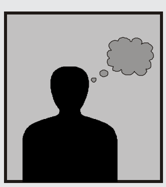
ESE GATE PSUs

State Engg. Exams

MADE EASY

Workbook 2024



Detailed Explanations of Try Yourself *Questions*

Electronics Engineering

Electromagnetics

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Publications

1 Vector Analysis

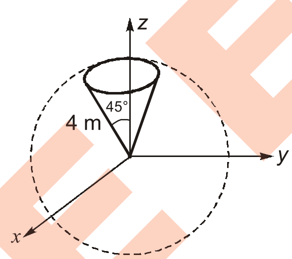
T1. Sol.

$$\bar{A} = \nabla f = 4xyz\hat{a}_x + 2x^2 z\hat{a}_y + 2x^2 y\hat{a}_z$$

$$(0,0,0) \xrightarrow{dx\,\hat{a}_x} (2,0,0) \xrightarrow{dy\,\hat{a}_y} (2,7,0) \xrightarrow{dz\,\hat{a}_z} (2,7,4)$$

$$\therefore \quad \int \bar{A}\cdot d\bar{l} = \int 4xyz\,dx \,@\, \begin{matrix} y=0 \\ z=0 \end{matrix} + \int 2x^2 z\,dy \,@\, \begin{matrix} z=0 \\ x=2 \end{matrix} + \int_{z=0}^{4} 2x^2 y\,dz \,@\, \begin{matrix} x=2 \\ y=7 \end{matrix} = 224$$

T2. Sol.



$\oint \bar{D}\cdot d\bar{s}$:

Spherical co-ordinate system : $\hat{a}_r\ \hat{a}_\theta\ \hat{a}_\phi$ / $dr\ d\theta\ d\phi$ / $r\sin\theta$

$$\int \frac{5r^2}{4}\cdot r^2 \sin\theta\, d\theta\, d\phi \,@\, \begin{matrix} \theta = 0, \dfrac{\pi}{4} \\ d = 0, 2\pi \end{matrix}$$

$$= 589.1\ \text{C}$$

$\int (\nabla \cdot F)\, dV$:

$$\nabla\cdot\bar{D} = 5r$$

$$dV = r^2 \sin\theta\ d\theta\ d\theta\ dr$$

T3. Sol.

$$\nabla\cdot F = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}[\rho(\sin^2\phi)] + \frac{\partial}{\partial z}(-z)$$

$$= 2 + 2\sin\phi\cos\phi - 1 = 1 + \sin 2\phi$$

$$\nabla\cdot F = 1 + \sin 2\phi$$

If $\phi = 0$, $\quad \nabla\cdot F = 1$

If $\phi = \dfrac{\pi}{2}$, $\quad \nabla\cdot F = 1$

If $\phi = \frac{\pi}{4}$, $\qquad \nabla \cdot F = 2$

Hence, option (d) satisfied.

T4. Sol.

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{1}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(-r^2\sin\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(10\cos\phi)$$

$$\nabla \cdot \vec{A} = \frac{1}{r^2} - 2r\cos\theta - \frac{10\sin\phi}{r\sin\theta}$$

$$\nabla \cdot \vec{A}\Big|_{\left(2,\frac{\pi}{4},\frac{\pi}{2}\right)} = \frac{1}{4} - \frac{4}{\sqrt{2}} - \frac{10}{2 \times \frac{1}{\sqrt{2}}} = \frac{1}{4} - 7\sqrt{2} = -9.65$$

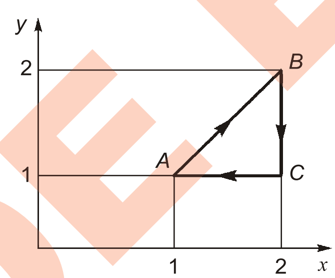
T5. Sol.

$$\nabla \cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot Kr^n) = \frac{1}{r^2}K(n+2)\,r^{n+2}$$

$$\nabla \times A = 0$$

Hence, option (d) is correct.

