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Important Questions *for* GATE 2022

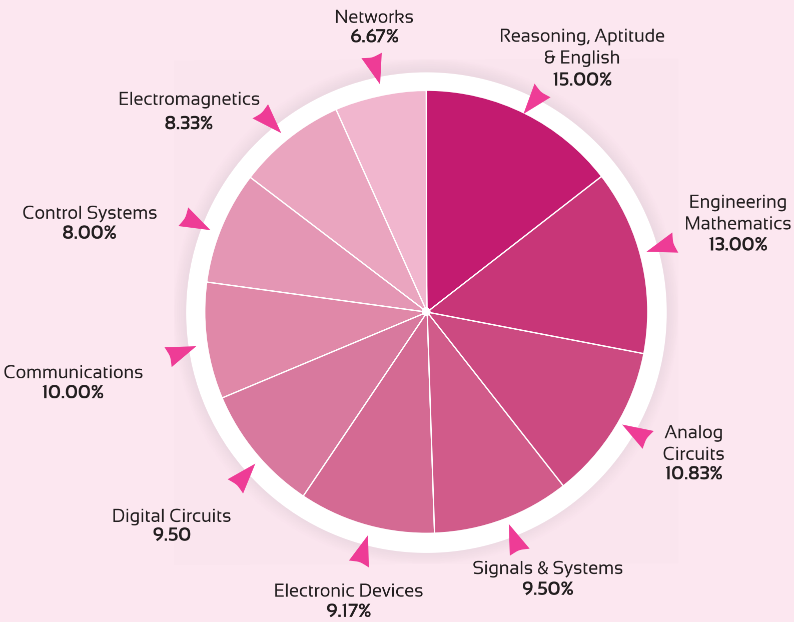
ELECTRONICS ENGINEERING

Day 2 of 8

Q.26 - Q.50 (Out of 200 Questions)

Networks and Control Systems

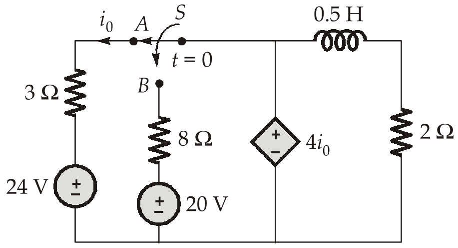
SUBJECT-WISE WEIGHTAGE ANALYSIS OF GATE SYLLABUS



Subject	Average % (last 5 yrs)*
Reasoning, Aptitude & English	15.00%
Engineering Mathematics	13.00%
Analog Circuits	10.83%
Signals & Systems	9.50%
Electronic Devices	9.17%
Digital Circuits	9.50%
Communications	10.00%
Control Systems	8.00%
Electromagnetics	8.33%
Networks	6.67%
Total	**100%**

Networks and Control Systems

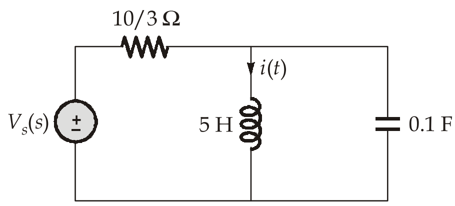
Q.26 For the circuit shown in the figure below (the switch 'S' moves from A to B at $t = 0$). The voltage across 2 Ω resistor for $t > 0$ will be



(a) $96e^{-3.2t}\, u(t)$ V
(b) $48e^{-4t}\, u(t)$ V
(c) $96e^{-4t}\, u(t)$ V
(d) $48e^{-t/3.2t}\, u(t)$ V

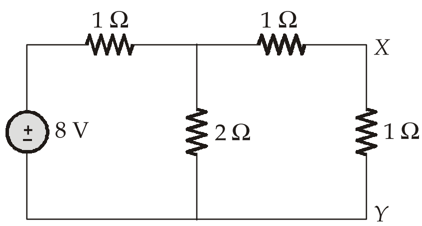
Q.27 The Thevenin equivalent impedance of a source is $Z_{Th} = (120 + j60)$ Ω and the peak Thevenin voltage is $V_{Th} = (150 + j0)$. The maximum average available power from the source is _______ W.

Q.28 Consider the circuit shown in the figure below. Assuming the value of $V_s(t) = 10u(t)$ V and assume that at $t = 0$, $i(0) = -1$ A flows through the inductor and +5 V is across the capacitor, the voltage across the capacitor will be
for $t > 0$,

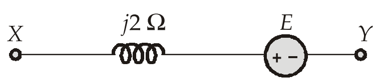


(a) $(30e^{-t} + 7e^{-2t})\, u(t)$ V
(b) $(1 - e^{-t} + 7e^{-2t})\, u(t)$ V
(c) $(3 - 7e^{-t} + 3e^{-2t})\, u(t)$ V
(d) $(35e^{-t} - 30e^{-2t})\, u(t)$ V

Q.29 For the circuit shown in below figure, we are interested in replacing the branch X-Y by a voltage generator with series impedance of $j2$ Ω as shown in figure (b), without affecting the circuit response.



(a)

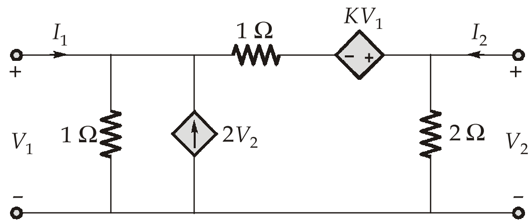


(b)

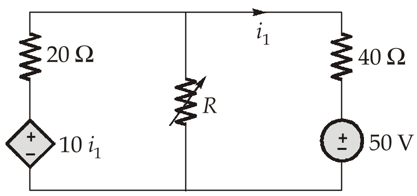
The value of voltage of the generator E will be

(a) $2\angle 90°$ V
(b) $2\sqrt{5}\angle -63.43°$ V
(c) $2\angle -180°$ V
(d) $\sqrt{5}\angle 35°$ V

Q.30 The circuit shown in the below figure is reciprocal if K = __________.

