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**WORKBOOK 2024**



**Detailed Explanations of  
Try Yourself Questions**

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**Electronics Engineering**  
Signals and Systems



**MADE EASY**  
Publications

# 1

## Introduction



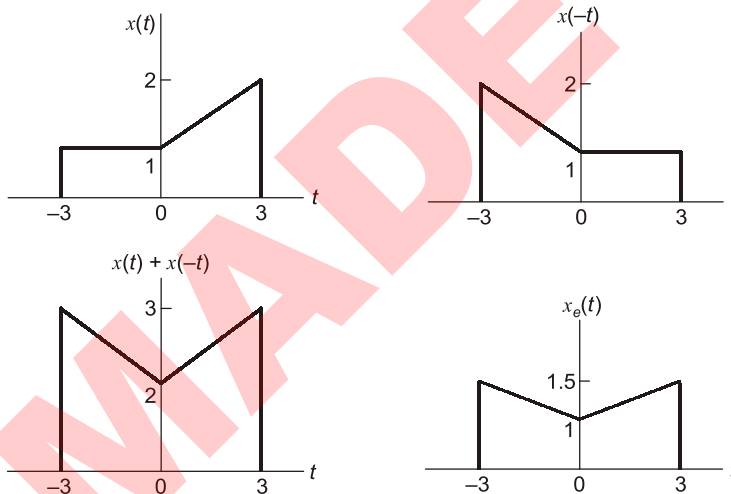
### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(a)

Even part of  $x(t)$ ,  $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Signal  $x_e(t)$  is obtained as follows:



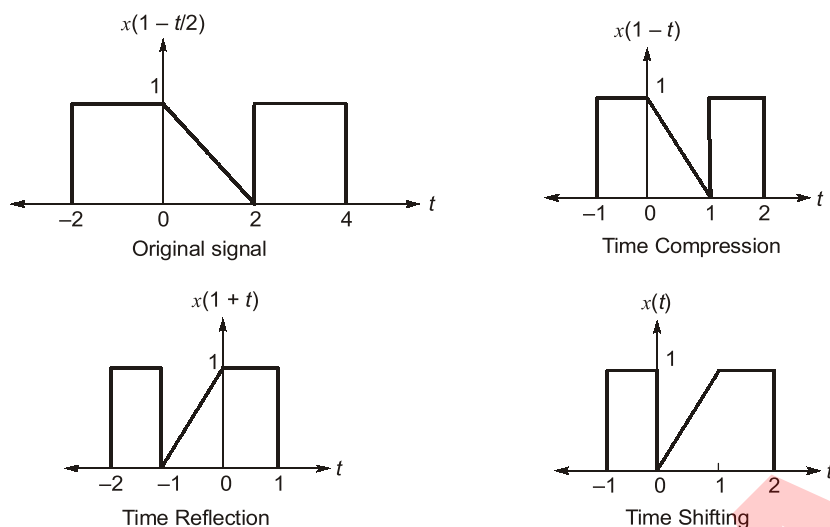
#### T2 : Solution

(c)

We can perform following sequence of transformation.

$$x\left(1 - \frac{t}{2}\right) \xrightarrow[\text{time compression}]{t \rightarrow 2t} x(1 - t) \xrightarrow[\text{folding}]{t \rightarrow -t} x(t + 1) \xrightarrow[\text{time shifting}]{t \rightarrow t - 1} x(t)$$

Graphically it is obtained as



**T3 : Solution**

(a)

The expression of  $x(t)$  is  $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 4k) - \delta(t - 4k - 1)$ .

So  $x(t)$  is a subtraction of two signals each periodic with period 4. So  $x(t)$  is periodic with period 4.

**T4 : Solution**

The signal is,

$$x(t) = 3e^{-t} u(t)$$

Now, energy of signal will be

$$E_x = \int_0^{\infty} [3e^{-t}]^2 dt = 4.5$$

**T5 : Solution**

(d)

$$\begin{aligned} y(t) &= 4^2 \cos^2\left(200t + \frac{\pi}{6}\right) \\ &= 4^2 \frac{1 + \cos 2\left(200t + \frac{\pi}{6}\right)}{2} \\ &= 8 + 8 \cos\left(400t + \frac{\pi}{3}\right) \end{aligned}$$

Thus the DC component is 8.

