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India's Best Institute for IES, GATE & PSUs

Important Questions *for* GATE 2022

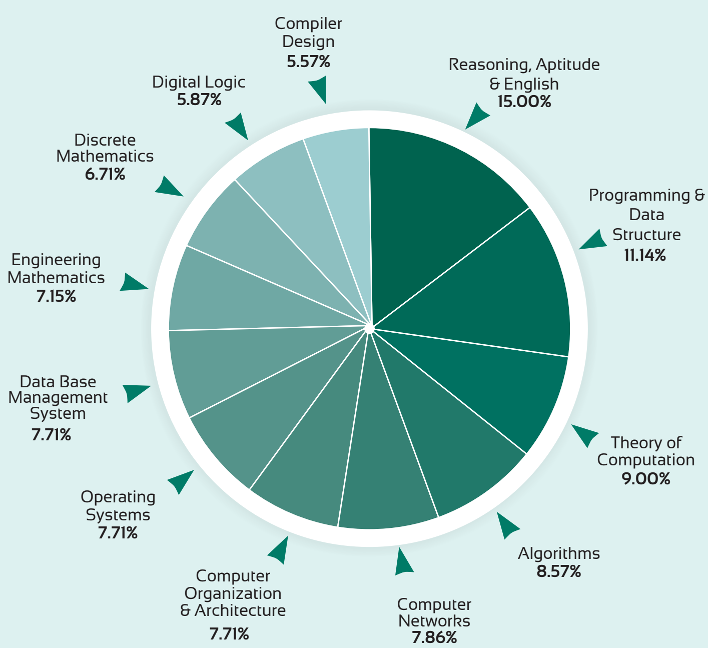
COMPUTER SCIENCE & IT

Day 3 of 8

Q.51 - Q.75 (Out of 200 Questions)

Theory of Computation

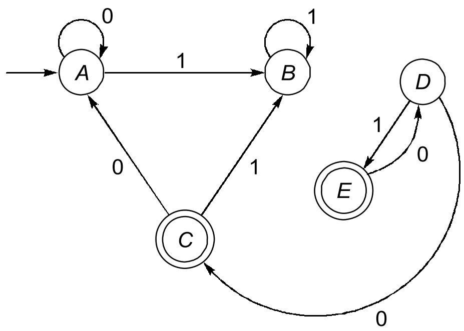
SUBJECT-WISE WEIGHTAGE ANALYSIS OF GATE SYLLABUS



Subject	Average % (last 5 yrs)
Reasoning, Aptitude & English	15.00%
Programming & Data Structure	11.14%
Theory of Computation	9.00%
Algorithms	8.57%
Computer Networks	7.86%
Operating Systems	7.71%
Computer Organization & Architecture	7.71%
Data Base Management System	7.71%
Engineering Mathematics	7.15%
Discrete Mathematics	6.71%
Digital Logic	5.87%
Compiler Design	5.57%
Total	**100%**

Theory of Computation

Q.51 Consider the following DFA:



If above DFA accepts the language denoted by the regular expression (0 + 1)*10(0 + 1), find the missing transitions.

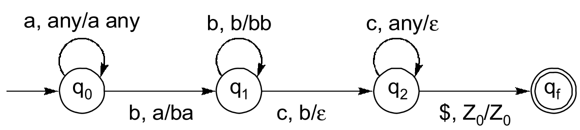
(a) $\delta(B, 0) = A$, $\delta(E, 1) = C$
(b) $\delta(B, 0) = B$, $\delta(E, 1) = B$
(c) $\delta(B, 0) = D$, $\delta(E, 1) = B$
(d) $\delta(B, 0) = A$, $\delta(E, 1) = B$

Q.52 Which of the following is a non-regular language?

(a) $L = \{wxwy \mid x, y, w \in (a + b)^+\}$
(b) $L = \{xwyw \mid x, y, w \in (a + b)^+\}$
(c) $L = \{wxyw \mid x, y, w \in (a + b)^+\}$
(d) All of the above

Q.53 Consider the following PDA:

$$\text{PDA} = (Q, \Sigma, \delta, \Gamma, q_0, Z_0, q_f)$$



Identify the language accepted by the above PDA?

