
- $$\text{Power} = EI = E\left(\frac{V-E}{R}\right) = \frac{EV}{R} - \frac{E^2}{R}$$

$$\frac{dP}{dE} = \frac{V}{R} - \frac{2E}{R}$$

for maximum power,

$$\frac{dP}{dE} = 0 \qquad \text{i.e. } E = \frac{V}{2}$$

Maximum mechanical power occurs when back emf is equal to half of the applied voltage.

27. (d)

Current in each lamp, $I = \frac{P}{V} = \frac{100}{120} = \frac{5}{6}$ A

Total load current, $I_L = \frac{5}{6} \times 60 = 50$ A

Field current, $I_{sh} = \frac{120}{60} = 2$ A

Armature current, $I_a = I_{fL} + I_{sh} = 50 + 2 = 52$ A

A wave winding has 2 parallel paths, so current in each armature conductor $= \frac{52}{2} = 26$ A

28. (b)

For dc generator,

$$\text{generated emf, } E = \frac{P\phi N}{60}\frac{Z}{A}$$

$$\text{Speed, } N \propto \frac{E_g}{\phi} \quad \text{...(i)}$$

As a generator armature current,

$$I_a = \frac{2 \times 1000}{400} = 5 \text{ A}$$

$$E_g = 400 + 0.4 \times 5 = 402 \text{ V}$$

As a motor, $E_m = 400 - 0.4 \times 5 = 398$ V

Since flux per pole is increased by 20% when operated as motor, we can write

$$\phi_m = 1.2\ \phi_g$$

Using equation (i),

$$\frac{N_g}{N_m} = \frac{E_g\,\phi_m}{E_m\,\phi_g} = \frac{402 \times 1.2\phi_g}{398 \times \phi_g} = 1.21$$

29. (d)

We know,

Torque developed in dc motor,

$$\tau = K\,\phi\,I_a \quad \text{...(i)}$$

If operated in new condition,

$$\phi' = 0.8\ \phi$$

$$I_a' = 1.4\ I_a$$

$$\tau' = 240 = K\phi'\,I_a'$$

$$240 = K \times 0.8 \times 1.4\,\phi\,I_a \quad \text{...(ii)}$$

Taking ratio of equation (i) and (ii),

$$\frac{\tau}{240} = \frac{K\phi I_a}{K \times 0.8 \times 1.4 \times \phi I_a}$$

$$\tau = \frac{240}{0.8 \times 1.4} = 214.29 \text{ N-m}$$

Hence option (d) is correct.

30. (c)

Let the generator A and B supply load currents I_A and I_B respectively at common bus voltage V_b

$$I_{LA} + I_{LB} = 300 \quad \text{...(i)}$$

Shunt field current for both generators $= \dfrac{V_b}{150}$

For generator A,

$$\text{Armature current, } I_{aA} = I_{LA} + \frac{V_b}{150}$$

For generator B,

$$\text{Armature current, } I_{aB} = I_{LB} + \frac{V_b}{150}$$

For generator A, using voltage equation,

$$E_{bA} = V_b + I_{aA} r_A$$

$$275 = V_b + I_{aA}(0.2)$$

$$275 = V_b + (0.2)\left(I_{LA} + \frac{V_b}{150}\right) \quad \text{...(ii)}$$

For generator B,

$$E_{bB} = V_b + I_{aB} r_B$$

$$270 = V_b + 0.2 \times \left(I_{LB} + \frac{V_b}{150}\right) \quad \text{...(iii)}$$

Adding equation (ii) and (iii),

$$545 = 2V_b + 0.2\left(I_{LA} + I_{LB} + \frac{2V_b}{150}\right)$$

Using equation (i),

$$545 = 2V_b + 0.2\left(300 + \frac{2V_b}{150}\right)$$

$$545 = 2V_b + 60 + \frac{0.4V_b}{150}$$

$$485 = 2V_b + \frac{4V_b}{1500}$$

$$485 = \frac{3004V_b}{1500}$$

$$V_b = \frac{485 \times 1500}{3004} = 242.18 \text{ V}$$

31. (c)

- Interpoles located in interpolar region are connected in series with armature to reduce armature mmf by producing interpole mmf to counter it.
- Armature mmf is maximum at q-axis and has zero value at direct axis.

