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ESE 2021

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-II

EXAM DATE : 21-11-2021 | 2:00 PM to 5:00 PM

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ANALYSIS

Mechanical Engineering Paper-II
ESE 2021 Main Examination

Sl.	Subjects	Marks
1.	Engineering Mechanics & Strength of Materials	72
2.	Mechanisms and Machines	104
3.	Design of Machine Elements	52
4.	Manufacturing Engineering & Engg. Materials	96
5.	Industrial & Maintenance Engineering	64
6.	Mechatronics & Robotics	92
		Total 480

Scroll down for detailed solutions

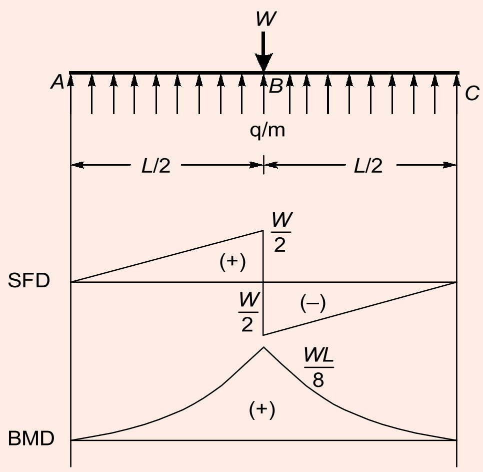
Section-A

Q.1 (a) A light wooden log of length L is floating on water with a concentrated load W acting at the mid-point. Write the equations and draw the diagram for shearing force and bending moment.

[12 marks : 2021]

Solution:

The force on the log will be as shown in figure



From force balance, $$qL = W$$

$\Rightarrow$ $$q = \frac{W}{L}$$

SF in section AB [x from A]

$$S_x = q\,x = \frac{W}{L}x$$

SF in section BC [x from C]

$$S_x = -\,q x = -\frac{W}{L}x$$

BM in section AB [x from A]

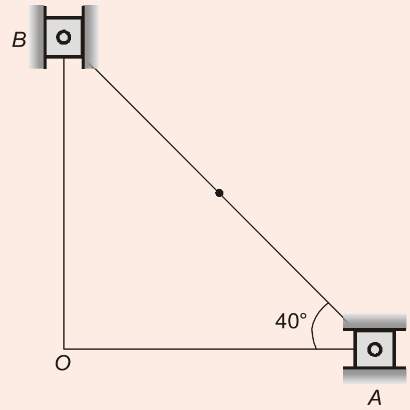
$$M_x = q{\cdot}x\cdot\frac{x}{2} = \frac{W}{2L}x^2$$

BM in section BC [x from C]

$$M_x = q{\cdot}x\cdot\frac{x}{2} = \frac{Wx^2}{2L}$$

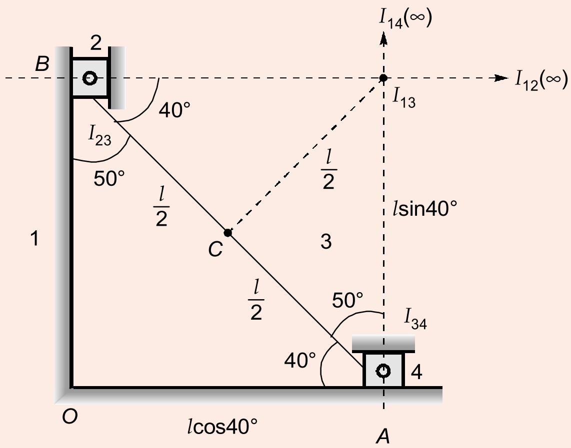
End of Solution

Q.1 (b) The slider A, of an elliptical trammel shown in the figure moves towards *O* with a velocity of 3 m/s at the instant when AB makes an angle of 40° with the horizontal. Determine the velocity of the mid-point of the link AB at this instant.



[12 marks : 2021]

Solution:



AB link motion (I_{13}),

$$\frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B} = \frac{V_C}{I_{13}C}$$

$$\frac{V_A}{I_{13}A} = \frac{V_C}{I_{13}C} \quad \text{...(i)}$$

$$(I_{13}C)^2 = \left(\frac{l}{2}\right)^2 + (l\sin 40°)^2 - 2\left(\frac{l}{2}\right)(l\sin 40°)\cos 50°$$

$$= \frac{l^2}{4} + l^2\sin^2 40° - l^2\sin^2 40°\cos 50°$$

$$= l^2\left[\frac{1}{4} + \sin^2 40° - \sin 40°\cos 50°\right]$$

$$= (0.25)l^2$$

$$I_{13}C = (0.2)l = \frac{l}{2} \qquad \text{...(ii)}$$

$$\frac{3}{l\sin 40^\circ} = \frac{V_c}{\frac{l}{2}} \qquad \text{[From equation (i) and (ii)]}$$

$$\frac{3}{l\sin 40^\circ} = 2V_c$$

$$V_c = \frac{3}{2\sin 40^\circ}$$

$$V_c = 2.3333 \text{ m/s}$$

End of Solution

Q.1 (c) State and prove the law of gearing.

[12 marks : 2021]

Solution:

A positive drive is the one which transmits an absolute uniform angular velocity ratio during even a small fraction of a revolution. In these days of high speed machines, a gear drive must be positive else vibrations and dynamic forces would develop causing immature failure.

During the period of engagement of two teeth, there is some sliding of one tooth over another. While the sliding occurs, the gears should continue to transmit a constant velocity ratio. Thus, for a gear drive to be positive, it is not enough to have interlocking teeth between a gear pair. The contacting surfaces of a pair of meshing teeth must be so shaped that during the entire period of engagement between a pair of teeth, angular velocity ratio remains constant. Such surfaces of mating teeth are called conjugate. The law of gearing thus provides a basis to decide conjugate tooth surfaces.

The law states that for transmitting constant angular velocity ratio, common normal to the contacting surfaces of mating teeth, at every instantaneous point of contact, must pass through a fixed point on the line of centres of the two gears. The fixed point is called the pitch point which divides the line of centres in inverse ratio of the angular velocities of the mating gears.

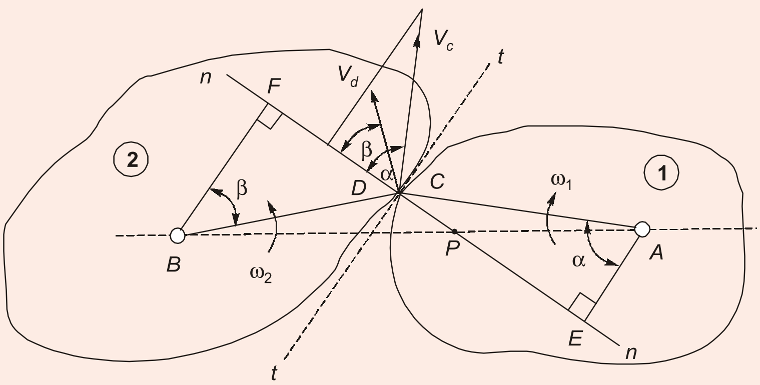


Illustration for law of gearing

The law of gearing states the condition which must be fulfilled by the gear tooth profiles to maintain a constant angular velocity ratio between two gears. Fig shows two bodies 1 and 2 representing a portion of the two gears in mesh.

A point *C* on the tooth profile of the gear 1 is in contact with a point *D* on the tooth profile of the gear 2. The two curves in contact at points C or D must have a common normal at the point. Let it be $n-n$.

Let ω_1 = instantaneous angular velocity of the gear 1 (clockwise)

ω_2 = instantaneous angular velocity of the gear 2 (counter-clockwise)

v_c = linear velocity of *C*.

v_d = linear velocity of *D*.

Then $v_c = \omega_1 . AC$ in a direction perpendicular to *AC* or at an angle α to $n-n$.

$v_c = \omega_2 . BD$ in a direction perpendicular to *BD* or at an angle β to $n-n$.

Now, if the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent $t-t$. The relative motion between the surfaces along the common normal $n-n$ must be zero to avoid the separation, or the penetration of the two teeth into each other.

Component of v_c along $n-n = v_c \cos\alpha$

Component of v_d along $n-n = v_d \cos\beta$

Relative motion along $n-n = v_c \cos\alpha - v_d \cos\beta$

Draw perpendiculars *AE* and *BF* on $n-n$ from points *A* and *B* respectively.

Then $\angle CAE = \alpha$ and $\angle DBF = \beta$. For proper contact,

$$v_c \cos\alpha - v_d \cos\beta = 0$$

or $$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$$

or $$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

or $$\omega_1 AE - \omega_2 BF = 0$$

or $$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP} \qquad [\because DEP \text{ and } BEP \text{ are similar}]$$

Thus, it is seen that the centre line *AB* is divided at *P* by the common normal in the inverse ratio of the angular velocities of the two gears. If it is desired that the angular velocities of two gears remain constant, the common normal at the point of contact of the two teeth should always pass through a fixed point *P* which divides the line of centres in the inverse ratio of angular velocities of two gears.

As seen earlier, P is also the point of contact of two pitch circles which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point. Thus, for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Also, as the Δs *AEP* and *BFP* are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

or $$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \text{ or } \omega_1\, EP = \omega_2\, FP$$

In order that a pair of curved surface (tooth profiles) may transmit a constant angular velocity ratio, the shape of contacting tooth profiles must be such that the common normal passes through a fixed point P on the line of centres. The point P divides the line of centres in an inverse proportion as the ratio of angular velocities. The fixed point P is called the pitch point and the line EF (common normal to contacting surface) is called the line of action.

End of Solution

Q.1 (d) Explain with suitable illustration the S-N curve of a ferrous material and briefly discuss its significant in the design of machine elements.

[12 marks : 2021]

Solution:

The basis of the Stress-Life method is the Wohler S-N diagram, shown schematically for two materials in Figure 1. The S-N diagram plots nominal stress amplitude S versus cycles to failure N. There are numerous testing procedures to generate the required data for a proper S-N diagram. S-N test data are usually displayed on a log-log plot, with the actual S-N line representing the mean of the data from several tests.

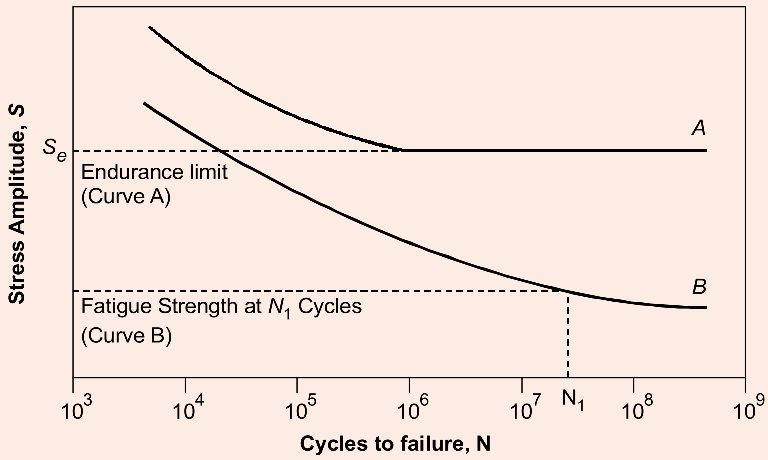


Fig 1. Typical S-N Curves

Endurance Limit : Certain materials have a fatigue limit or endurance limit which represents a stress level below which the material does not fail and can be cycled infinitely. If the applied stress level is below the endurance limit of the material, the structure is said to have an infinite life. This is characteristic of steel and titanium in benign environmental conditions. A typical S-N curve corresponding to this type of material is shown Curve A in Figure 1.

