

**ESE**

**GATE**

**State Engg. Exams**

**MADE EASY**  
**WORKBOOK 2027**



**Detailed Explanations of  
Try Yourself *Questions***

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**Electronics Engineering  
Communication Systems**



# 1

## Amplitude Modulation



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

The signal

$$s(t) = A_C [1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$$

The signal can be represented as

$$s(t) = \operatorname{Re} \left[ A_C e^{j\omega_c t} + \frac{A_C \mu}{2} (e^{j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t}) \right]$$

$$s(t)|_{\text{complex}} = \left[ A_C e^{j\omega_c t} + \frac{A_C \mu}{2} (e^{j(\omega_c + \omega_m)t} + e^{j(\omega_c - \omega_m)t}) \right]$$

$$s(t)|_c = [s(t)|_{ce} e^{-j\omega_c t}]$$

(where,  $s(t)|_c$  = the complex signal  $s(t)$  and  $s(t)|_{ce}$  = the complex low pass equal of the signal  $s(t)$ )

$\therefore$

$$s(t)|_{ce} = A_C + \frac{A_C \mu}{2} [\cos \omega_m t + j \sin \omega_m t] + \frac{A_C \mu}{2} [\cos \omega_m t - j \sin \omega_m t]$$

Putting the conditions given in the questions we get:

$$s(t)|_{ce} = 1 + \frac{1}{8} [\cos \omega_m t + j \sin \omega_m t] + \frac{1}{4} [\cos \omega_m t - j \sin \omega_m t]$$

$$s(t)|_{ce} = 1 + \frac{3}{8} \cos \omega_m t - j \frac{1}{8} \sin(\omega_m t)$$

$\therefore$

$$\text{A envelop} = \left[ \left( 1 + \frac{3}{8} \cos(\omega_m t) \right)^2 + \left( \frac{1}{8} \sin(\omega_m t) \right)^2 \right]^{1/2}$$

#### T2. Sol.

Expression for AM signal

$$V_{AM}(t) = A_C \cos \omega_c t + A_C m_a \cos(\omega_c + \omega_m)t + A_C m_a \cos(\omega_c - \omega_m)t$$

$\therefore$

$$P_C = 100 = \frac{A_C^2}{2}$$

$\therefore$

$$A_C = 14.14 \text{ V}$$

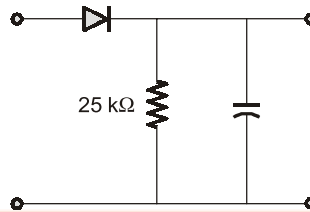
Also 
$$\eta = \frac{m_a^2}{2 + m_a^2} = 40\%$$

or 
$$0.8 + 0.4 m_a^2 = m_a^2$$

$$m_a = 1.154$$

$\therefore B = A_C m_a / 2 = 8.16$

**T3. Sol.**



$$RC \leq \frac{1}{\omega_n} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \leq \frac{1}{R\omega_n} \frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \leq \frac{1}{10^4 \times 2\pi \times 25 \times 10^3} \cdot \frac{\sqrt{1-(0.5)^2}}{0.5}$$

$$C \leq 1.1 \text{ nF}$$

**T4. (c)**

$$x(t) = m(t) + \cos \omega_c t$$

$$y(t) = 4(m(t) + \cos \omega_c t) + 10[m^2(t) + \cos^2 \omega_c t + 2m(t) \cos \omega_c t]$$

$$= 4m(t) + 10m^2(t) + 4 \cos \omega_c t + \frac{10}{2} + \frac{10}{2} \cos 2\omega_c t + 20m(t) \cos \omega_c t$$

after passing through filter

$$y(t) = 4 \cos \omega_c t + 20m(t) \cos \omega_c t$$

$$= 4[1 + 5m(t)] \cos \omega_c t$$

$$\mu = 5 \times M$$

$$0.8 = 5 \times M$$

$$M = \frac{0.8}{5} = 0.16$$



# 2

## Angle Modulation



### Detailed Explanation of Try Yourself Questions

**T1. Sol.**

Maximum instantaneous frequency

$$f_i = f_c + \frac{K_p}{2\pi} \dot{m}(t)$$

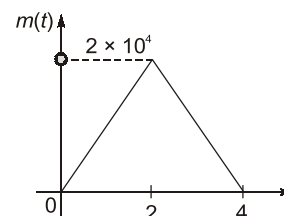
$$115.95 \times 10^3 = \frac{10^5}{2\pi} + \left(\frac{K_p}{2\pi}\right) \times 10^4$$

