MADE EASY
India's Best Institute for IES, GATE & PSUs

ESE 2021

Main Exam
Detailed Solutions

CIVIL ENGINEERING

PAPER-I

EXAM DATE : 21-11-2021 | 9:00 AM to 12:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/ error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

Corporate Office : 44-A/1, Kalu Sarai, Near Hauz Khas Metro Station, New Delhi-110016

Delhi | Hyderabad | Bhopal | Jaipur | Lucknow | Bhubaneswar | Pune | Kolkata | Patna

9021300500 www.madeeasy.in

ANALYSIS

Civil Engineering Paper-I
ESE 2021 Main Examination

Sl.	Subjects	Marks
1.	Building Materials and Construction	52
2.	Strength of Materials	104
3.	Structural Analysis	76
4.	Steel Structures	52
5.	RCC	144
6.	CTPM and Equipments	52
	Total	**480**

Scroll down for detailed solutions

Section-A

Q.1 (a) (i) What are the functions of cement mortar in brick masonry work?

(ii) What are the purposes of making frog in bricks during moulding? Explain with the help of neat sketches.

[6 + 6 = 12 Marks]

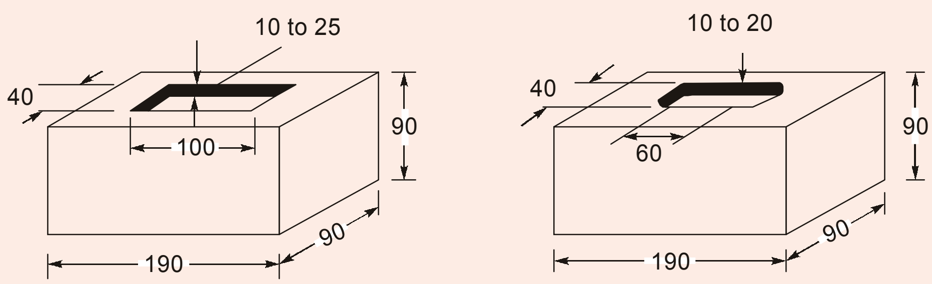
Solution:

(i) Uses of cement mortar:

- To bind the building units such as bricks.
- To carry out pointing and plaster work on exposed surfaces of masonry.
- To form an even and soft bedding layer for building units.
- To form joints of pipes.
- To hide the open joints of brickwork.
- To improve the general appearance of structure.
- To fill up the cracks detected in the structure during maintenance process etc.

(ii) Frog or kick: A frog is an indentation in the face of a brick to form a key for holding the mortar. It serves two purposes:

1. It indicates the trade name of the manufacturer.
2. In brickwork, the bricks are laid with frog uppermost. It thus affords a key for mortar when the next brick is placed over it.



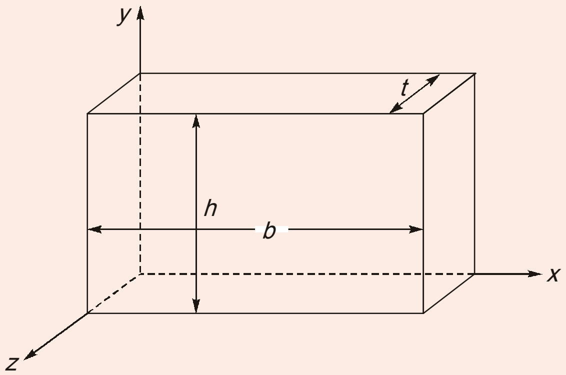
End of Solution

Q.1 (b) A rectangular plate in biaxial stress is subjected to normal stress σ_x = 65 MPa and σ_y = –20 MPa. The plate ($b \times h \times t$) has dimensions 200 × 300 × 15 and is made of aluminium with E = 75 GPa an v = 0.33. All dimensions are in mm. Determine

(i) The maximum in-plane shear strain (γ_{max}) in the plate,

(ii) The change in the thickness of the plate (Δt), and

(iii) The change in volume of the plate (Δv).



[12 Marks]

Solution:

Strain in x-direction

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{1}{75 \times 10^3}[(65) - (0.33)(-20)]$$

$$\epsilon_x = 9.54 \times 10^{-4}$$

Strain in y-direction

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{1}{75 \times 10^3}[(-20) - (0.33)(65)]$$

$$\epsilon_y = -5.52 \times 10^{-4}$$

Strain in z-direction

$$\epsilon_z = \frac{1}{E}[0 - \nu(65) - \nu(-20)]$$

$$\epsilon_z = -1.98 \times 10^{-4}$$

(i) Maximum in-plane shear strain

$$\gamma_{max} = \epsilon_{p\,max} - \epsilon_{p\,min}$$
$$= (9.54 \times 10^{-4}) - (-5.52 \times 10^{-4})$$
$$\gamma_{max} = 15.06 \times 10^{-4}$$

(ii) Change in thickness of plate

$$\Delta t = \epsilon_z(t)$$
$$= (-1.98 \times 10^{-4})\,(15)$$
$$\Delta t = -2.97 \times 10^{-3}\text{ mm}$$

(iii) Change in volume of plate

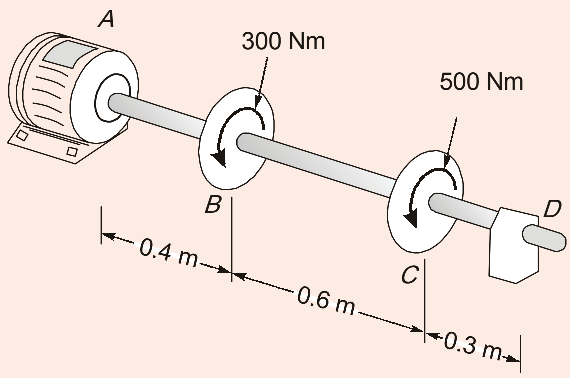
$$\epsilon_v = \frac{\Delta V}{\text{Volume}} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\Delta v = [(9.54 - 5.52 - 1.98) \times 10^{-4}]\,[200 \times 300 \times 15]$$

$$\Delta v = 183.6\text{ mm}^3$$

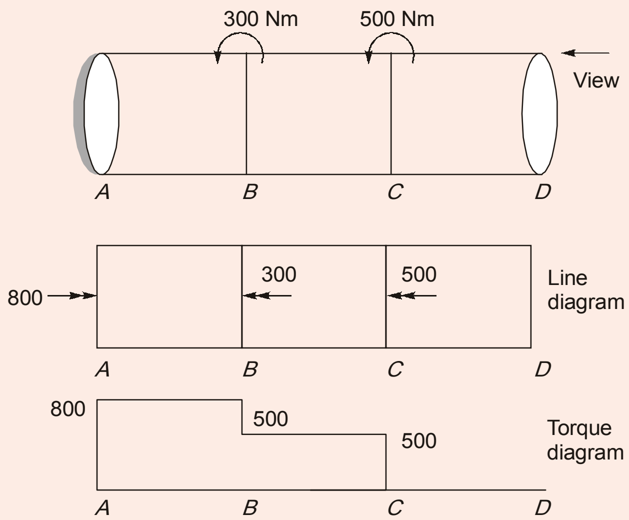
End of Solution

Q.1 (c) The electric motor exerts a torque of 800 N on the steel shaft ABCD when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from A to D and that the angle of twist between A and D not exceed 1.5°. Knowing that $\tau_{max} \geq 60$ MPa and G = 77 GPa, determine the diameter of the shaft that my be used.



[12 Marks]

Solution:



For maximum shear stress

$$\tau_{max} = \frac{16T_{max}}{\pi D^3}$$

$$60 = \frac{16(800 \times 10^3)}{\pi D^3}$$

$$D = 40.49 \text{ mm}$$

angle of twist

$$\theta_{D/A} = \theta_{D/C} + \theta_{C/B} + \theta_{B/A} = \left(1.5 \times \frac{\pi}{180}\right)$$

$$\theta_{D/C} = 0$$

$$\theta_{C/B} = \frac{(500 \times 10^3)(0.6 \times 10^3)}{(77 \times 10^3)\frac{\pi}{32}D^4}$$

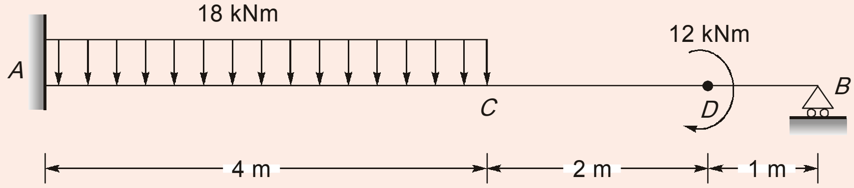
$$\theta_{B/A} = \frac{(800 \times 10^3)(0.4 \times 10^3)}{(77 \times 10^3)\left(\frac{\pi}{32}D^4\right)}$$

$$D = 42.08 \text{ mm}$$

$$\text{So, design dia.} = 42.08 \text{ mm} \simeq 43 \text{ mm}$$

End of Solution

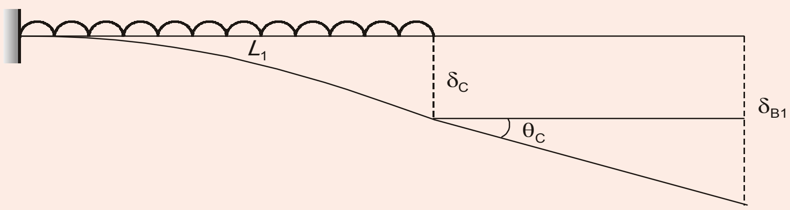
Q.1 (d) Analyse the propped cantilever beam shown in the figure. During loading the rop B sinks by 6 mm. E = 2 × 10^5 MPa, I = 86.04 × 10^6 mm^4. Draw shear force and bending moment diagrams. Find the maximum B Mn value of its location from either of the supports.



[12 Marks]

Solution:

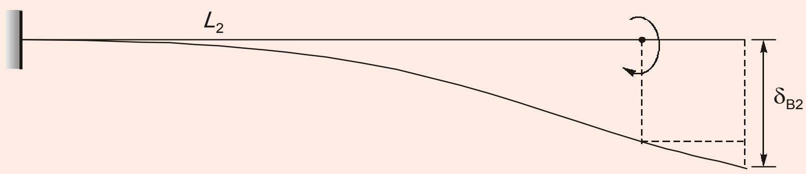
Dflection at support B due to UDL



$$\delta_{B1} = \frac{WL_1^4}{8EI} + \frac{WL_1^3}{6EI} \times 3(\downarrow)$$

$(L_1 = 4 \text{ m})$

Deflection at support B due to concentrated moment



$$\delta_{B2} = \frac{ML_2^2}{2EI} + \frac{ML_2}{EI} \times 1(\downarrow)$$

$(L_2 = 6\text{m})$

Deflection at support B dueto R_B

$$\delta_{B3} = \frac{R_B L^3}{3EI}(\uparrow)$$

$$\delta_{B_1} + \delta_{B_2} - 6 \times 10^{-3} = \delta_{B_3}$$

$$\frac{WL_1^4}{8EI} + \frac{WL_1^3}{3EI} \times 3 + \frac{ML_2^2}{2EI} + \frac{ML_2}{EI} \times 1 - 6 \times 10^{-3} = \frac{R_B 7^3}{3EI}$$

$$= \frac{(18 \times 10^3) \times 4^4}{8EI} + \frac{(18 \times 10^3) \times 4^3}{6EI} \times 3 + \frac{(12 \times 10^3) \times 6^2}{2EI} + \frac{12 \times 6}{EI} \times 1 - 6 \times 10^{-3}$$

$$= \frac{R_B \times 7^3}{3EI}$$

$$\Rightarrow \quad \frac{576 \times 10^3}{EI} + \frac{576 \times 10^3}{EI} + \frac{216 \times 10^3}{EI} + \frac{72 \times 10^3}{EI} - 6 \times 10^{-3} = \frac{R_B \times 114.33}{EI}$$

$$\Rightarrow \quad 1440 \times 10^3 - 6 \times 10^{-3} \times EI = R_B \times 114.33$$

$$EI = \left[2 \times 10^5 \times 10^6 \times 86.04 \times 10^6 \times 10^{-12}\right]$$

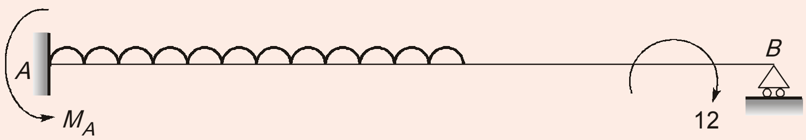
$$= [172.08 \times 10^5]$$

$$1440000 - 103248 = R_B \times 114.33, \quad R_B = 11.69 \text{ kN}$$

$$\Sigma F_y = 0$$

$$R_A + R_B = 18 \times 4$$

$$R_A = 60.31 \text{ kN}$$

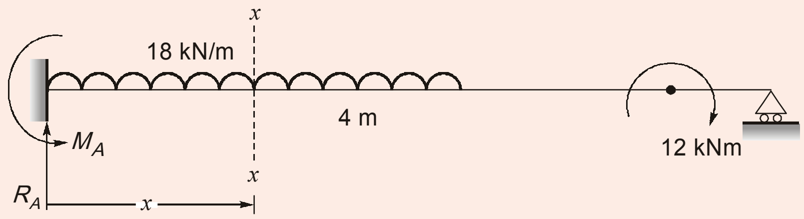


$$\Sigma M_B = 0$$

$$-M_A + 60.31 \times 7 - 18 \times 4 \times 5 + 12 = 0$$

$$M_A = 74.17 \text{ kNm}$$

BMD



$$M_x = -M_A - R_A x - \frac{Wx^2}{2} = -74.17 + 60.31x - \frac{18x^2}{2}$$

$$\frac{dM_x}{dx} = 60.31 - 18x = 0$$

$$x = 3.35 \text{ m}$$

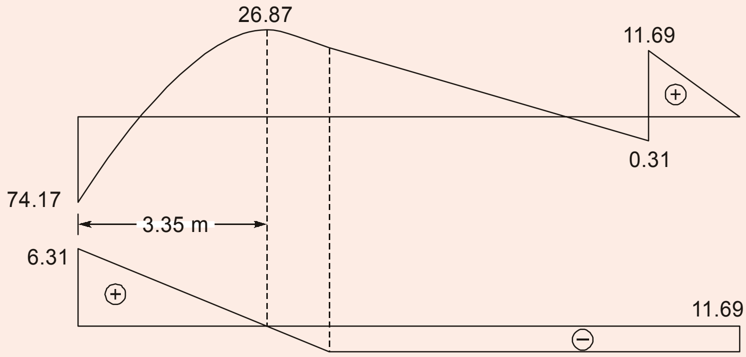
$$BM_{\text{at support A }(x = 0)} = -M_A = -74.17$$

$$\text{BM at C } (x = 0) = -74.17 + 241.24 - 144 = 23.07$$

$$BM_{\text{at left of D}} = -74.17 + 60.31 \times 6 - (18 \times 4) \times 4$$

$$= -0.31 \text{ kNm}$$

$$BM_{\text{at right of D}} = -0.31 + 12 = 11.69 \text{ kNm}$$



End of Solution

Q.1 (e) A simply supported rectangular beam (cross-section 300 mm × 400 mm) with effective span of 6 meters is carrying the following characteristic load:

(i) Characteristic Dead load (including self-weight) = 15 kN/m

(ii) Characteristic Imposed load (not fixed) = 10 kN/m

(iii) Characteristic equivalent wind load (acting downward) = 5 kN/m

Calculate the Design Bending Moment and Design shear force for most critical load combination for limit state of collapse and limit state of serviceability.

Partial Safety Factor (γ_f) for load

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	Dead load	Imposed load	Wind load	Dead load	Imposed load	Wind load
Dead load + Imposed Load	1.5	1.5	—	1.0	1.0	—
Dead Load + Wind load	0.9	—	1.5	1.0	—	1.0
Dead Load + Imposed Load + Wind load	1.2	1.2	1.2	1.0	0.8	0.8

[12 Marks]

Solution:

Most critical load combination for limit state of collapse:

Case-1 $= 1.5\,DL + 1.5\,IL$ kN/m

$= 1.5 \times 15 + 1.5 \times 10 = 37.5$

Case-2 $= 0.9 \times DL + 1.5\,IL$

$= 0.9 \times 15 + 1.5 \times 10 = 28.5$

Case-3 $= 1.2\,DL + 1.2\,LL + 1.2\,WL$

$= 1.2 \times (15 + 10 + 5) = 36$ kN/m

Most critical is case -1

(a) Maximum bending moment for limit state of collapse

$$= \frac{w_u \times L_e^2}{8} = \frac{37.5 \times 6^2}{8}$$

For simply supported beam

BMU $= 168.75$ kN-m

(b) Maximum SF. $V_u = \dfrac{w_u \times L_{cl}}{2}$

(Note: Maximum SF shall be calculated from clear span $= L_{cl} = (L_{eff} - d)$)

$$L_{clear} = 6.0 - 0.35 = 5.65 \text{ m}$$

$$D = 400 \text{ mm}$$

Assuming conv. $= 50$ mm

$$d = 350 \text{ mm}$$

Maximum SF at face of support,

$$V_u = \frac{w_u \times L_{cl}}{2} = \frac{37.5 \times 5.65}{2}$$

$= 105.94$ kN

Load combinations for limit state of serviceability

Case-1 $= 1.0\,DL + 1.0\,IL$

$= 1.0 \times (15 + 10) = 25$ kN/m

Case-2 $= 1.0 \times DL + 1.0 \times WL$

$= 1.0 \times (15 + 5) = 21$ kN/m

Case-3 $= 1.0 \times DL + 0.8\,LL + 0.8\,WL$

$= 1.0 \times 15 + 0.8 \times (10 + 5) = 27$ kN/m

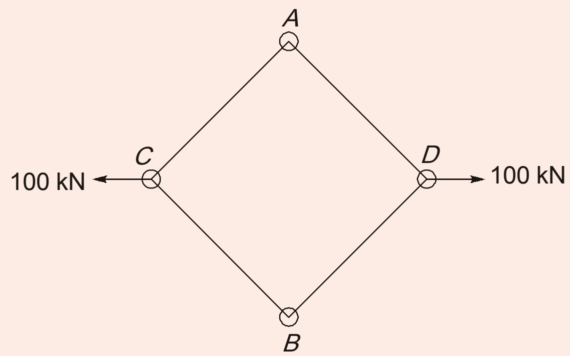
For simply supported beam:

(a) Maximum bending moment $= \dfrac{wL_e^2}{8} = \dfrac{27 \times 5^2}{8} = 121.5$ kNm

(b) Maximum SF. $= \dfrac{wL_{cl}}{2} = \dfrac{27 \times 5.65}{2} = 76.275$ kN

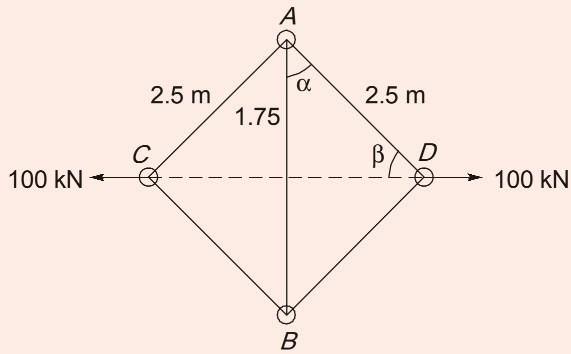
End of Solution

Q.2 (a) A steel bar AB of length 3.5 m and diameter 25 mm is connected by four inextensible cables + of length 2.5 m each, forming a rhombus with AB ad diagonal. A 100 kN force acts at the × = points C and D. Determine the decrease in length of the member AB and increase in the length between points C and D. Take modulus of steel as 201 GPa.



