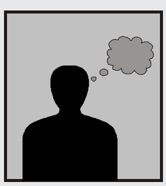
ESE GATE PSUs

State Engg. Exams

MADE EASY Workbook 2024



Detailed Explanations of Try Yourself *Questions*

Electronics Engineering

Communication Systems

MADE EASY Publications

1 Amplitude Modulation

Detailed Explanation of Try Yourself Questions

T1. Sol.

The signal $$s(t) = A_C[1 + \mu \cos(\omega_m t)] \cos(\omega_c t)$$

The signal can be represented as $$s(t) = \text{Re}\left[A_C e^{j\omega_c t} + \frac{A_C\mu}{2}\left(e^{j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t}\right)\right]$$

$$s(t)|_{\text{complex}} = \left[A_C e^{j\omega_c t} + \frac{A_C\mu}{2}\left(e^{j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t}\right)\right]$$

$$s(t)|_c = \left[s(t)_{ce} e^{-j\omega_c t}\right]$$

(where, $s(t)|_c$ = the complex signal $s(t)$ and $s(t)|_{ce}$ = the complex low pass equal of the signal $s(t)$)

$$\therefore \quad s(t)|_{ce} = A_C + \frac{A_C\mu}{2}[\cos\omega_m + j\sin\omega_m t] + \frac{A_C\mu}{2}[\cos\omega_m - j\sin\omega_m t]$$

Putting the conditions given in the questions we get:

$$s(t)|_{ce} = 1 + \frac{1}{8}[\cos\omega_m + j\sin\omega_m t] + \frac{1}{4}[\cos\omega_m - j\sin\omega_m t]$$

$$s(t)|_{ce} = 1 + \frac{3}{8}\cos\omega_m t - j\frac{1}{8}\sin(\omega_m t)$$

$$\therefore \quad \text{A envelop} = \left[\left(1 + \frac{3}{8}\cos(\omega_m t)\right)^2 + \left(\frac{1}{8}\sin(\omega_m t)\right)^2\right]^{1/2}$$

T2. Sol.

Expression for AM signal

$$V_{AM}(t) = A_C \cos\omega_c t + A_C m_a \cos(\omega_c + \omega_m)t + A_C m_a \cos(\omega_c - \omega_m)t$$

$$\because \quad P_C = 100 = \frac{A_C^2}{2}$$

$$\therefore \quad A_C = 14.14 \text{ V}$$

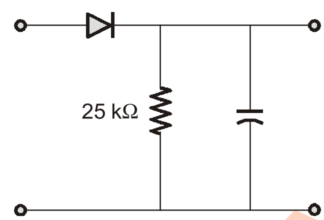
Also $\eta = \dfrac{m_a^2}{2 + m_a^2} = 40\%$

or $0.8 + 0.4\, m_a^2 = m_a^2$

$m_a = 1.154$

$\therefore \quad B = A_C m_a/2 = 8.16$

T3. Sol.



$$RC \le \frac{1}{\omega_n}\frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \le \frac{1}{R\omega_n}\frac{\sqrt{1-\mu^2}}{\mu}$$

$$C \le \frac{1}{10^4 \times 2\pi \times 25 \times 10^3}\cdot\frac{\sqrt{1-(0.5)^2}}{0.5}$$

$$C \le 1.1 \text{ nF}$$

T4. (c)

$$x(t) = m(t) + \cos\omega_c t$$

$$y(t) = 4\left(m(t) + \cos\omega_c t\right) + 10\left[m^2(t) + \cos^2\omega_c t + 2m(t)\cos\omega_c t\right]$$

$$= 4m(t) + 10m^2(t) + 4\cos\omega_c t + \frac{10}{2} + \frac{10}{2}\cos 2\omega_c t + 20\,m(t)\cos\omega_c t$$

after passing through filter

$$y(t) = 4\cos\omega_c t + 20\,m(t)\cos\omega_c t$$

$$= 4[1 + 5\,m(t)]\cos\omega_c t$$

$$\mu = 5 \times M$$

$$0.8 = 5 \times M$$

$$M = \frac{0.8}{5} = 0.16$$

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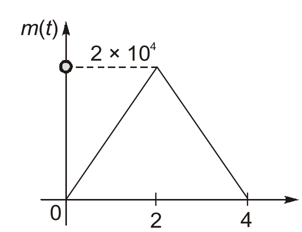
2 Angle Modulation

Detailed Explanation of Try Yourself Questions

T1. Sol.