$$10^5 = \left(\frac{K_p}{2\pi}\right) \times 10^4$$

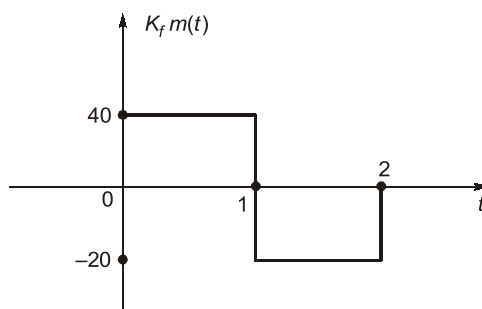
$$10 = \left(\frac{K_p}{2\pi}\right)$$

$$K_p = 2\pi \times 10 \text{ Hertz/Volt}$$

$$K_p = 10 \text{ rad/volt}$$



**T2. Sol.**



$$s(t) = 10 \cos [2\pi \times 10^6 t + 20\pi [4r(t) - 6r(t-1) + 2r(t-2)]]$$

Standard FM expression is given by:

$$s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int m(t) dt \right]$$

$$2\pi k_f \int m(t) dt = 20\pi (4r(t) - 6r(t-1) + 2r(t-2))$$

$$k_f m(t) = 10[4u(t) - 6u(t-1) + 2r(t-2)]$$

$$\Delta f = \max |k_f m(t)| = 40 \text{ Hz}$$

**T3. Sol.**

Maximum frequency deviation

$$\Delta f_{\max} = \frac{K_p}{2\pi} \left| \frac{d}{dt} m(t) \right|_{\max} = \frac{K_p}{2\pi} 2t e^{-t^2}$$

$$= \frac{8000}{2\pi} \cdot 2 \cdot \frac{1}{\sqrt{2}} \cdot e^{-1/2} \quad \left( \because \max 2 + e^{-t^2} \text{ is at } t = \frac{1}{\sqrt{2}} \right)$$

$$= 3.43 \text{ kHz}$$

**T4. Sol.**

Comparing the equation with the standard equation.

$$s(t) = A \cos[\omega_c t + k_p m(t)]$$

$$\therefore k_p m(t) = 0.1 \sin(10^3 \pi t)$$

$$m(t) = \frac{0.1}{k_p} \sin(10^3 \pi t)$$

$$= 0.01 \sin(10^3 \pi t)$$

Similarly

$$s(t) = A \cos \left[ \omega_c t + K_f \int m(t) dt \right]$$

$$K_f \int m(t) dt = 0.1 \sin(10^3 \pi t)$$

$$\int m(t) dt = \frac{0.1}{10\pi} \sin(10^3 \pi t) = \frac{0.1 \times 10^3 \pi}{10\pi} \cos(10^3 \pi t) = 10 \cos(10^3 \pi t)$$

**T5. Sol.**

$A_m = 5 \text{ V}, f_m = 100 \text{ Hz} \} \Delta f = k_f A_m = 1 \text{ kHz}$   
 $A_m = 10 \text{ V}, f_m = 50 \text{ Hz} \} \Delta f = 2 \text{ kHz}$   
 To get  $\Delta f = 30 \text{ kHz}$   
 frequency multiplication factor should be 15.

**T6. (a)**

$$BW = 2[\beta + 1]f_m$$

$$\beta = k_p A_m = 5$$

$A_m$  is doubled  $\Rightarrow \beta = 10 ; f_m = \frac{1}{2} \text{ kHz}$

$$BW = 2[10 + 1] \cdot \frac{1}{2} = 11 \text{ kHz}$$

**T7. Sol.**

$$\beta_f = \frac{k_f \max \{m(t)\}}{f_m} = \frac{100 \text{ k} \times 1}{1 \text{ k}} = 100$$

$$BW_f = 2(100 + 1) 1 \text{ k} = 202 \text{ kHz.}$$

$$\begin{aligned} \beta_p &= k_p \max \{m(t)\} \\ &= 10 \times 1 = 10 \end{aligned}$$

$$BW_p = 2(10 + 1) 1 \text{ k} = 22 \text{ kHz}$$

Bandwidth required for channel

$$= 202 + 22 = 224 \text{ kHz}$$

**T8. Sol.**

The phase modulated signal can be given by,

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] = A_c \cos[\theta(t)]$$

The instantaneous frequency of the modulated signal,

$$f_i = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

Given that,

$$m(t) = 100 \text{ sinc}(1000t) \text{ V} = 100 \frac{\sin(1000\pi t)}{1000\pi t}$$

$$\frac{dm(t)}{dt} = 100 \left[ \frac{1000\pi \cos(1000\pi t)}{1000\pi t} - \frac{\sin(1000\pi t)}{1000\pi t^2} \right]$$