Many non-ferrous metals and alloys, such as aluminum, magnesium, and copper alloys,

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do not exhibit well-defined endurance limits. These materials instead display a continuously decreasing S-N response, similar to Cuve B in Figure 1. In such cases a fatigue strength Sf for a given number of cycles must be specified. An effective endurance limit for these materials is sometimes defined as the stress that causes failure at 1×10^8 or 5×10^8 loading cycles.

The concept of an endurance limit is used in infinite-life or safe stress designs. It is due to interstitial elements (such as carbon or nitrogen in iron) that pin dislocations, thus preventing the slip mechanism that leads to the formation of microcracks. Care must be taken when using an endurance limit in design applications because it can disappear due to:

- Periodic overloads (unpin dislocations)
- Corrosive environments (due to fatigue corrosion interaction)
- High temperatures (mobilize dislocations)

The endurance limit is not a true property of a material, since other significant influences such as surface finish cannot be entirely eliminated. However, a test values (S_e') obtained from polished specimens provide a baseline to which other factors can be applied. Influences that can affect the endurance limit include:

- Surface Finish
- Temperature
- Stress Concentration
- Notch Sensitivity
- Size
- Environment
- Reliability

Such influences are represented by reduction factors, k, which are used to establish a working endurance strength Se for the material:

$$S_e = kS_e'$$

Power Relationship : When plotted on a log-log scale, an S-N curve can be approximated by a straight line as shown in figure 2. A power law equation can then be used to define the S-N relationship:

$$N_1 = N_2\left(\frac{S_1}{S_2}\right)^{1/b}$$

where b is the slope of the line, sometimes referred to as the Basquin slope, which is given by :

$$b = \frac{-(\log S_1 - \log S_2)}{\log N_2 - \log N_1}$$

Given the Basquin slope and any coordinate pair (N,S) on the S-N curve, the power law equation calculates the cycles to failure for a known stress amplitude.

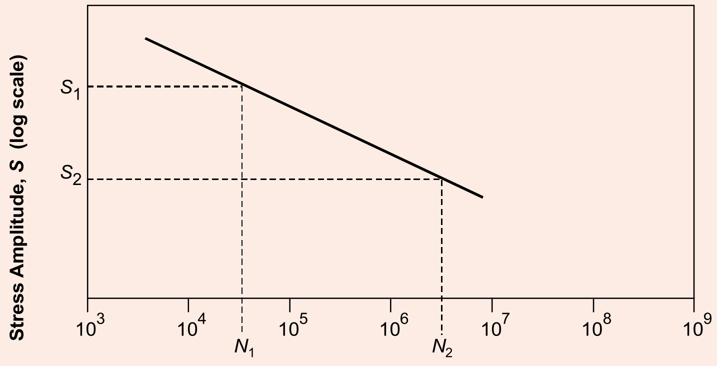


Fig 2. Idealized S-N Curve

The power relationship is only valid for fatigue lives that are on the design line. For ferrous metals this range is from 1×10^3 to 1×10^6 cycles. For non-ferrous metals, this range is from 1×10^3 to 5×10^8 cycles. Note the empirical relationships and equations described above are only estimates. Depending on the level of certainty required in the fatigue analysis, actual test data may be necessary.

End of Solution

Q.1 (e) A shaft made of mild steel is required to transmit 100 kW at 300 rpm. The supported length of the shaft is 3 meters. It carries two pulleys, each weighing 1500 N supported at a distance of 1 meter from the end respectively. Assuming the safe value of stress to be 60 N/mm^2, determine the diameter of the shaft.

[12 marks : 2021]

Solution:

Assumptions:

1. Given permissible stress is considered as permissible shear stress.
2. Oversafe design is considered. Hence, M .S.S.T. is used for design.
3. Weight of shaft is neglected.

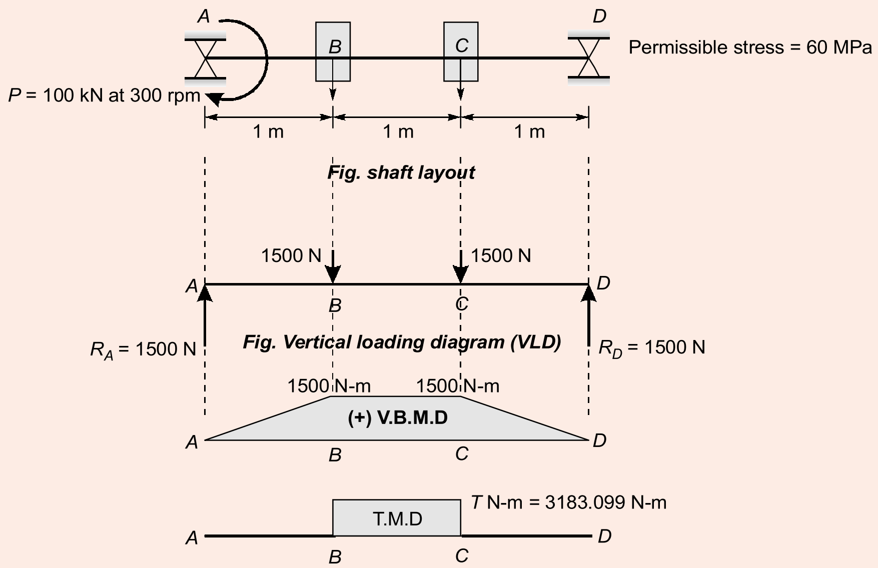


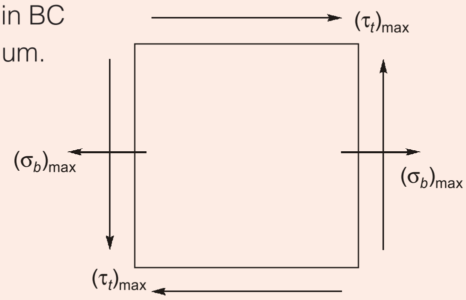
Fig. shaft layout

Fig. Vertical loading diagram (VLD)

1. T = Torque to be transmitted by the shaft

$$T = \frac{P \times 60}{2\pi N} \times 10^3 = \frac{100 \times 60}{2\pi(300)}(10)^3 = 3183.099 \text{ N-m}$$

2. From vertical bending moment diagram, maximum bending moment on the shaft (M) = 1500 N-m
3. Critical portion is the x-cross section of shaft in BC portion because both B.M and TM are maximum.
4. Diagram of shaft is determined by using M.S.S.T because shafts are made of ductile material and critical point on the critical cross-section is under combined stresses as shown in figure.



5. As per M.S.S.T

$$\text{Te} = \sqrt{M^2 + T^2} = \frac{\pi}{16} d^3 \tau_{per}$$

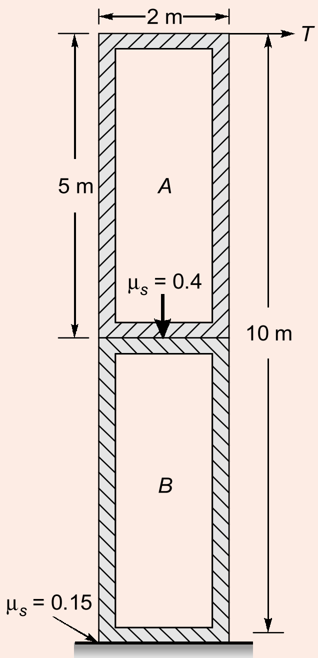
$$(10)^3 \sqrt{(1500)^2 + (3183.099)^2} = \frac{\pi}{16}(d^3)(60 \times 1)$$

$$d = 66.845 \text{ mm}$$

$\therefore$ Diameter of the shaft is 70 mm.

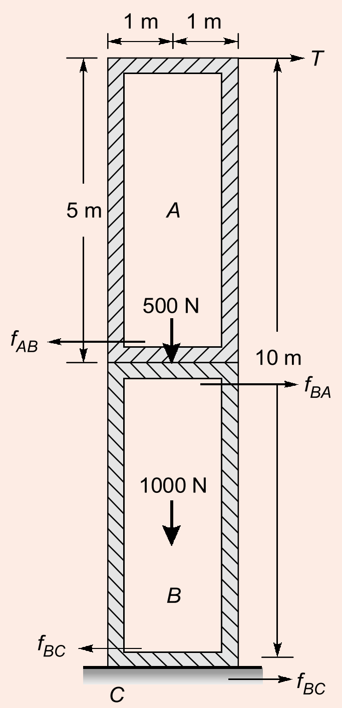
End of Solution

Q.2 (a) A 500 N crate *A* rests on a 1000 N crate *B*. The centres of gravity of the crates are at the geometric centres. The coefficients of static friction between contact surfaces are shown in the diagram. The force T is increased from zero. What is the first action to occur?



[20 marks : 2021]

Solution:



$$(f_{AB})_{max} = 0.4 \times 500 = 200 \text{ N}$$
$$(f_{BC})_{max} = 0.15 \times 1500 = 225 \text{ N}$$

$\Rightarrow$ Book A may slip off from right and before it slips

$$\Sigma \vec{M}_0 = 0$$
$$T \times 5 = 500 \times 1$$
$$T = \frac{500}{5} = 100 \text{ N}$$

So, till the value of T is 100 N on crate A, it will be in rest and but at 100 N, it will be at verge to tip off about A, so the first action will be tipping off crate A on crate B about O.