(a) $\{a^m b^n c^k \mid m = n = k, m, n, k \geq 1\}$
(b) $\{a^m b^n c^k \mid m, n, k \geq 1, m = n + k\}$
(c) $\{a^m b^n c^k \mid m, n, k \geq 1, m + n = k\}$
(d) $\{a^m b^n c^k \mid m, n, k \geq 1\}$

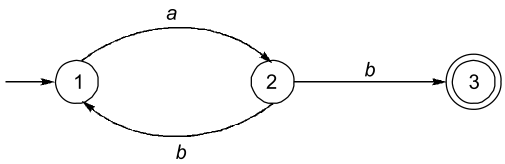
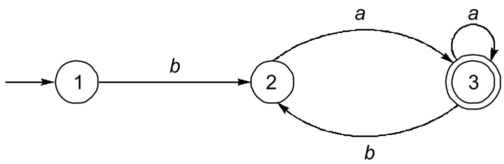
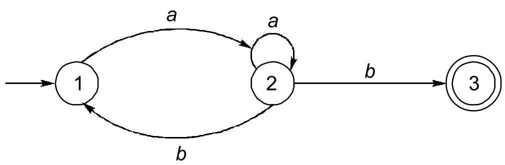
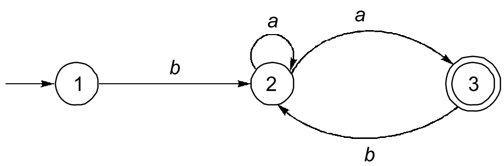
Q.54 Find the grammar that generates inherently ambiguous context free language.

(a) $S \rightarrow AB \mid CD$
$A \rightarrow aAb \mid \varepsilon$
$B \rightarrow dB \mid \varepsilon$
$C \rightarrow aC \mid \varepsilon$
$D \rightarrow bDd \mid \varepsilon$

(b) $S \rightarrow AB \mid CD$
$A \rightarrow aAb \mid \varepsilon$
$B \rightarrow bB \mid \varepsilon$
$C \rightarrow aC \mid bC \mid \varepsilon$
$D \rightarrow bB \mid \varepsilon$

(c) Both (a) and (b)
(d) Neither (a) nor (b)

Q.55 Match the following groups:

List-I (Finite Automata)	**List-II (Regular Expression)**
A. 	**1.** b(a + ab)*a
B. 	**2.** a(a + ba)*b
C. 	**3.** ba(a + ba)*
D. 	**4.** a(ba)*b

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	4	3	2	1
(d)	4	2	3	1

Q.56 Assume $(n + 2)$ languages are used in the composition with union operation as follows:

$$A \cup L_1 \cup L_2 \cup \ldots L_n \cup B = Y$$

Resultant language is Y. If $A = \Sigma^*$ and $B = \phi$ then what is the language represented by Y? All languages are defined over same input alphabet $\Sigma = \{a, b\}$

(a) Y is regular but not finite
(b) Y is regular and finite
(c) Y is non-regular and infinite
(d) None of these

Q.57 Which of the following CFG generates the language $L = \{a^m b^n \mid m >= n\}$?

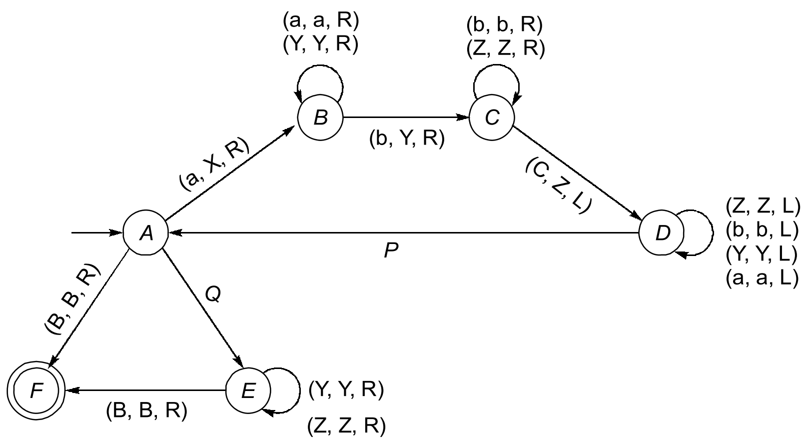
(a) $S \rightarrow aA \mid \varepsilon$
$A \rightarrow Sb \mid Ab$