Maximum instantenious frequency

$$f_i = f_c + \frac{K_p}{2\pi}\dot{m}(t)$$

$$115.95 \times 10^3 = \frac{10^5}{2\pi} + \left(\frac{K_p}{2\pi}\right) \times 10^4$$

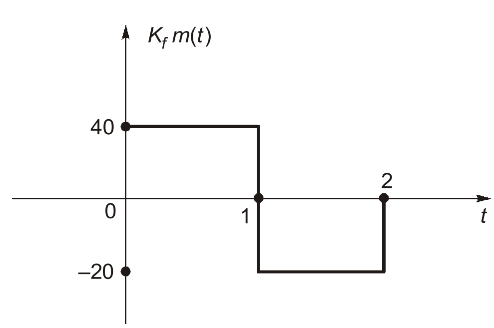
$$10^5 = \left(\frac{K_p}{2\pi}\right) \times 10^4$$

$$10 = \left(\frac{K_p}{2\pi}\right)$$

$$K_p = 2\pi \times 10 \text{ Hrtz/Volt}$$

$$K_p = 10 \text{ rad/volt}$$

T2. Sol.



$$s(t) = 10\cos[2\pi \times 10^6 t + 20\pi[4r(t) - 6r(t-1) + 2r(t-2)]]$$

Standard FM expression is given by:

$$s(t) = A_c \cos\left[2\pi fct + 2\pi k_f \int m(t)dt\right]$$

$$2\pi k_f \int m(t)dt = 20\pi\,(4r(t) - 6r(t-1) + 2r(t-2))$$

$$k_f\, m(t) = 10[4\, u(t) - 6u\,(t-1) + 2r\,(t-2)]$$

$$\Delta f = \max|k_f m(t)| = 40 \text{ Hz}$$

T3. Sol.

Maximum frequency deviation

$$\Delta f_{max} = \frac{K_p}{2\pi}\left|\frac{d}{dt}m(t)\right|_{max} = \frac{K_p}{2\pi}2t\,e^{-t^2}$$

$$= \frac{8000}{2\pi}.2.\frac{1}{\sqrt{2}}.e^{-1/2} \qquad \left(\because \max 2 + e^{-t^2} \text{ is at } t = \frac{1}{\sqrt{2}}\right)$$

$$= 3.43 \text{ kHz}$$

T4. Sol.

Compairing the equation with the standard equation.

$$s(t) = A\cos[\omega_c t + k_p\, m(t)]$$

$$\therefore \quad k_p\, m(t) = 0.1 \sin(10^3\pi t)$$

$$m(t) = \frac{0.1}{k_p}\sin(10^3\pi t)$$

$$= 0.01 \sin(10^3\pi t)$$

Similarly

$$s(t) = A\cos\left[\omega_c t + K_f \int m(t)dt\right]$$

$$K_f \int m(t)\,dt = 0.1 \sin(10^3\pi t)$$

$$\int m(t)dt = \frac{0.1}{10\pi}\sin(10^3\pi t) = \frac{0.1\times 10^3\pi}{10\pi}\cos(10^3\pi t) = 10\cos(10^3\pi t)$$

T5. Sol.

$A_m = 5$ V, $f_m = 100$ Hz } $\Delta f = k_f A_m = 1$ kHz

$A_m = 10$ V, $f_m = 50$ Hz } $\Delta f = 2$ kHz

To get $\Delta f = 30$ kHz

frequency multiplication factor should be 15.

T6. (a)

$$BW = 2[\beta + 1]f_m$$

$$\beta = k_p A_m = 5$$

A_m is doubled $\Rightarrow \beta = 10$; $f_m = \frac{1}{2}$ kHz

$$BW = 2[10 + 1]\cdot\frac{1}{2} = 11 \text{ kHz}$$

T7. Sol.

$$\beta_f = \frac{k_f \max\{m(t)\}}{f_m} = \frac{100\text{ k} \times 1}{1\text{ k}} = 100$$

$$BW_f = 2\,(100 + 1)\,1\text{ k} = 202\text{ kHz.}$$

$$\beta_p = k_p \max\{m(t)\}$$

$$= 10 \times 1 = 10$$

$$BW_p = 2\,(10 + 1)\,1\text{k} = 22\text{ kHz}$$

Bandwidth required for channel

$$= 202 + 22 = 224\text{ kHz}$$

T8. Sol.