32. (c)

$$\frac{\text{Rotor Gross Output}}{\text{Rotor Input}} = 1 - s = \frac{N}{N_s}$$

$$\frac{1800}{2000} = 1 - s \Rightarrow s = 0.1 \text{ or } 10\%$$

$$1 - 0.1 = \frac{810}{N_s} \Rightarrow N_s = 900 \text{ rpm}$$

$$f' = sf \quad \Rightarrow 6 = 0.1 \times f \Rightarrow f = 60 \text{ Hz}$$

$$900 = \frac{120 \times (6/0.1)}{P}$$

$$P = \frac{120 \times 60}{900} = 8$$

33. (d)

$$\text{Synchronous speed, } N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\text{Slip, } s = \frac{E_{(\text{injected})}}{E_{(\text{rotor})}} = \frac{18}{54} = \frac{1}{3} = 0.33$$

$$\text{Rotor speed, } N_r = N_s(1 - s)$$

$$= 1500\,(1 - 0.33) = 1000 \text{ rpm}$$

34. (c)

Power output, $P_0 = 380$ watts

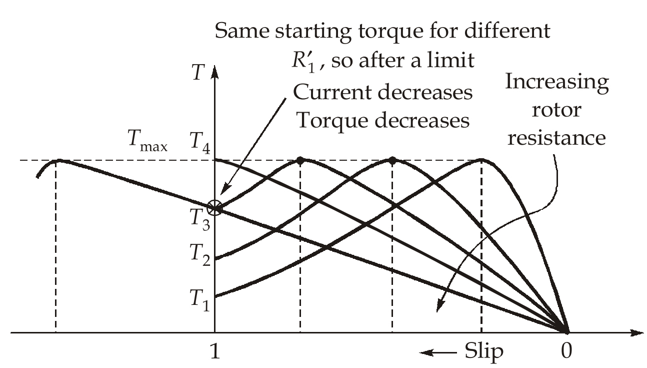
Power input, $P_{in} = VI\cos\phi$

$= 230 \times 3 \times 0.7 = 483$ W

Motor efficiency, $\eta = \frac{P_o}{P_{in}} \times 100 = \frac{380}{483} \times 100$

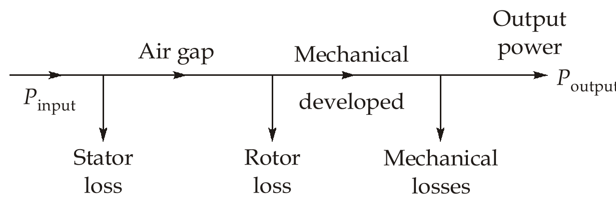
$= 78.67\%$

35. (c)



As R increases, starting torque increases upto maximum torque ($T_4 = T_{max} > T_3 > T_2 > T_1$) but rotor consumes more power so copper loss increases and efficiency decreases.

36. (c)



Given, Slip, $s = 0.03$

Per phase output power,

$$P_0 = \frac{30}{3} = 10 \text{ kW}$$

Now, $\text{Mechanical losses} = \frac{10 \times 1.5}{100} = 0.15 \text{ kW}$

Mechanical developed power,

$$= 10.15 \text{ kW}$$

$$\text{Per phase rotor copper loss} = s(\text{air gap power})$$

$$= \frac{s(\text{Mechanical developed power})}{1-s}$$

$$P_{\text{cu (rotor)}} = \frac{0.03}{0.97} \times 10.15 = 314 \text{ W}$$

37. (a)

We know that,

$$\text{Torque, } T = \frac{3}{\omega_{sm}} \times \frac{V^2}{R_2'} s \qquad \text{(for low slip)}$$