T6. Sol.



$$\oint \bar{A} \cdot d\bar{l} = \left[\int_A^B + \int_B^C + \int_C^A\right]\bar{A} \cdot d\bar{l}$$

$$\left.\begin{aligned}\bar{A} &= 3x^2y^3\hat{a}_x - x^3y^2\,\hat{a}_y \\ d\bar{l} &= dx\,\hat{a}_x + dy\,\hat{a}_y\end{aligned}\right\}\ \bar{A} \cdot d\bar{l} = 3x^2y^3dx - x^3y^2dy$$

Path AB : $y = x \Rightarrow dy = dx$

$$\int \bar{A} \cdot d\bar{l} = \int 3x^2y^3dx - x^3y^2dy = \int 3x^5 - x^5\,dx = \int_{x=1}^{2} 2x^5\,dx = 2 \cdot \frac{x^6}{6}\bigg|_1^2 = 21$$

Path BC : $x = 2 \quad \Rightarrow dx = 0$

$$\int \bar{A} \cdot d\bar{l} = -\int x^3y^2\,dy = -x^3 \int_{y=2}^{1} y^2\,dy \ @\ x = 2 = -8 \times \frac{y^3}{3}\bigg|_2^1 = +\frac{56}{3}$$

Path CA : $y = 1 \quad \Rightarrow dy = 0$

$$\int \bar{A} \cdot d\bar{l} = \int 3x^2 y^3 \, dx = 3y^3 \int_{x=2}^{1} x^2 \, dx \ @ \ y = 1 = 3 \cdot \left. \frac{x^3}{3} \right|_2^1 = -7$$

$\therefore$

$$\oint \bar{A} \cdot d\bar{l} = 21 + \frac{56}{3} - 7 = \frac{98}{3}$$

$\int (\nabla \times \bar{A}) \cdot d\bar{s}$:

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3 & -x^3 y^2 & 0 \end{vmatrix} = -12x^2y^2\hat{a}_z$$

$$d\bar{s} = dx \, dy \, (-\hat{a}_z)$$

$\therefore$

$$\int \nabla \times \bar{A} \cdot d\bar{s} = 12 \int_{x=1}^{2} x^2 \, dx \int_{y=1}^{x} y^2 \, dy = 12 \int_{x=1}^{2} x^2 \, dx \left. \frac{y^3}{3} \right|_{y=1}^{x} = \frac{12}{3} \int_{x=1}^{2} x^2 \, dx (x^3 - 1)$$

$$= 4 \left[\int_1^2 x^5 dx - \int_1^2 x^2 \, dx \right] = \frac{98}{3}$$

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2 Maxwell's Equations and Boundary Conditions

T1. Sol.

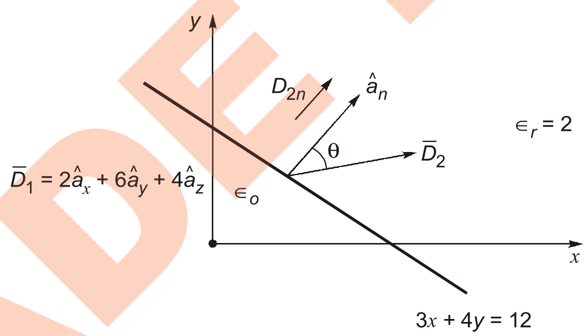
$$\phi = Q_{enc} = \int \rho_v \, dV = \int (\nabla \cdot \bar{D})\, dV$$

$$\Rightarrow \quad \phi = \int_v (y + x + z) \cdot dx\, dy\, dz = \int_{y=-2}^{2} y dy \int_{x=1}^{4} dx \int_{z=-1}^{2} dz + \int_{x=1}^{4} x dx \int_{y=-2}^{2} dy \int_{z=-1}^{2} dz + \int_{z=-1}^{2} z dz \int_{x=1}^{4} dx \int_{y=-2}^{2} dy$$

$$\Rightarrow \quad \phi = \left.\frac{y^2}{2}\right|_{-2}^{2} \cdot x\Big|_1^4 \cdot z\Big|_{-1}^{2} + \left.\frac{x^2}{2}\right|_1^4 \cdot y\Big|_{-2}^{2} \cdot z\Big|_{-1}^{2} + \left.\frac{z^2}{2}\right|_{-1}^{2} \cdot x\Big|_1^4 \cdot y\Big|_{-2}^{2}$$

$$\Rightarrow \quad \phi = 0 + \frac{15}{2} \cdot 4.3 + \frac{3}{2} \cdot 3.4 = 90 + 18 = 108 \text{ C}$$

T2. Sol.



$$3x + 4y = 12$$

@ $x = 0$; $y = 3$

@ $y = 0$; $x = 4$

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{3\hat{a}_x + 4\hat{a}_y}{5}$$