[20 Marks]

Solution:

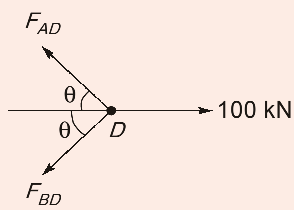


$$\sin\theta = \frac{1.75}{2.5} = 44.43°$$

$$\cos\alpha = \frac{1.75}{2.5} = 45.57°$$

At joint D:

$$\Sigma f_x = 0$$



$$F_{AD}\cos\theta + F_{BD}\cos\theta = 100$$

$$f_y = 0$$

$$F_{AD} \sin\theta = F_{BD} \sin\theta$$

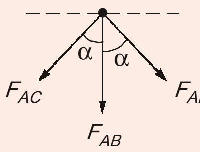
$$F_{AD} = F_{BD}$$

$$2F_{AD}\cos\theta = 100$$

$$F_{AD} = \frac{50}{\cos 44.43} = 70.02 \text{ kN}$$

At joint A:

$$\Sigma f_x = 0$$



$$F_{AC} = F_{AD}$$

$$\Sigma f_y = 0$$

$$F_{AB} + 2F_{AD}\cos\alpha = 0$$

$$F_{AB} = -2 \times 70.02 \times \cos 45.57° = -98.03 \text{ kN}$$

$$F_{AB} = 98.03 \text{ (compression)}$$

Decrease in the length of member (Δ_{AB}) $AB = \dfrac{PL}{AE}$

$$A = \frac{\pi}{4}(0.025)^2 = 4.91 \times 10^{-4} \text{ m}^2$$

$$E = 201 \times 10^6 \text{ kN/m}^2$$

$$= \frac{98.03 \times 3.5}{4.91 \times 10^{-4} \times 201 \times 10^6}$$

$$= 0.347 \times 10^{-2} \text{ m} = 3.47 \text{ mm}$$

As cables are inextensible, total work done by the external force will be equal to the strain energy stored in the bar *AB*.

Strain energy stored in bar $AB = \dfrac{1}{2} F_{AB}\,\Delta_{AB}$

$$= \left[\frac{1}{2} \times 98.02 \times 3.47\right] \quad \text{...(i)}$$

Work done = Strain energy stored

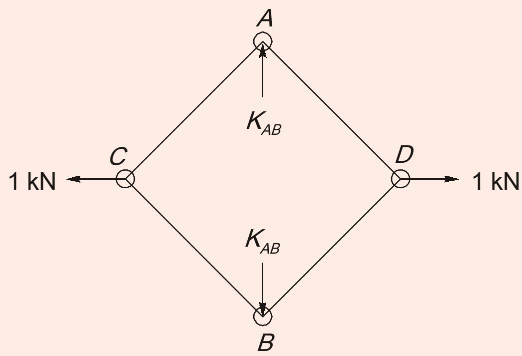
$$\frac{1}{2} \times 100 \times \Delta_{CD} = \frac{1}{2} \times 98.02 \times 3.47$$

$$\Delta_{CD} = 3.401 \text{ mm}$$

Alternative solution:

Unit load method

$$\delta_{CD} = \Sigma K_i \Delta_i$$



$$\Delta_{AC} = \Delta_{CB} = \Delta_{BD} = \Delta_{AD} = 0$$

$$\Delta_{AB} = -3.47 \text{ mm}$$

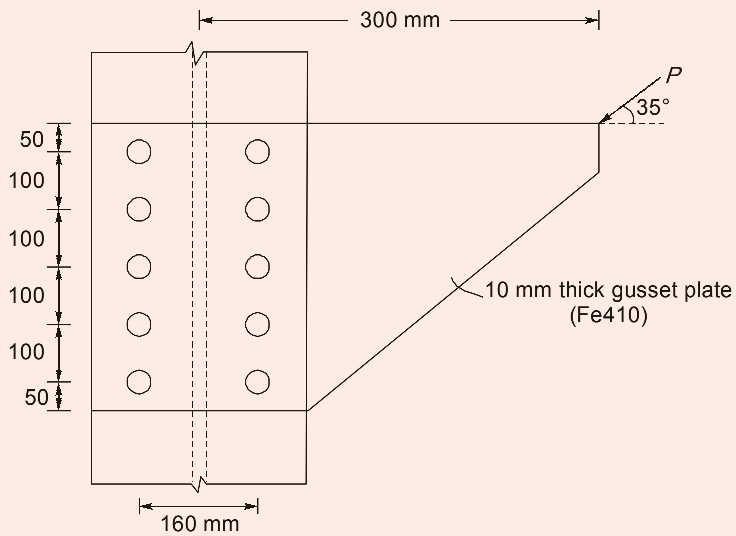
$$K_{AB} = \frac{F_{AB}}{100} = \left(-\frac{98.03}{100}\right) = -0.9803$$

$$\delta_{CD} = K_{AB}\Delta_{AB}$$

$$= (-0.9803)\,(-3.47) = 3.401 \text{ mm}$$

End of Solution

Q.2 (b) Determine the maximum load P the joint can carry if bolts of M 20, grade 4.6 are used.



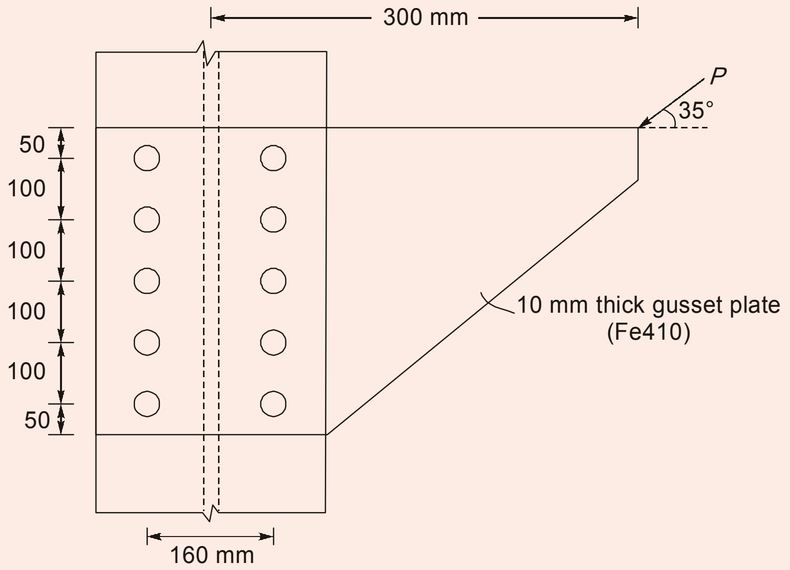
Design shear strength of bolt $V_{nsb} = \frac{f_{ub}}{\sqrt{3}} \cdot \frac{A_{nb}}{\gamma_{mb}}$

Bearing strength of bolt = $\frac{1}{\gamma_{mb}} 2.5\, k_b \cdot d \cdot t \cdot f_u$ d

Where k_b = Least of $\left[\frac{e}{3d_0}, \left(\frac{p}{3d_0} - 0.25\right), \frac{f_{ub}}{f_u}, 1.0\right]$

[20 Marks]

Solution:



(1) Bolt strength (V_{db})

(a) Design shear capacity of Bolt, V_{dsb},

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\gamma_{mb}}(n_s A_{sb} + n_n A_{nb})$$

(Here $n_s = 0$ given in question)

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\gamma_{mb}} n_n \; A_{nb}$$

$$= \frac{400}{\sqrt{3} \times 1.25} \times 1 \times 0.78 \times \frac{\pi}{4} \times 20^2 = 45.27 \text{ kN}$$

(b) Design bearing capacity of Bolt,

$$k_b = \frac{e}{3d_0} = \frac{50}{3(20+2)} = 0.757$$

$$= \frac{p}{3d_0} - 0.25 = \frac{100}{3(20+2)} - 0.25 = 1.265$$

$$= \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.97$$

$$= 1$$

k_b is minimum of all above $k_b = 0.757$

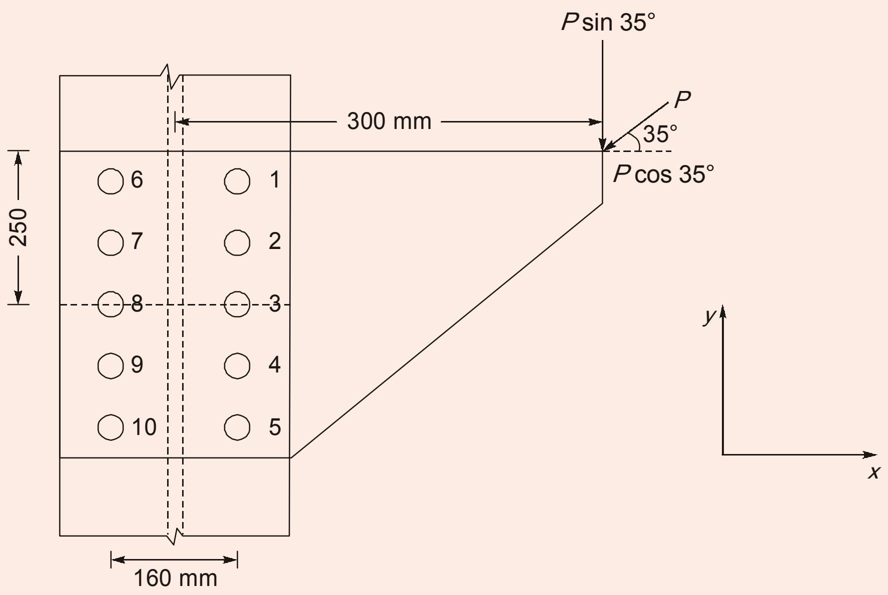
$$V_{dpb} = \frac{2.5 \, k_b dt \, f_u}{\gamma_{mb}}$$

$$= \frac{2.5 \times 0.757 \times 20 \times 10 \times 410}{1.25} = 124.148 \text{ kN}$$

Bolt strength is minimum out of V_{dsb} and V_{dpb}

$$\therefore \quad V_{db} = 45.27 \text{ kN}$$

(2) Maximum resultant shear force (F_{Rmax})



Due to inclined loading, two direct force/loads will be induced in x and y direction. Along with a torsional moment.

(a) Direct shear force in x direction.

$$F_{D1} = \frac{P\cos 35°}{10} = 0.082\text{P kN}$$

(b) Direct shear force in y direction.

$$F_{D2} = \frac{P\sin 35°}{10} = 0.05736 \text{ P kN}$$

(c) Torsional moment on bolted connection.

Torsional moment = P sin 35 × 0.3 – P cos 35° × 0.25 = 0.0327 P.

Net torsional moment is 0.0327 P kNm in anticlockwise direction.

(d) Maximum resultant shear force will be generated at the bolt on which radial distance from C. G is maximum and inclination between direct shear force and torsional shear force is Minimum.

$\therefore$ Maximum radial distance (r_{max})

$$r_{max} = \sqrt{(100+100)^2 + 80^2} = 215.40 \text{ mm}$$

Radial distance for bolts,

$$r_1 = r_5 = r_6 = r_{10} = r_{max} = 215.40 \text{ mm}$$

$$r_2 = r_4 = r_7 = r_9 = \sqrt{100^2 + 80^2} = 128.06 \text{ mm}$$

$$r_3 = r_8 = 80 \text{ mm}$$

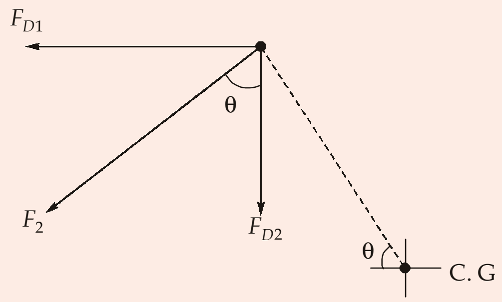
Maximum torsional shear force,

$$F_2 = \frac{(TM) \times r_{max}}{\sum_{i=1}^{n} r_i^2}$$

$$= \frac{0.0327\,P \times 10^3 \times 215.40}{4 \times 215.40^2 + 4 \times 128.06^2 + 2 \times 80^2}$$

$$= 0.02668 \text{ P kN}$$

(e) Total force acting along on bolt 6



$$\theta = \tan^{-1}\frac{200}{80} = 68.2°$$

$$F_x = F_{D1} + F_2 \sin\theta = 0.082\text{ P} + 0.02668\text{ P} \sin 68.2$$

$$= 0.10678 \text{ P kN}$$

$$F_y = F_{D2} + F_2 \cos\theta$$

$$= 0.05736\text{ P} + 0.02668\text{ P} \times \cos 68.2°$$

$$= 0.0673 \text{ P kN}$$

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.10678P)^2 + (0.0673P)^2}$$

$$= 0.12622 \text{ P kN}$$

3. For safe design, $F_R \ngtr$ Bolt strength (V_{db})

$$0.12622\,P_{max} = 45.27 \text{ kN}$$

$$P_{max} = 358.662 \text{ kN}$$

Safe load, $$P_{safe} = \frac{P_{max}}{\gamma_f} = \frac{358.662}{1.5} = 239.108 \text{ kN}$$

End of Solution

Q.2 (c) (i) How do super plasticizers increase the mobility and make the cement concrete flow? What is the advantage of using fly ash in concrete for massive dam construction work?

(ii) What are the disadvantages of destructive methods of testing concrete and advantages of nondestructive testing of concrete?

[10 + 10 = 20 Marks]

Solution:

(i) Superplasticisers are hydrodynamic lubricants which impart high workability by reducing friction between the grains or by reducing the amount of water to be added. Superplasticizers are principally surface reactive agents (surfactants). They confer negative charge on individual cement particles (and also its hydrated particles) such that they are kept in a dispersed or suspended state due to inter-particle repulsion. Thus they confer high mobility to the particles.

Following are the advantages of using fly ash in concrete for massive dam construction work.

- Improved workability with lesser amount of water.
- Lower heat of hydration and thermal shrinkage.
- Improved resistance to attack from salts and sulfates from soils and sea water.
- Reduced susceptibility to dissolution and leaching of calcium hydroxide.
- Reduce permeability
- Lower costs

(ii) **Disadvantages of destructive test**:

1. A large number of specimens are required which could be tested to destruction, at various ages.
2. Test results are not very reliable as all specimens are not identical in quality with the entire mass of concrete.
3. There is much delay in obtaining the results.
4. It is impossible to obtain requisite information on in-situ concrete without damaging the concrete.
5. Crushed samples used for destructive testing creates high amount of debris which in-turn degrades the environment.

Advantages of destructive test:

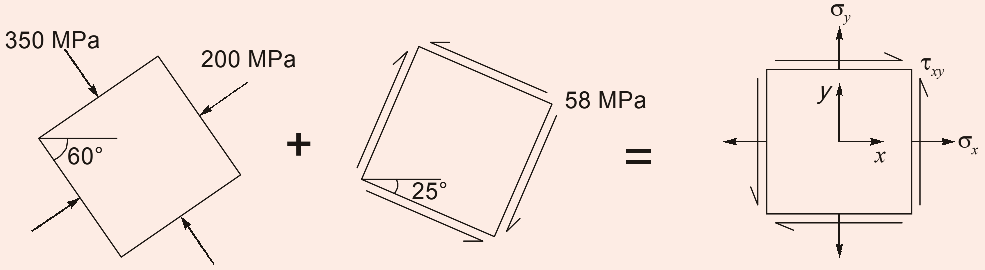
1. The measurement can be done on concrete in-situ and thus representative samples are not required.
2. Non-destructive testing makes its possible to study the variation in quality of concrete with time and external influences.
3. In N.D.T method the concrete is not loaded to destruction. Its quality is judged by measuring certain of its physical properties, which are related to its quality.
4. In N.D.T there is no wastage of material as in destructive methods of testing.
5. Non-destructive methods are quick and can be performed both in laboratory and in-situ with convenience.
6. Non-destructive test (NDT) can be performed on fresh (green) as well as hardened concrete with equal ease and are described as follows.

End of Solution

Q.3 (a) A point on a thin plate is subjected to the two successive states of stress as shown in the figure below. Using Mohr's circle method, determine

(i) The resultant stress of stress represented on the element oriented as shown on the right.