End of Solution

Q.2 (b) A single cylinder, single acting 4-stroke engine develops 20 kW at 250 rpm. For the sake of simplicity, the turning moment curve is represented by two isosceles triangles during compression and expansion stroke. The turning moment can be assumed zero for suction and exhaust stroke. The work done by the gases is 3 times more than the work done on the gases. If the flywheel has a mass of 1500 kg and has radius of gyration of 0.6 m, find the coefficient of fluctuation of speed

[20 marks : 2021]

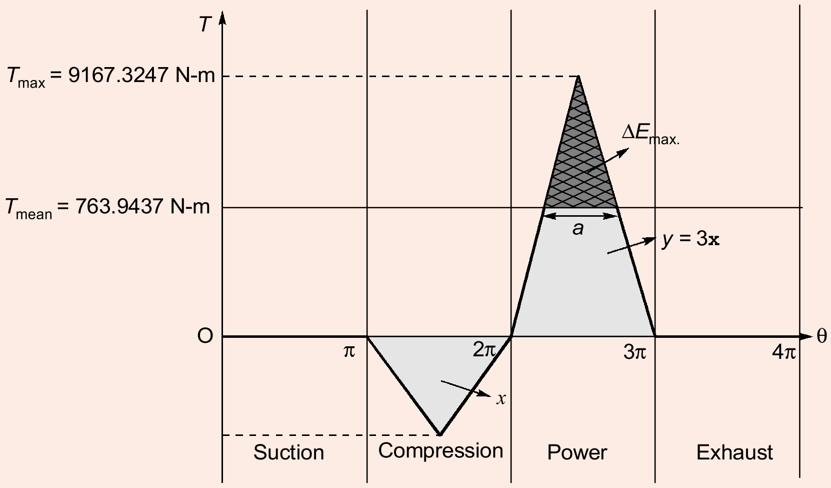
Solution:

$$P = 20 \text{ kW at } 250 \text{ rpm}$$
$$P = T_{mean} \times \omega$$

$$20 \times 10^3 = T_{mean} \times \frac{2\pi \times 250}{60}$$

$$\Rightarrow \quad T_{mean} = \frac{20 \times 10^3 \times 60}{2\pi \times 250}$$

$$T_{mean} = 763.9437 \text{ N-m}$$



$$T_{mean} = \frac{W_{cycle}}{4\pi}$$

$$W_{cycle} = T_{mean} \times 4\pi = 763.9437 \times 4\pi$$

$$W_{cycle} = 9600 \text{ Joules}$$

$$(3x - x) = W_{cycle} = 9600$$

$$2x = 9600$$

$$x = 4800 \text{ Joules}$$

$$3x = 14400 \text{ Joules} = \frac{T_{max} \times \pi}{2}$$

$$T_{max} = 9167.324722 \text{ N-m}$$

$$\frac{T_{max}}{\pi} = \frac{T_{max} - T_{mean}}{a}$$

$$a = \left(\frac{9167.3247 - 763.9437}{9167.3247}\right) \times \pi = 2.8797 \text{ rad}$$

$$\Delta E_{max} = a \times \frac{(T_{max} - T_{mean})}{2} = \frac{2.8797}{2} \times (9167.3247 - 763.9437)$$

$$= 12099.60813 \text{ Joules} = I\omega^2 C_s$$

$$12099.60813 = 1500 \times (0.6)^2 \times (26.1799)^2 \times C_s$$

$$\Rightarrow \quad C_s = 0.03269$$

End of Solution

Q.2 (c) A single-plate clutch is used to rotate a machine from a shaft rotating at a uniform speed of 300 rpm. Both sides of the clutch are effective, friction lining is 140 mm inner diameter and 220 mm outer diameter, respectively. The coefficient of friction between friction lining and flywheel surface is 0.28. Assuming uniform wear theory for clutch and maximum intensity of pressure 0.1 MPa, determine the time required to attain full speed by the machine if moment of inertia of the rotating parts is 7.2 kg-m^2. How much energy has been lost during slipping of the clutch?

[20 marks : 2021]

Solution:

Input Data

Type of clutch: Single plate clutch with both sides effective,

Inner diameter of clutch (D_i) = 140 mm

Outer diameter of clutch (D_o) = 220 mm

Coefficient of friction (μ) = 0.28

Maximum pressure (P_{max}) = 0.1 MPa

MOI of rotating parts of the machine (I) = 7.2 kg/m^2

Final speed $(N_2)_f$ = 300 rpm

Energy lost during slipping of clutch , (E) = ?

Type of theory to be used = Uniform wear theory (UWT)

n = Number of frictional contact surfces = 2

(i) T_f = Frictional torque transmitted by clutch by using UWT.

$$T_f = (n \mu \pi p_{per})(R_i)(R_o^2 - R_i^2)$$
$$= (2)(0.25)(\pi)(0.1)(70)(1102 - 70^2)$$
$$= 88704 \text{ N/mm} = 88.704 \text{ N-m}$$

(ii) $T_f = I\alpha$ where α = angular acceleration

$$\alpha = \frac{T_f}{I} = 12.32 \text{ rad/sec}^2$$

(iii) Time required to engage the clutch,

$$(\omega_2)_f = (\omega_2)_i + \alpha t$$
$$\frac{2\pi \times 300}{60} = 0 + \alpha t$$
$$t = 2.55 \text{ sec}$$

(iv) θ_1 = angle turned by driver shaft during 't' sec

$$\omega_1 t = (\omega_2)_f t$$
$$= \frac{2\pi \times 300}{60}(2.55) = 80.121 \text{ rad}$$

θ_2 = Angle turned by driven shaft during 't' sec

$$= \frac{1}{2}\alpha t^2 = 40.055 \text{ rad}$$

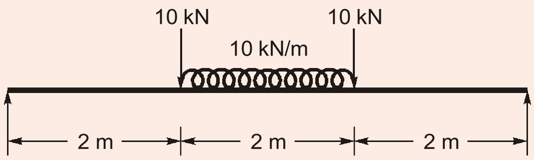
Energy lost, $E = (T_f)(\theta_1 - \theta_2) = 3554.01$ J

End of Solution

Q.3 (a) A beam is loaded as shown in the figure. If $E = 200 \times 10^6$ kN/m^2 and $I = 10 - 4$ m^4, determine

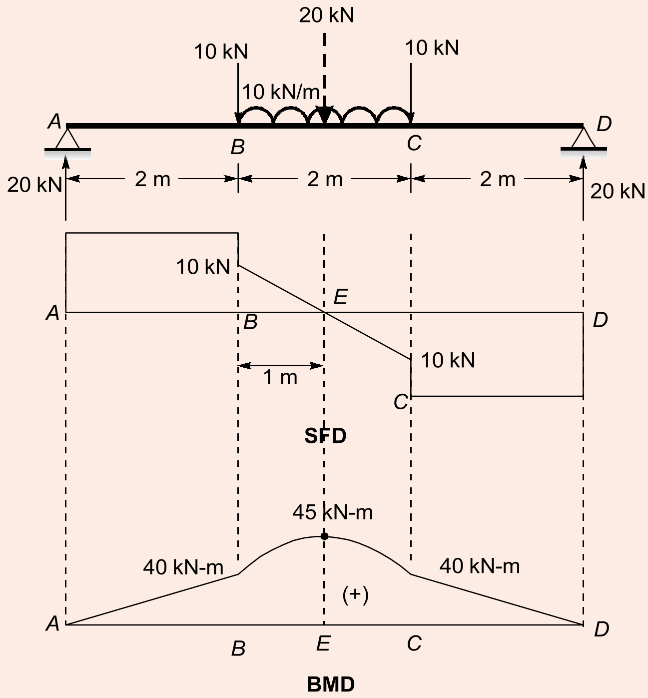
(i) slope at the ends, and

(ii) maximum deflection of the beam.



[20 marks : 2021]

Solution:



$$M_A = 0;\ M_B = 20 \times 2 = 40 \text{ kN-m}$$

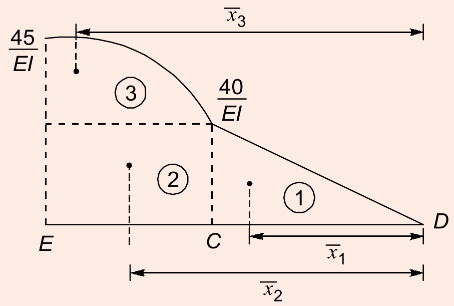
$$M_E - M_B = \frac{1}{2} \times 10 \times 1$$

$\Rightarrow$ $$M_E = 45 \text{ kN-m}$$

Moment area method is used to determine slope at supports and the maximum deflection.

Considering 'D' as origin (i.e. point of non-zero slope) and 'E' as reference point (i.e. point of zero slope)

by using I-theorem,



$\frac{M}{EI}$ diagram between E & D

$$\theta_D - \theta_E = \frac{1}{EI}(A)_{ED}$$

$$\theta_D - 0 = \frac{1}{EI}[A_1 + A_2 + A_3]$$

$$\theta_D = \frac{1}{EI}\left[\left(\frac{1}{2}\times 40\times 2\right) + (40\times 1) + \frac{2}{3}(1)(5)\right]$$

$$\theta_D = \frac{1}{EI}\left[40 + 40 + \frac{10}{3}\right] = \frac{250}{3EI}$$

$$\theta_D = \frac{250}{3\times 200\times 10^6\times 10^{-4}} = 0.00417 \text{ radians (ACW)}$$

$$\theta_D = \theta_D = 0.00417 \text{ radians (CW)}$$

by using II-theorem,

$$Y_D - Y_E = \frac{1}{EI}[A\bar{x}]_{ED} = \frac{1}{EI}[A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3]$$

$$0 - Y_{max} = \frac{1}{EI}\left[\left(40\times\frac{2}{3}\times 2\right) + (40\times 2.5) + \left(\frac{10}{3}\times\frac{21}{8}\times 1\right)\right]$$

$$Y_{max} = \frac{-1}{200\times 10^6\times 10^{-4}}\left[\frac{160}{3} + 100 + \frac{210}{24}\right]\times 10^3$$

$$Y_{max} = -8.1 \text{ mm}$$

$$Y_{max} = 8.1 \text{ mm } (\downarrow)$$

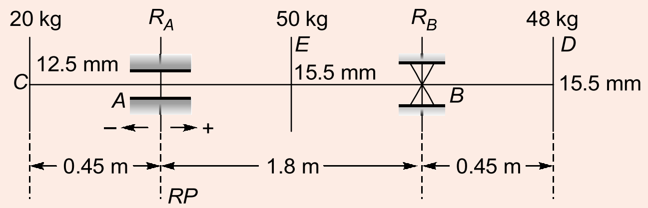
End of Solution

Q.3 (b) A shaft rotates in two bearings A and B, 1.8 m apart and projects 0.45 m beyond A and B. At the extreme ends of the shaft, two pulleys of masses 20 kg and 48 kg are attached with eccentricity of 12.5 mm and 15.5 mm respectively. In the middle of the bearings, another pulley of mass 50 kg is attached with an eccentricity of 15.5 mm.

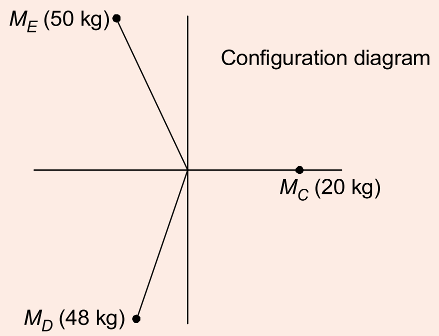
If the three pulleys have been arranged so as to obtain static balance, determine the dynamic forces produced on the bearings when shaft speed is 300 rpm.