**T6 : Solution**

(b)

Cosine function is a periodic signal. As all periodic signals are power signals, therefore the given signal is power signal.

**T7 : Solution**

(a)

$$\int_{-\infty}^{\infty} \delta(t) \cos\left(\frac{3t}{2}\right) dt = f(0) = \cos\left(\frac{3 \times 0}{2}\right) = \cos 0 = 1$$

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# 2

## Fourier Series



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(a)

Given that Fourier series coefficient of  $x(t)$  is  $a_k$

So,  $x(t) \xrightarrow{\text{F.S.}} a_k$

Now, real part of  $x(t)$  is  $\frac{x(t) + x^*(t)}{2}$

and if

$x(t) \xrightarrow{\text{F.S.}} a_k$

then

$x^*(t) \xrightarrow{\text{F.S.}} a_{-k}^*$

So real part of  $x(t)$ ,  $\frac{x(t) + x^*(t)}{2} \xrightarrow{\text{F.S.}} \frac{a_k + a_{-k}^*}{2}$

#### T2 : Solution

(d)

#### T3 : Solution

(b)

#### T4 : Solution

Power of signals is

$$\sum_{-\infty}^{\infty} |C_n|^2 \Rightarrow \sum_{-2}^2 |C_n|^2$$

So power is

$$= \sum_{-2}^2 |C_n|^2 = (2)^2 + (8)^2 + (8)^2 + (2)^2 = 136$$

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# 3

## Fourier Transform



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(b)

The Fourier transform is  $X(\omega) = u(\omega) - u(\omega - 2)$ , we know that

- If signal is real then  $X(\omega)$  is conjugate symmetric.
- If signal is imaginary then  $X(\omega)$  is conjugate anti-symmetric

The given  $X(\omega)$  is neither conjugate symmetric nor conjugate anti-symmetric.

So  $x(t)$  is complex signal.

#### T2 : Solution

(c)

Fourier transform of  $x(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t)\cos(\omega t)dt - j \int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

If  $x(t)$  is odd, then  $x(t)\sin \omega t$  is an even function and  $x(t)\cos \omega t$  is an odd function.

So, 
$$\int_{-\infty}^{\infty} x(t)\cos(\omega t)dt = 0$$

and, 
$$X(j\omega) = j \int_{-\infty}^{\infty} x(t)\sin(\omega t)dt$$

or, 
$$X(j\omega) = -2j \int_0^{\infty} x(t)\sin(\omega t)dt$$

#### T3 : Solution

(a)

Given  $X(j\omega)$  is real and odd, so  $x(t)$  is imaginary and odd.

**T4 : Solution**

(a)

Fourier transform is  $G(\omega) = \frac{\omega^2 + 21}{\omega^2 + 9}$

So,  $G(\omega) = \frac{\omega^2}{\omega^2 + 9} + \frac{21}{\omega^2 + 9} = 1 + \frac{12}{\omega^2 + 9}$

As we know that Fourier transform of  $e^{-a|t|}$  is  $\frac{2a}{a^2 + \omega^2}$

So  $g(t) = \delta(t) + 2\exp(-3|t|)$

**T5 : Solution**

(d)

If,  $x(t) \xrightarrow{F} X(j\omega)$

then,  $\frac{dx(t)}{dt} \xrightarrow{F} (j\omega)X(j\omega)$  (Time differentiation property)

and,  $\frac{d^2x(t)}{dt^2} \xrightarrow{F} -\omega^2 X(j\omega)$

$\frac{d^2[x(t-2)]}{dt^2} \xrightarrow{F} -\omega^2 e^{-j2\omega} X(j\omega)$  (Time-shifting property)

**T6 : Solution**

(a)

We have,  $y(t) = \int_{-\infty}^t x(\tau) d\tau$

$\int_{-\infty}^t x(\tau) d\tau \xrightarrow{F} \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$  (Time integration property)

So,  $Y(j\omega) = \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$

$= \frac{1}{j\omega} \left( \frac{j\omega}{5 + \frac{j\omega}{10}} \right) + 0 = \frac{1}{\left( 5 + \frac{j\omega}{10} \right)}$   $X(0) = 0$

Now, area under  $y(t)$ ,  $\int_{-\infty}^{\infty} y(t) dt = Y(0)$

Thus,  $Y(0) = \frac{1}{5+0} = \frac{1}{5}$

**T7 : Solution**

(c)

Properties of distortionless system are:

- Magnitude should be constant w.r.t. frequency.
- Phase should depend linearly on frequency.