At  $t = 1 \text{ ms}$ ,

$$\frac{dm(t)}{dt} = \frac{100 \cos(\pi)}{10^{-3}} = -10^5 \text{ V/s}$$

So,

$$f_i = f_c + \frac{1}{2\pi} (-10^5 k_p) = 100 - \frac{100 \times 2}{2\pi} \text{ kHz}$$

$$= 100 - \frac{100}{\pi} \text{ kHz} = 68.17 \text{ kHz}$$



# 3

## Radio Receivers



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

Given  $88.5 \text{ MHz} < f_c < 108 \text{ MHz}$

Also,

$$f_{LO} - f_c = 10.8 \text{ MHz}$$

or,

$$f_{LO} = 10.8 \text{ MHz} + f_c$$

$\therefore$

$$f_{LO1} = 10.8 + 88.5 = 99.3 \text{ MHz}$$

$\therefore$

$$f_{LO2} = 10.8 + 108 = 118.8 \text{ MHz}$$

$$\text{range} = 99.3 \text{ MHz} - 118.8 \text{ MHz}$$

#### T2. Sol.

$$C = \frac{C_{\max}}{C_{\min}} = \left( \frac{f_{\max}}{f_{\min}} \right)^2 = 1.45$$

Where

$$f_{\max} = f_{m2} + IF \quad \& \quad f_{\min} = f_{m2} - IF$$

$$\frac{110.5 + IF}{90 + IF} = \sqrt{1.45} = 1.204$$

$$110.5 + IF = 90 \times 1.204 + IF \times 1.204$$

$$2.126 = 0.204 IF$$

or

$$IF = 10.42 \text{ MHz}$$

Also

Image frequency

$$= f_s + 2 IF$$

$$125 = f_s + 2 \times 10.42$$

$$f_s = 104.16 \text{ MHz}$$



# 4

## Sampling and Pulse Code Modulation



### Detailed Explanation of Try Yourself Questions

**T1. (d)**

$$f_m = 100 \text{ Hz}$$

$$Q_e = \pm \frac{\Delta}{2}$$

$$f_s = 1.5 \times f_m \times 2 = 300 \text{ Hz}$$

$$\frac{\Delta}{2} \leq \frac{0.1}{100} \times A_m$$

$$\frac{2A_m}{2^n \times 2} \leq \frac{0.1}{100} \times A_m$$

⇒

$$n = 10$$

$$r_b = N n f_s = 8 \times 10 \times 300 = 24000 \text{ Hz} = 24 \text{ kbits/sec}$$

**T2. Sol.**

$$\begin{aligned} \text{Sampling frequency } (f_s) &= 1.5 \times 2 \times 4 \\ &= 12 \text{ kHz} \end{aligned}$$

$$\text{step size } (\Delta) = 10 \text{ mV}$$

To avoid slope overload distortion in Delta modulation;

$$\frac{\Delta}{T_s} \geq \left| \frac{dm(t)}{dt} \right|_{\max}$$

i.e.,

$$\frac{\Delta}{T_s} \geq 2\pi f_m \cdot A_m$$

...for sinusoidal message signal

$$A_m \leq \frac{\Delta}{T_s(2\pi f_m)}$$

$$\begin{aligned}(A_m)_{\max} &= \frac{\Delta}{T_s(2\pi f_m)} = \frac{\Delta \cdot f_s}{2\pi f_m} \\ &= \frac{10 \times 10^{-3} \times 12 \times 10^3}{2\pi \times 10^3} = 19.09 \times 10^{-3} \approx 19.1 \text{ mV}\end{aligned}$$

**T3. (c)**

To prevent slope overload

$$\begin{aligned}\delta f_s &\geq \max \left| \frac{dm(t)}{dt} \right| \\ \delta \times 200 \times 10^3 &\geq 2\pi A_m f_m \\ \delta &\geq \frac{2 \times \pi \times (10 \times 10^3) \times \frac{1}{2}}{200 \times 10^3} \\ \delta &\geq 0.157 \text{ Volts}\end{aligned}$$