(b) $S \rightarrow Ab \mid \varepsilon$
$A \rightarrow aS \mid aA$

(c) $S \rightarrow aAb \mid \varepsilon$
$A \rightarrow aB \mid aA \mid \varepsilon$
$B \rightarrow Bb \mid \varepsilon$

(d) None of these

Q.58 Consider the following Turing Machine:



If the above TM accepts a language $L = \{a^n b^n c^n \mid n \geq 0\}$, then what are the missing transitions at P and Q respectively

(a) (Y, Y, R) and (X, X, R)
(b) (X, X, R) and (Y, Y, R)
(c) (X, X, L) and (Y, Y, R)
(d) (Y, Y, R) and (X, X, L)

Q.59 How many of the following CFGs generates a CFL but not regular languages?

(*i*) $S \rightarrow AB$
$A \rightarrow aA \mid bA \mid \varepsilon$
$B \rightarrow aBb \mid \varepsilon$

(*ii*) $S \rightarrow AaB$
$A \rightarrow aC \mid \varepsilon$
$B \rightarrow aB \mid bB \mid \varepsilon$
$C \rightarrow aCb \mid \varepsilon$

(*iii*) $S \rightarrow AB$
$A \rightarrow aA \mid \varepsilon$
$B \rightarrow aBb \mid \varepsilon$

Q.60 Consider the following three languages:

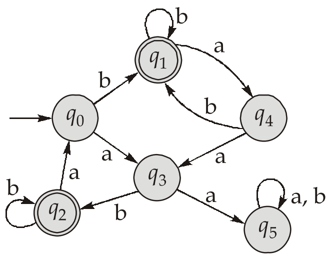
- $L_1 = \{w \mid w \in \{a, b\}^* \text{ and } w = w^R\}$
- $L_2 = \{ww^R \mid w \in \{a, b\}^*\}$
- $L_3 = \{w\,(a + b)\,w^R \mid w \in \{a, b\}^*\}$

What is the relation between L_1, L_2 and L_3?

(a) $L_2 \subset L_1$ and $L_3 \subset L_1$ and $L_1 = L_2 \cup L_3$
(b) $(L_2 = L_3) \subset L_1$
(c) $L_2 \cap L_1 = L_3$
(d) $L_2 \subset L_1$ and $L_3 \subset L_1$ but $L_1 \neq L_2 \cup L_3$

Q.61 The number of states in minimal NFA, which accepts all strings in which the 3rd last bit is b is ________. [Assume $\Sigma = \{a, b\}$]

Q.62 Consider the following DFA:



The number of states in the minimal DFA obtained by applying minimization algorithm on the above DFA is equal to ________.

Q.63 Consider the following statements:

S_1 : Pumping lemma can be used to prove given language is regular.
S_2 : Given a grammar, checking if the grammar is not regular is decidable problem.
S_3 : If L is a regular and M is not a regular language then L.M. is necessarily non-regular.
S_4 : The number of derivations step for any strings W of length n is grammar is CNF and GNF form is $(2n - 1)$ and (n) respectively.

Which of the following statement is correct ?

(a) Only S_1, S_3 is correct
(b) Only S_2, S_4 is correct
(c) Only S_3 is correct
(d) Only S_2, S_3 is correct

Q.64 Identify the language generated by the following grammar where S is start variable?