The phase modulated signal can be given by,

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] = A_c \cos[\theta(t)]$$

The instantaneous frequency of the modulated signal,

$$f_i = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

Given that, $$m(t) = 100\,\text{sinc}(1000t)\text{ V} = 100\frac{\sin(1000\pi t)}{1000\pi t}$$

$$\frac{dm(t)}{dt} = 100\left[\frac{1000\pi\cos(1000\pi t)}{1000\pi t} - \frac{\sin(1000\pi t)}{1000\pi t^2}\right]$$

At $t = 1$ ms, $$\frac{dm(t)}{dt} = \frac{100\cos(\pi)}{10^{-3}} = -10^5\text{ V/s}$$

So, $$f_i = f_c + \frac{1}{2\pi}(-10^5 k_p) = 100 - \frac{100 \times 2}{2\pi}\text{ kHz}$$

$$= 100 - \frac{100}{\pi}\text{ kHz} = 68.17\text{ kHz}$$

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3 Radio Receivers

Detailed Explanation of Try Yourself Questions

T1. Sol.

Given 88.5 MHz < f_c < 108 MHz

Also,

$$f_{LO} - f_c = 10.8 \text{ MHz}$$

or,
$$f_{LO} = 10.8 \text{ MHz} + f_c$$

$\therefore$
$$f_{LO_1} = 10.8 + 88.5 = 99.3 \text{ MHz}$$

$$f_{LO_2} = 10.8 + 108 = 118.8 \text{ MHz}$$

$\therefore$
$$\text{range} = 99.3 \text{ MHz} - 118.8 \text{ MHz}$$

T2. Sol.

$$C = \frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2 = 1.45$$

Where
$$f_{max} = f_{m_2} + IF \;\&\; f_{min} = f_{m_2} + IF$$

$$\frac{110.5 + IF}{90 + IF} = \sqrt{1.45} = 1.204$$

$$110.5 + IF = 90 \times 1.204 + IF \times 1.204$$

$$2.126 = 0.204\, IF$$

or
$$IF = 10.42 \text{ MHz}$$

Also Image frequency

$$= f_s + 2\, IF$$

$$125 = f_s + 2 \times 10.42$$

$$f_s = 104.16 \text{ MHz}$$

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4 Sampling and Pulse Code Modulation

Detailed Explanation of Try Yourself Questions

T1. (d)

$$f_m = 100 \text{ Hz}$$

$$Q_e = \pm\frac{\Delta}{2}$$

$$f_s = 1.5 \times f_m \times 2 = 300 \text{ Hz}$$

$$\frac{\Delta}{2} \leq \frac{0.1}{100} \times A_m$$

$$\frac{2A_m}{2^n \times 2} \leq \frac{0.1}{100} \times A_m$$

$$\Rightarrow \quad n = 10$$

$$r_b = N\,n\,f_s = 8 \times 10 \times 300 = 24000 \text{ Hz} = 24 \text{ kbits/sec}$$

T2. Sol.

Sampling frequency (f_s) $= 1.5 \times 2 \times 4$

$= 12$ kHz

step size (Δ) $= 10$ mV

To avoid slope overload distortion in Delta modulation;

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

i.e., $$\frac{\Delta}{T_s} \geq 2\pi f_m \cdot A_m$$ ...for sinusoidal message signal

$$A_m \leq \frac{\Delta}{T_s(2\pi f_m)}$$

$$(A_m)_{max} = \frac{\Delta}{T_s(2\pi f_m)} = \frac{\Delta \cdot f_s}{2\pi f_m}$$

$$= \frac{10\times10^{-3}\times12\times10^{3}}{2\pi\times10^{3}} = 19.09\times10^{-3} \approx 19.1\,\text{mV}$$

T3. (c)

To prevent slope overload

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

$$\delta\times200\times10^{3} \geq 2\pi A_m f_m$$

$$\delta \geq \frac{2\times\pi\times(10\times10^{3})\times\frac{1}{2}}{200\times10^{3}}$$

$$\delta \geq 0.157\text{ Volts}$$

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5 Digital Carrier Modulation Schemes

Detailed Explanation of Try Yourself Questions

T1. Sol.