# 5

## Digital Carrier Modulation Schemes



### Detailed Explanation of Try Yourself Questions

**T1. Sol.**

$$\begin{aligned} \text{Average energy} &= \frac{1}{16} [4(\sqrt{2}a)^2 + 8(\sqrt{10}a)^2 + 4(\sqrt{18}a)^2] \\ &= \frac{1}{4} [2a^2 + 20a^2 + 18a^2] \\ &= 10a^2. \end{aligned}$$

**T2. Sol.**

Let signal I be represented as

$$S_1(t) = \begin{cases} A_1 \sin \frac{\pi t}{T} & ; 0 \leq t \leq T \\ 0 & ; 0 \leq t \leq T \end{cases}$$

and signal II be represented as

$$S_2(t) = \begin{cases} A_2 \sin \frac{\pi t}{T} & ; 0 \leq t \leq T \\ -A_2 \sin \frac{\pi t}{T} & ; 0 \leq t \leq T \end{cases}$$

The average energy of signal will be

$$P_{\text{avg}_1} = \frac{1}{2} \left( \frac{A_1^2}{2} \right) + \frac{1}{2} (0) = \frac{A_1^2}{4}$$

Average energy of signal (ii)

$$P_{\text{avg}_2} = \frac{1}{2} \left( \frac{A_2^2}{2} \right) + \frac{1}{2} \left( \frac{A_2^2}{2} \right) = \frac{A_2^2}{2}$$

$$\therefore \frac{A_1^2}{4} = \frac{A_2^2}{2}$$

$$\Rightarrow \frac{A_1}{\sqrt{2}} = A_2$$

**T3. (b)**

$$\begin{aligned} \text{Sampling frequency } (f_s) &= 1.25 \times (2f_m) + \text{Guard band} \\ &= 1.25 \times 2 \times 10 + 1 \\ &= 26 \text{ kHz} \end{aligned}$$

$$\begin{aligned} \text{Bit rate } (R_b) &= n.f_s = 4 \times 26 \quad \dots [L \leq 2^n] \\ &= 104 \text{ kHz} \end{aligned}$$

$\therefore$  Bandwidth of channel is 100 kHz i.e.,

$$\text{B.W} \leq 100 \text{ kHz}$$

$$R_s(1 + \alpha) \leq 100 \quad \dots [\text{For M-ary PSK } (BW)_{\min} = R_s(1 + \alpha)]$$

$$\therefore R_s = \frac{R_b}{\log_2 M} \quad \dots [R_s = \text{symbol rate}]$$

$$\therefore \frac{R_b}{\log_2 M} (1 + \alpha) \leq 100$$

$$\frac{104}{\log_2 M} (1 + 0.3) \leq 100$$

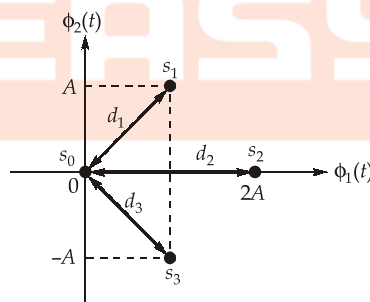
$$\log_2 M \geq 1.352$$

$$M \geq 2^{1.352}$$

$$M_{\min} = 4$$

$$(\because M = 2^n)$$

**T4. (d)**



Let the energy associated with the symbols  $s_0, s_1, s_2$  and  $s_3$  are  $E_0, E_1, E_2$  and  $E_3$  respectively.

$$E_i = (d_i)^2; \quad i = 0, 1, 2, 3$$

From the above diagram,

$$d_0 = 0$$

$$d_1 = d_3 = \sqrt{A^2 + A^2} = \sqrt{2A^2}$$

$$d_2 = 2A$$

So,

$$E_0 = 0$$

$$E_1 = E_3 = 2A^2$$

$$E_2 = 4A^2$$

The average symbol energy of the modulation scheme can be given as,

$$\begin{aligned} E_s &= \sum_{i=0}^3 E_i P(s_i) ; & P(s_i) &= \text{probability of occurrence of the symbol } s_i \\ &= 0(0.3) + 2A^2(0.2) + 4A^2(0.4) + 2A^2(0.1) \\ &= (0.4 + 1.6 + 0.2)A^2 \\ &= 2.2 A^2 \end{aligned}$$

