2019

RANK IMPROVEMENT WORKBOOK

Mechanical Engineering

Strength of Materials

Answer Key of Objective & Conventional Questions

MADE EASY Publications

1 Mechanical Properties of Materials & Elastic Constants

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(b)
4.	(c)
5.	(b)
6.	(b)
7.	(c)
8.	(b)
9.	(b)
10.	(a)
11.	(d)

LEVEL 2 Objective Questions

12.	(c)
13.	(b)
14.	(c)
15.	(d)
16.	(d)
17.	(b)
18.	(a)
19.	(c)
20.	(0.4)
21.	(c)
22.	(b)

■■■■

LEVEL 3 Conventional Questions

Solution : 23

$$\begin{aligned} \text{Elastic modulus, } E &= 199.289 \approx 199.3 \text{ GPa} \\ \text{Modulus of Rigidity, } G &= 78.44 \text{ GPa} \\ \nu &= 0.27 \text{ (Poisson's ratio)} \\ \text{Bulk modulus, } K &= 144.42 \text{ GPa} \end{aligned}$$

Solution : 24

Hence

$$\sigma_2 = \frac{2}{3}\frac{\mu}{1-\mu}\sigma_1$$

$$e_1' = \frac{\sigma_1}{E}\left[\frac{3-3\mu-4\mu^2}{3(1-\mu)}\right] \text{ compressive.}$$

Solution : 25

$$\begin{aligned} \mu &= 0.267 \\ K &= 1.359 \times 10^5 \text{ N/mm}^2 \\ \text{Maximum \% error in } \mu &= 9.475\% \end{aligned}$$

Solution : 26

$$G_B = \frac{1.01E_AG_A}{1.01E_A - 3(1.01-1)G_A} = \frac{101E_AG_A}{101E_A - 3G_A}$$

Solution : 27

$$\begin{aligned} m &= 3.126 \text{ and } \frac{1}{m} \approx 0.32 \\ E &= 1.32 \times 10^5 \text{ N/mm}^2 \end{aligned}$$

Solution : 28

$$\begin{aligned} \text{\% reduction in volume} &= 0.00249 \\ \delta V &= 321 \text{ mm}^3 \end{aligned}$$

Solution : 29

$$\begin{aligned} G &= 40 \text{ GPa} \\ K &= 66.7 \text{ GPa} \\ \text{Change in volume} &= dV = 1588.56 \text{ mm}^3 \end{aligned}$$

Ans.

■■■■

2 Stress and Strain

LEVEL 1 Objective Questions

1.	(b)
2.	(a)
3.	(d)
4.	(c)
5.	(b)
6.	(c)
7.	(c)
8.	(a)
9.	(a)
10.	(c)
11.	(b)

LEVEL 2 Objective Questions

12.	(d)
13.	(d)
14.	(c)
15.	(a)
16.	(b)
17.	(b)
18.	(b)
19.	(d)

■■■■

LEVEL 3 Conventional Questions

Solution : 20

$$\sigma_{max} = -240 \text{ MPa} \quad \text{(in portion } AB\text{)}$$
$$\text{Change in length} = -2.817 \text{ mm (contraction)}$$

Solution : 21

$$\sigma_c = -33.86 \text{ N/mm}^2$$
$$\sigma_s = 1.543 \times \sigma_c = 52.26 \text{ N/mm}^2$$

Solution : 22

$$d = \sqrt{d_1 d_2}$$
$$\sigma_{bar} = \frac{4P}{\pi d_1^2}$$
$$\sigma_{avg} = \frac{4P}{\pi d^2}$$

Solution : 23

$$d_2 = 41.2 \text{ mm}$$
$$L_2 = 188.62 \text{ mm}$$
$$L_1 = 155.69 \text{ mm}$$

Solution : 24

$$\text{Total elongation} = \Delta_1 + \Delta_2 = 23.41 + 4.30 = 27.71 \text{ mm}$$

Solution : 25

$$\Delta = \frac{PL}{\pi E}\left[\frac{1}{d_1^2} + \frac{1}{d_2^2}\right]$$

Solution : 26

$$t = 25.45 \text{ mm, say } 26 \text{ mm.}$$

Solution : 27

$$\delta l_c = 0.2 \text{ mm}$$
$$\delta l_1 = 0.05 \text{ mm}$$
$$\delta l_B = 0.15 \text{ mm}$$

Solution : 28

$$P_{Cu} = 36.93 \text{ kN}$$
$$P_{Zn} = 42.61 \text{ kN}$$
$$P_{Al} = 45.45 \text{ kN}$$
$$\sigma_{Cu} = 147.72 \text{ MPa}$$
$$\sigma_{Zn} = 113.63 \text{ MPa}$$
$$\sigma_{Al} = 90.9 \text{ MPa}$$

■■■■

3 Shear force & Bending Moment

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(c)
4.	(a)
5.	(a)
6.	(d)
7.	(c)
8.	(d)
9.	(d)
10.	(c)
11.	(d)
12.	(c)
13.	(a)

LEVEL 2 Objective Questions

14.	(a)
15.	(a)
16.	(c)
17.	(c)
18.	(b)
19.	(c)
20.	(a)
21.	(d)
22.	(c)
23.	(b)
24.	(b)
25.	4.47 (4.40-4.50)
26.	(50)
27.	(d)
28.	(d)
29.	150 (148 to 153)
30.	(a)
31.	(a)
32.	(b)
33.	(c)