Only function given in option (c) follow the given conditions.

**T8 : Solution**

(a)

The signal  $x(t) = (2 + e^{-3t}) u(t)$  then final value i.e.  $x(\infty)$  will be 2.

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# 4

## Laplace Transform



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(b)

Convolution in time domain is multiplication in  $s$ -domain.

$$\therefore L[h(t)] = L[f(t)] \times L[g(t)] = \frac{1}{s+3}$$

#### T2 : Solution

(c)

$$r(t) \xleftrightarrow{\text{L.T.}} \frac{1}{s^2}$$

$$r(t-a) \xleftrightarrow{\text{L.T.}} e^{-as} \times \frac{1}{s^2} = \frac{e^{-as}}{s^2}$$

#### T3 : Solution

(c)

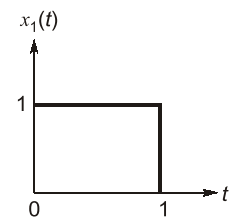
$$\lim_{t \rightarrow \infty} i(t) = \lim_{s \rightarrow 0} sI(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{2}{s(1+s)} = 2$$

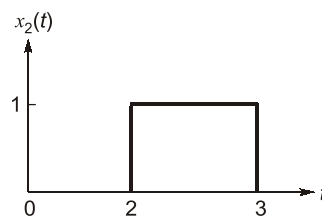
#### T4 : Solution

(b)

We can express the given function in terms of unit step function as follows:



$$x_1(t) = u(t) - u(t-1)$$



$$x_2(t) = u(t-2) - u(t-3)$$

Thus,

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) + x_3(t) + \dots \\ &= u(t) - u(t-1) + u(t-2) - u(t-3) + \dots \end{aligned}$$

We know that

$$u(t) \xrightarrow{L} \frac{1}{s}$$

$$u(t - t_0) \xrightarrow{L} \frac{1}{s} e^{-st_0}$$

(time-shifting)

The Laplace transform of  $x(t)$  is

$$\begin{aligned} X(s) &= \frac{1}{s} - \frac{1}{s} e^{-s} + \frac{1}{s} e^{-2s} - \frac{1}{s} e^{-3s} + \frac{1}{s} e^{-4s} - \frac{1}{s} e^{-5s} + \dots \\ &= \frac{1}{s} [1 + e^{-2s} + e^{-4s} + \dots] - \frac{1}{s} [e^{-s} + e^{-3s} + e^{-5s} + \dots] \\ &= \frac{1}{s} \left[ \frac{1}{1 - e^{-2s}} \right] - \frac{1}{s} \left[ \frac{e^{-s}}{1 - e^{-2s}} \right] \\ &= \frac{1}{s} \left[ \frac{1 - e^{-s}}{1 - e^{-2s}} \right] \\ &= \frac{1}{s} \left[ \frac{1}{1 + e^{-s}} \right] \end{aligned}$$

#### T5 : Solution

(d)

From the time integration property of Laplace transform

$$x(t) \xrightarrow{L} X(s)$$

$$\int_0^t x(\tau) d\tau \xrightarrow{L} \frac{1}{s} X(s)$$

Time integration Property

$$\int_0^t x(\tau) d\tau \xrightarrow{L} \frac{(s+1)}{s(s^2 + 4s + 5)}$$

#### T6 : Solution

(d)

Given,

$$H(s) = \frac{k(s^2 + \omega_0^2)}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$

So value of  $H(s)$  at  $s \rightarrow \infty$  is k  
and value of  $H(s)$  at  $s \rightarrow 0$  is k.  
So the filter is a band stop filter or notch filter.