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# 6

## Information Theory and Error Control Coding



### Detailed Explanation of Try Yourself Questions

**T1. Sol.**

$$P(y) = [0.5 \ 0.5] \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix} = [0.4 \ 0.2 \ 0.4]$$

$$= [y_1 \ y_2 \ y_3]$$

$$\left[ \because P(y) = P(x).P\left(\frac{y}{x}\right) \right]$$

**T2. Sol.**

(i) We know that

$$P\left(\frac{r_0}{m_0}\right)P(m_0) > P\left(\frac{r_0}{m_1}\right)P(m_1) > P\left(\frac{r_0}{m_2}\right)P(m_2)$$

$$\Rightarrow (0.6)(0.3) > (0.1)(0.5) > (0.1)(0.2)$$

Hence, we select  $m_0$  wherever  $r_0$  is received.

$$\text{We also find that } P\left(\frac{r_1}{m_1}\right)P(m_1) > P\left(\frac{r_1}{m_0}\right)P(m_0) > P\left(\frac{r_1}{m_2}\right)P(m_2)$$

$$\Rightarrow (0.5)(0.5) > (0.3)(0.3) > (0.1)(0.2)$$

Hence, we select  $m_1$  wherever  $r_1$  is received.

We also find that

$$P\left(\frac{r_2}{m_1}\right)P(m_1) > P\left(\frac{r_2}{m_2}\right)P(m_2) > P\left(\frac{r_2}{m_0}\right)P(m_0)$$

$$\Rightarrow (0.4)(0.5) > (0.8)(0.2) > (0.1)(0.3)$$

Hence, we select  $m_1$  whenever  $r_2$  is received.

(ii) The probability of being correct is

$$P(c) = P(m_0) \cdot P\left(\frac{r_0}{m_0}\right) + P(m_1) P\left(\frac{r_1}{m_1}\right) P(m_1) \cdot P\left(\frac{r_2}{m_1}\right)$$

$$= (0.6)(0.3) + (0.5)(0.5) + (0.5)(0.4) = 0.63$$

Hence probability of error,  $P(e) = 1 - P(c)$

$$P(e) = 0.37$$

### T3. Sol.

For a binary symmantric channel for wrong transmission let the probability be  $p$

Thus

mutual information

$$= I(X; Y) = H(Y) - H(Y|X)$$

and

$$H(Y|X) = -p \log_2 p - (1-p) \log_2 (1-p)$$

$\therefore$

$$I(X; Y) = H(Y) + p \log_2 p + (1-p) \log_2 (1-p)$$

$$C_{\max} = I(X; Y)_{\max}$$

$$= 1 + p \log_2 p + (1-p) \log_2 (1-p)$$

### T4. (a)

$$C = (c_1, c_2, c_3, c_4, c_5, c_6) = (x_1, x_2, x_3, c_4, c_5, c_6)$$

$$c_4 = c_1 \oplus c_2 = x_1 \oplus x_2$$

$$c_5 = c_2 \oplus c_3 = x_2 \oplus x_3$$

$$c_6 = c_1 \oplus c_3 = x_1 \oplus x_3$$

$$[c_4 \ c_5 \ c_6] = [x_1 \ x_2 \ x_3] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\bar{G} = \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array} \right]$$

### T5. Sol.

It is given that "1001100" is a valid code word.

$$\Rightarrow C = 1001100$$

$$H^T = \begin{bmatrix} 1 & b & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ a & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

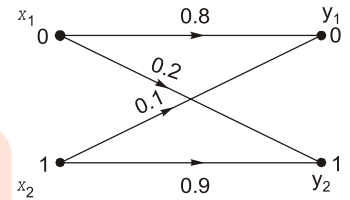
$$CH^T = 0 \quad \Rightarrow \quad [1001100] \begin{bmatrix} 1 & b & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ a & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$CH^T = [a \ b \oplus 1 \ 0] = [0 \ 0 \ 0] \quad [\oplus \rightarrow \text{modulo-2 addition}]$$

Hence on comparing,  $a = 0, b = 1$

**T6. Sol.**

$$\begin{aligned} [P(y|x)] &= \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} \\ P[y] &= P[x] P[y|x] \\ &= [0.4 \ 0.6] \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} = [0.38 \ 0.62] \\ P_Y(0) &= 0.38 \\ P_Y(1) &= 0.62 \end{aligned}$$



