GATE PSUs

State Engg. Exams

MADE EASY

Workbook 2025

Detailed Explanations of Try Yourself *Questions*

Computer Science & IT

Algorithms

MADE EASY PUBLICATIONS

1 Loops Analysis, Asymptotic Notations, Recursive Algorithm and Methods to Solve Recurrence Relations

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

$$f(n) = \Omega(n),\ g(n) = O(n),\ h(n) = \Theta(n)$$

Then $[f(n) \cdot g(n)] + h(n)$

$$f(n) = \Omega(n)$$

i.e. $f(n)$ should be anything greater than or equal to 'n' lets take n.

$$g(n) = O(n)$$

i.e. $g(n)$ should be less than or equal to 'n' lets take n.

$$h(n) = \Theta(n)$$

i.e. $h(n)$ should be equal to n.

So $[f(n) \cdot g(n)] + h(n)$

$[n \cdot n] + n$

$$= \Theta n^2 + \Theta n = \Omega(n)$$

Here we only comment about lower bound. Upper bound depend an the $g(n)$ value i.e. n^2, n^3, n^4... etc.

T2 : Solution

The increasing order of given five functions are: $f_4 < f_2 < f_5 < f_1 < f_3$.

T3 : Solution

(b)

```
find (int n)
{
    if (n < 2) then return;
    else
    {
        sum = 0;
        for (i = 1; i ≤ 4; i++) find(n/2);    → O(log n)
        for (i = 1; i ≤ n * n; i++)           → O(n²)
        sum = sum +1;
    }
}
```

Since first for loop run 4log n times for which second for loop run n^2 times for every value of (4 log n).
So total time complexity = $O(4\log n \times n^2) = O(n^2 \log n)$.

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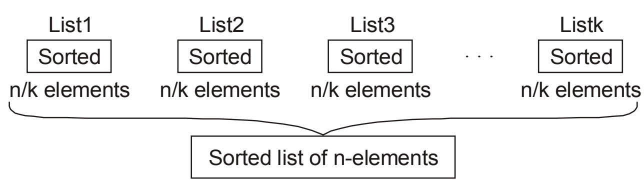
2 Divide and Conquer

Detailed Explanation of Try Yourself Questions

T1 : Solution

$O(n \log k)$



(*i*) Remove the smallest element from each list and build min heap with k-elements $\Rightarrow O(k)$.

(*ii*) Extract the minimum elements from this heap that will be the next smallest in the resulted list $\Rightarrow O(\log k)$.

(*iii*) Remove the elements from original list where we have extracted next smallest element and insert into the heap $\Rightarrow O(\log k)$.

Repeat step2 and step3 until all elements are in the resulted list

$$= O(k) + [O(\log k) + O(\log k)] * O(n)$$
$$= O(n \log k)$$

T2 : Solution

$O(mk + m \log (m/k))$

Insertion sort takes $O(k^2)$ time per k-element list in worst case. Therefore sorting n/k lists of k-element each take $O(k^2 n/k) = O(nk)$ time in worst case.

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3 Binary Trees, Binary Heaps and Greedy Algorithms

Detailed Explanation of Try Yourself Questions

T1 : Solution

O(E + V)

In adjacency list representation of directed graph to find the out degree of each vertex will take O(E+V) time in worst case i.e. for an element we have to search n time.

T2 : Solution

O(V + E)

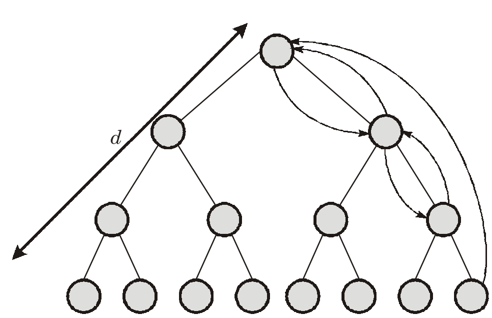
In adjacency matrix representation of directed graph to find universal sink will take O(V + E) time i.e. for every entry in adjacency matrix we have to check n time.

T3 : Solution

(b)

Heap is implemented using array. If 'i' is parent element then '$2i$' is left child and '$2i + 1$' is right child. So if an element is delete from last level of the heap then it will take O(1) time. Since element can be deleted from any level of heap tree in worst case root element is deleted then at every level one element is exchange.

Example:



Minimum O(d) time will take if a element is deleted in heap tree but not O(1).

T5 : Solution

O(n)

Sorting the array using **binary search tree** will take O(n) time i.e. inorder sequence.

Sorting the array using **min heap tree** will take O(n log n) time i.e. O(n) time to build and log n time to get every minimum element. So O(n) + O(n log n) = O(n log n).

In the giving question **binary search tree is better than min heap tree by O(n) time**.

T6 : Solution

(b)

```
max-heapify (int A[ ], int n, int i)
{
    int P, m;
    P = i;
    while (2P ≤ n) for checking whether left child is present.
    {
        if (2P + 1 ≤ n && A[2P + 1] > A[2P]) for checking if right child is present or not and finding between
        left and right child which is greater.
        n = 2P+1;
        else m = 2P;
        whichever is greater, swap that child with its parent
        if (A[P] < A[m])
        {
            Swap (A[P], A[m]);
            P = m;
        }
        else
    return;
    }
}
```

T7 : Solution

(7)

Kruskal's algorithm: AE, AG, AB, CE, FI, FH, CD, CF

Prim's algorithm: CF, CE, EA, AG, AB, FI, FH, CD (since we have to find maximum difference so start with edge CF)

$$\max\left|(e_{p_i})_{\text{Prim's}} - (e_{p_i})_{\text{Kruskal's}}\right| = |8 - 1| = 7$$

T8 : Solution

[O(m)]

Kruskal's algorithm:

(i) Sorting $\Rightarrow O(m \log m)$

(ii) Union $\Rightarrow O(n \log n)$

(iii) Find $\Rightarrow O(m \log n)$

$\Rightarrow$ Running time $= O(m \log m)$

Now edges are already sorted.