**T7 : Solution**

(c)

$$X(s) = L[x(t)] = \frac{s}{s^2 + 1}$$

$$H(s) = L[h(t)] = \frac{1}{s^2 + 1}$$

$$y(t) = x(t) * h(t)$$

$$Y(s) = L[x(t) * h(t)] = X(s)H(s) = \frac{s}{(s^2 + 1)^2}$$

Using partial fractional,

$$Y(s) = \frac{-j/4}{(s-j)^2} + \frac{j/4}{(s+j)^2}$$

We know that  $te^{-at}u(t) \xleftrightarrow{L} \frac{1}{(s+a)^2}$

so,  $\frac{1}{(s-j)^2} \xleftrightarrow{L^{-1}} te^{jt}$

$$\frac{1}{(s+j)^2} \xleftrightarrow{L^{-1}} te^{-jt}$$

so,  $y(t) = \frac{j}{4}[-te^{jt} + te^{-jt}] = \frac{j}{4}t[e^{-jt} - e^{jt}] = \frac{t}{2}\sin t, \quad t \geq 0$

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# 5

## Sampling Theorem and Discrete Time System



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(c)

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$

#### T2 : Solution

(c)

$$\begin{aligned} y[n] &= \sum_{n=-\infty}^{\infty} n^2 \delta[n+2] \\ &= n^2 \Big|_{n=-2} \\ &= (-2)^2 = 4 \end{aligned}$$

$$\sum_{n=-\infty}^{\infty} x[n] \delta[n - n_0] = x[n_0]$$

#### T3 : Solution

(d)

(A)

$$\begin{aligned} y[n] &= x[n^2] \\ x_1[n] \rightarrow y_1[n] &= x_1[n^2] \\ x_2[n] \rightarrow y_2[n] &= x_2[n^2] \\ ax_1[n] + bx_2[n] &\rightarrow ax_1[n^2] + bx_2[n^2] \\ &= ay_1[n] + by_2[n] \end{aligned}$$

Hence the system is linear.

(B)

$$y[n] = x^2[n - 1]$$

For a delayed input  $x[n - n_0]$ , output is

$$y[n, n_0] = x^2[n - n_0 - 1]$$

The delayed output

$$y[n - n_0] = x^2[n - n_0 - 1]$$

Since

$$y[n, n_0] = y[n - n_0]$$

Hence the system is time-invariant.

(C)

$$y[n] = x[n] + n$$

$y[n]$  depends on present value of  $x[n]$ , so the system is causal.

(D)

$$y[n] = x[3n]$$

$$y[-1] = x[-3]$$

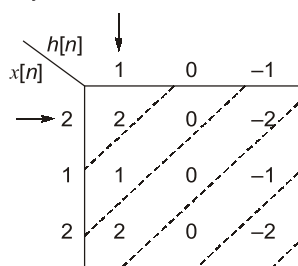
$$y[1] = x[3]$$

System has memory, therefore it is a dynamic system.

#### T4 : Solution

(c)

Since  $x[n]$  is even symmetric about mid point ( $n = 1$ ) and  $h[n]$  is odd symmetric about mid point ( $n = 1$ ) so  $y[n]$  will be odd symmetric about its mid point  $n = 2$ .



$$y[n] = x[n] * h[n] = \{2, 1, 0, -1, -2\}$$

$y[n]$  is odd symmetric about  $n = 2$ .

#### T5 : Solution

(a)

Causality:

$$h[n] = 0, n < 0$$

The system is causal.

Stability:

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |h[n]| &= 2 \sum_{n=0}^{\infty} (0.4)^n - \sum_{n=0}^{\infty} (0.2)^n \\ &= 2 \left[ \frac{1}{1-0.4} \right] - \frac{1}{(1-0.2)} < \infty \end{aligned}$$

The system is stable.