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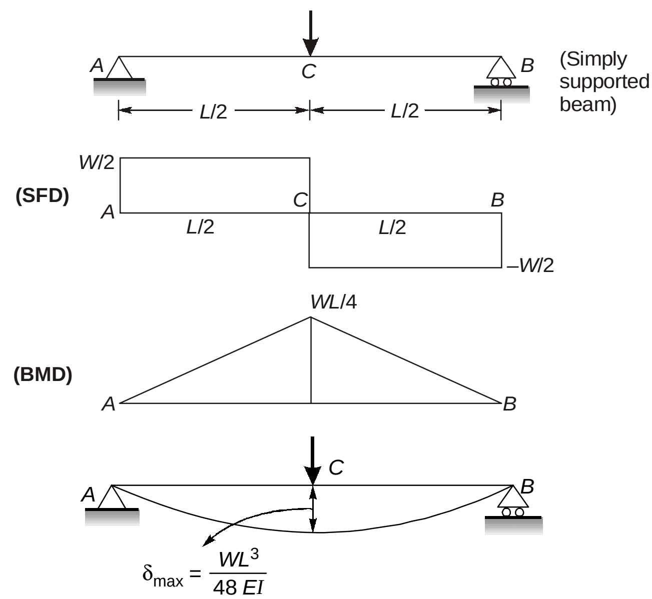
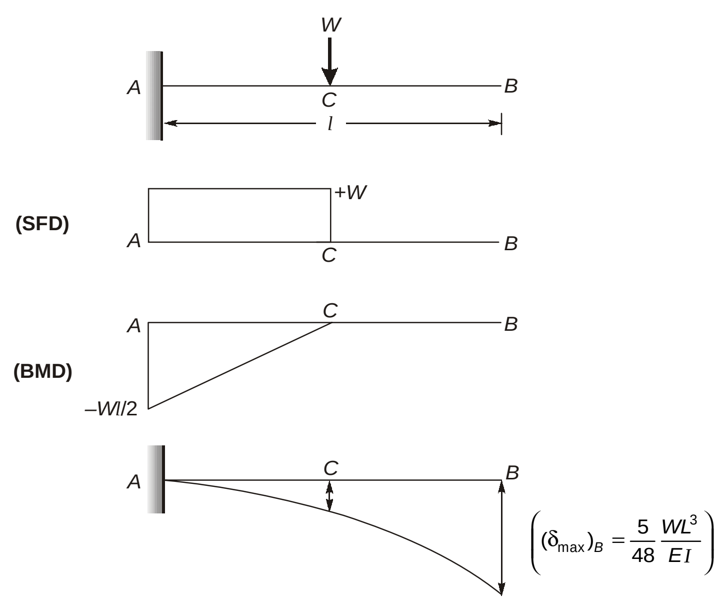
LEVEL 3 Conventional Questions

Solution : 34



Deflection diagram:



Deflection diagram:

(ii) Structural members subjected to compression and which are relatively long compared to their lateral dimensions are called columns or struts. Generally, the term column is used to denote vertical members and the term strut denotes inclined members.

Columns are generally fixed at the both ends while strut can have any end fixation conditions like both end fixed, both ends hinged, one end fixed other end free, etc.

According to Rankine's formulae,

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

where P_e = buckling load

P_c = crushing load

For short column $P_e >>>> Pc$

or $\frac{1}{P_e} <<< \frac{1}{P_c} \Rightarrow \frac{1}{P_e}$ can be neglected ,

$$\frac{1}{P_R} = \frac{1}{P_c}$$

$$P_R \approx P_c \approx A\sigma_c$$

For long columns $P_c >>> P_e$

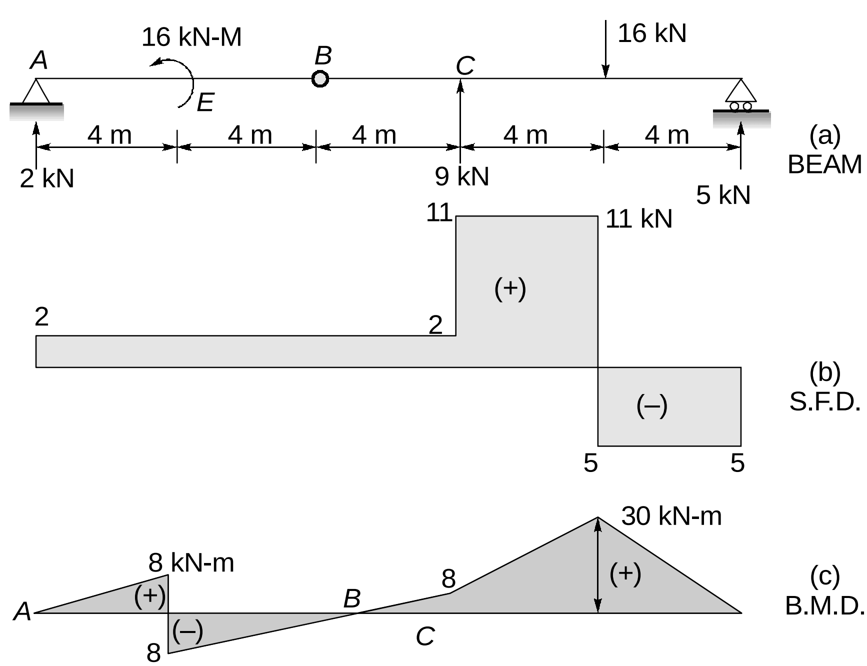
$\frac{1}{P_c} <<< \frac{1}{P_e} \Rightarrow \frac{1}{P_c}$ can be neglected

$$P_R \approx P_e = \frac{\pi^2 E I_{min}}{L_e^2}$$

$\therefore$ $$P_R = \frac{\sigma_c A}{1 + c(Se)^2}$$

where, c = Rankine's constant and Se = slenderness ratio

Solution : 35



Considering L.H.S $M_B = 0 = R_A \times 8 - 16$

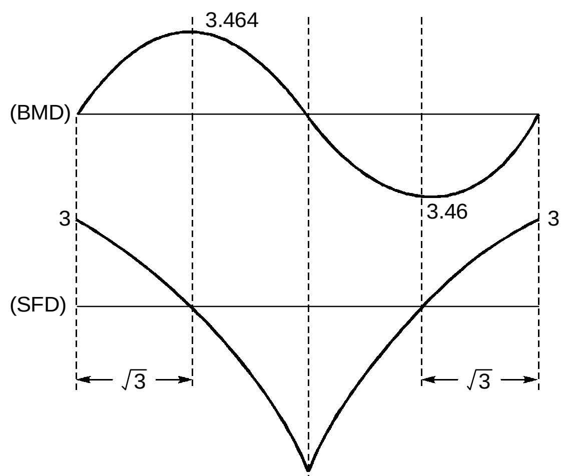
$$R_A = \frac{16}{8} = 2 \text{ kN}(\uparrow)$$