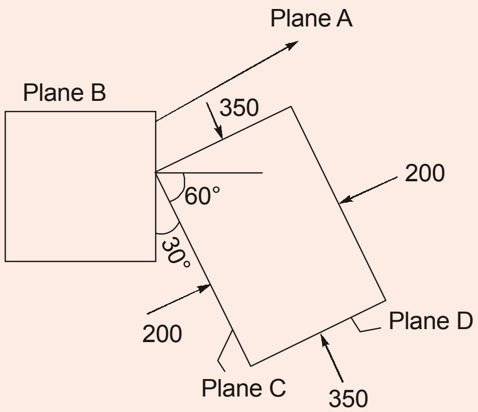
(ii) The principal stresses and principal planes caused by the superposition of these two stress states.

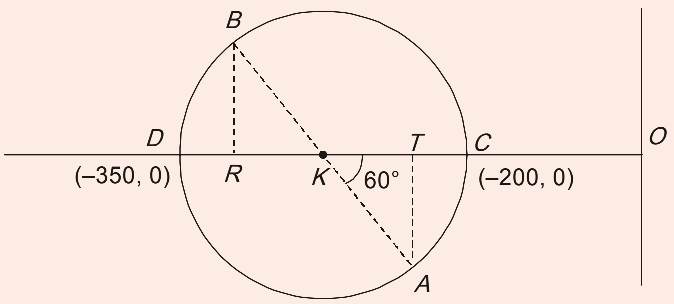


[20 Marks]

Solution:

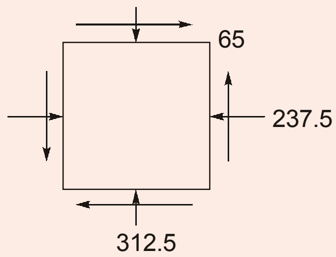
Case-1



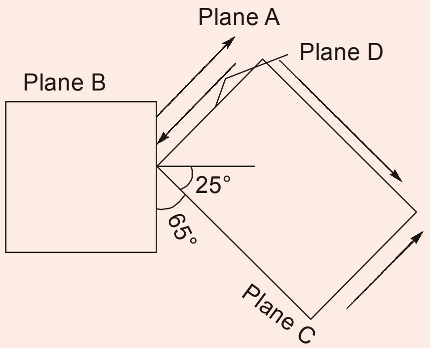


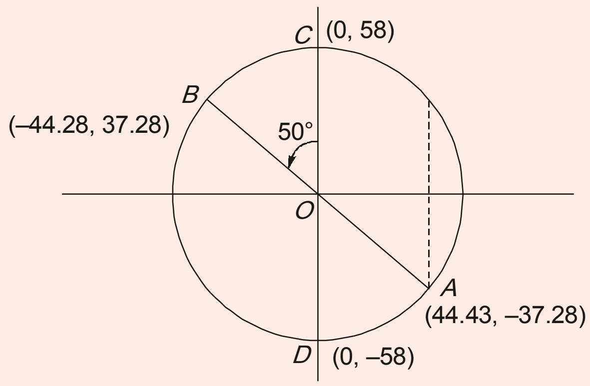
$$KC = \frac{350 - 200}{2} = 75$$

$$\sigma_{x1} = -275 + 75 \cos 60° = -237.5 \text{ MPa}$$
$$\sigma_{x2} = -275 - 75 \cos 60° = -312.5 \text{ MPa}$$
$$\tau_{x1\,y1} = 75 \sin 60° = 65 \text{ MPa}$$

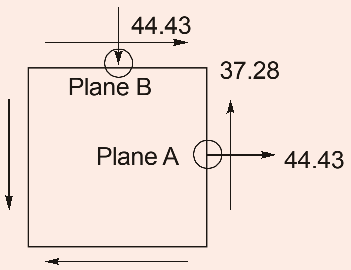


Case-2

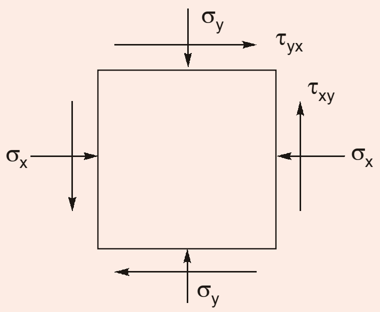




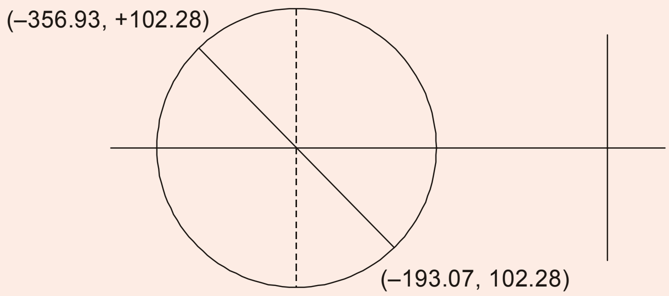
$$\sigma_{x2} = 58 \sin 50° = 44.43 \text{ MPa}$$
$$\sigma_{y2} = -44.43 \text{ MPa}$$
$$\tau_{x2y2} = 58 \cos 50° = 37.28 \text{ MPa}$$



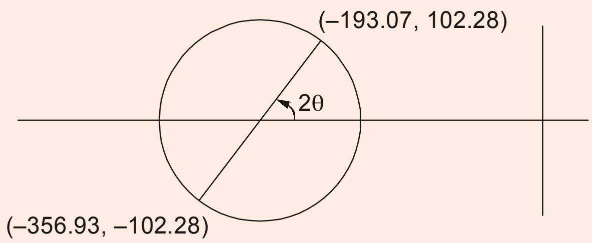
Total stress



$$\sigma_x = (-237.5) + (+44.43) = -193.07 \text{ MPa}$$
$$\sigma_y = (-312.5) + (-44.43) = -356.93 \text{ MPa}$$
$$\tau_{xy} = 65 + 37.28 = 102.28 \text{ MPa}$$



Principal stress



$$\text{Centre} = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$= \left(\frac{-193.07 - 356.93}{2}, 0\right)$$

$$= (-275, 0)$$

$$R = \sqrt{\left(\frac{-356.93 + 193.07}{2}\right)^2 + 102.28^2}$$

$$= 131.05$$

$$\sigma_1 = -275 - R = -275 - 131.05 = -406.5$$

$$\sigma_2 = -275 + R = 275 + 131.05 = -143.95$$

$$\sin 2\theta = \frac{\tau}{R} = \frac{102.28}{131.05} = 0.78$$

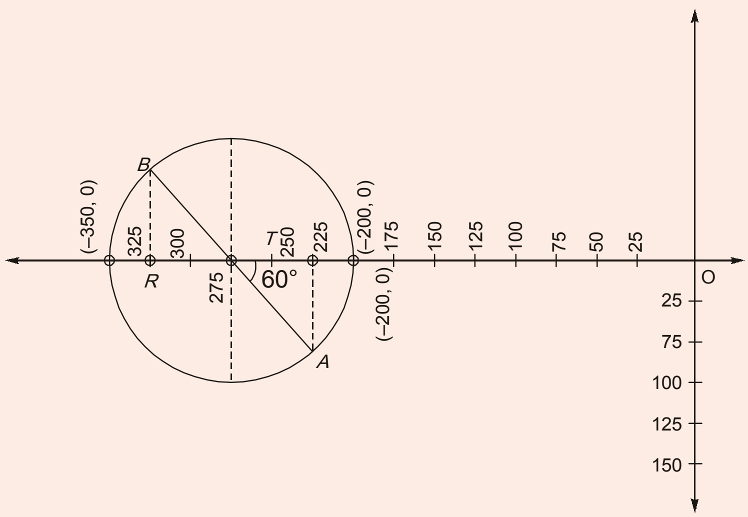
$$2\theta = 51.26$$

$$\theta = 25.63 \text{ (Anticlockwise from } x\text{-axis)}$$

IInd Method

By Graphical Mohr's circle method:

Scale: 1 cm = 25 MPa



From scale

Distance, OR = 12.5 cm

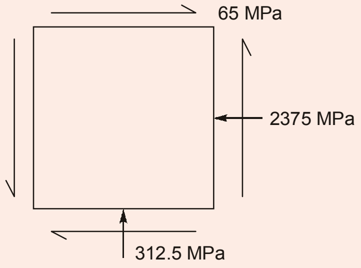
So stress, OR = 12.5 × 25 = 312.5 MPa (C)

Distance, OT = 9.5 cm

So, stress, OT = 9.5 × 25 = 237.5 MPa (C)

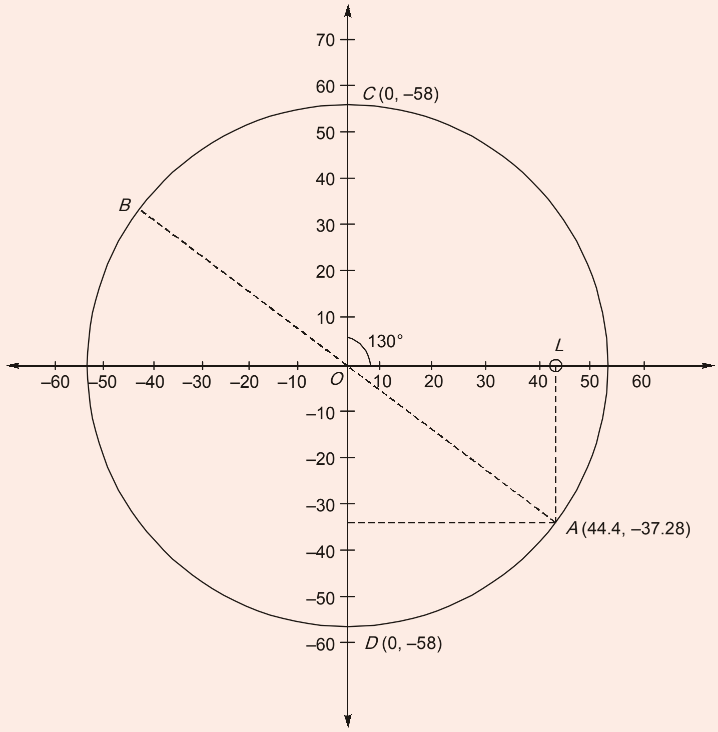
Distance, TA = 2.6 cm

Stress, TA = 2.6 × 25 = 65 MPa



Case-2

Scale, 1 cm = 10 MPa



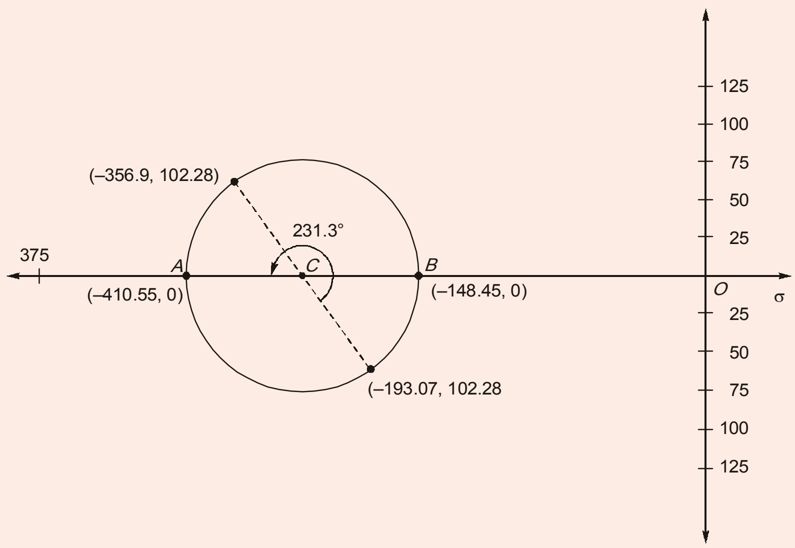
Distance, OL = 4.44 cm

Stress, OL = 4.44 × 10 = 44.4 MPa

Distance, LA = 3.728 cm

Stress, LA = 37.28 MPa

Total stress, 1 cm = 25 cm



$$\sigma_x = -237.5 + (44.4) = -193.1 \text{ MPa} = 7.724 \text{ cm}$$
$$\sigma_y = -312.5 - 44.4 = -356.9 \text{ MPa} = 14.276 \text{ cm}$$
$$\tau_{xy} = 65 + 37.28 = 102.28 \text{ MPa} = 4.09 \text{ cm}$$

From graph:

Major principal stress, $\sigma_1 = OA = 16.242 \text{ cm} \times 25 = -406.05 \text{ MPa}$

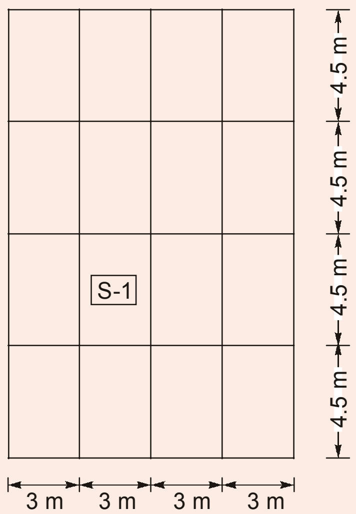
Minor principal stress, $\sigma_2 = OB = 5.758 \text{ cm} \times 25 = -143.95 \text{ MPa}$

Principal angles: From Graph

$$2\theta_{p2} = 51.31°$$
$$\Rightarrow \quad \theta_{p2} = 25.65°$$
$$2\theta_{p1} = 231.31°$$
$$\Rightarrow \quad \theta_{p1} = 115.65°$$

End of Solution

Q.3 (b) A solid RCC slab is having effective span of $L_x = 3.0$ m and $L_y = 4.5$ m. Position of slab S - 1 is shown in the figure below.



Centre line Top plan of slab panels

Slab carrying total design load of 20 kN/m^2. In slab S - 1, calculate and provide the spacing of all main reinforcement only, by limit state method of design, for 8 mm diameter high strength deformed bars. Check these spacings must not exceed the standard guidelines of IS 456 : 2000.

Effective thickness of slab = 120 mm

Grade of concrete M 20

Grade of reinforcement Fe 415

$\frac{M}{bd^2}$	0.30	0.40	0.512	0.60	0.65	0.662
p_L	0.085	0.114	0.143	0.172	0.187	0.203

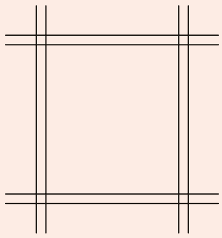
p_t is the percentage of reinforcement.

Case No.	Type of Panel and Moments Considered	Short Span Coefficient α_x (Values of l_y/l_x)								Long Span Coefficients α_y for All Values of l_y/l_x
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1.	*Interior Panels:* Negative moment at continuous edge	0.032	0.037	0.043	0.047	0.051	0.053	0.060	0.065	0.032
	Positive moment at mid-span	0.024	0.028	0.032	0.036	0.039	0.041	0.045	0.049	0.024
9.	*Four Edges: Discontinuous:* Positive moment at mid-span	0.056	0.064	0.072	0.079	0.085	0.089	0.100	0.107	0.056

[20 Marks]

Solution:

M20/Fe415 Steel



$$L_{xeff} = 3.0\text{ m}$$

$$L_{yeff} = 4.5\text{ m}$$

Step 1

$$\frac{L_{yeff}}{L_{xeff}} = \frac{4.5}{3.0} = 1.50 < 2.0$$

It is a two way slab.

It is a Interior panel.

Step 2 Total Design load = 20 kN/m^2

$$\text{factored load} = W_u = 1.5 \times 20$$

$$= 30\text{ kN/m}^2$$

Step 3 Effective depth of slab (given) = 120 mm.