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# 6

## Z-Transform



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(b)

Given that,

$$x(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots$$

$$x(n) = u(n)$$

$$X(z) = \text{Z.T.}[u(n)]$$

$$X(z) = \frac{z}{z-1}$$

#### T2 : Solution

(c)

z-transform of  $x[n]$ ,

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \alpha^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty} \alpha^{-n} z^{-n} u[n]$$

$$= \sum_{n=0}^{\infty} (\alpha z^{-1})^n + \sum_{n=0}^{\infty} (\alpha z)^{-n} = \underbrace{\frac{1}{1-\alpha z^{-1}}}_I + \underbrace{\frac{1}{1-(\alpha z)^{-1}}}_{II}$$

Series I converges, if  $\alpha z^{-1} < 1$  or  $|z| > |\alpha|$

Series II converges, if  $(\alpha z)^{-1} < 1$  or  $\alpha z > 1$  or  $|z| > \frac{1}{|\alpha|}$

So, ROC is intersection of both

$$\text{ROC : } |z| > \max\left(|\alpha|, \frac{1}{|\alpha|}\right)$$

**T3 : Solution**

(b)

$$X(z) = \frac{z+1}{z(z-1)}$$

$$= -\frac{1}{z} + \frac{2}{z-1} = -\frac{1}{z} + 2z^{-1} \left( \frac{z}{z-1} \right) \quad \text{By partial fraction}$$

Taking inverse z-transform

$$x[n] = -\delta[n-1] + 2u[n-1]$$

$$x[0] = -0 + 0 = 0$$

$$x[1] = -1 + 2 = 1$$

$$x[2] = -0 + 2 = 2$$

**T4 : Solution**

(c)

By taking z-transform of  $x[n]$  and  $h[n]$

$$H(z) = 1 + 2z^{-1} - z^{-3} + z^{-4}$$

$$X(z) = 1 + 3z^{-1} - z^{-2} - 2z^{-3}$$

From the convolution property of z-transform

$$Y(z) = H(z) X(z)$$

$$Y(z) = 1 + 5z^{-1} + 5z^{-2} - 5z^{-3} - 6z^{-4} + 4z^{-5} + z^{-6} - 2z^{-7}$$

$$y[n] = \{1, 5, 5, -5, -6, 4, 1, -2\}$$

$$y[4] = -6$$

Sequence is

**T5 : Solution**

(d)

Given that  $x(n)$  is right sided and real,  $X(z)$  has two poles, two zeros at origin and one pole at  $e^{j\pi/2}$ ,  $X(1) = 1$ . Since  $x(n)$  is real so poles of  $X(z)$  should be in conjugate pairs so other pole will be at  $e^{-j\pi/2}$ .

So,

$$X(z) = \frac{k z^2}{(z - e^{-j\pi/2})(z - e^{+j\pi/2})} = \frac{k z^2}{z^2 + 1}$$

Since,

$$X(1) = 1 \quad \text{so, } k = 2$$

So,

$$X(z) = \frac{2z^2}{z^2 + 1} \quad \text{and } |z| > 1$$

**T6 : Solution**

(a)

We know that,

$$\alpha^n u[n] \xleftrightarrow{z} \frac{z}{z - \alpha}$$

$$\alpha^{n-10} u[n-10] \xleftrightarrow{z} \frac{z^{-10} z}{z - \alpha} \quad \text{(time shifting property)}$$

**T7 : Solution**

(b)

$$Y(z) - \frac{1}{3}z^{-1}Y(z) = X(z)$$

$$Y(z) \left[ 1 - \frac{1}{3}z^{-1} \right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

where,

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

 $\therefore$ 

$$Y(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)} = \frac{3}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}}$$

$$y[n] = \left[ 3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n \right] u[n]$$

**T8 : Solution**

(a)

We know that convolution of  $x[n]$  with unit step function  $u[n]$  is given by

$$x[n] * u[n] = \sum_{k=-\infty}^{\infty} x[k]$$

so,

$$y[n] = x[n] * u[n]$$

Taking z-transform on both sides

$$Y(z) = X(z) \frac{z}{(z-1)} = X(z) \frac{1}{(1-z^{-1})}$$

Transfer function,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-z^{-1})}$$