$$R_D = 5 \text{ kN}(\uparrow) \text{ and } R_C = 9 \text{ kN}(\uparrow)$$

Question : 36

$$R_c = \frac{1600}{216} = \frac{200}{27} \text{ kN}$$

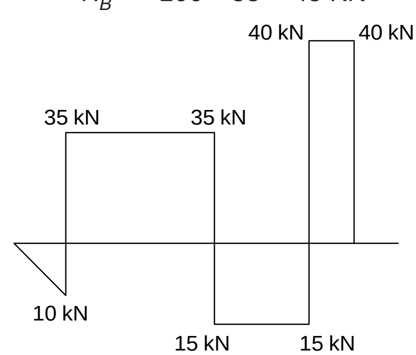
Solution : 37

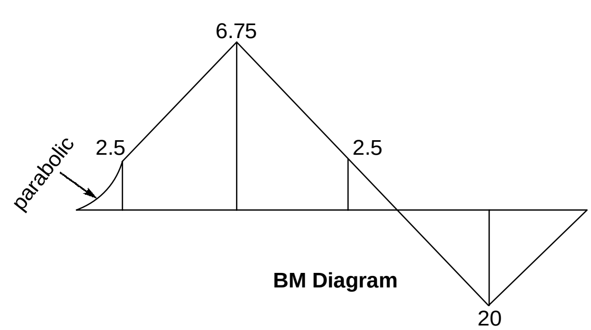


Solution : 38

$$R_E = 55 \text{ KN}$$

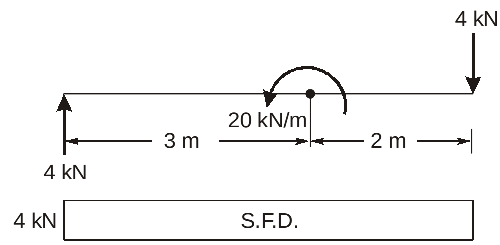
$\therefore$ $$R_B = 100 - 55 = 45 \text{ KN}$$





BM Diagram

Solution : 39



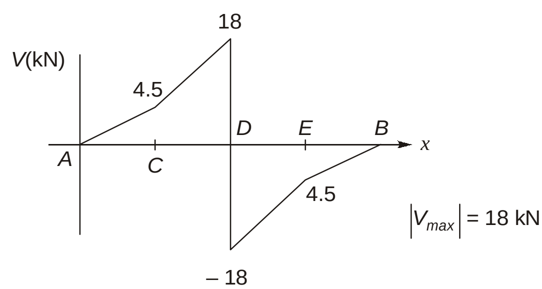
Solution : 40

$$w = 15 \text{ kN/m}$$

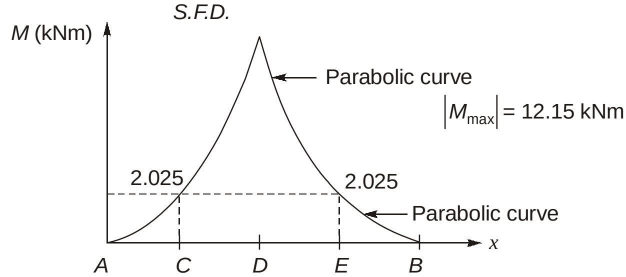
At D,

$$V = 15 \times 1.8 - 9 = 18 \text{ kN}$$

$$M = 12.15 \text{ kNm}$$



S.F.D.



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4 Shear Stress and Bending Stress

LEVEL 1 Objective Questions

1.	(a)
2.	(c)
3.	(b)
4.	(c)
5.	(c)
6.	(b)
7.	(d)
8.	(a)
9.	(a)
10.	(c)
11.	(b)
12.	(c)
13.	(b)
14.	(a)
15.	(c)
16.	(b)
17.	(d)
18.	(a)
19.	(b)

LEVEL 2 Objective Questions

20.	(c)
21.	(d)
22.	(b)
23.	(b)
24.	(b)
25.	1.36 (1.30 to 1.40)
26.	89.05 (88 to 90)
27.	(b)
28.	56.25 (56-57)
29.	(d)
30.	6.25 (6.21 to 6.27)
31.	266.67 (264 to 268)
32.	(b)
33.	166.67 (166-167)
34.	(a)
35.	(c)
36.	120 (119 to 121)

■■■■

LEVEL 3 Conventional Questions

Solution : 37

$$Z_1 : Z_2 : Z_3 = 0.9025 : 0.20529 : 0.122845$$
$$= 7.346 : 1.671 : 1$$

Solution : 38

$$y = \frac{10\sqrt{2}}{8} = 1.76 \text{ cm from neutral axis.}$$

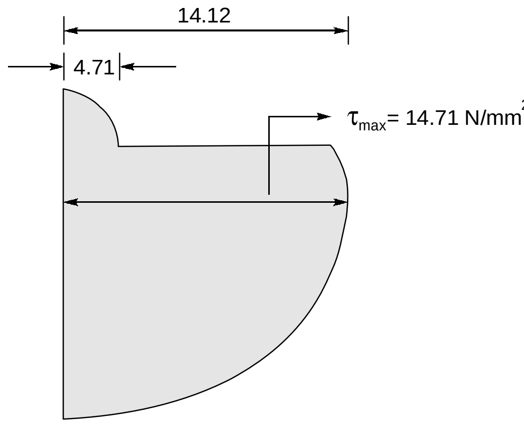
Solution : 39

$$W = 413.76 \text{ kN}$$
$$\text{Minimum length of plate} = 5 - (0.942 \times 2) = 3.116 \text{ m}$$

Solution : 40

$$\left.\begin{array}{ll} \text{Breadth of beam,} & b = 71.8 \text{ mm} \\ \text{Depth of beam,} & d = 316.7 \text{ mm} \end{array}\right\} \text{Ans.}$$

Solution : 41



Shear Stress Distribution

$$\tau_{max} = 14.71 \text{ N/mm}^2$$

Shear stress in the flange at the junction of flange and web = 14.12 N/mm^2

The shear stress distribution is shown in figure.