Normal component :

$$\therefore \quad D_{1n} = \bar{D}_1 \cdot \hat{a}_n = 2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z \cdot \frac{3\hat{a}_x + 4\hat{a}_y}{5} = \frac{6 + 24}{5} = 6$$

$$\therefore \quad \bar{D}_{1n} = D_{1n} \cdot \hat{a}_n$$

$$\Rightarrow \quad D_{1n} = 6\left\{\frac{3\hat{a}_x + 4\hat{a}_y}{5}\right\} = \boxed{3.6\hat{a}_x + 4.8\hat{a}_y = \bar{D}_{2n}}$$

Tangential component:

$$\bar{E}_{1t} = \bar{E}_{2t}$$

$$\Rightarrow \quad \frac{\bar{D}_{1t}}{\epsilon_1} = \frac{\bar{D}_{2t}}{\epsilon_2}$$

$$\Rightarrow \quad \bar{D}_{2t} = \frac{\epsilon_2}{\epsilon_1}\bar{D}_{1t} = \frac{2\epsilon_o}{\epsilon_o}\{\bar{D}_1 - \bar{D}_{1n}\} = 2\{(2, 6, 4) - (3.6, 4.8, 0)\}$$

$$= 2\{-1.6, 1.2, 4\}$$

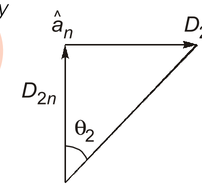
$$= -3.2, 2.4, 8$$

$$\therefore \quad \bar{D}_{2t} = -3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z$$

Hence,

$$\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2n} = -3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z + 3.6\hat{a}_x + 4.8\hat{a}_y$$

$$\Rightarrow \quad \bar{D}_2 = 0.4\hat{a}_x + 7.2\hat{a}_y + 8\hat{a}_z \text{ V/m}$$



$$\cos\theta_2 = \frac{|D_{2n}|}{|D_2|} = \frac{6}{\sqrt{0.4^2 + 7.2^2 + 8^2}} = 56.14^\circ$$

T3. Sol.

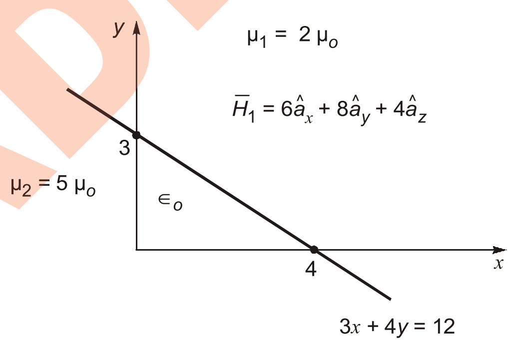
Given $\rho_s = 0$

$\therefore$ From boundary conditions;

$\bar{E}_{2t} = \bar{E}_{1t}$ } a is wrong, b is correct

$\bar{D}_2n = \bar{D}_{1n}$ } c is correct, d is wrong

T4. Sol.



$$3x + 4y = 12$$

@ $x = 0$; $y = 3$

@ $y = 0$; $x = 4$

As $\bar{H}_1 = 6\hat{a}_x + 8\hat{a}_y + 4\hat{a}_z$

$$\Rightarrow \quad \bar{H}_{1n} = (\bar{H}, \hat{a}_n)\,\hat{a}_n = \left\{(6, 8, 4)\cdot\left(\frac{3, 4, 0}{5}\right)\right\}\left(\frac{3, 4, 0}{5}\right)$$

$$= \left\{\frac{18 + 32}{25}\right\}(3\hat{a}_x + 4\hat{a}_y) = 6\hat{a}_x + 8\hat{a}_y$$

$\therefore \quad \bar{H}_{1t} = \bar{H}_1 - H_{1n} = (6, 8, 4) - (6, 8, 0) = 4\hat{a}_z$

Now, @ $\bar{K} = 0$, $\bar{H}_{1t} = \bar{H}_{2t} \Rightarrow \bar{H}_{2t} = 4\hat{a}_z$

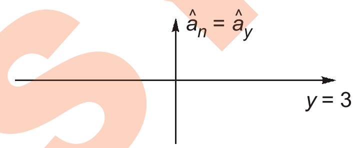
Also, $\quad \bar{B}_{1n} = B_{2n}$

$\Rightarrow \quad \mu_1 \bar{H}_{1n} = \mu_2 \bar{H}_{2n}$

$\Rightarrow \quad \bar{H}_{2n} = \dfrac{\mu_1}{\mu_2}\bar{H}_{1n} = \dfrac{2}{5}\{6\hat{a}_x + 2\hat{a}_y\} = 2.4\hat{a}_x + 0.8\hat{a}_y$

$\therefore \quad \bar{H}_2 = \bar{H}_{2t} + \bar{H}_{2n} = 2.4\hat{a}_x + 0.8\hat{a}_y + 4\hat{a}_z$