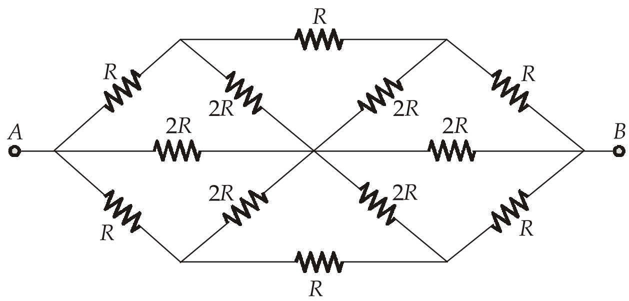


Q.31 For the circuit shown in the figure below, the resistance R and the maximum power transfer to it will be



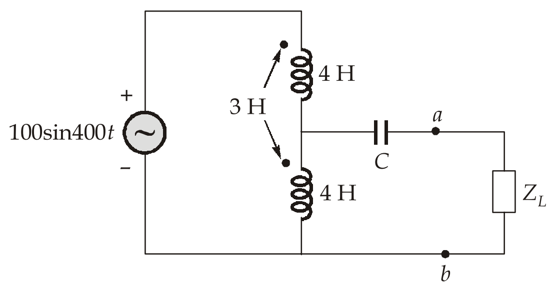
(a) 4 Ω and 1.56 W
(b) 16 Ω and 17.36 W
(c) 16 Ω and 1.56 W
(d) 4 Ω and 7.36 W

Q.32 A rectangular hexagon is formed from six wires of R Ω each as shown in the figure below:



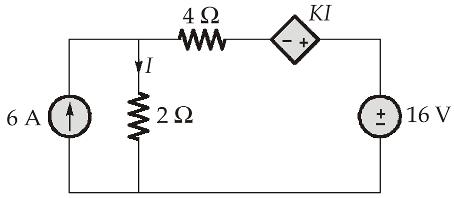
The equivalent resistance seen across the terminal A and B is ___________ R Ω.

Q.33 Consider the circuit shown below:



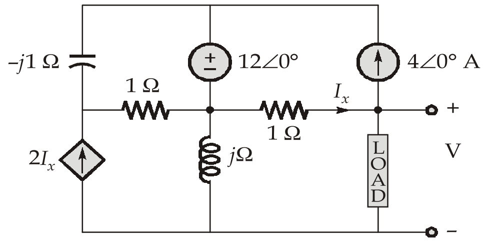
If the voltage across Z_L is to be independent of the value of Z_L, then the value of C is ______ μF.

Q.34 Consider the circuit shown in the figure below:

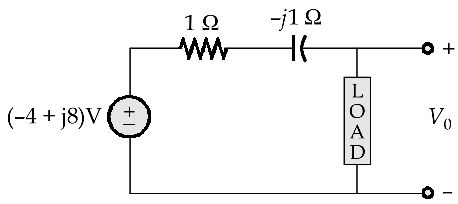


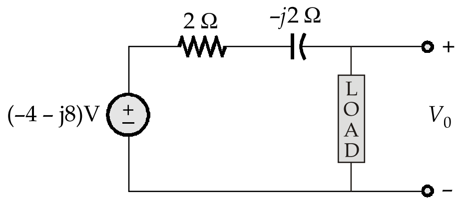
The value of 'K' such that the power dissipated by 2 Ω resistor does not exceed 50 W is ________.

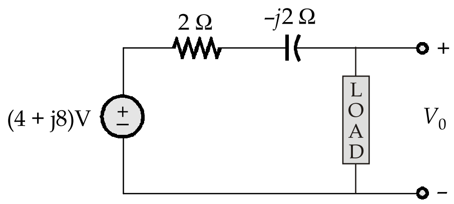
Q.35 Consider the circuit shown in the figure below:

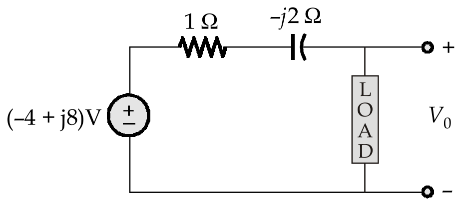


Then the Thevenin's equivalent circuit with connected one Ω load can be represented as

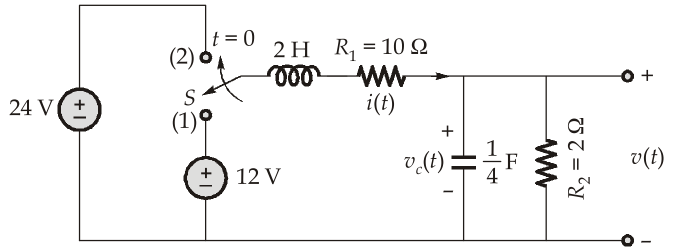
(a) 

(b) 

(c) 

(d) 

Q.36 Consider the circuit shown in the figure below:

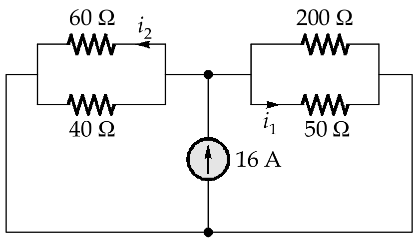


The switch 'S' is moved from position (1) to position (2) at t = 0. Then which of the following differential equations does not satisfy the circuit working condition.

(a) $\frac{di(t)}{dt} + 5i(t) + 0.5v(t) = 12 \text{ for } t > 0$ (b) $\frac{dv(0)}{dt} = 4i(0) - 2v(0)$

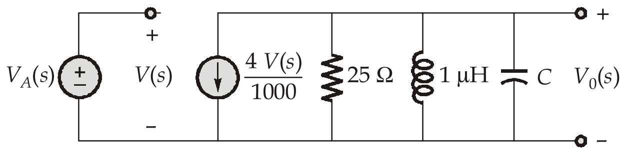
(c) $\frac{dv(t)}{dt} + 2v(t) = 4i(t) \text{ for } t > 0$ (d) $\frac{d^2v(t)}{dt} + 7\frac{dv(t)}{dt} + 12v(t) = 48 \text{ for } t > 0$

Q.37 Consider the circuit shown in the figure below:



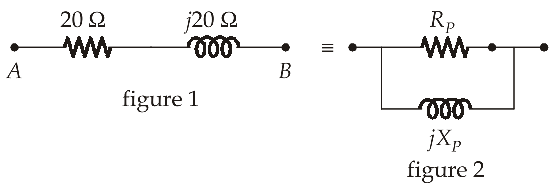
Then the ratio $\frac{i_1}{i_2}$ is equal to ________.

Q.38 Consider the RLC circuit shown in the figure below:



Let the input voltage be $V_A(s)$ and output voltage be $V_0(s)$, then the value of 'C' for which the circuit will produce maximum gain at 91.1 MHz is equal to ________ pF.