$S \to S_1 | S_2$
$S_1 \to S_1c | A$
$A \to aAb | \in$
$S_2 \to aS_2 | B$
$B \to bBc | \in$

(a) $\{a^n b^n c^m | n, m \geq 0\}$
(b) $\{a^n b^m c^k | n, m, k \geq 0\}$
(c) $\{a^n b^m c^m | n, m \geq 0\}$
(d) $\{a^n b^n c^m | n, m \geq 0\} \cup \{a^n b^m c^m | n, m \geq 0\}$

Q.65 Consider the following problems:

P_1 : {<M, x, k> | M is a TM and M does not halt on x within k steps}.
P_2 : {<M> | M is a TM and M accepts atleast two strings of different length}.
P_3 : {<M> | M is a TM and there exist an input whose length is less than 100, on which M halts}.

The number of problems which is RE but not REC is _______.

Q.66 Consider the following CFG:

$$S \to aSa | bSb | a | b | \varepsilon$$

For the above CFG, the total number of strings generated whose length is less than or equal to 6 is ________.

Q.67 Consider the following languages:

L_1 : {<M, k> | M is a Turing Machine and $|\{w \in L(M) : w \in a^*b^*\}| \leq k$}.

L_2 : {<M> | there exist a Turing Machine M' such that $\langle M \rangle \neq \langle M' \rangle$ and $L(M) = L(M')$}.

Which of the language is recursive?

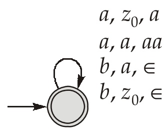
(a) L_1 is recursive but not L_2.
(b) L_2 is recursive but not L_1.
(c) L_1 and L_2 both are recursive.
(d) L_1 and L_2 both are not recursive.

Q.68 We're given a grammar G which is in CNF form and a string $w \in L(G)$. If $|w| = x$, then the number of steps in derivation of w in G has 41 steps. Then x is equal to

(a) 10
(b) 19
(c) 20
(d) 21

Q.69 Consider the following PDA given below where z_0 represent stack symbol:

P:



Which of the following represent L(P)?

(a) $L(P) = \{w \in \{a, b\}^* \mid \text{in every } w,\ n_a(w) \geq n_b(w)\}$
(b) $L(P) = \{w \in \{a, b\}^* \mid \text{in every suffix of } w,\ n_a(w) \geq n_b(w)\}$
(c) $L(P) = \{w \in \{a, b\}^* \mid \text{in every prefix of } w,\ n_a(w) \geq n_b(w)\}$
(d) None of the above

Q.70 Let M be a Turing Machine having $Q = \{q_0, q_1, q_2, q_3, q_4\}$ a set of states, input alphabet {0, 1}. The tape alphabet are $\{0, 1, B, X, Y\}$. The symbol B is used to represent end of input string. The initial and the final states is q_0 and q_4 respectively. The transitions are as follows.

1. $(q_0, 0) = (q_1, X, R)$
2. $(q_0, Y) = (q_3, Y, R)$
3. $(q_1, 0) = (q_1, 0, R)$
4. $(q_1, 1) = (q_2, Y, L)$
5. $(q_1, Y) = (q_1, Y, R)$
6. $(q_2, 0) = (q_2, 0, L)$
7. $(q_2, X) = (q_0, X, R)$
8. $(q_2, Y) = (q_2, Y, L)$
9. $(q_3, Y) = (q_3, Y, R)$
10. $(q_3, B) = (q_4, B, R)$

Which of the following is true about M?

(a) M accepts on L having 011 as substring
(b) M accepts on L having 010 as substring
(c) M accepts on $\{L = 0^n1^n, n > 0\}$
(d) M accepts on L not having 1100 substring

Q.71 Which of the following problems are undecidable?

I. {<G, w> | G is a CFG, $w \in \Sigma^*$ and $w \in$ L(G)}.
II. {<G> | G is a CFG and G is ambiguous}.
III. {<M> | M is a Turing Machine and M accepts ∈}.