T5. Sol.



$$(\bar{H}_{1t} - \bar{H}_{2t}) \times \hat{a}_{n12} = \bar{K}$$

$\therefore \quad [(4\hat{a}_x + 7\hat{a}_y - 5\hat{a}_z) - (8\hat{a}_x + 14\hat{a}_y - 5\hat{a}_z)] \times \hat{a}_y = \bar{K}$

$\Rightarrow \quad [-4\hat{a}_x - 7\hat{a}_y] \times \hat{a}_y = \bar{K}$

$\therefore \quad -4\hat{a}_z = \bar{K}$

T6. (c)

Relative motion always causes induced emf which is absent in option (c).

T7. Sol.

Source free space, $\rho_v = 0$, $\bar{J} = 0$, $\sigma = 0$

(a) $\nabla \cdot \bar{D} = \rho_v = 0 \Rightarrow \nabla \cdot \bar{E} = 0$

(b) $\nabla \cdot \bar{B} = 0$

(c) $\nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t}$

(d) $\nabla \times \bar{H} = \bar{J} + \dfrac{\partial \bar{D}}{\partial t} = \sigma\bar{E} + \in \dfrac{\partial \bar{E}}{\partial t}$

$\Rightarrow \quad \nabla \times \bar{H} = \in \dfrac{\partial \bar{E}}{\partial t}$

$\Rightarrow \quad \nabla \times \bar{B} = \mu_o \in_o \dfrac{\partial E}{\partial t}$

$\Rightarrow \quad \nabla \times \bar{B} - \dfrac{1}{c^2}\dfrac{\partial \bar{E}}{\partial t} = 0$

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3 Electromagnetic Waves

T1. Sol.

(a) Attenuation in *Y*-direction and propagation in *ZX*-direction. (Invalid)

(b) Valid

(c) Valid

(d) Invalid- $\frac{\omega}{\beta} = 2 \times 10^8$ m/s not 3×10^8 m/s

(e) Invalid-*E* or *H* having phase shift

(f) Invalid-H is not orthogonal to propagation.

T2. (c)

Heaviest attenuation in case-3 and highest loss tangent.

T3. (c)

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu \cdot j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)} = j\omega\sqrt{\mu\epsilon}\left(\sqrt{1 + \frac{j\sigma}{\omega\epsilon}}\right)$$

T4. Sol.

$$V_p \text{ of the wave} = \frac{0.6}{5 \times 10^{-9}} = 1.20 \times 10^8 \text{ m/s}$$

$$\lambda = \frac{1.20 \times 10^8}{1} = 120 \text{ MHz}$$

T5. (b)

$$\delta = \frac{1}{\alpha} \quad \text{where } \alpha \text{ is zero (attenuation constant)}$$

$$\delta = \infty \text{ for } \sigma = 0$$

T6. Sol.

(i) $\beta = 250$ rad/m

(ii) $V_p = \frac{\omega}{\beta}$ $\quad \omega = 3 \times 10^8 \times 250 \quad ; \quad \omega = 75 \times 10^9$ rad/sec

(iii) $$\beta = 250 = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{\pi}{125} \text{ m}$$

(iv) $$\eta = 120\ \Omega$$

(v) $$H_s = \frac{200\angle 30°}{120\pi} e^{-j250z} a_y \text{ A/m}$$

T7. (d)

For a good conducting medium

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

∴ Phase velocity

$$v_p = \left(\frac{\omega}{\beta}\right) = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu\sigma}} = \sqrt{\frac{4\pi f}{\mu\sigma}} = 2\sqrt{\frac{\pi f}{\mu\sigma}}$$

T8. Sol.

$\frac{\sigma}{\omega\epsilon}$ decides type of material

$$\frac{12\times 10^2}{2\pi\times 10^7 \times \frac{1}{36\pi\times 10^9}} >> 1$$

T9. Sol.

$$\frac{J_c}{J_d} = 1 \quad ; \quad \frac{\sigma}{\omega\epsilon} = 1$$

$$n\text{'s phase} = E_x \text{ to } H_y \text{ phase} = \frac{\tan^{-1}(\sigma/\omega\epsilon)}{2} = 22.5°$$

T10. Sol.

$$P_{avg} = \frac{1}{2}(E\times H^*)$$

$$\frac{100}{2}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ j & 2 & -j \\ -1 & -j & 1 \end{vmatrix}$$

$$= 50(3\hat{a}_x + 3\hat{a}_z) = 150(\hat{a}_x + \hat{a}_z)$$

T11. (a)

Linear: In phase components.