Q.39 Consider the series R-L series circuit shown in the figure 1 below.

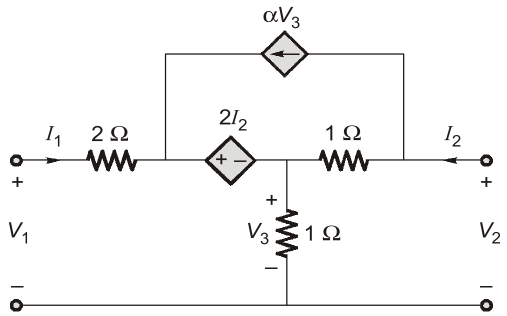


If the circuit shown in figure-2 is equivalent to figure-1 then the value of R_p and L_p is equal to

(a) R_p = 20 Ω and X_p = 40 Ω (b) R_p = 20 Ω and X_p = 20 Ω

(c) R_p = 40 Ω and X_p = 40 Ω (d) R_p = 40 Ω and X_p = 20 Ω

Q.40 Consider the circuit shown below:



The value of 'α' for which the circuit is said to be reciprocal is __________ ℧.

Q.41 The open-loop transfer function of a closed-loop system with unity negative feedback is given by,

$$G(s) = \frac{K(s^2+s+1)}{s(s^3+2s^2+s+1)}$$

Consider the following statements regarding the Nyquist plot of $G(s)$:

S_1 : For $K > 1$, the Nyquist plot encircles the point $(-1 + j0)$ two times in clockwise direction.

S_2 : For $0 < K < 1$, the Nyquist plot encircles the point $(-1 + j0)$ two times in counterclockwise direction.

S_3 : For $K = 1$, the Nyquist plot passes through the point $(-1 + j0)$.

Select the correct statement(s) using the codes given below.

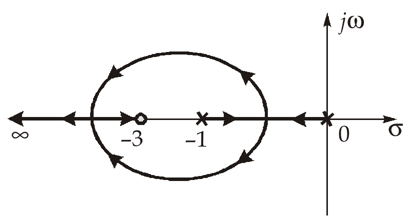
(a) S_1 and S_3 only
(b) S_2 and S_3 only
(c) S_1 and S_2 only
(d) S_3 only

Q.42 The open-loop transfer function of a closed-loop system with unity negative feedback is given by,

$$G(s) = \frac{K}{s(s+2)^2}$$

The root locus plot (for $K > 0$) of this system intersects the constant damping ratio (ξ) line, for $\xi = 0.50$, at $K = K_0$. The value of K_0 is ________.

Q.43 Consider the root locus diagram of a unity negative feedback system shown below:



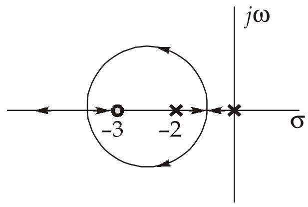
The corresponding open loop transfer function will be

(a) $\dfrac{K}{s(s+1)(s+3)}$
(b) $\dfrac{K(s+3)}{s(s+1)}$
(c) $\dfrac{K(s+1)}{s(s+3)}$
(d) $\dfrac{Ks}{(s+1)(s+3)}$

Q.44 The open-loop transfer function of negative feedback system is

$$G(s)H(s) = \frac{k(s+3)}{s(s+2)}$$

The root locus plot of the system consists a circle as shown in the figure below. The equation of this circle is



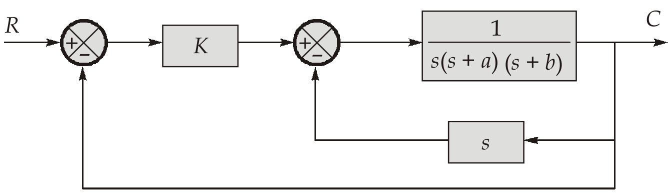
(a) $(\sigma + 4)^2 + \omega^2 = 4$
(b) $(\sigma - 3)^2 + \omega^2 = 9$
(c) $(\sigma + 3)^2 + \omega^2 = 3$
(d) $(\omega - 4)^2 + \omega^2 = (2)^2$

Q.45 Consider a unity negative feedback system having the characteristic equation,

$$1 + \frac{K}{(s+1)(s+1.5)(s+2)} = 0$$

It is desired that, all roots of the characteristic equation have real parts less than or equal to "–1". The maximum value of K, that satisfies the given condition is ___________.

Q.46 Consider the unity feedback system which employs rate feedback as shown in the figure.



The undamped oscillation frequency of this system is

(a) $\sqrt{ab} + 1$ rad/sec
(b) $\sqrt{ab}$ rad/sec
(c) $\sqrt{ab+1}$ rad/sec
(d) $ab + 1$ rad/sec

Q.47 A unity negative feedback control system has an amplifier with gain K = 10 and gain ratio $G(s) = \frac{1}{s(s+2)}$ in the forward path. A derivative negative feedback $H(s) = K_0 s$ is introduced as a minor loop around $G(s)$. The value of derivative feedback constant K_0 such that the system damping ratio is 0.6 will be _______.

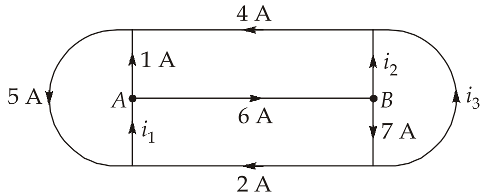
Q.48 The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{10}{s(s+A)}$$

If the step response of the closed loop system will have 0% overshoot and minimum settling time then, the value of 'A' is __________.

Multiple Select Questions (MSQ)

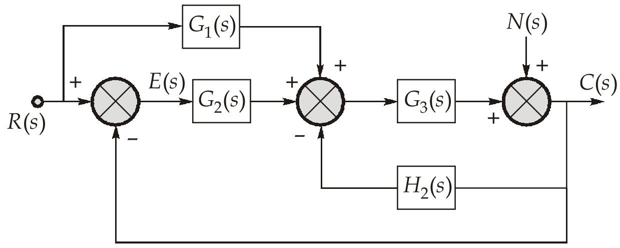
Q.49 For the given circuit :



Which of the following is correct?

(a) $i_1 = 7$ A
(b) $i_3 = 5$ A
(c) $i_2 = 1$ A
(d) $i_2 = -1$ A

Q.50 The block diagram of a control system is shown in figure.



Which of the following statements is/are true?