Step 4

Bending Moment Calculations

	α value	BMU	$\frac{MU}{B.d_2}$	P_t%
$M_{x(-)}$	0.053	0.053 × 270 = 14.31	0.99	Not given
$M_{x(+)}$	0.041	0.041 × 270 = 11.07	0.77	Not given
$M_{y(-)}$	0.032	0.032 × 270 = 8.64	0.60	0.172
$M_{y(+)}$	0.024	0.084 × 270 = 6.48	0.45	Can be calculated from given values

$$\text{Moments} = \alpha \cdot W_u \cdot L_x^2 e$$

$$= \alpha \times 30 \times 3.0^2 = 270\,\alpha$$

$$\frac{BM_u}{B \cdot d^2} = \frac{BM_u \times 10^6}{1000 \times 120^2}$$

Values for 0.99 and 0.77 not given in table

Use $$\text{Ast} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}}\right] \times Bd$$

$$\text{Ast}_{x(-)} = \frac{0.5 \times 20}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 14.31 \times 10^6}{20 \times 1000 \times 120^2}}\right] \times 1000 \times 120$$

$$= 352 \text{ mm}^2$$

$$\text{Ast}_{x(+)} = \frac{0.5 \times 20}{415} \times \left[1 - \sqrt{1 - \frac{4.6 \times 11.07 \times 10^6}{20 \times 1000 \times 120^2}}\right] \times 1000 \times 120$$

$$= 268 \text{ mm}^2$$

by same formula.

$$\text{Ast}_{y(-)} = \text{for } BM_u = 8.64 \text{ kN-m}$$

$$= 206.92 \text{ say } 207 \text{ mm}^2$$

Using P_t% = 0.172% also

$$\text{Ast}_{y(-)} = \frac{0.172}{100} \times 1000 \times 120 = 206.4 \text{ mm}^2$$

$$= 206.4 \text{ say } 207 \text{ mm}^2$$

$\text{Ast}_{y(+)}$ for BM_u = 6.48 kN-m

= 153.72 mm2

Step 5 Spacing of reinforcement using 8 mm φ.

$$\text{Ast}_{x(-)} = \frac{1000}{Ast} \times \frac{\pi}{4}(8)^2$$

$$= \frac{1000}{352} \times \frac{\pi}{4}(8)^2 = 142 \text{ mm} = \text{say } 140 \text{ mm}$$

$$Ast_{x(+)} = \frac{1000}{268} \times \frac{\pi}{4}(8)^2 = 187 \text{ mm} = \text{say } 180 \text{ mm}$$

$$Ast_{y(-)} = \frac{1000}{206.92} \times \frac{\pi}{4}(8)^2 = 242 \text{ mm} = \text{say } 240 \text{ mm}$$

$$Ast_{y(+)} = \frac{1000}{159.72} \times \frac{\pi}{4}(8)^2 = 326 \text{ mm} = \text{say } 270 \text{ mm}$$

Step 6 Minimum Reinforcement

Considering 30 mm eff cover

$$D = 120 + 30 = 150 \text{ mm}$$

$$Ast_{min} = \frac{0.12}{100} \times 1000 \times 150 = 180 \text{ mm}^2$$

Spacing of 8 mm ϕ shall not exceed

(a) Minimum steel = $\frac{1000}{180} \times \frac{\pi}{4}(8)^2 = 279$ mm

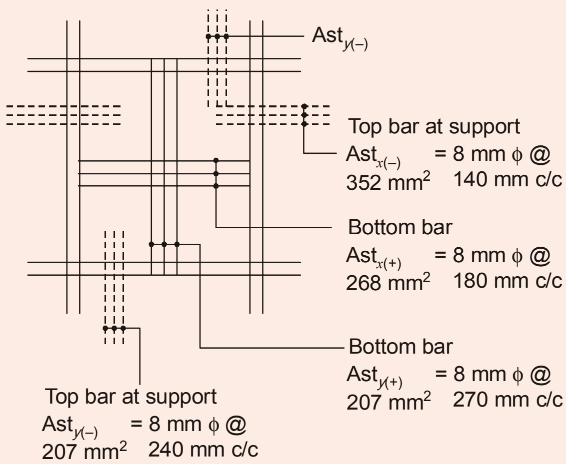
(b) 3d = 3 × 120 = 360 mm

(c) 300 mm

only in $Ast_{y(+)}$ – steel – spacing is more than 279 mm

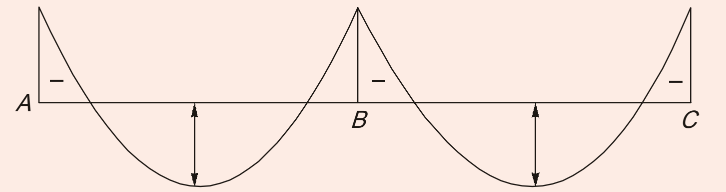
Provide $Asy_{y(+)}$ = 8 mm ϕ @ 270 mm c/c

Step 7 **Provided steel:**

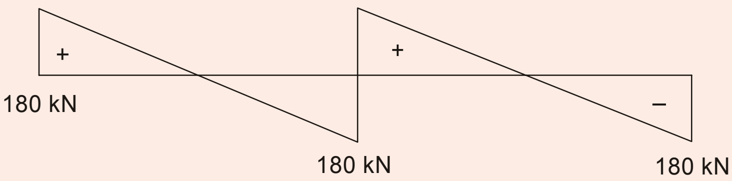


End of Solution

Q.3 (c) Design bending moment and shear force diagram have been given below for a two span continuous beam. Effective span of beam is 6.0 m each. Design a rectangular, singly reinforced RCC beam section at support 'B' only by LSM of design.



BM Diagram



SF Diagram

The following parameters may be used for design. All notations are as per IS 456 : 2000.

(1) Grade of concrete M 20

(2) Grade of reinforcement Fe 415

(3) Nominal cover to reinforcement – 25 Mm

(4) (Effective depth/width) ratio – 2

(5) Diameter of flexural bar – 20 mm

(6) Diameter of shear reinforcement – 8 mm (CS area 50 mm^2)

(7) Design shear strength of concrete τ_c = 0.88 N/m^2

(8) Maximum shear stress τ_{cmax} = 2.8 N/mm^2

(9) Round off effective (calculated) to next higher multiple of 100

(10) $M_{u,lim}/f_{ck}\, bd^2 = 0.138$

(11) $\left(\dfrac{x}{d}\right) = 1.2 - \left[(1.2)^2 - \dfrac{6.68\,M_u}{f_{ck}\, bd^2}\right]$

(12) Lever arm $z = d\left(1 - 0.42\dfrac{x}{d}\right)$

(13) $A_{st} = \dfrac{M_u}{0.87\, f_y \cdot z}$

(14) $V_{us} = \dfrac{0.87\, f_y \cdot A_{sv} \cdot d}{S_v}$

(15) Minimum shear reinforcement $= \dfrac{A_{sv}}{b \cdot S_v} = \dfrac{0.40}{087\, f_y}$

Check all spacing of reinforcement as per standard guidelines.

[20 Marks]

Solution:

Beam is to be designed at support '*B*' only by LSM where

$$\left.\begin{aligned} BM &= 180 \text{ kN-m} \\ SF &= 180 \text{ kN} \end{aligned}\right\}$$

Factored values : $BMU = 1.5 \times 180 = 270$ kN-w

$SF = Vu = (1.5 \times 180) = 270$ kN

(a) Design for Bending Moment

Using the value given in question,

$$\frac{d}{B} = 2.0 \quad | \quad M20 \,|\, Fe415$$

1. **Equating** for a limiting **section**.

$$BM_U = 0.138 \times f_{ck} \cdot B \cdot d^2$$

$$270 \times 10^6 = 0.138 \times 20 \times B \times (2B)^2$$

$$B = 290.30 \text{ mm}$$

$$d = 2 \times 290.30 \text{ mm} = 580.5 \text{ mm}$$

Let us provide, $b = 300$ mm

$d = 600$ mm

Keeping effective cover

$$= NC + \phi_{st} + \frac{1}{2}\phi_m = 25 + 8 + \frac{1}{2} \times 20 = 43 \text{ mm}$$

(say 50 mm)

Overall depth = d + effective cover

$$D = 600 + 50 = 650 \text{ mm}$$

2. Area of steel

$$\frac{x}{d} = 1.2 - \left[(1.2)^2 - \frac{6.68 \times MO}{f_{ck} \cdot bd^2}\right]^{1/2}$$

$$= 1.20 - \sqrt{(1.2)^2 - \frac{6.68 \times 270 \times 10^6}{20 \times 300 \times 600^2}} = 0.42218$$

$$x = 0.42218 \times 600 = 253.31 \text{ mm}$$

$$\text{LA (Lever arm)} = Z = d\left(1 - 0.42 \times \frac{x}{d}\right)$$

$$= 600 \times (1 - 0.42 \times 0.42218) = 493.60 \text{ mm}$$

$$A_{st} = \frac{BM_u}{0.87 \times f_y \times z} = \frac{270 \times 10^6}{0.87 \times 415 \times 493.60} = 1515 \text{ mm}^2$$

Number of 20 mmϕ bars

$$= \frac{1515}{\frac{\pi}{4}(20)^2} = 4.82 \quad \text{Provide 5 - 20 mm } \phi$$

Provide 5 - 20 mm ϕ bars at top face of beam for negative BM at '*B*'.

3. **Check for Shear**

Maximum shear force = $Vu = 270$ kN

Nominal shear stress

$$\tau_v = \frac{Vu}{Bd} = \frac{270 \times 10^3}{300 \times 600} = 1.50$$

$$\tau_c = 0.88 \text{ N/mm}^2$$

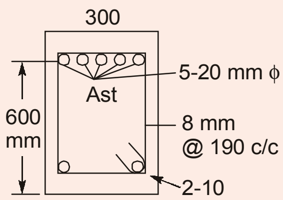
Shear force for design of shear reinforcement

$$V_{su} = (\tau_v - \tau_c)\, B.d$$

$$= (1.50 - 0.88) \times \frac{300 \times 600}{1000}$$

$$= 111.60 \text{ kN}$$

Spacing of 2 legged – 8 mm ϕ steel



$$= \frac{A_{sv} \times 0.87 f_y \times d}{V_{su}}$$

$$= \frac{2 \times 50 \times 0.87 \times 415 \times 600}{111.60 \times 1000}$$

$$= 194.11 \text{ mm}$$

Spacing as per minimum shear steel

$$S_v = \frac{0.87 f_y \times A_{sv}}{0.4 \times B} = \frac{0.8\tau \times 415 \times 2 \times 4\tau}{0.4 \times 300}$$

$$= 300.8 \text{ mm}$$

So provide 2 legged - 8 mm ϕ @ 190 mm c/c near support

Provide 2-10 mm bars at bottom also as hanger bars.

4. **Check for spacing of reinforcement**

(a) Maximum spacing of shear reinforcement

(i) $0.75d = 0.75 \times 600$

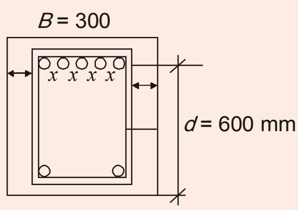
$= 450$ mm

(ii) 300 mm

Provided spacing of 190 mm is ok

(b) Spacing of Main bars

$B = 300$ mm



$$x = \frac{(300 - 2 \times 25 - 2 \times 8 - 5 \times 20)}{4}$$

$$= 33.5 \text{ mm}$$

Maximum spacing of horizontal reinforcement

(i) Maximum dia of main bar = 20 mm

(ii) 5 mm + maximum size of aggregates

= 5 + 20 = 25 mm

So, provided spacing of 33.5 mm is ok.

End of Solution

Q.4 (a) (i) Briefly explain the following with the help of neat sketches:

I. Structure of an exogenous tree

II. Heart shakes and star shakes defects in timber

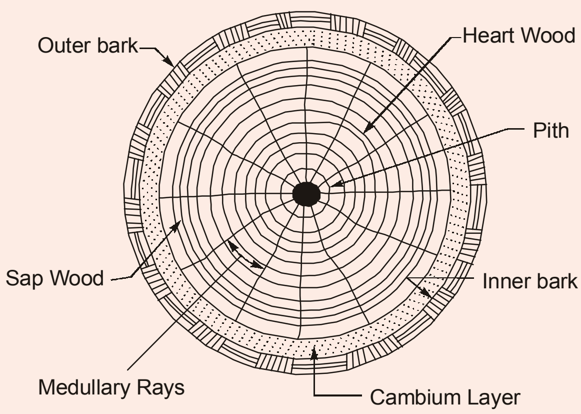
(ii) Determine the proportion of aggregates A (with Fineness Modulus FM = 7.83) and B (FM = 6.81) required to suitably combine to provide the following grading of such a combination:

IS sieve designation (mm)	80 mm	40 mm	20 mm	10 mm	4.75 mm	2.36 mm	1.18 mm	600 μm	300 μm	150 μm
Cumulative Percentage retained on each sieve	0	6	45	72	95	100	100	100	100	100

[10 + 10 = 20 Marks]

Solution:

(i) The structure of wood visible to the naked eye or at a small magnification is called the macrostructure.



1. **Pith:** The innermost central portion or core of the tree is called the pith or medulla.
2. **Heart Wood:** The inner annual rings surrounding the pith is known as heart wood. It is usually dark in colour.

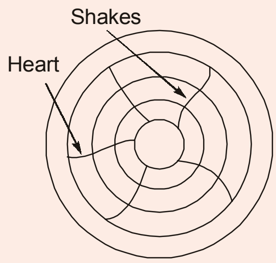
   It does not take active part in the growth of tree. But it imparts rigidity to tree and hence, it provides strong and durable timber for various engineering purposes.

3. **Sap Wood:** The outer annual rings between heart wood and cambium layer is known as sap wood. It is usually light in colour and weight. It indicates recent growth and it contains sap.

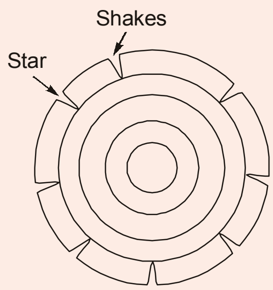
   It takes active part in the growth of tree and sap moves in an upward direction through it. Sap wood is also known as laburnum.

4. **Cambium Layer:** The thin layer of sap between sap wood and inner bark is known as cambium layer. It indicates sap which has yet not been converted into sap wood.
5. **Inner Bark:** It gives protection of cambium layer from any injury.
6. **Outer Bark:** It consists of cells of wood fibre and is also known as cortex.
7. **Medullary Rays:** The thin radial fibres extending from pith to cambium layer are known as *medullary rays*. The function of these rays is to hold together the annual rings of heart wood and sap wood.

(ii) **Heart Shakes:** These cracks occur in the centre of cross-section of tree and they extend from pith to sap wood in the direction of medullary rays as shown in **Fig.** These cracks occur due to shrinkage of interior part of tree which is approaching maturity. Heart shakes divide the tree cross-section into two to four parts.



Star Shakes: These are cracks which extend from bark towards the sap wood. They are usually confined up to the plane of sap wood. They are usually formed due to extreme heat or frost.



(ii) Fineness modulus of A = 7.83
Fineness modulus of B = 6.81

Fineness of modulus of combination of both A and B to be calculating as per given table to get the proportion.

Sieve	Cumulative retention each sieve
80 mm	0
40 mm	6
20 mm	45
10 mm	72
4.75 mm	95
2.36 mm	100
1.18 mm	100
600 µm	100
300 µm	100
150 µm	100
	Sum of cumulative retained = 718

Fineness modulus of combination,

$$= \frac{\text{Sum of \% cumulative wt. retained}}{100} = \frac{718}{100} = 7.18$$

Let the proportion of $A = x$

Then proportion of B in mix $= 1 - x$

$$x \times 7.83 + (1 - x) \times 6.81 = 7.18$$

$$7.83\,x + 6.81 - 6.81\,x = 7.18$$

$$(7.83 - 6.81)x = 7.18 - 6.81$$

$$1.02\,x = 37$$

$$x = 36.27\%$$

Proportion of A in mix $= 36.27\%$

Proportion of B in mix $= 63.72\%$

Proportion of A with respect to B in mix,

$$= \frac{36.27}{63.72} \times 100 = 56.92\%$$

Proportion of $A : B = 36.27 : 63.72 = 1 : 1.756.$

End of Solution

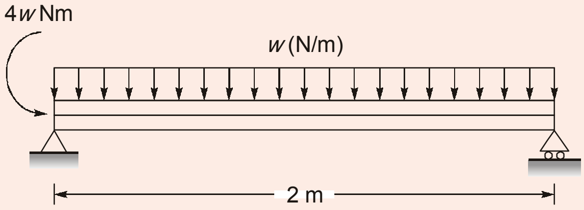
Q.4 (b) A T-section beam is constructed by ruling two pieces of wood together as shown in the figure. The maximum stress in the glue joints is to be limited to 2 MPa in tension and the maximum shear stress is to be limited to 1.7 MPa.

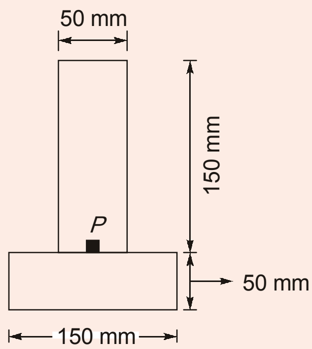
(i) Determine the stress components on element at pint 'P'. Point 'P' is located at glued joint.

(ii) Determine principal stresses at point 'P'.

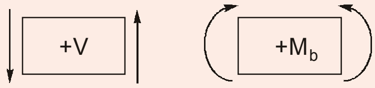
(iii) Show these stresses on properly oriented 2-D element.

(iv) Determine the maximum value for load w.