Now, consider the inverse system of  $H(z)$ , let impulse response of the inverse system is given by  $H_1(z)$ , then we can write

$$H(z)H_1(z) = 1$$

$$H_1(z) = \frac{X(z)}{Y(z)} = 1 - z^{-1}$$

$$(1 - z^{-1})Y(z) = X(z)$$

$$Y(z) - z^{-1}Y(z) = X(z)$$

Taking inverse z-transform

$$y[n] - y[n-1] = x[n]$$

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# 7

## DTFT, DTFS & DFT



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

(c)

Since

$$x[n] \xrightarrow{\text{DTFT}} X(e^{j\Omega})$$

Thus

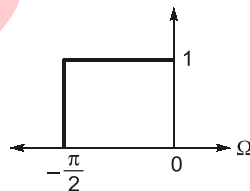
$$e^{j\Omega_0 n} x[n] \xrightarrow{\text{DTFT}} X(e^{j(\Omega - \Omega_0)})$$

(Frequency shifting property)

$$\Omega_0 = -\pi/4$$

$$e^{-j\frac{\pi}{4}n} x[n] \xrightarrow{\text{DTFT}} X(e^{j(\Omega + \pi/4)})$$

The graph of  $X(e^{j\Omega})$  is shifting to left by  $\frac{\pi}{4}$  units. So, DTFT of  $e^{-j\frac{\pi}{4}n} x[n]$  is



#### T2 : Solution

(a)

N-point DFT is given as

$$X_{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}}, k = 0, 1, \dots, N-1$$

$$X_{DFT}[k] = \sum_{n=0}^3 x[n] e^{-j\frac{\pi nk}{2}}$$

$$\because N = 4$$

For  $k = 0$ ,

$$\begin{aligned} X_{DFT}[0] &= \sum_{n=0}^3 x[n] \\ &= x[0] + x[1] + x[2] + x[3] \\ &= \cos 0 + \cos \pi + \cos 2\pi + \cos 3\pi \\ &= 1 - 1 + 1 - 1 = 0 \end{aligned}$$

For  $k = 1$ ,

$$\begin{aligned} X_{DFT}[1] &= \sum_{n=0}^3 x[n] e^{-j\frac{\pi n}{2}} \\ &= x[0]e^0 + x[1]e^{-j\frac{\pi}{2}} + x[2]e^{-j\pi} + x[3]e^{-j\frac{3\pi}{2}} \\ &= \cos 0 + \cos \pi(-j) + \cos 2\pi(-1) + \cos 3\pi(j) \\ &= 1 + (-1)(-j) + 1(-1) + (-1)(j) \\ &= 1 + j - 1 - j \\ &= 0 \end{aligned}$$

Similarly we can obtain  $X_{DFT}[2]$  and  $X_{DFT}[3]$  for  $k = 2$  and  $k = 3$  respectively,

$$\begin{aligned} X_{DFT}[2] &= 1 + 1 + 1 + 1 = 4 \\ X_{DFT}[3] &= 1 - j - 1 + j = 0 \\ X_{DFT}[k] &= \{0, 0, 4, 0\} \end{aligned}$$

**T3 : Solution**

(c)

$$\begin{aligned} X(e^{j\Omega}) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-2}^2 e^{-j\Omega n} \\ &= e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega} \\ &= e^{-j2\Omega} (1 + e^{j\Omega} + e^{j2\Omega} + e^{j3\Omega} + e^{j4\Omega}) \\ &= e^{-j2\Omega} \frac{(1 - e^{j5\Omega})}{1 - e^{j\Omega}} \quad \text{(Summation of finite GP)} \\ &= \frac{e^{-j5\pi/2} - e^{j5\Omega/2}}{e^{-j\pi/2} - e^{j\Omega/2}} = \frac{\sin 2.5\Omega}{\sin 0.5\Omega} \end{aligned}$$

**T4 : Solution**

(b)

$$\begin{aligned} X(e^{j\Omega}) &= j4 \sin 4\Omega - 1 \\ &= 2(e^{j4\Omega} - e^{-j4\Omega}) - 1 \end{aligned}$$

Taking inverse Fourier transform, we have

$$x[n] = 2\delta[n+4] - 2\delta[n-4] - \delta[n]$$

Since,  $\delta[n - n_0] \xrightarrow{DTFT} e^{-j\Omega n_0}$

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