(a) $\left.\dfrac{C(s)}{R(s)}\right|_{N(s)=0} = \dfrac{G_1G_3 + G_2G_3}{1 + G_3H_1 + G_2G_3}$

(b) $\left.\dfrac{C(s)}{N(s)}\right|_{R(s)=0} = \dfrac{G_1G_3 + G_2G_3}{1 + G_3H_1 + G_2G_3}$

(c) $\left.\dfrac{C(s)}{N(s)}\right|_{R(s)=0} = \dfrac{1}{1 + G_3H_1 + G_2G_3}$

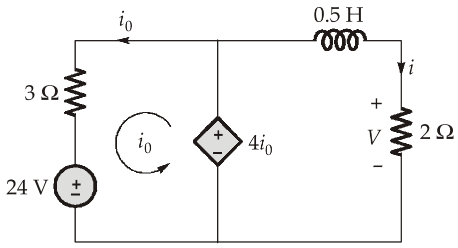
(d) $\left.\dfrac{C(s)}{R(s)}\right|_{N(s)=0} = \dfrac{1}{1 + G_3H_1 + G_2G_3}$

■■■■

Detailed Explanations

26. **(c)**

At $t < 0$, the circuit can be redrawn as



By using KVL in first loop, we get,

$$3i_0 + 24 - 4i_0 = 0$$

or $$i_0 = 24$$

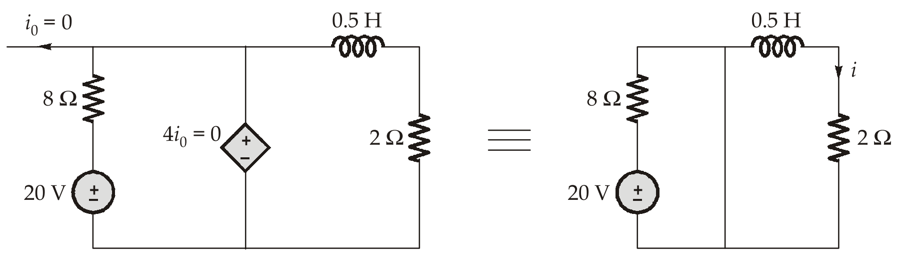
∵ Under steady state, the inductor behaves as a short circuit,

∴ $$v(0^-) = 4 \times i_0 = 96 \text{ V}$$

and $$i(0^-) = i(0^+) = \frac{96}{2} = 48 \text{ A} \quad \text{...(i)}$$

For $t > 0$, the switch moves to 'B',

∴ The circuit can be modified as



Due to short circuit the current $i(\infty) = 0$

∴ $$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

where $R_{Th} = 2\ \Omega$ and $\tau = \dfrac{L}{R_{Th}} = \dfrac{0.5}{2} = \dfrac{1}{4}$

∴ $$i(t) = 48e^{-4t}$$

and $$v(t) = 2i(t) = 96e^{-4t}\, u(t) \text{ V}$$

27. **23.44 (23.20 to 23.80)**

For maximum power transfer to the load,

$$Z_L = Z_{Th}^*$$

∴ $$Z_L = (120 - j60)\ \Omega$$

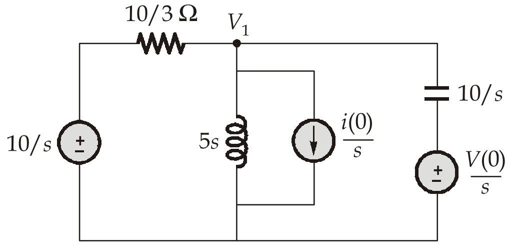
∴ $$I_{L\text{rms}} = \frac{150}{\sqrt{2} \times (Z_L + Z_{Th})} = \frac{150}{\sqrt{2} \times 240} = 0.442 \text{ A}$$

and $$P_{\text{avg}} = \left|I_{L\text{rms}}\right|^2 \times 120$$

$$= (0.442)^2 \times 120 = 23.44 \text{ W}$$

28. (d)

The circuit can be represent in s-domain as



Using nodal equation at (1), we get,

$$\frac{V_1 - 10/s}{10/3} + \frac{V_1 - 0}{5s} + \frac{i(0)}{s} + \frac{V_1 - (V(0)/s)}{10/s} = 0$$

$$0.1\left(s + 3 + \frac{2}{s}\right)V_1 = \frac{3}{s} + \frac{1}{s} + 0.5$$

where, $v(0) = 5$ V and $i(0) = -1$ A

Simplifying, we get,

$$(s^2 + 3s + 2)V_1 = 40 + 5s$$

or $$V_1 = \frac{40 + 5s}{(s+1)(s+2)}$$

Using partial fraction, we get,

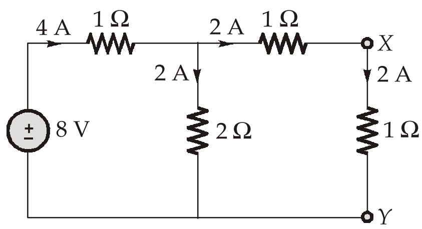
$$V_1 = \frac{35}{(s+1)} - \frac{30}{(s+2)}$$

Taking inverse Laplace transform, we get,

$$v_1(t) = v_c(t) = (35e^{-t} - 30e^{-2t})\, u(t) \text{ V}$$

29. (b)

Considering figure (a), we get,



Here, $I_{XY} = 2$ A and $V_{XY} = 2$ V

$\therefore$ $V_{XY} = Z_{XY} I_{XY} + E$

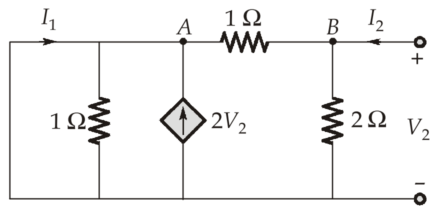
$2 = j2 \times 2 + E$

or $E = (2 - j4)$ V

$= 2\sqrt{5}\angle -63.43°$ V

30. (2)

Let us calculate the y-parameter, for this, considering $V_1 = 0$, the circuit can be redrawn as