T12. Sol.

$$\eta = \frac{25}{1.2}\angle 35^\circ = \frac{E_x}{H_y}$$

T13. Sol.

(0, 0, 0) to (1, 1, 1) - propagation directions

$$\beta_x = \beta_y = \beta_z = \frac{\beta}{\sqrt{3}}$$

Wave polarized in *YZ* plane

$$E \text{ direction} = K_1 \hat{a}_y + K_2 \hat{a}_z$$

$$K_1 = K_2 \text{ as } \beta_y = \beta z$$

$$\beta_y \cdot K_1 + \beta_z K_2 = 0$$

$$E(x, y, z, t)_{(y,z)} = E_o e^{j(\omega t - \beta/\sqrt{3}(x+y+z))} \frac{(\hat{a}_y - \hat{a}_z)}{\sqrt{2}} \text{ or } \frac{(-\hat{a}_y + \hat{a}_z)}{\sqrt{2}}$$

T14. Sol.

$E(y, z, t)_{(y,z)}$ – *E* direction and propagation direction in same plane.

T15. Sol.

(i) $$V_p = \frac{2\pi \times 10^7}{0.8} \neq 3 \times 10^8 \quad \text{Wrong}$$

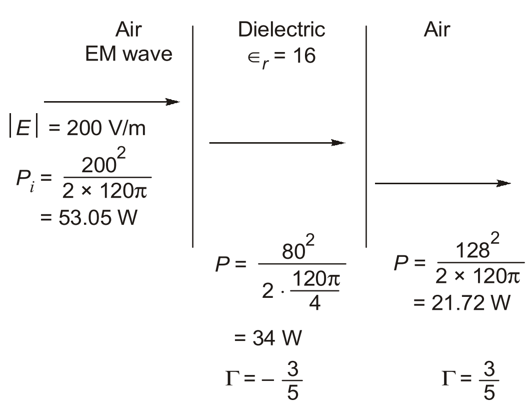
(ii) Wave has $\alpha = 0$

So, lossless medium but not conductor. As conductor has heavy loss.

(iii) $$V_p = \frac{2\pi \times 10^7}{0.8} = 0.78 \times 10^8 \text{ m/s}$$

(iv) $$\text{Power density} = \frac{1}{2} \times \frac{4^2}{120\pi} \times \sqrt{\epsilon_R} = 21\sqrt{\epsilon_R}\,\frac{\text{mW}}{\text{m}^2}$$

T16. Sol.



$$\tau_E = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2 \times \frac{1}{4}}{\frac{1}{4} + 1} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{2}{5}$$

$\therefore$ $$\frac{E_t}{E_i} = \tau_E \Rightarrow E_t = \frac{2}{5} \times 200 = 80 \text{ V/m}$$

$$\tau_E = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2}{\frac{1}{4} + 1} = \frac{2}{5} \times 4 = \frac{8}{5}$$

$\therefore$ $$E_t = \tau_E \times E_i = \frac{8}{5} \times 80 = 128 \text{ V/m}$$

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4 Transmission Lines

T1. Sol.

25 dB per 25 km

1 dB/km is the attenuation rate

2.5 dB corresponds to 2.5 km

$$D = 2.5 \text{ km}$$

T2. (d)

$$\Gamma \text{ in first case } = \frac{2Z - Z}{2Z + Z} = \frac{1}{3}$$

$$\Gamma \text{ in second case } = \frac{Z/2 - Z}{Z/2 + Z} = -\frac{1}{3}$$

$$\text{Power reflection coefficient } = \frac{1}{9}$$

Same power in both case.

T3. (c)

$$\frac{\beta}{\omega L} = \frac{\omega\sqrt{LC}}{\omega L} = \sqrt{\frac{C}{L}} = \frac{1}{Z_o}$$

T4. (b)

Distortionless line has $\frac{L}{R} = \frac{C}{G}$

$$\alpha = \sqrt{RG} = \sqrt{RG} = \sqrt{R \cdot \frac{RC}{L}} = R\sqrt{\frac{C}{L}}$$

T5. (a)

$$I(x) = I_L \cosh(rx) + \frac{V_L}{Z_o}\sinh(rx)$$

with $V_L = 0$ at short circuit, $I(x) = I_L \cosh(rx)$

T6. (c)

Z_0 depends on physical dimensions but never on the length of the line.