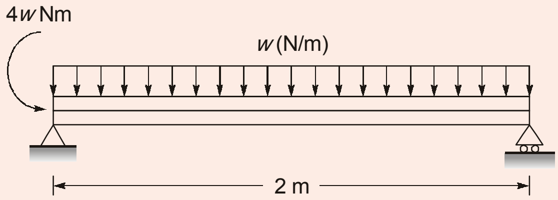


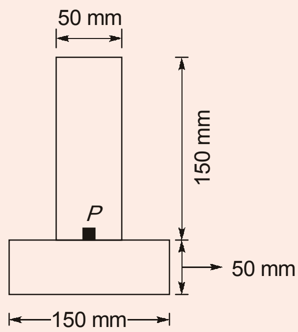
Sign convention for shear force and bending moment are as follows:



[20 Marks]

Solution:





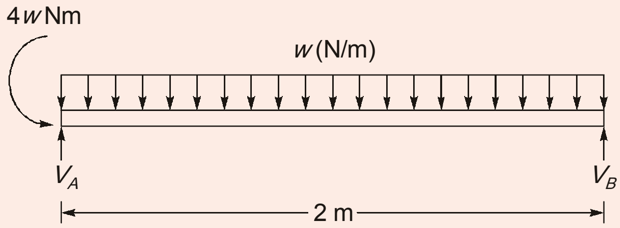
$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(150\times 50\times 25)+(150\times 50\times 125)}{(150\times 50)+(150\times 50)} = 75 \text{ mm}$$

$$I_{NA} = I_1 + I_2$$

$$= \left[\frac{150(50)^3}{12}+\left(150\times 50\times 50^2\right)\right]+\left[\frac{50(150)^3}{12}+\left(50\times 150\times 50^2\right)\right]$$

$$I_{NA} = \left(20.31\times 10^6\right)+\left(32.81\times 10^6\right) = 53.12\times 10^6 \text{ mm}^4$$



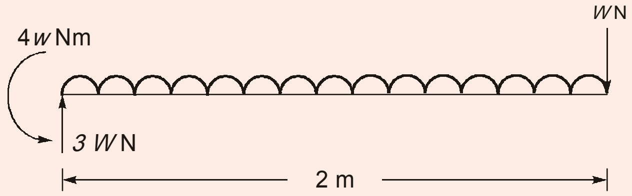
$$V_A + V_B = 2w \qquad \text{...(i)}$$

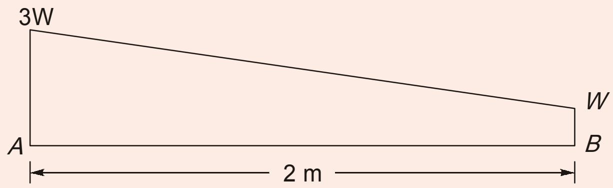
$$\Sigma M_A = 0$$

$$(-V_B \times 2) + (w \times 2 \times 1) + (-4w) = 0$$

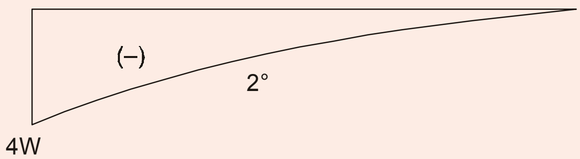
$$V_B = -w \text{ N}$$

$$V_A = 3w \text{ N}$$



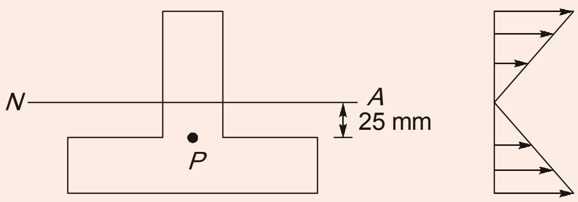


SFD



BMD

Bending stress at 'P'

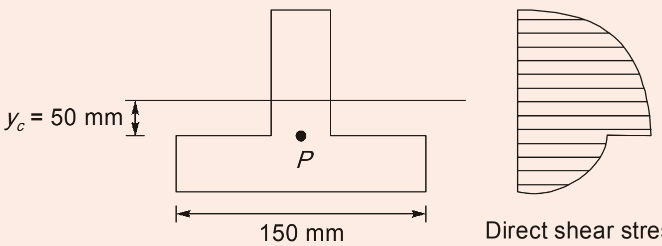


Bending stress distribution

$$\sigma = \frac{My}{I} = \frac{(4w)(25)\times 10^6}{53.12 \times 1.6}$$

$$\sigma = 18.82 \times 10^{-4}\, w$$

Direct shear stress at 'P'



Direct shear stress distribution (parabolic)

$$\tau = \frac{VQ}{It} = \frac{(3w)\times(150\times 50)(50)}{\left(53.12\times 10^6\right)(50)} = 4.23\times 10^{-4} w$$

$$(4.23 \times 10^{-4})\, w \le 1.7$$

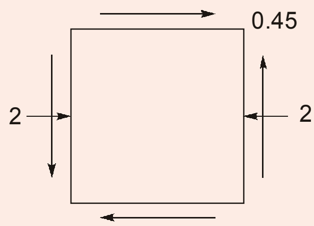
$$w \le 4.014 \text{ kN/m}$$

$$\sigma \le 2 \text{ MPa}$$

(It seems error in problem because in hogging moment below NA at point 'P' compressive bending stress will develop)

So, $w \le 1.062$ kN/m

(i) Stress element as 'P'



$$\tau_p = \frac{(3w)(150\times 50\times 50)}{\left(53.125\times 10^6\right)(50)} = 0.449\text{MPa} = 0.45 \text{ MPa}$$

$$\sigma_x = -2 \text{ MPa}$$

$$\sigma_y = 0$$

$$\tau_{xy} = 0.45 \text{ MPa}$$

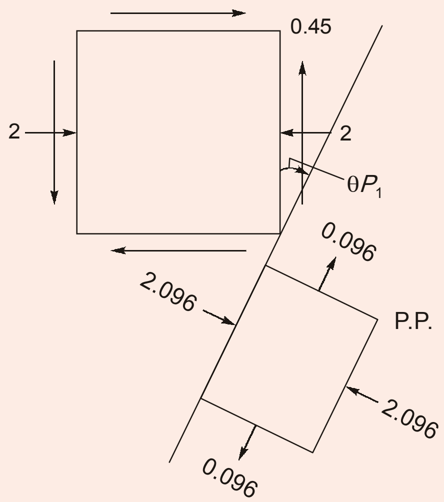
(ii)

$$\sigma_{p_1}/\sigma_{p_2} = \frac{(-2+0)}{2} \pm \frac{1}{2}\sqrt{(0+2)^2 + 4(0.45)^2}$$

$$\sigma_{p_1} = -2.096 \text{ MPa}$$

$$\sigma_{p_2} = +0.96 \text{ MPa}$$

(iii)



$$\tan 2\theta_{p1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(0.45)}{(-2)-(0)}$$

$$\theta_{p1} = -12.11°$$
$$\theta_{p2} = 77.86°$$

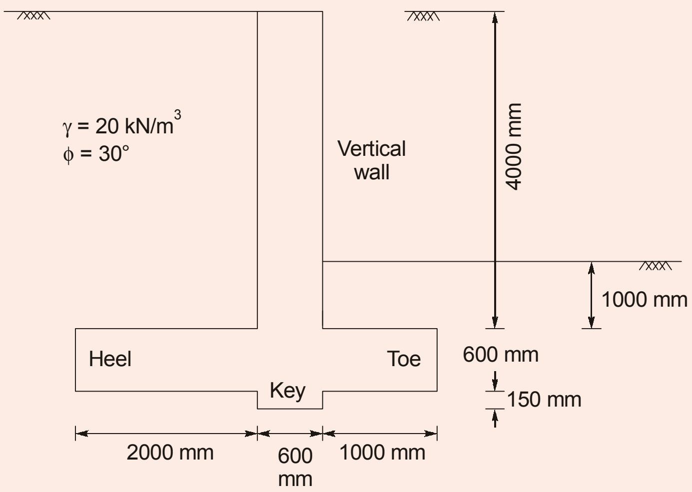
Crosscheck:

$$\theta = -12.11°$$

$$\sigma_x' = (-2)\cos^2(-12.11) + 2(0.45)\cos(-12.11)\sin(-12.11)$$
$$= -2.096 \text{ MPa}$$

End of Solution

Q.4 (c) An RCC cantilever retaining wall is to be designed to support the soil as shown below. Design and sketch the reinforcement for vertical wall only. Also sketch the position of main reinforcement in Toe and Heel slab. (Do not design Toe and Hell slab). Neglect the effect of passive earth pressure and self-weight of vertical wall. Water table is not affecting the moisture condition of retained soil.



(1) M 20 grade of concrete

(2) Fe 415 grade of reinforcement

(3) Diameter of main and distribution reinforcement : 8 mm

(4) Minimum effective thickness required : 400 mm

(5) $K_a = \dfrac{1-\sin\phi}{1+\sin\phi}$

For M 20 and Fe 415

Percentage reinforcement p_t in %

$\frac{M_u}{bd^2}$	0.4	0.5	0.6	0.667
p_t	0.114	0.142	0.172	0.204

Design shear strength of concrete τ_c in N/mm^2

p_t	0.15	0.25	0.5	0.75
τ_c	0.28	0.36	0.48	0.56

[20 Marks]

Solution:

Design of vertical wall of retaining wall----stem.

Height of wall = 4.0 m

Coefficient of active earth pressure

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

Active earth pressure at bottom

$$= k_a \cdot \gamma \cdot H = \frac{1}{3} \times 20 \times 4 = 26.67$$

Total active E/P

$$P_a = \frac{1}{2} k_a \cdot r \cdot H^2 = \frac{1}{2} \times \frac{1}{3} \times 20 \times 4^2 = 53.33 \text{ kN}$$

Maximum BM (factored)

$$\text{BMu} = 1.5 \times P_a \times \frac{H}{3} = 1.5 \times 53.33 \times \frac{4}{3} = 106.67 \text{ kN-m}$$

Say 107 kN-m

Let us provide thickness of

d = 400 mm as given in question

Total depth = d + 60 mm effective cover

= 400 + 60 = 460 mm

Eff. depth required $d = \sqrt{\frac{BM_u}{Q.B}} = \sqrt{\frac{107 \times 10^6}{0.138 \times 20 \times 1000}}$

= 197 mm < 400 mm available safe

$$A_{st\ min} = \frac{0.12}{100} \times 1000 \times 460 = 552 \text{ mm}^2$$

As per table

$$\frac{Mu}{Bd^2} = \frac{107 \times 10^6}{1000 \times 400^2} = 0.668$$

$$P_t\% = 0.204\%$$

$$A_{st} = \frac{0.204}{100} \times 1000 \times 400 = 816 \text{ mm}^2$$

$$\text{Spacing of 8 mm } \phi = \frac{1000}{816} \times \frac{\pi}{4}(8)^2 = 61.6 \text{ mm}$$

Provide 8 mm ϕ @ 60 mm c/c.

Vertical steel = 8 mmϕ @ 60 mm c/c

Horizontal steel = Minimum = 552 mm^2

Spacing of 8 mmϕ steel.

$$= \frac{1000}{552} \times \frac{\pi}{4}(8)^2 = 91 \text{ mm}$$

Provide 8 mmϕ @ 90 mm c/c

Check for shear:

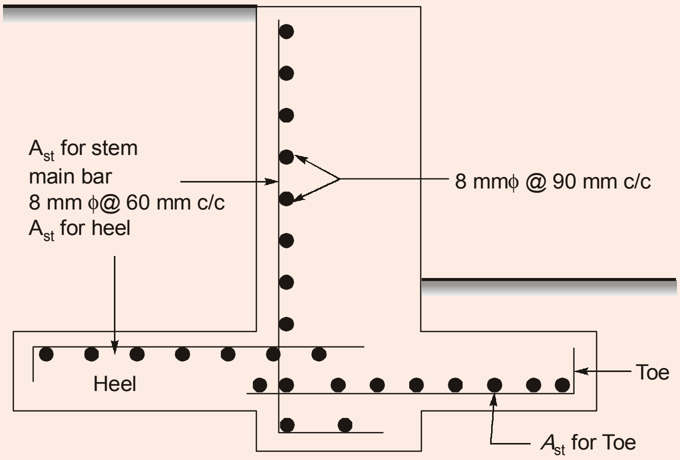
Maximum shear force at base= V_u = 1.5 Pa = 1.5 × 53.33 = 80 kN

$$\tau_v = \frac{V_u}{Bd} = \frac{80000}{1000 \times 400} = 0.20 \text{ N/mm}^2$$

$$P_t\% = 0.204\%$$

$$\tau_c = 0.28 + \frac{(0.36 - 0.28)}{(0.25 - 0.15)} \times (0.204 - 0.15) = 0.32$$

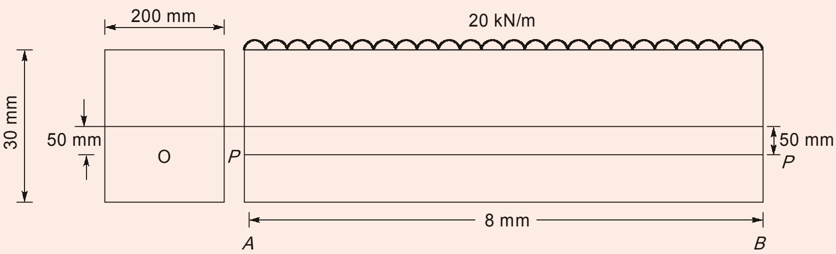
$$\tau_v < \tau_c \quad \text{Safe}$$



Section-B

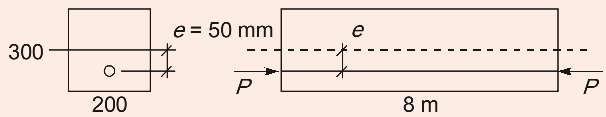
Q.5 (a) A rectangular prestress concrete beam has a cross-section of 200 mm × 300 mm. Its effective span is 8 meters. This beam is prestressed by a straight cable, 50 mm below the central longitudinal axis. This beam supports an imposed load of 20 kN/m.

Find the magnitude of prestressing force which can balance the stresses due to dead load and imposed load at bottom fibre of mid-span. Unit weight of concrete : 25 kN/m^3.



[12 Marks]

Solution:



1. Self wt of beam, DL = 0.20 × 0.30 × 1.0 × 25 = 1.5 kN/m
2. Imposed load w_L = 20 kN/m

At mid span:

$$\text{Maximum BM} = \frac{wL^2}{8} = (1.5+20)\times\frac{8^2}{8} = 172 \text{ kN-m}$$

To balance the stresses at bottom fibre at mid span

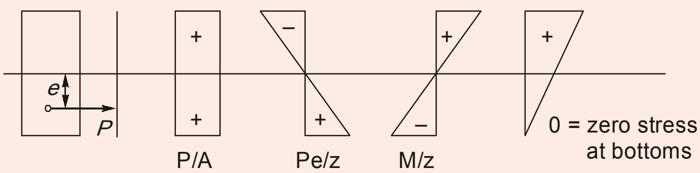
$$\text{Stress} = 0$$

$$\Rightarrow \quad \frac{P}{A}+\frac{Pe}{z}-\frac{M}{z} = 0$$

$$\Rightarrow \quad P\left(\frac{1}{A}+\frac{e}{z}\right) = \frac{M}{z}$$

$$\Rightarrow \quad P = \frac{M}{\left(\frac{z}{A}+e\right)} = \frac{M}{\left(\frac{D}{6}+e\right)}$$

$$= \frac{172\times10^6}{\left(\frac{300}{6}+50\right)\times1000} = \frac{172}{100\times10^{-3}} = 1720 \text{ kN}$$



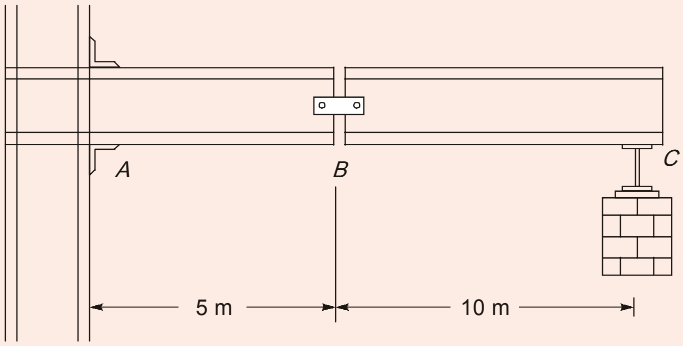
End of Solution

Q.5 (b) The compound beam is subjected to a uniform dead load of 1.5 kN/m and a single live load of 10 kN. Determine

(i) The maximum negative moment created by these load at 'A', and

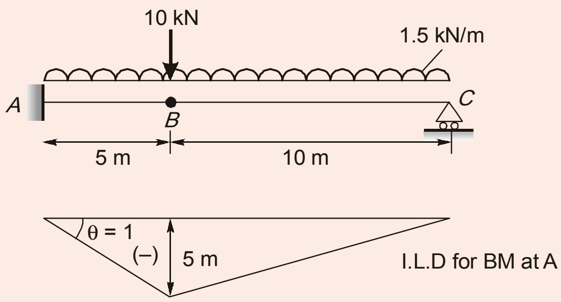
(ii) Maximum positive shear at 'B'.

Assume A is a fixed support, B is a pin and C is a roller.



[12 Marks]

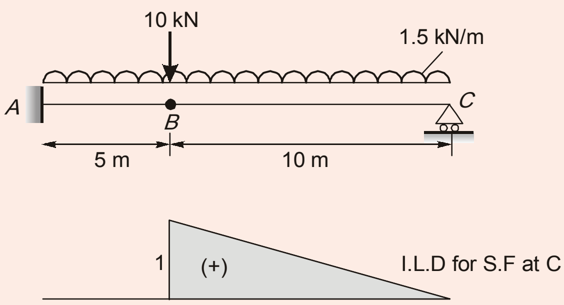
Solution:



$$\text{Maximum – ve BM at A} = -1.5\left(\frac{1}{2}\times 15\times 5\right) - 10\times 5$$

$$= -56.25 - 50 = -106.25 \text{ kN-m}$$

Maximum +ve SF at B,



$$\text{Maximum SF at } B = 1.5\left(\frac{1}{2}\times 10\times 1\right) + 10\times 1$$

$$= 7.5 + 10 = 17.5 \text{ kN}$$

End of Solution

Q.5(c) Derive the expression for displacement of an undamped free vibration of motion for a single degree of freedom system from first principles. Plot the undamped free vibration response.

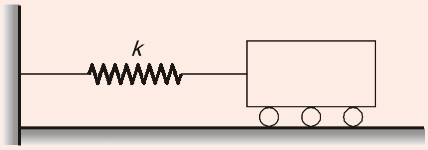
[12 Marks]

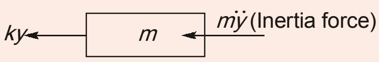
Solution:

Displacement of an undamped free vibration of motion for a single degree of freedom system.

(i) Differential equation for motion:

$$a = \ddot{y} = \frac{d^2y}{dt^2}$$





Body in dynamic equilibrium

$\Sigma x = 0 \Rightarrow \quad -ky - m\ddot{y} = 0$

$\Rightarrow \quad m\ddot{y} + ky = 0 \qquad \text{... (A)}$

(ii) Solutions to differential equation of motion:

Since it is a second order linear differential homogeneous equation with constant coefficients, a trial solution can be

$$y = A\cos\omega t \qquad \text{... (i)}$$

$$y = B\sin\omega t \qquad \text{... (ii)}$$

Substituting equation (i) into equation (A) gives,

$$(-m\omega^2 + k)A\cos\omega t = 0$$

To satisfy this equation at anytime,

$$-m\omega^2 + k = 0$$

i.e. $\omega^2 = \dfrac{k}{m}$

$\omega = \sqrt{\dfrac{k}{m}}$ is called natural frequency of the system.