[20 marks : 2021]

Solution:

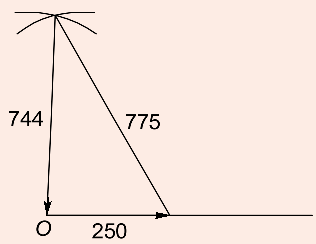


Planes	Mass (m) (kg)	*r* (mm)	*mr*	Distance from (RP) (*l*) (meter)	*mrL*
C	20	12.5	250	–0.45	–112.5
A	-	-	R_A/ω^2	0	0
E	50	15.5	775	0.9	697.5
B	-	-	R_B/ω^2	1.8	$1.8 \times R_B/\omega^2$
D	48	15.5	744	2.25	1674

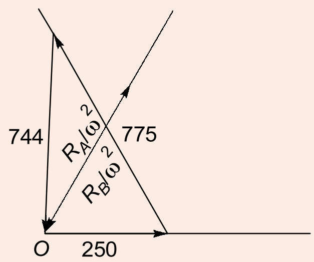


3 Pulley C, E, D are in static balance

Force polygon of 3 pulleys only:
Scale: 100 mm ≡ 1 cm

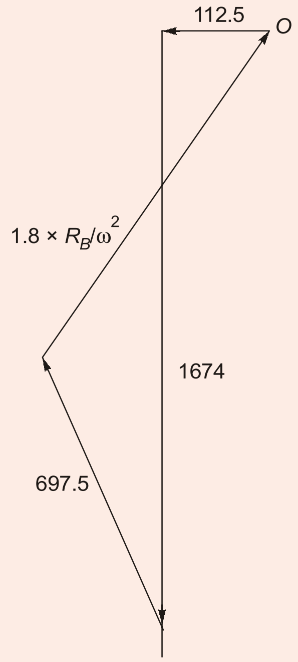


Force polygon of complete system:
Scale: 100 units ≡ 1 cm



Moment polygon complete system

Scale: 100 units = 1 cm



$$1.8 \times \frac{R_B}{\omega^2} = 10.85 \text{ cm} = 10.85 \times 100 \text{ units} = 1085$$

$$\frac{R_B}{\omega^2} = \frac{1085}{1.8} = 602.777 \text{ kg–mm}$$

$$\frac{R_B}{\omega^2} = \frac{602.777}{1000} \text{ kg-m}$$

$$R_B = \frac{602.777}{1000} \times \omega^2$$

$$= \frac{602.777}{1000} \times \left(\frac{2\pi \times 300}{60}\right)^2 \quad [N = 300 \text{ rpm}]$$

$$R_B = 594.917 \text{ N (Bearing reaction)}$$

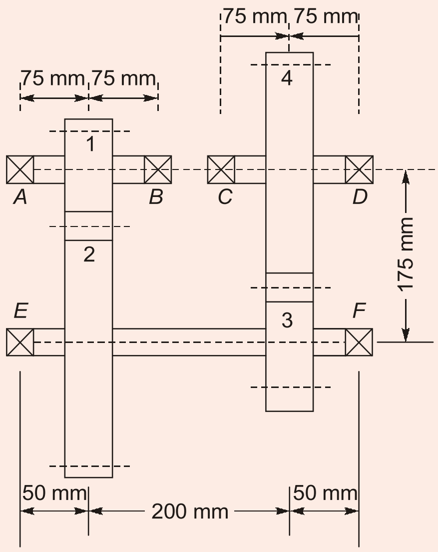
Now,
$$\frac{R_A}{\omega^2} = \frac{R_B}{\omega^2}$$

$\Rightarrow$
$$R_A = R_B$$

$$R_A = 594.917 \text{ N (Bearing reaction)}$$

End of Solution

Q.3 (c) The layout of a two-stage gear box is shown in the figure.



The number of teeth on the gears are as follows:

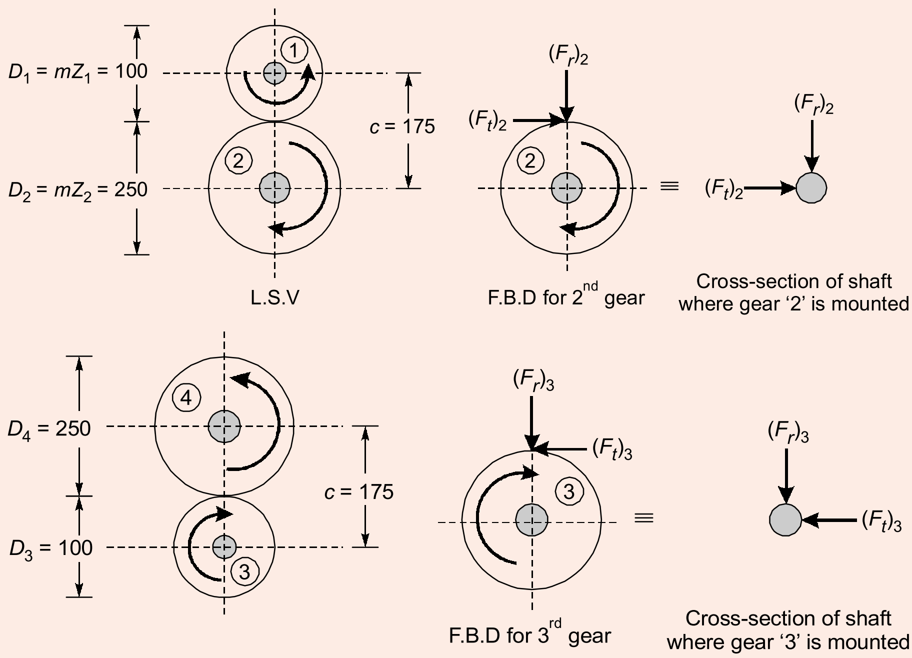
$z_1 = 20$, $z_2 = 50$, $z_3 = 20$, $z_4 = 50$

Pinion 1 rotates at 1440 rpm in the anti-clockwise direction when observed from the left side and transmits 10 kW power to the gear train. The pressure angle is 20°. Draw a free body diagram of the gear tooth forces and determine the reactions at bearings E and F.

[20 marks : 2021]

Solution:

1. Free body diagrams for the gear drives:



2. $$C = \frac{m}{2}(Z_1 + Z_2) = \frac{m}{2}(Z_3 + Z_4)$$

$$m = 5 \text{ mm}$$

3. $$T_1 = \frac{P \times 60}{2\pi N_1} \times 10^6 = \frac{10 \times 60 \times 10^6}{2\pi \times (1440)} = 66314.56 \text{ N-mm}$$

$$(F_t)_1 = (F_t)_2 = \frac{2T_1}{D_1} \text{ or } \frac{2T_2}{D_2} = 1326.291 \text{ N}$$

$$(F_r)_1 = (F_r)_2 = (F_t)_1 \tan\phi = 482.73 \text{ N}$$

4. $$\frac{T_2}{T_1} = \frac{Z_2}{Z_1} \Rightarrow T_2 = 165786.4 \text{ N-mm}$$

5. $$T_3 = T_2 = 165786.4 \text{ N-m} \quad (\because \; P_3 = P_2 \text{ and } N_3 = N_2)$$

6. $$(F_t)_3 = (F_t)_4 = \frac{2T_3}{D_3} \text{ or } \frac{2T_4}{D_4} = 3315.728 \text{ N}$$

$$(F_r)_3 = (F_r)_4 = (F_t)_3 \tan\phi = 1206.826 \text{ N}$$

7. Vertical and horizontal loading diagrams for the shaft is shown in figure.

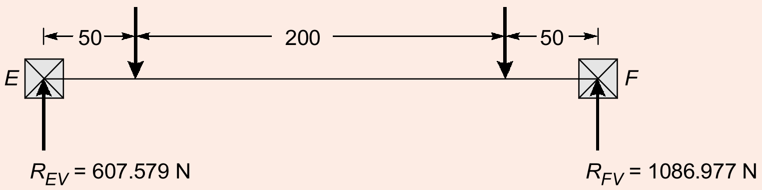


Fig : VLD

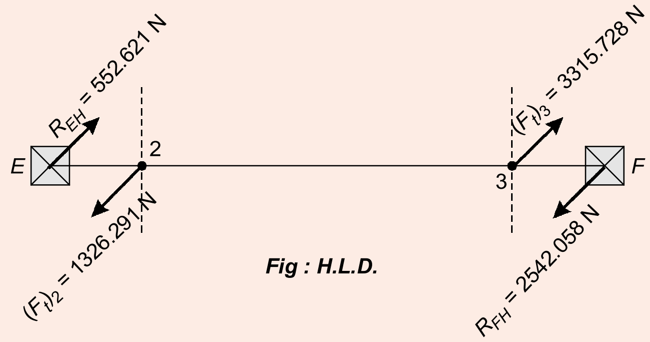
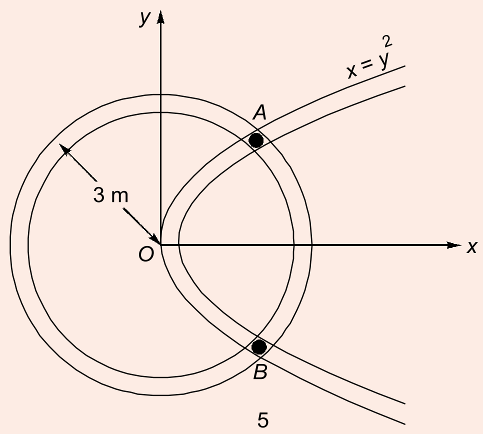


Fig : H.L.D.

Reactions	V(N)	H(N)	R(N)
Bearing	Vertical	Horizontal	Resultant
E	607.579(↑)	552.621(↗)	821.305
F	1086.977(↑)	2542.058(↙)	2764.701

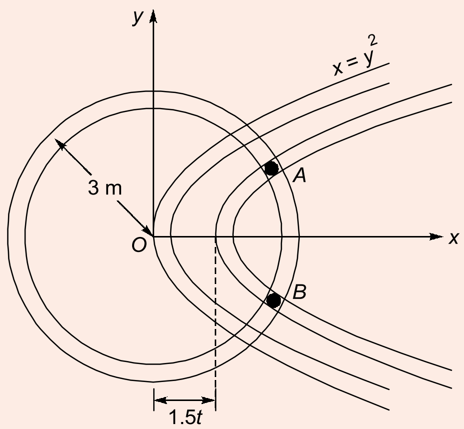
End of Solution

Q.4 (a) Particles A and B are confined to always be in a circular groove of radius 3 m. At the same time, these particles must also be in a slot that has the shape of a parabola. The slot is shown at time $t = 0$ with equation $x = y^2$. If the slot moves to the right at a constant speed of 1.5 m/s, what are the speed and rate of change of speed of the particles towards each other at $t = 1$ s?