Average energy
$$= \frac{1}{16}\left[4(\sqrt{2}a)^2 + 8(\sqrt{10}a)^2 + 4(\sqrt{18}a)^2\right]$$
$$= \frac{1}{4}[2a^2 + 20a^2 + 18a^2]$$
$$= 10\,a^2.$$

T2. Sol.

Let signal I be represented as

$$S_1(t) = \begin{cases} A_1 \sin\dfrac{\pi t}{T}; & 0 \le t \le T \\ 0 \quad ; & 0 \le t \le T \end{cases}$$

and signal II be represented as

$$S_2(t) = \begin{cases} A_2 \sin\dfrac{\pi t}{T}\ ; & 0 \le t \le T \\ -A_2 \sin\dfrac{\pi t}{T}; & 0 \le t \le T \end{cases}$$

The average energy of signal will be

$$P_{avg_1} = \frac{1}{2}\left(\frac{A_1^2}{2}\right) + \frac{1}{2}(0) = \frac{A_1^2}{4}$$

Average energy of signal (ii)

$$P_{avg_2} = \frac{1}{2}\left(\frac{A_2^2}{2}\right) + \frac{1}{2}\left(\frac{A_2^2}{2}\right) = \frac{A_2^2}{2}$$

$\therefore \qquad \frac{A_1^2}{4} = \frac{A_2^2}{2}$

$\Rightarrow \qquad \frac{A_1}{\sqrt{2}} = A_2$

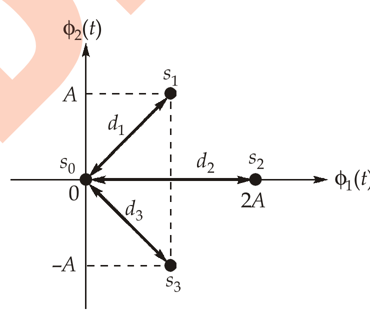
T3. (b)

$$\text{Sampling frequency } (f_s) = 1.25 \times (2f_m) + \text{Guard band}$$
$$= 1.25 \times 2 \times 10 + 1$$
$$= 26 \text{ kHz}$$
$$\text{Bit rate } (R_b) = n.f_s = 4 \times 26 \qquad ...[L \le 2^n]$$
$$= 104 \text{ kHz}$$

$\because$ Bandwidth of channel is 100 kHz i.e.,

$$\text{B.W} \le 100 \text{ kHz}$$
$$R_s(1 + \alpha) \le 100 \qquad ...[\text{For M-ary PSK} \quad (BW)_{min} = R_s(1 + \alpha)]$$
$$\because \quad R_s = \frac{R_b}{\log_2 M} \qquad ...[R_s = \text{symbol rate}]$$
$$\therefore \quad \frac{R_b}{\log_2 M}(1+\alpha) \le 100$$
$$\frac{104}{\log_2 M}(1+0.3) \le 100$$
$$\log_2 M \ge 1.352$$
$$M \ge 2^{1.352}$$
$$M_{min} = 4 \qquad (\because M = 2^n)$$

T4. (d)



Let the energy associated with the symbols s_0, s_1, s_2 and s_3 are E_0, E_1, E_2 and E_3 respectively.

$$E_i = (d_i)^2 ; \qquad i = 0, 1, 2, 3$$

From the above diagram,

$$d_0 = 0$$
$$d_1 = d_3 = \sqrt{A^2 + A^2} = \sqrt{2A^2}$$
$$d_2 = 2A$$

So,

$$E_0 = 0$$
$$E_1 = E_3 = 2A^2$$
$$E_2 = 4A^2$$

The average symbol energy of the modulation scheme can be given as,

$$E_s = \sum_{i=0}^{3} E_i P(s_i) \; ; \qquad P(s_i) = \text{probability of occurrence of the symbol } s_i$$

$$= 0(0.3) + 2A^2(0.2) + 4A^2(0.4) + 2A^2(0.1)$$

$$= (0.4 + 1.6 + 0.2)A^2$$

$$= 2.2\,A^2$$

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6 Information Theory and Error Control Coding

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$P(y) = [0.5 \quad 0.5]\begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix} = [0.4 \quad 0.2 \quad 0.4]$$

$$= [y_1 \quad y_2 \quad y_3] \qquad \left[\because P(y) = P(x).P\left(\frac{y}{x}\right)\right]$$

T2. Sol.