∴ KCL at node (A) results in

$$-I_1 - 2V_2 - \left(I_2 - \frac{V_2}{2}\right) = 0$$

or $$I_1 + I_2 = -\frac{3}{2}V_2 \quad \text{...(i)}$$

By KVL in outer loop,

$$V_2 = 1\times\left(I_2 - \frac{V_2}{2}\right)$$

⇒ $$\frac{3}{2}V_2 = I_2$$

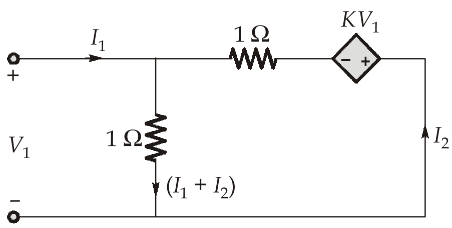
∴ $$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{3}{2}\ ℧ \quad \text{...(i)}$$

From equation (i), we get,

$$I_1 + \frac{3}{2}V_2 = -\frac{3}{2}V_2$$

or $$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -3\ ℧$$

Now by keeping $V_2 = 0$, the circuit can be redrawn as



Here, $$V_1 = I_1 + I_2 \quad \text{...(iii)}$$

and $$KV_1 + I_2 + V_1 = 0$$

or $$I_2 = -(1+K)V_1$$

∴ $$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -(1+K)$$

For reciprocal network,

$$y_{12} = y_{21}$$

$$-(1+K) = -3$$

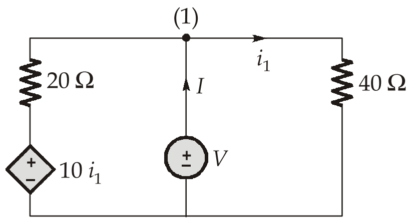
⇒ $$K = 2$$

31. **(c)**

For maximum power transfer,

$$R = R_{Th}$$

Finding R_{Th} in the circuit, by removing independent voltage source, as



By KCL at (1), we get,

$$\frac{V - 10i_1}{20} + \frac{V}{40} = I$$

but $$i_1 = \frac{V}{40}$$

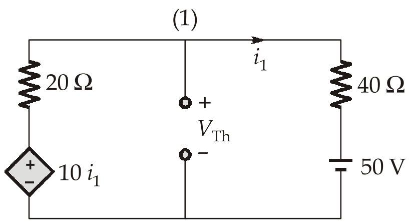
∴ $$\frac{V - 10\left(\frac{V}{40}\right)}{20} + \frac{V}{40} = I$$

or $$\frac{\frac{6\text{ V}}{4} + V}{40} = I$$

or $$\frac{10\text{ V}}{160} = I$$

or $$\frac{V}{I} = \frac{160}{10} = 16\ \Omega$$

Finding V_{Th}:



By KCL at (i),

$$\frac{V_{Th} - 10i_1}{20} + \frac{V_{Th} - 50}{40} = 0$$

$$\frac{V_{Th}}{20} - \frac{1}{2}\left(\frac{V_{Th} - 50}{40}\right) + \left(\frac{V_{Th} - 50}{40}\right) = 0$$

$$\frac{V_{Th}}{20} + \frac{V_{Th}}{80} - \frac{50}{80} = 0$$

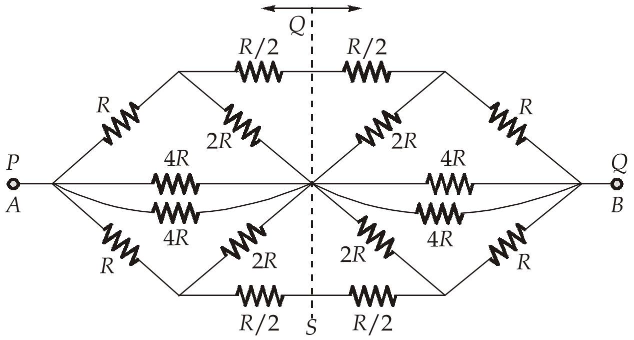
or $$5V_{Th} = 50$$

or $$V_{Th} = 10\text{ V}$$

∴ $$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{10^2}{4 \times 16} = 1.56\text{ W}$$

32. 1.037 (0.90 to 1.20)

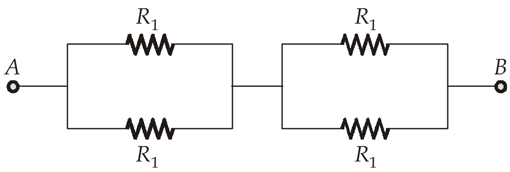
As the hexagon is symmetric, the equivalent resistance across first part (PQ) is



$$R_1 = (2R \| R/2 + R) \| 4R$$

$$= \frac{28}{27}R$$

$\therefore$



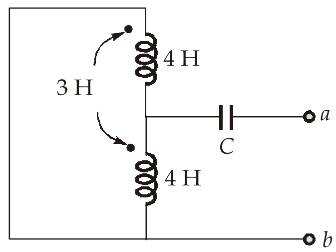
$$R_{AB} = (R_1 \| R_1) + (R_1 \| R_1) = 2\left(\frac{28}{27} \Big\| \frac{28}{27}\right)R = \frac{28}{27}R\ \Omega = 1.037R\ \Omega$$

33. 1.785 (1.50 to 1.90)

The Thevenins equivalent voltage with terminals a - b open circuited is obtained as

$$V_{\text{Th}} = V \times \frac{1}{2} = 50 \sin 400t \qquad \text{(By voltage division rule)}$$

In order to find Thevenin's equivalent impedance, the circuit can be redrawn as



$$Z_{\text{Th}} = j\omega\left[\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}\right] - \frac{j}{\omega C}$$

$$= j400\left[\frac{4 \times 4 - 3^2}{4 + 4 - 6}\right] - \frac{j}{\omega C} = \left(j1400 - \frac{j}{400C}\right)$$

$\therefore$ Current through load impedance,

$$I_L = \frac{V_{Th}}{Z_L + Z_{Th}} = \frac{50\sin 400t}{\left(j1400 - \frac{j}{400C}\right) + Z_L}$$

$\because$ $$V_L = I_L \times Z_L$$

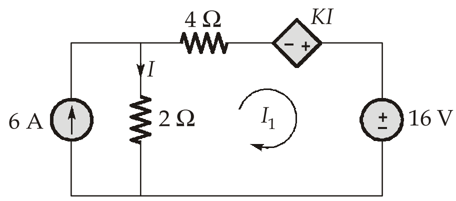
$\therefore$ The voltage will be independent of Z_L if

$$j1400 = \frac{j}{400C}$$

or $$C = \frac{1}{1400 \times 400} = 1.785\ \mu\text{F}$$

34. (2)

Redrawing the given circuit, we have,



By KVL, we get,

$$4I_1 - KI + 16 + 2(I_1 - 6) = 0$$

or $$I_1 = \left(\frac{KI - 4}{6}\right) \quad \text{...(i)}$$

Also, $$I = (6 - I_1)$$

$$= 6 - \left(\frac{KI - 4}{6}\right)$$

$$I = \frac{40 - KI}{6}$$

or $$I = \frac{40}{6 + K}$$

$\because$ Power dissipated in 2 Ω resistor is 50 W.