[20 marks : 2021]

Solution:



$$x^2 + y^2 = 9 \qquad \text{...(1)}$$

at $t = 1$ sec

Parabolic equation, $x - 1.5 = y^2$ ...(2)

By (1) and (2),

$$x^2 + x - 1.5 = 9$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 + 4 \times 10.5}}{2} = \frac{-1 + \sqrt{43}}{2} = 2.778 \text{ m} \simeq 2.78 \text{ m}$$

$$\Rightarrow \quad y = \pm\sqrt{2.78 - 1.5} = \pm 1.13 \text{m}$$

Also, at time 't'

Parabolic equation, $x - 1.5t = y^2$ ...(3)

(As parabola shift right with velocity 1.5 m/s)

Differentiating equation (1) w.r.t. time

$$\Rightarrow \qquad x\cdot\frac{dx}{dt}+y\cdot\frac{dy}{dt} = 0$$

$$\Rightarrow \qquad x\cdot u + y\cdot v = 0 \qquad \text{...(4)}$$

$$\left[\frac{dx}{dt}=u, \frac{dy}{dt}=v\right]$$

$$\Rightarrow \qquad 2.78u + 1.13v = 0 \qquad \text{...(4')}$$

Differentiating equation (3), w.r.t. time

$$\frac{dx}{dt}-1.5 = 2y\cdot\frac{dy}{dt}$$

$$\Rightarrow \qquad u-1.5 = 2y\cdot v \qquad \text{...(5)}$$

$$u-1.5 = 2\times 1.13v \qquad \text{...(5')}$$

Solving (4') and (5')

$$\Rightarrow \qquad u = 0.22865 \text{ m/s} \simeq 0.23 \text{ m/s}$$

$$v = -0.56 \text{ m/s}$$

$$\text{Speed} = \sqrt{(0.23)^2+(-0.56)^2} = 0.605 \text{ m/s}$$

Differentiating equation (4) w.r.t. time

$$\Rightarrow \qquad u^2 + x\cdot a_x + v^2 + y\cdot a_y = 0$$

$$\Rightarrow \qquad a_x = -\frac{1}{x}\left(y\cdot a_y + v^2 + u^2\right)$$

differentiating equation (5) w.r.t. time

$$a_x = 2v^2 + 2ya_y$$

$$-\frac{1}{x}\left(y\cdot a_y + v^2 + u^2\right) = 2v^2 + 2ya_y$$

$$a_y = -\frac{\left(2xv^2+v^2+u^2\right)}{y(1+2x)}$$

at t = 1 sec,

$$a_y = -\frac{\left(2\times 2.78\times(-0.56)^2+(-0.56)^2+(0.23)^2\right)}{1.13(1+2\times 2.78)} = -0.284 \text{ m/s}^2$$

$$a_x = -0.01464 \text{ m/s}^2$$

Rate of change of speed of particles towards each other = $|2a_y|$

$$= 2\times 0.284 = 0.568 \text{ m/s}^2$$

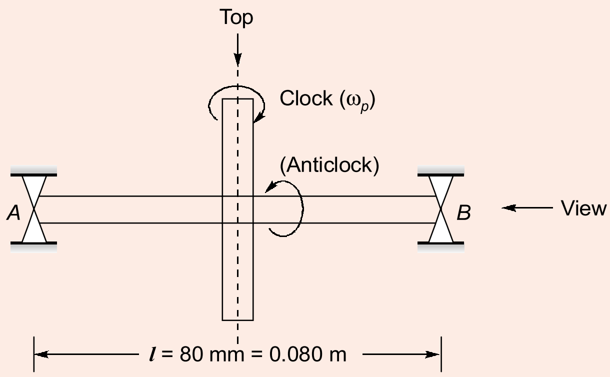
End of Solution

Q.4 (b) A uniform disc of radius of gyration 60 mm and a mass of 4 kg is mounted centrally on a horizontal axle of 80 mm length between the bearings. It spins at 800 rpm CCW when viewed from the right side bearing. The axle precesses about the vertical axis at 50 rpm in the clockwise direction when viewed from the top. Determine the reaction at each bearing due to the mass and gyroscopic effect.

[20 marks : 2021]

Solution:

Given: $k = 60$ mm $= 0.060$ m; $m = 4$ kg; $I = mk^2 = 4 \times (0.060)^2$; $I = 0.0144$ kg-m^2



Disc speed $\quad N = 800$ rpm

$$\omega = \frac{2\pi \times 800}{60} = 83.7758 \text{ rad/s}$$

Axle precision $\quad N_p = 50$ rpm

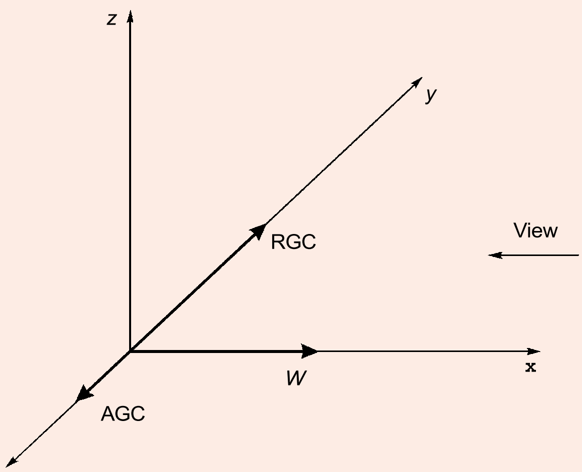
$$\omega_p = \frac{2\pi \times 50}{60} = 5.23598 \text{ rad/s}$$

Gyroscopic couple $\quad C = I\omega\omega_p = 0.0144 \times 83.7758 \times 5.23598$

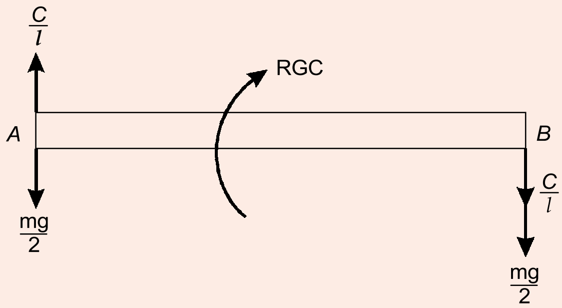
$$C = 6.3165 \text{ N-m}$$

Gyroscopic force $\quad \frac{C}{l} = \frac{6.3165}{0.080} = 78.9567 \text{ N}$

$$\frac{mg}{2} = \frac{4 \times 9.81}{2} = 19.62 \text{ N}$$



Forces by the shaft:



$$F_A = \left(\frac{C}{l} - \frac{mg}{2}\right) \quad \text{(upward)}$$

$$F_B = \left(\frac{C}{l} + \frac{mg}{2}\right) \quad \text{(downward)}$$

Bearing Reaction:

$$R_A = \left(\frac{C}{l} - \frac{mg}{2}\right) \text{(downward)} = 78.9562 - 19.62$$

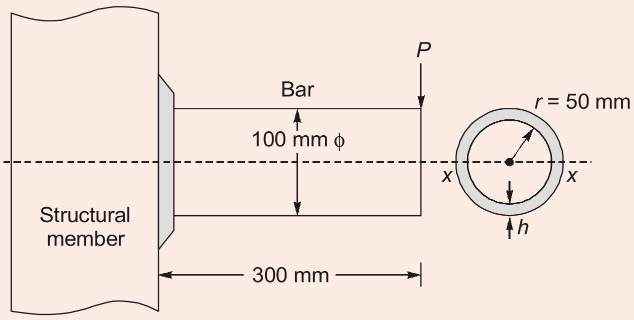
$$= 59.3362 \text{ N} \quad \text{(downward)}$$

$$R_B = \left(\frac{C}{l} + \frac{mg}{2}\right) \text{(upward)} = 78.9562 + 19.62$$

$$= 98.5767 \text{ N (upward)}$$

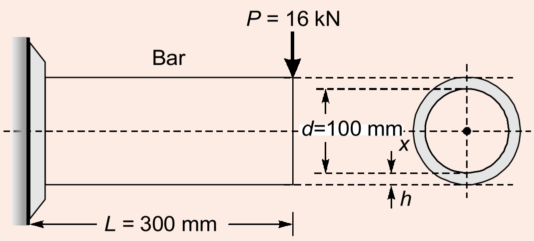
End of Solution

Q.4 (c) A solid circular bar of 100 mm diameter is welded to a structural member, as shown in the figure, by a fillet weld all round the bar. Determine the leg dimension of the fillet weld if P =1 6 kN and permissible shear stress in weld in 90 N/mm^2.



[20 marks : 2021]

Solution:



Given: Permissible shear stress for weld material = 90 MPa

(i) Circular fillet weld weld is subjected to eccentric load. The moment due to eccentric load is acting in a plane perpendicular to plane of welds. Hence, shear stress and bending stresses are developed at critical point on the weld.

(ii) To determine the size of weld (i.e. leg of weld) MSST is used because of combined streses.

(iii) $$T_S = \frac{P}{A_W} = \frac{P}{(h = 0.707t)(L_e = \pi d)} = \frac{16000}{(0.707t)(\pi)(100)}$$

$$T_S = \frac{72.04}{t}\text{ MPa}$$

(iv) $$(\sigma_b)_{max} = \frac{M = PL}{Z_W = \frac{\pi d^2}{4}(h = 0.707t)} = \frac{4PL}{\pi(100)^2 \times (0.707t)}$$

$$(\sigma_b)_{max} = \frac{4 \times 16000 \times 300}{\pi \times 100^2 \times 0.707t} = \frac{864.434}{t}\text{ MPa}$$

(v) Size of the weld (t) by using MSST, $\tau_{max} \le \tau_{per}$

$$\sqrt{\sigma_x^2 + 4\tau_{xy}^2} \le \tau_{per}$$

$$\sqrt{(\sigma_b)_{max}^2 + 4\tau_s^2} \le 90$$

$$\sqrt{\left(\frac{864.434}{t}\right)^2 + 4\left(\frac{72.04}{t}\right)^2} \le 90$$

$$\Rightarrow \quad t \ge 9.737 \text{ mm}$$

Leg of weld = 10 mm ...Ans

End of Solution

Section-B

Q.5 (a) A slab of aluminium of dimensions 25 cm × 20 cm × 5 cm is to be cast along with a side cylindrical riser. The riser is not insulated on any surface. If the volume shrinkage of aluminium during solidification is 5% find

(i) the relationship between diameter and height of cylindrical riser for longest solidification time.

(ii) the minimum volume of riser required to compensate the shrinkage volume of casting. (Assume volume of riser = 3 × Shrinkage volume of casting)

[12 marks : 2021]

Solution:

Given, casting dimensions: 25 cm × 20 cm × 5 cm.

Volume shrinkage = 5%

(i) For side cylindrical riser,

$$\text{Surface area, } A = 2\times\frac{\pi}{4}D^2 + \pi Dh$$

and

$$\text{Volume, } V = \frac{\pi}{4}D^2 h \Rightarrow h = \frac{4V}{\pi D^2}$$

$\therefore$

$$A = 2\times\frac{\pi}{4}D^2 + \pi D\left(\frac{4V}{\pi D^2}\right)$$

$$A = \frac{\pi}{2}D^2 + \left(\frac{4V}{D}\right)$$

Since longest solidification time will be for minimum surface area, and for minimum surface area,

$$\frac{dA}{dD} = 0$$

$\Rightarrow$

$$\pi D - \frac{4V}{D^2} = 0$$

$\Rightarrow$

$$V = \frac{\pi D^3}{4} \quad \text{[For longest solidification time.]}$$

Also,

$$V = \frac{\pi}{4}D^2 h = \frac{\pi D^3}{4}$$

$$h = D$$

(ii) Minimum volume of riser = 3 × Percentage shrinkage volume of casting

$$= 3\times\left(\frac{5}{100}\times 25\times 20\times 5\right) = 375 \text{ cm}^3$$

End of Solution

Q.5 (b) To minimize the total processing time, arrange the sequence of operations of the jobs on the following five machines using Johnson's rule. The processing times (in minutes) of different jobs on individual machines are given below. Also find the total idle time on each machine.