**T7. (b)**

**Condition 1:** When  $r_0$  is received, decision is made in favour of  $m_0$ .

So,  $P(r_0 | m_0) P(m_0) > P(r_0 | m_1) P(m_1)$   
 $(0.7)(1-p) > (0.3)(q)$   
 $0.7p + 0.3q < 0.7$   
 $7p + 3q < 7$  ... (i)

**Condition 2:** When  $r_1$  is received, decision is made in favour of  $m_1$ .

So,  $P(r_1 | m_1) P(m_1) > P(r_1 | m_0) P(m_0)$   
 $(0.3)(1-q) > (0.7)(p)$   
 $0.7p + 0.3q < 0.3$   
 $7p + 3q < 3$  ... (ii)

If condition 2 is satisfied, then condition - 1 will be satisfied automatically.

So, the sufficient condition to be satisfied is condition-2, i.e.  $7p + 3q < 3$



# 7

## Random Variables and Random Process



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

The A.C power of the signal is given as  $\sigma_X^2$   
where  $\sigma_X^2$  is the standard deviation

where

now,

and

$\therefore$

$$\begin{aligned}\sigma_X^2 &= E[X^2] - (E[X])^2 \\ E[X^2] &= \text{second moment} \\ [E(X)]^2 &= (\text{mean})^2 \\ (E[X])^2 &= \lim_{\tau \rightarrow \infty} R_{XX}(\tau) = 6 \\ E[X^2] &= \lim_{\tau \rightarrow 0} R_{XX}(\tau) = 10 \\ \sigma_X^2 &= 10 - 6 = 4 \text{ W.}\end{aligned}$$

#### T2. Sol.

$$\begin{aligned}E[\gamma] &= \int_{-\infty}^{\infty} f_x(x) \gamma dx \\ &= \int_{-\infty}^{\infty} e^x dx = \int_0^1 e^x dx \\ &= -(1 - e) = (e - 1)\end{aligned}$$

#### T3. Sol.

$$(i) \quad \int_{-\infty}^{\infty} f_X(x) dx = 1$$

$$2 \int_0^{\infty} a e^{-bx} dx = 1$$

$$\Rightarrow \quad \frac{2a}{b} e^{-bx} \Big|_0^{\infty} = 1$$

$$\Rightarrow \quad 2a = b$$

$$(ii) \quad \begin{aligned} \text{c.d.f} &= \int_0^x f_x(d) dx \\ &= \int_0^x a e^{-bx} dx = 1 - \frac{1}{2} e^{-bx} \end{aligned}$$

for  $x \geq 0$  and  $\int_{-\infty}^x f_0(x) dx$  for  $x < 0$

$$f_x(x) = \frac{1}{2} e^{bx}$$

$x < 0$

$$(iii) \quad \begin{aligned} P(1 \leq X \leq 2) &= \int_1^2 f_X(x) dx \\ &= \int_1^2 \left(1 - \frac{1}{2} e^{-bx}\right) dx = \frac{1}{2} [e^{-b} - e^{-2b}] \end{aligned}$$

**T4. (b)**

$$\begin{aligned} Y(t) &= X(t) \cos(2\pi f_c t + \theta) \\ \text{power of } Y(t) &= E[Y^2(t)] \\ &= E[X^2(t) \cdot \cos^2(2\pi f_c t + \theta)] \\ &= E[X^2(t)] \cdot E[\cos^2(2\pi f_c t + \theta)] \\ E[X^2(t)] &= R_{XX}(\tau)|_{\tau=0} \\ E[\cos^2(2\pi f_c t + \theta)] &= \frac{1}{2\pi} \int_0^{2\pi} \cos^2(2\pi f_c t + \theta) d\theta = \frac{1}{2} \\ \text{Power of } y(t) &= R_{XX}(\tau)|_{\tau=0} \cdot \frac{1}{2} \\ &= \text{sinc}(0) \cdot \frac{1}{2} = \frac{1}{2} \end{aligned}$$

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## Detailed Explanation of Try Yourself Questions

**T1. Sol.**

$$P_e = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

$$E_b = (10)^2 \times 100 \times 10^{-6} = 10^{-2}$$

$$N_0 = 2 \times 10^{-4}$$

$\therefore$

$$P_e = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{10^{-2}}{2 \times 10^{-4}}} \right) = \frac{1}{2} \operatorname{erfc}(\sqrt{50})$$