T7. Sol.

$$l = 2.75\lambda$$

$$\text{Periodicity of } Z(x) = \frac{\lambda}{2}$$

$$Z\left(x + \frac{\lambda}{2}\right) = Z(x)$$

$$Z(2.7\lambda) = Z(0.25\lambda) = Z\left(\frac{\lambda}{4}\right)$$

(i) Z_{in} due to 25 Ω = $\frac{50^2}{25} = 100\ \Omega$

(ii) 100 Ω - Source
100 Ω - input impedance
Γ is zero at the input.

(iii) 0.5 A – divides into 100 Ω of the source and line.

$$I_{in} = \frac{0.5}{2} = 0.25\text{ A} = I_L$$

$$\text{Due to lossless line,}\quad \Gamma \text{ at load} = \frac{25-50}{25+50} = \frac{-1}{3}$$

$$\Gamma \text{ of powers at load} = \frac{-1}{9}$$

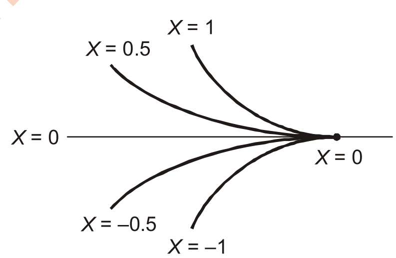
$$\text{Power dissipated in load} = \frac{8}{9} \times (0.25)^2 \times 25 = 1.4\text{ W}$$

T8. (a)

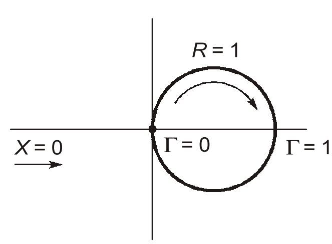
$$\Gamma = \frac{j20-50}{j20+50} = 1\angle 180 - \tan^{-1}\left(\frac{2}{5}\right) = 1\angle 136^\circ$$

T9. (a, b, c)

Symmetry of lines are clearly as shown below



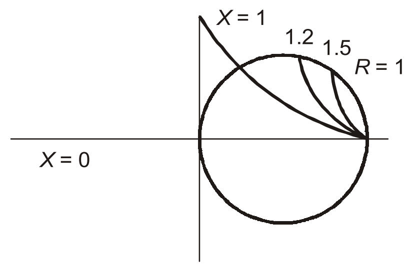
(b) and (c) True



Hence, a, b and c are correct.

T10. (a)

$$\frac{Z_L}{Z_o} = R + jX \text{ with } R > 1$$



T11. Sol.

The problem is a case of λ/4 quarter wave-transformer

$$Z_o' = \sqrt{120\pi \times 30\pi} = 60\pi = \frac{120\pi}{\sqrt{\epsilon_R}}$$

$$\epsilon_R = 4$$

$$\lambda = \frac{3\times10^8}{4\times2.5\times10^9} = 3 \text{ cm}$$

$$\text{Thickness} = \frac{\lambda}{4} = 0.75 \text{ cm}$$

T12. (a)

$$\lambda = \frac{3\times10^8}{2\times10^9} = 0.15 \text{ m} = 15 \text{ cm}$$

$$\frac{\lambda}{2} = 7.5 \text{ cm} \Rightarrow 1^{st} \text{ minimum is at the load}$$

Z_L is resistive and less than Z_o.

T13. Sol.

$$Z_{SC} = jZ_o \tan\beta l = j25$$

Inductive reactance to cancel the load reactance

$$j50 \tan\beta l = j25$$

$$\tan\beta l = \frac{1}{2} \qquad \beta l = 0.46 \text{ radians}$$

$$l = \frac{0.46}{2\times3.14} \times \frac{3\times10^8}{10\times10^9} = 0.22 \text{ cm}$$

T14. Sol.

$$Z_{max} = 75\ \Omega$$

$$Z_{min} = \frac{50}{1.5} = 33.3\ \Omega$$

T15. **(b)**

At the input when $t = 0$,

$$V_{in} = \frac{120}{300} \times 300 = 120 \text{ V}$$

$$\Gamma \text{ at load} = \frac{100 - 120}{100 + 120} = \frac{-1}{11}$$

The reflected voltage cancels to 120 V and reduces to less than 120 V.