(i) We know that

$$P\left(\frac{r_0}{m_0}\right)P(m_0) > P\left(\frac{r_0}{m_1}\right)P(m_1) > P\left(\frac{r_0}{m_2}\right)P(m_2)$$

$\Rightarrow$ (0.6) (0.3) > (0.1) (0.5) > (0.1) (0.2)

Hence, we select m_0 wherever r_0 is received.

We also find that $P\left(\frac{r_1}{m_1}\right)P(m_1) > P\left(\frac{r_1}{m_0}\right)P(m_0) > P\left(\frac{r_1}{m_2}\right)P(m_2)$

$\Rightarrow$ (0.5) (0.5) > (0.3) (0.3) > (0.1) (0.2)

Hence, we select m_1 wherever r_1 is received.

We also find that

$$P\left(\frac{r_2}{m_1}\right)P(m_1) > P\left(\frac{r_2}{m_2}\right)P(m_2) > P\left(\frac{r_2}{m_0}\right)P(m_0)$$

$\Rightarrow$ (0.4) (0.5) > (0.8) (0.2) > (0.1) (0.3)

Hence, we select m_1 whenever r_2 is received.

(ii) The probability of being correct is

$$P(c) = P(m_0)\cdot P\left(\frac{r_0}{m_0}\right) + P(m_1)P\left(\frac{r_1}{m_1}\right)P(m_1)\cdot P\left(\frac{r_2}{m_1}\right)$$

$$= (0.6)(0.3) + (0.5)(0.5) + (0.5)(0.4) = 0.63$$

Hence probability of error, $P(e) = 1 - P(c)$

$$P(e) = 0.37$$

T3. Sol.

For a binary symmantric channel for wrong transmission let the probability be p

Thus

mutual information

$$= I(X;Y) = H(Y) - H(Y/X)$$

and $H(Y/X) = -p\log_2 p - (1-p)\log_2(1-p)$

$\therefore$ $I(X;Y) = H(Y) + p\log_2 p + (1-p)\log_2(1-p)$

$$C_{max} = I(X;Y)_{max}$$

$$= 1 + p\log_2 p + (1-p)\log_2(1-p)$$

T4. (a)

$$C = (c_1, c_2, c_3, c_4, c_5, c_6) = (x_1, x_2, x_3, c_4, c_5, c_6)$$

$$c_4 = c_1 \oplus c_2 = x_1 \oplus x_2$$

$$c_5 = c_2 \oplus c_3 = x_2 \oplus x_3$$

$$c_6 = c_1 \oplus c_3 = x_1 \oplus x_3$$

$$[c_4 \ c_5 \ c_6] = [x_1 \ x_2 \ x_3]\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$$

$$\bar{P} = \begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$$

$$\bar{G} = \left[\begin{array}{ccc|ccc}1&0&0&1&0&1\\0&1&0&1&1&0\\0&0&1&0&1&1\end{array}\right]$$

T5. Sol.

It is given that “1001100” is a valid code word.

$\Rightarrow$ $C = 1001100$

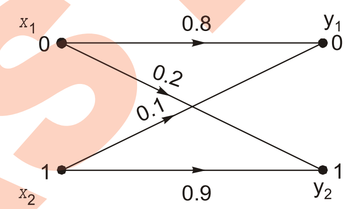
$$H^T = \begin{bmatrix}1&b&1\\1&0&1\\1&1&0\\a&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$CH^T = 0 \quad \Rightarrow \quad [1001100]\begin{bmatrix} 1 & b & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ a & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$CH^T = [a \quad b \oplus 1 \quad 0] = [0 \quad 0 \quad 0] \quad [\oplus \rightarrow \text{modulo-2 addition}]$$

Hence on comparing, $a = 0$, $b = 1$

T6. Sol.



$$[P(y|x)] = \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}$$

$$P[y] = P[x]\,P[y|x]$$

$$= [0.4 \quad 0.6]\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} = [0.38 \quad 0.62]$$

$$P_Y(0) = 0.38$$

$$P_Y(1) = 0.62$$

T7. (b)

Condition 1: When r_0 is received, decision is made in favour of m_0.