Solution : 42

$$d = 86.3 \text{ mm say } 90 \text{ mm}$$

Solution : 43

$$\text{Allowable load, } W = 15 \text{ kN/m (taking lower of two)}$$

Solution : 44

$$D = 0.0248 \text{ m} = 24.78 \text{ mm}$$

Solution : 45

$$\sigma_{max} = 964.63 \text{ MPa}$$
$$M = 20455.627 \text{ Nmm}$$
$$M_{max} = 20.455 \text{ Nm}$$

Solution : 46

$$M = 80.24 \times 10^3 \text{ Nm}$$

Solution : 47

$$R_A = \frac{13}{3} \text{kN}$$
$$R_B = \frac{17}{3} \text{kN}$$
$$\sigma_1 = -23.3674 \text{ N/mm}^2$$
$$\sigma_2 = 0.8586 \text{ N/mm}^2$$

■■■■

5 Torsion of Shafts

LEVEL 1 Objective Questions

1.	(b)
2.	(d)
3.	(a)
4.	(d)
5.	(d)
6.	(a)
7.	(b)
8.	(b)
9.	(d)
10.	(b)
11.	(b)
12.	(d)
13.	(d)
14.	(c)
15	(b)
16.	(d)
17.	(b)

LEVEL 2 Objective Questions

18.	(b)
19.	20.00 (20.00 to 20.00)
20.	(400)
21.	(b)
22.	(b)
23.	0.12 (0.11 to 0.15)
24.	(b)
25.	(d)
26.	34.17 (34.00 to 34.50)

■■■■

LEVEL 3 Conventional Questions

Solution : 27

$$T = 1.335 \text{ kNm}$$

Solution : 28

Weight of hollow shaft is 0.6431 times the weight of solid shaft.

Solution : 29

$$d_2 = 184.16 \text{ mm}, \quad d_1 = 69.06 \text{ mm}$$

Solution : 30

$$D_1 = 95.5 \text{ mm} \quad \textbf{Ans.}$$

$$\text{Ratio of torsional rigidity} = 2.093$$

Solution : 31

$$d = 0.108 \text{ m}$$

$$\text{Hence percentage savings in weight} = 38.78\%$$

Solution : 32

$$D = 137.57 \text{ mm}$$

Hence minimum external diameter of shaft = 137.57 mm (taking bigger one value).

Solution : 33

$$\frac{Z_s}{Z_h} = \frac{D \times D_0}{\left(2D_0^2 - D^2\right)}$$

Solution : 34

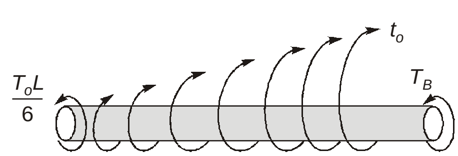
Maximum shear stress in the shaft = 42.56 N/mm^2

Twist of the end D with respect to the end A = 0.03089 radian = 1.77 degrees

Solution : 35

$$T_A = -\frac{t_oL}{6}$$

$$T_B = -\frac{t_oL}{3}$$



$$\text{Expression for } \phi(x) = \frac{1}{GJ}\left[-\frac{t_oL}{6}\cdot x + \frac{t_o}{2L} \times \frac{x^3}{3}\right]$$

$$\phi \text{ is } \phi_{max} \text{ at } x = \frac{L}{\sqrt{3}}$$

$$\phi_{max} = \frac{-\sqrt{3}t_o L^2}{27GJ}$$

Solution : 36

$$\text{Torque} = 1.1 \text{ kNm}$$

Solution : 37

$$\text{Power} = 98 \text{ kW}$$

■■■■

6 Principle Stress & Strain, Mohr's Circle

LEVEL 1 Objective Questions

1.	(d)
2.	(c)
3.	(a)
4.	(c)
5.	(d)
6.	(c)
7.	(a)
8.	(b)
9.	(c)
10.	(b)
11.	(d)
12.	(a)
13.	(c)
14.	(c)
15.	(c)
16.	(c)
17.	(d)
18.	(c)

LEVEL 2 Objective Questions

19.	(a)
20.	(70)
21.	(d)
22.	(75)
23.	(b)
24.	(a)
25.	(a)
26.	(c)
27.	(b)
28.	(a)
29.	(b)
30.	6.98 (6.8 to 7.2)
31.	(0)
32.	(86.60)

■■■■

LEVEL 3 Conventional Questions

Solution : 33

Hooke's Law, $\sigma_x = 91.6$ MPa

Solution : 34

Given: σ_1, σ_2 are the principal stresses.

(i) According to plane stress transformation equations

$$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{... (i)}$$

$$\sigma_y' = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{... (ii)}$$

Adding equation (i) and (ii),

$$\sigma_x' + \sigma_y' = \sigma_x + \sigma_y$$

Similarly, $$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (i)}$$

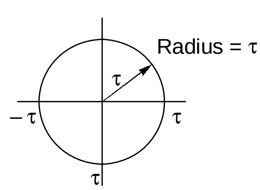
$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (ii)}$$

Adding, (i) and (ii),

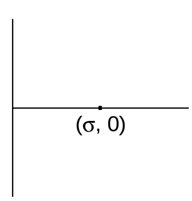
$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$\therefore \quad \sigma_x + \sigma_y = \sigma_x' + \sigma_y' = \sigma_1 + \sigma_2$$

(ii) Mohr's circle for pure shear stress state.



Mohr's circle for hydrostatic state.



Solution : 35

$$\sigma_1 = 12.73 \text{ N/mm}^2, \theta = 45°$$

$$\sigma_2 = -12.73 \text{ N/mm}^2, \theta = 135°$$

Solution : 36

Principal stress, $\sigma_1 = 38.55 \text{ N/mm}^2$

$\sigma_2 = -38.55 \text{ N/mm}^2$

$\theta_p = -14.3°, 75.7°$

Solution : 37

$$\theta_1 = 36.6992°, \quad \theta_2 = 126.6992°$$

Maximum shear stress, $\tau_{max} = 70$ MPa

Solution : 38

If $\sigma_{11}, \sigma_{22}, \sigma_{33}$ be the three principle stresses, then the principle strains t_{11}, t_{22}, t_{33} obtained by Generalized Hooke's law as:

$$\in_{11} = \frac{\sigma_{11}}{E} - \frac{\sigma_{22}+\sigma_{33}}{mE} \quad ...(i)$$

$$\in_{22} = \frac{\sigma_{22}}{E} - \frac{\sigma_{33}+\sigma_{11}}{mE} \quad ...(ii)$$

$$\in_{33} = \frac{\sigma_{33}}{E} - \frac{\sigma_{11}+\sigma_{22}}{mE} \quad ...(iii)$$

where, E = Modulus of elasticity

and $\frac{1}{m}$ = Poisson's Ratio

From equation (i), we have

$$E\in_{11} = \sigma_{11} - \frac{\sigma_{22}}{m} - \frac{\sigma_{33}}{m} \quad ...(iv)$$

From equation (ii), we have

$$E\in_{22} = \sigma_{22} - \frac{\sigma_{33}}{m} - \frac{\sigma_{11}}{m} \quad ...(v)$$