$\therefore$ $$P_{2\,\Omega} = I^2 \times 2 = 50$$

or $$I = \sqrt{\frac{50}{2}} = 5\text{ A}$$

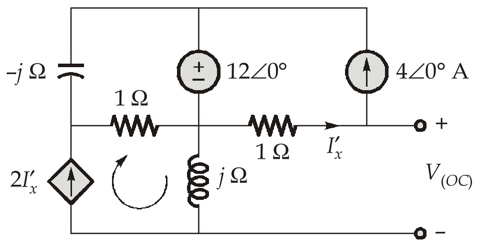
$\Rightarrow$ $$\frac{40}{6 + K} = 5$$

or $$5K = 10$$

or $$K = 2$$

35. **(a)**

The value of $V_{(OC)}$ can be calculated by removing 1 Ω resistor

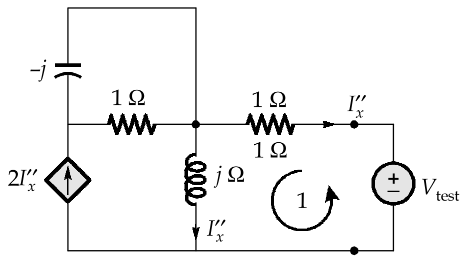


thus, the value of $I'_x = 4\angle 0°$ and current flowing through j Ω = $2I'_x$

thus, $V_{OC} = -4\angle 0° + j(2I'_x)$

$= -4 + 8j$ V

now, applying a test voltage to find R_{th} we get,



Applying RCL in loop (1)

$$jI''_x - I''_x - V_{test} = 0$$

$$I''_x = \frac{-V_{test}}{1-j}$$

$$\therefore \quad Z_{th} = \frac{-V_{test}}{I''_x} = 1 - j\ \Omega$$

36. **(d)**

Applying KCL at node $V(t)$ we get,

$$i(t) = C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} \qquad \text{...(1)}$$

$$\therefore \quad 4i(t) = \frac{dv(t)}{dt} + 2v(t) \text{ for } t > 0$$

for $t > 0$, the 24 V is connected to the circuit.

Thus applying KVL we get

$$\frac{Ldi(t)}{dt} + R_1 i(t) + v(t) = 24 \qquad \text{...(2)}$$

$$\frac{di(t)}{dt} + \frac{R_1}{L}i(t) + \frac{v(t)}{L} = \frac{24}{L}$$

$$\therefore \quad \frac{di(t)}{dt} + 5i(t) + 0.5v(t) = 12 \text{ for } t > 0$$

Combining eqn. (1) and (2) we get,

$$\frac{d^2v(t)}{dt^2}+\left(\frac{1}{R_1C}+\frac{R_1}{L}\right)\frac{dv(t)}{dt}+\frac{R_1+R_2}{R_2LC}v(t)=\frac{24}{LC}$$

$$\therefore \quad \frac{d^2v(t)}{dt^2}+7\frac{dv(t)}{dt}+12v(t) = 48 \quad \text{for } t > 0.$$

Thus option 'd' is incorrect.

37. **(1.2)**

Since all the branches are connected in parallel to the current source. Thus,

$$\frac{i_1}{i_2} = \frac{\dfrac{1/50}{R_{eq}}}{\dfrac{1/60}{R_{eq}}}=\frac{60}{50}=\frac{6}{5}=1.2$$

where $$R_{eq} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}$$

38. **(3.05) (2 to 4)**

Now, $$\frac{V_0(s)}{V_A(s)} = \frac{-4}{1000}\times\left[\frac{s/C}{s^2+s/RC+\frac{1}{LC}}\right]$$

at resonance, $$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{(for maximum value of gain)}$$

$$\therefore \quad 2\pi(91.1\times10^6) = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad C = 3.05 \text{ pF}$$

39. **(c)**

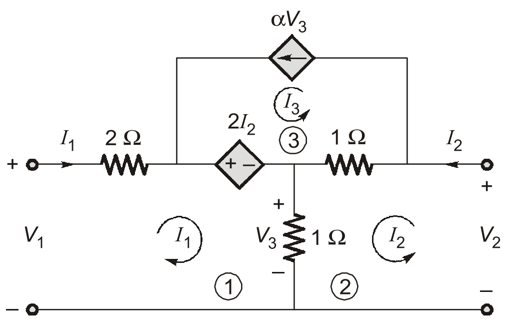
$$R_p = R(1+Q^2)$$

Now, $$Q = \frac{|X_L|}{R_L}=\frac{20}{20}=1$$

$$\therefore \quad R_p = 20\times2 = 40\ \Omega$$

and $$jX_p = j20\left(1+\frac{1}{Q^2}\right)=j40\ \Omega$$

40. **(–2)**



Writing the KVL equation in loop (1), we get,

$$V_1 = 2I_1 + 2I_2 + I_1 + I_2$$

$$V_1 = 3I_1 + 3I_2 \quad \text{...(i)}$$

Writing the KVL equation for loop (2), we get,

$$V_2 = 2I_2 + I_1 + (-\alpha V_3)$$

Also, $$V_2 = V_3 + (I_2 - \alpha V_3)$$

$$V_2 = I_2 + (1 - \alpha)V_3 \quad \text{...(ii)}$$

$\because$ $$V_3 = (I_1 + I_2) \times 1\ \Omega \quad \text{...(iii)}$$

$\therefore$ From equations (ii) and (iii),

$$V_2 = I_2 + (1 - \alpha)(I_1 + I_2)$$

$$= I_2 + (1 - \alpha)I_2 + (1 - \alpha)I_1$$

$$= (1 - \alpha)I_1 + (2 - \alpha)I_2 \quad \text{...(iv)}$$

From equations (i) and (iv),

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ (1-\alpha) & (2-\alpha) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the circuit to be reciprocal,

$$Z_{12} = Z_{21}$$

$$(1 - \alpha) = 3$$

or $$-\alpha = 3 - 1 = 2$$

or, $$\alpha = -2$$

41. **(d)**

The number of encirclements (N) of the Nyquist plot about the point $(-1 + j0)$ can be given by,

$$N = P - Z$$

Here, P = Number of open-loop poles in the RHS of s-plane

Z = Number of closed-loop poles in the RHS of s-plane

$s(s^3 + 2s^2 + s + 1) = 0$ does not produce any roots in the RHS of s-plane. So, $P = 0$.