So, $P(r_0 \mid m_0)\,P(m_0) > P(r_0 \mid m_1)\,P(m_1)$

$$(0.7)(1-p) > (0.3)(q)$$

$$0.7p + 0.3q < 0.7$$

$$7p + 3q < 7 \quad \text{... (i)}$$

Condition 2: When r_1 is received, decision is made in favour of m_1.

So, $P(r_1 \mid m_1)\,P(m_1) > P(r_1 \mid m_0)\,P(m_0)$

$$(0.3)(1-q) > (0.7)(p)$$

$$0.7p + 0.3q < 0.3$$

$$7p + 3q < 3 \quad \text{... (ii)}$$

If condition 2 is satisfied, then condition - 1 will be satisfied automatically.

So, the sufficient condition to be satisfied is condition-2, i.e. $7p + 3q < 3$

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7 Random Variables and Random Process

Detailed Explanation of Try Yourself Questions

T1. Sol.

The A.C power of the signal is given as σ_X^2

where σ_X^2 is the standard deviation

$$\sigma_X^2 = E[X^2] - (E[X])^2$$

where $E[X^2]$ = second moment

$[E(X)]^2 = (\text{mean})^2$

now, $$(E[X])^2 = \lim_{\tau\to\infty} R_{XX}(\tau) = 6$$

and $$E[X^2] = \lim_{\tau\to 0} R_{XX}(\tau) = 10$$

$\therefore$ $$\sigma_X^2 = 10 - 6 = 4 \text{ W.}$$

T2. Sol.

$$E[Y] = \int_{-\infty}^{\infty} f_x(x) Y \, dx$$

$$= \int_{-\infty}^{\infty} e^x \, dx = \int_0^1 e^x dx$$

$$= -(1 - e) = (e - 1)$$

T3. Sol.

(i) $$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

$$2\int_0^{\infty} a\, e^{-bx} dx = 1$$

$\Rightarrow$ $$\frac{2a}{b} e^{-bx} \Big|_{\infty}^{0} = 1$$

$\Rightarrow$ $$2a = b$$

(ii)
$$\text{c.d.f} = \int_0^x f_x(d)dx$$
$$= \int_0^x a\,e^{-bx}dx = 1-\frac{1}{2}e^{-bx}$$

for $x \geq 0$ and $\int_{-\infty}^x f_0(x)dx$ for $x < 0$

$$f_X(x) = \frac{1}{2}e^{bx}$$

$x < 0$

(iii)
$$P(1 \leq X \leq 2) = \int_1^2 f_X(x)dx$$
$$= \int_1^2\left(1-\frac{1}{2}e^{-bx}\right)dx = \frac{1}{2}\left[e^{-b}-e^{-2b}\right]$$

T4. (b)

$$Y(t) = X(t)\cos(2\pi f_c t+\theta)$$
$$\text{power of } Y(t) = E[Y^2(t)]$$
$$= E[X^2(t)\cdot\cos^2(2\pi f_c t+\theta)]$$
$$= E[X^2(t)]\cdot E[\cos^2(2\pi f_c t+\theta)]$$
$$E[X^2(t)] = R_{XX}(\tau)\big|_{\tau=0}$$
$$E[\cos^2(2\pi f_c t+\theta)] = \frac{1}{2\pi}\int_0^{2\pi}\cos^2(2\pi f_c t+\theta)d\theta = \frac{1}{2}$$
$$\text{Power of } y(t) = R_{XX}(\tau)\big|_{\tau=0}\cdot\frac{1}{2}$$
$$= \text{sin}c(0)\cdot\frac{1}{2} = \frac{1}{2}$$

T5. (a)

To maximize the entropy, all the decision boundaries should be equiprobable

$$\int_1^5 \rho_x(x)dx = \frac{1}{3} \quad ; \quad \int_1^5 b\,dx = \frac{1}{3} \quad ; \quad b[x]_1^5 = \frac{1}{3}$$
$$b[5-1] = \frac{1}{3} \quad ; \quad 4b = \frac{1}{3}$$
$$b = \frac{1}{12}$$
$$\int_{-1}^1 \rho_x(x)dx = \frac{1}{3} \quad ; \quad \int_{-1}^1 a\,dx = \frac{1}{3} \quad ; \quad a[x]_{-1}^1 = \frac{1}{3}$$
$$a[1-(-1)] = \frac{1}{3} \quad ; \quad 2a = \frac{1}{3}$$
$$a = \frac{1}{6}$$