Now, T = constant $\qquad T \propto V^2 s$

(or) $\qquad V_2^2 s_2 = V_1^2 s_1$

(or) $\qquad s_2 = \left(\frac{V_1}{V_2}\right)^2 s_1$

(or) $\qquad s_2 = 4s_1$,

hence slip increases 4 times,

Also, $\qquad T = \frac{3I_2'^2}{\omega_{sm}} \times \frac{R_2'}{s} = \text{const.}$

(or) $\qquad I_2'^2 \propto s$

$$\frac{I_2'}{I_1'} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{4}{1}} = 2$$

Hence, current increases by 2 times.

38. (a)

$$\text{Power output, } P_0 = 10 \text{ kW}$$
$$\text{Frequency, } f = 50 \text{ Hz}$$
$$\text{Poles, } P = 6$$
$$\text{Slip, } s = 0.04$$
$$\text{Friction and windage losses} = 0.4 \text{ kW}$$
$$\text{Mechanical power developed} = 10.4 \text{ kW}$$
$$\text{Air gap power, } P_g = \frac{\text{Mechanical power developed}}{1-s} = \frac{10.4}{1-0.04} = 10.83 \text{ kW}$$
$$\text{Synchronous speed} = \frac{120 \times 50}{6} = 1000 \text{ rpm or } 104.72 \text{ rad/sec}$$

Full load electromagnetic torque,

$$T_e = \frac{P_g}{\omega_s} = \frac{10.83 \times 10^3}{104.72} = 103.42 \text{ N-m}$$

39. (d)

$$\frac{T_{st}}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl}$$

For $T_{st} = T_{fl}$

$$\frac{I_{st}}{I_{fl}} = \sqrt{\frac{1}{s_{fl}}} = \frac{1}{\sqrt{0.01}} = 10$$

40. (c)

Both the statements are correct.

41. (b)

Both statements are correct as lower leakage flux means lower per unit reactance and hence lower regulation or better and desirable regulation.

43. (d)

Statement-I is false although speed of dc shunt motor can be increased by increasing field resistance but large variation can lead to very small field current, which will lead to low holding force for electromagnet of three point starter and hence disconnected motor from line. This is major drawback of 3 point starter.

44. (c)

Damper winding resistance of synchronous machine is made larger, the starting (zero speed) torque increases, but for hunting purpose, it is kept lower, but not for starting purpose.

45. (c)

At starting, a large portion of starting current flows in the top cage due to large leakage reactance of inner (bottom cage) at starting.

Section B : Control Systems-1 + Engineering Mathematics-1

46. (d)

$$\text{Transfer function} = \frac{2\left(1+\frac{1}{\tau_1 s}\right)\left(\frac{5}{s(s+1)}\right)}{1+2\left(1+\frac{1}{\tau_1 s}\right)\left(\frac{5}{s(s+1)}\right)}$$

Characteristic equation,

$$\tau_1 s^3 + \tau_1 s^2 + 10\tau_1 s + 10 = 0$$

s^3	τ_1	$10\tau_1$
s^2	τ_1	10
s^1	$10\tau_1 - 10$	0
s^0	10	

$$10\tau_1 - 10 > 0$$

$$\therefore \quad \tau_1 > 1$$

47. (c)

Method I: (Analytical method)

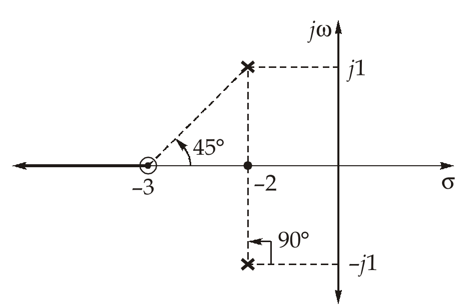
$$\arg\left|G(s)H(s)\right|_{s=-2+j1} = \arg\left|\frac{K(s+3)}{(s+2+j1)}\right|_{s=-2+j1}$$

$$= \arg\left|\frac{K(1+j1)}{2j}\right| = \frac{\angle 45^\circ}{\angle 90^\circ} = \angle -45^\circ$$

$$\theta_d = 180^\circ + \arg\left|G(s)H(s)\right| \text{ for poles}$$

$$\theta_d = 180^\circ - 45^\circ = 135^\circ$$

Method II: (Graphical method)



$$\theta_d = 180^\circ - (90^\circ - 45^\circ)$$
$$= 135^\circ$$

48. (d)