Since the differential equation is linear,

$y = A\cos\omega t + B\sin\omega t$ is also a solution

So, $\dot{y} = -A\omega\sin t + B\omega\cos\omega t$

At $t = 0$, $y = y_o$, $v = v_o$, $\dot{y} = v = v_o$

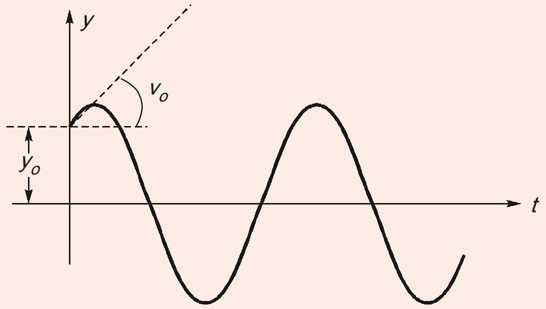
$$y_o = A\cos 0 + B\sin o \Rightarrow A = y_o$$

$$v_o = -A\omega\sin 0 + B\omega\cos 0 \Rightarrow v_o = B\omega \Rightarrow B = \frac{v_o}{\omega}$$

Substituting A and B in the equation of motion, we get,

$$y = y_o\cos\omega t + \frac{v_o}{\omega}\sin\omega t$$

Free vibration response:

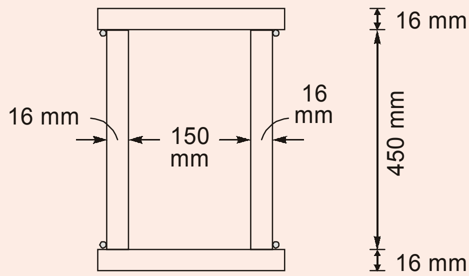


Undamped free vibration response

End of Solution

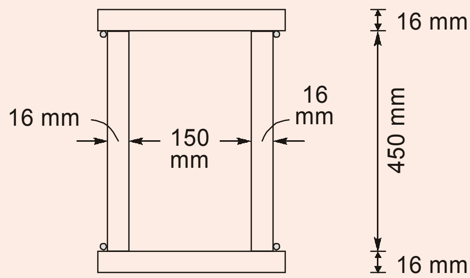
Q.5 (d) Calculate the safe load of a compression member made up of a welded box section as shown in the figure. The effective lengths along major axis and minor axis are 5 m and 2 m respectively. The yield stress of the material is 340 MPa.

$\frac{KL}{r}$	Design Compressive Stress f_{cd} (MPa)
20	299
30	278
40	256



[12 Marks]

Solution:



(i) Moment of Inertia about principle axis.

$$I_{xx} = \left[\frac{182\times16^3}{12}+(182\times16)(225+8)^2\right]\times2+\left[\frac{16\times450^3}{12}\right]\times2$$

$$= 559.304\times10^6\ \text{mm}^4$$

$$I_{yy} = \left[\frac{16\times182^3}{12}\right]\times2+\left[\frac{450\times16^3}{12}+(450\times16)(75+8)^2\right]\times2$$

$$= 115.585\times10^6\ \text{mm}^4$$

As $I_{xx} > I_{yy}$

Major principle axis → x-x axis.

Minor principle axis → y-y axis.

(ii) Maximum slenderness ratio

about x-x axis

$$\lambda_{xx} = \frac{L_{ex}}{r_{xx}}$$

Where, $$r_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{559.304 \times 10^6}{(450 \times 16) \times 2 + (182 \times 16) \times 2}} = 166.3 \text{ mm}$$

$$\lambda_{xx} = \frac{5000}{166.3} = 30.066$$

About y-y axis $$\lambda_{yy} = \frac{L_{ey}}{r_{yy}}$$

Where, $$r_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{115.585 \times 10^6}{20224}} = 75.6 \text{ mm}$$

Hence, $$\lambda_{yy} = \frac{2000}{75.6} = 26.455$$

As, $$\lambda_{xx} > \lambda_{yy},$$

$$\lambda_{max} = \lambda_{xx}$$

Also, Buckling strength will be based on x-x axis.

(iii) Compressive strength

$\frac{KL}{r}$	f_{cd} (MPa)
20	299
30	278
40	256

(a) Using linear interpolation, to determine f_{cd} corresponding to $\lambda = 30.066$

$$\frac{30.066 - 30}{40 - 30} = \frac{f_{cd} - 278}{256 - 278}$$

$$f_{cd} = 277.855 \text{ MPa}$$

(b) Design compressive strength,

$$P_d = A_e f_{cd} = 20244 \times 277.855 = 5619.34 \text{ kN}$$

(c) Safe compressive load $= \dfrac{(P_d)}{\gamma_F} = \dfrac{5619.34}{1.5} = 3746.23 \text{ kN}$

End of Solution

Q.5 (e) (i) What information is generally needed to be provided in tender documents?
(ii) Differentiate between 'Lump sum contract' and 'Unit price contract'.

[6 + 6 = 12 Marks]

Solution:

(i) Tender documents are meant to keep the tenders informed about the general and specific conditions applicable for tenderers. Tender documents usually consist of the following:

(a) A letter of invitation to the tenderers
(b) Specimen tender form
(c) General instruction to the tenders
(d) Details of civil/structural work along with complete set of civil/structural drawings.
(e) Details and specifications of machinery/equipment to be supplied, if any
(f) Draft contract agreement
(g) Arbitration authority who will decide in case of dispute
(h) Time schedule for completion of work ..
(i) Amount of earnest money to be deposited and the form in which it is to be deposited.

(ii) **Lumpsum contract (Fixed price):** This is a single fixed price contact. In this contact, contractor agrees to perform specified job for fixed sum. The owner provides the contractor exact specification of the work. In this contract following are the advantages of the fixed price contract.

(a) Owner in aware of the cost of the project before the project construction starts.
(b) It avoids a lot of details and accounting by both owner and contractor
(c) Contractor gets free hand to execute the work
(d) In this contract is used with design contract method of delivery, contractor gets opportunity to use value engineering.

Unit price contract: In this type of contract, the price is paid per unit of the work carried out. There are different variations of this type of contract. Some of them are mentioned below.

Bill of quantities contract: In this type of contract owner provide the drawing, quantities of work to be done and specification. The contractor bid based on the unit cost of the items of construction. The contractor overhead, profit and other expanses can be included in the unit cost of the item of work. Sometimes contractor quotes the unit price of the work and lump-sum amount separately as profit overhead. The estimated quantities of the work to be done called Bill of the quantities is fixed.

This type of construction is usually followed in government sector for large infrastructure construction. This type of contract provides owner a competitive bid. Disadvantages of the methods are:

(a) Owner needs to measure the quantity of work done in the field, hence requires owner presence at the site.

(b) Final price of the construction is not known precisely until last price of work is completed. If there is significant difference between the estimated quantities and the reality of the situation, owner is put in adverse situation. Mistaken quantities is called unbalanced bid. Significant unbalanced bid now considered as unethical.

End of Solution

Q.6 (a) Design an RCC square column with the following data. Provide the main reinforcement and stirrups. Check the standard guidelines of IS 456 : 2000.

(i) Column size : 500 mm × 500 mm

(ii) Concrete grade M 20

(iii) Steel reinforcement Fe 415

(iv) Design load: 1600 kN

(v) Design moment $M_{ux} = M_{uy} = 100$ kN

(vi) Eccentricities $e_x = e_y = 20$ mm

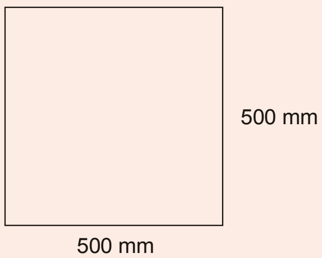
(vii) Minimum reinforcement : 1.2%

(viii) Check $\left[\frac{M_{ux}}{M_{ux_1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy_1}}\right]^{\alpha_n} < 1.0$, where $\alpha_n = \frac{P_u}{P_{uz}}$

[20 Marks]

Solution:

Design of column



$$B = 500 \text{ mm} - \text{square}$$

$$P_u = 1600 \text{ kN} = \text{Consider factored}$$

$$M_{ux} = M_{uy} = 100 \text{ kN}$$

Minimum eccentricity

$$e_{x,min} = e_{y,min} = \left(\frac{L}{500} + \frac{B}{30}\right) \text{ or } 20 \text{ mm}$$

$$= 20 \text{ mm} \quad \text{(given in question)}$$

Minimum moments, $M_{ux} = M_{uy} = P_u \cdot e_{min}$

$$= 1600 \times \frac{20}{1000} = 32 \text{ kN-m}$$

Trial 1 Consider A_{st} = Minimum = 1.20% (given in question)

$$A_{sc} = \frac{1.2}{100} \times 500 \times 500 = 3000\ \text{mm}^2$$

$$P_{uz} = 0.45 \times f_{ck} \cdot A_c + 0.75\ f_y \cdot A_{sc}$$
$$= [0.45 \times 20 \times (500 \times 500 - 3000) + 0.75 \times 415 \times 3000]/1000$$
$$= 3156.75\ \text{kN}$$

P_{uz} can be found from figure also.

For $P_t\% = 1.20\%$ (M20 | Fe 415)

$$\frac{P_{uz}}{Ag} = 12.50$$

$$P_{uz} = 12.50 \times A_g$$

$$= 12.50 \times \frac{500^2}{1000} = 3125\ \text{kN}$$

$$\frac{P_u}{p_{uz}} = \frac{1600}{3156.75} = 0.507$$

$$\alpha_n = \text{Calculated based on } \frac{P_u}{P_{uz}}$$

$$= 1.0 + \frac{(2.0 - 1.0)}{(0.8 - 0.2)} \times (0.507 - 0.20) = 1.5116$$

$$\frac{P_t}{f_{ck}} = \frac{1.20}{20} = 0.06$$

$$\frac{P_u}{f_{ck}Bd} = \frac{1600 \times 1000}{20 \times 500 \times 500} = 0.32$$

From interaction curve - value of

$$\frac{M_{ux1}}{f_{ck}BD^2} = 0.10$$

$$M_{ux1} = 0.10 \times 20 \times 500 \times \frac{500^2}{10^6} = 250\ \text{kN-m}$$

Since column is square,

$$M_{ux1} = M_{uy1}$$

$$\text{Check}\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} = \left(\frac{100}{250}\right)^{1.5116} + \left(\frac{100}{250}\right)^{1.5116}$$

$$= 0.500 < 1.0 - \text{Safe}$$

So we can provide minimum steel 1.2% as suggested in question.

Area of steel $= A_{sc} = 3000\ \text{mm}^2$

Number of 20 mmϕ bars

$$\eta = \frac{3000}{\frac{\pi}{4}(20)^2} = 9.50$$

Provide 10 bars of 20 mmϕ.

Lateral ties

Diameter $= 8$ mm (given in question)

or $\frac{20}{4} = 5$ mm

Spacing minimum,

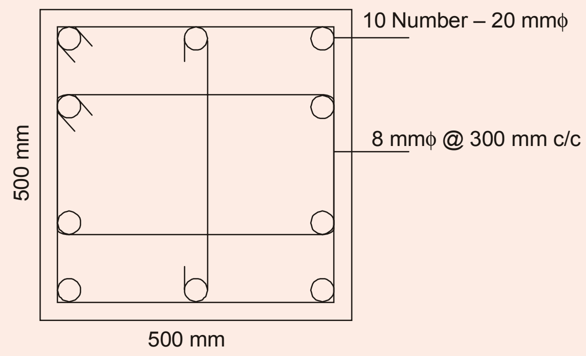
Least lateral dimension $= 500$ mm

$16\,\phi_{longi,\,min.} = 16 \times 20 = 320$ mm

$= 300$ mm

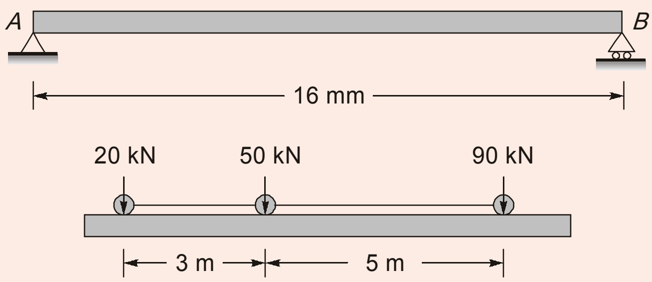
So, provide 8 mmϕ @ 300 mm c/c.

Reinforcement details:



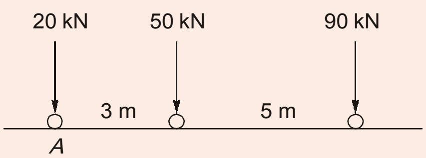
End of Solution

Q.6 (b) Determine the absolute bending moment in a 16 m long beam AB due to the truck loading shown in the figure.



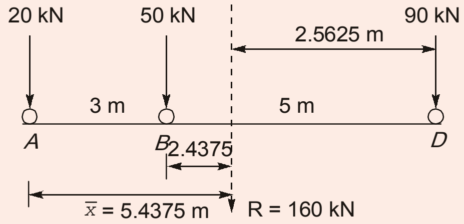
[20 Marks]

Solution:



Absolute maximum BM occurs near centre.

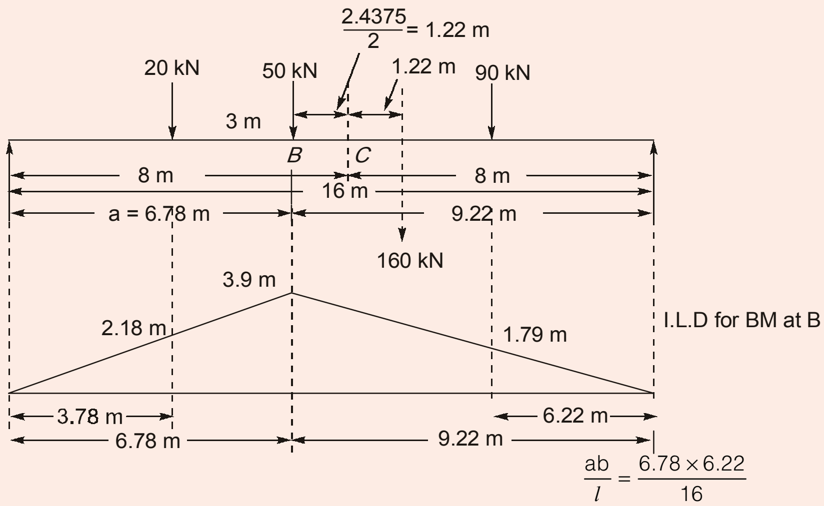
Resultant of load system



$$R \times \bar{x} = 20 \times 0 + (50 \times 3) + 90 \times 8$$

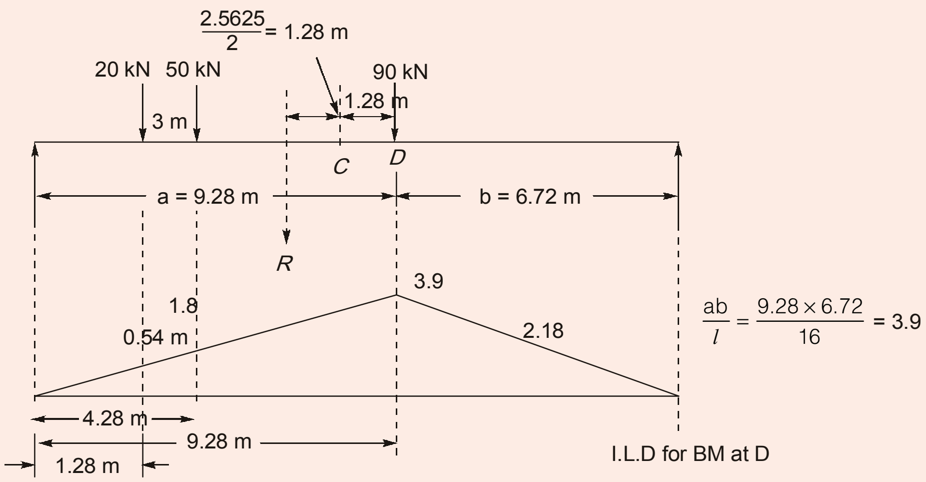
$$\bar{x} = \frac{150 + 120}{160} = 5.4375\,\text{m}$$

I^st^-possibility: Keep 50 kN and resultant at equal distance from centre



$$\text{Maximum BM under 50 kN load} = 20 \times 2.18 + 50 \times 3.91 + 90 \times 1.79$$
$$= 400.2\ \text{kN-m}$$

2^nd^ possibility maximum BM under 90 kN load



$$\text{Maximum BM at D} = 20 \times 0.54 + 50 \times 1.8 + 90 \times 3.9$$
$$= 451.8 \text{ kNm}$$
$$\therefore \quad \text{Absolute maximum BM} = 451.8 \text{ kNM}$$

End of Solution

Q.6 (c) (i) Briefly discuss various factors affecting the output of power shovel to excavate earth.

(ii) What are the precautions to be taken for labour safety during excavation works?

[10 +10 = 20 Marks]

Solution:

(i) The various factors affecting the output of power shovel:

1. **Class of Material:** Production decreases with increase in hardness of material.
2. **Height of Cut:** If the depth of fall from which a shovel is excavating material is too shallow, cycle time increases and it will be difficult or impossible to fill the bucket in one pass up the face, consequently the production is reduced.
3. **Angle of Swing :** The angle of swing of a shovel is the horizontal angle, expressed in degrees, between the position of the bucket when it is excavating and the position where it discharges the load. The total time in a cycle includes digging, swinging to the dumping position, dumping, and returning to the digging position. If the angle of swing is increased, the time for a cycle will be increased, whereas if the angle of swing is decreased, the time for a cycle will be decreased.
4. **Operator's Skill**
5. **Condition of the Shovel**
6. **Haul Unit Exchange:** Haul-unit exchange refers to the total time required for a loaded truck to clear its loading position under the excavator and for the next empty truck to be positioned for loading.
7. **Size of Hauling Units:** Size of hauling units like trucks affect the output of a shovel, for optimum output, size of hauling units should be governed by size of a power shovel. If the shovel used is of smaller size, the size of hauling units also must be small while for a large shovel, hauling units must be of larger size.
8. **Handling of Oversize Material:** When handling shot rock, carefully evaluate the amount of oversize material to be moved. A machine with a bucket whose bite width and pocket are satisfactory for the average-size pieces may spend too much time handling individual oversize pieces. A large bucket, or a larger machine, or changing the blasting pattern should be considered when there is a large percentage of oversize material.