$\therefore$ Running time $= O(m)$

T9 : Solution

O(|V|)

Let e be an edge of G but not in T

(i) Run DFS on $T \cup \{e\}$

(ii) Find cycle

(iii) Trace back edges and find edge e′ thus has maximum weight.

(iv) Remove e′ from $T \cup \{e\}$ to get MST

In $T \cup \{e\}$ $\Rightarrow$ Number of eges = Number of vertices

$\therefore$ Running time of DFS $= O(|V|+|E|) = O(|V|)$

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4 Sorting Algorithms, Graph Traversals and Dynamic Programming

Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

```
Insertion-sort (A)
{
    for j ← 2 to length (A)
    {
        key ← A[j]
        i = [j – 1]
        while (i > 0 && A[i] > key)
        {
            A[i + 1] ← A[i]
            i = [i – 1]
        }
    A[i + 1] ← key;
    }
}
```

T2 : Solution

O(n)

The length of array A is n which stores the integers. We need two additional arrays B[0...k] and C[0...k]. Initialize B and C with 0. It requires O(k). For each element of A increment B[A[i]]. It will take O(n) time B[j] contain the number of elements of A having value j.

Do C[1] = B[1] and for each element i of array C do

$$C[i] = B[i] + [i - 1]$$

It will take $O(k)$ for getting answer compute $C[b] - C[a] + B[a]$.

The preprocessing time takes = $O(n)$

T3 : Solution

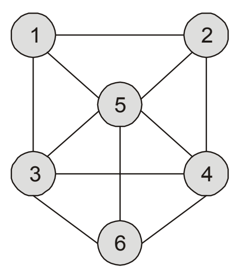
G is the connected graph with $n - 1$ edges $\Rightarrow$ G don't have any cycle.

Therefore statement 1 and 3 implies 2.

T4 : Solution

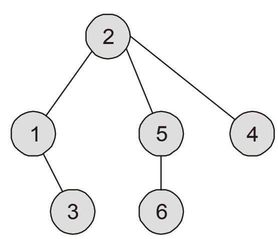
(d)

Lets take a undirected graph



(G) Graph

and 2 is source, after performing BFS on graph we obtain the following tree.



(T) Tree

Now missing edges are: 1 to 5, 4 to 5, 4 to 6, 3 to 6, 3 to 5, 3 to 4 for 1 to 5 = $d(u) - d(v)$

(Distance from 2 to 1) – (Distance from 2 to 5)

$$= 1 - 1 = 0$$

$$\text{for 4 to 5} = d(u) - d(v)$$

$$= 1 - 1 = 0$$

$$\text{for 4 to 6} = d(u) - d(v)$$

$$= 1 - 2$$

$$= -1 \text{ or } 1$$

$$\text{for 3 to 6} = d(u) - d(v)$$

$= 2 - 2 = 0$

for 3 to 5 $= d(u) - d(v)$

$= 2 - 1$

$= 1 \text{ or } -1$

for 3 to 4 $= d(u) - d(v)$

$= 2 - 1$

$= -1 \text{ or } 1$

So 2 is not possible

So answer is (d)

T5 : Solution

(64)

In question restriction on BST is height should be '6'. So we need 7 levels (given that root at height '0'). In creation of BST we have to use all element without repeatation.

At **1 level** = We have 2 choice i.e., either take 1 or 7.

At **2 level** = We have 2 choice for root 1 and 7 each. If 1 is root then 2 choice will be 6 and 2. If 7 is root then 2 choice will be 1 and 6.

At **3 level** = If we take 1 at root, 6 at 2nd level than we have 2 choice i.e., 5 and 2 at 3rd level.

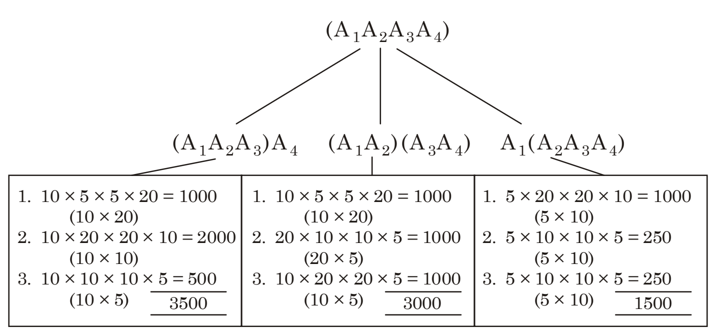
If we take 1 at root, 2 at 2nd level than we have 2 choice i.e., 6 and 3 at 3rd level. Similarly if we take 7 as root element.

So till 6th level, we have two choice at every level and for last level we left with only 1 element. So number of BST with height 6

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 = 2^6 = 64$$

T7 : Solution

(1500)



The minimum number of multiplication required using basic matrix multiplication method will be 1500.

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