T6. (d)

Signal power = s;

$$= \int_{-5}^{-1} x^2 \rho_x(x)dx + \int_{-1}^{1} x^2 \rho_x(x)dx + \int_{1}^{5} x^2 \rho_x(x)dx$$

$$s = \int_{-5}^{1} x^2 b dx + \int_{-1}^{1} x^2 a dx + \int_{1}^{5} x^2 b dx$$

$$s = \frac{1}{12}\int_{-5}^{-1} x^2 dx + \frac{1}{6}\int_{-1}^{1} x^2 dx + \frac{1}{12}\int_{1}^{5} x^2 dx$$

$$s = \frac{1}{2}\left[\frac{x^3}{3}\right]_{-5}^{-1} + \frac{1}{6}\left[\frac{x^3}{3}\right]_{-1}^{1} + \frac{1}{12}\left[\frac{x^3}{3}\right]_{1}^{5}$$

$$s = \frac{1}{36}[-1-(-125)] + \frac{1}{18}[1-(-1)] + \frac{1}{36}[125-1]$$

$$s = \frac{124}{36} + \frac{2}{18} + \frac{124}{36} = \frac{124}{18} + \frac{2}{18} = \frac{126}{18}$$

$$s = 7 \text{ volt}^2$$

$$\boxed{\text{step size} \;=\; \Delta = \frac{V_{p-p}}{L}}$$

$$\Delta = \frac{5-(-5)}{6} = \frac{10}{6} = \frac{5}{3}$$

$$\boxed{\text{QNP} = \frac{\Delta^2}{12}}$$

$$\text{QNP} = \frac{\left(\frac{5}{3}\right)^2}{12} = \frac{25}{9} \times \frac{1}{12}$$

$$\text{QNP} = 0.23 \cong 0.25$$

$$\boxed{\text{SQNR} = \frac{s}{\text{QNP}}}$$

$$\text{SQNR} = \frac{7}{0.25} = 28$$

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8 Noise

Detailed Explanation of Try Yourself Questions

T1. Sol.

$$Pe = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$E_b = (10)^2 \times 100 \times 10^{-6} = 10^{-2}$$

$$N_0 = 2 \times 10^{-4}$$

$\therefore$ $$Pe = \frac{1}{2}erfc\left(\sqrt{\frac{10^{-2}}{2\times10^{-4}}}\right) = \frac{1}{2}erfc\left(\sqrt{50}\right)$$

T2. Sol.

$$P_e = \frac{1}{2}erfc\sqrt{\frac{E_b}{N_0}}$$

$$erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \le 2 \times 10^{-4}$$

$$1 - erf\left(\sqrt{\frac{E_b}{N_0}}\right) \le 2 \times 10^{-4}$$

$$erf\left(\sqrt{\frac{E_b}{N_0}}\right) \ge 0.9998$$

$$\sqrt{\frac{E_b}{N_0}} \ge 2.6$$

$\Rightarrow$ $$\frac{E_b}{N_0} \ge 6.76$$

$$E_b \ge 1.352 \times 10^{-9} \text{ Joule}$$

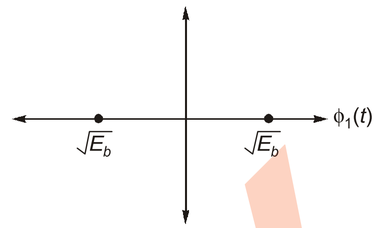
$$E_b = P \times T_b$$

$\Rightarrow$ $$P \ge (1.352 \times 10^{-9})\,(10^6)$$

$$P \ge 1.352 \text{ mW}$$

T3. Sol.

For BPSK signal



$$E_b = \frac{A^2 T_b}{2}$$

and
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$10^{-5} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\sqrt{\frac{A^2 T_b}{N_0}} = 4.27$$

$$A^2 = (4.27)^2 \times 9.6 \times 10^{-8} = 1.75 \times 10^{-6} = 1.32 \times 10^{-3}$$

$$A = 1.32 \text{ mV}$$

T4. Sol.