Characteristic equation is,

$$s^4 + 8s^3 + 29s^2 + 52s + K = 0$$

s^4	1	29	K
s^3	8	52	
s^2	22.5	K	
s^1	$52 - 0.35556K$	0	
s^0	K		

To determine $K_{marginal}$ we put,

$$52 - 0.35556\,K = 0$$

$$K = 146.25$$

Auxiliary equation is,

$$22.5s^2 + 146.25 = 0$$

49. (d)

Of the four choices, the first two are ruled out because they possess pole/zero in the right half of plane.

For $\dfrac{e^{-3s}}{s}$,

$$\Rightarrow \quad \frac{e^{-3j\omega}}{j\omega} = -3\omega - \frac{\pi}{2}$$

For minimum phase,

$$\angle F(j\omega)\big|_{\omega=\infty} = -(P-Z)\frac{\pi}{2}$$

The above function is not minimum phase function because

at $\omega = \infty$, $\quad \angle F(j\omega) = -3(\infty) - \dfrac{\pi}{2} = -\infty$ radian $\neq -(P-Z)\dfrac{\pi}{2}$

For $\dfrac{s}{(s+1)(s+2)}$,

$$\Rightarrow \quad \frac{j\omega}{(j\omega+1)(j\omega+2)}$$

$$\angle F(j\omega) = 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$\angle F(j\omega)\big|_{\omega=\infty} = 90^\circ - 90^\circ - 90^\circ = -90^\circ$$

$$= -(P-Z)\frac{\pi}{2}$$

Hence option (d) is correct.

50. (c)

There are two forward paths and one loop. So, we have

$$P_1 = a, \qquad P_2 = b$$
$$L_1 = c$$
$$\Delta_1 = 1 = \Delta_2$$
$$\Delta = 1 - c$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{y_2}{y_1} = \frac{a+b}{1-c}$$

51. (a)

The signal flow graph contains two forward paths and one loop,

$$P_1 = \frac{-1}{s}, \qquad P_2 = -K$$
$$L_1 = \frac{-K}{s}$$
$$\Delta = 1 + \frac{K}{s}, \qquad \Delta_1 = 1 = \Delta_2$$
$$\frac{Y(s)}{X(s)} = \frac{-\frac{1}{s} - K}{1 + \frac{K}{s}} = \frac{-(1+Ks)}{s+K}$$

52. (b)

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Applying, $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ and $R_4 \to R_4 - 6R_1$

We have, $$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Now applying $R_4 \to R_4 - R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

Now applying $R_4 \to R_4 - R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By elementary transformations, the given matrix is reduced to an equivalent upper triangular matrix. Since all the elements of the fourth row are zero its rank $\rho(A) < 4$.
By inspection we see that one of its minor.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & 0 & -3 \end{vmatrix} = 1 \times (-4) \times (-3) = 12 \neq 0$$

$\therefore$ $\rho(A) = 3$

53. (a)

The augmented matrix,

$$C = [A \mid B]$$

$$C = \left[\begin{array}{cccc|c} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 4 & -5 & 8 & -9 & K \end{array}\right]$$

Applying $R_3 \rightarrow R_3 - 2R_1$

$$= \left[\begin{array}{cccc|c} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 0 & 1 & -4 & 1 & K-6 \end{array}\right]$$

Applying $R_3 \rightarrow R_3 - R_2$

$$= \left[\begin{array}{cccc|c} 2 & -3 & 6 & -5 & 3 \\ 0 & 1 & -4 & 1 & 1 \\ 0 & 0 & 0 & 0 & K-7 \end{array}\right]$$