The characteristic equation of the closed-loop system is,

$$s(s^3 + 2s^2 + s + 1) + K(s^2 + s + 1) = 0$$

$$s^4 + 2s^3 + (1 + K)s^2 + (1 + K)s + K = 0$$

Using the RH criteria,

s^4	1	$(1+K)$	K
s^3	2	$(1+K)$	0
s^2	$\frac{(1+K)}{2}$	K	0
s^1	$\frac{(1-K)^2}{(1+K)}$	0	0
s^0	K	0	0

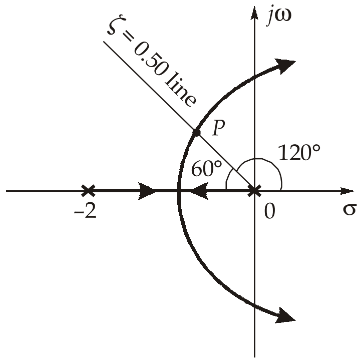
- For $K = 1$, row of zeros occurs and the system becomes marginally stable.
- For $K > 1$, system becomes stable.
- For $0 < K < 1$, system becomes stable.

So, for $K = 1$, the Nyquist plot passes through $(-1 + j0)$ point. Hence, statement S_3 is correct.
For $K > 1$, the system is stable. So, $Z = 0$ and $N = P - Z = 0 - 0 = 0$.
For $0 < K < 1$, the system is stable. So, $Z = 0$ and $N = P - Z = 0 - 0 = 0$.
Hence, statements S_1 and S_2 are incorrect.

42. **(3)**

$$\cos(\phi) = \zeta = 0.50 \Rightarrow \phi = \cos^{-1}(0.50) = 60°$$

Let, the $\xi = 0.50$ line intersects the root locus plot at point "P" in the s-plane.



So, $$P = r\angle 120° = -\frac{r}{2} + j\sqrt{3}\frac{r}{2}$$

From angle condition,

$$\angle G(s)\big|_{s=P} = \pm 180°$$

$$G(s)\big|_{s=P} = \frac{K}{(r\angle 120°)\left(2 - \frac{r}{2} + j\sqrt{3}\frac{r}{2}\right)^2}$$

$$-\left[120° + 2\tan^{-1}\left(\frac{\sqrt{3}\,r}{4-r}\right)\right] = \pm 180°$$

$$\tan^{-1}\left(\frac{\sqrt{3}\,r}{4-r}\right) = 30°$$

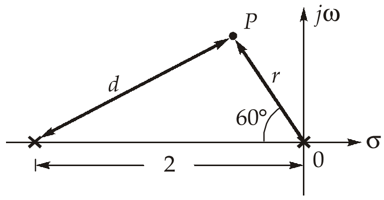
$$\frac{\sqrt{3}\,r}{4-r} = \tan(30°) = \frac{1}{\sqrt{3}}$$

$$3r = 4 - r$$

$$r = 1$$

So, the ξ = 0.50 line intersects the root locus plot at a radial distance of 1 from the origin.

From cosine rule,



$$d^2 = (2)^2 + r^2 - 2(2)(r)\cos(60°)$$
$$= 4 + (1)^2 - (2)(1) = 3$$
$$d = \sqrt{3}$$

From magnitude condition,

$$K_0 = \frac{\Pi(\text{Vector distances from OL poles})}{\Pi(\text{Vector distances from OL zeros})} = \sqrt{3}\times\sqrt{3}\times 1 = 3$$

43. **(b)**

The root loci starts from s = –1 and 0 and ends at –3 and ∞. Hence poles are at –1, 0 and zeros are at –3, ∞. Thus, the transfer function will be $\frac{K(s+3)}{s(s+1)}$.

44. **(c)**

For root locus plot

$$\angle G(s)H(s) = 180°$$

∴ substituting $s = \sigma + j\omega$ we get

$$G(\sigma + j\omega)H(\sigma + j\omega) = \frac{k(\sigma+3+j\omega)}{(\sigma+j\omega)(\sigma+2+j\omega)}$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180° + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right)$$

$$\frac{\frac{\omega}{\sigma+3} - \frac{\omega}{\sigma}}{1+\left(\frac{\omega}{\sigma+3}\right)\left(\frac{\omega}{\sigma}\right)} = \frac{\omega}{\sigma+2}$$

$$\frac{-3\omega}{\sigma(\sigma+3)+\omega^2} = \frac{\omega}{\sigma+2}$$

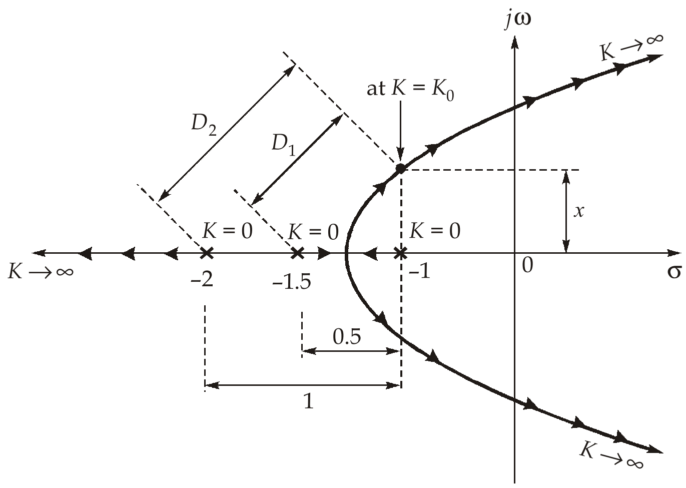
$$-3(\sigma + 2) = \sigma(\sigma + 3) + \omega^2$$
$$(\sigma^2 + 6\sigma + 9) + \omega^2 = -6 + 9$$
$$(\sigma + 3)^2 + \omega^2 = 3$$

45. 0.75 (0.60 to 0.90)

The loop transfer function,

$$G(s)H(s) = \frac{K}{(s+1)(s+1.5)(s+2)}$$

Drawing the root locus plot of the corresponding system,



From root locus diagram, it is clear that, for $K \le K_0$, all roots of the characteristic equation have real parts less than or equal to "–1"

To determine the value of K_0:

If we know the value of 'x', then it is very easy to determine the value of K_0.