Jobs	MC 1	MC 2	MC 3	MC 4	MC 5
A	10	8	5	6	12
B	9	9	7	8	13
C	8	7	8	9	11
D	11	6	6	5	9

[12 marks : 2021]

Solution:

Johnson's algorithm:

We need to convert these 5 original machines into 2 hypothetical machines before Johnson's rule can be applied but before that we need to check two conditions out of those atleast if one is satisfied then only we can convert.

1. Minimum Machine 1 ≥ Maximum of [M2, M3, M4]

OR

2. Minimum Machine 5 ≥ Maximum of [M2, M3, M4]

Now here 2nd condition is satisfied.

Minimum of M5 is (9), and maximum of M2, M3 and M4 is (9)

Now, we will convert into 2 machine let X and Y

X = M1 + M2 + M3 + M4

Y = M2 + M3 + M4 + M5

Jobs	X	Y
A	29	31
B	33	37
C	32	35
D	28	26

Sequence will be A → C → B → D

Jobs	Machine 1		Machine 2		Machine 3		Machine 4		Machine 5	
	In	Out	In	Out	In	Out	In	Out	In	Out
A	0	10	10	18	18	23	23	29	29	41
B	10	18	18	25	25	33	33	42	42	53
C	18	27	27	36	36	43	43	51	53	66
D	27	38	38	44	44	50	51	56	66	75

MST = 75 min.

Idle time = MST – Working time of machine

Working time of machine 1 = 38 min

Machine 2 = 30 min

Machine 3 = 26 min

Machine 4 = 28 min

Machine 5 = 45 min

Idle time – Machine 1 = 75 – 38 = 37 min

Machine 2 = 75 – 30 = 45 min

Machine 3 = 75 – 26 = 49 min

Machine 4 = 75 – 28 = 47 min

Machine 5 = 75 – 45 = 30 min

End of Solution

Q.5 (c) What are the basic types of corrosion? Briefly describe any one of them.
[12 marks : 2021]

Solution:

Corrosion is defined as deterioration or destruction of material due to its interaction with environment or in other words it is reverse of metallurgy. In metallurgy, metals are extracted from their ores (oxides or hydrides of the metals) and refined and then converts into slabs, tubes or sheets etc. The material then starts transforming into again oxides or hydrides due to interaction of the environment, therefore, in the material world nothing is permanent. Corrosion is classified in number of ways, based on temperature such as low temperature corrosion, high temperature corrosion, and also like dry corrosion and wet corrosion but most of the time based on the *appearance* of the corrosive products and surface of the material it has classified in to eight forms.

- ***Uniform attack***
- ***Galvanic or Two-Metal Corrosion***
- ***Crevice Corrosion***
- ***Filliform Corrosion***
- ***Pitting***
- ***Intergranular Corrosion***
- ***Erosion Corrosion***
- ***Stress corrosion***

Crevice Corrosion : It is highly localized corrosion generally appears in shielded areas like gasket surfaces, lap joints, under bolts and rivet heads where small amount of stagnant solutions (like dust), surface deposits, are settled, then, these are the means to create an acidic atmosphere to automatically propagate the corrosion into thickness. Furthermore, the resulted corrosive productives are prone to improve the acidic nature more and more. It is also called as deposit or gasket carrsion.

Crevice corrosion under the nuts due to the gaskets

Prevention:

1. Use but joint instead of lap-joints.
2. Clean thoroughly to remove stagnant.
3. Avoid sharp corners while designing.
4. Use non-absorbable solid gaskets such as Teflon

End of Solution

Q.5 (d) For an electronic components, the failure density function is defined as

$$f(t) = \begin{cases} 0.002e^{-0.002t} & t \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

Determine the reliability of the component at 346.6 hours. Also determine the MTTF of the same component.

[12 marks : 2021]

Solution:

Failure density function, $f(t) = \begin{cases} 0.002e^{-0.002t}, & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Now,

Cumulative density function $F(t) = \int_0^t 0.002e^{-0.002t}\,dt$

$$= 1 - e^{-0.002t}$$

So, Reliability, $R(t) = 1 - F(t)$

$$R(t) = 1 - (1 - e^{-0.002t})$$

$$R(t) = e^{-0.002t}$$

Now, at $t = 346.6$ hours

$$R(346.6) = e^{-0.002 \times 346.6}$$

$\Rightarrow$ $R(346.6) \simeq 0.5$

Now, $\text{MTTF} = \int_0^\infty R(t)\,dt$

$$= \int_0^\infty e^{-0.002t}\,dt = \left.\frac{e^{-0.002t}}{-0.002}\right|_0^\infty$$

$$= \frac{1}{-0.002}(0-1)$$

$\Rightarrow$ $\text{MTTF} = \dfrac{1}{0.002} = 500$

End of Solution

Q.5 (e) What kind of the motors are used in paper-feed and print-head-positioning motors in printers and plotters? Justify your answers with reason. Show a schematic diagram of a two-phase permanent magnet stepper motor.

[12 marks : 2021]

Solution:

A stepper motor is generally used in printhead positioning motor in a printer and plotter as stepper motor is a pulse-driven motor which changes the angular position of shaft in steps (precise steps) i.e. a stepper motor move in precise repeatable steps which helps in positioning of pointer or any object precisely and by counting the number of steps, we can easily identify the position of plotter and print head which makes multi-dimensional printing or 3D printing precise and accurate.

The step angle i.e. angle turned during each step can be measured by using following formula,

Where,

- α is the step angle
- m_s is the number of stator phases
- N_r is the number of rotor teeth

$$\alpha = \frac{360°}{m_s N_r}$$

Permanent magnet (PM) stepper motor

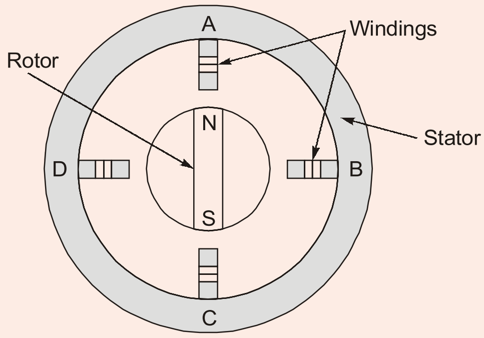
In this type of motor, the rotor is a permanent magnet. Unlike the other stepping motors, the PM motor rotor has no teeth and is designed to be magnetized at a right angle to its axis.

Figure shows a simple, 90° PM motor with four phases (A-D).

Applying current to each phase in sequence will cause the rotor to rotate by adjusting to the changing magnetic fields.

Although it operates at fairly low speed, the PM motor has a relatively high torque characteristic.

These are low cost motors with typical step angle ranging between 7.5° to 15°.



End of Solution

Q.6 (a) During an orthogonal cutting of a metal, the following data were observed:

Chip thickness = 0.5 mm; Width of cut = 3 mm;
Depth of cut = 0.3 mm; Feed rate = 0.3 mm/rev;
Cutting force = 1200 N; Feed thrust force = 300 N;
Cutting speed = 3 m/sec; Rake angle = 10°

(i) Shear -force on the shear plane,
(ii) Coefficient of friction on chip-tool interface and friction angle, and
(iii) Percentage of total energy that goes into overcoming friction at chip-tool interface.

[20 marks : 2021]

Solution:

Given: t_c = 0.5, F_c = 1200 N, w = 3 mm, F_T = 300 N, t = 0.3 mm, V = 3 m/s, $r = \frac{t}{t_c} = \frac{0.3}{0.5} = 0.6$, $\alpha = 10°$

(i) F_s = ?

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} = \frac{0.6\cos 10}{1 - 0.6\sin 10}$$

$$\phi = 33.409°$$

$$F_s = F_c\cos\phi - F_T\sin\phi = 836.53\text{N}$$

(ii)

$$\mu = \tan\beta = \frac{\frac{F_T}{F_c} + \tan\alpha}{1 - \frac{F_T}{F_c}\tan\alpha}$$

$$\beta = 24.036°$$

(iii)

$$F = F_T\cos\alpha + F_c\sin\alpha = 503.82\text{ N}$$

$$\frac{V}{\cos(\phi - \alpha)} = \frac{V_c}{\sin\phi}$$

$$V_c = 1.799\text{ m/s}$$

or

$$\frac{V_c}{V} = \frac{t}{t_c} = r$$

$\Rightarrow$

$$V_c = 3 \times 0.6 = 1.8\text{ m/s}$$

Percentage loss of energy to overcome friction $= \frac{FV_c}{F_C \times V} \times 100\% = 25.19\%$

End of Solution

Q.6 (b) (i) What are the different structures of ceramic compounds? Explain with neat sketches and examples.

(ii) The lattice constant of a metal with cubic lattice is 2.88Å. The density of the metal is 7200 kg m^{-3}. Calculate the number of unit cells present in 1 kg of the metal.

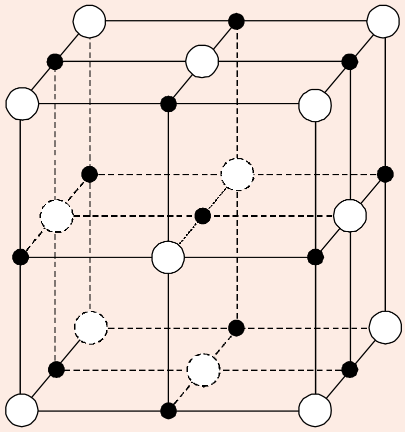
[12+8 marks : 2021]

Solution:

(i)

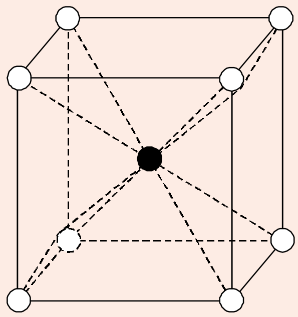
Structures of ceramic compounds

- AX-Type Crystal Structures of Ceramics : If there are equal numbers of cations and anions in structure, then they are often referred to as AX compounds, where A denotes the cation and X the anion. Perhaps the most common AX crystal structure is the **sodium chloride (NaCl)**, or rock salt, type. The coordination number for both cations and anions is 6. A unit cell for this crystal structure is generated from an FCC arrangement of anions with one cation situated at the cube center and one at the center of each of the 12 cube edges.



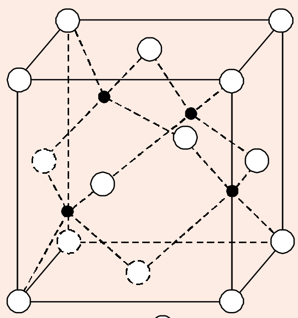
A unit cell for the rock salt, or sodium chloride (NaCl), crystal structure

- Cesium Chloride Structure : Figure shows a unit cell for the cesium chloride (CsCl) crystal structure the coordination number is 8 for both ion types. The anions are located at each of the corners of a cube, whereas the cube center is a single cation. Interchange of anions with cations, and vice versa, produces the same crystal structure. This is not a BCC crystal structure because two different kinds of ions are involved.