There are infinite number of solution,

If $\rho(A) = \rho(C) = 2$

$\therefore$ $K - 7 = 0$

$K = 7$

54. (a)

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 7\lambda^2 + 36 = 0$$

From here,

$$x + y + z = 7$$

$$xy + yz + zx = 0$$

$$xyz = -36$$

$$\therefore \quad xy + yz + zx - xyz = 36$$

Note : For a cubic equation

$$ax^3 + bx^2 + cx + d = 0$$

Let, p, q and r be its roots

then, $$p + q + r = \frac{-b}{a}$$

$$pq + qr + rq = \frac{c}{a}$$

$$pqr = \frac{-d}{a}$$

55. (c)

$$x = (N)^{1/N}$$

$$x^N = N$$

$$x^N - N = 0$$

$$f(x) = x^N - N$$

$$f'(x) = N \cdot x^{N-1}$$

Newton-Raphson Iteration,

$$x_{K+1} = x_K - \frac{f(x_K)}{f'(x_K)} = x_K - \frac{x_K^N - N}{N x_K^{N-1}}$$

$$= \frac{Nx_K^N - x_K^N + N}{N x_K^{N-1}}$$

$$x_{K+1} = \left(\frac{N-1}{N}\right) x_K + x_K^{1-N}$$

56. (b)

Given, $$\lim_{x \to a} \frac{g(x) f(a) - g(a) f(x)}{x - a} = \frac{0}{0}$$

Applying L'hospitals rule,

$$\lim_{x \to a} \frac{g'(x) f(a) - g(a) f'(x)}{1} = \frac{2 \times 2 - (-1) \times 1}{1} = 5$$

57. (d)

Statements 1, 2 and 3 are correct.

$$f'(c) = 0 \text{ for Rolle's theorem.}$$

58. (c)

By Simpson's $\frac{1}{3}$ rule, with 4 equally spaced intervals.

The integral value, $$I = \frac{h}{3}[f_0 + 4f_1 + 2f_2 + 4f_3 + f_4]$$

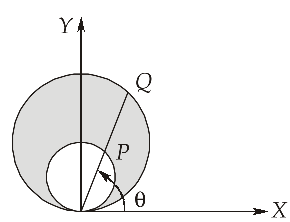
Given : $h = 1$ and $f_0 = 3$, $f_1 = 10$, $f_2 = 21$, $f_3 = 36$ and $f_4 = 55$

Therefore, $$I = \int_0^4 f(x)\,dx = \frac{1}{3}(3 + 4 \times 10 + 2 \times 21 + 4 \times 36 + 55)$$

$$= 94.67$$

59. (c)

Given circles, $r = 2\sin\theta$ and $r = 4\sin\theta$



If we integrate first with respect to r, then its limit are from $P(r = 2\sin\theta)$ to $Q(r = 4\sin\theta)$ and to cover the whole region θ varies from 0 to π. Thus the required integral is

$$I = \int_0^{\pi} \int_{2\sin\theta}^{4\sin\theta} r^3 dr\, d\theta = \int_0^{\pi} \left[\frac{r^4}{4}\right]_{2\sin\theta}^{4\sin\theta}$$

$$= 60\int_0^{\pi} \sin^4\theta\, d\theta = 60 \times 2 \int_0^{\pi/2} \sin^4\theta\, d\theta$$

$$= 120 \times \frac{3 \times 1}{4 \times 2} \times \frac{\pi}{2} = 22.5\pi$$

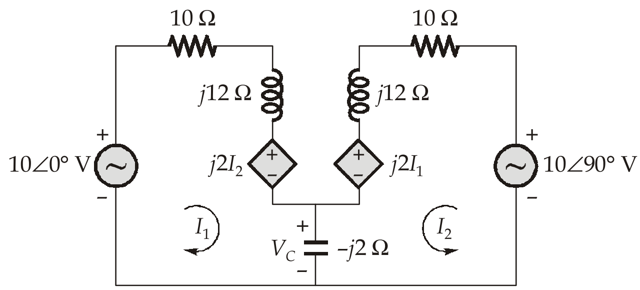
60. (c)

PD control improves transient part and *PI* control improves the steady state part.