From equation, (iii) we have

$$E\in_{33} = \sigma_{33} - \frac{\sigma_{11}}{m} - \frac{\sigma_{22}}{m} \quad ...(vi)$$

Subtracting equation (v) from equation (iv), we get

$$E(\in_{11} - \in_{22}) = (\sigma_{11} - \sigma_{22})\left(1 + \frac{a}{m^2}\right) \quad ...(vii)$$

From equations (i) and (iii), (vi), we get

$$E(\in_{11} + \in_{33}) = \sigma_{11}\left(1 - \frac{1}{m^2}\right) - \sigma_{22}\left(1 + \frac{1}{m^2}\right)\frac{1}{m} \quad ...(viii)$$

$$\sigma_{11} = \frac{E\left(1-\frac{1}{m}\right)\in_{11} + \frac{1}{m}(\in_{22} + \in_{33})}{\left(1+\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Similarly,

$$\sigma_{22} = \frac{E\left(1-\frac{1}{m}\right)\in_{22} + \frac{1}{m}(\in_{33} + \in_{22})}{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Similarly,

$$\sigma_{33} = \frac{E\left(1-\frac{1}{m}\right)\in_{33} + \frac{1}{m}(\in_{11} + \in_{22})}{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Solution : 39

$$\text{Hence major axis of the ellipse} = 100 + 0.839 = 100.839 \text{ mm}$$
$$\text{Minor axis of the ellipse} = 100 - 0.633 = 99.367 \text{ mm}$$
$$\theta_1 = 12.22°$$
$$\theta_2 = 102.22°$$

Solution : 40

$$\sigma_{max}(\sigma_1) = 280.62 \text{ MPa}$$
$$\sigma_{min}(\sigma_2) = 119.38 \text{ MPa}$$
$$\theta_1 = 14.87°$$
$$\theta_2 = 104.87°$$
$$\text{Maximum shearing stress} = \pm 80.62 \text{ MPa}$$

i.e., 59.87° and 149.87°

Solution : 41

$$\sigma_1 = 205.625 \text{ MPa}, \quad \sigma_2 = -155.625 \text{ MPa}$$
$$\theta_P = 20.815° \text{ and } 110.815°$$

Position of the plane on which normal stress is zero,

$$\theta = 69.8°$$

Solution : 42

Same as question 40.

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7 Strain Energy and Thermal Stress

LEVEL 1 Objective Questions

1. (b)
2. (a)
3. (a)
4. (d)
5. (a)
6. (*)
7. (d)
8. (a)
9. (80)
10. (d)

LEVEL 2 Objective Questions

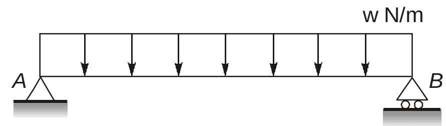
11. (c)
12. (1)
13. 187.5 (185 to 190)
14. (d)
15. (c)
16. (a)
17. (b)
18. (d)
19. (b)

■■■■

LEVEL 3 Conventional Questions

Solution : 20

Given:

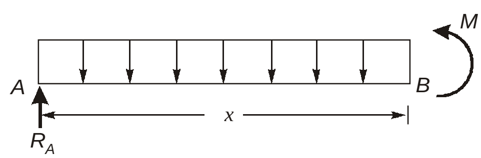


Simple supported beam loaded by uniformly distributed load.

Let U_e be the elastic strain energy due to bending of beam.

$$U_e = \int_0^L \frac{M^2 dx}{2EI}$$

$$R_A = \frac{wL}{2}$$



$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$U_e = \frac{1}{2EI}\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx$$

$$U_e = \frac{1}{2EI}\times\frac{w^2}{4}\int_0^L (L^2x^2 + x^4 - 2Lx^3)dx$$

$$U_e = \frac{w^2}{8EI}\left[\frac{L^5}{3} + \frac{L^5}{5} - \frac{2L^5}{4}\right] = \frac{w^2}{8EI}\times\left[\frac{20+12-30}{60}\right]L^5$$

$$U_e = \left(\frac{w^2L^5}{240\ EI}\right) = \frac{w^2L^5}{240\times E\times\frac{1}{12}bh^3} = \frac{w^2L^5}{20\,E\,bh^3}$$

Now,
$$\sigma_{max} = \frac{My}{I} = \frac{6M}{bd^2} = \frac{6\times wL^2}{8\times bh^2} = \frac{3wL^2}{4bh^2}$$

$$\text{Volume of beam} = A\times L = (b\times h)L$$

$$= \left(\frac{\sigma_{max}^2}{2E}\right)\times\left(\frac{8}{45}\times \text{volume of beam}\right)$$

$$= \frac{9w^2L^4}{32b^2h^4E}\times\frac{8}{45}\times(b\times h)\times L = \frac{w^2L^5}{240\ Ebh^3} = U_e$$

This expression is same as obtained by equation (i).

Solution : 21

$$\sigma_{b1} = 15 \text{ N/mm}^2$$
$$\sigma_{s1} = 4\,\sigma_{b1} = 60 \text{ N/mm}^2$$
$$\sigma_{b2} = 9.33 \text{ N/mm}^2,\ \sigma_{s2} = 37.33 \text{ N/mm}^2$$
$$\sigma_b = 24.33 \text{ N/mm}^2 \quad \text{(compressive)}$$
$$\sigma_b = 24.33 \text{ N/mm}^2 \quad \text{(tensile)}$$

Solution : 22

$$\sigma_s = 91.77 \text{ MPa} \quad \text{(tensile)}$$
$$\sigma_c = 108.3 \text{ MPa} \quad \text{(compressive)}$$

Solution : 23

$$\sigma_c = 198.486 \text{ MPa}$$
$$\sigma_a = 49.62 \text{ MPa}$$
$$\sigma_s = 198.486 \text{ MPa}$$
$$\sigma_s = 94.309 \text{ MPa}$$
$$\sigma_a = 23.577 \text{ MPa}$$

Solution : 24

$$\text{Vertical deflection} = \frac{PL^3}{6EI}$$

Solution : 25

$$A_s = A_b = \frac{\pi}{4}\times\left(6\times10^{-3}\right)^2 = 28.27 \times 10^{-6} \text{ m}^2$$
$$\sigma_{s1} = \sigma_{b1} = 123.8 \text{ MPa}$$
$$\sigma_{s2} = 29.62 \text{ MPa (tensile)}$$
$$\sigma_{b2} = 16.36 \text{ MPa (compressive)}$$
$$\sigma_{sf} = 153.42 - 56.84 = 96.58 \text{ MPa (tensile)}$$
$$\sigma_{bf} = 107.44 - 56.84 = 50.6 \text{ MPa (tensile)}$$