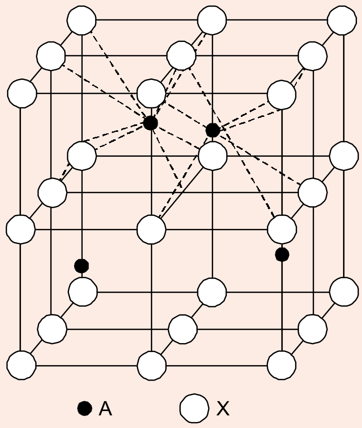
A unit cell for the cesium chloride (CsCl) crystal structure

- Zinc Blende Structure : The coordination number is 4 and all ions are tetrahedrally coordinated.



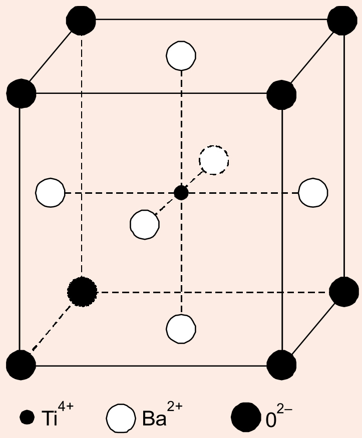
A unit cell for the zinc blende (ZnS) crystal structure

- A_mX_P - Type Crystal Structures : If the charges on the cations and anions are not the same, compound ions exist with the chemical formula A_mX_p and such structure is found in fluorite (CaF_2). Calcium ions are positioned at the centers of cubes, with fluorine ions at the corners.



A unit cell for the fluorite (A_mX_p) crystal structure

- $A_mB_nX_p$ - Type Crystal Structures



A unit cell for the $A_mB_nX_p$ type crystal structure

It is also possible for ceramic compounds to have more than one type of cation; for two types of cations (represented by A and B), their chemical formula may be designated as $A_mB_nX_p$. Barium titanate ($BaTiO_3$), having both Ba^{2+} and Ti^{4+} cations, falls into this classification.

(ii)

$$a = 2.88\text{Å} = 2.88 \times 10^{-8} \text{ cm} \quad 1 \text{ kg}$$
$$\rho = 7200 \text{ kg/m}^3 = 7.2 \text{ g/cc}$$

Number of unit cells,

$$\text{Density } (g/cc) = \frac{g/\text{unit cell}}{\text{Volume of unit cell}}$$

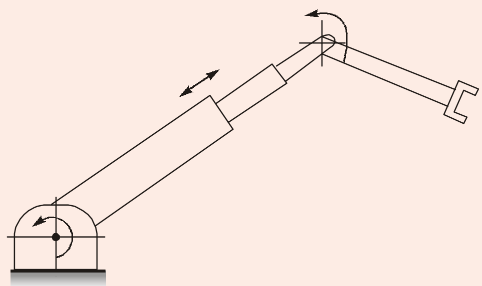
$$7.2 = \frac{g/\text{unit cell}}{(2.88\times10^{-8})^3}$$

$$g/\text{unit cell} = 7.2\times(2.88\times10^{-8})^3$$

$$\text{Number of unit cells/kg} = \frac{1000}{7.2\times(2.88\times10^{-8})^3} = 5.814\times10^{24}$$

End of Solution

Q.6 (c) Consider the three-link planar manipulator of the figure given below. Derive the forward kinematic equations using the D-H Algorithm. Assume suitable link and joint parameters.

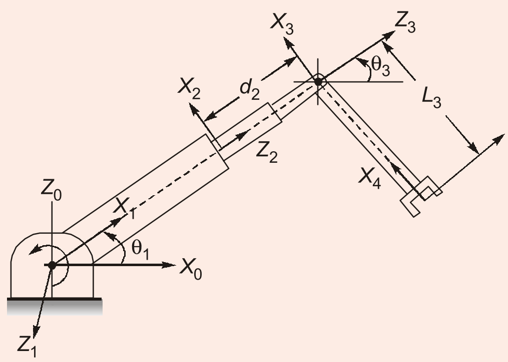


Where,

Link length = a_i ; Link twist = α_i ; Joint angle = θ_i ; Joint displacement = d_i

[20 marks : 2021]

Solution:



For given RPR-Planar robot

DH - Parameters are

Link	a_i	α_i	d_i	θ_i
1	0	+90°	0	θ_1
2	0	–90°	α_2	0
3	L_3	0	0	θ_3

$$_{i}^{i-1}T = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i - S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,
$$ {}^0_1T = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So, forward kinematics solution will be

$$ {}^0_3T = {}^0_1T\,{}^1_2T\,{}^2_3T $$

Let $\cos\theta_i = C_i$

$\sin\theta_i = S_i$

$$ {}^0_3T = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = \left[\begin{array}{ccc:c} C_1C_3 - S_1S_3 & -C_1S_3 - S_1C_3 & 0 & L_3C_1C_3 - L_3S_1S_3 + d_2S_1 \\ S_1C_3 + C_1S_3 & -S_1S_3 + C_1C_3 & 0 & L_3C_3S_1 + L_3S_3C_1 + d_2C_1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] $$

End of Solution

Q.7 (a) The production rates (units/day) of the five workers on five different machine are given in the following table. To maximize the total production, assign the workers on specific machines using Hungarian method. Also find the total production per day.

Production Units per day

		MC 1	MC 2	MC 3	MC 4	MC 5
Workers	A	24	37	18	34	34
	B	18	37	20	31	42
	C	26	25	14	37	42
	E	22	25	26	40	50
		24	34	30	37	46

[20 marks : 2021]

Solution:

As the problem is given for maximization, we need to convert first of all into minimization problem as Hungarian method can only be applied to minimum.

24	37	18	34	34
18	37	20	31	42
26	25	14	37	42
24	25	26	40	50
24	34	30	37	46

Subtracting from the highest element all the elements of matrix. (i.e. 50 - Whole matrix)

26	13	32	16	16
32	13	30	19	8
24	25	36	13	8
28	25	24	10	0
26	16	20	13	4

⇒

13	0	19	3	3
24	5	22	11	0
16	17	28	5	0
28	25	24	10	0
22	12	16	9	0

$\boxed{0}$	$\xcancel{0}$	3	$\xcancel{0}$	3
11	5	6	8	$\boxed{0}$
3	17	12	2	$\xcancel{0}$
15	25	8	7	$\xcancel{0}$
9	12	$\boxed{0}$	6	$\xcancel{0}$

As current solution is not optimum, so performing optimality.

$\boxed{0}$	$\xcancel{0}$	3	$\xcancel{0}$	5
9	3	4	6	$\boxed{0}$
1	15	10	$\boxed{0}$	$\xcancel{0}$
13	23	6	5	$\xcancel{0}$
9	12	$\boxed{0}$	6	2

⇓

$\boxed{0}$	$\xcancel{0}$	3	$\xcancel{0}$	8
6	$\boxed{0}$	1	3	$\xcancel{0}$
1	15	10	$\boxed{0}$	3
10	20	3	2	$\boxed{0}$
9	12	$\boxed{0}$	6	5

As the number of allocations are exactly equal to size of matrix. So the current solution is optimal.

$24 + 37 + 37 + 50 + 30 = 178$

Total production per day will be 178 units.

End of Solution

Q.7 (b) In a roll forming process, a 20 mm thick plate is rolled to 16 mm in a four high mill. Determine the coefficient of friction
considering it as maximum possible reduction. The diameter of the roll is 480 mm. Also find neutral section, backward and forward slips and maximum pressure. It is given that the value of σ_o = 120 N/m^2 for hot rolls of mild steel at about 1100°C.

[20 marks : 2021]

Solution:

Given: $h_0 = 20$ mm, $h_f = 16$ mm, $D = 480$ mm, $\sigma_0 = 120$ N/mm^2

Maximum possible Reduction,

$$\Delta h_{max} = h_0 - h_f = 20 - 16 = 4 \text{ mm}$$

(i)
$$(\Delta h_{max}) = \mu^2 R$$

$$\mu = \sqrt{\frac{\Delta h_{max}}{R}} = \sqrt{\frac{4}{240}} = 0.1291$$

$$\mu = 0.1291$$

Now,
$$\cos\alpha = 1 - \frac{\Delta h}{D} = 1 - \frac{4}{480}$$

$$\alpha = 7.4316°$$

$$\alpha = 0.1297 \text{ rad}$$

$$H_0 = 2\sqrt{\frac{R}{h_f}}\cdot\tan^{-1}\left(\frac{R}{h_f}\cdot\alpha\right) \qquad [\alpha \text{ is in radians}]$$

$$= 2\sqrt{\frac{240}{16}}\cdot\tan^{-1}\left(\frac{240}{16}\times 0.1297\right)$$

$$= 8.4896 \text{ (Constant)}$$

$$H_n = \frac{1}{2}\left[H_0 - \frac{1}{\mu}\ln\left(\frac{h_0}{h_f}\right)\right]$$

$$= \frac{1}{2}\left[8.4896 - \frac{1}{0.1291}\ln\left(\frac{20}{16}\right)\right] = 3.38057 \text{ mm}$$

$$\theta_n = \sqrt{\frac{h_f}{R}}\cdot\tan^{-1}\left(\sqrt{\frac{h_f}{R}}\times\frac{H_n}{2}\right)$$

$$\theta_n = \sqrt{\frac{16}{240}}\cdot\tan^{-1}\left(\sqrt{\frac{16}{240}}\times\frac{3.38057}{2}\right) = 0.10625$$

$$\theta = 6.0878°$$

$$h_n = h_f + D(1 - \cos\theta_n)$$

$$= 16 + 480\,(1 - \cos 6.0878)$$
$$= 18.707 \text{ mm}$$
$$h_0 v_0 = h_n v_n = h_f v_f$$
$$v_n = v_r = \text{velocity of roller}$$
$$h_0 v_0 = h_n v_r$$
$$\frac{v_0}{v_r} = \frac{h_n}{h_0} = \frac{18.707}{20} = 0.935$$
$$\frac{v_f}{v_r} = \frac{h_n}{h_f} = \frac{18.707}{16} = 1.169$$
$$\text{Forward slip} = \frac{v_f - v_r}{v_r} = \left(\frac{v_f}{v_r} - 1\right) = (1.169 - 1) \times 100 = 16.9\%$$
$$\text{Backward slip} = \frac{v_r - v_0}{v_r} = \left(1 - \frac{v_0}{v_r}\right) = (1 - 0.9353) \times 100 = 6.47\%$$
$$P_{max} = \sigma_0' \frac{h_n}{h_f} e^{\mu H_n} = \frac{2}{\sqrt{3}} \sigma_0 \frac{h_n}{h_f} e^{\mu H_n}$$
$$P_{max} = \frac{2}{\sqrt{3}} \times 120 \times \frac{18.707}{16} \times e^{0.1291 \times 3.38057}$$
$$P_{max} = 250.654 \text{ N/mm}^2$$

End of Solution

Q.7 (c) The kinematic parameters of a 3R planar articulated robot are given below:

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	0	d_3	θ_3

Determine the kinematic model of the robot using D-H Algorithm and relation between adjacent frames.