(a) III only
(b) II and III only
(c) I, II and III only
(d) I and III only

Q.72 Which of the following languages over the alphabet $\Sigma = \{a, b, 0\}$ is regular?

$L_1 = \{a^n\ 0\ b^n \mid n \geq 0\}$
$L_2 = \{0ww^R \mid w \in (a, b)^*\}$
$L_3 = \{w\ 0w^R \mid w \in (a, b)^*\}$
$L_4 = \{w_1 w_2^R 0 \mid w \in (a, b)^*\}$

(a) L_1 and L_2
(b) Only L_4
(c) Only L_3
(d) Only L_2

Q.73 Consider the ambiguous grammar given below:

$S \rightarrow aA \mid bB$
$A \rightarrow aAA \mid bS \mid b$
$B \rightarrow bBB \mid aS \mid a$

The minimum length of string for which more than one derivation trees are possible ________.

Q.74 Consider languages L_1 and L_2 over alphabet $\Sigma = \{a, b\}$.

L_1 is known to be a context-free language.
$L_2 = \{w \mid w \text{ is prefix of } w' \in L_1\}$

Which of the following is true?

(a) L_2 need not be CFL
(b) L_2 will be regular
(c) L_2 will be CFL
(d) None of the above

Multiple Select Question (MSQ)

Q.75 Consider the two languages L_1 and L_2 where L_1 is regular and L_2 is DCFL. Then the language $L_3 = (L_1 \cap L_2^*)'$ is/are true about L_3?

(a) CSL
(b) CFL
(c) Recursive
(d) Recursively enumerable

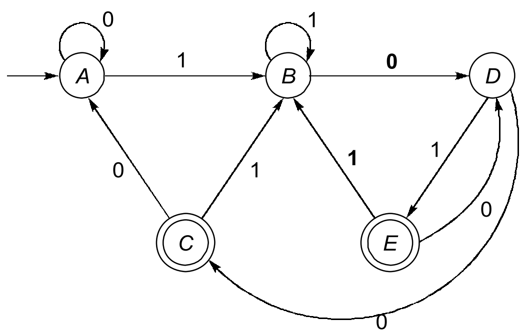
■■■■

Detailed Explanations

51. **(c)**

$$RE = (0 + 1)^* \, 10 \, (0 + 1)$$

Every string either end with 100 or 101.



52. **(c)**

(a) $L = \{wxwy \mid x, y, w \in (a + b)^+\}$

$L = [a(a + b)^+ a(a + b)^+] + [b(a + b)^+ b(a + b)^+] \Rightarrow L$ is regular language.

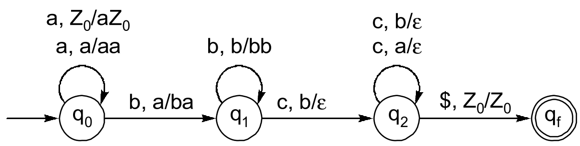
(b) $L = \{xwyw \mid x, y, w \in (a + b)^+\}$

$L = [(a + b)^+ a(a + b)^+ a] + [(a + b)^+ b(a + b)^+ b] \Rightarrow L$ is regular language.

(c) $L = \{wxyw \mid x, y, w \in (a + b)^+\} \Rightarrow L$ is non-regular language.

53. **(c)**

Given PDA can be redrawn as following:



At q_0, all a's are pushed.

At q_1, all b's are pushed.

At q_2, all c's are matched with b's and a's.

If #c's = #a's + #b's then goes to final state.

$\Rightarrow$ $L = \{a^m b^n c^k \mid m, n, k \geq 1, k = m + n\}$

$\therefore$ Option (c) is correct.

54. **(a)**

(a) $L = \{a^m b^n c^k \mid m = n \text{ or } n = k\}$

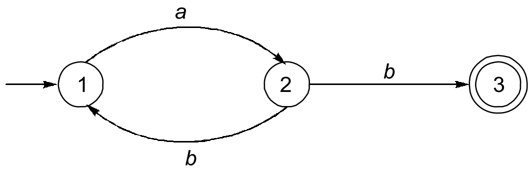
(b) $L = (a + b)^*$

Option (a) is inherently ambiguous language, because no equivalent unambiguous grammar exist for the language.