Solution : 26

$$E = 79.6 \text{ GPa}$$
$$U_T = 8 \text{ N-m}$$
$$U = 7.9 \text{ N-m}$$

Solution : 27

Resultant stress in aluminium = 25.3 MPa (tensile)

Resultant stress in steel = 79.3 MPa (tensile)

$\Delta T = 85.138°\text{C}$

Solution : 28

Length of circular portion = 0.12 m or 120 mm

d = 0.0255 m or 25.5 mm

strain energy = 1.4 J

■■■■

8 Deflection of Beams

1.	(b)
2.	(d)
3.	(d)
4.	(c)
5.	(a)
6.	(b)
7.	(c)
8.	(a)
9.	(d)
10.	(a)
11.	(d)
12.	(d)
13.	(d)
14.	(b)
15.	(d)
16.	(d)
17.	(0.67)
18.	0.78
19.	(0.21)
20.	2.2 (2.15-2.25)
21.	(b)
22.	(a)
23.	(a)
24.	(b)
25.	(d)
26.	(b)
27.	(34.91)
28.	(c)
29.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 30

Equation of elastic curve:

$$EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3}{24}x$$

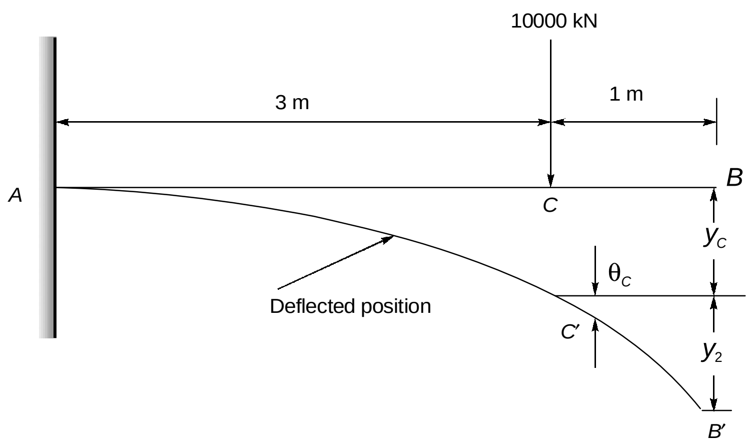
$$y_{max} = -\frac{5wL^4}{384EI}$$

Solution : 31

$$y_C = \frac{5wL^4}{384EI} \text{ (downwards)}$$

$$y_C = -\frac{wa^2L^2}{16\,EI} \text{ (upward direction)}$$

Solution : 32



$$y_C = 1.125 \text{ cm}$$

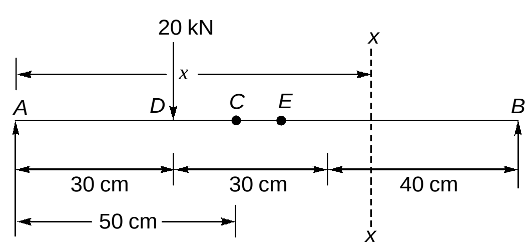
$$\theta_B = \theta_C = \frac{9}{1600} \text{radians}$$

$$Y_B = 1.6875 \text{ cm}$$

Solution : 33

Total deflection of B = 8.69 mm

Solution : 34



Taking moment about A,

$$20 \text{ kN} \times 30 = R_B \times 100$$
$$R_B = 6 \text{ kN}$$
$$R_A = 20 - 6 = 14 \text{ kN}$$

At a distance x from A in the DB portion,

$$EI\frac{d^2y}{dx^2} = R_A \cdot x - W(x-30) \quad \text{...(1)}$$

Integrating, (For BD section)

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{W}{2}(x-30)^2 + C_1 \quad \text{...(2)}$$

Again integrating, (For BD section)

$$EIy = R_A\frac{x^3}{6} - \frac{W}{2\times 3}(x-30)^3 + C_1x + C_2 \quad \text{...(3)}$$

Boundary conditions,

At $x = 0$, $y = 0$, So, $C_2 = 0$

At $x = L$, $y = 0$ $\qquad 0 = R_A\frac{L^3}{6} - \frac{W}{6}\times 70^3 + C_1L + 0$

$$\left[\frac{20}{6}\times 70^3 - \frac{14\times 100^3}{6}\right]\frac{10^3}{100}\times 10^{-4} = C_1 \quad [C_1 = -1190]$$

Putting value of C_1 in equation (2) and (3),

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{W}{2}(x-30)^2 - 1190 \quad \text{...(4)}$$

$$EIy = R_A\frac{x^3}{6} - \frac{W}{6}(x-30)^3 - 1190x \quad \text{...(5)}$$

For slopes at different points,

Slope at point, A, $\theta_A = -\frac{1190}{EI}$ radian

Slope at point B, $\theta_B = \frac{1}{EI}\left[14\times 10^3 \times \frac{100^2}{2}\times 10^{-4} - \frac{20\times 10^3}{2}\times (70)^2 \times 10^{-4} - 1190\right]$

$$= \frac{1}{EI}[7000 - 4900 - 1190] = \frac{910}{EI} \text{ radian}$$

Slope at point C, $\theta_C = \frac{1}{EI}\left[14\times 10^3 \times \frac{50^2}{2}\times 10^{-4} - \frac{20\times 10^3}{2}\times 20^2 \times 10^{-4} - 1190\right]$

$$= \frac{1}{EI}[1750 - 400 - 1190]$$

$$\theta_C = +\frac{160}{EI} \text{ radian}$$

Slope at point D, $\theta_D = \frac{1}{EI}\left[\frac{14}{2}\times 10^3 \times 30^2 \times 10^{-4} - 1190\right] = -\frac{560}{EI}$ radian

$$\text{Slope at point } E,\ \theta_E = \frac{1}{EI}\left[\frac{14\times10^3}{2}\times0.6^2-\frac{20}{2}\times10^3\times0.3^2-1190\right]$$

$$= \frac{430}{EI}\text{ radian}$$

Deflection at different points,

Deflection of point A, $y_A = 0$

Deflection of point B, $y_B = 0$

$$\text{Deflection of point C},\ y_C = \frac{1}{EI}\left[14\times10^3\times\frac{0.5^3}{6}-\frac{20\times10^3}{2}\times0.2^3-1190\times0.5\right]$$