[20 marks : 2021]

Solution:

For 3R planar articulated robot,

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2
3	0	0	d_3	θ_3

Let $c_i = \cos \theta_i$

$s_i = \sin \theta_i$

Now,
$$ {}^1_0T = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_1T = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_2T = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Now, final frame with respect to base frame,

$$ {}^3_0T = {}^1_0T\,{}^2_1T\,{}^3_2T $$

$$ {}^3_0T = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_0T = \begin{bmatrix} \begin{Bmatrix} c_1c_2c_3 + s_1s_2c_3 \\ -s_3s_2c_1 - s_1s_3c_2 \end{Bmatrix} & \begin{Bmatrix} s_1s_2s_3 - c_1c_2s_3 \\ -c_1c_3s_2 - s_1c_2c_3 \end{Bmatrix} & 0 & \begin{Bmatrix} a_1c_1c_2 - a_2s_1s_2 \\ +a_1c_1 \end{Bmatrix} \\ \begin{Bmatrix} c_3s_1c_2 + c_1s_2c_3 \\ -s_1s_2s_3 + c_1c_2s_3 \end{Bmatrix} & \begin{Bmatrix} c_1s_2s_3 - c_2s_1s_3 \\ -c_3s_1s_2 + c_1c_2c_3 \end{Bmatrix} & 0 & \begin{Bmatrix} a_2c_2s_1 + a_2s_2c_1 \\ +a_1s_1 \end{Bmatrix} \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

End of Solution

Q.8 (a) (i) There are three coordinate frames $o_1\, x_1\, y_1\, z_1$, $o_2\, x_2\, y_2\, z_2$ and $o_2x_2y_3z_3$ and relation between frames are given below:

$$ {}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \quad {}^1R_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} $$

Find the matrix ${}^2R_3$ that relates frames (2) and (3).

(ii) The homogeneous coordinates of a point are $1_P = [0\ 0\ 10\ 1]^T$. The coordinate frame is translated relative to the fixed (F) coordinate frame, by 6 units along x-axis and –2 units along y-axis. What is the homogeneous transformation matrix that represents the operation? What is the description of point P relative to the reference frame (F) following the translation?

[10+10 marks : 2021]

Solution:

(i)

$$ {}^3_2R = ? $$

We know
$$ {}^1_3R = {}^1_2R\,{}^2_3R $$

$$ {}^1_2R^{-1}\,{}^1_3R = {}^1_2R^{-1}\,{}^1_2R\,{}^2_3R $$

$$ {}^1_2R^{-1}\,{}^1_3R = {}^2_3R $$

Since,
$$ {}^1_2R^{-1} = {}^1_2R^T $$

Then,
$$ {}^2_3R = {}^1_2R^T\,{}^1_3R $$

$$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} $$

$$ {}^2_3R = \begin{bmatrix} 0 & 0 & -1 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 \end{bmatrix} $$

(ii)

$$ 1_P = [0\ \ 0\ 10\ 1]^T $$

Translated 6 units $\rightarrow$ x-axis

–2 units $\rightarrow$ y-axis

Homogeneous transformation matrix:

$$ T = \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Now,

Description of P relative to frame {F}

$\Rightarrow$ $[J]\,[1_P]$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 10 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 6 \\ -2 \\ 10 \\ 1 \end{bmatrix} = [6 \quad -2 \quad 10 \quad 1]^T$$

End of Solution

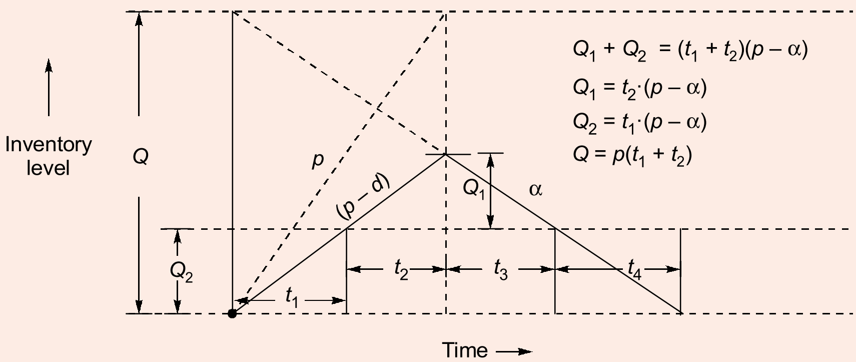
Q.8 (b) The demand for an item is observed as 24000 units per year. The production capacity of the plant is 3000 units per month. The item cost is Rs. 40 per unit and inventory carrying cost is 12% of the item cost per unit per year. The cost of one set-up is Rs. 480. The shortage cost of one unit is Rs. 240 per year. Supply of the item is non-instantaneous (gradual).

Determine the economic order quantity, optimal number of shortages production line required for each cycle and the number of the set-ups in a year.

[20 marks : 2021]

Solution:

Given : D = 24000 units/h, P = 3000 units/mete, C = ₹40/unit,
c_h = 12% of C = 0.12 × 40 = ₹4.8/unit/year
C_0 = ₹480/setup, C_b = ₹240/unit/year, d = 2000 units/month



(i)
$$Q^* = \sqrt{\frac{2DC_0}{C_h}} \sqrt{\frac{P}{p-d}} \cdot \sqrt{\frac{C_b + C_h}{C_b}}$$

$$= \sqrt{\frac{2 \times 24000 \times 480}{4.8}} \cdot \sqrt{\frac{3000}{3000-2000}} \cdot \sqrt{\frac{240+4.8}{240}}$$

$$Q^* = 3832.49 \text{ units/setup}$$
$$\simeq 3832 \text{ units/setup}$$

(ii)

$$Q_2 = ?$$
$$\text{Holding} = \text{Stockout cost}$$
$$Q_1C_h = Q_2C_b$$
$$Q_2 = 50 \times Q_2$$
$$Q_1 = 50 \times Q_2 \quad \text{...(a)}$$
$$t_2 = \frac{Q_1}{p-d}, t_1 = \frac{Q_2}{p-d}$$

On adding,
$$t_1 + t_2 = \frac{Q_1+Q_2}{p-d} \quad \text{...(b)}$$
$$t_1 + t_2 = \frac{Q}{p} = \frac{3832}{2000} = 1.916 \text{ month}$$

Putting
$$t_1 + t_2 = 1.916 \text{ in equation (b)}$$
$$Q_1 + Q_2 = (3000 - 2000) \times 1.916$$
$$Q_1 + Q_2 = 1916$$

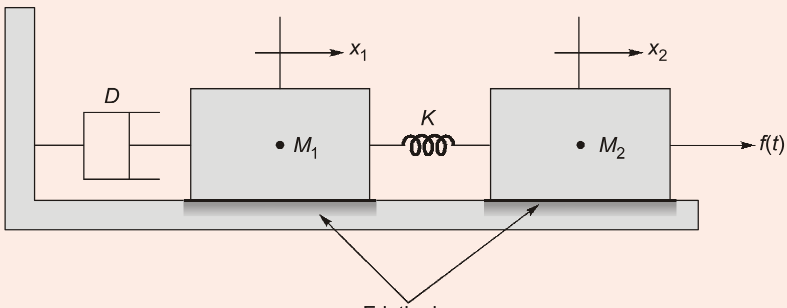
From equation (a),
$$Q_1 = 50Q_2$$
$$Q_2 = 37.56 \text{ units/setup}$$
$$Q_1 = 1878.44 \text{ units/setup}$$

Optimum number of units short per cycle will be $Q_2 = 37.56 \simeq 38$ units

Production time required for each cycle is $t_1 + t_2 = 1.916$ month.

End of Solution

Q.8 (c) Find the state equations in matrix form for the translational mechanical system as shown above. Assume zero initial conditions,



where

M_1 = Mass of block 1; M_2 = Mass of block 2; K = Spring stiffness constant

D = Coefficient of viscous friction; x_1 = Displacement of block 1

x_2 = Displacement of block 2

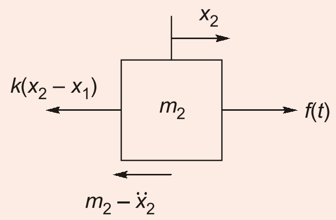
[20 marks : 2021]

Solution:

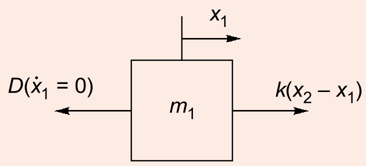
Step:1

Form a detailed differential equation for each free body.

$$m_2\ddot{x}_2 + k(x_2 - x_1) = f(t) \quad \text{...(i)}$$



$$m_1\ddot{x}_1 + k(x_1 - x_2) + dx_1 = 0 \qquad \text{...(ii)}$$



Step:2

Assign states sequentially to each element of the derivative family (except the element with highest order)

Elements of derivative family,

$x_1, \dot{x}_1$ and $\ddot{x}_1$

$$q_1 = x_1$$
$$q_2 = \dot{x}_1 = \dot{q}_1$$
$$\ddot{x}_2 = \dot{q}_2$$

$x_2, \dot{x}_2$ and $\ddot{x}_2$

$$q_3 = x_2$$
$$q_4 = \dot{x}_2 = \dot{q}_3$$
$$\ddot{x}_2 = \dot{q}_4$$

Where, q_1, q_2, q_3 and q_4 are state variables and $\dot{q}_1, \dot{q}_2, \dot{q}_3$ and $\dot{q}_4$ are differential state variables. (two state variables for each differential equation)

Now, $$\ddot{x}_2 = -\frac{k}{m_2}(x_2 - x_1) + \frac{f(t)}{m_2}$$

$$\dot{q}_4 = -\frac{k}{m_2}(q_3 - q_1) + \frac{f(t)}{m_2}$$

While, $$\ddot{x}_1 = -\frac{k}{m_1}(x_2 - x_1) - \frac{D}{m_1}\dot{x}_1$$

$$q_2 = -\frac{k}{m}(q_3 - q_1) - \frac{D}{m_1}q_2$$

So, $$\dot{q}_1 = q_2$$

$$\dot{q}_2 = \frac{k}{m_1}(q_1 - q_3) - \frac{D}{m_1}q_2$$

$$\dot{q}_3 = q_4$$

$$\dot{q}_4 = \frac{k}{m_2}(q_1 - q_3) + \frac{f(t)}{m_2}$$

Now, using state equation i.e.,

$$[\dot{X}] = [A][X]+[B][U]$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k}{m_1} & -\frac{D}{m_1} & -\frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} f(t)$$

Now, output is generated at $x_2(t)$ end,

Hence, $y(t) = q_3(t)$

[Let $x_2(t) = y(t)$]

$$[y] = [0\,0\,1\,0]\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + O\,u(t)$$

[Using $[y] = CX + Du$ equation]

End of Solution

■■■■