The probability density function of the input variable R can be given by,

$$f_R(r|s_0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r-2+1)^2}{2}} \quad \text{; When "–1" is transmitted}$$

$$f_R(r|s_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(r-2-1)^2}{2}} \quad \text{; When "+1" is transmitted}$$

Given that, $P(s_0) = 2/3$ and $P(s_1) = 1/3$.

According to MAP criteria, the rule for determination of threshold is,

$$P(s_0) f_R(r \mid s_0) \underset{H_1}{\overset{H_0}{\gtrless}} P(s_1) f_R(r \mid s_1)$$

At optimum threshold ($r = r_{th}$),

$$P(s_0)\, f_R(r_{th}|s_0) = P(s_1)\, f_R(r_{th}|s_1)$$

$$\frac{2/3}{\sqrt{2\pi}} e^{-\frac{(r_{th}-1)^2}{2}} = \frac{1/3}{\sqrt{2\pi}} e^{-\frac{(r_{th}-3)^2}{2}}$$

$$(r_{th} - 1)^2 - (r_{th} - 3)^2 = 2\ln(2)$$

$$-2r_{th} + 1 + 6r_{th} - 9 = 2\ln(2)$$

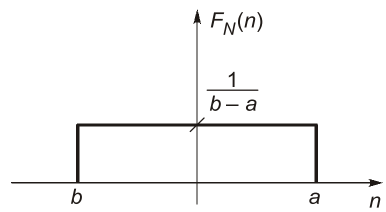
$$4r_{th} = 8 + 2\ln(2)$$

$$r_{th} = 2 + \frac{1}{2}\ln(2) = 2 + \ln\left(\sqrt{2}\right)$$

$$r_{th} = 2.3466 \approx 2.35$$

T5. (c)

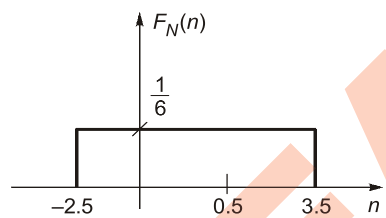
- N is a uniformly distributed noise variable with a variance by 3.



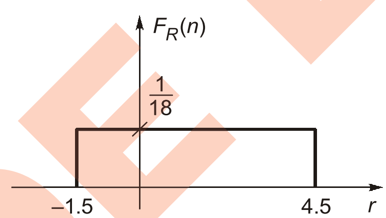
$$\text{Variance} = \frac{(b-a)^2}{12}$$

- Mean of noise variable 'N' is βX.

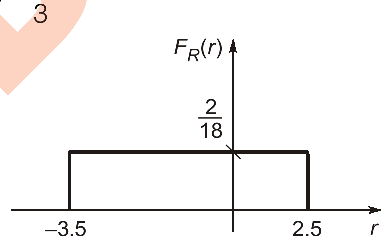
i.e. $E[N] = E[\beta X] = \beta = 0.5$



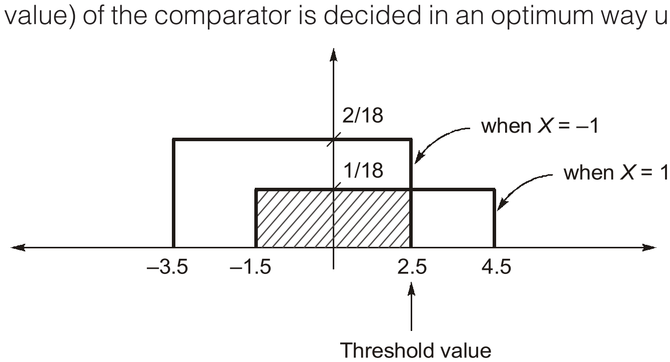
- When $X = +1 \Rightarrow P(X = +1) = \frac{1}{3}$



- When $X = -1 \Rightarrow P(X = -1) = \frac{2}{3}$



$\because$ r_{th} (threshold value) of the comparator is decided in an optimum way using MAP criteria.



$\therefore \quad r_{th} = 2.5$

when $\quad R > 2.5 \ldots X = 1$ detected

$\quad R < 2.5 \ldots X = -1$ detected

$$\therefore \quad \text{Probability of error} = \frac{1}{18} \times 3 \ldots \text{area of shaded portion}$$

$$= \frac{1}{6}$$

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