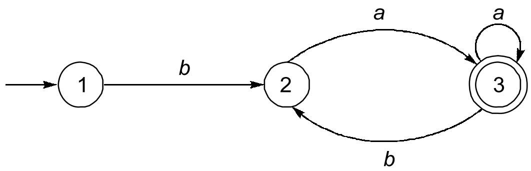
Option (b) is unambiguous language, because many unambiguous grammars exist for the language.

55. **(c)**

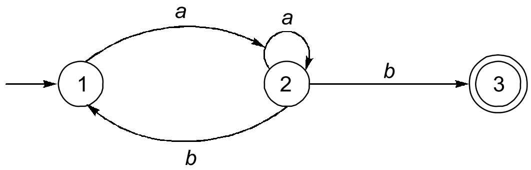
A–4, B–3, C–2, D–1

(a) 

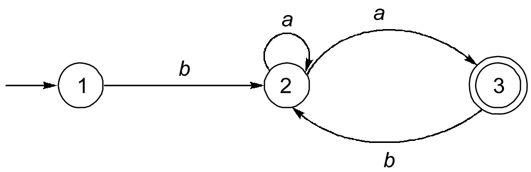
Equivalent regular expressions are: (*i*) (ab)*ab and (*ii*) a(ba)*b ...(4)

(b) 

Equivalent regular expressions are: (*i*) b(aa*b)*aa* and (*ii*) ba(a + ba)* ...(3)

(c) 

Equivalent regular expressions are: (*i*) a(a + ba)*b and (*ii*) (aa*b)*aa*b ...(2)

(d) 

Equivalent regular expressions are: (*i*) b(a + ab)*a and (*ii*) ba*a(ba*a)* ...(1)

56. **(a)**

$$Y = A \cup L_1 \cup L_2 \cup \ldots L_n \cup B$$
$$Y = \Sigma^* \cup L_1 \cup L_2 \cup \ldots L_n \cup \phi$$
$$Y = \Sigma^* = (a + b)^*$$

$\therefore$ Y is regular and infinite language.

57. **(b)**

(a) $S \rightarrow aA \mid \varepsilon$

$A \rightarrow Sb \mid Ab$

$\Rightarrow L = \{a^m b^n \mid m <= n\}$

(b) $S \rightarrow Ab \mid \varepsilon$

$A \rightarrow aS \mid aA$

$\Rightarrow L = \{a^m b^n \mid m >= n\}$

(c) $S \rightarrow aAb \mid \varepsilon$

$A \rightarrow aB \mid aA \mid \varepsilon$

$B \rightarrow Bb \mid \varepsilon$

$\Rightarrow$ aabbb string is possible.

$\therefore$ Option (b) is correct.

58. **(b)**

P = (X, X, R); To find the left most 'a', state B will move left until X appears. When X appears, it moves to the right position to read symbol 'a' if exists.

Q = (Y, Y, R); If all a's are written by X's then A will find Y that indicates all b's and all c's are written by Y's and Z's respectively. A $\Rightarrow$ E $\Rightarrow$ F path to read all Y's then all Z's, finally goes to final state.

59. **(1)**

(*i*) $S \rightarrow AB$

$A \rightarrow aA \mid bA \mid \varepsilon$

$B \rightarrow aBb \mid \varepsilon$

$\Rightarrow L = (a+b)^{*}.a^nb^n = (a+b)^*$ is regular.

(*ii*) $S \rightarrow AaB$

$A \rightarrow aC \mid \varepsilon$

$B \rightarrow aB \mid bB \mid \varepsilon$

$C \rightarrow aCb \mid \varepsilon$

$\Rightarrow L = a(a+b)^*$ is regular.