T16. **Sol.**

$$2\beta Z_{max} = 2n\pi + \theta$$

$$\Rightarrow \quad 2 \cdot \frac{2\pi}{150} \cdot 500 = 2n\pi - 150$$

$$\Rightarrow \quad \frac{40\pi}{3} = 2n\pi - \frac{6\pi}{6}$$

$$\Rightarrow \quad \frac{40}{3} = 2n - \frac{5}{6}$$

$$\Rightarrow \quad n = 7.08 \simeq 7$$

$$2\beta Z_{max} = 2\pi - 150$$

$$Z_{max} = 43 \text{ m}$$

$$\Rightarrow \quad 1^{st} \text{ max} = 43$$

$$\Rightarrow \quad 2^{nd} \text{ max} = 43 + \{\text{distortion between two successive max}\}$$

$$= 43 + \frac{\lambda}{2} = 43 + \frac{159}{2} = 43 + 75 = 118$$

Number of maximas on the line, $n = 7$

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5 Waveguides

T1. Sol.

$$V_g = \frac{d\omega}{d\beta} = \sqrt{A\omega}$$

$$\frac{d\omega}{\sqrt{\omega}} = \sqrt{A}\, d\beta$$

Integrate on both sides, $\frac{(\omega)^{1/2}}{1/2} = \sqrt{A}\beta$

$$2\sqrt{\omega} = \sqrt{A}\beta$$

Divide with ω on both sides, $\frac{2\sqrt{\omega}}{\omega} = \sqrt{A}\frac{\beta}{\omega}$

$$\frac{\omega}{\beta} = V_p = \sqrt{\frac{A\omega}{2}} = \frac{V_g}{2} \quad \text{As } \beta \propto \omega^{1/2}$$

T2. Sol.

$$\sin\theta = \frac{f_c}{f}$$

First mode $f_c = \frac{c}{2a} = \frac{3\times10^8}{\sqrt{9}\times2\times3\times10^{-2}}$

$$f_c = \frac{5}{3}\text{ GHz} = 1.67 \text{ GHz}$$

$$\sin\theta = \frac{1.67}{2}$$

$$\theta = 56^\circ$$

T3. Sol.

f_c for dominant mode TE_{10} $\quad f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times7\times10^{-2}} = 2.14 \text{ GHz}$

$$TE_{20}\ f_c = 4.28 \text{ GHz}$$

$$TE_{01}\ f_c = \frac{c}{2b} = \frac{3\times10^8}{2\times4\times10^{-2}} = 3.75 \text{ GHz}$$

$$\frac{TE_{11}}{TM_{11}}\ f_c = 4.3 \text{ GHz}$$

Total 5 modes $TE_{10} - TE_{20} - TE_{01} - TE_{11} - TM_{11}$.

T4. (b)

T5. (c, d)

Maximum single mode operational bandwidth when $a \le 2b$.

T6. (c)

TM_{12}.

T7. Sol.

(b) and (c).

T8. Sol.

$$TE_1 \; f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 3\times 10^{-2}} = 5 \text{ GHz}$$

7 modes: TE_1, TM_1, TE_2, TM_2, TE_3, TM_3 an TEM

T9. Sol.

$$\text{Power dissipated} = \frac{1}{2}\frac{E_0^2}{n} a\cdot b = \frac{E_0^2}{120\pi}\cos\theta \, a\cdot b$$

$$= \frac{1}{2}\times\frac{4\times 4\times 10^6}{377}\sqrt{1-\frac{1}{4}}\times 2\times 10^{-4} = 3.6 \text{ W}$$

T10. Sol.

Least possible TM mode is TM_{11}

$$f_c = \left(\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{a}\right)^2}\right)\frac{c}{2} = \frac{c}{\sqrt{2}\,a} = 21.2 \text{ GHz}$$

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6 Antennas

T1. **(d)**

A small loop has an approximate gain of 1.5.

$$\text{Gain} = \frac{4\pi}{\lambda^2} \cdot Ae$$

$$\lambda = \sqrt{\frac{4\pi}{1.5}\frac{3}{32\pi}} = 0.5\text{ m} = 500\text{ mm}$$

T2. **(c)**

$$W_r = \frac{W_t \cdot G_t \cdot G_r}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$d = 1000$ m ; $f = 300$ kHz ; $\lambda = \dfrac{3\times10^8}{300\times10^6} = 1\text{m}$; $G_t = 10$; $G_r = 10^{0.8}$

$$\therefore \quad W_r = \frac{25\times10\times10^{0.8}}{\left(\frac{4\pi\times10\times10^3}{1}\right)^2} = 99.8\text{ nW}$$

T3. **(a)**

$$\text{Gain} = \frac{4\pi V_o \sin\theta \sin^2\phi}{\int\limits_{\theta=0}^{\pi}\int\limits_{\phi=0}^{\pi} V_o \sin\theta\sin^2\phi\sin\theta\, d\theta\, d\phi}$$

$$\int\limits_{\theta=0}^{\pi} \sin^2\theta\, d\theta = \int\limits_{\theta=0}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{\pi}{2}$$

$$D = \frac{4\pi}{\frac{\pi}{2}\times\frac{\pi}{2}} = 5.1$$

T4. **(c)**

$\lambda/2$ dipole open circuit at one end and other end is also open. It is a shunt LC circuit with radiation resistance of 73 Ω.