**T2. Sol.**

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

$$\operatorname{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right) \leq 2 \times 10^{-4}$$

$$1 - \operatorname{erf} \left( \sqrt{\frac{E_b}{N_0}} \right) \leq 2 \times 10^{-4}$$

$$\operatorname{erf} \left( \sqrt{\frac{E_b}{N_0}} \right) \geq 0.9998$$

$$\sqrt{\frac{E_b}{N_0}} \geq 2.6$$

$$\Rightarrow \frac{E_b}{N_0} \geq 6.76$$

$$E_b \geq 1.352 \times 10^{-9} \text{ Joule}$$

$$E_b = P \times T_b$$

$$\Rightarrow P \geq (1.352 \times 10^{-9}) (10^6)$$

$$P \geq 1.352 \text{ mW}$$

**T3. Sol.**

For BPSK signal

$$E_b = \frac{A^2 T_b}{2}$$

and

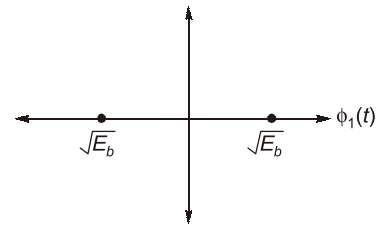
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$10^{-5} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\sqrt{\frac{A^2 T_b}{N_0}} = 4.27$$

$$A^2 = (4.27)^2 \times 9.6 \times 10^{-8} = 1.75 \times 10^{-6} = 1.32 \times 10^{-3}$$

$$A = 1.32 \text{ mV}$$



**T4. Sol.**

The probability density function of the input variable  $R$  can be given by,

$$f_R(r | s_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r-2)^2}{2}} ; \text{When "-1" is transmitted}$$

$$f_R(r | s_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r-2)^2}{2}} ; \text{When "+1" is transmitted}$$

Given that,  $P(s_0) = 2/3$  and  $P(s_1) = 1/3$ .

According to MAP criteria, the rule for determination of threshold is,

$$P(s_0) f_R(r | s_0) \underset{H_1}{\overset{H_0}{>}} P(s_1) f_R(r | s_1)$$

At optimum threshold ( $r = r_{th}$ ),

$$P(s_0) f_R(r_{th} | s_0) = P(s_1) f_R(r_{th} | s_1)$$

$$\frac{2/3}{\sqrt{2\pi}} e^{-\frac{(r_{th}-1)^2}{2}} = \frac{1/3}{\sqrt{2\pi}} e^{-\frac{(r_{th}-3)^2}{2}}$$

$$(r_{th} - 1)^2 - (r_{th} - 3)^2 = 2 \ln(2)$$

$$-2r_{th} + 1 + 6r_{th} - 9 = 2 \ln(2)$$

$$4r_{th} = 8 + 2 \ln(2)$$

$$r_{th} = 2 + \frac{1}{2} \ln(2) = 2 + \ln(\sqrt{2})$$

$$r_{th} = 2.3466 \approx 2.35$$

T5. (c)

$$\begin{aligned}
 x(t) &= -\sqrt{2} \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] \sin\left(\pi t - \frac{\pi}{4}\right) \\
 &= -\sqrt{2} \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] \left[ \sin\pi t \cos\frac{\pi}{4} - \cos\pi t \sin\frac{\pi}{4} \right] \\
 &= \sqrt{2} \underbrace{\left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right]}_{x_i(t)} \sin\frac{\pi}{4} \cos\pi t - \sqrt{2} \underbrace{\left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right]}_{x_q(t)} \cos\frac{\pi}{4} \sin\pi t \\
 x(t) &= x_i(t)\cos 2\pi f_c t - x_q(t)\sin 2\pi f_c t \\
 \text{where, } f_c &= \frac{1}{2} \text{ Hz} \\
 x_{ce}(t) &= x_i(t) + jx_q(t) \\
 &= \sqrt{2} \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] \sin\frac{\pi}{4} + j \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] \cos\frac{\pi}{4} = j\sqrt{2} \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] \left[ \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} \right] = j\sqrt{2} \left[ \frac{\sin\left(\frac{\pi t}{5}\right)}{\frac{\pi t}{5}} \right] e^{-j\pi/4}
 \end{aligned}$$