(*iii*) $S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow aBb \mid \varepsilon$

$\Rightarrow L = \{a^mb^n \mid m \geq n\}$ is non-regular but CFL. So only (*iii*) is CFL but not regular.

60. **(a)**

L_2 is even palindrome on $\{a, b\}^*$

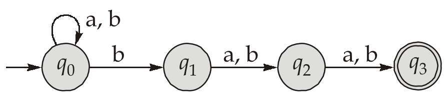
L_3 is odd palindrome on $\{a, b\}^*$

L_1 is any palindrome on $\{a, b\}^*$

Clearly, $L_2 \subset L_1$ and $L_3 \subset L_1$ and $L_1 = L_2 \cup L_3$.

61. **(4)**

Minimal NFA:



62. **(4)**

Partition-1: $\{\underline{q_1, q_2}\}, \{\underline{q_0, q_3, q_4, q_5}\}$

Partition-2: $\{q_1, q_2\}, \{\underline{q_0, q_3, q_4}\}, \{\underline{q_5}\}$

Partition-3: $\{q_1, q_2\}, \{\underline{q_0, q_4}\}, \{\underline{q_3}\}\{q_5\}$

Partition-4: $\{q_1, q_2\}, \{q_0, q_4\}, \{q_3\}\{q_5\}$

Therefore 4 states will be required.

63. (b)

S_1 : Pumping lemma can prove that language is not regular but can't prove that the language is regular. Hence this is false.

S_2 : We can check regular grammar by following productions $V \to T^* V + T^*$ or $V \to V T^* + T^*$.

S_3 : Consider 'L' to be ϕ and 'M' to $\{a^n b^n \mid n \leq 0\}$

L.M. = ϕ, which is regular

S_4 : In case of CNF, $(n - 1)$ derivations are required to generate a string with (n) Non-Terminals, since only one Non-Terminals is added during each derivation.

Further, (n) derivations are required to convert those Non-Terminals to terminals.

So, in total, to generate a string of n terminals:

$$\underbrace{(n-1)}_{\substack{\text{To generate} \\ \text{string with } n \\ \text{Non-Terminals}}} + \underbrace{(n)}_{\substack{\text{To convert} \\ \text{NT} \to \text{T}}} = \underbrace{(2n-1)}_{\text{Total}}$$

However, in case of GNF: In a single derivation, we get a terminal in addition to our Non-Terminals. **S → T(NT)***

Therefore, no need for $(n - 1)$ derivations to increase length.

Hence, only (n) derivations are required.

64. (d)

$L_1 : S_1 \to S_1 c \mid A \Leftarrow \{a^n b^n c^m \mid n, m \geq 0\}$

$A \to aAb \mid \in \; = \{a^n b^n \mid n \geq 0\}$

$L_2 : S_2 \to aS_2 \mid B \Leftarrow \{a^n b^m c^m \mid n, m \geq 0\}$

$B \to bBc \mid \in \; \Rightarrow \{b^m c^m \mid m \geq 0\}$

So, $L = L_1 \cup L_2 = \{a^n b^n c^m \mid n, m \geq 0\} \cup \{a^n b^m c^m \mid n, m \geq 0\}$.

65. (2)

P_1 : T_{Yes} : When machine does not halt on x until k steps.

T_{No} : When machine halt on x within k steps.

So, recursive.

P_2 : T_{Yes} : When machine accepts atleast two strings of different length.

T_{No} : Not exist, since machine may go into infinite loop

So, Re but not REC.

P_3 : T_{Yes} : Run all strings till 100 steps, if machine halt.

T_{No} : Does not exist, since machine may go into infinite loop.

So, RE but not REC.

66. **(29)**

The grammar generates the set of all palindromes possible over $\{a, b\}$.