$$= \frac{-1150}{3EI}\text{ units}$$

$$\text{Deflection of point } D,\ y_D = \frac{1}{EI}\left[14\times10^3\times\frac{0.3^3}{6}-1190\times0.3\right] = \frac{-294}{EI}\text{ units}$$

$$\text{Deflections of point } E,\ y_E = \frac{1}{EI}\left[14\times10^3\times\frac{0.6^3}{6}-\frac{20000}{6}\times0.3^3-1190\times0.6\right]$$

$$= \frac{-300}{EI}\text{ units}$$

Solution : 35

$$\theta_D = \frac{72W}{EI}$$

$$\delta_D = \frac{301.33W}{EI}$$

Slope of the free end, $\theta_D = 3.6\times10^{-3}$ rad

Deflection at the free end, $\delta_D = 15.06$ mm

■■■■

9 Theories of Failure & Springs

LEVEL 1 Objective Questions

1.	(c)
2.	4.5 (4.45-4.55)
3.	(a)
4.	(b)
5.	(d)
6.	(d)
7.	(c)
8.	(b)
9.	(a)
10.	(c)
11.	(d)
12.	(b)
13.	(d)
14.	(d)
15.	(b)
16.	(c)

LEVEL 2 Objective Questions

17.	(d)
18.	112.51 (112 to 113.5)
19.	(b)
20.	(d)
21.	(d)
22.	(b)
23.	(d)
24.	(c)
25.	(b)
26.	(a)
27.	(d)
28.	(d)
29.	(d)

■■■■

LEVEL 3 Conventional Questions

Solution : 30

Strain energy theory, $d = 13.48$ or 14 mm

Shear strain energy theory, $d = 13.68$ mm

Solution : 31

(i) According to Von-misses criterion.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \leq 2(\sigma_y)^2 \qquad \text{[Considering bi-axial stress]}$$

For stress state,

$$\sigma_3 = 0$$
$$\sigma_1 = \tau$$
$$\sigma_2 = -\tau$$
$$[\tau - (-\tau)]^2 + \tau^2 + \tau^2 \leq \sigma_y^{\ 2}$$
$$6\tau_{ys}^{\ 2} \leq 2\sigma_{yt}^{\ 2}$$
$$\frac{\sigma_{yt}}{\sqrt{3}} = \tau_{ys} \qquad \text{[Relation between tensile and shear yield stress]}$$
$$\tau = \text{Shear yield stress}$$
$$\sigma_y = \text{Tensile stress}$$

(ii) Accroding to Tresca criteria:

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_y}{2}$$

For Stress State,

$$\sigma_1 = \tau \qquad \text{[Considering bi-axial stress]}$$
$$\sigma_2 = -\tau, \sigma_3 = 0$$
$$\frac{\tau - (-\tau)}{2} \leq \frac{\sigma_y}{2}$$
$$\tau_{ys} = \frac{\sigma_y}{2} \qquad \text{[Relation between tensile and shear yield stress.]}$$

Solution : 32

$d = 5.229$ or 6 mm (say)

Total length of wire = 1922.65 mm or 1.92 m

Solution : 33

According to maximum principal stress theory

$$d = 2.029 \times 10^{-2} \text{ m} = 20.29 \text{ mm}$$

According to shear strain energy theory

$$d = 0.02143\ m = 21.43 \text{ mm}$$

Solution : 34

Given, $W = 120$ N, $d = 80$ mm, $T = 500$ N.mm, $\theta = 90°$, $D = 25$ mm

$$\delta(\text{Deflection of close coiled spring}) = \frac{8WD^3n}{Gd^4}$$

or $$1 = \frac{8WD^3n}{Gd^4} \text{ (for unit deflection)}$$

$$\theta = \frac{64\,T\,Dn}{E\,d^4}$$

for unit angular rotation, $$1 = \frac{64\,T\,Dn}{E\,d^4}$$

$$\therefore \quad \frac{8WD^3n}{Gd^4} = \frac{64\,T\,Dn}{E\,d^4}$$

$$\frac{T}{W} = \frac{D^2E}{8G} = \frac{D^2 \times 2G(1+\nu)}{8G} = \frac{D^2(1+\nu)}{4}$$

Torque/unit angular rotation, $$T = \frac{500}{90 \times \frac{\pi}{180}} = 318.3 \text{ N-mm/rad}$$

$$\nu = 0.358$$

Solution : 35

$$T = 2.947 \text{ kNm}$$
$$T = 2.872 \text{ kNm}$$

Solution : 36

$$d = 13 \text{ mm} \quad \text{(wire diameter)}$$
$$D\text{(mean diameter)} = 10\,d = 130 \text{ mm}$$

Solution : 37

$$N = 1.6$$

■■■■

10 Euler's theory of column

LEVEL 1 Objective Questions

1.	(b)
2.	(b)
3.	(c)
4.	(c)
5.	(a)
6.	(a)
7.	(c)
8.	(d)
9.	(a)
10	(c)

LEVEL 2 Objective Questions

11.	(b)
12.	(d)
13.	(c)
14.	(c)
15.	(a)
16.	(b)
17.	(d)

■■■■

LEVEL 3 Conventional Questions

Solution : 18

Structural members subjected to compression and which are relatively long compared to their lateral dimensions are called columns or struts. Generally, the term column is used to denote vertical members and the term strut denotes inclined members.

Columns are generally fixed at the both ends while strut can have any end fixation conditions like both end fixed, both ends hinged, one end fixed other end free, etc.