Section C : Electrical Circuits-2 + Digital Circuits-2

61. (a)

Using decoupled technique



Applying KVL in loop-1 and 2

$$10\angle 0° = I_1[10 + j12 - j2] + I_2(j2 - j2)$$

$$10\angle 0° = [10 + j10]I_1 \quad \text{...(i)}$$

$$10\angle 90° = I_2[10 + j12 - j2] + I_1[j2 - j2]$$

$$= I_2[10 + j10] \quad \text{...(ii)}$$

Adding equation (i) and (ii),

$$10 + j10 = (I_1 + I_2)(10 + j10)$$

$$I_1 + I_2 = 1 \text{ A}$$

Voltage across capacitor,

$$V_C = -j2 \text{ V}$$

$$= 2\angle -90° \text{ V}$$

62. (d)

Number of trees, T = Determinant $[A]\,[A^T]$

$$[A]\,[A^T] = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\text{Determinant } [A]\,[A^T] = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(6 - 1) + 1(-2) + 0 = 8$$

63. (c)

$$V_1 = h_{11}I_1 + h_{12}V_2$$

and $$I_2 = h_{21}I_1 + h_{22}V_2$$

∴ $$V_1 = I_1 + 2V_2 \quad \text{...(i)}$$

$$I_2 = -2I_1 + V_2 \quad \text{...(ii)}$$

also $V_2 = -R_L I_2 = -I_2$ ...(iii)

and $I_L^2 R_L = 100$

$-I_2 = I_L = 10$ A

Putting, $V_2 = 10$ V in equation (ii)

$-10 = -2I_1 + 10$

$I_1 = 10$ A

Putting, $I_1 = 10$ A and $V_2 = 10$ V

$V_1 = 10 + 20 = 30$ V

$V_S = 2I_1 + V_1$

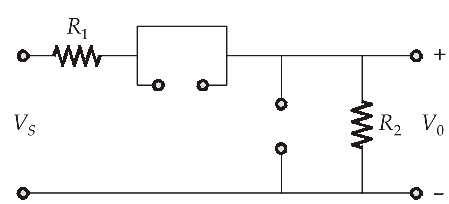
$= 20 + 30 = 50$ V

Hence the source voltage is 50 V.

64. (d)

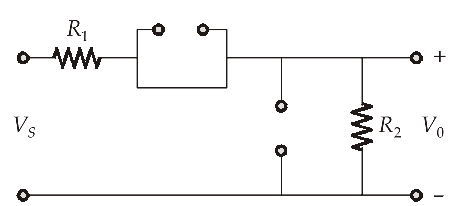
Analyzing the circuit for ω = 0 and ω = ∞

At ω = 0,



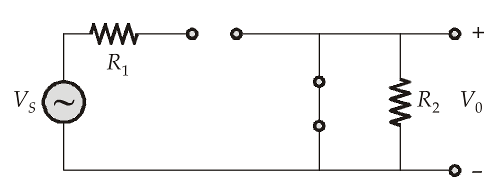
$$\frac{V_0}{V_S} = \frac{R_2}{R_1 + R_2}$$

At ω = ∞,



$$\frac{V_0}{V_S} = \frac{R_2}{R_1 + R_2}$$

At $\omega = \frac{1}{\sqrt{LC}}$,



$V_0 = 0$

65. (a)