(ii) Safety measures for excavation

(a) Prior to excavation work, a complete knowledge of underground structures such as sewers, waterpipes, gas mains etc. is essential so as to proper precaution to prevent any accidents.

(b) The workers must be provided with all protective devices.

(c) When depth of excavation exceeds 2 m, the trenches should be securely shored and timbered.

(d) Sheathing should be placed against the side of the trench. In case of loose or soft soil, sheathing should be driven into the bottom of the trench.

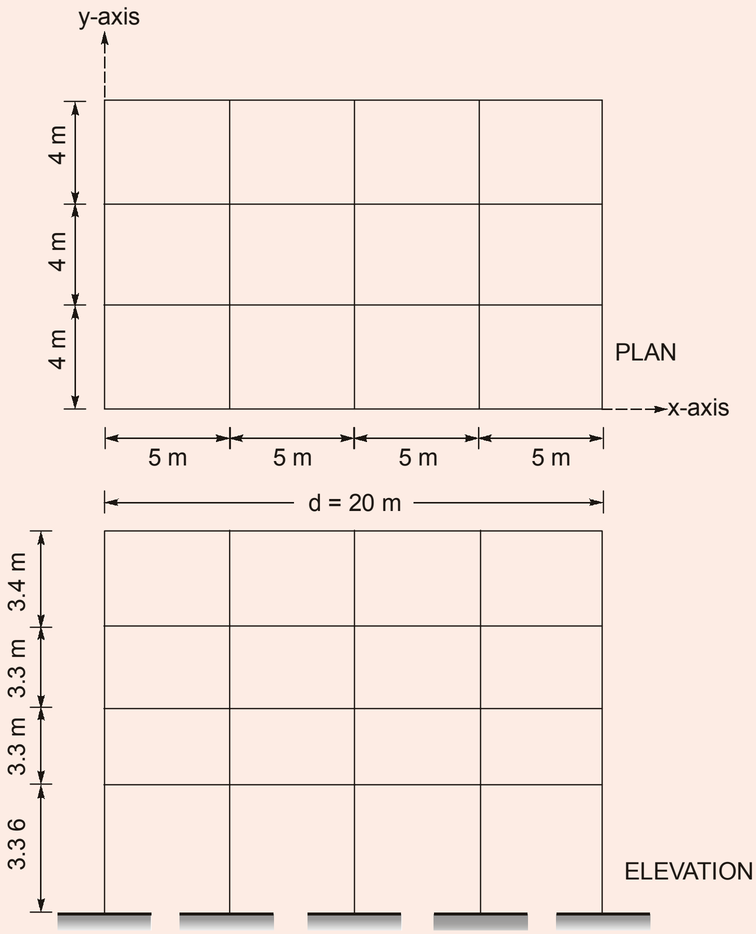
(e) Excavated materials should be kept away from the edge of the trench.

(f) Heavy equipments such as excavating machineries, trucks etc. should be kept away at a suitable distance from the excavated sides.

(g) A fence or barricade should be erected and at night the area must be properly lighted.

End of Solution

Q.7 (a) A four-storey reinforced concrete (RC) office building located in seismic zone IV is shown in the figure. The RC frames are infilled with brick masonry. The lumped weights due to dead loads is 15 kN/m^2 on the floors and 12 kN/m^2 on the roof. The floors have to cater to a live load of 4 kN/m^2 on the floors and 1.5 kN/m^2 on The roof. Calculate the design seismic load on the structure at different floors using Linear static (Equivalent static) analysis, along x-axis.



Zone factor Z = 0.24

Importance factor I = 1.2

Response Reduction factor R = 5

Fundamental translational natural period $T_a = \dfrac{0.09h}{\sqrt{d}}$

$$\frac{S_a}{g} = \begin{cases} 2.5 & 0 < T \leq 0.55\ s \\ \dfrac{1.36}{T} & 0.55 \leq T \leq 4.0s \\ 0.34 & T > 4.0\ s \end{cases}$$

[20 Marks]

Solution:

$$\text{Zone-IV} = Z = 0.24$$

$$\text{Importance factor, } I = 1.20$$

$$\text{Response reduction, } R = 5.00$$

$$\text{Fundamental time period, } T_a = \frac{0.09 \times h}{\sqrt{d}} = \frac{0.09 \times 13.6}{\sqrt{20}} = 0.274$$

$$h = 3.6 + 3.3 + 3.3 + 3.4 = 13.60 \text{ m}$$

$$\text{Along x-axis, } d = L = 4 \times 5 \text{ m} = 20 \text{ m}$$

$$\frac{S_a}{g} = 2.50 \quad \text{for } T_a < 0.55 \text{ sec}$$

Seismic weights on floors

$$W_1 = (12 \text{ m} \times 20 \text{ m}) \times (\text{DL} + \text{Part of LL})$$

$$= 12 \times 20 \times (15 + 4 \times 0.5)$$

$$= 4080 \text{ kN} = W_1 = W_2 = W_3$$

Seismic weight of roof $W_4 = 12 \times 20 \times (12 + 0) = 2880$ kN

Live load on roof not considered.

Total seismic weight of building $= W_1 + W_2 + W_3 + W_4$

$$= 4080 + 4080 + 4080 + 2880 = 15120 \text{ kN}$$

Total lateral force due to earthquake

$$V_B = A_H \times \Sigma W$$

$$= \left(\frac{Z}{2}\right) \times \left(\frac{I}{R}\right) \times \left(\frac{S_a}{g}\right) \times \Sigma W$$

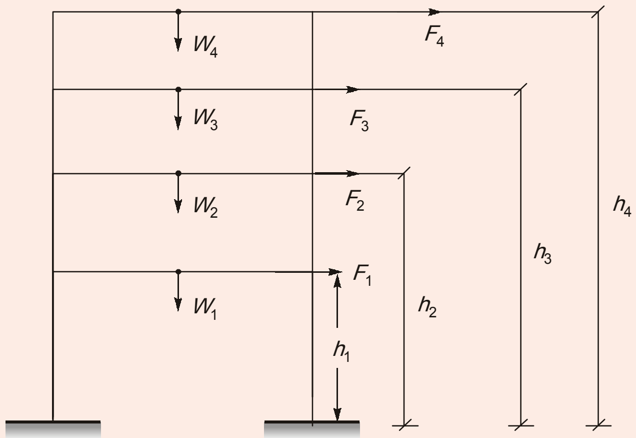
$$= \frac{0.24}{2} \times \frac{1.20}{5.0} \times 2.50 \times 15120$$

$$V_B = 1088.64 \text{ kN}$$

Say 1089 kN

Seismic load on each floor:

$$\Sigma Wh^2 = 4080 \times 3.60^2 + 4080 \times 6.9^2 + 4080 \times 10.2^2 + 2880 \times 13.6^2$$
$$= 1204293.60$$

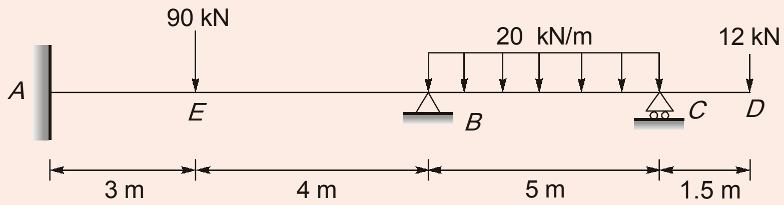


$$\text{Distribution of lateral force} = \frac{W_1 h_1^2}{\Sigma W h^2} \times V_B$$

Floor	W	h	Wh^2	Distribution of lateral forces
1	4080	3.6	52876.8	$47.80\,\text{kN} = f_1$
2	4080	6.9	194248.8	$175.6\,\text{kN} = f_2$
3	4080	10.2	424483.5	$383.8\,\text{kN} = f_3$
4	2880	13.6	532684.8	$481.8\,\text{kN} = f_4$
		ΣWh^2	1204293.6	Total = 1089 kN

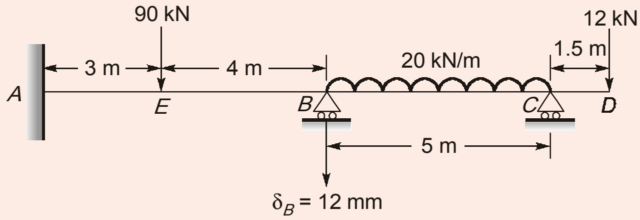
End of Solution

Q.7 (b) Analyse a continuous beam shown in the figure. During loading the support B sinks by 12 mm. E = 210 GPa, I = 5131.6 × 10^4 mm^4. EI is constant. Draw BMD and Elastic curve.



[20 Marks]

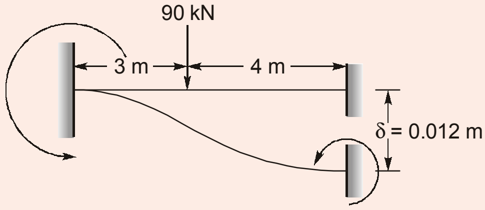
Solution:



$$E = 210 \text{ GPa}, I = 5131.6 \times 10^4 \text{ mm}^4 = \frac{5131.6 \times 10^4}{10^{12}}$$

$$EI = 210 \times 10^6 \times \frac{5131.6 \times 10^4}{10^{12}} = 10776.36 \text{ kN–m}^2$$

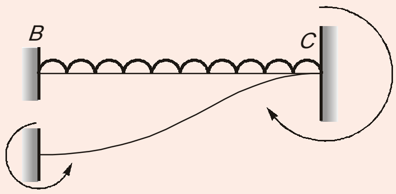
Step-1 FEMS due to applied loads and sinking of supports.



$$m_{fab} = -\frac{90 \times 3 \times 4^2}{7^2} - \frac{6 \times 10776.36 \times 0.012}{7^2} = -104 \text{ kNm}$$

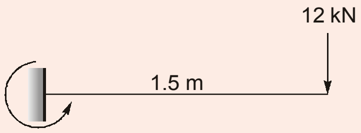
$$m_{fba} = +\frac{90 \times 3^2 \times 4}{7^2} - \frac{6 \times 10776.36 \times 0.012}{7^2} = 50.29 \text{ kNm}$$

$$m_{fbc} = -\frac{20 \times 5^2}{12} + \frac{6 \times 10776.36 \times 0.012}{5^2} = -10.63 \text{ kNm}$$



$$m_{fcb} = +\frac{20 \times 5^2}{12} + \frac{6 \times 10776.36 \times 0.012}{5^2} = 72.7 \text{ kNm}$$

$$m_{fcd} = -18 \text{ kNm}$$



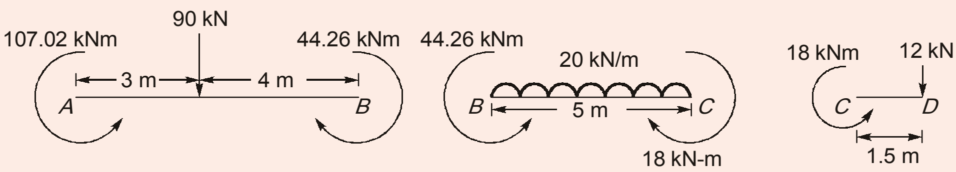
Step-2: Distribution factors

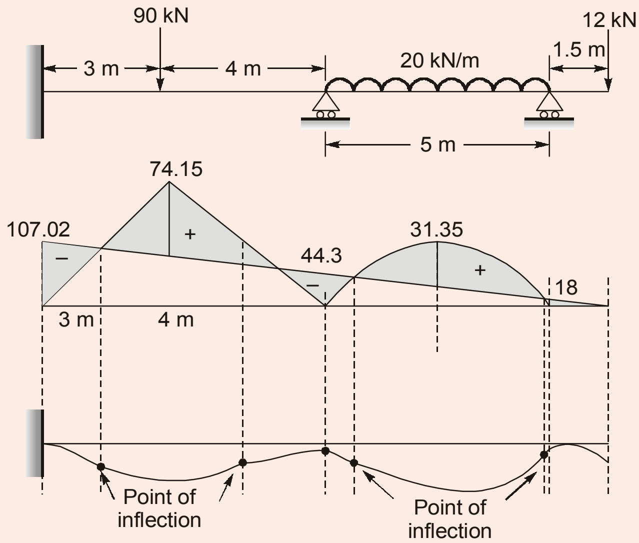
Joint	Member	k	Σk	$DF=\frac{k}{\Sigma k}$
	BA	$I/7$		0.49
B			$0.293I$	
	BC	$\frac{3}{4}\times\frac{I}{5}=\frac{3I}{20}$		0.51

Step-3: End Moment distribution

Joint	A	B		C		D
DF	0	0.49	0.51	1	0	–
FEMS	–104	50.29	–10.63	72.7	–18	–
Release C COM			–27.35	–54.7	0	
Net FEMS	–104	50.29	–37.98	+18.0	–18	
Bal B		–6.03	–6.28			
COM	–3.02			0		
Final and moments	–107.02	44.26	–44.26	+18	–18	–

Step-4: BMD





End of Solution

Q.7 (c) Differentiate between optimistic time estimate and pessimistic time estimate in a PERT network.

A construction company has an opportunity to submit a bid for the construction of a residential building and a commercial building. The 3 time estimates (in months) for completion of each building are as follows:

	Optimistic time (in months)	Most likely time (in months)	Pessimistic time (in months)
Residential Building	3	4	6
Commercial Building	4	6	8

Determine the expected time for completion of each building. Also analyse which building has more reliable time estimate.

[20 Marks]

Solution:

Optimistic Time (t_o) :

This is the shortest possible time in which an activity can be completed, under ideal conditions, This particular time estimates represents the time in which we could complete the activity or job if everything went along perfectly with no problems or adverse conditions. Better than normal conditions are assumed to prevail.

This time estimate is demanded by to.

Pessimistic time (t_p):

It is the best given of maximum time that would be required to complete the activity. This particular time estimate represents the time, it might take us to complete a particular activity if everything went wrong and abnormal situations prevailed. However, this estimates does not include possible effects of highly unusual catastrophic conditions such as earthquakes, floods, drawings, fire etc.

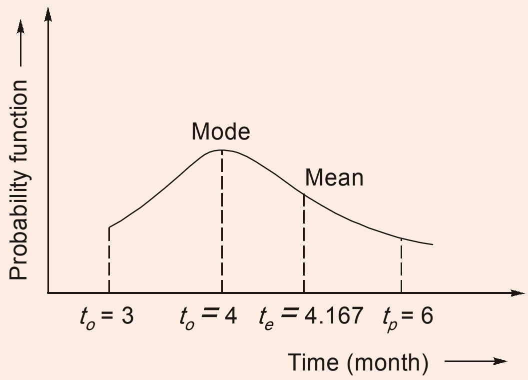
This time is denoted by t_p.

Expected completion time for residential building is given by

$$t_{e1} = \frac{t_0 + 4t_L + t_p}{6} = \frac{3 + 4 \times 4 + 6}{6} = 4.167 \text{ months}$$

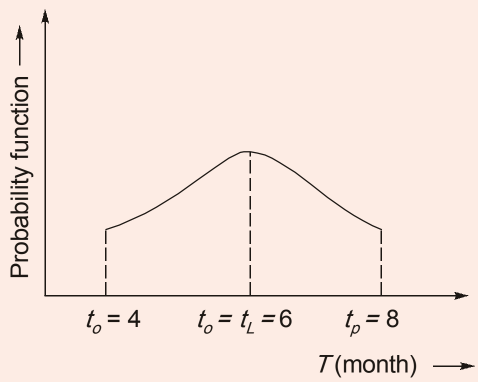
Variance for residential building is given by

$$\sigma_1^2 = \left(\frac{t_p - t_o}{6}\right)^2 = \left(\frac{6-3}{6}\right)^2 = 0.25 \text{ months}^2$$



Expected completion time for commercial building is given by

$$t_{e2} = \frac{t_0 + 4t_L + t_p}{6} = \frac{4 + 4 \times 6 + 8}{6} = 6 \text{ months}$$



Variance for commercial building is given by

$$\sigma_2^2 = \left(\frac{t_p - t_0}{6}\right)^2 = \left(\frac{8-4}{6}\right)^2 = \left(\frac{4}{6}\right)^2 = 0.444 \text{ month}^2$$

As variance for residential building is less than variance for commercial building hence uncertainty is less for completion of project.

Time estimates for residential building is more reliable.

End of Solution

Q.8 (a) Design an open cylindrical water tank of 350 m^3 capacity. This tank will rest on ground and have a free-flexible joint at base. Overall height of tank is 4.0 m, including the free board of 200 mm.

Design the vertical cylindrical wall of tank and sketch the details. Consider only maximum hoop tension for entire height.

The following parameters may be used for design, if required.