Lets first find the number of even palindromes of length atmost 6 (0, 2, 4, 6 length respectively).

$$\begin{aligned} \text{0 length palindromes} &= 2^{0/2} = 1 \\ \text{2 length palindromes} &= 2^{2/2} = 2 \\ \text{4 length palindromes} &= 2^{4/2} = 4 \\ \text{6 length palindromes} &= 2^{6/2} = 8 \end{aligned}$$

So total number of even palindromes of length atmost 6 = 1 + 2 + 4 + 8 = 15

Similarly number of odd palindromes of length atmost 6 = 2 + 4 + 8 = 14

So total palindromes = 29

67. **(b)**

- L_1 is not recursive but recursive enumerable since string from a^*b^* belongs to $L(M)$ with length less than K is semidecidable, we can say "yes" when machine halt within K steps but if machine is in infinite loop, the we an not say anything. Hence not recursive.
- L_2 is recursive since for every language there are more than one Turing Machine exist, so it is trivially decidable. Hence recursive

68. **(d)**

In CNF, [number of steps for $|w| = x$] = $(2x - 1)$

Given, $2x - 1 = 41 \Rightarrow 2x = 42$

$x = 21$

69. **(c)**

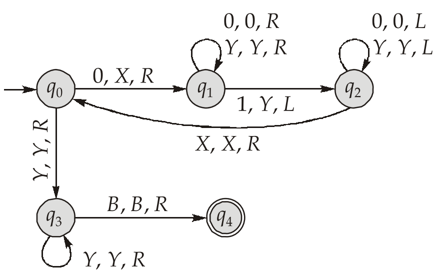
(a) Generate string '*bbaa*' which is not accepted by PDA 'P'.

(b) Generate string '*baaa*' which is not accepted by PDA 'P'.

(c) Generate string '*a*', '*aa*', '*aaa*' ..... *ab*, *aab*, *aaabb*, etc. which is generate by PDA.

70. **(c)**

Drawing the equivalent Turing Machine we have



This is a standard Turing Machine program for $\{0^n 1^n \mid n > 0\}$.

Hence (c) is the correct option.

71. **(b)**

I $\rightarrow$ membership problem for CFGs; decidable.

II $\rightarrow$ ambiguity problem for CFGs; undecidable.

III $\rightarrow$ {<M> | M is a Turing Machine, null string belongs to L(M)}.

- Applying Rice's theorem, since this is a non trivial question on RE language, therefore undecidable.

72. **(b)**

L_1: is CFL since their is comparison between a and b. To make same number of b we have to the remember number of a's. So it is CFL (DCFL).

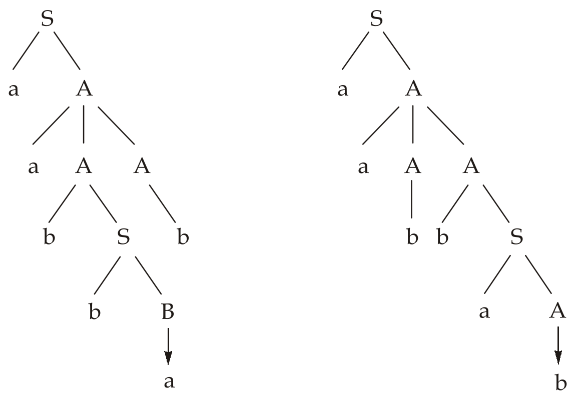
L_2: is CFL since their is comparison between w and w^R.

L_3: is CFL more appropriately said DCFL.

73. **(6)**

Language contain string: ab, ba, aabb, abab, abba, baab, bbaa, baba

For string "aabbab" we get 2 derivation tree



[**Note:** Grammar generate all strings that are of even length, so, check only for 2, 4, 6 .....]

74. **(c)**

Let $L_2 = \text{INIT}(L_1)$

Both regular and CFL are closed under INIT operation.

Therefore L_2 will be CFL.

Hence option (c) is correct.

75. **(a, c, d)**

$$\begin{aligned} L_3 &= (L_1 \cap L_2^*)' = (\text{DCFL}^* \cap \text{Regular})' \\ &= (\text{Regular} \cap \text{CFL})' \\ &= (\text{CFL})' = \text{CSL } \{\text{using the closure property}\} \end{aligned}$$

■■■■