Applying mesh analysis,

$$-120 + (20 + 30)I_1 - 30I_2 + V_1 = 0$$

$$50I_1 - 30I_2 + V_1 = 120 \quad \text{...(i)}$$

$$-V_2 + (10 + 30)I_2 - 30I_1 = 0$$

$$-30I_1 + 40I_2 - V_2 = 0 \quad \text{...(ii)}$$

At the transformer terminals,

$$V_2 = \frac{-1}{2}V_1 \text{ and } I_2 = -2I_1$$

$$-55I_2 - 2V_2 = 120 \quad \text{...(iii)}$$

$$15I_2 + 40I_2 - V_2 = 0$$

$$V_2 = 55I_2 \quad \text{...(iv)}$$

$$-165\, I_2 = 120$$

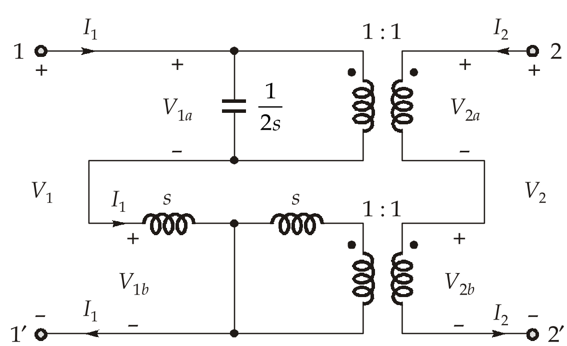
$$I_2 = \frac{-120}{165} = -0.7272 \text{ A}$$

The power absorbed by the 10 Ω resistor,

$$P = (-0.7272)^2 \times 10$$

$$= 5.3 \text{ W}$$

66. (c)



For top portion of circuit :

Since transformer turns ratio is 1 : 1, the primary and secondary currents and voltages are equal in magnitude

$$\therefore \quad V_{1a} = V_{2a} = \frac{1}{2s}(I_1 + I_2)$$

For bottom portion of circuit,

$$V_{1b} = sI_1 + 0I_2$$

$$V_{2b} = 0I_1 + sI_2$$

Now,
$$V_1 = V_{1a} + V_{1b} = \left(s + \frac{1}{2s}\right)I_1 + \frac{1}{2s}I_2$$

$$V_2 = V_{2a} + V_{2b} = \frac{1}{2s}I_1 + \left(s + \frac{1}{2s}\right)I_2$$

Hence, the z-parameters are given by comparing with the following equations

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

and $$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$\therefore$ $$Z_{11} = Z_{22} = s + \frac{1}{2s}$$

and $$Z_{12} = Z_{21} = \frac{1}{2s}$$

$\therefore$ The network is symmetrical and reciprocal.

67. (d)

The time period of clock pulse $= \dfrac{1}{500\,\text{kHz}} = 2\ \mu\text{sec}$

Maximum conversion time, T_c = 2 μsec × no. of clock pulses required

For 8 bit ADC, 256 clock pulses are required

$\therefore$ Maximum conversion time,

$$T_{c\,(\max)} = 2\ \mu\text{sec} \times 256$$
$$= 512\ \mu\text{sec}$$

Conversion rate, $$\frac{1}{T_{c\max}} = \frac{1}{512\ \mu\text{sec}} = 0.00195312 \times 10^{+6}$$
$$= 1953\ \text{counts/sec}$$

68. (b)

Left shift register is multiplication by 2 and right shift register is division by 2 network.

69. (b)

$$D = \bar{B}Q_n + A$$

which is equal to the excitation equation of a SR-flip flop

$$D = \bar{R}Q_n + S$$

70. (a)

$$Q^+ = Q \oplus X = Q\bar{X} + \bar{Q}X$$

Hence, it represents a T-flip-flop.

71. (c)

Total stable states = 14

$\therefore$ $$f_{out} = \frac{50 \times 10^3}{14} = 3.57\ \text{kHz}$$

72. (d)

The minimum number of comparators required for n bit flash ADC

$$= 2^n - 1$$
$$= 2^8 - 1 = 255$$

73. (c)

$Z_{11} = Z_{22}$: symmetrical network
$Z_{12} = Z_{21}$: Reciprocal network

74. (c)

Low cost of dual slope *ADC* is achieved as precision components are not required which are costly.

75. (a)

Both statement-I and II are correct and statement II is the correct explanation of statement-I.

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