- Using angle condition, to determine the value of "x".

$$\angle G(s)\,H(s)\big|_{s=-1+jx} = -180°$$

$$-90° - \tan^{-1}(2x) - \tan^{-1}(x) = -180°$$

$$\tan^{-1}\left(\frac{3x}{1-2x^2}\right) = 90°$$

$$1 - 2x^2 = 0$$

$$x = \frac{1}{\sqrt{2}}\text{ rad/sec}$$

- Using magnitude condition, to determine the value of K_0

$$|G(s)\,H(s)|_{s=-1+jx} = 1$$

or, $$K_0 = (D_1)\,(D_2)\,(x)$$

$$x = \frac{1}{\sqrt{2}};\ D_1 = \sqrt{0.25+x^2} = \sqrt{0.75}\ ;\ D_2 = \sqrt{1+x^2} = \sqrt{1.50}$$

So, $$K_0 = \frac{1}{\sqrt{2}} \times \sqrt{0.75} \times \sqrt{1.50} = 0.75$$

46. (c)

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+a)(s+b)+s+K}$$

Characteristic equation $\Rightarrow s^3 + (a+b)s^2 + (ab+1)\,s + K = 0$

Routh Table

s^3	1	$ab+1$
s^2	$a+b$	K
s^1	$\dfrac{(a+b)(ab+1)-K}{a+b}$	
s^0	K	

For undamped oscillations, s^1 Row must has all zeros.

$$(a+b)(ab+1) = K$$

Also auxiliary equation

$A(s) \Rightarrow (a+b)s^2 + K = 0$

at $s = j\omega$

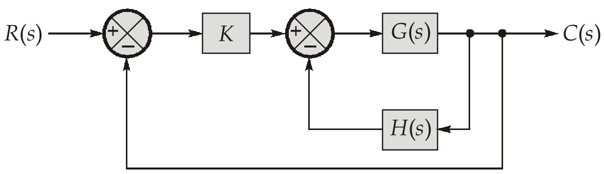
$$(a+b)(-\omega^2)+K = 0$$

$$\omega^2 = \frac{K}{a+b} = \frac{(a+b)(ab+1)}{(a+b)}$$

$$\omega = \left(\sqrt{ab+1}\right)\text{rad/sec}$$

47. 1.794 (1.60 to 2.00)

The block diagram representation of the given system is



Where $K = 10$, $G(s) = \dfrac{1}{s(s+2)}$ and $H(s) = K_0 s$

The closed-loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{K\dfrac{G(s)}{1+G(s)H(s)}}{1+\dfrac{KG(s)}{1+G(s)H(s)}} = \frac{KG(s)}{1+KG(s)+G(s)H(s)}$$

$$= \frac{\dfrac{10}{s(s+2)}}{1+\dfrac{10}{s(s+2)}+\dfrac{K_0 s}{s(s+2)}} = \frac{10}{s(s+2)+10+K_0 s}$$

or $$\frac{C(s)}{R(s)} = \frac{10}{s^2 + (2+K_0)s + 10}$$

Comparing the above equation with standard second order equation, we get,

and $2\xi\omega_n = (2 + K_0)$

Here $\omega_n = \sqrt{10} = 3.162 \text{ rad/sec}$

given, $\xi = 0.6$

$$2 \times 0.6 \times \sqrt{10} = 2 + K_0$$

$$K_0 = [2 \times 0.6 \times 3.162 - 2] = 1.794$$

48. (6.325)(6.25 to 6.45)

Minimum settling time and 0% overshoot means 100% damping ratio i.e., $\xi = 1$.

From given system,

$$T(s) = \frac{10}{s^2 + As + 10}$$

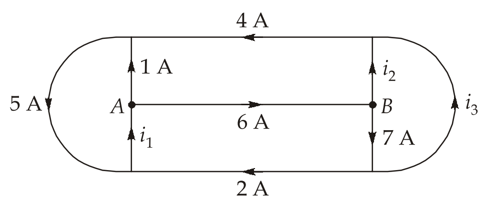
Here, $\omega_n = \sqrt{10}$

and $2\xi\omega_n = A$

$\therefore$ $A = 2 \times 1 \times \sqrt{10}$

$= 2\sqrt{10} = 6.3245$

49. (a, b, d)



Applying KCL at A

$$1 + 6 = i_1$$
$$i_1 = 7 \text{ A}$$

Applying KCL at B

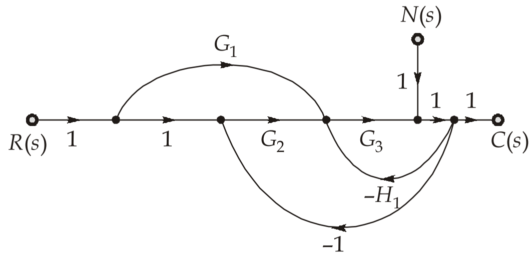
$$i_2 + 7 = 6$$
$$i_2 = -1 \text{ A}$$

Applying KCL at C

$$i_2 + i_3 = 4$$
$$-1 + i_3 = 4$$
$$i_3 = 5 \text{ A}$$

50. (a, c)

Signal flow diagram



To find $\frac{C(s)}{R(s)}$ when $N(s) = 0$

The two forward paths are

$$P_1 = G_1G_3$$
$$P_2 = G_2G_3$$

The closed loop touches both the forward paths, therefore path factors are

$$\Delta_1 = 1 \quad \text{and} \quad \Delta_2 = 1$$

The individual loops are $L_1 = -G_3H_1$ and $L_2 = -G_2G_3$

Applying Mason's gain formula the closed loop transfer function.

$$\left.\frac{C(s)}{R(s)}\right|_{N(s)=0} = \frac{G_1G_3 + G_2G_3}{1 + G_3H_1 + G_2G_3}$$

To find $\left.\frac{C(s)}{N(s)}\right|_{R(s)=0}$

There is single forward path having gain

$$P_1 = 1$$

The path factor $\Delta_1 = 1$.

The individual loops are

$$L_1 = -G_3H_1 \text{ and } L_2 = -G_2G_3$$
$$R(s) = 0$$
$$\frac{C(s)}{N(s)} = \frac{1}{1 + G_3H_1 + G_2G_3}$$

■■■■