T5. (d)

Most monopoles are used to produce vertical polarized waves suitable for ground waves.

T6. (c)

E and *H* fields in a dipole antenna exists as induction and radiation terms.
H or *E* depending as $1/r$ is radiation field.
E_θ has single $1/r$ term.
H or *E* depending as $1/r^2$ and $1/r^3$ and induction.
E_r, E_θ and H_ϕ have such fields.

T7. Sol.

$$\text{Gain} = 5\sin 2\theta \qquad \text{Directivity} = 5$$

$$\text{In half power direction gain} = 2.5$$

$$\text{Power density} = \frac{E_{rms}^2}{120\pi} = \frac{W_t G_t}{4\pi d^2}$$

$$E_{rms} = \frac{\sqrt{30\times 1\times 10^3\times 2.5}}{5\times 10^3} = 0.054 \text{ V/m}$$

T8. Sol.

Any antenna is a resonant device under oscillations of *V* and *I*.
Marconi antenna is a $\lambda/4$ monopole and resonant at any length and any side.

T9. Sol.

$$\text{Broadside array means } \alpha = 0$$

$$\frac{\sin(N\psi/2)}{\sin(\psi/2)} = \text{Array pattern}$$

$$\text{For null directions } \frac{N\psi}{2} = 2n\pi \text{ with } \frac{\psi}{2} \neq 2n\pi \text{ as denominator } \sin\left(\frac{\psi}{2}\right) \neq 0$$

$$\psi \neq 4n\pi$$

$$\psi = \frac{4n\pi}{N} = \frac{4n\pi}{6} = \frac{2n\pi}{3}$$

$$\psi = \beta d\cos\theta = \frac{2n\pi}{3}$$

$$\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\theta = \frac{2\pi}{3} \quad \text{when } n = 1$$

$$\cos\theta = \frac{2}{3} \Rightarrow \theta_{NP} = 42$$

$$\text{Beam width first Nulls} = \frac{\text{HPBW}}{2} = 84°$$

T10. (b)

$$\psi = 0 + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\cos\theta = \frac{\pi}{2}\cos\theta$$

For maximas $\frac{\pi}{2}\cos\theta = 0$ $\quad\theta_{max} = 90° \text{ or } 270°$

For minimas, $\psi = \pi$ $\quad\frac{\pi}{2}\cos\theta = \pi$ this is not possible

For half power points $\quad\frac{\pi}{2}\cos\theta = \frac{\pi}{2}$ $\quad\theta = 0° \text{ or } 180°$

T11. Sol.

As per multiplication of patterns

$$\sin\theta\cos(\psi/2) = 0 \text{ towards } 45° \text{ direction}$$

$$\frac{\psi}{2} = \frac{\pi}{2} \text{ or } (2n+1)\frac{\pi}{2}$$

$$\alpha + \beta d\cos\theta = \pi \text{ with } \alpha = \pi \text{ due to image}$$

The next solution $\quad\pi + \frac{2\pi}{\lambda}d\frac{1}{\sqrt{2}} = 3\pi$

$$d = \lambda\sqrt{2} \text{ is possible when } \theta = 45°$$

For maxima direction

$$\pi + \frac{2\pi}{\lambda}\sqrt{2\lambda}\cdot\cos\theta = 2\pi$$

$$\cos\theta = \frac{1}{2\sqrt{2}}$$

$$\theta = 70°$$

T12. (a)

$$E_{rms} = \frac{\sqrt{30W_t}}{d} = \frac{\sqrt{30\times 3\times 10^3}}{3\times 10^3} = 0.1 \text{ V/m}$$

T13. (a)

$$\text{Gain} = \frac{4\pi}{\Omega_A} = \frac{4\pi}{(HPBW)^2} = \frac{4\pi}{\lambda^2}\cdot\pi R^2$$

$$\text{HPBW} \propto \frac{\lambda}{D}$$

T14. Sol.

$$\text{Gain} = 6 \text{ dB over isotropic} = 4$$

250 mW with gain 4

With isotrpic antenna, $\quad\text{Power} = \frac{250}{4} = 62.5 \text{ mW}$

T15. (a)

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