(i) Permissible direct tensile stress of concrete σ_{ct} = 1.2 MPa

(ii) Permissible tensile stress of steel σ_{st} = 150 MPa up to 225 mm from water fall
= 190 MPa beyond 225 mm from water all

(iii) Molecular ratio M = 13

(iv) Minimum thickness required = 180 mm

(v) Minimum % of reinforcement up to 100 mm thickness 0.24% linearly reduces to 0.16% for more than 450 mm thickness

(vi) Use 16 mm diameter, high strength deformed bar for hoop reinforcement

(vii) Use 8 mm diameter, high strength deforced bar for vertical distribution reinforcement

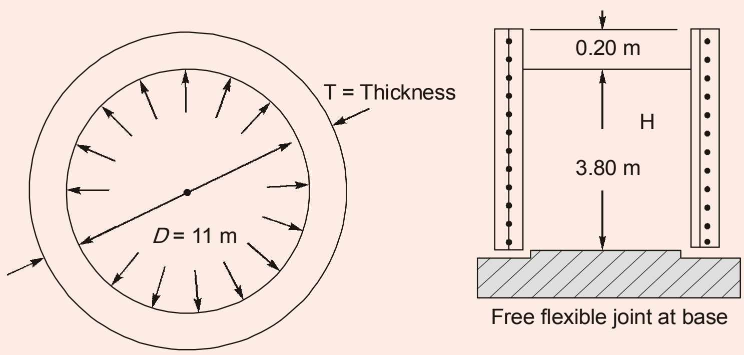
(viii) Minimum diameter of tank required = 11.0 m

(ix) Unit weight of water = 10 kN/m^3

(x) Tensile stress in concrete = $\dfrac{T}{A_c + (m-1)\,A_s}$

[20 Marks]

Solution:



1. Minimum dia. of tank = 11 m

$$D = 11 \text{ m}$$

Volume,
$$V = 350 \text{ m}^3 = \frac{\pi}{4}(D)^2 \times H$$

$$H = \frac{350}{\frac{\pi}{4}(11)^2} = 3.68 \text{ m} \qquad \text{say } 3.70 \text{ m}$$

Tank height given = 4.0 m

free board = 0.2 m

$$H \text{ available} = 4.0 - 0.2 = 3.80 \text{ m}$$
$$> 3.70 \text{ m required}$$

Let us design for $D = 11$ m

$H = 3.80$ m

2. Maximum hoop tension in tank at bottom.

$$= T_H = \frac{p.D}{2} = \frac{\gamma_w H.D}{2}$$

$$T_H = \frac{10 \times 3.80 \times 11}{2} = 209 \text{ kN}$$

3. Area of steel required. (Considered cracked)

$$\text{Ast} = \frac{T_H}{f_{st}} = \frac{209000}{150} = 1394 \text{ mm}^2$$

Using 16 mm ϕ bars for hoop height spacing

$$= \frac{1000}{1394} \times \frac{\pi}{4}(16)^2 = 144 \text{ mm}$$

Provide 16 mm ϕ @ 140 mm c/c in single layer.

4. Minimum thickness of tank = 180 mm

Check, stress developed in concrete if

$T = 180$ mm is provided

$$f_{ct} = \frac{T_H}{1000 \times T + (m-1) A_{st}}$$

[Considering uncracked section]

$$= \frac{209000}{(1000 \times 180 + (13-1) \times 1394)}$$

$$= 1.06 \text{ N/mm}^2 < (f_{ct}(\text{pev})) = 1.20 \text{ N/mm}^2$$

So safe in direct tension.

5. Minimum reinforcement for $T = 180$ mm

$$= 0.24 - \frac{(0.24 - 0.16)}{(450 - 100)} \times (180 - 100) = 0.222\%$$

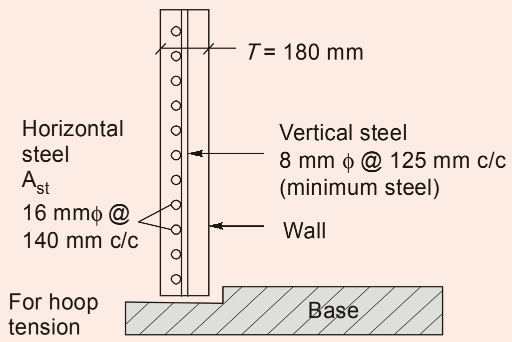
$$A_{st} = \frac{0.222}{100} \times 1000 \times 180$$

$$= 399.6 \text{ say } 400 \text{ mm}^2$$

Spacing of 8 mm ϕ for vertical reinforcement.

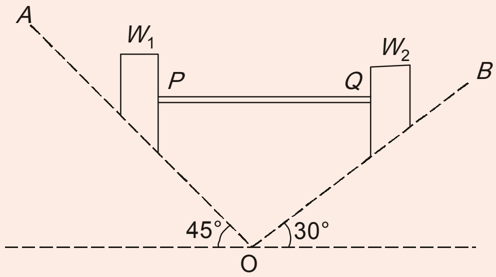
$$= \frac{1000}{399.6} \times \frac{\pi}{4} 8^2 = 125.7 \text{ mm}$$

Provide 8 mmϕ @ 125 mm c/c



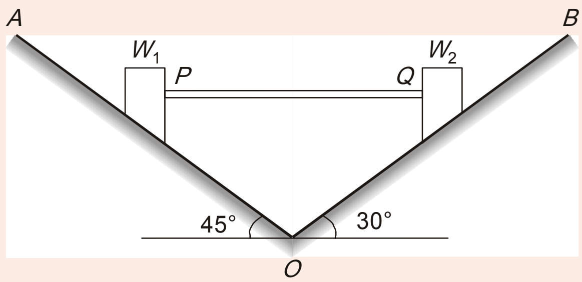
End of Solution

Q.8 (b) Two loads W_1 (equal to 1.5 kN) and W_2, resting on two inclined rough planes OA and Ob are connected by a horizontal link *PQ* as shown in the figure. Find the maximum and minimum values of W_2 for which the equilibrium can exist. Take angle of friction for both the planes as 20°.



[20 Marks]

Solution:

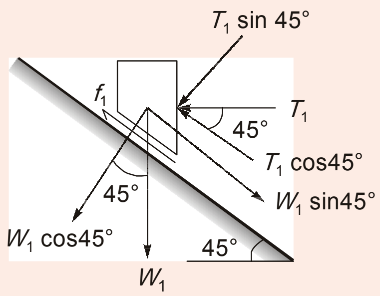


Given, $W_1 = 1.5$ kN

Angle of friction, $\phi = 20°$

$\mu = \tan\phi = \tan 20° = 0.364$

FBD of W_1 (For minimum value of W_2)



Normal reaction, $$N_1 = T_1 \sin 45° + W_1 \cos 45°$$

$$N_1 = 1.06 + 0.707\, T_1 \quad ..(i)$$

For equilibrium of load,

$$f_1 + T_1 \cos 45° = W_1 \sin 45°$$

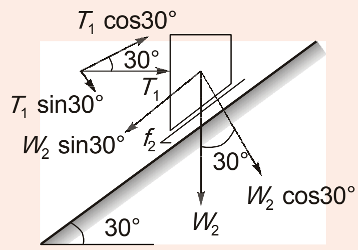
$$\mu N_1 + T_1 \cos 45° = 1.06$$

$$(0.364)\,(1.06 + 0.707\, T_1) + 0.707 T_1 = 1.06$$

$$0.385 + 0.257 T_1 + 0.707 T_1 = 1.06$$

$$T_1 = 0.7 \text{ kN}$$

FBD of W_2, (for minimum value of W_2)



Normal reaction, $$N_2 = T_1 \sin 30° + W_2 \cos 30°$$

$$= 0.7 \sin 30° + W_2 \cos 30°$$

$$= 0.35 + 0.866 W_2$$

For equilibrium of W_2 load,

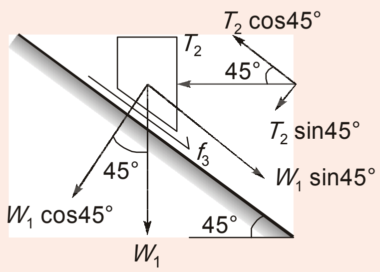
$$f_2 + W_2 \sin 30° = T_1 \cos 30°$$

$$\mu N_2 + W_2 \sin 30° = 0.7 \cos 30°$$

$$0.364(0.35 + 0.866 W_2) + W_2 \times 0.5 = 0.606$$

$$W_2 = \frac{0.7786}{0.8152} = 0.587 \text{ kN}$$

FBD of W_1 (for maximum value of W_2)



Normal reaction,

$$N_3 = W_1\cos 45° + T_2\sin 45°$$

$$N_3 = 1.06 + 0.707T_2$$

For equilibrium of load W_1,

$$f_3 + W_1\sin 45° = T_2\cos 45°$$

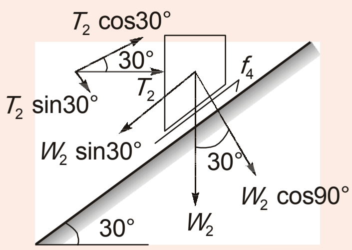
$$\mu N_3 + (1.5)\sin 45° = T_2\cos 45°$$

$$0.364(1.06 + 0.707T_2) + 1.06 = T_2 \times 0.707$$

$$1.445 = 0.4496\, T_2$$

$$\mathbf{T_2 = 3.21\ kN}$$

FBD of W_2, (for maximum value of W_2)



Normal reaction,

$$N_4 = T_2\sin 30° + W_2\cos 30°$$

$$N_4 = 3.213\sin 30° + W_2\cos 30°$$

$$N_4 = 1.606 + 0.866W_2 \quad \text{...(ii)}$$

For equilibrium of W_2 load,

$$f_4 + T_2\cos 30° = W_2\sin 30°$$

$$\mu N_4 + 3.213\cos 30° = W_2\sin 30°$$

$$0.364(1.606 + 0.866W_2) + 2.782 = 0.5W_2$$

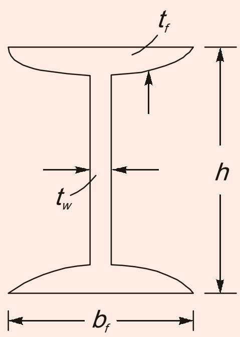
$$0.5845 + 0.315W_2 + 2.782 = 0.5W_2$$

$$0.184W_2 = 3.366$$

$$W_2 = 18.293 \text{ kN}$$

End of Solution

Q.8 (c) A rolled steel joint ISMB 450 is used as beam for the roof of a hall 7.5 m × 12 m. Thickness of RC slab is 125 mm. The rolled steel joists are spaced at 3 m centre to centre. The floor finishing load is 1.5 kN/m^2 and the roof slab has to support a live load of 4 kN/m^2. Assume the self-weight of the beam as 1 kN/m. Take the width of bearing for the beam as 300 mm. The limiting deflection for the beam is span/240. γ_{mo} = 1.1, f_y = 250 MPa. Check the adequacy of the section against any two modes of failure.



Elastic section modulus = 30390.8 × 10^3 mm^3

Plastic section modulus Z_p = 1533.36 × 10^3 m^3

Depth of section h = 450 mm

Width of flange b_f =150 mm

Thickness of flange t_f = 17.4 mm

Thickness of web t_w = 9.4 mm

Radius at root = 15 mm

Shear capacity $V_d = \dfrac{A_v \cdot f_y}{\sqrt{3}\,\gamma_{mo}}$

Design bending strength $M_d = \dfrac{\beta_b \cdot Z_p \cdot f_y}{\gamma_{mo}}$

Slenderness ratio	Design Compressive Stress (f_{cd})
100	107
110	94.6
120	83.7

Buckling strength $F_b = (b_1 + n_1)\, tw \cdot f_{cd}$

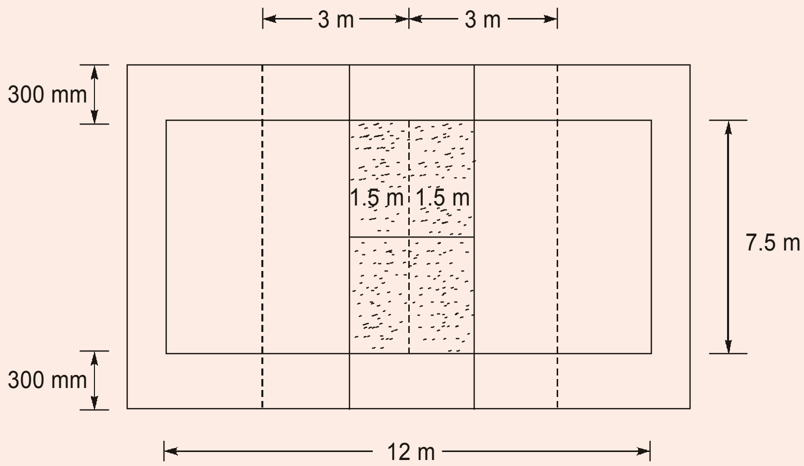
Capacity of web at its connection to the flange $F_w = \dfrac{(b_1 + n_2) t_w \cdot f_{yw}}{\gamma_{mo}}$

The section ISMB 450 may be assumed as plastic.

[20 Marks]

Solution:

In this question, Z_e value given in question is greater than Z_p value. Which is not correct. Still solution is done using same values given in question.



Shaded region is tributory area for which joist will be designed.

(1) Load's on Beam

$w_s \rightarrow$ Self - *wt* of slab $= 0.125 \times 3 \times 24$ kN/m^3

(Assuming unit-*wt* of *RCC* slab = 24 kN/m^3)

$= 9$ kN/m

$w_f \rightarrow$ Weight of finishing $= 1.5$ kN/m$^2 \times 3$ m $= 4.5$ kN/m

$w_b \rightarrow$ Self - *wt* of Beam $= 1$ kN/m

$w_L \rightarrow$ Live load $= 4$ kN/m$^2 \times 3$ m $= 12$ kN/m

Total service load, $w = w_s + w_t + w_b + w_L = 9 + 4.5 + 1 + 12 = 26.5$ kN/m.

Factored load or design load

$$w_u = \gamma_F \times w = 1.5 \times 26.5 = 39.75 \text{ kN/m}$$

(2) Maximum bending moment and maximum shear force

Effective span of beam = c/c distance between support

$$= 7.5 + \frac{0.3}{2} + \frac{0.3}{2} = 7.8 \text{ m}$$

$$\text{Maximum BM} = (\text{B.M})_{max} = M_u = \frac{w_u \cdot L^2}{8} = \frac{39.75 \times 7.8^2}{8}$$

$= 302.3$ kNm

Maximum applied shear force,

$$V_u = \frac{w_u \cdot L}{2} = \frac{39.75 \times 7.8}{2} = 155.025 \text{ kN}$$

(3) Adequacy check for ISMB 450.

(i) Shear strength of the beam

Shear capacity $$V_d = \frac{A_v \cdot f_y}{\sqrt{3} \cdot \gamma_{mo}}$$

$$= \frac{(ht_w) \cdot f_y}{\sqrt{3}\ \gamma_{mo}} = \frac{(450 \times 9.4) \times 250}{\sqrt{3} \times 1.1} = 555.044 \text{ kN}$$

As $V_d > V_u$, safe in shear

(ii) Bending strength of beam

$$M_d = \frac{\beta_b Z_p f_y}{\gamma_{mo}} \le 1.2\, Z_e \frac{f_y}{\gamma_{mo}}$$

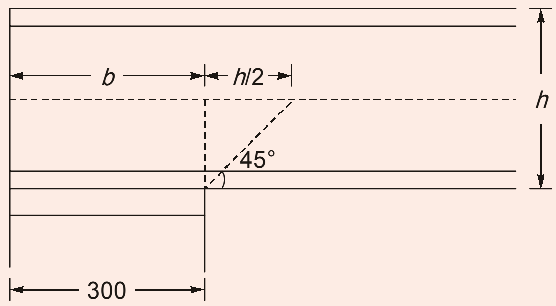
$$= \frac{1 \times 1533.36 \times 10^3 \times 250}{1.1} \le \frac{1.2 \times 30390.8 \times 10^3 \times 250}{1.1}$$

$$= 348.49 \text{ kNm} \le 8288.4 \text{ kNm.}$$

Also, $V_u < 0.6\, V_d$. So no reduction in bending strength required as it is a low shear case.

Hence, $M_d > M_u$ (Beam is safe in Bending)

(iii) Web buckling strength



$$P_{WB} = A_e \cdot f_{cd} = (b + n_1) \cdot t_w \cdot f_{cd}$$

Where, f_{cd} is design compressive stress which depends upon slenderness ratio of web:

$$\lambda = 2.45 \frac{d}{t_w} = 2.45 \times \frac{h - 2(t_F + R_1)}{t_w}$$

$$= 2.45 \times \frac{450 - 2(17.4 + 15)}{9.4} = 100.4$$

Using table given

Slenderness Ratio	f_{cd} (N/mm^2)
100	107
110	94.6
120	83.7

Using linear interpolation to determine f_{cd} value corresponding to $\lambda = 100.4$.

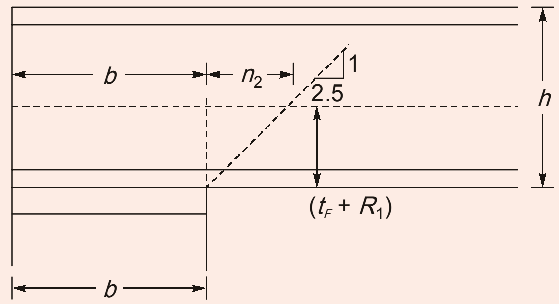
$$\frac{100.4 - 100}{110 - 100} = \frac{f_{cd} - 107}{94.6 - 107}$$

$$f_{cd} = 106.54 \text{ N/mm}^2$$

$$\therefore \quad P_{WB} = \left(300 + \frac{450}{2}\right) \times 9.4 \times 106.54 = 525.6 \text{ kN}$$

As $P_{WB} > V_u$, safe in web buckling.

(iv) Web crippling strength



$$P_{wc} = A_e \cdot \frac{f_y}{\gamma_{mo}} = \frac{(b + n_2)\, t_w\, f_y}{\gamma_{mo}}$$

$$= \frac{\left[300 + 2.5(17.4 + 15)\right] \times 9.4 \times 250}{1.1} = 813.96 \text{ kN}$$

As $P_{wc} > V_u$, safe in web crippling.

(v) Check for deflection

$$\delta_{cal} = \frac{5}{384} \frac{wL^4}{EI} \quad (\text{Where } I = Z_e \times \frac{h}{2})$$

$$= \frac{5}{384} \times \frac{26.5 \times \left(7.8 \times 10^3\right)^4}{2 \times 10^5 \times 30390.8 \times 10^3 \times \frac{450}{2}} = 0.934 \text{ mm}$$

$$\text{Limiting deflection} = \frac{\text{Span}}{240} = \frac{7800}{240} = 32.5 \text{ mm}$$

As δ_{cal} < limiting deflection.

Hence sec is safe in deflection.

Conclusion: ISMB 450 satisfies all the criteria's hence, it is Adequate.

■■■■