According to Rankine's formulae,

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

where P_e = buckling load

P_c = crushing load

For short column $P_e >>>> Pc$

or $$\frac{1}{P_e} <<< \frac{1}{P_c} \Rightarrow \frac{1}{P_e} \text{ can be neglected ,}$$

$$\frac{1}{P_R} = \frac{1}{P_c}$$

$$P_R \approx P_c \approx A\sigma_c$$

For long columns $P_c >>> P_e$

$$\frac{1}{P_c} <<< \frac{1}{P_e} \Rightarrow \frac{1}{P_c} \text{ can be neglected}$$

$$P_R \approx P_e = \frac{\pi^2 EI_{min}}{L_e^2}$$

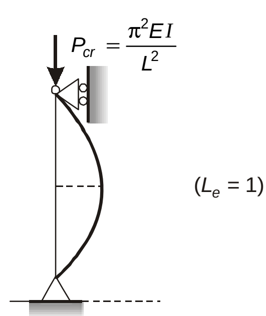
$\therefore$ $$P_R = \frac{\sigma_c A}{1 + c(Se)^2}$$

where, c = Rankine's constant and Se = slenderness ratio

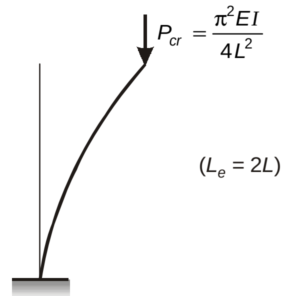
Solution : 19

(i) Buckling of columns under axial load with four different end conditions

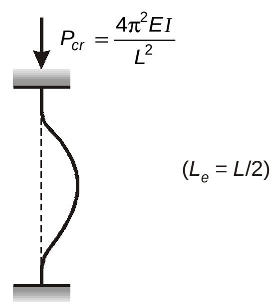
(a) Pinned Ends:



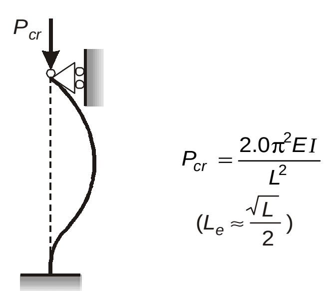
(b) One end fixed, other end free



(c) Both ends fixed



(d) One end fixed, other end pinned



$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

$$L_e \approx \frac{L}{\sqrt{2}}$$

Solution : 20

$$d_0 = 74.9 \text{ mm}$$

Solution : 21

$$P_{cr} = 55.51 \times 10^3 \text{ N}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = 37.01 \text{ N/mm}^2$$

$L = 769.5$ mm ≈ 0.77 m which is minimum length.

Solution : 22

Euler's buckling load = 8576 kN.

Solution : 23

$$P_{cr} = 8216.33 \text{ kN}$$
$$P_{cr} = 4108.167 \text{ kN}$$

Solution : 24

$$L = 11252 \text{ mm}$$
$$\text{Safe load} = 1185.12 \text{ kN}$$

■■■■

11 Pressure Vessel

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(a)
4.	(b)
5.	(c)
6.	(b)
7.	(b)
8.	(b)
9.	(b)
10.	(b)
11.	(b)

LEVEL 2 Objective Questions

12.	(c)
13.	(d)
14.	(a)
15.	(c)
16.	(c)
17.	(6.68)
18.	(34.17)
19.	(b)
20.	(d)

■■■■

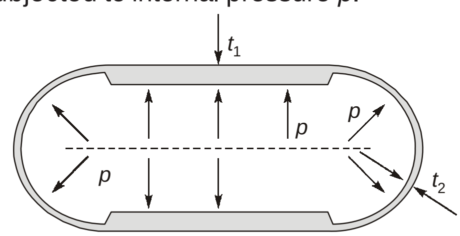
LEVEL 3 Conventional Questions

Solution : 21

In a pressure vessel, D is internal diameter and t is wall thickness, if $\frac{D}{t} > 20$, then it is a thin shell. When the shell is subjected to internal pressure, the hoop stress developed in the shell does not vary much across the thickness. If $\frac{D}{t} < 20$. Then it is thick shell; i.e., thickness of shell is considerable in comparison to diameter, then there is variation of hoop and radial stresses across the thickness of thick shell.

Figure shows a thin cylindrical shell with hemispherical ends, subjected to internal pressure p.

In cylindrical portion:



$$\text{Hoop stress, } \sigma_{h_c} = \frac{pD}{2t_1}$$

$$\text{Axial stress, } \sigma_{a_c} = \frac{pD}{4t_1}$$

$$\text{Hoop strain, } \varepsilon_{h_c} = \frac{pD}{2t_1E} - \frac{\nu pD}{4t_1E} = \frac{pD}{4t_1E}(2-\nu)$$

In hemispherical portion at junction

$$\text{Hoop stress, } \sigma_{h_s} = \frac{pD}{4t_2}$$

$$\text{Hoop strain, } \varepsilon_{h_s} = \frac{pD}{4t_2E}(1-\nu)$$

For no distortion

$$\varepsilon_{h_c} = \varepsilon_{h_s}$$

$$\frac{pD}{4t_1E}(2-\nu) = \frac{pD}{4t_2}(1-\nu)\frac{1}{E}$$

or

$$\frac{t_2}{t_1} = \frac{1-\nu}{2-\nu} = \frac{1-0.3}{2-.07} = \frac{0.7}{1.7} = \frac{7}{17}$$

$$\text{Maximum stress in cylindrical portion} = \frac{pD}{2t_1} = \sigma_{h_c}$$

$$\text{Maximum stress in hemispherical portion} = \frac{pD}{4t_2} = \sigma_{h_s}$$

$$\sigma_{h_c} = \sigma_{h_s}$$

$$\frac{pD}{2t_1} = \frac{pD}{4t_2}; \quad \frac{t_2}{t_1} = 0.5$$

Solution : 22

$$\text{Poisson's ratio, } \nu = 0.308$$

Solution : 23

$$\text{Force required to push the rod} = 91.25 \text{ kN}$$

Solution : 24

$$\text{Change in length} = 2.25 \times 10^{-4} \text{ m}$$
$$\text{Change in diameter} = 3.188 \times 10^{-4} \text{ m}$$
$$\text{Maximum shear stress} = 38.25 \text{ MPa}$$
$$\delta V = 1.67 \times 10^{-3} \text{ m}^3$$

Solution : 25

$$(\sigma_w)_P = 184.95 \text{ MPa}$$
$$(\sigma_c)_P = 184.95 - 82.575 = 102.37 \text{ MPa}$$

Solution : 26

$$\text{Thickness of the shell} = 7.55 \text{ mm}$$

Solution : 27

$$K = 2.497 \text{ Gpa}$$

Solution : 28

$$P = 2.86 \text{ N/mm}^2$$

Solution : 29

$$\tau_{max} = \frac{\sigma_t}{2} = 30 \text{ MPa}$$
$$\delta d = 0.199 \text{ mm}$$
$$\delta L = 0.117$$
$$\delta V = 559050 \text{ mm}^3 \text{ (increase)}$$

Solution : 30

$$I = 0.12784 \text{ mm}$$

Solution : 31

$$\text{Factor of safety} = 4.32$$

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